Takehome Final

Michael Anderson

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CS533

Prof. Fern

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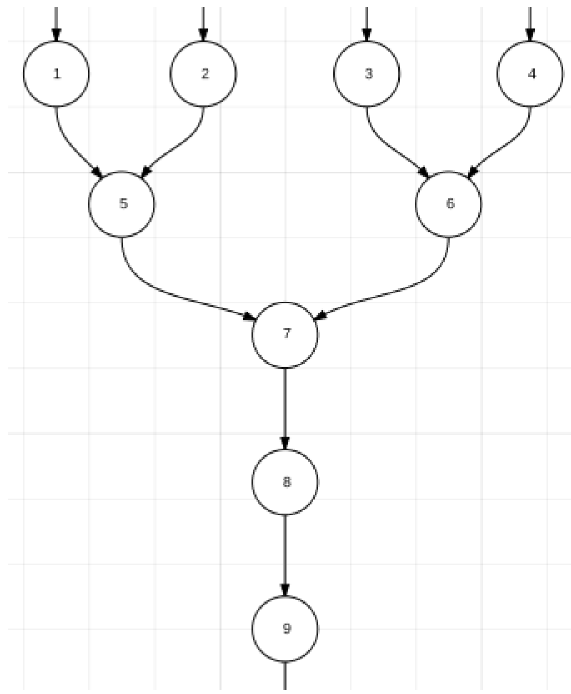
Imagine a k-order MDP in which all states have at most one parent state. In other words, for all s' that are states in the MDP, there exists at most one s such that $T(s, a, s') > 0$. Such a a k-order MDP would be directly convertible to a first-order MDP, because each state would only have one possible vector of ancestor states.

However, in an average k-order MDP where states can have multiple parents, a state could have a number of different ancestor vectors, and that information is not really accounted for in a normal first-order MDP.

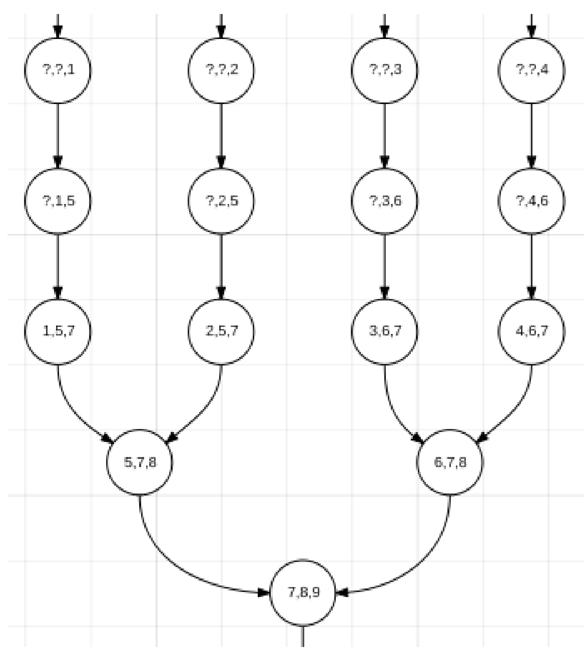
So, to convert a k-order MDP M to a first-order MDP M', traverse M cloning states, actions, rewards, and transitions over to M' exactly, except whenever a state s is found with multiple parents, make a set of states in M', $s_1, s_2, ...$, one state for each of the parents to transition to. Also duplicate all of the descendants of s, so that $s_1, s_2, ...$ each have their own copy. Finally, when copies of descendants are made in this way, check to see if they can be merged back together farther down the tree when they have the same ancestor vectors again (see picture).

The handling of A', R', and T' are straightforward. For states without multiple parents, they are identical to A, R, and T. But supposing that for some state s' in S there exists multiple s such that s is in S and $T(s, a, s') > 0$, then for each such s with a corresponding a that reaches s', make a a member of A' and make $T'(s, a, s_i) = T(s, a, s')$. For the rewards, make $R'(s_i) = R(s')$.

Below is an example of part of a 3-order MDP M and the corresponding part of its equivalent first-order MDP M'. This MDP is deterministic for simplicity, but without loss of generality. States are labeled 1 through 9 in M, and in M' states are labeled by their grandfather, their father, and then the state themselves. Notice that since 5 and 6 have two parents, they are each expanded into two states in M'. 7 is expanded into four states, because it has four combinations of grandfather/father pairs. 8 only has two such combinations and 9 only has one, so we see in the picture an example of the merging of descendant states mentioned above.



M



M'

2

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Assume the worst case, which is that the maximum reward R_{max} is obtained at each level of the tree. Then at search depth h, the reward is $\beta^h R_{max}$, because R_{max} is discounted by a factor of β at each level. The infinite horizon reward becomes an easily simplifiable geometric series:

$$Q_\pi(s,a) = \sum_{i=1}^{\infty} \beta^i R_{max} = \frac{1}{1-\beta} R_{max}$$

The finite horizon reward then, for a horizon of h, is

$$Q_\pi(s,a,h) = \sum_{i=1}^{h} \beta^i R_{max} = \frac{1-\beta^h}{1-\beta} R_{max}$$

Since this is the worst case, the difference between $Q_\pi(s,a)$ and $Q_\pi(s,a,h)$ is maximized by:

$$\frac{1}{1-\beta} R_{max} - \frac{1-\beta^h}{1-\beta} R_{max} = \frac{\beta^h}{1-\beta} R_{max}$$

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(a)

(b)

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