ST561: Homework 3

Due: Monday, Oct.17, 2011

Note: Exercise 8 is not required to be handed in, but if you provide a correct answer to it, you can earn credits to compensate your possible mistakes in Exercises 1–7.

1. Textbook, Page 57, Exercise 1.6.3.
2. Let $f(x)$ be a p.d.f. and let a be a number such that, for all $\epsilon > 0$, $f(a+\epsilon) = f(a-\epsilon)$. Such a p.d.f. is said to be symmetric about the point a. Show that if a random variable X has a p.d.f. which is symmetric about a, then the median of X equals to a.
3. Let $X \sim N(\mu, \sigma^2)$. Suppose the median of X is 50 and $P(X > 100) = 0.025$. Can μ and σ be evaluated uniquely? If yes, find the values of μ and σ.
4. Let X be a continuous random variable with the c.d.f. $F(x)$ and p.d.f. $f(x)$. Find the c.d.f. and p.d.f. of $|X|$.
5. Textbook, Page 63, Exercise 1.7.21.
6. Compute the mean and variance for random variables with each of the following p.d.f.'s:
   (a) $f(x) = ax^{a-1}$, for $0 < x < 1$ and $a > 0$;
   (b) $f(x) = (3/2)(x-1)^2$, for $0 < x < 2$.
7. For any c.d.f. $F(x)$ of a continuous random variable, show that $\int_{-\infty}^{\infty}[F(x+b) - F(x+a)]\mathrm{d}x = b - a$ for any $a < b$.
8. Let X be a random variable having a continuous c.d.f. $F(x)$. Show that $Y = F(X)$ has the uniform distribution $U(0,1)$.