ECE565: Estimation Detection and Filtering (Fall 2011) - Dr. Raviv Raich
Homework assignment 1 (due in two weeks)

1. **(LS)** Consider the following model:

$$y(n) = \begin{cases} a & n \text{ is even} \\ \\ b & n \text{ is odd} \end{cases} + e(n) \tag{1}$$

You are asked to estimate a and b using LS from $y(1), y(2), \ldots, y(N)$.

   (a) Determine θ and H.

   (b) Write the expression for the LS estimator of a and of b in closed-form.

   (c) Consider the change of parameters from the original m element parameter vector θ to another m element parameter vector θ' by $\theta' = M\theta$, where M is an invertible $m \times m$ matrix. Express the LS estimator of θ' (i.e., $\hat{\theta}'_{LS}$) in terms of the LS estimator of θ (i.e., $\hat{\theta}_{LS}$). This part should be solved in general and not only for the model above.

   (d) Let $c = (a + b)/2$ and $d = (a - b)/2$. Find the LS estimators for c and d. (Hint: note that the vector $[c, d]^T$ can be expressed as a linear transformation of the vector $[a, b]^T$.)

2. (**WLS**) Consider the following model:

$$y(n) = A + e(n) \tag{2}$$

You are asked to estimate A from the latest N samples $y(n), y(n-1), \ldots, y(n-(N-1))$. To deal with a slowly time-varying A, one could consider a small window N. Another approach is to consider the following cost to minimize:

$$\sum_{i=n-N+1}^{n} \lambda^{i-n} e(i)^2, \tag{3}$$

where λ is a real-valued scalar.

   (a) Find W, y, H, and θ in the WLS framework for this problem.

   (b) Find the WLS estimator of A. Simplify it for $N \to \infty$.