

Leire Garcia

-o 1-

5. gaia 2. ARIKETA

a) Ekuazio bidezko espezifikazioa:

⊗ Eragiketa mota:

$$\text{ezabatu} :: (t, [t]) \rightarrow [t]$$

⊗ Eragiketa definitzen duten ekuazioak:

$$\text{ezabatu}(x, []) = [] \quad (1)$$

$$\text{ezabatu}(x, s:r) = \text{ezabatu}(x, r) \quad (2)$$

$$| x == s$$

=

$$\text{ezabatu}(x, r) \quad (2)$$

$$| x \neq s$$

=

$$s : \text{ezabatu}(x, r) \quad (3)$$

b) Ekuazio bidezko espezifikazioa:

⊗ Eragiketa mota:

$$++ :: ([t], [t]) \rightarrow [t]$$

⊗ Eragiketa definitzen duten ekuazioak:

$$[] ++ s = s \quad (4)$$

$$(x:r) ++ s = x : (r ++ s) \quad (5)$$

c) Ekuazio bidezko espezifikazioa:

⊗ Eragiketa mota:

$$\text{alder} :: ([t]) \rightarrow [t]$$

⊗ Eragiketa definitzen duten ekuazioak:

$$\text{alder}([]) = [] \quad (6)$$

listaren 1. elementu/hondarra

$$\text{alder}(x:r) = \text{alder}(r) ++ (x:[]) \quad (7)$$

2. ARIKETA

Leire Garcia

- 2 -

d) Frogatu induktiboa erabiliz, s zerrendaren gainean aplikatu behar da.

$$\text{ezabatu}(x, s) = \text{alder}(\text{ezabatu}(x, \text{alder}(s)))$$

⊗ Oinarizko kasua: $s = []$

$$\text{ezabatu}(x, []) = \text{alder}(\text{ezabatu}(x, \text{alder}([]))) ?$$

$$\text{⊗ } \text{ezabatu}(x, []) \stackrel{(1)}{=} []$$

$$\text{⊗ } \text{alder}(\text{ezabatu}(x, \text{alder}([]))) \stackrel{(6)}{=}$$

$$\text{alder}(\text{ezabatu}(x, [])) \stackrel{(1)}{=}$$

$$\text{alder}([]) \stackrel{(6)}{=} []$$

Alde bietan $[]$ kasua lortu dute. Bera? propietatea bete egiten da oinarizko kasuan.

⊗ Kasu errekorra: $s = z:w$

$$\text{ezabatu}(x, z:w) = \text{alder}(\text{ezabatu}(x, \text{alder}(z:w)))$$

⊗ Induktibo-hipotesia (ih): (x elementuarentzat eta w zerrendarentzat betetzen da)

$$\text{ezabatu}(x, w) = \text{alder}(\text{ezabatu}(x, \text{alder}(w)))$$

Frogatu nahi denera itzuliz:

$$\text{ezabatu}(x, z:w) = \text{alder}(\text{ezabatu}(x, \text{alder}(z:w))) ?$$

Aurreko atalean ikusi den bezala ezabatu funtzioaren definizioan bi kasu desberdin aipatu behar dira zerrenda hutsa denean. Hori dela eta, eta bi kasu horiek bakartza bere abstrakzio kontradikzioa behar da:

Jene Garu

• $X = z$

$$\textcircled{*} \text{ ezabatu}(x, z:w) =^{(2)}$$

$$= \text{ezabatu}(x, w) =^{(iv)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)))$$

$$\textcircled{*} \text{ alder}(\text{ezabatu}(x, \text{alder}(z:w))) =^{(7)}$$

$$= \text{alder}(\text{ezabatu}(x, (\text{alder}(w) ++ (z:[])))) =^{(\text{prop } 4)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)) ++ \text{ezabatu}(x, (z:[]))) =^{(2)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)) ++ \text{ezabatu}(x, [])) =^{(1)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)) ++ []) =^{(\text{prop } 2)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)))$$

Beraz, kasu inuktibosoan ere propietatea bete egiten da $\textcircled{*}$ eta $\textcircled{2}$ berdinak direnean.

• $X \neq z$

$$\textcircled{*} \text{ ezabatu}(x, z:w) =^{(3)}$$

$$= z: \text{ezabatu}(x, w) =^{(iv)}$$

$$= z: \text{alder}(\text{ezabatu}(x, \text{alder}(w)))$$

$$\textcircled{*} \text{ alder}(\text{ezabatu}(x, \text{alder}(z:w))) =^{(7)}$$

$$= \text{alder}(\text{ezabatu}(x, (\text{alder}(w) ++ (z:[])))) =^{(\text{prop } 4)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)) ++ \text{ezabatu}(x, (z:[]))) =^{(3)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)) ++ z: \text{ezabatu}(x, [])) =^{(1)}$$

$$= \text{alder}(\text{ezabatu}(x, \text{alder}(w)) ++ z:[]) =^{(\text{prop } 3)}$$

$$= \text{alder}(z:[]) ++ \text{alder}(\text{ezabatu}(x, \text{alder}(w))) =^{(7)}$$

$$= (\text{alder}([]) ++ (z:[])) ++ \text{alder}(\text{ezabatu}(x, \text{alder}(w))) =^{(4)}$$

$$= z:[] ++ \text{alder}(\text{ezabatu}(x, \text{alder}(w))) =^{(5)}$$

$$= z:([] ++ \text{alder}(\text{ezabatu}(x, \text{alder}(w)))) =^{(4)}$$

$$= z: \text{alder}(\text{ezabatu}(x, \text{alder}(w)))$$

Beraz, kasu inuktibosoan propietatea bete egiten da $\textcircled{*}$ eta $\textcircled{2}$ desberdinak direnean ere.