CS 517
Due: Wednesday, 20 Apr

Homework #3
Collatz Problem

We mentioned in class that the function defined by

$$f(x) \;=\; \begin{cases} 1 & \text{if } x = 1 \\ x/2 & \text{if } x \text{ is even} \\ 3x+1 & \text{if } x \text{ is odd and } x > 1 \end{cases}$$

has two fixed point 0 and 1 in the naturals.
The Collatz problem is : For the above f does every natural number $x \;\geq\; 1$ eventually map to 1, where by eventually we mean that we get 1 after f is applied recursively to the output $f(x)$?
We use the notation $f^{(n)}(x)$ for this n-fold iteration, that is, $f^{(0)}(x) \;=\; x$ and $f^{(n+1)}(x) \;=\; f(\,f^{(n)}(x)\,)$.

1. Argue that the Collatz problem is trivial by showing that there is a one line program which correctly solves this problem.

2. To get a <u>feel</u> for the above iteration, write a program that for each number x, finds n so that $f^{(n)}(x) \;=\; 1$.

3. Plot the results of your program.
   Are there any "regularities" that you can notice in these plots??

4. Discuss: " $g(x) \;=\;$ the least n so that $f^{(n)}(x) \;=\; 1$, is a primitive recursive function."

5. Let
   $$S \;=\; \{\, x \mid \; \exists n f^{(n)}(x) \;=\; 1\}.$$
   Describe an acceptor for S.
   Discuss whether or not S has a recognizer.