CS 517
Due: Wed 27 Apr

Homework #4

1. Use the web to find a reasonable Turing machine simulator. Use the Turing machine simulator that you have located to simulate several Turing machine programs. Start with some easy problems, for example, binary addition, and
determining if an input string is a palindrome.
Next do *at least one* harder problem, for example,
computing Fibonacci numbers,
or multiplying natural numbers (in base notation),
or determining if an input number is a perfect square,
or determining if an input number is a prime.
(You can either hand in some prints outs indicating that your simulations were successful, or give me a demonstration to show that you know how this Turing machine simulator works on a variety of problems.)

2. Show that if $f(n)$ is computable by a Turing machine which uses *quintuple* instructions, then $f(n)$ is computable by another Turing machine which uses *triplet* instructions.

3. Let A be an infinite set of strings over the alphabet Σ. Assume that A is *recognizable* by a Turing machine that uses at most $T(n)$ steps to process a string of length n.
Given any natural number K, show that there is another Turing machine which recognizes A and uses at most $T(n)/K$ steps to process a string from Σ^n (at least for all BIG enough n).

4. Prove that there are sets which *cannot* be accepted by any Turing machine by counting the number of sets over Σ^* and the number of Turing machines.
(HINT: Use Cantor's theorem: The power set of any set is strictly larger than the set.)

5. **Algebraic** A real number r is *algebraic* iff there is a polynomial with rational coefficients which has r as a root.
Computable A real number r in $(0, 1)$ is *computable* iff there is a Turing machine which given n as input, prints out the first n digits of r. You may make this decimal digits or binary digits.
Which set is bigger, properly contains the other?
Show that both sets are the same size.

6. Show that an infinite set $\mathbf{S}$ is Recursive if and only if it has a strong generator $\mathrm{GEN}_{\mathbf{S}}(n)$ which satisfies $\mathrm{GEN}_{\mathbf{S}}(n) < \mathrm{GEN}_{\mathbf{S}}(n+1)$ for all n.