MATEMATIKA APLIKATUA

2. ORRIA (200 puntu)

A) Kalkulatu: $\int \ln(\operatorname{sen} x)\cdot \operatorname{sen} x\, dx \qquad \int \frac{1}{x^5\cdot\sqrt{1+\frac{1}{x^2}}}\,dx$

(60 p)

Ebazpena

$$I=\int \ln(\operatorname{sen} x)\cdot \operatorname{sen} x\, dx = \left\| \begin{matrix} u=\ln(\operatorname{sen} x) & du=\frac{\cos x}{\operatorname{sen} x}dx \\ dv=\operatorname{sen} x\,dx & v=-\cos x \end{matrix} \right\| =$$

$$=-\ln(\operatorname{sen} x)\cdot\cos x+\int\frac{\cos^2 x}{\operatorname{sen} x}dx=-\ln(\operatorname{sen} x)\cdot\cos x+J$$

$$J=\int\frac{\cos^2 x}{\operatorname{sen} x}dx=\int\frac{1-\operatorname{sen}^2 x}{\operatorname{sen} x}dx=\int\left(\frac{1}{\operatorname{sen} x}-\operatorname{sen} x\right)dx=\int\frac{1}{\operatorname{sen} x}dx-\int\operatorname{sen} x\,dx=H+\cos x$$

$$H=\int\frac{1}{\operatorname{sen} x}dx=\left\| \begin{matrix} t=\operatorname{tg}\frac{x}{2} & dx=\frac{2}{1+t^2}dt \\ \operatorname{sen} x=\frac{2t}{1+t^2} & \end{matrix} \right\| =\int\frac{1+t^2}{2t}\cdot\frac{2}{1+t^2}dt=\int\frac{1}{t}dt=$$

$$=\ln|t|+K=\ln\left|\operatorname{tg}\frac{x}{2}\right|+K_1$$

$$\boxed{I}=-\ln(\operatorname{sen} x)\cdot\cos x+J=-\ln(\operatorname{sen} x)\cdot\cos x+\cos x+H=$$

$$=\boxed{-\ln(\operatorname{sen} x)\cdot\cos x+\cos x+\ln\left|\operatorname{tg}\frac{x}{2}\right|+K}$$

$$I=\int\frac{1}{x^5\cdot\sqrt{1+x^{-2}}}dx=\int x^{-5}\left(1+x^{-2}\right)^{-1/2}dx=\begin{bmatrix} m=-5 & n=-2 \\ p=-\frac{1}{2}\notin\mathbb{Z} & \frac{m+1}{n}=2\in\mathbb{Z} \end{bmatrix}=\begin{pmatrix}\text{binomia}\\ 2^{\text{o}}\text{ caso}\end{pmatrix}=$$

$$= \left\| \begin{matrix} x^{-2} = t \to x = t^{-1/2} \\ dx = -\frac{1}{2} t^{-3/2} dt \end{matrix} \right\| = \int t^{5/2} (1+t)^{-1/2} \left(-\frac{1}{2} \right) t^{-3/2} dt = -\frac{1}{2} \int t (1+t)^{-1/2} \, dt =$$

$$= \left\| \begin{matrix} 1+t = z^2 \\ dt = 2z\, dz \end{matrix} \right\| = -\frac{1}{2} \int (z^2 - 1) z^{-1} \, 2z\, dz = -\int (z^2 - 1) \, dz =$$

$$= z - \frac{z^3}{3} + K = (1+t)^{1/2} - \frac{(1+t)^{3/2}}{3} + K = \boxed{(1+x^{-2})^{1/2} - \frac{(1+x^{-2})^{3/2}}{3} + K}$$

B) Zehaztu a konstante erreal positiboaren balioa, hurrengo domeinuak

$$D = \left\{ (x,y) \in \mathbb{R}^2 \;/\; x^2 + y - 4 \le 0 \;\wedge\; y \ge a \cdot x^2 \;\wedge\; y \ge 0 \right\}$$

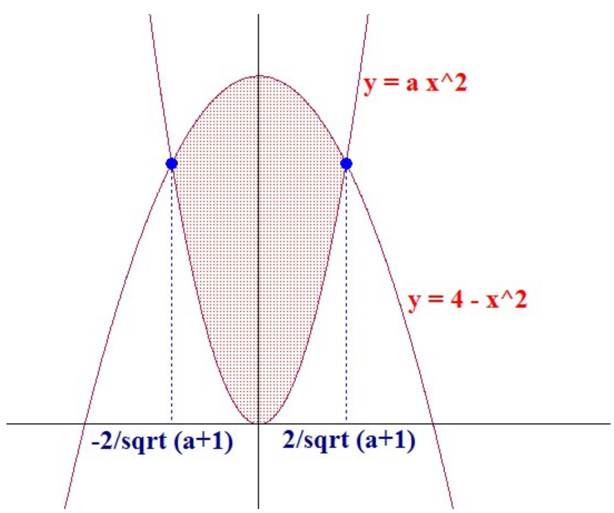
definitutako azalera $A = \frac{16}{3} \quad u^2$ izan dadin.

(80 p)

Ebazpena

Bi parabolak ditugu ardatz bertikala dutenak. Ebakidura puntuak hurrengoak dira:

$$\left. \begin{matrix} y = a\,x^2 \\ y = 4 - x^2 \end{matrix} \right\} \Rightarrow \quad x = \pm \frac{2}{\sqrt{a+1}}$$



Simetria kontuan izanda, azalera horrela kalkulatuko dugu:

$$A = 2\left[\int_0^{\frac{2}{\sqrt{a+1}}} (4 - x^2) \, dx - \int_0^{\frac{2}{\sqrt{a+1}}} (ax^2) \, dx \right] = 2\left[\left(4x - \frac{a+1}{3} x^3 \right) \Bigg|_0^{\frac{2}{\sqrt{a+1}}} \right] = \frac{16}{3} \quad \to$$

$$\frac{8}{\sqrt{a+1}}-\frac{8}{3\sqrt{a+1}}=\frac{8}{3} \rightarrow \frac{2}{\sqrt{a+1}}=1 \rightarrow 2=\sqrt{a+1} \rightarrow \boxed{a=3}$$

C) Hurrengo integral inpropioak kalkulatu:

$$\int_4^{\infty}\frac{1}{x(\ln x)^2}\,dx \qquad \int_0^6\frac{2x}{\left(x^2-4\right)^{2/3}}\,dx$$

(60 p)

Ebazpena

$$\int_4^{\infty}\frac{1}{x(\ln x)^2}\,dx=\lim_{b\to\infty}\int_4^b\frac{1}{x(\ln x)^2}\,dx \underset{*}{=} \lim_{b\to\infty}\left[-\frac{1}{\ln b}+\frac{1}{\ln 4}\right]=\frac{1}{\ln 4}$$

$$\int_4^b\frac{1}{x(\ln x)^2}\,dx=\left[\ln x=t \quad \frac{1}{x}dx=dt\right]=\int_{\ln 4}^{\ln b}\frac{1}{t^2}\,dt=-\frac{1}{t}\bigg|_{\ln 4}^{\ln b}=-\frac{1}{\ln b}+\frac{1}{\ln 4} \quad (*)$$

$$\int_0^6\frac{2x}{\left(x^2-4\right)^{2/3}}\,dx=\int_0^2\frac{2x}{\left(x^2-4\right)^{2/3}}\,dx+\int_2^6\frac{2x}{\left(x^2-4\right)^{2/3}}\,dx=$$

$$=\lim_{\varepsilon\to 0}\left[\int_0^{2-\varepsilon}\frac{2x}{\left(x^2-4\right)^{2/3}}\,dx+\int_{2+\varepsilon}^6\frac{2x}{\left(x^2-4\right)^{2/3}}\,dx\right]\underset{*}{=}$$

$$=\lim_{\varepsilon\to 0}\left[3\left(\sqrt[3]{(2-\varepsilon)^2-4}-\sqrt[3]{-4}\right)+3\left(\sqrt[3]{32}-\sqrt[3]{(2+\varepsilon)^2-4}\right)\right]=3\left(\sqrt[3]{4}+\sqrt[3]{32}\right)=3\left(\sqrt[3]{4}+2\sqrt[3]{4}\right)=9\sqrt[3]{4}$$

$$\int_0^{2-\varepsilon}\frac{2x}{\left(x^2-4\right)^{2/3}}\,dx=3\left(x^2-4\right)^{1/3}\bigg|_0^{2-\varepsilon}=3\left[\sqrt[3]{(2-\varepsilon)^2-4}-\sqrt[3]{-4}\right] \quad (*)$$

$$\int_{2+\varepsilon}^{6} \frac{2x}{\left(x^2-4\right)^{2/3}}\,dx = 3\left(x^2-4\right)^{1/3}\Big|_{2+\varepsilon}^{6} = 3\left[\sqrt[3]{32} - \sqrt[3]{\left(2+\varepsilon\right)^2-4}\right] \quad (*)$$