ST561: Homework 1

Due: Monday, Oct.3, 2011

Note: Exercises 1–4 are not required to be handed in. But please make sure you can solve them.

1. Find the union $A_1 \cup A_2$ and the intersection $A_1 \cap A_2$ of the two sets A_1 and A_2 where

   (a) $A_1 = \{x : 0 < x < 1\}$, $A_2 = \{x : 1 \le x < 3\}$;
   (b) $A_1 = \{(x, y) : 0 \le x < 3, 0 < y \le 3\}$, $A_2 = \{(x, y) : 1 < x \le 4, 1 \le y < 4\}$;
   (c) $A_1 = \{(x, y) : x+y \le 1, x \ge 0, y \ge 0\}$, $A_2 = \{(x, y) : x^2+y^2 \le 1/2, x \ge 0, y \ge 0\}$.

2. Find $\cup_{k=1}^{\infty} A_k$ if

   (a) $A_k = \{x : 1/k \le x < 2 - 1/k\}$, $k = 1, 2, 3, \ldots$;
   (b) $A_k = \{(x, y) : 1/k < x^2 + y^2 \le 4 - 1/k\}$, $k = 1, 2, 3, \ldots$.

3. Find $\cap_{k=1}^{\infty} A_k$ if

   (a) $A_k = \{x : 2 - 1/k < x \le 2\}$, $k = 1, 2, 3, \ldots$;
   (b) $A_k = \{x : 3 < x \le 3 + 1/k\}$, $k = 1, 2, 3, \ldots$;
   (c) $A_k = \{(x, y) : 0 \le x^2 + y^2 < 1/k\}$, $k = 1, 2, 3, \ldots$.

4. Textbook, page 52, 1.3.5 (iv).

5. Textbook, page 52, 1.3.6.

6. A man and a woman (unrelated) each have two children. Suppose we know that at least one of the man's children is a boy, and the woman's younger child is a boy. Does the chance that the man has two boys equal the chance that the woman has two boys?

7. The following procedure was used in a survey of cheating on taxes. A taxpayer is selected at random among all Oregon taxpayers to be a subject in the survey. The subject is given a six-sided die with 4 green sides and 2 red sides, and is given the following instructions:

   – Roll the die (the outcome of the roll is not seen by anyone except the subject);
   – If the outcome is green, say "Yes" if you cheated on your Oregon tax return last year, and say "No" if you did not;
   – If the outcome is red, say "Yes".

   (a) Suppose that 20% of Oregon taxpayers cheated last year. If a subject says "Yes", what is the probability that he/she cheated on last year's Oregon tax return?
   (b) Suppose we do not know the percentage of Oregon taxpayers who cheated last year, and we perform this survey to estimate it. We select a random sample of 120 taxpayers and put them through the procedure as described above. It turns out that 54 of them say "Yes". Estimate the percentage of Oregon taxpayers who cheated last year.