ST561: Homework 4
Due: Monday, Oct.24, 2011

1. Textbook, Page 94, Exercise 2.3.15.

2. Textbook, Page 94, Exercise 2.3.16.

3. Textbook, Page 161, Exercise 3.2.12.

4. Prove that
$$f(x, y) = \frac{1}{2} \sin(x + y), \quad 0 < x, y < \frac{\pi}{2}$$
   is a joint p.d.f. What are the marginal density functions?

5. Suppose (X, Y) follows the uniform distribution in the region $D = \{(x, y) : 0 < x^2 < y < x < 1\}$.

   (a) What is the marginal density function of X?
   (b) Find $P(Y > 1/2)$.

6. Let $f_X(x)$ be the p.d.f. of a positive continuous random variable X. Prove that

$$f(x, y) = \frac{f_X(x + y)}{x + y}, \quad x, y > 0$$

   is a joint p.d.f.

7. Let X be a continuous random variable with p.d.f. $f_1(x)$ and c.d.f. $F_1(x)$, and Y be another continuous random variable with p.d.f. $f_2(y)$ and c.d.f. $F_2(y)$. Prove that

   (a) for any $\alpha \in (-1, 1)$,

$$f_\alpha(x, y) = f_1(x)f_2(y)[1 + \alpha\{2F_1(x) - 1\}\{2F_2(y) - 1\}]$$

      is a joint p.d.f.;
   (b) $f_\alpha(x, y)$ has the marginal density functions $f_1(x)$ and $f_2(y)$.