4.2 JARDUERAREN EBAZPENA:

Izan bedi ondoko PO problema:

$$\begin{aligned}&\max\ Z = 3x_1 + 5x_2\\&\text{non}\ x_1 + 5x_2 \le 11\\&\quad x_1 - x_2 \le 2\\&\quad x_1, x_2 \ge 0\ \ \text{eta osoak}\end{aligned}$$

Lortu problema honen soluzioa osoa Adartze- eta bornatze metodoa erabiliz.

<u>1. Pausua:</u> Hasieraketa

Problema osoaren erlaxazio lineala ebatzi:

$$P1:\begin{cases}\max\ z = 3x_1 + 5x_2\\ \text{non}\ x_1 + 5x_2 \le 11\\ \quad x_1 - x_2 \le 2\\ \quad x_1, x_2 \ge 0\end{cases}$$

Simplex metodoa aplikatuz problema hau ebatziko dugu. Horretarako lehenengo eta behin problema era estandarrean idatziko dugu:

$$\begin{aligned}&\max\ Z = 3x_1 + 5x_2\\&\text{non}\ x_1 + 5x_2 + x_3 = 11\\&\quad x_1 - x_2 + x_4 = 2\\&\quad x_1, x_2, x_3, x_4 \ge 0\end{aligned}$$

Hasierako oinarrizko soluzio bideragarria: $x_B = (x_3, x_4) = (11, 2)$ eta Simplex metodoaren taula eraiki:

			3	5	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4
0	x_3	11	1	5	1	0
0	x_4	2	1	-1	0	1
Z=0		z_j	0	0	0	0
		W_j	-3	-5	0	0

$\exists W_j < 0 \Rightarrow$ jarraitu

<u>Sartze-irizpidea:</u> $W_j = \min\{z_k - c_k\} = -5 \Rightarrow$ x_2 sartu

Irtetze-irizpidea: $\frac{x_{B_i}}{y_{ij}} = \min\left\{ \frac{x_{B_k}}{y_{kj}} \ / \ y_{kj} > 0 \right\} = \frac{11}{5} \Rightarrow$ x_3 irten

Simplex-en taula berria: $\begin{cases} e_1 \leftarrow e_1 / 5 \\ e_2 \leftarrow e_2 + e_1 \end{cases}$

			3	5	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4
5	x_2	11/5	1/5	1	1/5	0
0	x_4	21/5	6/5	0	1/5	1
Z=11		z_j	1	5	1	0
		W_j	-2	0	1	0

$\exists W_j < 0 \Rightarrow$ jarraitu

Sartze-irizpidea: $W_j = \min\{z_k - c_k\} = -2 \Rightarrow$ x_1 sartu

Irtetze-irizpidea: $\frac{x_{B_i}}{y_{ij}} = \min\left\{ \frac{x_{B_k}}{y_{kj}} \ / \ y_{kj} > 0 \right\} = \min\left\{ \frac{11/5}{1/5}, \frac{21/5}{6/5} \right\} = \frac{21}{5} \Rightarrow$ x_4 irten

Simplex-en taula berria: $\begin{cases} e_2 \leftarrow \frac{5}{6}\cdot e_2 \\ e_1 \leftarrow e_1 - \frac{1}{5}\cdot e_2 \end{cases}$

			3	5	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4
5	x_2	3/2	0	1	1/6	-1/6
3	x_1	21/6	1	0	1/6	5/6
Z=18		z_j	3	5	4/3	5/3
		W_j	0	0	4/3	5/3

$W_j \geq 0 \quad \forall j \Rightarrow$ amaitu

Problema osoaren erlaxazio linealaren soluzio optimoa: $(x_1, x_2) = \left(\frac{21}{6}, \frac{3}{2} \right); \ z_1 = 18$ da. Behe-bornea $\underline{z} = -\infty$

2. Pausua: Adarkatzea

$$x_1 = \frac{21}{6} \Rightarrow \left\lfloor \frac{21}{6} \right\rfloor = 3.5 \Rightarrow \begin{cases} P2: \ x_1 \leq 3 \\ P3: \ x_1 \geq 4 \end{cases}$$

$$P2: \begin{cases} \max \ z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 \leq 11 \\ \quad x_1 - x_2 \leq 2 \\ \quad x_1 \leq 3 \\ \quad x_1, x_2 \geq 0 \end{cases} \qquad P3: \begin{cases} \max \ z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 \leq 11 \\ \quad x_1 - x_2 \leq 2 \\ \quad x_1 \geq 4 \\ \quad x_1, x_2 \geq 0 \end{cases}$$

3. Pausua: Bornatzea

$$P2: \begin{cases} \max \ z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 \leq 11 \\ \quad x_1 - x_2 \leq 2 \\ \quad x_1 \leq 3 \\ \quad x_1, x_2 \geq 0 \end{cases} \quad \Rightarrow \quad P2: \begin{cases} \max \ z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 + x_3 = 11 \\ \quad x_1 - x_2 + x_4 = 2 \\ \quad x_1 + x_5 = 3 \\ \quad x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

Murrizketa bat gehitu dugunez, sentikortasun-analisia erabiliz:

			3	5	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5
5	x_2	3/2	0	1	1/6	-1/6	0
3	x_1	21/6	1	0	1/6	5/6	0
0	x_5	3	1	0	0	0	1
Z=18		z_j	3	5	4/3	5/3	0
		W_j	0	0	4/3	5/3	0

Beharrezkoak diren eragiketak eginez: $e_3 \leftarrow e_3 - e_2$

			3	5	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5
5	x_2	3/2	0	1	1/6	-1/6	0
3	x_1	21/6	1	0	1/6	5/6	0
0	x_5	-1/2	0	0	-1/6	-5/6	1
Z=18		z_j	3	5	4/3	5/3	0
		W_j	0	0	4/3	5/3	0

$\exists X_{D_K} < 0 \Rightarrow$ Bideragarritasuna galdu da $\Rightarrow$ Simplex dual metodoa erabiliz:

Irtetze-irizpidea: $\max\left\{ \left| X_{D_K} \right| / X_{D_K} < 0 \right\} = \frac{1}{2} \Rightarrow$ x_5 irten

Sartze-irizpidea: $\min\limits_{k, a_{ik}<0}\left\{\frac{|z_k - c_k|}{|a_{ik}|}\right\} = \min\left\{\frac{4/3}{1/6}, \frac{5/3}{5/6}\right\} = \min\{8,2\} = 2 \Rightarrow$ x_4 sartu

Taula berria: $\begin{cases} e_3 \leftarrow e_3 \cdot \left(\frac{-6}{5}\right) \\ e_1 \leftarrow e_1 + \frac{1}{6} \cdot e_3 \\ e_2 \leftarrow e_2 - \frac{5}{6} \cdot e_3 \end{cases}$

			3	5	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5
5	x_2	8/5	0	1	1/5	0	-1/5
3	x_1	3	1	0	0	0	1
0	x_4	3/5	0	0	1/5	1	-6/5
Z=17		z_j	3	5	1	0	2
		W_j	0	0	1	0	2

$X_{D_K} \geq 0 \Rightarrow$ Bideragarritasuna lortu da. Ondorioz, P2 problemaren soluzio optimoa lortu dugu:

$$z_2 = 17;\ x_1 = 3,\ x_2 = 8/5$$

$$P3: \begin{cases} \max\ z = 3x_1 + 5x_2 \\ x_1 + 5x_2 \leq 11 \\ x_1 - x_2 \leq 2 \\ x_1 \geq 4 \\ x_1, x_2 \geq 0 \end{cases} \Rightarrow P3: \begin{cases} \max\ z = 3x_1 + 5x_2 \\ x_1 + 5x_2 + x_3 = 11 \\ x_1 - x_2 + x_4 = 2 \\ -x_1 + x_5 = -4 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

Murrizketa bat gehitu dugunez, sentikortasun-analisia erabiliz:

			3	5	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5
5	x_2	3/2	0	1	1/6	-1/6	0
3	x_1	21/6	1	0	1/6	5/6	0
0	x_5	-4	-1	0	0	0	1
Z=18		z_j	3	5	4/3	5/3	0
		W_j	0	0	4/3	5/3	0

Beharrezkoak diren eragiketak eginez: $e_3 \leftarrow e_3 + e_2$

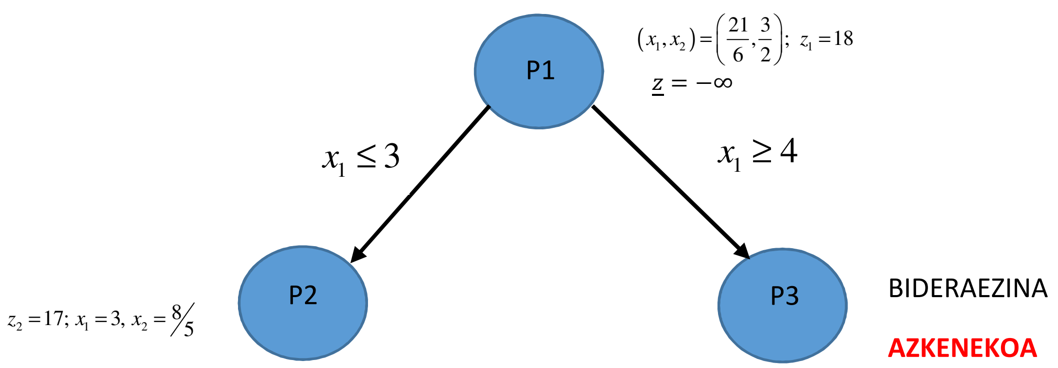
			3	5	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5
5	x_2	3/2	0	1	1/6	-1/6	0
3	x_1	21/6	1	0	1/6	5/6	0
0	x_5	-1/2	0	0	1/6	5/6	1
Z=18		z_j	3	5	4/3	5/3	0
		W_j	0	0	4/3	5/3	0

$\exists X_{D_K} < 0 \Rightarrow$ Bideragarritasuna galdu da $\Longrightarrow$ Simplex dual metodoa erabiliz:

<u>Irtetze-irizpidea:</u> $\max\left\{\left|X_{D_K}\right| / X_{D_K} < 0\right\} = \frac{1}{2} \Rightarrow$ x_5 irten

<u>Sartze-irizpidea:</u> $\min\limits_{k, a_{ik}<0}\left\{\frac{\left|z_k - c_k\right|}{\left|a_{ik}\right|}\right\},\ a_{3k} > 0\ \ \forall k = 1,2,3,4 \Rightarrow$ Problema duala bornatugabea da, ondorioz, problema primala bideraezina da.

<u>4. pausua:</u> Azkeneko problemak:



$x_2 = 8/5$ osoa ez denez $\Rightarrow$ 2. pausura joan.

<u>2. Pausua:</u> Adarkatzea

$$x_2 = 8/5 \Rightarrow \left\lfloor \frac{8}{5} \right\rfloor = 1.6 \Rightarrow \begin{cases} P4:\ x_2 \leq 1 \\ P5:\ x_2 \geq 2 \end{cases}$$

$$P4: \begin{cases} \max\ z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 \leq 11 \\ \quad x_1 - x_2 \leq 2 \\ \quad x_1 \leq 3 \\ \quad x_2 \leq 1 \\ \quad x_1, x_2 \geq 0 \end{cases} \qquad P5: \begin{cases} \max\ z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 \leq 11 \\ \quad x_1 - x_2 \leq 2 \\ \quad x_1 \leq 3 \\ \quad x_2 \geq 2 \\ \quad x_1, x_2 \geq 0 \end{cases}$$

<u>3. Pausua:</u> Bornatzea

$$P4:\begin{cases} \max\ z = 3x_1 + 5x_2 \\ x_1 + 5x_2 \le 11 \\ x_1 - x_2 \le 2 \\ x_1 \le 3 \\ x_2 \le 1 \\ x_1, x_2 \ge 0 \end{cases} \Rightarrow P4:\begin{cases} \max\ z = 3x_1 + 5x_2 \\ x_1 + 5x_2 + x_3 = 11 \\ x_1 - x_2 + x_4 = 2 \\ x_1 + x_5 = 3 \\ x_2 + x_6 = 1 \\ x_1, x_2, x_3, x_4, x_5, x_6 \ge 0 \end{cases}$$

Murrizketa bat gehitu dugunez, sentikortasun-analisia erabiliz:

			3	5	0	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5	x_6
5	x_2	8/5	0	1	1/5	0	-1/5	0
3	x_1	3	1	0	0	0	1	0
0	x_4	3/5	0	0	1/5	1	-6/5	0
0	x_6	1	0	1	0	0	0	1
Z=17		z_j	3	5	1	0	2	0
		W_j	0	0	1	0	2	0

Beharrezkoak diren eragiketak eginez: $e_4 \leftarrow e_4 - e_1$

			3	5	0	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5	x_6
5	x_2	8/5	0	1	1/5	0	-1/5	0
3	x_1	3	1	0	0	0	1	0
0	x_4	3/5	0	0	1/5	1	-6/5	0
0	x_6	-3/5	0	0	-1/5	0	1/5	1
Z=17		z_j	3	5	1	0	2	0
		W_j	0	0	1	0	2	0

$\exists X_{D_K} < 0 \Rightarrow$ Bideragarritasuna galdu da $\Longrightarrow$ Simplex dual metodoa erabiliz:

<u>Irtetze-irizpidea:</u> $\max\left\{\left|X_{D_K}\right| / X_{D_K} < 0\right\} = \dfrac{3}{5} \Rightarrow$ x_6 irten

<u>Sartze-irizpidea:</u> $\min\limits_{k, a_{ik} < 0}\left\{\dfrac{\left|z_k - c_k\right|}{\left|a_{ik}\right|}\right\} = \min\left\{\dfrac{1}{1/5}\right\} = 5 \Rightarrow$ x_3 sartu

Taula berria: $\begin{cases} e_4 \leftarrow e_4 \cdot \left(\dfrac{1}{-5} \right) \\ e_1 \leftarrow e_1 - \dfrac{1}{5} \cdot e_4 \\ e_2 \leftarrow e_2 \\ e_3 \leftarrow e_3 - \dfrac{1}{5} \cdot e_4 \end{cases}$

			3	5	0	0	0	0
C_{oin}	A_{oin}	$B^{-1} \cdot b$	x_1	x_2	x_3	x_4	x_5	x_6
5	x_2	1	0	1	0	0	0	1
3	x_1	3	1	0	0	0	1	0
0	x_4	0	0	0	0	1	-1	1
0	x_3	3	0	0	1	0	-1	-5
Z=14		z_j	3	5	0	0	3	5
		W_j	0	0	0	0	3	5

$X_{D_K} \geq 0 \Rightarrow$ Bideragarritasuna lortu da. Ondorioz, P4 problemaren soluzio optimoa lortu dugu:

$$z_4 = 17;\ x_1 = 3,\ x_2 = 1$$

$$P5: \begin{cases} \max z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 \leq 11 \\ \quad x_1 - x_2 \leq 2 \\ \quad x_1 \leq 3 \\ \quad x_2 \geq 2 \\ \quad x_1, x_2 \geq 0 \end{cases} \qquad P5: \begin{cases} \max z = 3x_1 + 5x_2 \\ \quad x_1 + 5x_2 + x_3 = 11 \\ \quad x_1 - x_2 + x_4 = 2 \\ \quad x_1 + x_5 = 3 \\ \quad -x_2 + x_6 = -2 \\ \quad x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{cases}$$

Murrizketa bat gehitu dugunez, sentikortasun-analisia erabiliz:

			3	5	0	0	0	0
C_{oin}	A_{oin}	$B^{-1} \cdot b$	x_1	x_2	x_3	x_4	x_5	x_6
5	x_2	8/5	0	1	1/5	0	-1/5	0
3	x_1	3	1	0	0	0	1	0
0	x_4	3/5	0	0	1/5	1	-6/5	0
0	x_6	-2	0	-1	0	0	0	1
Z=17		z_j	3	5	1	0	2	0
		W_j	0	0	1	0	2	0

Beharrezkoak diren eragiketak eginez: $e_4 \leftarrow e_4 + e_1$

			3	5	0	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5	x_6
5	x_2	8/5	0	1	1/5	0	-1/5	0
3	x_1	3	1	0	0	0	1	0
0	x_4	3/5	0	0	1/5	1	-6/5	0
0	x_6	-2/5	0	0	1/5	0	-1/5	1
Z=17		z_j	3	5	1	0	2	0
		W_j	0	0	1	0	2	0

$\exists X_{D_K} < 0 \Rightarrow$ Bideragarritasuna galdu da $\Longrightarrow$ Simplex dual metodoa erabiliz:

Irtetze-irizpidea: $\max\left\{\left|X_{D_K}\right| / X_{D_K} < 0\right\} = \dfrac{2}{5} \Rightarrow$ x_6 irten

Sartze-irizpidea: $\min\limits_{k, a_{ik} < 0}\left\{\dfrac{|z_k - c_k|}{|a_{ik}|}\right\} = \min\left\{\dfrac{2}{1/5}\right\} = 10 \Rightarrow$ x_5 sartu

Taula berria: $\begin{cases} e_4 \leftarrow e_4 \cdot \left(\dfrac{1}{-5}\right) \\ e_1 \leftarrow e_1 + \dfrac{1}{5} \cdot e_4 \\ e_2 \leftarrow e_2 - e_4 \\ e_3 \leftarrow e_3 + \dfrac{6}{5} \cdot e_4 \end{cases}$

			3	5	0	0	0	0
C_{oin}	A_{oin}	$B^{-1}\cdot b$	x_1	x_2	x_3	x_4	x_5	x_6
5	x_2	2	0	1	0	0	0	-1
3	x_1	1	1	0	1	0	0	5
0	x_4	3	0	0	-1	1	0	-6
0	x_5	2	0	0	-1	0	1	-5
Z=13		z_j	3	5	3	0	0	10
		W_j	0	0	3	0	0	10

$X_{D_K} \geq 0 \Rightarrow$ Bideragarritasuna lortu da. Ondorioz, P5 problemaren soluzio optimoa lortu dugu:

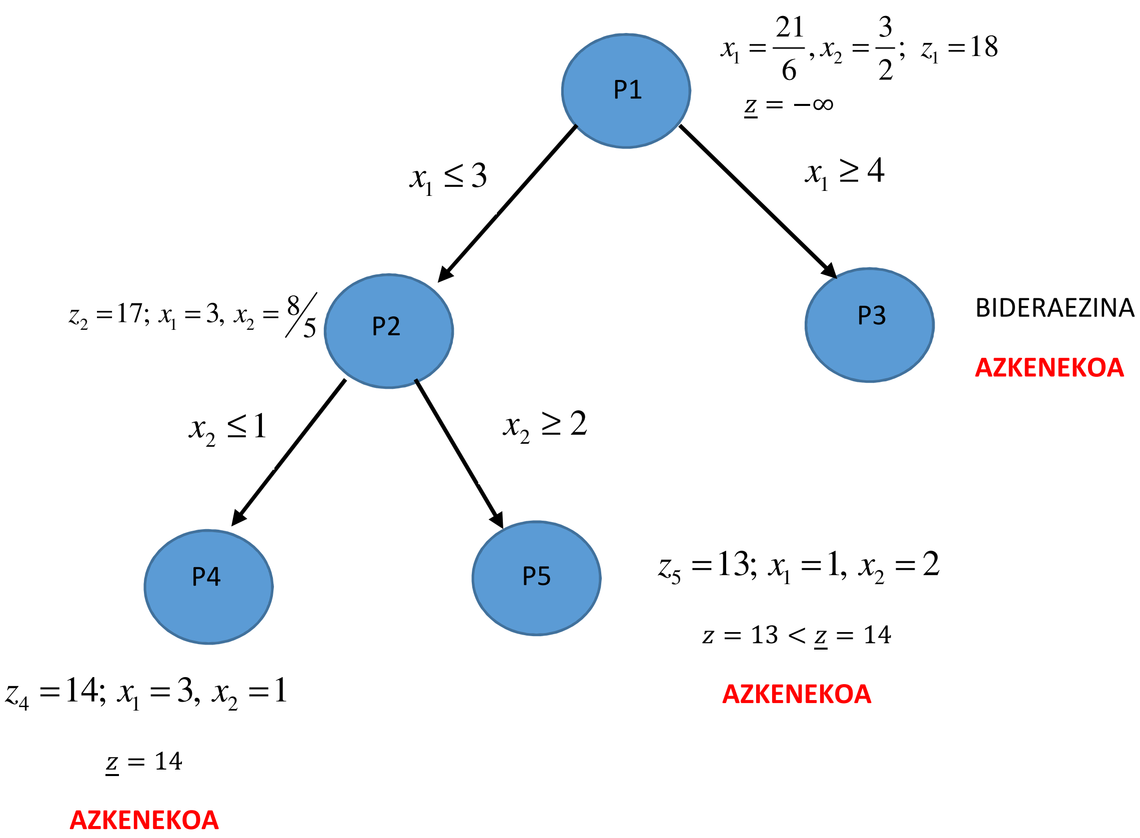
$$z_5 = 13;\ x_1 = 1,\ x_2 = 2$$

<u>4. pausua:</u> Azkeneko problemak

P4: Azkeneko problema da.

Problemaren soluzioa osoa da eta $z_4 = 14 > \underline{z} = -\infty \Rightarrow$ Behe-bornea $\underline{z} = z_4 = 14$

P5: Azkeneko problema da: $z_5 = 13 < \underline{z} = 14$



Problema guztiak azkenekoak dira $\Longrightarrow$ Soluziogaia problema osoaren optimoa da, hau da, P4 problemaren soluzio optimoa problema osoaren soluzio optimoa da:

$$z = 14;\ x_1 = 3,\ x_2 = 1$$