IKERKETA OPERATIBOA

Talde lana – 7. Ariketa

Camilo Agudelo
Adei Arias
Martin Amezola
Jon Barbero

Ebatzi Beharreko Problema

$$Max\ \ z = -2x_1 - 4x_2 - 2x_3$$
$$3x_1 + x_2 + 2x_3 \geq 8$$
$$x_1 + 3x_2 + 4x_3 \geq 12$$
$$x_1, x_2, x_3 \geq 0$$

Ebatzi Beharreko Problema

$$Max\ \ z = -2x_1 - 4x_2 - 2x_3$$
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$$x_1, x_2, x_3 \geq 0$$

Zerotik gertuen dagoen zenbaki negatiboa!

Erabiliko Diren Ebazpideak

1. Simplex Dual Metodoa

2. Bi Fase Metodoa

3. Dualtasuna - Ebazpen Grafikoa
   Osagarrizko Lasaitasuna

Simplex Dual Metodoa

Simplex Dual Metodoa

1. Murrizketak '*-1*'-ekin biderkatu, '*B*' matrizea '*I*' identitatea izateko.
2. Forma estandarrera pasatu (lasaiera aldagaiak sartu)

$$\begin{aligned} Max\ \ z = & -2x_1 - 4x_2 - 2x_3 \\ & \left.\begin{aligned} 3x_1 + x_2 + 2x_3 &\geq 8 \\ x_1 + 3x_2 + 4x_3 &\geq 12 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}\right\} \end{aligned} \xrightarrow{*\,(-1)} \begin{aligned} Max\ \ z = & -2x_1 - 4x_2 - 2x_3 \\ & \left\{\begin{aligned} -3x_1 - x_2 - 2x_3 &\leq -8 \\ -x_1 - 3x_2 - 4x_3 &\leq -12 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}\right. \end{aligned}$$

$\leq \rightarrow$ *lasaiera aldagaiak batzen*

$$\boxed{\begin{aligned} Max\ \ z = & -2x_1 - 4x_2 - 2x_3 \\ -3x_1 - x_2 - 2x_3 + x_4 &= -8 \\ -x_1 - 3x_2 - 4x_3 + x_5 &= -12 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}}$$

Simplex Dual Metodoa

Hasierako oinarrizko soluzioa lortzen: $B \cdot x_B = b$

$$A = \begin{array}{ccccc} \left(\begin{matrix} -3 & -1 & -2 \\ -1 & -3 & -4 \end{matrix} \right. & \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} \left. \right) \\ x_1 \quad x_2 \quad x_3 & x_4 \quad x_5 \end{array} \Longrightarrow B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} -8 \\ -12 \end{pmatrix}$$

$$B \cdot x_B = b \quad \rightarrow \quad B^{-1} \cdot B \cdot x_B = B^{-1} \cdot b \quad \rightarrow \quad \{B = I = B^{-1}\}$$

$$I \cdot x_B = I \cdot b \quad \rightarrow \quad x_B = b$$

$$x_B = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -8 \\ -12 \end{pmatrix}$$

Simplex Dual: 1. Iterazioa

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
0	X_4	-8	-3	-1	-2	1	0
0	X_5	-12	-1	-3	-4	0	1
Z = 0		z_j	0	0	0	0	0
		$w_j = z_j - c_j$	2	4	2	0	0

$\exists K:\ x_{DK} < 0 \ \rightarrow$ Jarraitu

Simplex Dual: 1. Iterazioa

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
0	X_4	-8	-3	-1	-2	1	0
0	X_5	-12	-1	-3	-4	0	1
Z = 0		z_j	0	0	0	0	0
		$w_j = z_j - c_j$	2	4	2	0	0

Irtetze-irizpidea:

$$\max_k\{\,|X_{DK}|\ /\ X_{DK}\ < 0\} = \{|-8|, |-12|\} = 12 \rightarrow\ x_5\ \text{irten}$$

Sartze-irizpidea:

$$\min_{k,a_{ik}<0}\left\{\frac{|z_k - c_k|}{|a_{ik}|}\right\} = \left\{\frac{|2|}{|-1|}, \frac{|4|}{|-3|}, \frac{|2|}{|-4|}\right\} = \left\{2, \frac{4}{3}, \frac{1}{2}\right\} = \frac{1}{2}\ \rightarrow\ x_3\ \text{sartu}$$

Simplex Dual: 1. Iterazioa

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
0	X_4	-8	-3	-1	-2	1	0
0	X_5	-12	-1	-3	-4	0	1
Z = 0		z_j	0	0	0	0	0
		$w_j = z_j - c_j$	2	4	2	0	0

$$e_{2B} \leftarrow \frac{e_2}{-4}$$

$$e_{1B} \leftarrow e_1 + 2\, e_{2B}$$

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
0	X_4	-2	-5/2	1/2	0	1	-1/2
-2	X_3	3	1/4	3/4	1	0	-1/4
Z = -6		z_j	-1/2	-3/2	-2	0	1/2
		$w_j = z_j - c_j$	3/2	5/2	0	0	1/2

Simplex Dual: 2. Iterazioa

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
0	X_4	-2	-5/2	1/2	0	1	-1/2
-2	X_3	3	1/4	3/4	1	0	-1/4
Z = -6		z_j	-1/2	-3/2	-2	0	1/2
		$w_j = z_j - c_j$	3/2	5/2	0	0	1/2

$\exists K\colon\ x_{DK} < 0 \ \rightarrow$ Jarraitu

Simplex Dual: 2. Iterazioa

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
0	X_4	-2	-5/2	1/2	0	1	-1/2
-2	X_3	3	1/4	3/4	1	0	-1/4
Z = -6		z_j	-1/2	-3/2	-2	0	1/2
		$w_j = z_j - c_j$	3/2	5/2	0	0	1/2

Irtetze-irizpidea:

$$\max_k\{\,|X_{DK}|\ /\ X_{DK}\ <0\} = \{\,|-2|\} = 2 \rightarrow\ x_4\ \text{ irten}$$

Sartze-irizpidea:

$$\min_{k,a_{ik}<0}\left\{\frac{|z_k-c_k|}{|a_{ik}|}\right\} = \left\{\frac{\left|\frac{3}{2}\right|}{\left|\frac{-5}{2}\right|},\frac{\left|\frac{1}{2}\right|}{\left|\frac{-1}{2}\right|}\right\} = \left\{\frac{3}{5},1\right\} = \frac{3}{5}\ \rightarrow\ x_1$$

sartu

Simplex Dual: 2. Iterazioa

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
0	X_4	-2	-5/2	1/2	0	1	-1/2
-2	X_3	3	1/4	3/4	1	0	-1/4
Z = -6		z_j	-1/2	-3/2	-2	0	1/2
		$w_j = z_j - c_j$	3/2	5/2	0	0	1/2

$$e_{1B} \leftarrow -\frac{2}{5} e_1$$

$$e_{2B} \leftarrow e_2 - \frac{1}{4} e_{1B}$$

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
-2	X_1	4/5	1	-1/5	0	-2/5	1/5
-2	X_3	14/5	0	4/5	1	1/10	-3/10
$Z = -\frac{36}{5}$		z_j	-2	-6/5	-2	3/5	1/5
		$w_j = z_j - c_j$	0	14/5	0	3/5	1/2

Simplex Dual: 3. Iterazioa

C_{oin}	A_{oin}	$B^{-1} * b$	-2	-4	-2	0	0
			x_1	x_2	x_3	x_4	x_5
-2	X_1	4/5	1	-1/5	0	-2/5	1/5
-2	X_3	14/5	0	4/5	1	1/10	-3/10
$Z = -\frac{36}{5}$		z_j	-2	-6/5	-2	3/5	1/5
		$w_j = z_j - c_j$	0	14/5	0	3/5	1/2

$\forall K\colon\ x_{DK} \geq 0 \ \rightarrow$ Gelditu

Simplex Dual: Soluzioa

Optimoa lortu dugu eta soluzio bakarra da, ez oinarrizko kostu murriztuak zeroren desberdinak direlako:

$$x_1^{\,*} = \frac{4}{5}; \quad x_2^{\,*} = 0; \quad x_3^{\,*} = \frac{14}{5}; \quad x_4^{\,*} = 0; \quad x_5^{\,*} = 0; \quad Z^* = -\frac{36}{5}$$

Bi Fase Metodoa

Bi Fase Metodoa

1. Minimizaziora pasatu → Helburu funtzioa '-*1*'-ekin biderkatu.

$$Max\ \ z = -2x_1 - 4x_2 - 2x_3$$
$$3x_1 + x_2 + 2x_3 \geq 8$$
$$x_1 + 3x_2 + 4x_3 \geq 12$$
$$x_1, x_2, x_3 \geq 0$$

$* (-1)$

$$\boxed{\begin{array}{c} Min\ \ z = 2x_1 + 4x_2 + 2x_3 \\ 3x_1 + x_2 + 2x_3 \geq 8 \\ x_1 + 3x_2 + 4x_3 \geq 12 \\ x_1, x_2, x_3 \geq 0 \end{array}}$$

Bi Fase Metodoa

2. Forma estandarrera pasatu (lasaiera aldagaiak sartu)

3. Aldagai artifizialak gehitu

$$Min \;\; z = 2x_1 + 4x_2 + 2x_3$$
$$\left.\begin{array}{l} 3x_1 + \;\; x_2 \; + 2x_3 \geq 8 \\ x_1 \; + \; 3x_2 \; + 4x_3 \geq 12 \\ x_1, x_2, x_3 \geq 0 \end{array}\right\}$$

$\geq \rightarrow$ lasaiera aldagaiak kentzen

$$Min \;\; z = 2x_1 + 4x_2 + 2x_3$$
$$\left\{\begin{array}{l} 3x_1 + \;\; x_2 + 2x_3 - x_4 = 8 \\ x_1 + \; 3x_2 + \; 4x_3 - x_5 \; = 12 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}\right.$$

$$Min \;\; z = 2x_1 + 4x_2 + 2x_3$$
$$3x_1 + \;\; x_2 + 2x_3 - x_4 + q_1 = 8$$
$$x_1 + 3x_2 \; + \; 4x_3 - x_5 + q_2 \;\; = 12$$
$$x_1, x_2, x_3, x_4, x_5, q_1, q_2 \geq 0$$

Bi Fase Metodoa: 1.Fasea

Ebatziko den problema:

$$Min\ \ z = \ q_1 + q_2$$

$$3x_1 + \quad x_2 \ + 2x_3 \ - x_4 + q_1 = 8$$

$$x_1 + \ 3x_2 \ + \ 4x_3 - x_5 + q_2 = 12$$

$$x_1, x_2, x_3, x_4, x_5, q_1, q_2 \geq 0$$

- Helburu funtzioa aldagai artifizialen batukaria izango da.

- Bigarren faseko lehenengo oinarrizko soluzio bideragarria bilatuko da.

Bi Fase Metodoa: 1.Fasea

Taularako matrizeak:

$$A = \begin{pmatrix} 3 & 1 & 2 & -1 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 8 \\ 12 \end{pmatrix} \; ; \; B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \; ; \; x_B = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \; ; \; x_n = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

$$\boxed{x_B = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}}$$

Bi Fase Metodoa: 1.Fasea, 1. Taula

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
1	q_1	8	3	1	2	-1	0	1	0
1	q_2	12	1	3	4	0	-1	0	1
Z = 20		z_j	4	4	6	-1	-1	1	1
		$w_j = z_j - c_j$	4	4	6	-1	-1	0	0

$\exists w_j > 0 \rightarrow \text{Jarraitu}$

Bi Fase Metodoa: 1.Fasea, 1. Taula

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
1	q_1	8	3	1	2	-1	0	1	0
1	q_2	12	1	3	$\boxed{4}$	0	-1	0	1
Z = 20		z_j	4	4	6	-1	-1	1	1
		$w_j = z_j - c_j$	4	4	6	-1	-1	0	0

Sartze-irizpidea:

$$\max\{ z_j - c_j \} = \{4,4,6\} = 6 \ \rightarrow \ x_3 \text{ sartu}$$

Irtetze-irizpidea:

$$\min\left\{ \frac{x_{Bk}}{y_{kj}} \ / \ y_{kj} > 0 \right\} = \left\{ \frac{8}{2}, \frac{12}{4} \right\} = \{4,3\} = 3 \ \rightarrow \ q_2 \ \text{ irten}$$

Bi Fase Metodoa: 1.Fasea, 2. Taula

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
1	q_1	8	3	1	2	-1	0	1	0
1	q_2	12	1	3	4	0	-1	0	1
Z = 20		z_j	4	4	6	-1	-1	1	1
		$w_j = z_j - c_j$	4	4	6	-1	-1	0	0

$$e_{2B} \leftarrow \frac{e_2}{4}$$

$$e_{1B} \leftarrow e_1 - 2\, e_{2B}$$

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
1	q_1	2	5/2	-1/2	0	-1	1/2	1	-1/2
0	X_3	3	1/4	3/4	1	0	-1/4	0	1/4
Z = 2		z_j	5/2	-1/2	0	-1	1/2	1	-1/2
		$w_j = z_j - c_j$	5/2	-1/2	0	-1	1/2	0	-3/2

Bi Fase Metodoa: 1.Fasea, 2. Taula

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
1	q_1	2	5/2	-1/2	0	-1	1/2	1	-1/2
0	X_3	3	1/4	3/4	1	0	-1/4	0	1/4
Z = 2		z_j	5/2	-1/2	0	-1	1/2	1	-1/2
		$w_j = z_j - c_j$	5/2	-1/2	0	-1	1/2	0	-3/2

$\exists w_j > 0 \quad \rightarrow$ Jarraitu

Bi Fase Metodoa: 1.Fasea, 2. Taula

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
1	q_1	2	5/2	-1/2	0	-1	1/2	1	-1/2
0	X_3	3	1/4	3/4	1	0	-1/4	0	1/4
Z = 2		z_j	5/2	-1/2	0	-1	1/2	1	-1/2
		$w_j = z_j - c_j$	5/2	-1/2	0	-1	1/2	0	-3/2

Sartze-irizpidea:

$$\max\left\{ z_j - c_j \right\} = \left\{\frac{5}{2}, \frac{1}{2}\right\} = \frac{5}{2} \rightarrow x_1 \text{ sartu}$$

Irtetze-irizpidea:

$$\min\left\{ \frac{x_{Bk}}{y_{kj}} / y_{kj} > 0 \right\} = \left\{\frac{2}{\frac{5}{2}}, \frac{3}{\frac{1}{4}}\right\} = \left\{\frac{4}{5}, 12\right\} = \frac{4}{5} \rightarrow q_1 \text{ irten}$$

Bi Fase Metodoa: 1.Fasea, 3. Taula

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
1	q_1	2	5/2	-1/2	0	-1	1/2	1	-1/2
0	X_3	3	1/4	3/4	1	0	-1/4	0	1/4
Z = 2		z_j	5/2	-1/2	0	-1	1/2	1	-1/2
		$w_j = z_j - c_j$	5/2	-1/2	0	-1	1/2	0	-3/2

$$e_{1B} \leftarrow \frac{2}{5} e_1$$

$$e_{2B} \leftarrow e_2 - \frac{1}{4} e_{1B}$$

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
0	x_1	4/5	1	-1/5	0	-2/5	1/5	2/5	-1/5
0	X_3	14/5	0	4/5	1	1/10	-3/10	-1/10	3/10
Z = 0		z_j	0	0	0	0	0	0	0
		$w_j = z_j - c_j$	0	0	0	0	0	-1	-1

Bi Fase Metodoa: 1.Fasea, 3. Taula

C_{oin}	A_{oin}	$B^{-1} * b$	0	0	0	0	0	1	1
			x_1	x_2	x_3	x_4	x_5	q_1	q_2
0	x_1	4/5	1	-1/5	0	-2/5	1/5	2/5	-1/5
0	X_3	14/5	0	4/5	1	1/10	-3/10	-1/10	3/10
Z = 0		z_j	0	0	0	0	0	0	0
		$w_j = z_j - c_j$	0	0	0	0	0	-1	-1

$\forall w_j \leq 0 \rightarrow$ Gelditu

Bi Fase Metodoa: 1.Faseko Soluzioa

Z = 0 da, eta aldagai artifizialak oinarritik kanpo daudenez, bigarren faseko oinarrizko soluzio bideragarria lortu da:

$$x_1 = \frac{4}{5}; \quad x_2 = 0; \quad x_3 = \frac{14}{5}; \quad x_4 = 0; \quad x_5 = 0; \quad q_1 = 0; \quad q_2 = 0; \quad Z = 0$$

Lehenengo fasea bukatuta

Bi Fase Metodoa: 2.Fasea

C_{oin}	A_{oin}	$B^{-1} * b$	2	4	2	0	0
			x_1	x_2	x_3	x_4	x_5
2	x_1	4/5	1	-1/5	0	-2/5	1/5
2	X_3	14/5	0	4/5	1	1/10	-3/10
Z = 36/5		z_j	2	6/5	2	-3/5	-1/5
		$w_j = z_j - c_j$	0	-14/5	0	-3/5	-1/5

Bigarren fase honetan lortutako taula optimotik abiatuko da, hasierako helburu funtzioa erabiliz. Gainera, q1 eta q2 desagertuko dira.

$$\forall w_j \leq 0 \rightarrow \text{Gelditu}$$

Bi Fase Metodoa: 2.Faseko Soluzioa

Optimoa lortu da. Oinarrizkoak ez diren aldagaien kostu murriztuak zeroren desberdinak direrenez, soluzioa bakarra da:

$$x_1{}^* = \frac{4}{5}; \quad x_2{}^* = 0; \quad x_3{}^* = \frac{14}{5}; \quad x_4{}^* = 0; \quad x_5{}^* = 0; \quad Z^* = \frac{36}{5}$$

Optimoa lortu da. Oinarrizkoak ez diren aldagaien kostu murriztuak zeroren desberdinak direnez, soluzioa bakarra da:

$$Z_{emaitza} = -\frac{36}{5}$$

Dualtasuna – Osagarrizko Lasaitasuna

Dualtasuna eta Osagarrizko lasaitasuna

Lehenik murrizketen zeinua aldatuko dugu prozedura errazteko.

$$\begin{array}{l} Max\ \ z = -2x_1 - 4x_2 - 2x_3 \\ 3x_1 + x_2 + 2x_3 \geq 8 \\ x_1 + 3x_2 + 4x_3 \geq 12 \\ x_1, x_2, x_3 \geq 0 \end{array} \Longrightarrow \begin{array}{l} Max\ \ z = -2x_1 - 4x_2 - 2x_3 \\ -3x_1 - x_2 - 2x_3 \leq -8 \\ -x_1 - 3x_2 - 4x_3 \leq -12 \\ x_1, x_2, x_3 \geq 0 \end{array}$$

$$A = \begin{pmatrix} -3 & -1 & -2 \\ -1 & -3 & -4 \end{pmatrix} \Longrightarrow A^t = \begin{pmatrix} -3 & -1 \\ -1 & -3 \\ -2 & -4 \end{pmatrix}$$

$$C^t = \begin{pmatrix} -2 & -4 & -2 \end{pmatrix} \Longrightarrow C = \begin{pmatrix} -2 \\ -4 \\ -2 \end{pmatrix}$$

$$B = \begin{pmatrix} -8 \\ -12 \end{pmatrix} \Longrightarrow B^t = \begin{pmatrix} -8 & -12 \end{pmatrix}$$

Dualtasuna eta Osagarrizko lasaitasuna: Primal-Dual erlazioa

$$\max\ C^t \cdot X \qquad\qquad \min\ B^t \cdot U$$

$$A \cdot X \boxed{\geq} B \qquad\qquad A^t \cdot U \ ?\ C$$

$$X \boxed{\geq} 0 \qquad\qquad U \ ?\ 0$$

Dualtasuna eta Osagarrizko lasaitasuna

Taula hau aplikatuz, problema duala lortzen dugu.

Helburu funtzioa: max	⇔	Helburu funtzioa: min
i. murrizketa ≤	⇔	i. aldagaia ≥ 0
i. murrizketa =	⇔	i. aldagaia ez-murriztua
i. murrizketa ≥	⇔	i. aldagaia ≤ 0
i. aldagaia ≥ 0	⇔	i. murrizketa ≥
i. aldagaia ez-murriztua	⇔	i. murrizketa =
i. aldagaia ≤ 0	⇔	i. murrizketa ≤

PROBLEMA DUALA

$$\min z = -8u_1 - 12u_2$$

$$\begin{aligned} -3u_1 - u_2 &\geq -2 \\ -u_1 - 3u_2 &\geq -4 \\ -2u_1 - 4u_2 &\geq -2 \\ u_1, u_2 &\geq 0 \end{aligned}$$

Dualtasuna eta Osagarrizko lasaitasuna: Ebazpen Grafikoa

Problema dualaren soluzioa grafikoki ebatzi dezakegu geogebra erabiliz:

Zuzenak:

r → $-3x-y \geq -2$

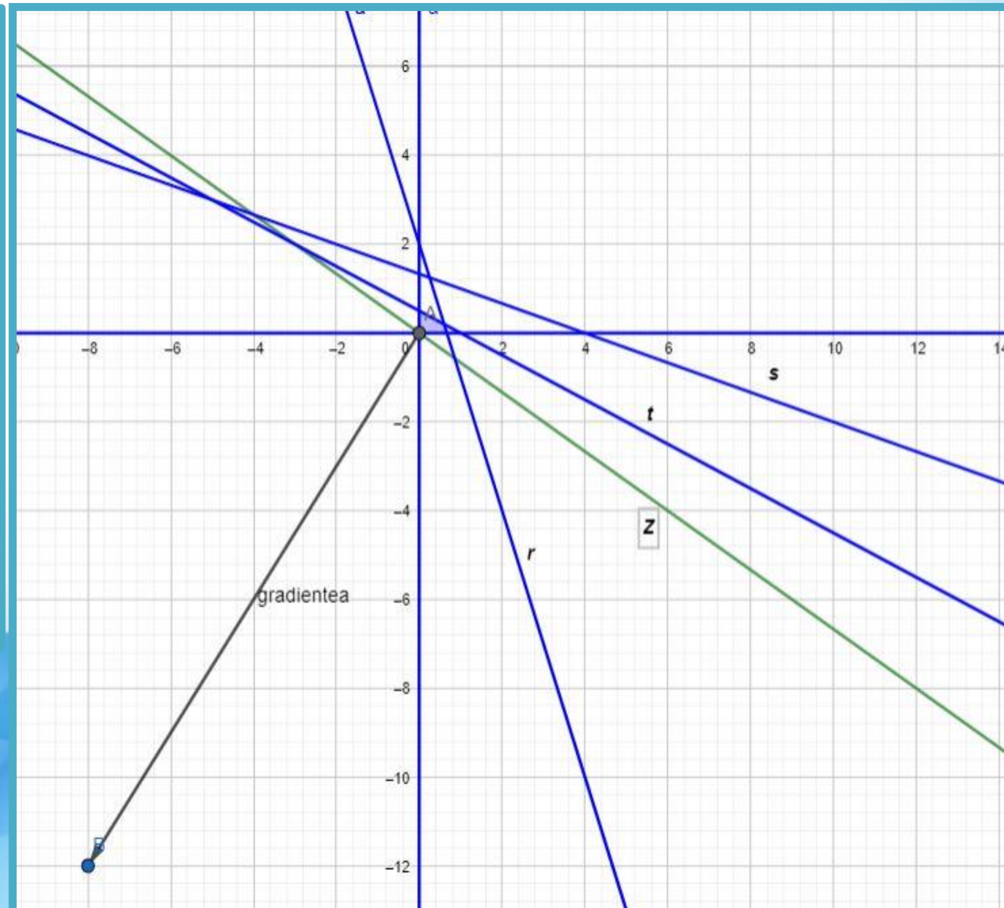
s → $-x-3y \geq -4$

t → $-2x -4y \geq -2$

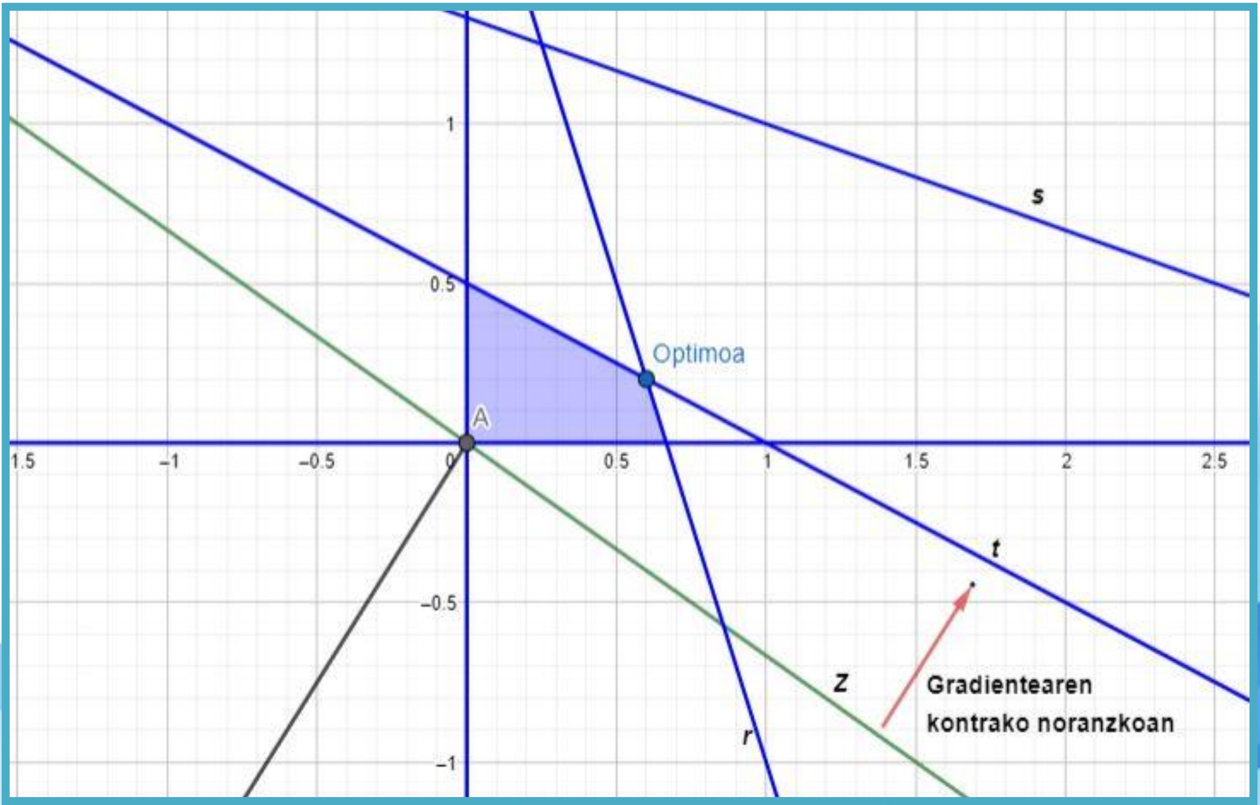
f → $x \geq 0$

g → $y \geq 0$

Gradientea → $(\frac{\partial Z}{\partial x}, \frac{\partial Z}{\partial y}) = (-8 , -12)$



Dualtasuna eta Osagarrizko lasaitasuna: Ebazpen Grafikoa



Helburu Funtzioa gradientearen kontrako bidean mugitzen badugu optimoa ukisten dugu r eta t –ren arteko ebakidura puntua dela

Sistema bakanduz…

Soluzio Optimoa

$Z^* = -36/5$, $u_1{}^* = 3/5$, $u_2{}^* = 1/5$

Dualtasuna eta Osagarrizko lasaitasuna: Problema dualaren optimoa

r: $-3u_1 - u_2 \;= -2$

s: $-2u_1 - 4u_2 \;= -2$

$\Rightarrow$ $10\,u_1 = 6$ $\Rightarrow$ $\boxed{\mathbf{u_1 = \frac{6}{10} = \frac{3}{5}}}$

$-3\left(\frac{3}{5}\right) - u_2 = -2$ $\Rightarrow$ $-\frac{9}{5} - u_2 = -2 \rightarrow u_2 = \frac{10-9}{5} \rightarrow \boxed{\boldsymbol{u_2 = \frac{1}{5}}}$

Problema dualaren optimoa lortu da:

$$\boxed{\boldsymbol{u_1}^* = \frac{\mathbf{3}}{\mathbf{5}};\quad \boldsymbol{u_2}^* = \frac{\mathbf{1}}{\mathbf{5}};\quad \boldsymbol{Z}^* = -\frac{\mathbf{36}}{\mathbf{5}}}$$

Dualtasuna eta Osagarrizko lasaitasuna: Osagarrizko lasaitasuna

Problema primalaren soluzioa kalkulatzeko, osagarrizko lasaitasuna aplikatuko dugu.

$$Max\ \ z = -2x_1 - 4x_2 - 2x_3$$

$$(1)\ -3x_1 - x_2 - 2x_3 + x_4 = -8$$
$$(2)\ -x_1 - 3x_2 - 4x_3 + x_5 = -12$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

$$min\ \ z = 8u_1 + 12u_2$$

$$(3)\ -3u_1 - u_2 - u_3 = -2$$
$$(4)\ -u_1 - 3u_2 - u_4 = -4$$
$$(5)\ -2u_1 - 4u_2 - u_5 = -2$$
$$u_1, u_2, u_3, u_4, u_5 \geq 0$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix};\ X^h = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix};\ \ U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix};\ \ U^h = \begin{pmatrix} u_3 \\ u_4 \\ u_5 \end{pmatrix} \Rightarrow$$

$$(6)\ x_1 \cdot u_3 = 0$$
$$(7)\ x_2 \cdot u_4 = 0$$
$$(8)\ x_3 \cdot u_5 = 0$$
$$(9)\ u_1 \cdot x_4 = 0$$
$$(10)\ u_2 \cdot x_5 = 0$$

Dualtasuna eta Osagarrizko lasaitasuna: Ekuazio sistema askatzen

- $-(3)$ ekuazioa erabiliz:

$$-3\left(\frac{3}{5}\right)-\left(\frac{1}{5}\right)-u_3 \;=-2 \;\rightarrow\; u_3=2-\frac{9}{5}-\frac{1}{5}=\frac{10-10}{5}=0 \;\rightarrow\; \boldsymbol{u_3=0}$$

- $-(4)$ ekuazioa erabiliz:

$$-\left(\frac{3}{5}\right)-3\left(\frac{1}{5}\right)-u_4=-4 \rightarrow\; u_4=4-\frac{3}{5}-\frac{3}{5}=\frac{20-6}{5}=\frac{14}{5} \;\rightarrow\; \boldsymbol{u_4=\frac{14}{5}}$$

- $-(5)$ ekuazioa erabiliz:

$$-2\left(\frac{3}{5}\right)-4\left(\frac{1}{5}\right)-u_5 \;=-2 \;\rightarrow\; u_5=2-\frac{6}{5}-\frac{4}{5}=\frac{10-10}{5}=0 \rightarrow\; \boldsymbol{u_5=0}$$

Dualtasuna eta Osagarrizko lasaitasuna: Problema primalaren soluzioa

–(7) ekuazioa erabiliz:

$$x_2 \cdot u_4 = 0 \rightarrow x_2 \cdot \frac{14}{5} = 0 \rightarrow x_2 = 0$$

–(9) ekuazioa erabiliz:

$$u_1 \cdot x_4 = 0 \rightarrow \frac{3}{5} \cdot x_4 = 0 \rightarrow x_4 = 0$$

–(10) ekuazioa erabiliz:

$$u_2 \cdot x_5 = 0 \rightarrow \frac{1}{5} \cdot x_5 = 0 \rightarrow x_5 = 0$$

–(1) eta (2) ekuazioa erabiliz:

$$\begin{cases} -3x_1 - (0) - 2x_3 + (0) = -8 \\ -x_1 - 3(0) - 4x_3 + (0) = -12 \end{cases}$$

$$\begin{cases} -3x_1 - 2x_3 = -8 \\ -x_1 - 4x_3 = -12 \end{cases} \xrightarrow{\text{'-2'-rekin biderkatuz}} \begin{cases} 6x_1 + 4x_3 = 16 \\ -x_1 - 4x_3 = -12 \end{cases} \xrightarrow{\text{Batu}} 5x_1 = 4 \rightarrow \boxed{x_1 = \frac{4}{5}}$$

$$-\left(\frac{4}{5}\right) - 4x_3 = -12 \rightarrow -\frac{1}{5} - x_3 = -3 \rightarrow x_3 = 3 - \frac{1}{5} = \frac{15-1}{5} = \frac{14}{5} \rightarrow \boxed{x_3 = \frac{14}{5}}$$

Dualtasuna eta Osagarrizko lasaitasuna: Primalaren soluzioa

Problema primalaren soluzio osoa hurrengoa da:

$$x_1{}^* = \frac{4}{5}; \quad x_2{}^* = 0; \quad x_3{}^* = \frac{14}{5}; \quad x_4{}^* = 0; \quad x_5{}^* = 0; \quad Z^* = -\frac{36}{5}$$

Soluzioa konprobatuko da:

$$-2x_1 - 4x_2 - 2x_3 = -\frac{36}{5} \rightarrow -2\left(\frac{4}{5}\right) - 4\,(0) - 2\left(\frac{14}{5}\right) = -\frac{36}{5} \rightarrow -\frac{8}{5} - \frac{28}{5} = -\frac{36}{5}$$

$$-\frac{36}{5} = -\frac{36}{5}$$

PHPSimplex Proba

PHPSimplex Proba

PHPSimplex

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Método Simplex

Operaciones intermedias **(mostrar/ocultar detalles)**

Tabla 1			-2	-4	-2	0	0
Base	**C_b**	**P_0**	**P_1**	**P_2**	**P_3**	**P_4**	**P_5**
P_1	-2	4 / 5	1	-1 / 5	0	-2 / 5	1 / 5
P_3	-2	14 / 5	0	4 / 5	1	1 / 10	-3 / 10
Z		-36 / 5	0	14 / 5	0	3 / 5	1 / 5

☑ Mostrar resultados como fracciones.

La solución óptima es Z = -36 / 5
X_1 = 4 / 5
X_2 = 0
X_3 = 14 / 5

La solución óptima es Z = -36 / 5
X_1 = 4 / 5
X_2 = 0
X_3 = 14 / 5

Eskerrik asko,
Segi Ondo!