IKERKETA OPERATIBOA TALDE LANA

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PROBLEMA

$$Max\ z = 2x_1 + 4x_2 + \frac{5}{2}x_3$$

$$non\ 3x_1 + 4x_2 + 2x_3 \leq 600$$

$$2x_1 + x_2 + 2x_3 \leq 400$$

$$x_1 + 3x_2 + 3x_3 \leq 300$$

$$x_1, x_2, x_3 \geq 0$$

- Ebatzi problema *Simplex* algoritmoa erabiliz.
- Problema primalaren taula optimoa erabiliz, problema dualaren soluzioa lortu.
- Problema duala idatzi eta osagarrizko lasaitasuna erabiliz ebatzi.
- Posiblea al da *Simplex* metodoa erabiliz problema duala ebaztea?

1. Ebazpena *Simplex* algoritmoa erabiliz

$$
\begin{aligned}
&Max\ z = 2x_1 + 4x_2 + \frac{5}{2}x_3 \\
&non\ 3x_1 + 4x_2 + 2x_3 \leq 600 \\
&2x_1 + x_2 + 2x_3 \leq 400 \\
&x_1 + 3x_2 + 3x_3 \leq 300 \\
&x_1, x_2, x_3 \geq 0
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
&Max\ z = 2x_1 + 4x_2 + \frac{5}{2}x_3 \\
&non\ 3x_1 + 4x_2 + 2x_3 + x_4 = 600 \\
&2x_1 + x_2 + 2x_3 + x_5 = 400 \\
&x_1 + 3x_2 + 3x_3 + x_6 = 300 \\
&x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}
$$

SIMPLEX ALGORITMOA

1. Problema forma estandarrean adierazi

SIMPLEX ALGORITMOA

2. Hasierako soluzio bideragarria

$$\begin{array}{cc} & \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ A = & \left(\begin{array}{ccc|ccc} 3 & 4 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 1 & 3 & 3 & 0 & 0 & 1 \end{array}\right) \\ & \qquad\qquad\qquad\qquad \mathbf{B} \end{array}$$

$$x_B = (x_4, x_5, x_6) \rightarrow B = I \rightarrow x_B = B^{-1} \cdot b = b = (600, 400, 300)$$
$$x_N = (x_1, x_2, x_3) = (0, 0, 0)$$

C_{oin}	A_{oin}	$B^{-1} \cdot b$	2	4	2,5	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
0	x_4	600	3	4	2	1	0	0
0	x_5	400	2	1	2	0	1	0
0	x_6	300	1	3	3	0	0	0
z = 0		z_j	0	0	0	0	0	0
		$z_j - c_j$	-2	-4	-2,5	0	0	0

SIMPLEX ALGORITMOA

3. Hasierako *Simplex* taula

SIMPLEX ALGORITMOA

4. Irizpideak

$Coin$	$Aoin$	$B^{-1}\cdot b$	2	4	2,5	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
0	x_4	600	3	4	2	1	0	0
0	x_5	400	2	1	2	0	1	0
0	x_6	300	1	3	3	0	0	0
z = 0		z_j	0	0	0	0	0	0
		$z_j - c_j$	-2	-4	-2,5	0	0	0

$\exists W_j < 0 \ \rightarrow \boldsymbol{JARRAITU}$

$SARTZE - IRIZPIDEA\text{: } W_j = \min k \ = z_K - C_K = \min\{-2, -4\ , -2.5\} \ = -4$
$\rightarrow \boldsymbol{x_2\ oinarrira\ sartzen\ da}.$

$IRTETZE - IRIZPIDEA\text{: } \min k \ \left\{\frac{x_{Bk}}{y_{ik}}/y_{ik} > 0\right\} \ = \left\{{}^{600}/_{4}, {}^{400}/_{1}, {}^{300}/_{3}\right\}$
$= 100 \ \rightarrow \boldsymbol{x_6\ irtetzen\ da}.$

SIMPLEX ALGORITMOA

5. *Simplex* taulan beharrezkoak diren aldaketak egin

Bigarren taula

$$\begin{cases} e_{3B} \leftarrow \dfrac{1}{3} e_3 \\ e_{2B} \leftarrow e_2 - e_{3B} \\ e_{1B} \leftarrow e_1 - 4e_{3B} \end{cases}$$

$\boldsymbol{Coin}$	$\boldsymbol{Aoin}$	$\boldsymbol{B^{-1} \cdot b}$	2	4	2,5	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
0	x_4	200	1,667	0	-2	1	0	-1,33
0	x_5	300	1,667	0	1	0	1	-0,33
4	x_2	100	0,333	1	1	0	0	0,333
z = 400		z_j	1,333	4	4	0	0	1,333
		$z_j - c_j$	-0,67	0	1,5	0	0	1,333

SIMPLEX ALGORITMOA

6. Irizpideak

Coin	**Aoin**	$B^{-1}\cdot b$	2	4	2,5	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
0	x_4	200	1,667	0	-2	1	0	-1,33
0	x_5	300	1,667	0	1	0	1	-0,33
4	x_2	100	0,333	1	1	0	0	0,333
z = 400		z_j	1,333	4	4	0	0	1,333
		$z_j - c_j$	-0,67	0	1,5	0	0	1,333

$\exists W_j < 0 \ \rightarrow \boldsymbol{JARRAITU}$

$SARTZE - IRIZPIDEA\text{: } W_j = \min k \ = z_K - C_K = \min\{-0.67, 1.5, 1.33\} \ = -0.67$
$\rightarrow \boldsymbol{x_1 \text{ oinarrira sartzen da}}.$

$IRTETZE - IRIZPIDEA\text{: } \min k \ \left\{\frac{x_{Bk}}{y_{ik}} / y_{ik} > 0\right\} \ = \left\{200 \cdot {}^3/_5, 300 \cdot {}^3/_5, 100 \cdot 3\right\}$
$= 120 \ \rightarrow \boldsymbol{x_4 \text{ irtetzen da}}.$

SIMPLEX ALGORITMOA

7. *Simplex* taulan beharrezkoak diren aldaketak egin

Hirugarren taula

$$\begin{cases} e_{1B} \leftarrow \frac{3}{5} e_1 \\ e_{2B} \leftarrow e_2 - \frac{5}{3} e_{1B} \\ e_{3B} \leftarrow e_3 - \frac{1}{3} e_{1B} \end{cases}$$

$\boldsymbol{Coin}$	$\boldsymbol{Aoin}$	$\boldsymbol{B^{-1} \cdot b}$	2	4	2,5	0	0	0
			$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{x_4}$	$\boldsymbol{x_5}$	$\boldsymbol{x_6}$
2	$\boldsymbol{x_1}$	120	1	0	-1,2	0,6	0	-0,8
0	$\boldsymbol{x_5}$	100	0	0	3	-1	1	1
4	$\boldsymbol{x_2}$	60	0	1	1,4	-0,2	0	0,6
z = 480		$\boldsymbol{z_j}$	2	4	3,2	0,4	0	0,8
		$\boldsymbol{z_j - c_j}$	0	0	0,7	0,4	0	0,8

SIMPLEX ALGORITMOA

8. Soluzioa

$Coin$	$Aoin$	$B^{-1} \cdot b$	2	4	2,5	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
2	x_1	120	1	0	-1,2	0,6	0	-0,8
0	x_5	100	0	0	3	-1	1	1
4	x_2	60	0	1	1,4	-0,2	0	0,6
z = 480		z_j	2	4	3,2	0,4	0	0,8
		$z_j - c_j$	0	0	0,7	0,4	0	0,8

$\forall W_j \geq 0 \; \rightarrow$ ***GELDITU, optimoa lortu dugu.***

Gainera, ***soluzio optimoa bakarra da*** *oinarrizkoak ez diren aldagaien kostu murriztuak* $\neq 0$ *direlako.*

Soluzioa:

$$x_1^* = 120;\; x_2^* = 60;\; x_3^* = 0;\; x_4^* = 0;\; x_5^* = 100;\; x_6^* = 0;\; z^* = 480$$

2. Ekuazio primaletik dualaren soluzioa lortu

TAULA OPTIMOA ERABILIZ

$$Max\ z = 2x_1 + 4x_2 + \frac{5}{2}x_3$$
$$non\ 3x_1 + 4x_2 + 2x_3 \leq 600$$
$$2x_1 + x_2 + 2x_3 \leq 400$$
$$x_1 + 3x_2 + 3x_3 \leq 300$$
$$x_1, x_2, x_3 \geq 0$$

$\longrightarrow$

$$Min\ z = 600y_1 + 400y_2 + 300y_3$$
$$non\ 3y_1 + 2y_2 + y_3 \geq 2$$
$$4y_1 + y_2 + 3y_3 \geq 4$$
$$2y_1 + 2y_2 + 3y_3 \geq {}^5/_2$$
$$y_1, y_2, y_3 \geq 0$$

TAULA OPTIMOA

1. Primaletik duala lortu

TAULA OPTIMOA

2. *Simplex* metodoaren azken taula

$Coin$	$Aoin$	$B^{-1} \cdot b$	2	4	2,5	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
2	x_1	120	1	0	-1,2	0,6	0	-0,8
0	x_5	100	0	0	3	-1	1	1
4	x_2	60	0	1	1,4	-0,2	0	0,6
z = 480		z_j	2	4	3,2	0,4	0	0,8
		$z_j - c_j$	0	0	0,7	0,4	0	0,8

Soluzio optimoa:

$$y_1^* = {}^2/_5\,; y_2^* = 0; y_3^* = {}^4/_5\,;\; z^* = 600 * {}^2/_5 + 300 * {}^4/_5 = 480$$

3. Ekuazio primaletik dualaren soluzioa lortu

OSAGARRIZKO LASAITASUNA

$$
\begin{aligned}
&Max\ z = 2x_1 + 4x_2 + \frac{5}{2}x_3 \\
&non\ 3x_1 + 4x_2 + 2x_3 \leq 600 \\
&2x_1 + x_2 + 2x_3 \leq 400 \\
&x_1 + 3x_2 + 3x_3 \leq 300 \\
&x_1, x_2, x_3 \geq 0
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
&Min\ z = 600y_1 + 400y_2 + 300y_3 \\
&non\ 3y_1 + 2y_2 + y_3 \geq 2 \\
&4y_1 + y_2 + 3y_3 \geq 4 \\
&2y_1 + 2y_2 + 3y_3 \geq 5/2 \\
&y_1, y_2, y_3 \geq 0
\end{aligned}
$$

OSAGARRIZKO LASAITASUNA

1. Primaletik duala lortu

$$
\begin{aligned}
&Min\, z = 600y_1 + 400y_2 + 300y_3\\
&non\; 3y_1 + 2y_2 + y_3 - y_4 = 2\\
&4y_1 + y_2 + 3y_3 - y_5 = 4\\
&2y_1 + 2y_2 + 3y_3 - y_6 = {}^5/_2\\
&y_1, y_2, y_3, y_4, y_5, y_6 \geq 0
\end{aligned}
$$

OSAGARRIZKO LASAITASUNA

2.Dulala lasaiera aldagaiekin

OSAGARRIZKO LASAITASUNA

3.Soluzio duala planteatu

$$x^T = (x_1, x_2, x_3) \qquad y^T = (y_1, y_2, y_3)$$
$$(x^h)^T = (x_4, x_5, x_6) \qquad (y^h)^T = (y_4, y_5, y_6)$$

- Ekuazioak:

$$1.\ x_1 \cdot y_4 = 0$$
$$2.\ x_2 \cdot y_5 = 0$$
$$3.\ x_3 \cdot y_6 = 0$$
$$4.\ x_4 \cdot y_1 = 0$$
$$5.\ x_5 \cdot y_2 = 0$$
$$6.\ x_6 \cdot y_3 = 0$$

OSAGARRIZKO LASAITASUNA

4.1. Primalaren soluziotik abiatuz, ahal diren ekuazioak ebatzi

$$\boldsymbol{x_1^* = 120;\ x_2^* = 60;\ x_3^* = 0;\ x_4^* = 0;\ x_5^* = 100;\ x_6^* = 0;\ z^* = 480}$$

1. $x_1 \cdot y_4 = 0 \ \rightarrow 120 \cdot y_4 = 0 \ \rightarrow \boldsymbol{y_4 = 0}$
2. $x_2 \cdot y_5 = 0 \ \rightarrow 60 \cdot y_5 = 0 \ \rightarrow \boldsymbol{y_5 = 0}$
3. $x_3 \cdot y_6 = 0$
4. $x_4 \cdot y_1 = 0$
5. $x_5 \cdot y_2 = 0 \ \rightarrow 100 \cdot y_2 = 0 \ \rightarrow \boldsymbol{y_2 = 0}$
6. $x_6 \cdot y_3 = 0$

OSAGARRIZKO LASAITASUNA

4.2. Dualean ordezkapenak egin eta faltadiren balioak lortu

$$Min\ z = 600y_1 + 400y_2 + 300y_3$$
$$non\ 3y_1 + 2y_2 + y_3 - y_4 = 2$$
$$4y_1 + y_2 + 3y_3 - y_5 = 4$$
$$2y_1 + 2y_2 + 3y_3 - y_6 = {}^5/_2$$
$$y_1, y_2, y_3, y_4, y_5, y_6 \geq 0$$

$$\boldsymbol{y_2^* = 0;\ y_4^* = 0;\ y_5^* = 0}$$

$$\begin{cases} 3y_1 + y_3 = 2 \\ 4y_1 + 3y_3 = 4 \\ 2y_1 + 3y_3 - y_6 = {}^5/_2 \end{cases} \xrightarrow{\cdot(-3)} \begin{cases} -9y_1 - 3y_3 = -6 \\ 4y_1 + 3y_3 = 4 \end{cases}$$

$$-5y_1 = -2 \rightarrow \boldsymbol{y_1 = {}^2/_5}$$

1. $3 \cdot {}^2/_5 + y_3 = 2 \rightarrow \boldsymbol{y_3 = {}^4/_5}$
2. $2 \cdot {}^2/_5 + 3 \cdot {}^4/_5 - y_6 = {}^5/_2 \rightarrow \boldsymbol{y_6 = {}^7/_{10}}$

Soluzio optimoa:

$$\boldsymbol{y_1^* = {}^2/_5;\ y_2^* = 0;\ y_3^* = {}^4/_5;\ y_4^* = 0;\ y_5^* = 0;\ y_6^* = {}^7/_{10};\ z^* = 480}$$

4. Dualetik primalera

SIMPLEX DUAL METODOA

$$\begin{aligned} &Min\, z = 600y_1 + 400y_2 + 300y_3 \\ &non\ 3y_1 + 2y_2 + y_3 \geq 2 \\ &4y_1 + y_2 + 3y_3 \geq 4 \\ &2y_1 + 2y_2 + 3y_3 \geq 5/2 \end{aligned}$$

$$\longrightarrow$$

$$\begin{aligned} &Min\, z = 600y_1 + 400y_2 + 300y_3 \\ &non\ 3y_1 + 2y_2 + y_3 - y_4 = 2 \\ &4y_1 + y_2 + 3y_3 - y_5 = 4 \\ &2y_1 + 2y_2 + 3y_3 - y_6 = 5/2 \\ &y_1, y_2, y_3, y_4, y_5, y_6 \geq 0 \end{aligned}$$

LASAIERA ALDAGAIAK GEHITU

1.Dulala lasaiera aldagaiekin

$$
\begin{aligned}
&Min\ z = 600y_1 + 400y_2 + 300y_3\\
&non\ 3y_1 + 2y_2 + y_3 - y_4 = 2\\
&4y_1 + y_2 + 3y_3 - y_5 = 4\\
&2y_1 + 2y_2 + 3y_3 - y_6 = {}^5/_2\\
&y_1, y_2, y_3, y_4, y_5, y_6 \geq 0
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
&Min\ z = 600y_1 + 400y_2 + 300y_3\\
&non\ -3y_1 - 2y_2 - y_3 + y_4 = -2\\
&-4y_1 - y_2 - 3y_3 + y_5 = -4\\
&-2y_1 - 2y_2 - 3y_3 + y_6 = -{}^5/_2\\
&y_1, y_2, y_3, y_4, y_5, y_6 \geq 0
\end{aligned}
$$

PROBLEMA OROKORTU

2.Dulala lasaiera aldagaiekin, identitatea lortu ahal izateko

TAULA OPTIMOA

3. *Simplex* metodoaren lehen taula

$Coin$	$Aoin$	$B^{-1} \cdot b$	600	400	300	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	-2	-3	-2	-1	1	0	0
0	y_5	-4	-4	-1	-3	0	1	0
0	y_6	-2,5	-2	-2	-3	0	0	1
z = 0		z_j	0	0	0	0	0	0
		$z_j - c_j$	-600	-400	-300	0	0	0

$$\exists k:\ x_{DK} < 0 \rightarrow JARRAITU$$

* IRTETZE-IRIZPIDEA : max { $|x_{DK}|$ / $x_{DK} < 0$ } = max { |-2|, |-4|, |-5/2| } = 4 ---> y_5 *atera*

* SARTZE-IRIZPIDEA : min { $\frac{|Z_K - C_K|}{|a_{in}|}$ / a_{in}<0} = min { $\frac{|-600|}{|-4|}, \frac{|-400|}{|-1|}, \frac{|-300|}{|-3|}$ } = 100 --> y_3 *sartu*

$$\text{Eragiketak:} \begin{cases} e_{2b} = -\frac{1}{3} * e_3 \\ e_{1b} = e_1 + e_{2b} \\ e_{3b} = e_3 + 3 * e_{2b} \end{cases}$$

TAULA OPTIMOA

4. *Simplex* metodoaren bigarren taula

Coin	Aoin	$B^{-1}\cdot b$	600	400	300	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	- 2/3	- 5/3	- 5/3	0	1	- 1/3	0
300	y_3	4/3	4/3	1/3	1	0	- 1/3	0
0	y_6	3/2	2	-2	0	0	-1	1
z = 400		z_j	400	100	300	0	-100	0
		$z_j - c_j$	-200	-300	0	0	-100	0

$$\exists k:\ x_{DK} < 0 \ \rightarrow JARRAITU$$

* <u>IRTETZE-IRIZPIDEA</u> : max { $|x_{DK}|$ / $x_{DK} < 0$ } = max { |-2/3| } = 2/3 ---> y_4 *atera*

* <u>SARTZE-IRIZPIDEA</u> : min { $\frac{|Z_K - C_K|}{|a_{in}|}$ / a_{in}<0} = min { $\frac{|-200|}{|-5/3|}, \frac{|-300|}{|-5/3|}, \frac{|-100|}{|-1/3|}$ } = 100 --> y_1 *sartu*

Eragiketak: $$\begin{cases} e_{1b} = -\frac{3}{5} * e_1 \\ e_{2b} = e_2 - \frac{4}{3} * e_{1b} \\ e_{3b} = e_3 - 2 * e_{1b} \end{cases}$$

TAULA OPTIMOA

5. *Simplex* metodoaren azken taula

Coin	Aoin	$B^{-1}\cdot b$	600	400	300	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6
600	y_1	2/5	1	1	0	- 3/5	1/5	0
300	y_3	4/5	0	-1	1	4/5	- 3/5	0
0	y_6	7/10	0	0	-2	- 8/5	1/5	1
z = 480		z_j	600	300	300	-120	-60	0
		$z_j - c_j$	0	-100	0	-120	-60	0

$$\forall\, x_{DK} > 0 \rightarrow GELDITU$$

Bideragarritasuna lortu egin da.
Ondorioz,problema dualaren soluzio optimoa lortu dugu:

$$z^* = 480,$$

$$y_1 = \frac{2}{5}, \quad y_3 = \frac{4}{5}, \quad y_6 = \frac{7}{10}, \quad y_2 = 0, \quad y_4 = 0, \quad y_5 = 0$$