Homework 3

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ECE565

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(a) Exponential densities can be written in the form $f_\theta(x) = a(\theta)b(x)e^{c(\theta)t(x)}$

Let c and t equal 0. There is no way to separate the expression into the product of a pure function of θ and a pure function of x.

(d) The score s is:

$$\frac{d\log f}{d\theta} = \frac{d}{d\theta}\log\left[\frac{1}{2}\left(\frac{1+3\theta x^2}{1+\theta}\right)\right] = \frac{3x^2}{1+3\theta x^2} - \frac{2}{2+2\theta}$$

The score is not writable in the form $c(\theta)(g(x)-\theta)$, confirming that the is not part of the exponential family.

$$FIM = -E\left[\frac{ds}{d\theta}\right] = -E\left[\frac{1}{(1+\theta)^2} - \frac{9x^4}{(3\theta x^2-1)^2}\right] =$$

$$\int_{-\infty}^{\infty}\left[\left(\frac{9x^4}{(3\theta x^2-1)^2} - \frac{1}{(1+\theta)^2}\right)\frac{1}{2}\left(\frac{1+3\theta x^2}{1+\theta}\right)\right]dx = \text{infinity?}$$

$$\text{CRLB} = 1/FIM = 0$$

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(a)

$$s = \frac{d\log f}{d\theta} = \frac{d}{d\theta}\left[\log\left(\frac{1}{\Gamma(\theta)}\right) + \log(x^{\theta-1}) + \log(e^{-x}))\right] = -\psi(\theta) + \log(x)$$

Cannot write in the form $c(\theta)(g(x)-\theta)$, so no efficient estimator.

$$FIM = -E\left[\frac{ds}{d\theta}\right] = -E\left[\frac{d}{d\theta}[\log(x)-\psi(\theta)]\right] = -E[\psi'(\theta)] =$$

$$-\int_{-\infty}^{\infty}\psi'(\theta)\left(\frac{1}{\Gamma(\theta)}x^{\theta-1}e^{-x}\right)dx$$