ECE565: Estimation Detection and Filtering (Fall 2011) - Dr. Raviv Raich

Project Assignment 2

This project deals with Doppler estimation in the frequency domain. Consider a vehicle driving along the x-axis, at speed v in the x-direction. Assuming uncorrelated far-field sources are transmitting at known frequency f_c, the statistical model for the independent observations in the frequency domain $Y(f_1), \ldots, Y(f_n)$ is given by:

$$Y(f_i) \quad \sim \quad \mathcal{CN}(0, \frac{E_s}{\sqrt{f_d^2 - f_i^2}} + \sigma^2) \tag{1}$$

where f_d is the Doppler spectrum frequency (given by $f_d = f_c \frac{v}{c}$ where f_c is carrier frequency v is the car speed and c is wave propagation speed), E_s is the signal energy, and σ^2 is the noise variance. Let $f_i = (i - \frac{N+1}{2})\frac{2f_o}{N}$ and assume that $f_d > f_o > 0$.

1. Consider the parameter vector $\boldsymbol{\theta} = [f_d, \text{SNR}, \sigma^2]$, where $\text{SNR} = \frac{E_s}{\sigma^2}$. Identify the observation vector $\mathbf{y}$ and write the PDF $f(\mathbf{y}|\boldsymbol{\theta})$.

2. Compute the FIM and the CRLB. To simplify the derivation, use the result of the CRLB for a Complex Gaussian process (see "Fundamentals of Statistical Processing, Volume I: Estimation Theory" by Steven M. Kay).

3. Find the ML estimator of $\boldsymbol{\theta}$.

4. Let $N = 50$, $f_d = 1$, $\sigma^2 = 1$, and $f_o = 0.8$. Using 500 Monte-Carlo runs, compute the ML estimate of f_d for the realization at each run. From the 500 estimates, find the empirical MSE. Repeat this process while varying SNR from 0[dB] (SNR = 1) to 20[dB] (SNR = 100). Plot the empirical MSE of the ML estimator of f_d against SNR on a logarithmic scale (for both SNR and the MSE). Include the CRLB as a function of SNR. Separately, plot the empirical bias of the ML estimator against SNR. For what range of SNR would you consider, the ML estimator approximately unbiased?