$$H\left[f(-1,-2)\right]=3>0; \quad \frac{\partial^2 f}{\partial x^2}(-1,-2)=4>0 \quad \wedge \quad f(-1,-2)=14$$

$M(-1,-2,14)$ puntua **minimo erlatiboa** da.

D) Integrazio-limiteak bi era desberdinetan jarri integral honetan $I=\iint_D f(x,y)\,dx\,dy$, hurrengo $[D]$ domeinuarentzat:

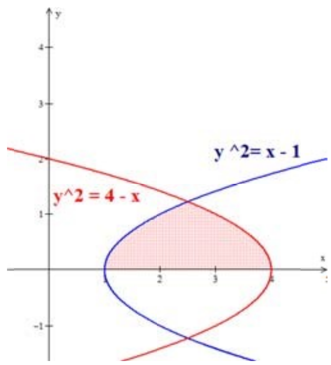
$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, x\geq 0\ ;\ \ y\geq 0\ ;\ \ y^2\leq x-1\ ;\ \ y^2\leq 4-x\right\}$$

Kalkulatu $[D]$ domeinua x ardatzaren inguruan biratzekoan sorrarazten den bolumena.

(6 p)

Ebazpena

Domeinua bat dator lehenengo koadrantean 0X ardatzeko $y^2=x-1$; $y^2=4-x$ parabolek mugatutako eskualdearekin.

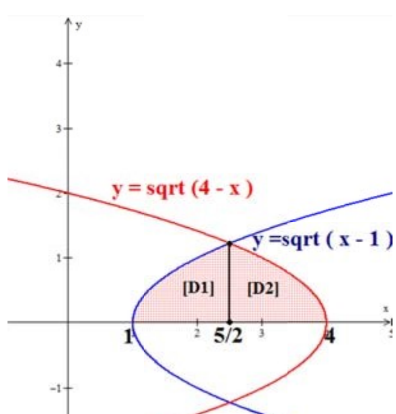


Bi kurba horien mozketa puntua lehenengo koadrantean:

$$\begin{cases} y^2=x-1 \\ y^2=4-x \end{cases} \rightarrow \quad x-1=4-x \quad \rightarrow \quad x=\frac{5}{2} \quad \rightarrow \quad P\left(\frac{5}{2},\sqrt{\frac{3}{2}}\right)$$

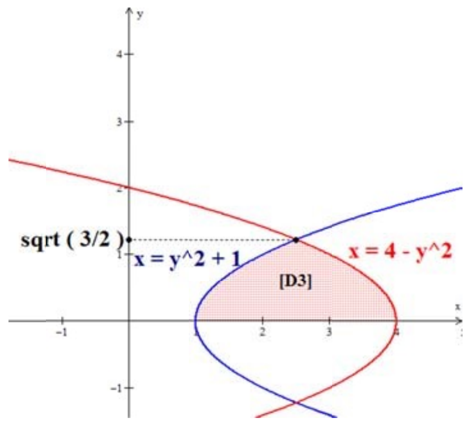
- Lehenengo integrazio aldagaitzat (y) hartuz:

$$I=\int_1^{5/2}dx\int_0^{\sqrt{x-1}}f(x,y)\,dy+\int_{5/2}^{4}dx\int_0^{\sqrt{4-x}}f(x,y)\,dy$$



- Lehenengo integrazio aldagaitzat (x) hartuz:

$$I=\int_0^{\sqrt{\frac{3}{2}}} dy\int_{y^2+1}^{4-y^2} f(x,y)\,dx$$



$[D]$ domeinua x ardatzaren inguruan biratzekoan sorrarazten den bolumena:

$$V=\pi\left[\int_1^{5/2}\left(\sqrt{x-1}\right)^2 dx+\int_{5/2}^{4}\left(\sqrt{4-x}\right)^2 dx\right]=\pi\left[\frac{x^2}{2}-x\Big|_1^{5/2}+4x-\frac{x^2}{2}\Big|_{5/2}^{4}\right]=\frac{9}{4}\pi\quad u^3$$

Existitzen den simetria kontuan hartuz, bolumena honela geratzen da:

$$V=2\pi\left[\int_1^{5/2}\left(\sqrt{x-1}\right)^2 dx\right]=2\pi\left[\frac{x^2}{2}-x\Big|_1^{5/2}\right]=\frac{9}{4}\pi\quad u^3$$

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2. ORRIA (20 puntu)

A) Klasifikatu eta ebatzi hurrengo EDA: $\left(xy-2y^2\right)dx-\left(x^2-3xy\right)dy=0$

(4 p)

Ebazpena

EDA **homogeneoa**:

$$y'=\frac{xy-2y^2}{x^2-3xy}\quad\rightarrow\quad y'=\frac{\frac{xy-2y^2}{x^2}}{\frac{x^2-3xy}{x^2}}=\frac{\frac{y}{x}-2\left(\frac{y}{x}\right)^2}{1-3\left(\frac{y}{x}\right)}$$

Hurrengo aldaketarekin: $\frac{y}{x}=u\quad\Rightarrow\quad y=xu\quad\Rightarrow\quad y'=u+xu'$

$$u+xu'=\frac{u-2u^2}{1-3u}\quad\rightarrow\quad x\frac{du}{dx}=\frac{u-2u^2}{1-3u}-u=\frac{u-2u^2-u+3u^2}{1-3u}=\frac{u^2}{1-3u}$$

Aldagai banangarrien EDA lortzen dugu: $\dfrac{1-3u}{u^2}du = \dfrac{1}{x}dx$

Integratuz:

$$-\frac{1}{u} - 3\ln|u| = \ln|x| + C \quad \rightarrow \quad -\frac{x}{y} = \ln|x| + 3\ln\left|\frac{y}{x}\right| + C \quad \rightarrow \quad \boxed{-\frac{x}{y} = \ln\left|\frac{y^3}{x^2}\right| + C}$$

B) Ebatzi hurrengo koefiziente aldakorreko ekuazioa

$$\left(x^2-1\right)y'' - 2x\,y' + 2xy = \left(x^2-1\right)^2$$

jakinda $y_1(x) = x$ ekuazio homogeneoaren soluzio partikularra dela. ***(6 p)***

Ebazpena

$$\left(x^2-1\right)y'' - 2x\,y' + 2xy = \left(x^2-1\right)^2 \quad \rightarrow \quad y'' - \frac{2x}{x^2-1}y' + \frac{2x}{x^2-1}y = x^2-1$$

Elkartutako ekuazio homogeneoaren beste soluzio partikularra (y_2), (y_1)-ekin linealki independentea dena, hurrengo formularen bidez lor daiteke:

$$y_2 = y_1 \int \frac{\exp\left(-\int P(x)dx\right)}{y_1^{\,2}}dx$$

$$P(x) = \frac{-2x}{x^2-1} \quad \Rightarrow \quad \int P(x)\,dx = -\int \frac{2x}{x^2-1}\,dx = -\ln\left|x^2-1\right| + cte$$

$$\exp\left(-\int P(x)\,dx\right) = e^{\ln\left|x^2-1\right|} = x^2-1 \quad \Rightarrow$$

$$\int \frac{\exp\left(-\int P(x)dx\right)}{y_1^{\,2}}dx = \int \frac{(x^2-1)}{x^2}dx = \int \left(1 - \frac{1}{x^2}\right)dx = x + \frac{1}{x} + cte$$

$$y_2 = y_1 \int \frac{\exp\left(-\int P(x)dx\right)}{y_1^{\,2}}dx = x\left[x + \frac{1}{x}\right] = x^2 + 1$$

Beraz, elkartutako ekuazio homogeneoaren soluzio orokorra hurrengoa da:

$$y_h = C_1 \cdot x + C_2(x^2+1)$$

Parametroen aldakuntzaren metodoa aplikatuko dugu:

$$y = L_1(x)\cdot x + L_2(x)\cdot(x^2+1) \qquad [*]$$

$L'_1(x)$ y $L'_2(x)$ hurrengo sistema ebatziz lortuko dira:

$$\begin{cases} L_1' \cdot x + L_2' \cdot (x^2+1) = 0 \\ L_1' \cdot 1 + L_2' \cdot 2x = x^2 - 1 \end{cases}$$

$$L_1'(x) = \frac{\begin{vmatrix} 0 & x^2+1 \\ x^2-1 & 2x \end{vmatrix}}{\begin{vmatrix} x & x^2+1 \\ 1 & 2x \end{vmatrix}} = \frac{-(x^2-1)(x^2+1)}{2x^2-x^2-1} = \frac{-(x^2-1)(x^2+1)}{x^2-1} = -(x^2+1)$$

$$L_2'(x) = \frac{\begin{vmatrix} x & 0 \\ 1 & x^2-1 \end{vmatrix}}{\begin{vmatrix} x & x^2+1 \\ 1 & 2x \end{vmatrix}} = \frac{x(x^2-1)}{x^2-1} = x$$

$$L_1(x) = -\int (x^2+1)\,dx = -\frac{x^3}{3} - x + A \qquad ; \qquad L_2(x) = \int x\,dx = \frac{x^2}{2} + B$$

[*] adierazpenean ordezkatuz, soluzio orokorra lortzen da:

$$\boxed{y} = \left[-\frac{x^3}{3} - x + A\right] \cdot x + \left[\frac{x^2}{2} + B\right] \cdot (x^2+1) = Ax + B(x^2+1) - \frac{x^4}{3} - x^2 + \frac{x^4}{2} + \frac{x^2}{2} =$$

$$= \boxed{Ax + B(x^2+1) + \frac{x^4}{6} - \frac{x^2}{2}}$$

C) Hurrengo EDA ebatzi**:** $y'' + 4y = (t-1)^2 u_1 \quad ; \quad y(0) = y'(0) = 0$

(6 p)

Ebazpena

Laplace transformatua aplikatuko da.

Hurrengo propietatea kontuan hartuz $\mathfrak{L}\,[f(t) \cdot u_a] = e^{-pa} \mathfrak{L}\,[f(t+a)]$

$$\mathfrak{L}\,[(t-1)^2 \cdot u_1] = e^{-p} \mathfrak{L}\,[(t+1-1)^2] = e^{-p} \mathfrak{L}\,[t^2] = e^{-p} \cdot \frac{2}{p^3}$$

$$\left[p^2 Y(p) - p y(0) - y'(0)\right] + 4Y(p) = \frac{2e^{-p}}{p^3}$$