AZTERKETA PARTZIALA

1 ARIKETA: Klasifikatu ondoko matrizeak:

a) $A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$ (puntu 1)

$$A^2 = A \cdot A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

A simetrikoa da → $A = A^T$

A inbolutiboa da → $A^2 = I$

A ortogonala da → $A \cdot A = A \cdot A^T = I \Rightarrow \boxed{A^T = A^{-1}}$

b) $B = \begin{pmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{pmatrix}$ (puntu 1)

$$B^2 = B \cdot B = \begin{pmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{pmatrix} \begin{pmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{pmatrix} = \begin{pmatrix} 6/9 & 3/9 \\ 6/9 & 3/9 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{pmatrix} = B$$

$B^2 = B$ → B idempotentea da

c) $C = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$ (puntu 1) $\Longrightarrow C^T = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} = C$ → C simetrikoa da

$$C^2 = C \cdot C = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$C^2 = I$ → inbolutiboa da

$C \cdot C = I = C \cdot C^T \to C^T = C^{-1}$ → ortogonala da

$\boxed{\text{2. ARIKETA}}$ Sailkatu eta ebatzi (bateragarri denean) ondoko sistema $a \in \mathbb{R}$ parametroaren balioaren arabera:

$$x\cdot\bar{a}_1 + y\cdot\bar{a}_2 + z\cdot\bar{a}_3 = \bar{b} \quad \text{non} \qquad (3 \text{ puntu})$$

$\bar{a}_1=(1,3,4)$; $\bar{a}_2=(2,-1,1)$; $\bar{a}_3=(-3,5,a^2-14)$; $\bar{b}=(4,2,a+2)$

Sistema ondokoa da:

$$\left\{\begin{array}{l} x+2y-3z=4 \\ 3x-y+5z=2 \\ 4x+y+(a^2-14)z=a+2 \end{array}\right. \xrightarrow{\text{matrize zabaldua}} AM=\left(\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & (a+2) \end{array}\right)$$

Gauss-en metodoa aplikatuz:

$$AM=\left(\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & (a+2) \end{array}\right) \overset{E_2-3E_1}{\underset{E_3-4E_1}{\sim}} \left(\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right) \sim$$

$$\overset{E_3-E_2}{\sim} \underbrace{\left(\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right)}_{AM'}$$

A'

$|A'| = -7(a^2-16) \rightarrow |A'|=0$ baldin $\left\langle\begin{array}{l} a=4 \\ a=-4 \end{array}\right.$

Baldin $\boxed{a\neq 4,-4} \rightarrow h(A')=h(AM')=3 \rightarrow \boxed{S.B.D}$

Baldin $\boxed{a=4} \rightarrow A'=\left(\begin{array}{ccc} 1 & 2 & -3 \\ 0 & -7 & 14 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right) \Longrightarrow h(A')=2$; $\neq 0$

$AM'=\left(\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right) \rightarrow h(AM')=2$

$\Big\}\ \boxed{S.B.I.}$

Baldin $\boxed{a=-4}$

$$A' = \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -7 & 14 \\ 0 & 0 & 0 \end{array}\right) \overset{\neq 0}{} \Rightarrow h(A') = 2$$

$$Am' = \left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & 0 & -8 \end{array}\right) \xrightarrow[\text{minore bat}]{\text{3. ordenako}} \begin{vmatrix} 1 & 2 & 4 \\ 0 & -7 & -10 \\ 0 & 0 & -8 \end{vmatrix} \neq 0 \rightarrow h(Am') = 3$$

$\boxed{\text{S. Bateraezinak}}$

Orain, sistema ebatziko dugu (bateragarria denean)

$a \neq 4, -4 \rightarrow$ S.B.D.

$$Am' = \left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right) \longrightarrow \begin{cases} x + 2y - 3z = 4 & ① \\ -7y + 14z = -10 & ② \\ (a^2-16)z = a-4 & ③ \end{cases}$$

③ $(a^2-16)z = a-4$

$$\cancel{(a-4)}(a+4)z = \cancel{(a-4)} \rightarrow \boxed{z = \frac{1}{a+4}}$$

② $-7y + 14\left(\frac{1}{a+4}\right) = -10$

$$-7y = -10 - \frac{14}{a+4} \rightarrow \boxed{y = \frac{10}{7} + \frac{2}{a+4}}$$

① $x + 2\left(\frac{10}{7} + \frac{2}{a+4}\right) - 3\left(\frac{1}{a+4}\right) = 4$

$$x + \frac{20}{7} + \frac{4}{a+4} - \frac{3}{a+4} = 4 \rightarrow \boxed{x = \frac{8}{7} - \frac{1}{a+4}}$$

$a = 4 \longrightarrow$ S.B.I.

$$Am' = \left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{cases} x + 2y - 3z = 4 & ① \\ -7y + 14z = -10 & ② \end{cases}$$

$$\boxed{z \in \mathbb{R}}$$

② $\boxed{y = \frac{10 + 14z}{7} = \frac{10}{7} + 2z}$

① $x + 2\left(\frac{10}{7} + 2z\right) - 3z = 4$

$$\boxed{x = 4 + 3z - \frac{20}{7} - 4z = \frac{8}{7} - z}$$

3. ARIKETA Izan bitez $\mathbb{R}^4$-ko ondoko azpiespazio bektorialak:

$$T=\{(x,y,z,t)\in\mathbb{R}^4 / y=0,\ z=10x-8t\}$$

$$S=L\{(3,3,1,0),(-1,1,-2,0),(1,5,-3,0)\}$$

a) Lortu T-ren oinarri bat eta bere dimentsioa (puntu 1)

b) Lortu S-ren oinarri bat eta bere dimentsioa (0'5 puntu)

c) Lortu S-ren ekuazio inplizituak (~~0'5 puntu~~) (puntu 1)

d) Lortu $S\cap T$ azpiespazioa (puntu 1)

e) Lortu $S+T$ azpiespazioa. Batura zuzena da? (0'5 puntu)

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a) T-ren bektore orokorra:

$$(x,0,10x-8t,t)=x(1,0,10,0)+t(0,0,-8,1)$$

$$\boxed{B_T=\{(1,0,10,0),(0,0,-8,1)\}}\to\boxed{\dim(T)=2}$$

b)

$$A=\left(\begin{array}{ccc|c}3&3&1&\cancel{0}\\-1&1&-2&\cancel{0}\\1&5&-3&\cancel{0}\end{array}\right)\xrightarrow[\text{bat}]{\text{3. ordenako minore}}\begin{vmatrix}3&3&1\\-1&1&-2\\1&5&-3\end{vmatrix}=-9-6-5-1-9+30=0$$

2. ordenako minore bat $\to\begin{vmatrix}3&3\\-1&1\end{vmatrix}=3+3\neq0\to r(A)=2\to$ 2 bektore linealki independente

$$\boxed{B_S=\{(3,3,1,0),(-1,1,-2,0)\}}\to\boxed{\dim(S)=2}$$

c) S-ren ekuazio inplizituak:

$$A=\begin{pmatrix}3 & 3 & 1 & 0\\ -1 & 1 & -2 & 0\\ x & y & z & t\end{pmatrix} \longrightarrow$$ A matrizearen heina = 2 izan behar da.

(3. ordenako minore guztiak nuluak izan behar dira).

$$\begin{vmatrix}3 & 3 & 1\\ -1 & 1 & -2\\ x & y & z\end{vmatrix}=0 \rightarrow 3z-6x-y-x+3z+6y=0$$

$$-7x+6z+5y=0$$

$$\boxed{7x-5y-6z=0}$$

$$\begin{vmatrix}3 & 3 & 0\\ -1 & 1 & 0\\ x & y & t\end{vmatrix}=0 \rightarrow 3t+3t=0 \rightarrow \boxed{t=0}$$

d) $(S\cap T)$ azpiespazioan S-ren ekuazioak eta T-ren ekuazioak beteko dira:

$$\begin{cases}\boxed{y=0}\\ 10x-8t-z=0\\ 7x-5y-6z=0\\ \boxed{t=0}\end{cases} \rightarrow \begin{matrix}10x-z=0 \rightarrow z=10x\\ 7x-6z=0 \longrightarrow z=\frac{7}{8}x\end{matrix} \Bigg| \begin{matrix}\boxed{x=0}\\ \boxed{z=0}\end{matrix}$$

Beraz, → $\boxed{S\cap T=\{\bar{0}\}}$ eta $\boxed{\dim(S\cap T)=0}$

e) $\dim(S+T)=\underset{\underset{2}{\shortparallel}}{\dim(S)}+\underset{\underset{2}{\shortparallel}}{\dim(T)}-\underset{\underset{0}{\shortparallel}}{\dim(S\cap T)}=4=\dim(\mathbb{R}^4)$

Beraz, $\boxed{S+T=\mathbb{R}^4}$. Gainera $\left\{\begin{matrix}S\cap T=\{\bar{0}\}\\ S+T=\mathbb{R}^4\end{matrix}\right. \Rightarrow$ Batura zuzena da