Homework Set 3

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CS533

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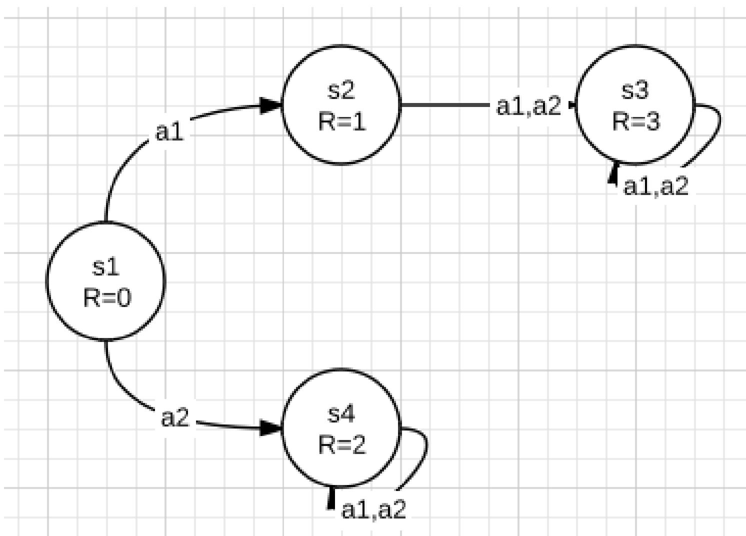
1. Consider the following MDP:

$S = \{s_1, s_2, s_3, s_4\}$
$A = \{a_1, a_2\}$
$R = \{(s_1, 0), (s_2, 1), (s_3, 3), (s_4, 2)\}$
$T = \{(s_1, a_1, s_2, 1), (s_1, a_2, s_4, 1)(s_2, a_1, s_3, 1), (s_2, a_2, s_3, 1),$
$(s_3, a_1, s_3, 1), (s_3, a_2, s_3, 1), (s_4, a_1, s_4, 1), (s_4, a_2, s_4, 1)\}$

This MDP looks like:



For the optimal stationary policy, if an agent starts at s_1, then it should clearly take a_1. In that way once the agent moves to s_2, and then further to s_3, they will end up trapped in a state with a lot of reward.

For the optimal non-stationary policy, suppose that the finite horizon h is only one. Now if the agent is at s_1, it should clearly take a_2, to grab the largest reward reachable in one action, which is at s_4.

2. a) Let the STRIPS problem be called S, and the MDP be called M.

There will be a state in M for each true/false combinations of the propositions in S. E.g., if S has two propositions p_1 and p_2, then M has the four states $\{p_1p_2, p_1\neg p_2, \neg p_1p_2, \neg p_1\neg p_2\}$.

Every single action in S maps directly to a single action in M.

States in M that have each of the propositions in the goal of S will have a reward of 1. All other states have a reward of 0.

The transition function of M is determined by the PRE, ADD, and DEL effects of each action in S. Iff a state s in M has the preconditions necessary for some action a, there will be some entry (s, a, s') in the transition function such that s' is simply s with all of the ADD effects of a set to true, and the DEL effects of a set to false. Since STRIPS actions are deterministic, $T(s, a, s') = 1$ for all such actions added in this way.

b) Since the state space of M is the enumeration of the possible true/false combinations of propositions of S, there are 2^n states in M.

c) Each of the 2^n states has a tree of width m (the number of actions), and of depth h, leading to a time complexity of $2^n mh$ using dynamic programming.

3. a) Make the recurrence relation of the policy evaluation algorithm:

$$V_\pi^k(s) = R(s) + \sum_{s' \; in \; NEXT(s,\pi(s,k))} T(s, \pi(s,k), s') V_\pi^{k-1}(s')$$

In other words, instead of considering all possible s's that could result from the action recommended by π, only consider s's that are reachable from (s,a) with non-zero probability. This reduces the time complexity of the algorithm from $O(hmn^2)$ to $O(hmnr)$, which is strictly better if $r < n$.

b) Make the recurrence relation of the value iteration algorithm:

$$V^k(s) = R(s) + \max_{a \; in \; LEGAL(s)} \sum_{s' \; in \; NEXT(s,a))} T(s, a, s') V^{k-1}$$

In other words, only consider actions that are actually takeable from s, and as in part a) only consider s's that are reachable from (s,a) with non-zero probability. This reduces the time complexity of the algorithm from $O(hmn^2)$ to $O(hknr)$, which is strictly better if $r < n$ and $k < m$.