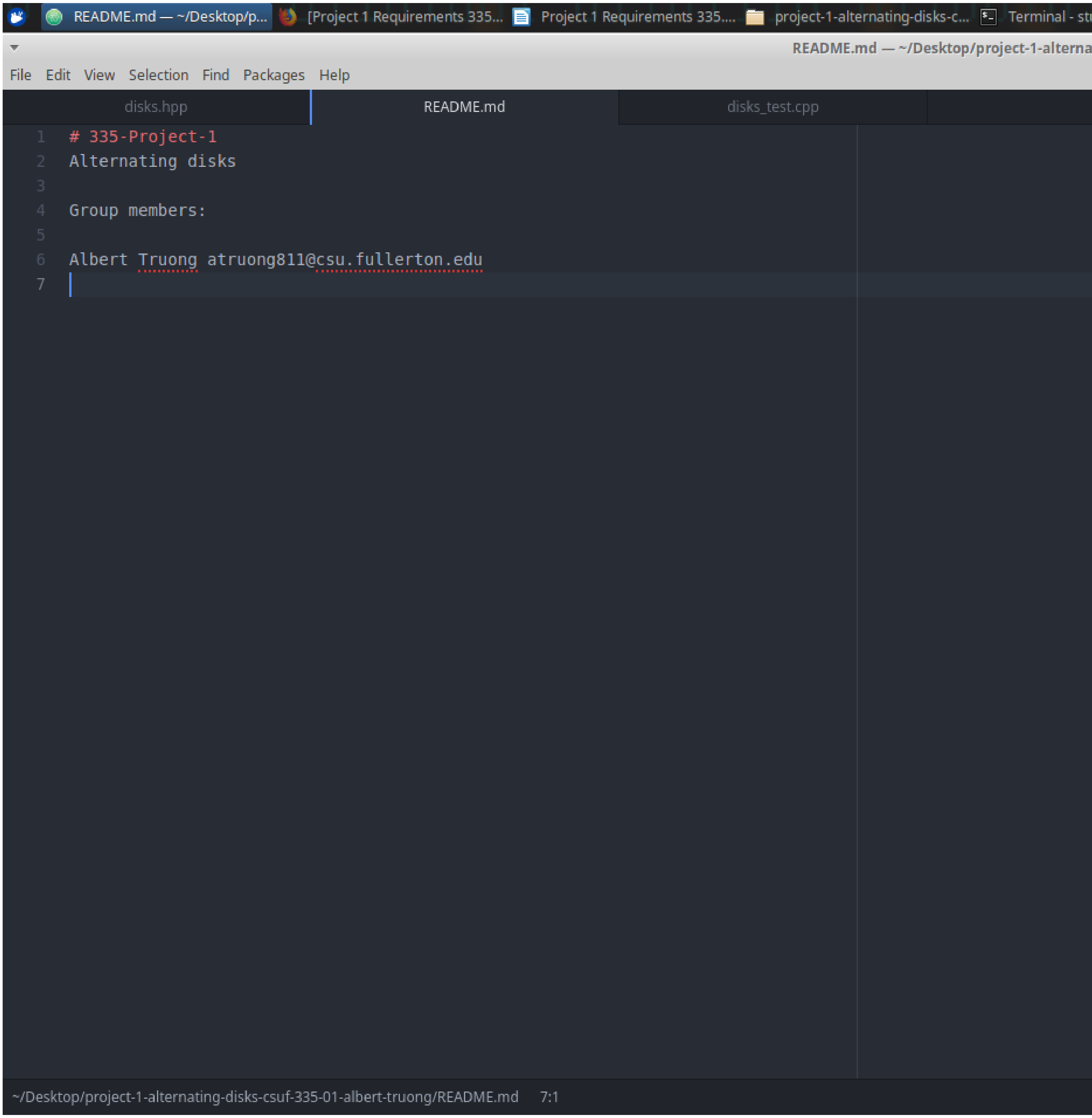
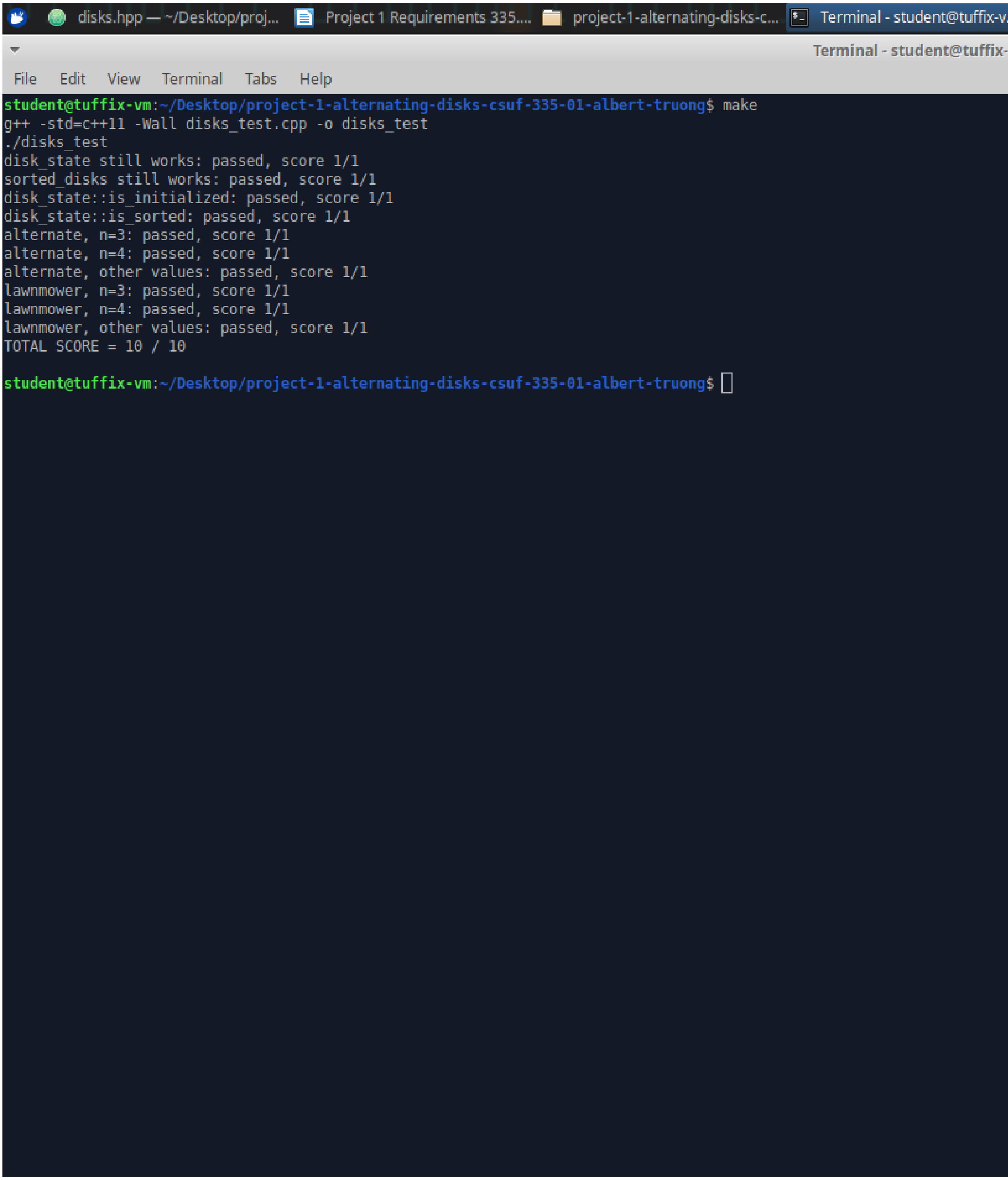
Project 1: Implementing Algorithms

CPSC 335 - Algorithm Engineering

Fall 2020

Albert Truong atruong811@csu.fullerton.edu




disks.hpp — ~/Desktop/proj...
Project 1 Requirements 335....
project-1-alternating-disks-c...
Terminal - student@tuffix-v...
Terminal - student@tuffix-
File
Edit
View
Terminal
Tabs
Help
student@tuffix-vm:~/Desktop/project-1-alternating-disks-csuf-335-01-albert-truong$ make
g++ -std=c++11 -Wall disks_test.cpp -o disks_test
./disks_test
disk_state still works: passed, score 1/1
sorted_disks still works: passed, score 1/1
disk_state::is_initialized: passed, score 1/1
disk_state::is_sorted: passed, score 1/1
alternate, n=3: passed, score 1/1
alternate, n=4: passed, score 1/1
alternate, other values: passed, score 1/1
lawnmower, n=3: passed, score 1/1
lawnmower, n=4: passed, score 1/1
lawnmower, other values: passed, score 1/1
TOTAL SCORE = 10 / 10
student@tuffix-vm:~/Desktop/project-1-alternating-disks-csuf-335-01-albert-truong$

Pseudocode:

Generic Swap Function:

A = T

B= (T+1)

T = B

(T+1) = A

Algorithm 1 (Lawnmower Algorithm):

Count=0

For k = 0 to n/2

For i = k to 2n do

If i= light && (i+1)= dark then

swap

Count++

End if

End for

For j = 2n to k do

If j= dark && (j-1)= light then

swap

Count++

End if

End for

End for

Algorithm 2 (Alternating Algorithm):

COUNT = 0

For i = 0 to n do

If i%2==1 then

k==0

Else

k==1

End if

For j = k to 2n-1 step 2 do

    If j= light && (j+1)= dark then

    Swap

    COUNT++

    End if

End for

End for

Mathematical Analysis for each pseudocode on Time Complexity (Step Count)

SC = Step Count

Generic Swap Function

A = T // SC = 1

B = (T+1) // SC = 2

T = B // SC = 1

(T+1) = A // SC = 2

SC= 1+2+1+2 = 6

Algorithm 1 (Lawnmower Algorithm) $\boldsymbol{\theta(n^2)}$:

COUNT = 0 // SC =1

For k = 0 to n/2 do // SC =$\sum_{k=0}^{\frac{n}{2}}$

For i = k to 2n do // SC = $\sum_{j=k}^{2n}$

If i= light && (i+1)= dark then // SC = 4

Swap // SC = 6

COUNT++ // SC= 1

End if

End for

For j = 2n to k do // SC = $\sum_{j=2n}^{k}$

If j= dark && (j-1)= light then // SC = 4

Swap // SC = 6

COUNT++ // SC= 1

End if

End for

End for

SC = 1 + $\sum_{k=0}^{\frac{n}{2}}(\sum_{i=k}^{2n}(4 + max(6+1,0)) + \sum_{j=2n}^{k}(4 + \max(6+1,0)))$

= 1 + $\sum_{k=0}^{\frac{n}{2}}(\sum_{i=k}^{2n}(11) + \sum_{j=2n}^{k}(11))$

= 1 + $\sum_{k=0}^{\frac{n}{2}}(\sum_{i=k}^{2n}(11) + \sum_{j=2n}^{k}(11))$

= 1 + $\sum_{k=0}^{\frac{n}{2}}(2\sum_{i=k}^{2n}(11))$ = 1+$\sum_{k=0}^{\frac{n}{2}}(\sum_{i=k}^{2n}(22))$

= 1 + $\sum_{k=0}^{\frac{n}{2}}(\sum_{i=0}^{2n}(22) - \sum_{i=0}^{k-1}(22))$

$\sum_{i=0}^{2n}(22) - \sum_{i=0}^{k-1}(22)$) = 22 + 44n – (22(k-1) + 22) = 22 + 44n – (22k – 22 + 22) = 44n -22k +22

= 22(2n – k + 1)

= 1 + $\sum_{k=0}^{\frac{n}{2}}$(22(2n – k + 1)) = 1 + 22 $\sum_{k=0}^{\frac{n}{2}}$(2n – k + 1) =1 + 22 $\sum_{k=0}^{\frac{n}{2}}$(2n + 1) + 22 $\sum_{k=0}^{\frac{n}{2}} k$

= 1 + 22(2n + 1) + 22(2n + 1)($\frac{n}{2}$) + 22($\frac{\frac{n}{2}(\frac{n}{2}+1)}{2}$)

= 1 + 44n + 22 + 22n^2 + 11n + 22($\frac{\frac{n^2}{4}+\frac{n}{2}}{2}$) = 22$n^2$ + 55n + 23 +$\frac{11n^2}{4} + \frac{11n}{2}$

$= \frac{99n^2}{4} + \frac{121n}{2} + 23$

Proof $\boldsymbol{\theta}(\boldsymbol{n^2})$

$$\frac{99n^2}{4} + \frac{121n}{2} + 23 \in \theta(n^2)$$

Find a value for C > 0 and $n_0 \geq 0$ such that $\frac{99n^2}{4} + \frac{121n}{2} + 23 \leq$ c*n^2 $\forall$ n $\geq n_0$

C = 23 + $\frac{99}{4} + \frac{121}{2} = \frac{92}{4} + \frac{99}{4} + \frac{484}{4} = \frac{675}{4}$

$\frac{99n^2}{4} + \frac{121n}{2} + 23 \leq \frac{675}{4}n^2$

$-\frac{675}{4}n^2 + \frac{99n^2}{4} + \frac{121n}{2} + 23 \leq 0$

$-\frac{666}{4}n^2$ + $\frac{89}{2}$n + 23 $\leq$ 0

Divide by $\frac{666}{4}$

-n^2 + $\frac{356}{1332}$n + $\frac{92}{666} \leq 0$

n $\leq n^2$

n $\leq n^2$ True

n $\geq$ 1 n_0=1

Proof by Definition $\boldsymbol{\theta}(\boldsymbol{n^3})$

$\frac{99n^2}{4} + \frac{121n}{2} + 23 \in \theta(n^3)$

Proof by Definition

Assume that the statement is true (by contradiction)

Find C > 0 and $n_0 \geq 0$ such that $\frac{99n^2}{4} + \frac{121n}{2} + 23 \leq$ c*n^3 $\forall$ n $\geq n_0$

Divide by n^3

$$\frac{\frac{99n^2}{4}}{n^3} + \frac{\frac{121n}{2}}{n^3} + \frac{23}{n^3} \leq c*\frac{n^3}{n^3}$$

0 + 0 + 0 $\leq$ c

Since C > 0, this is true. $\frac{99n^2}{4} + \frac{121n}{2} + 23 \in \theta(n^3)$ is false by ways of Contradiction.

Thus, $\frac{99n^2}{4} + \frac{121n}{2} + 23 \in \theta(n^2)$

The algorithm efficiency class for this pseudocode of the Lawnmower Algorithm is $\boldsymbol{\theta(n^2)}$

Algorithm 2 (Alternating Algorithm) $\boldsymbol{\theta(n^2)}$

```
COUNT = 0                        // SC= 1
For i = 0 to n do                // SC= (n-0)+1 = n+1

If i%2==1 then                   // SC= 2
k==0                             // SC= 1
Else
k==1                             // SC= 1
End if

For j = k to 2n-1 step 2 do      // SC = (2n-1-k)/2 +1 = n - 1/2 k - 1/2
If j=light && j+1=dark then      // SC = 4
        Swap                     // SC = 6
        COUNT++                  // SC= 1
        End if
        End for
End for
```

SC= 1+ ((n+1) (2+max(1,1)+ (n - $\frac{1}{2}$k - $\frac{1}{2}$) (4+max(6+1,0)))

= 1+ ((n+1) (2+1 + (n - $\frac{1}{2}$k - $\frac{1}{2}$) (4 + 7))

= 1 + ((n+1) (4 + (n - $\frac{1}{2}$k - $\frac{1}{2}$)(11))

= 1 + ((n+1) (4 + 11n - $\frac{11}{2}$k - $\frac{11}{2}$))

= 1 + (4n + $11n^2$ – $\frac{11}{2}$k*n + $\frac{11}{2}n$ + 11n - $\frac{11}{2}$k - $\frac{11}{2}$)

= $11n^2$ - $\frac{11}{2}$k*n + $\frac{41}{2}n$ -$\frac{11}{2}k$ - $\frac{11}{2}$

Proof $\boldsymbol{\theta(n^2)}$

$11n^2$ - $\frac{11}{2}$k*n + $\frac{41}{2}n$ -$\frac{11}{2}k$ - $\frac{11}{2}$ $\in \theta(n^2)$

Find a value for C > 0 and $n_0 \geq 0$ such that $11n^2$ - $\frac{11}{2}$k*n + $\frac{41}{2}n$ -$\frac{11}{2}k$ - $\frac{11}{2}$ $\leq$ c*n^2 $\forall$ n $\geq n_0$

K = 0 or 1

For K = 0

$11n^2$ + $\frac{41}{2}n$ - $\frac{11}{2}$ $\leq$ c*n^2

C=11 + $\frac{41}{2}$ + $\frac{11}{2}$ = $\frac{22}{2}$ + $\frac{41}{2}$ + $\frac{11}{2}$ = $\frac{74}{2}$ = 37

$11n^2$ + $\frac{41}{2}n$ - $\frac{11}{2}$ $\leq$ $37n^2$

$11n^2$ - $37n^2$ + $\frac{41}{2}n$ - $\frac{11}{2}$ ≤ 0

- $27n^2$ + + $\frac{41}{2}n$ - $\frac{11}{2}$ ≤ 0

Divide by 27

$\frac{41}{54}n$ - $\frac{11}{54}$ $\leq n^2$

n $\leq n^2$ True

n $\geq$ 1 n_0=1

For K = 1

$11n^2$ - $\frac{11}{2}$n + $\frac{41}{2}n$ -$\frac{11}{2}$ - $\frac{11}{2}$ $\leq$ c*n^2

$11n^2$ + $\frac{31}{2}n \leq$ c*n^2

C = 11 + $\frac{31}{2}$ = $\frac{22}{2}$ + $\frac{31}{2}$ = $\frac{53}{2}$

$11n^2$ + $\frac{31}{2}n \leq$ $\frac{53}{2}n^2$

$22n^2$ - $\frac{53}{2}n^2$ + $\frac{31}{2}n \leq 0$

$-\frac{9}{2}n^2$ + $\frac{31}{2}n \leq 0$

Divide by $\frac{9}{2}$

$\frac{31}{9}$n $\leq n^2$

n $\leq n^2$ True

n $\geq$ 1 n_0=1

Proof by Definition $\boldsymbol{\theta(n^3)}$

$11n^2 - \frac{11}{2}$k*n + $\frac{41}{2}n - \frac{11}{2}k - \frac{11}{2} \in \theta(n^3)$

Assume that the statement is true (by contradiction)

Find C > 0 and $n_0 \geq 0$ such that $11n^2 - \frac{11}{2}$k*n + $\frac{41}{2}n - \frac{11}{2}k - \frac{11}{2} \leq$ c*n^3 $\forall$ n $\geq n_0$

Divide by n^3

$$\frac{11n^2}{n^3} - \frac{\frac{11}{2}k*n}{n^3} + \frac{\frac{41}{2}n}{n^3} - \frac{\frac{11}{2}k}{n^3} - \frac{\frac{11}{2}}{n^3} \leq c*\frac{n^3}{n^3}$$

0   +   0   +   0   -   0   + 0 $\leq$ c

Since C > 0, this is true. Thus, $11n^2 - \frac{11}{2}$k*n + $\frac{41}{2}n - \frac{11}{2}k - \frac{11}{2} \in \theta(n^3)$ is false by ways of Contradiction.

Thus, $11n^2 - \frac{11}{2}$k*n + $\frac{41}{2}n - \frac{11}{2}k - \frac{11}{2} \in \theta(n^2)$

The algorithm efficiency class for this pseudocode of the Alternating Algorithm is $\boldsymbol{\theta(n^2)}$