$$\frac{\partial z}{\partial y}(0,0) = 2 \cdot e^{-1} \cdot 0 + (-1) \cdot e^{-1} \cdot (1) = -\frac{1}{e}$$

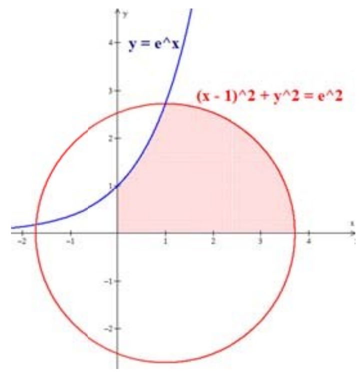
D) Jarri integrazio-limiteak bi era desberdinetan $I = \iint_D f(x,y)\,dx\,dy$ integralean, hurrengo $[D]$ eremurako:

$$D = \left\{(x,y) \in \mathbb{R}^2 \,/\, x \geq 0 \;;\; y \geq 0 \;;\; y \leq e^x \;;\; (x-1)^2 + y^2 \leq e^2\right\}$$

$[D]$ eremu lauaren azalera kalkulatu.

(6 p)

Ebazpena



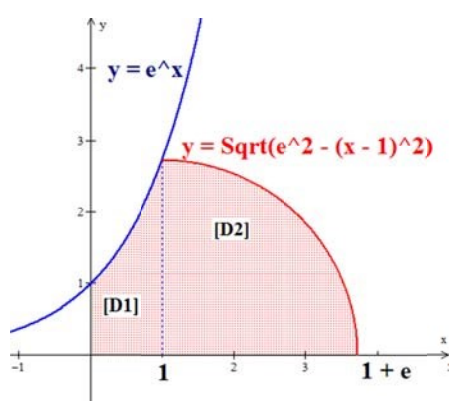
Domeinua lehenengo koadrantean dago. $y = e^x$ funtzioa eta $(x-1)^2 + y^2 = e^2$ zirkunferentzia (zentroa: (1,0); erradioa: e) domeinuaren mugak dira.

Ebakidura puntua:

$$\begin{cases} y = e^x \\ (x-1)^2 + y^2 = e^2 \end{cases} \rightarrow \quad (x-1)^2 + e^2 = e^2 \quad \rightarrow \quad x = 1 \quad P(1,e)$$

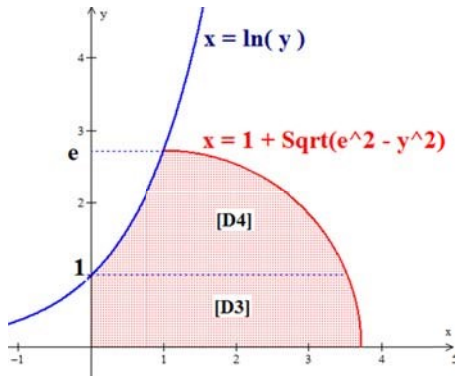
- (y) lehenengo integrazio-aldagaitzat hartuz:

$$I = \int_0^1 dx \int_0^{e^x} f(x,y)\,dy + \int_1^{1+e} dx \int_0^{\sqrt{e^2-(x-1)^2}} f(x,y)\,dy$$



- (x) lehenengo integrazio-aldagaitzat hartuz:

$$I=\int_0^1 dy\int_0^{1+\sqrt{e^2-y^2}} f(x,y)\,dx+\int_1^e dy\int_{\ln y}^{1+\sqrt{e^2-y^2}} f(x,y)\,dx$$



$[D]$ domeinuaren azalera $[D_1]$ domeinuaren azalera gehi $[D_2]$ domeinuaren azalera da:

$$A_T=A_1+A_2=\int_0^1 e^x dx+\frac{1}{4}\pi e^2=\left[e^x\right]_0^1+\frac{\pi e^2}{4}=e-1+\frac{\pi e^2}{4}=\frac{\pi e^2+4e-4}{4}\quad u^2$$

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2. ORRIA (20 puntu)

A) Klasifikatu eta ebatzi hurrengo EDA: $\left(e^x+\ln y+\dfrac{y}{x}\right)dx+\left(\dfrac{x}{y}+\ln x+\operatorname{sen} y\right)dy=0$

(4 p)

Ebazpena

$$\begin{cases} X(x,y)=e^x+\ln y+\dfrac{y}{x} \\ Y(x,y)=\dfrac{x}{y}+\ln x+\operatorname{sen} y \end{cases}$$

$$\frac{\partial X}{\partial y}=\frac{1}{y}+\frac{1}{x}=\frac{\partial Y}{\partial x}$$

Beraz, EDA **zehatza** da.

Soluzio orokorra hurrengoa da: $\displaystyle\int_a^x\left(e^x+\ln y+\frac{y}{x}\right)dx+\int_b^y\left(\frac{a}{y}+\ln a+\operatorname{sen} y\right)dy=C$

Kalkulua sinplifikatzeko $a=1$; $b=1$ aukeratzen da:

$$\int_1^x \left(e^x + \ln y + \frac{y}{x} \right) dx + \int_1^y \left(\frac{1}{y} + \operatorname{sen} y \right) dy = C$$

$$\left[e^x + x \ln y + y \ln x \right]_1^x + \left[\ln y - \cos y \right]_1^y = C$$

$$e^x + x \ln y + y \ln x - \left(e + \ln y \right) + \ln y - \cos y - \left(-\cos 1 \right) =$$

$$= e^x + x \ln y + y \ln x - e - \cos y + \cos 1 = C$$

Beraz, soluzio orokorra hurrengoa da:

$$\boxed{e^x + x \ln y + y \ln x - \cos y = K}$$

non $k = C + e - \cos 1$ den.

B) Ebatzi hurrengo EDA**:** $x^2 y'' - 3x\, y' + 3y = x + x^2 \cdot \ln x$

(5 p)

<u>*Ebazpena:*</u>

Euler-en EDA da.

$$y = x^r; \quad y' = r\, x^{r-1}; \quad y'' = r(r-1) x^{r-2}$$

$$x^2 y'' - 3xy' + 3y = x^2 r(r-1)\, x^{r-2} - 3x\, r\, x^{r-1} + 3x^r = x^r [r(r-1) - 3r + 3] = 0$$

$$r^2 - 4r + 3 = 0 \quad \rightarrow \quad r = \frac{4 \pm \sqrt{16-12}}{2} = \begin{cases} 3 \\ 1 \end{cases} \quad \Rightarrow \quad y = C_1 x + C_2 x^3$$

Parametroen aldakuntzaren metodoa erabiliz, soluzio orokorra hurrengoa da:

$$y = L_1(x) \cdot x + L_2(x) \cdot x^3$$

non $L_1(x)$ y $L_2(x)$ hurrengo sistemarekin kalkulatzen diren:

$$\begin{cases} L_1' \cdot x + L_2' \cdot x^3 = 0 \\ L_1' \cdot 1 + L_2' \cdot 3x^2 = \dfrac{x + x^2 \ln x}{x^2} = \dfrac{1}{x} + \ln x \end{cases}$$

$$L_1'(x) = \frac{\begin{vmatrix} 0 & x^3 \\ \frac{1}{x} + \ln x & 3x^2 \end{vmatrix}}{\begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix}} = \frac{-x^2 - x^3 \ln x}{2x^3} = -\frac{1}{2x} - \frac{1}{2}\ln x$$

$$L_2'(x) = \frac{\begin{vmatrix} x & 0 \\ 1 & \frac{1}{x} + \ln x \end{vmatrix}}{\begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix}} = \frac{1 + x\ln x}{2x^3} = \frac{1}{2x^3} + \frac{1}{2x^2}\ln x$$

Integratuz

$$L_1(x) = -\frac{1}{2}\int \frac{dx}{x} - \frac{1}{2}\int \ln x\, dx = -\frac{1}{2}\ln|x| - \frac{1}{2}\big(x\ln|x| - x\big) + A = -\frac{1}{2}\ln|x| - \frac{x}{2}\ln|x| + \frac{x}{2} + A$$

$$L_2(x) = \frac{1}{2}\int \frac{dx}{x^3} + \int \frac{1}{2x^2}\ln x\, dx = I + J$$

$$I = \frac{1}{2}\int \frac{dx}{x^3} = \frac{1}{2}\int x^{-3}dx = \frac{1}{2}\cdot\frac{x^{-2}}{(-2)} = -\frac{1}{4x^2} + cte$$

$$J = \int \frac{1}{2x^2}\ln x\, dx = \left\{ \begin{array}{lcl} \ln x = u & \Rightarrow & du = dx/x \\ \frac{1}{2x^2}dx = dv & \Rightarrow & v = -\frac{1}{2x} \end{array} \right\} = -\frac{1}{2x}\cdot\ln x + \int \frac{1}{2x^2}dx = -\frac{1}{2x}\cdot\ln x - \frac{1}{2x} + cte$$

$$L_2(x) = I + J = -\frac{1}{4x^2} - \frac{1}{2x}\cdot\ln x - \frac{1}{2x} + B$$

Orduan, soluzio orokorra hurrengoa da:

$$\boxed{y} = L_1(x)\cdot x + L_2(x)\cdot x^3 = \left(-\frac{1}{2}\ln|x| - \frac{x}{2}\ln|x| + \frac{x}{2} + A\right)\cdot x + \left(-\frac{1}{4x^2} - \frac{1}{2x}\cdot\ln x - \frac{1}{2x} + B\right)\cdot x^3 =$$

$$= -\frac{x}{2}\ln|x| - \frac{x^2}{2}\ln|x| + \frac{x^2}{2} + Ax - \frac{x}{4} - \frac{x^2}{2}\cdot\ln x - \frac{x^2}{2} + Bx^3 =$$

$$= \boxed{Ax + Bx^3 - \left(x^2 + \frac{x}{2}\right)\ln|x| - \frac{x}{4}}$$