ST 561 Homework Solution 2 (Total pts: 50)

Due Monday Oct. 10, 2011

1 (10 points).

(a) $1/\binom{52}{4} = 3.69 \times 10^{-6}$.

(b) $1/(52)(51)(50)(49) = 1.54 \times 10^{-7}$.

(c) If each card is replaced after it is drawn: (a') $1/(13)^4 = 3.5 \times 10^{-5}$; (b') $1/(52)^4 = 1.37 \times 10^{-7}$.

2 (10 points).

Treat textbooks of the same subject as a group, and we have 4! different arrangement for the positions of groups. For each group, we have 6!, 4!, 4! and 5! different ways to arrange books respectively. Thus, the total different ways are $4! \times 6! \times 4! \times 4! \times 5! = 1194393600$.

3 (10 points).

(a) Since 5 men are seated together, treat them as a "group" so that there are 5! ways to arrange men. For 5 women, we have 5! ways to arrange them. Also note that there can be 0,1,2,3,4,5 women sitting next to the men group, so in total we have $6 \times 5! \times 5! = 86400$ different ways for event (a).

(b) (The simplified version) Two general situations: MFMFMFMFMF or FMFMFMFMFM. For each case, men and women can be arranged by 5! and 5! ways respectively. Thus, we have $2 \times 5! \times 5! = 28800$ different ways for event (b).

(c) Treat a couple as a "group" so that there are 5! ways to arrange the groups. Also for each couple, there are 2 ways to arrange them, so we have $5! \times 2^5 = 3840$ different ways for event (c).

(d) The total number of arrangement for the 10 people is 10!, so using the result obtained in part (c), we have the probability equals $\frac{3840}{10!} = \frac{1}{945}$.

4 (10 points).

Since r different balls are to be randomly put into n different urns, we know the denominator in each of the following calculation is n^r. We just need to find out each numerator in the three situations.

(a) If r urns each contains exactly one ball, then the numerator is $r!$, so the probability is $P = \frac{r!}{n^r}$.

(b) If no urn contains more than one ball, then the numerator should be $n(n-1)\cdots(n-r+1)$. This is easily seen by the fact that the first ball can be put into one of the n urns so it has n choices. Once this first ball is put in one of the n urns, the second ball has only $n-1$ choices because each urn contains at most one ball. In this way, the r^{th} ball has $n-r+1$ choices. Therefore, the probability is $P = \frac{n(n-1)\cdots(n-r+1)}{n^r}$.

(c) If U_1 contains exactly m balls ($m \leq r$), then first we need to decide which m balls out of r balls are to be put into U_1: there are $\binom{r}{m}$ ways. Second, we need to put the rest of $r-m$ balls which are outside U_1 into the rest of the $n-1$ urns $U_2, \cdots, U_n$. Since there is no restriction, the total number of ways for this allocation is $(n-1)^{(r-m)}$. Therefore, the numerator is $\binom{r}{m} \cdot (n-1)^{(r-m)}$, and the probability is $P = \binom{r}{m} \cdot (n-1)^{(r-m)}/n^r$.

5 (10 points).

(a) P(A) is correct and P(B) is wrong. In calculating the probability of events A or event B, the denominator is the same, which is $\binom{52}{5}$.

For event A, we first choose 1 rank out of 13 ranks, and then choose 3 cards from that rank, which is $\binom{4}{3}$. Because the 2 cards to be selected are from a different rank, so there are 12 ranks left for us to choose from. Once the different rank is chosen, we have $\binom{4}{2}$ ways to choose the cards belonging to that rank. Thus, the numerator in P(A) is $(13)(12)\binom{4}{3}\binom{4}{2}$.

(b) For event B, since the two ranks picked up can be the same, which is different from event A, we have $\binom{13}{2}$ ways to select the ranks of 2 cards. For each selected rank, we have $\binom{4}{2}$ ways to pick up 2 cards. The 5th card comes from a different rank, so we choose the card out of the 52-4-4=44 cards left, which is $\binom{44}{1}$. Thus, the numerator in P(B) should be $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{44}{1}$.

(c) See the arguments in (a) and (b) above.

The grades are available on Blackboard. If you have any questions about these exercises, or if you find any grading errors, please stop by Kidder 113B, ***Thursday 1:00-2:00 pm****.*