

# CUÁDRICAS

Son las superficies más simples en  $\mathbb{R}^3$  después de los planos (corresponden a las cónicas de  $\mathbb{R}^2$ ). Su ecuación general es:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Kz + N = 0$$

## 1. Cuádricas con centro

### 1.1. Elipsoide

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

En particular, si  $a = b = c = r$ ,  $x^2 + y^2 + z^2 = r^2$  **esfera**

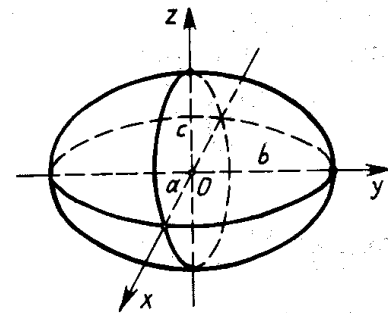
de radio  $r$ .

Secciones principales:

$$x = 0 \quad \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{elipse}$$

$$y = 0 \quad \frac{x^2}{a^2} + \frac{z^2}{c^2} = 1 \quad \text{elipse}$$

$$z = 0 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{elipse}$$



### 1.2. Hiperboloide de una hoja

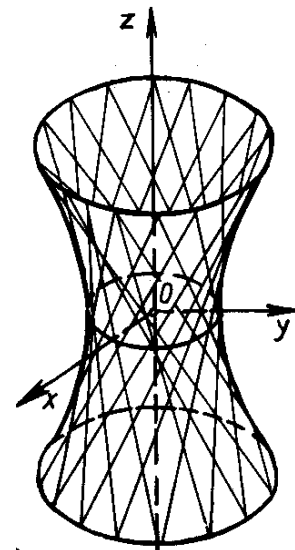
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Secciones principales:

$$x = 0 \quad \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \text{hipérbola}$$

$$y = 0 \quad \frac{x^2}{a^2} - \frac{z^2}{c^2} = 1 \quad \text{hipérbola}$$

$$z = 0 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{elipse}$$



### 1.3. Hiperboloide de dos hojas

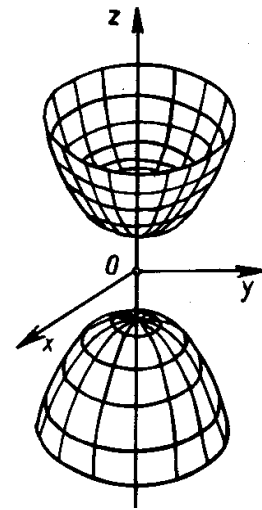
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Secciones principales:

$$x = 0 \quad -\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \text{no existe}$$

$$y = 0 \quad \frac{x^2}{a^2} - \frac{z^2}{c^2} = 1 \quad \text{hipérbola}$$

$$z = 0 \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{hipérbola}$$



## 2. Cuádricas sin centro

### 2.1 Paraboloide elíptico

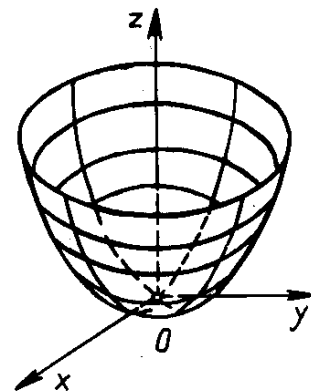
$$\frac{x^2}{2p} + \frac{y^2}{2q} = z$$

Secciones principales:

$$x = 0 \quad y^2 = 2qz \quad \text{parábola}$$

$$y = 0 \quad x^2 = 2pz \quad \text{parábola}$$

$$z = 0 \quad (0,0) \quad \text{un punto}$$



### 2.2. Paraboloide hiperbólico

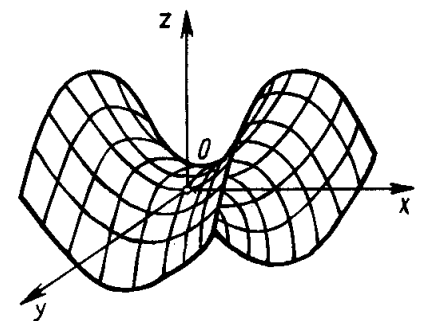
$$\frac{x^2}{2p} - \frac{y^2}{2q} = z$$

Secciones principales:

$$x = 0 \quad y^2 = -2qz \quad \text{parábola}$$

$$y = 0 \quad x^2 = 2pz \quad \text{parábola}$$

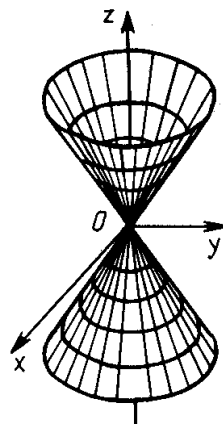
$$z = 0 \quad \frac{x}{\sqrt{2p}} + \frac{y}{\sqrt{2q}} = 0 \quad \frac{x}{\sqrt{2p}} - \frac{y}{\sqrt{2q}} = 0 \quad \text{dos rectas}$$



### 3. Cuádricas degeneradas

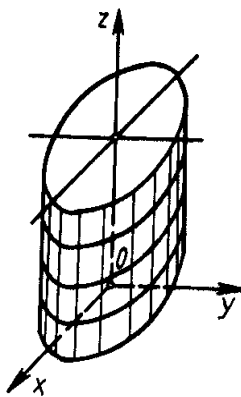
#### 3.1. Conos

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz = 0$$



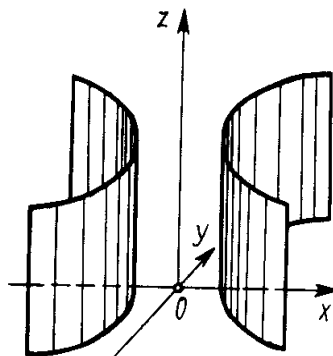
$$x^2 + y^2 = z^2$$

#### 3.2. Cilindros



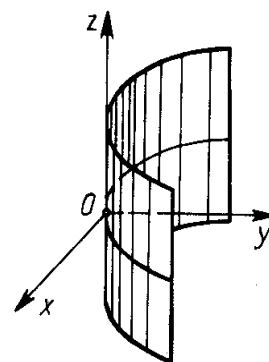
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Cilindro elíptico



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Cilindro hiperbólico



$$x^2 - 2ay = 1$$

Cilindro parabólico