KUDEAKETAREN ETA INFORMAZIO SISTEMEN INFORMATIKAREN INGENIARITZAKO GRADUA

ANALISIS MATEMATIKOA 2018ko urriaren 23an

1. ARIKETA

Izan bitez honako zenbaki konplexu hauek:

$$z_1 = \frac{-3+i}{1-2i}; \quad z_2 = \sqrt{2}_{\frac{3\pi}{4}}$$

Kalkulatu eta emaitza era binomikoan adierazi:

a) $z_1 \cdot z_2$

b) $\frac{z_1 - z_2}{z_1^4}$

Ebazpena:

a) $z_1 eta\ z_2$ era binomikoan idazten ditugu:

$$z_1 = \frac{-3+i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{-3-6i+i-2}{1+4} = \frac{-5-5i}{5} = -1-i$$

$$z_2 = \sqrt{2}\left(cos\left(\frac{3\pi}{4}\right) + isen\left(\frac{3\pi}{4}\right)\right) = \sqrt{2}\left(\frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -1+i$$

Orain, biderkadura egiten dugu:

$$z_1 \cdot z_2 = (-1-i)(-1+i) = 1 - i + i - i^2 = \boxed{2}$$

b) Eragiketak eginez:

$$\frac{z_1 - z_2}{z_1^4} = \frac{-1-i-(-1+i)}{\left(\sqrt{2}_{\frac{-\pi}{4}}\right)^4} = \frac{-1-i+1-i}{2^2_{-\pi}} = \frac{-2i}{4_\pi} = \frac{2_{\frac{3\pi}{2}}}{4_\pi} = \left(\frac{1}{2}\right)_{\frac{3\pi}{2}-\pi} =$$

$$\left(\frac{1}{2}\right)_{\frac{\pi}{2}} = \boxed{\frac{i}{2}}$$

2. ARIKETA

Honako segida hauen limitea kalkulatu:

a) $\lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{7^n}{3^{n+1}+7^{n-1}}$

b) $\lim_{n\to\infty} \frac{(n^2+n+1)\left(e^{\frac{1}{n}}-1\right)}{tan^2\left(\frac{1}{n}\right)}$

Ebazpena:

a) $\lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{7^n}{3^{n+1}+7^{n-1}} = A \cdot B$

$$A = \lim_{n\to\infty} \frac{n+1}{n} = 1$$

$$B = \lim_{n\to\infty} \frac{7^n}{3^{n+1}+7^{n-1}} = \lim_{n\to\infty} \frac{7^n}{7^{n+1}\left(\frac{3^{n+1}}{7^{n+1}}+\frac{1}{7^2}\right)}$$

$$= \lim_{n\to\infty} \frac{1}{7\left(\left(\frac{3}{7}\right)^{n+1}+\frac{1}{7^2}\right)} = \frac{1}{1/7} = 7$$

Beraz:

$$\lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{7^n}{3^{n+1}+7^{n-1}} = A \cdot B = 1 \cdot 7 = \boxed{7}$$

b) $\lim_{n\to\infty} \frac{(n^2+n+1)\left(e^{\frac{1}{n}}-1\right)}{tan^2\left(\frac{1}{n}\right)} \sim \lim_{n\to\infty} \frac{n^2 \cdot \left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)^2} = \lim_{n\to\infty} n^3 = \boxed{\infty}$

3. ARIKETA

Honako serie hauen konbergentzia aztertu:

a) $\sum_{n=1}^{\infty} \frac{1}{(3n+2)(3n+5)}$

b) $\sum_{n=1}^{\infty} \frac{a^n \sqrt{n}}{n^2+1} \qquad \forall a \in \mathbb{R}$

Ebazpena:

a) Gai positibozko serie bat da, beraz, zatiduraren irizpidea aplikatzen dugu:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{(3n+5)(3n+8)} \cdot \frac{(3n+2)(3n+5)}{1}$$
$$= \lim_{n\to\infty} \frac{9n^2+15n+6n+10}{9n^2+24n+15n+40}$$
$$= \lim_{n\to\infty} \frac{9n^2+21n+10}{9n^2+39n+40} \sim \lim_{n\to\infty} \frac{9n^2}{9n^2} = 1 \rightarrow Zalantza$$

Orain, Raabe-ren irizpidea aplikatzen dugu:

$$\lim_{n\to\infty} n\left(1 - \frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty} n\left(1 - \frac{9n^2+21n+10}{9n^2+39n+40}\right)$$
$$= \lim_{n\to\infty} n\left(\frac{9n^2+39n+40-9n^2-21n-10}{9n^2+39n+40}\right)$$
$$= \lim_{n\to\infty} n\left(\frac{18n+30}{9n^2+39n+40}\right) \sim \lim_{n\to\infty} \frac{18n^2}{9n^2} = 2 > 1$$

Beraz, **konbergentea** da.

b) Ez da gai positibozko serie bat, beraz, balio absolutua erabiltzen dugu:

$$|a_n| = \frac{|a|^n\sqrt{n}}{n^2+1} \sim \frac{|a|^n\sqrt{n}}{n^2} = \frac{|a|^n}{n^{3/2}}$$

Orain, zatiduraren irizpidea aplikatzen dugu:

$$\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|}=\lim_{n\to\infty}\frac{|a|^{n+1}}{(n+1)^{\frac{3}{2}}}\cdot\frac{(n)^{\frac{3}{2}}}{|a|^n}\sim|a|$$

$$\rightarrow\begin{cases}|a|<1\ (-1<a<1)\ \boldsymbol{Konbergente}\\ |a|>1\rightarrow\begin{cases}a>1\ \boldsymbol{Dibergente}\\ a<-1\ Ez\ abs.konbergente\end{cases}\\ |a|=1\ (a=1;a=-1)Zalantza\end{cases}$$

Zalantzazko kasuak aztertzen ditugu:

$\boxed{a<-1}$

$$\lim_{n\to\infty}\frac{a^n\sqrt{n}}{n^2+1}\sim\lim_{n\to\infty}\frac{a^n}{n^{3/2}}=\{a^n\gg n^{3/2}\}\neq 0$$

Baldintza beharrezkoa ez da betetzen, beraz, **dibergentea** da.

$\boxed{a=-1\ y\ a=1}$

$$\lim_{n\to\infty}\frac{(\pm1)^n\sqrt{n}}{n^2+1}\sim\lim_{n\to\infty}\frac{(\pm1)^n}{n^{3/2}}=0$$

Baldintza beharrezkoa betetzen da, beraz, konbergente edo dibergente izan daiteke.

Balio absolutua erabiliz:

$$|a_n|=\frac{1}{n^{3/2}}$$

Serie harmonikoa da eta berretzailea >1 da, beraz, **konbergentea** da.

4. ARIKETA

Honako serie konbergente honen batura kalkulatu:

$$\sum_{n=1}^{\infty} \left(\sqrt{2}\right)^{1-n}$$

Ebazpena:

a) $\sum_{n=1}^{\infty} \left(\sqrt{2}\right)^{1-n} = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\left(\sqrt{2}\right)^2} + \frac{1}{\left(\sqrt{2}\right)^3} + \cdots$

Serie geometrikoa da. Arrazoia $r = \frac{1}{\sqrt{2}}$ da, beraz, batura honako hau da:

$$S = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{\sqrt{2}}} = \boxed{\frac{\sqrt{2}}{\sqrt{2}-1}}$$