Homework 3

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CS517

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1

We could argue that since an n has been found for all of the trillions of natural numbers already tried, probably no natural number will be found that does not map to 1 eventually upon successive applications of the Collatz function. Therefore a program that returns an answer to the question: "Do all natural numbers eventually map to 1 upon successive applications of the Collatz function could be:

```
RETURN YES
```

If during the generation of the Collatz sequence for some x, a cycle is entered (which would preclude x ever reaching 1), then the program could return NO. So another possible one-liner (easily implementable as an actual one-liner in a language such as Haskell or J) would be:

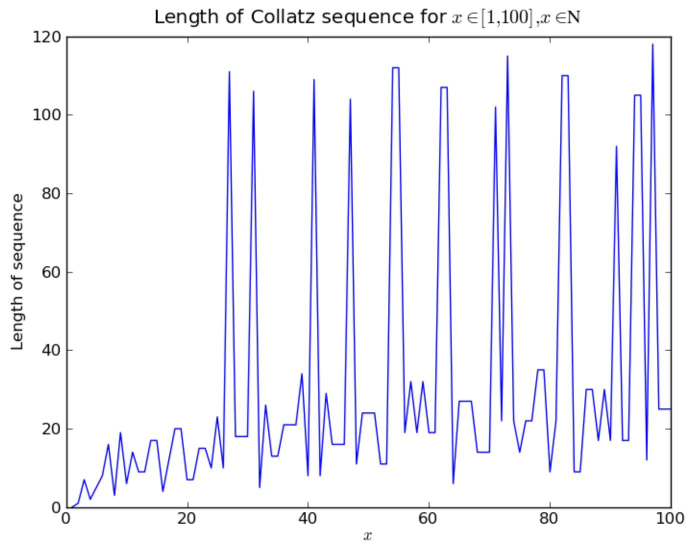
```
Iterate through the naturals until a Collatz sequence is found for one of them
 that enters into a cycle, and then return NO.
```

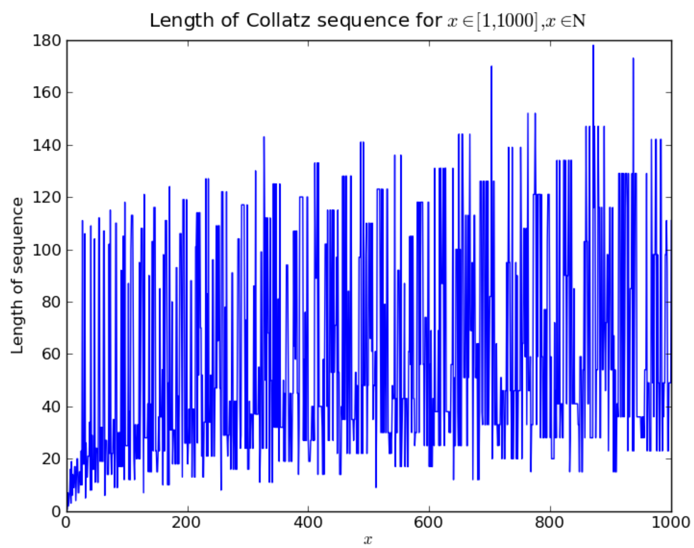
It turns out that in practice, the amount of time and space required to actually get an answer from the above program is very problematic.

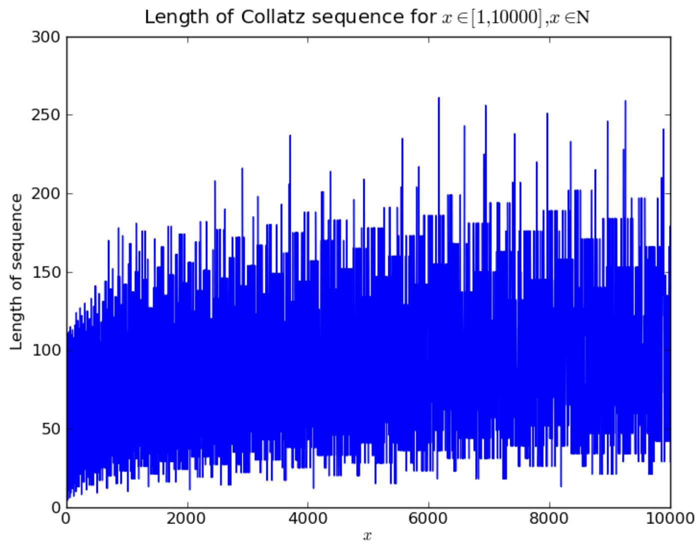
2

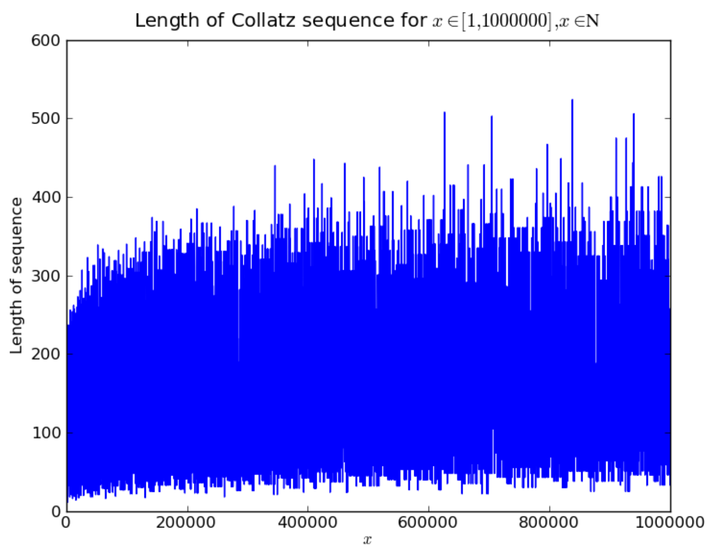
collatz.py is attached.

3


Length of Collatz sequence for $x \in [1,100], x \in \mathbb{N}$
Length of sequence
x
0
20
40
60
80
100
120


Length of Collatz sequence for $x \in [1,1000], x \in \mathbb{N}$
Length of sequence
x
0
200
400
600
800
1000
20
40
60
80
100
120
140
160
180





There are regularities in these plots. Moving from the interval of [1,100] to [1,1000000] looks almost like zooming out on a fractal. Each plot is punctuated by a dozen or so rather large spikes, indicating where particular values have an especially long Collatz sequence. Looking at any particular plot, the size of the spikes generally grows from left to right, but the growth is small. The spikes look to be fairly evenly far apart.

4

The given statement is false. $g(x)$ is not a primitive recursive function, because there is no way to bound the time required to compute n. As the Collatz function is applied successively, the input may grow or shrink in no predictable pattern. If $g(x)$ is implemented recursively, then there will be no steadily decreasing variable that is given as input into successive recursive calls, and therefore by definition of $PRIM$, $g(x)$ cannot be in $PRIM$.

5

An acceptor for S would successively perform the Collatz function. If it eventually reached 1, i.e. some n were found such that $f^{(n)}(x) = 1$, it would halt and return YES. Iff an x such that $x \notin S$ were given to it as input, it would not halt.

We can make no recognizer for S as long as the Collatz problem remains open. While an acceptor for S is easy to imagine, we can make no rejector (and therefore no recognizer) for it because we do not know if $\bar{S} = \emptyset$. Iff the nature of $\bar{S}$ is discovered, and it is computable, and a way to compute it is discovered, then a recognizer for S can be made. Therefore, whether or not $S \in \mathit{Recursive}$ is also an open problem.