ST561: Homework 2
Due: Monday, Oct.10, 2011

1. Four cards are drawn from a deck of 52 ordinary playing cards, without replacement.

    (a) What is the probability that all 4 cards are aces?

    (b) What is the probability of drawing, in order, the aces of clubs, diamonds, hearts, and spades?

    (c) Answer (a) and (b) above if each card is replaced (and the deck shuffled) after it is drawn.

2. Suppose that you own 6 different Spanish textbooks, 4 different History textbooks, 4 different Geology textbooks, and 5 different English textbooks. You have reserved a shelf on a bookcase on which you place all these textbooks. How many different ways are there to place all 19 textbooks on the shelf so that all the textbooks in a given subject are grouped together?

3. Suppose that 10 married couples, 5 men and 5 women, decided to purchase ten seats all in a row at the Majestic Theater for the performance of the Phantom of the Opera. If, after the tickets arrived, they were passed out completely at random to the ten people, how many different ways would it be possible for each of the following events to occur:

    (a) All of the men are seated together in contiguous seats;

    (b) Each person is seated next to a person of the opposite sex;

    (c) Each person is seated next to his or her spouse;

    (d) What is the probability that each person is seated next to his or her spouse?

4. (Urn Model) r different balls $B_1, \ldots, B_r$ are to be randomly put into n different urns $U_1, \ldots, U_n$ ($r \leq n$). What is the probability of the following events?

    (a) Urns $U_1, \ldots, U_r$ each contains exactly one ball;

    (b) No urn contains more than one ball;

    (c) Urn U_1 contains exactly m balls ($m \leq r$).

5. Consider the experiment of dealing a five card poker hand at random from a deck of 52 ordinary playing cards. Let A be the event that you obtain a full house (3 cards of one rank and 2 cards of a second rank, e.g., 3 kings and 2 eights). Let B be the event that you obtain two pairs (2 cards of one rank, 2 cards of a second rank, and a 5th card of a third rank, e.g., 2 tens, 2 threes and the Jack of diamonds). One of the following two probabilities is correct and the other probability is incorrect.

$$P(A) = \frac{(13)(12)\binom{4}{3}\binom{4}{2}}{\binom{52}{5}}, \quad P(B) = \frac{(13)(12)\binom{4}{2}\binom{4}{2}(44)}{\binom{52}{5}}.$$

(a) Determine which probability is correct;

(b) Correct the incorrect probability;

(c) Explain clearly how the correct probability is obtained and why the incorrect probability is wrongly calculated.