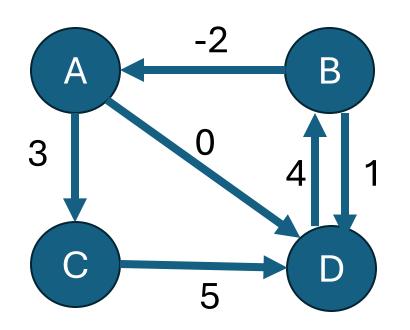
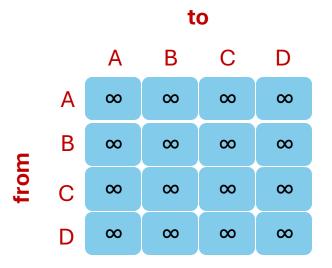
All Pairs Shortest Path

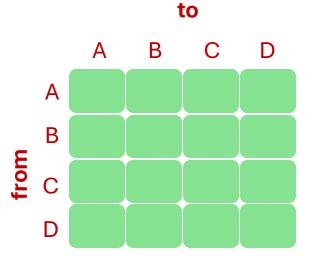
Floyd Warshall Algorithm

Floyd Warshall Algorithm

```
let dist be a |V| × |V| array of minimum distances initialized to ∞
for each edge (u, v)
    do dist[u][v] = w(u, v) // The weight of the edge (u, v)
for each vertex v do
    dist[v][v] = 0
for k from 1 to |V|
    for i from 1 to |V|
        for j from 1 to |V|
            if dist[i][j] > dist[i][k] + dist[k][j]
                dist[i][j] = dist[i][k] + dist[k][j]
            end if
```







let dist be a $|V| \times |V|$ array of minimum distances initialized to ∞

```
for each edge (u, v)

do dist[u][v] = w(u, v) // The weight of the edge <math>(u, v)

for each vertex v do

dist[v][v] = 0

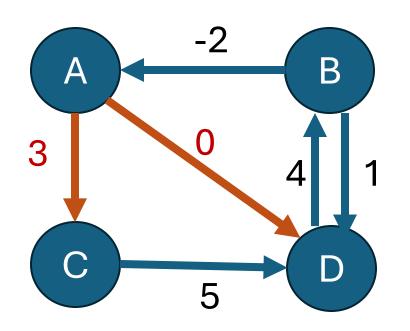
for k from 1 to |V|

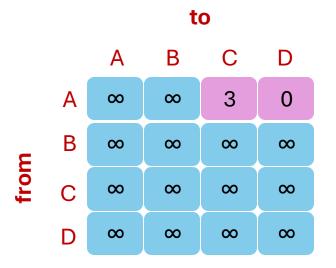
for i from 1 to |V|

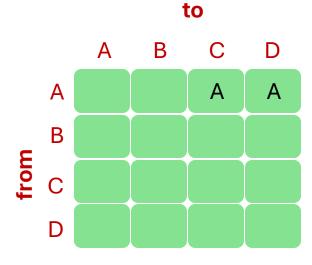
if dist[i][j] > dist[i][k] + dist[k][j]

dist[i][j] = dist[i][k] + dist[k][j]

end if
```







```
let dist be a |V| \times |V| array of minimum distances initialized to \infty for each edge (u, v)

do dist[u][v] = w(u, v) // The weight of the edge (u, v)

for each vertex v do

dist[v][v] = 0

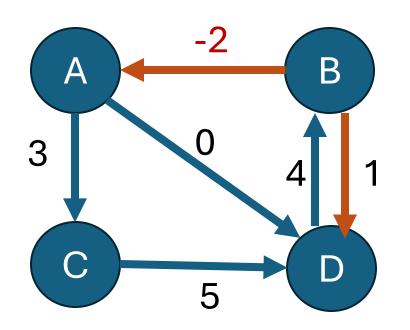
for k from 1 to |V|

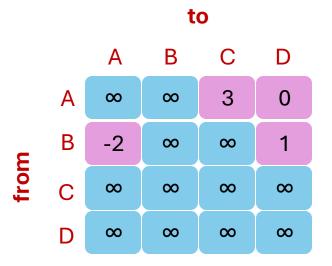
for i from 1 to |V|

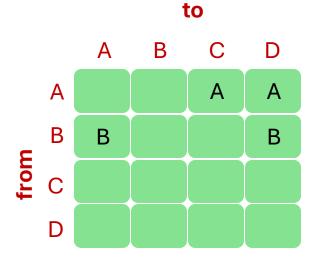
if dist[i][j] > \text{dist}[i][k] + \text{dist}[k][j]

dist[i][j] = \text{dist}[i][k] + \text{dist}[k][j]

end if
```







```
let dist be a |V| \times |V| array of minimum distances initialized to \infty for each edge (u, v)

do dist[u][v] = w(u, v) // The weight of the edge (u, v)

for each vertex v do

dist[v][v] = 0

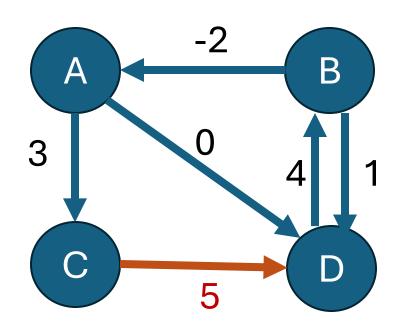
for k from 1 to |V|

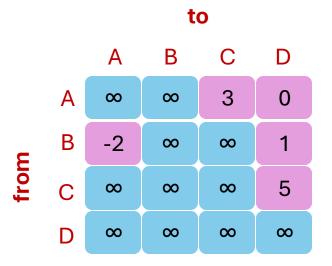
for i from 1 to |V|

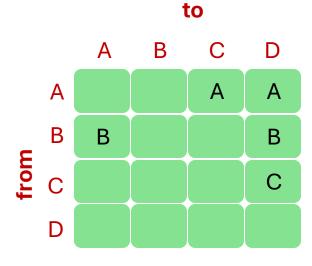
if dist[i][j] > \text{dist}[i][k] + \text{dist}[k][j]

dist[i][j] = \text{dist}[i][k] + \text{dist}[k][j]

end if
```







```
let dist be a |V| \times |V| array of minimum distances initialized to \infty for each edge (u, v)

do dist[u][v] = w(u, v) // The weight of the edge (u, v)

for each vertex v do

dist[v][v] = 0

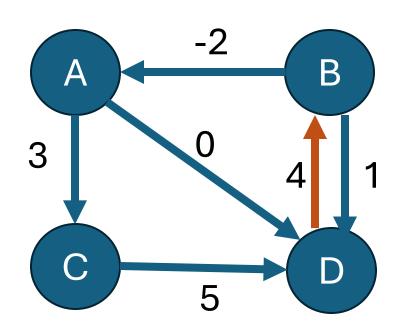
for k from 1 to |V|

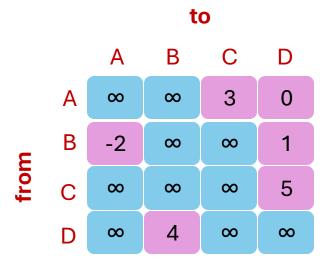
for i from 1 to |V|

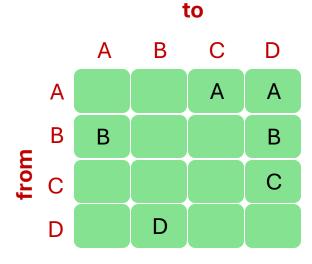
if dist[i][j] > \text{dist}[i][k] + \text{dist}[k][j]

dist[i][j] = \text{dist}[i][k] + \text{dist}[k][j]

end if
```







```
let dist be a |V| \times |V| array of minimum distances initialized to \infty for each edge (u, v)

do dist[u][v] = w(u, v) // The weight of the edge (u, v)

for each vertex v do

dist[v][v] = 0

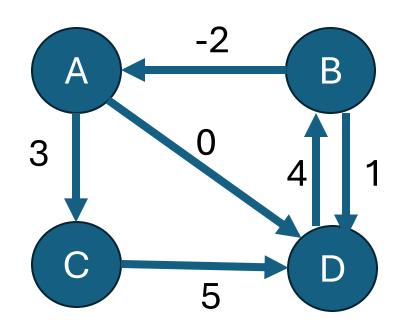
for k from 1 to |V|

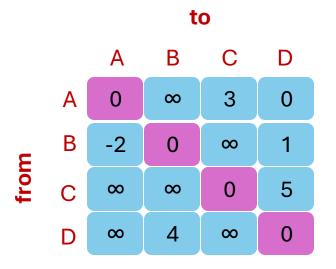
for i from 1 to |V|

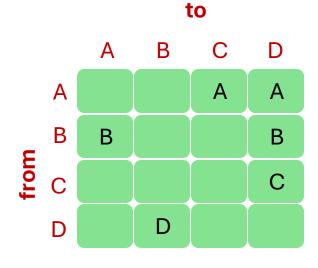
if dist[i][j] > \text{dist}[i][k] + \text{dist}[k][j]

dist[i][j] = \text{dist}[i][k] + \text{dist}[k][j]

end if
```







```
let dist be a |V| \times |V| array of minimum distances initialized to \infty for each edge (u, v)

do dist[u][v] = w(u, v) // The weight of the edge (u, v)

for each vertex v do

dist[v][v] = 0

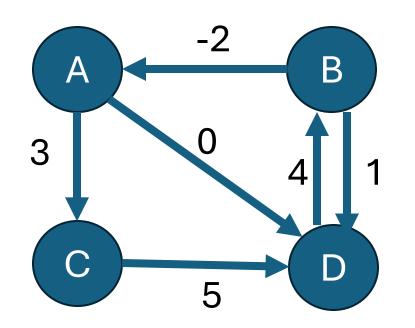
for k from 1 to |V|

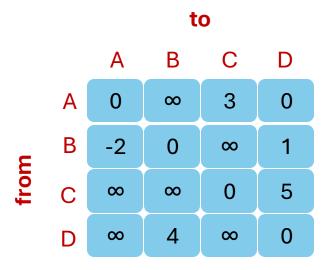
for j from 1 to |V|

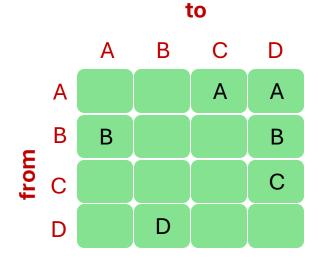
if dist[i][j] > \text{dist}[i][k] + \text{dist}[k][j]

dist[i][j] = \text{dist}[i][k] + \text{dist}[k][j]

end if
```







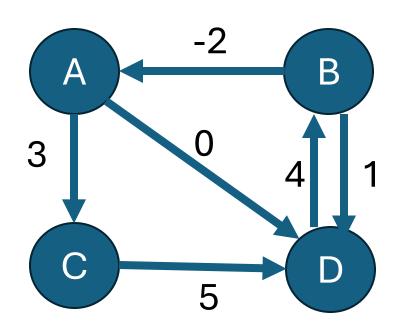
```
let dist be a |V| \times |V| array of minimum distances initialized to \infty

for each edge (u, v)

do dist[u][v] = w(u, v) // The weight of the edge (u, v)

for each vertex v do

dist[v][v] = 0
```



```
for k from 1 to |V|

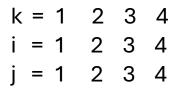
for i from 1 to |V|

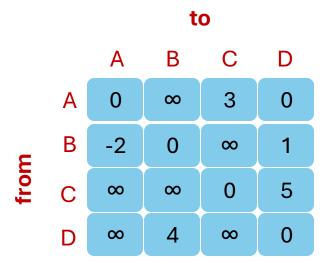
for j from 1 to |V|

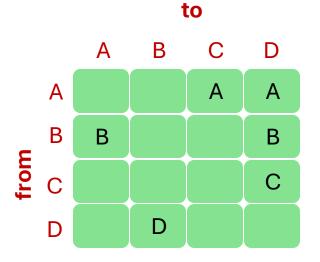
if dist[i][j] > dist[i][k] + dist[k][j]

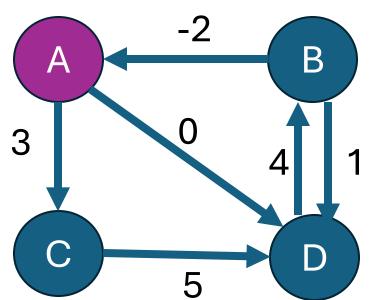
dist[i][j] = dist[i][k] + dist[k][j]

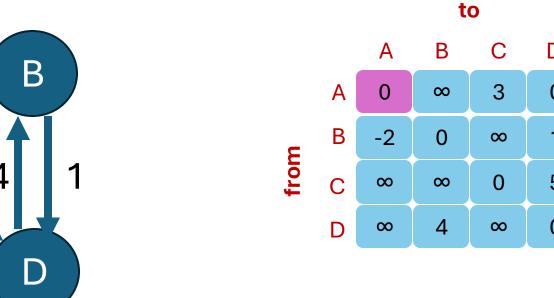
end if
```

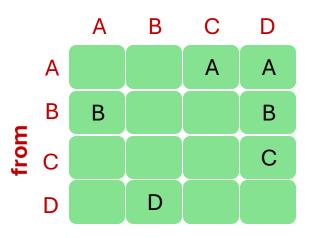




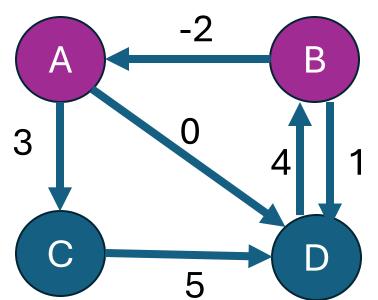


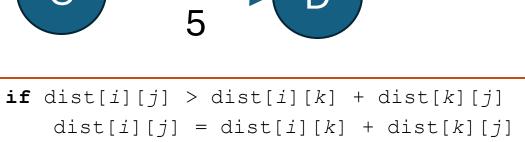


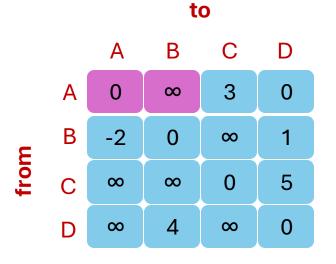




$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

```
AB = \infty > AA = 0 + AB = \infty

if dist[1][2] > dist[1][1] + dist[1][2]

dist[1][2] = dist[1][1] + dist[1][2]
```

C

Α

D

В

С

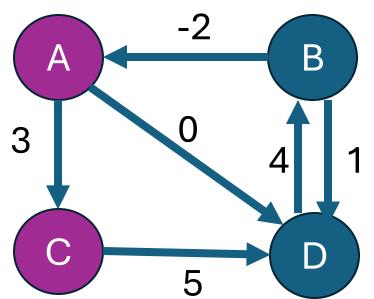
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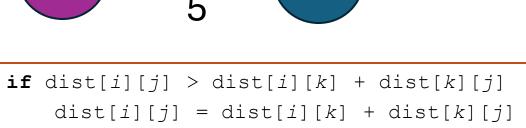
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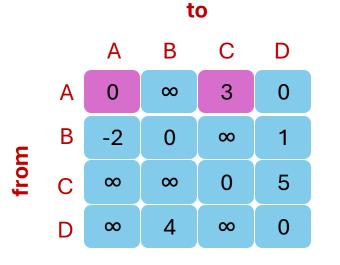
В

В

D







C

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В

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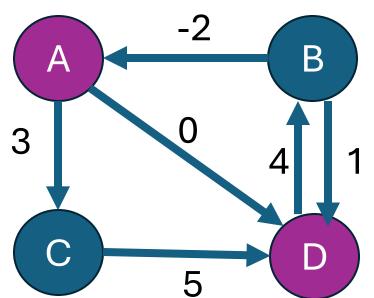
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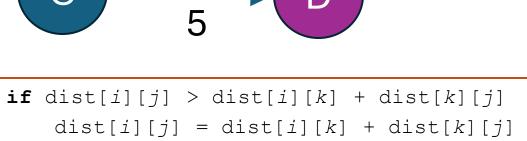
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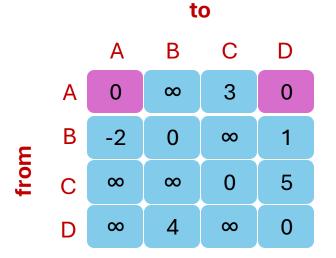
В

В

D







C

Α

D

В

С

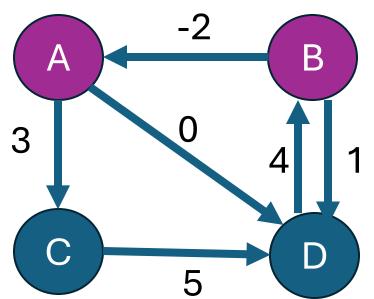
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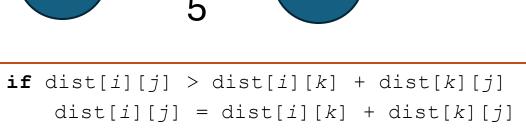
Α

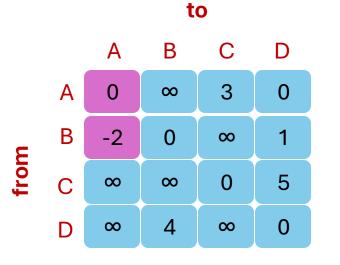
В

В

D







C

Α

D

В

С

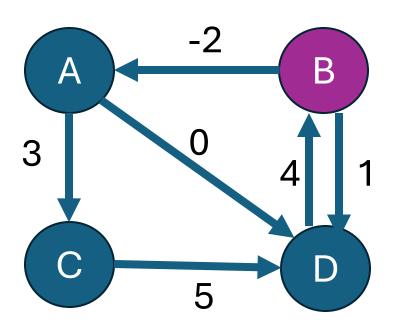
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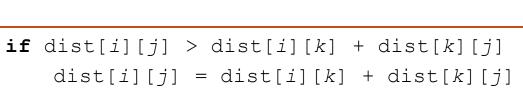
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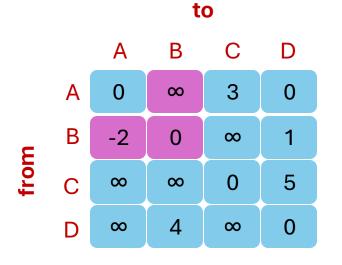
В

В

D







C

Α

D

В

С

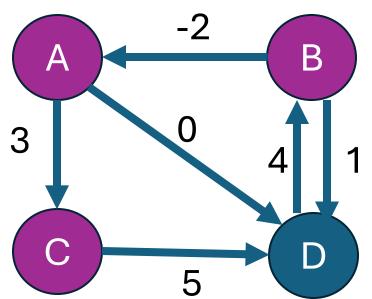
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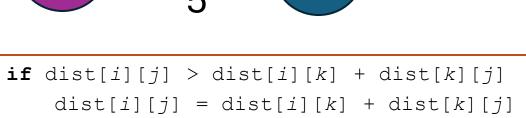
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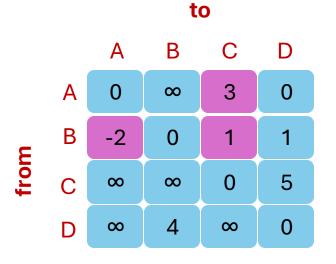
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

C

Α

Α

D

В

С

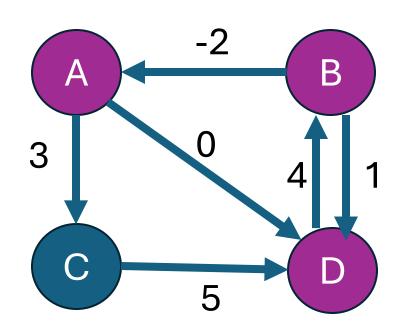
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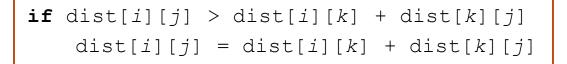
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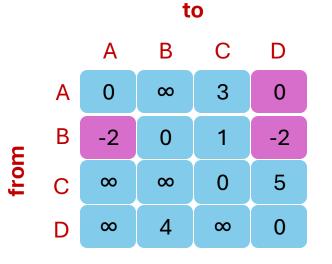
В

В

D







YES!!!!!!

to

C

Α

Α

D

Α

С

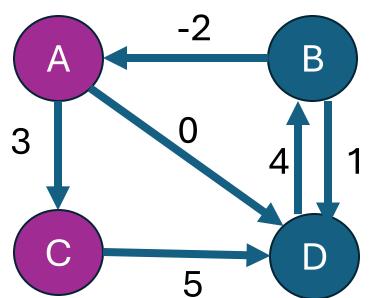
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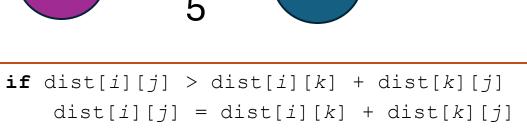
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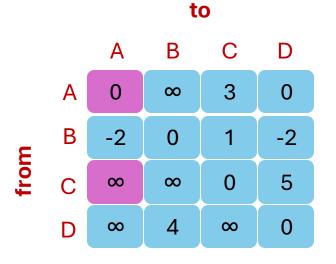
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

C

Α

Α

D

С

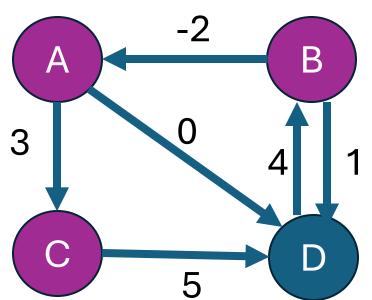
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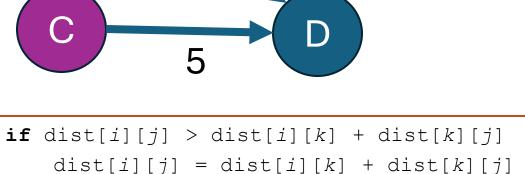
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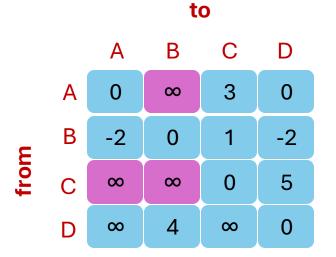
В

В

D

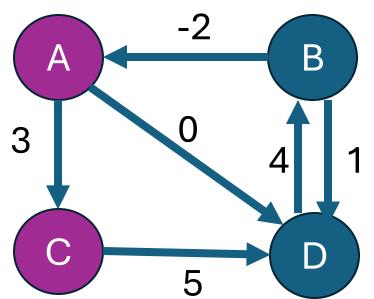


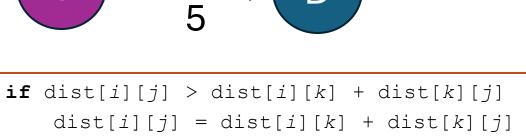


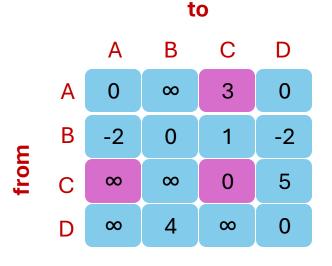


$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

$$CB = \infty$$
 > $CA = \infty$ + $AB = 0$
if dist[3][2] > dist[3][1] + dist[1][2]
dist[3][2] = dist[3][1] + dist[1][3]

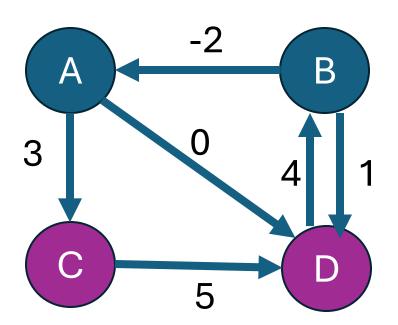


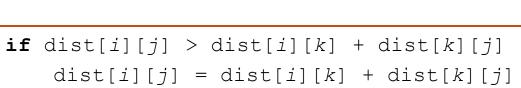


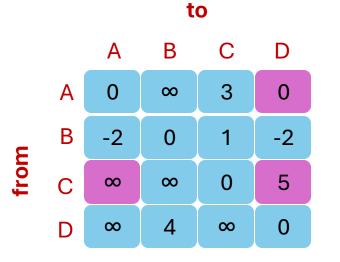


$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

$$CC = 0$$
 > $CA = \infty$ + $AC = 3$
if dist[3][3] > dist[3][1] + dist[1][3]
dist[3][3] = dist[3][1] + dist[1][3]







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

$$CD=5$$
 > $CA=\infty$ + $AD=0$
if dist[3][4] > dist[3][1] + dist[1][4]
dist[3][4] = dist[3][1] + dist[1][4]

C

Α

Α

D

C

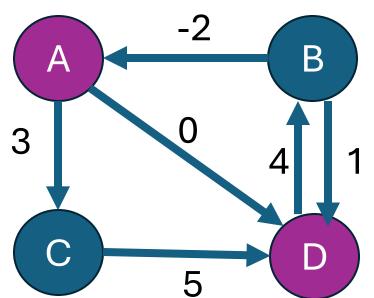
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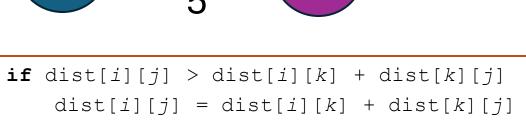
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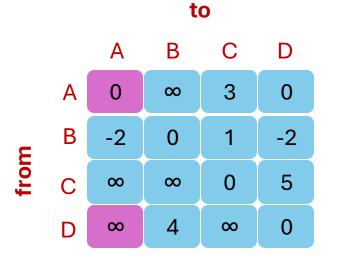
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

Α

Α

D

С

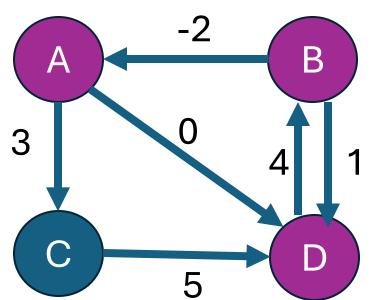
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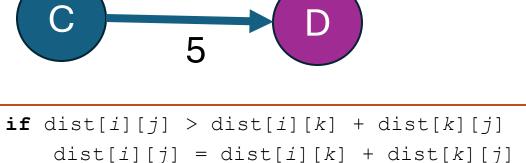
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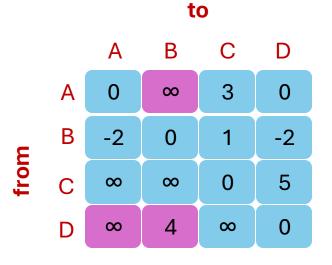
В

В

D







DB=4 > DA=
$$\infty$$
 + AB= ∞
if dist[4][2] > dist[4][1] + dist[1][2]
dist[4][2] = dist[3][1] + dist[1][2]

C

Α

Α

D

Α

С

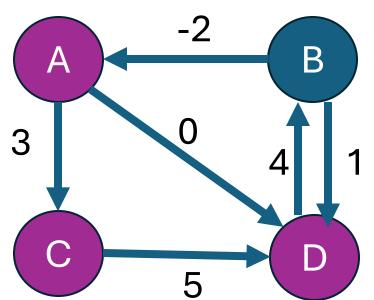
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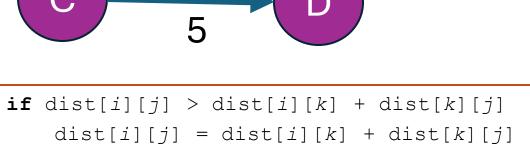
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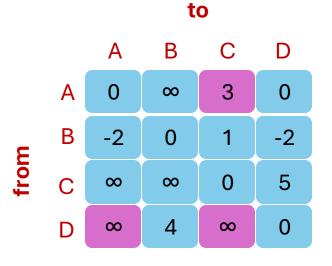
В

В

D







```
DC = ∞ > DA = ∞ + AC = 3

if dist[4][3] > dist[4][1] + dist[1][3]

dist[4][3] = dist[3][1] + dist[1][3]
```

C

Α

Α

D

С

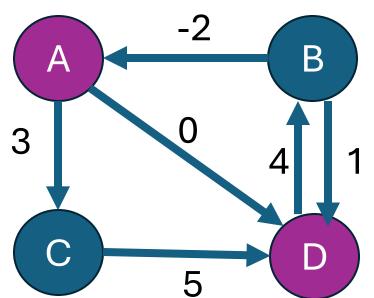
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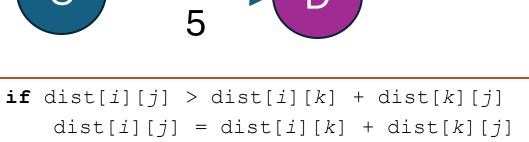
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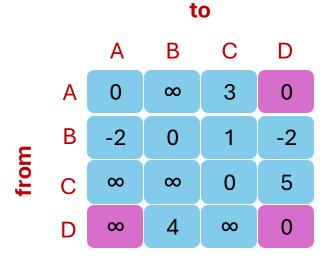
В

В

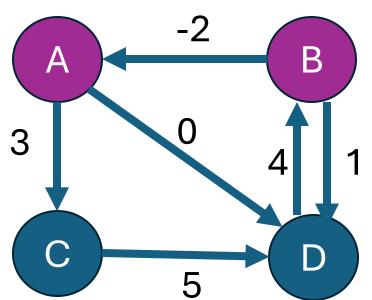
D

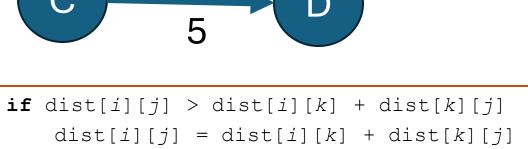


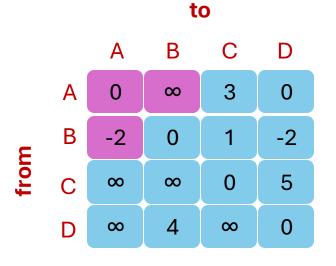




$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

$$AA = 0$$
 > $AB = \infty$ + $BA = -2$
if dist[1][1] > dist[1][2] + dist[2][1]
dist[1][1] = dist[1][2] + dist[2][1]

C

Α

Α

D

Α

С

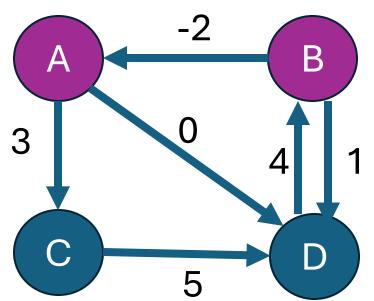
В

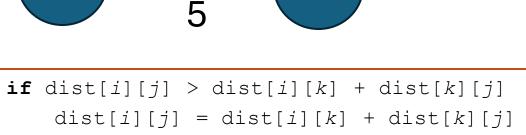
Α

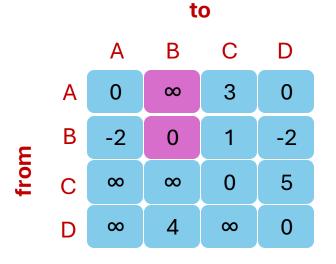
В

В

D







$$AB = \infty$$
 > $AB = \infty$ + $BB = 0$
if dist[1][2] > dist[1][2] + dist[2][2]
dist[1][2] = dist[1][2] + dist[2][2]

C

Α

Α

D

Α

С

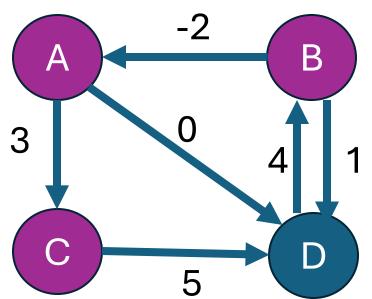
В

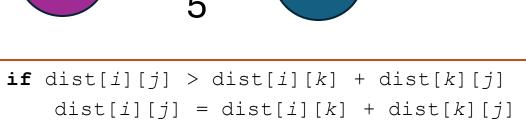
Α

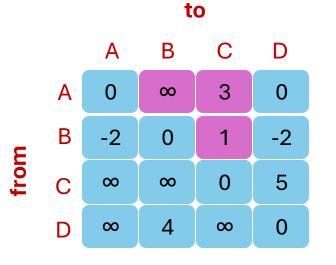
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

```
AC = 3 > AB = \infty + BC = 1

if dist[1][3] > dist[1][2] + dist[2][3]

dist[1][3] = dist[1][2] + dist[2][3]
```

C

Α

Α

D

Α

C

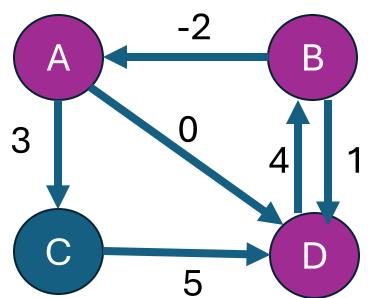
В

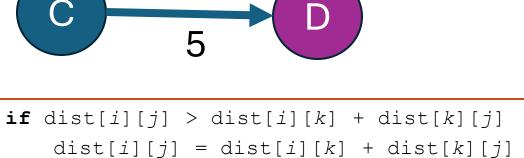
Α

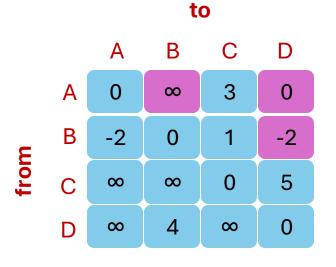
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

$$AD = 0$$
 > $AB = \infty$ + $BD = -2$
if dist[1][4] > dist[1][2] + dist[2][4]
dist[1][4] = dist[1][2] + dist[2][4]

C

Α

Α

D

Α

C

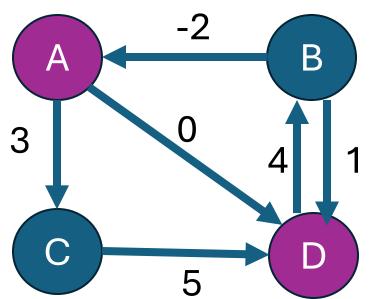
В

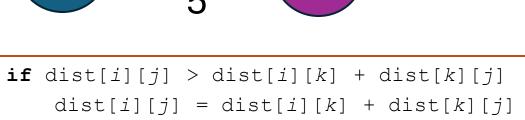
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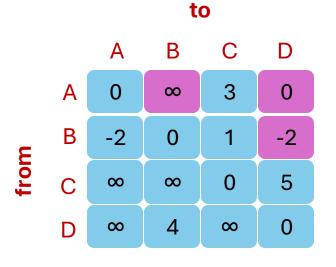
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

$$AD = 0$$
 > $AB = \infty$ + $BD = -2$
if dist[1][4] > dist[1][2] + dist[2][4]
dist[1][4] = dist[1][2] + dist[2][4]

C

Α

Α

D

Α

C

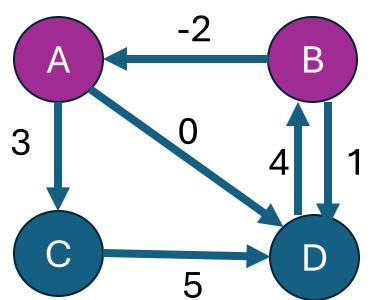
В

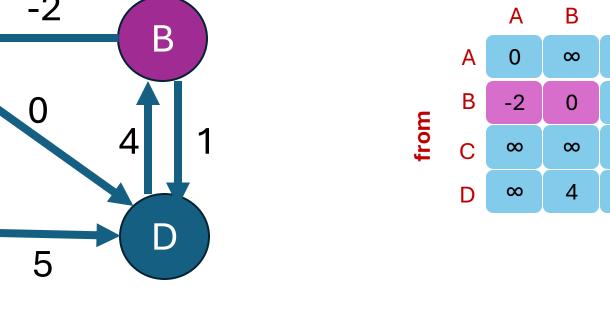
Α

В

В

D





$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

3

0

 ∞

-2

to

C

Α

Α

D

Α

C

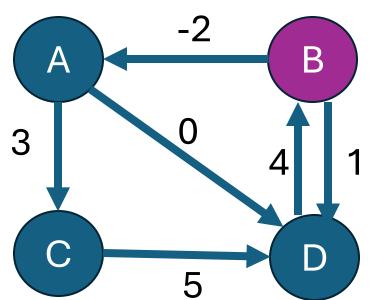
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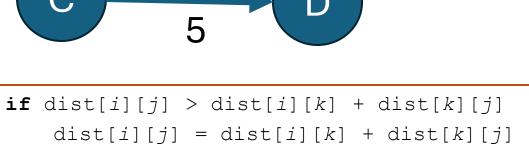
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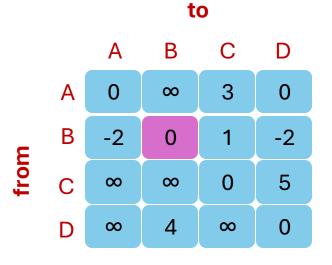
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

Α

Α

D

Α

C

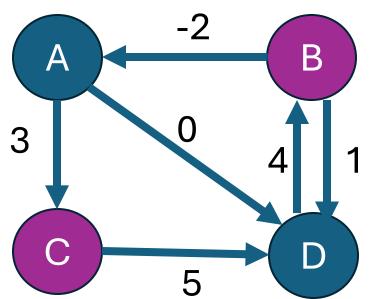
В

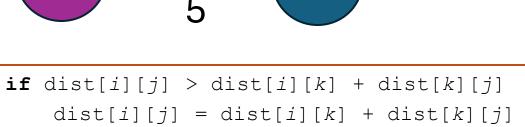
Α

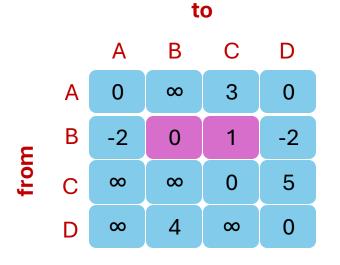
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

Α

Α

D

Α

C

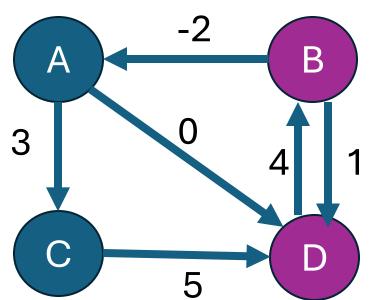
В

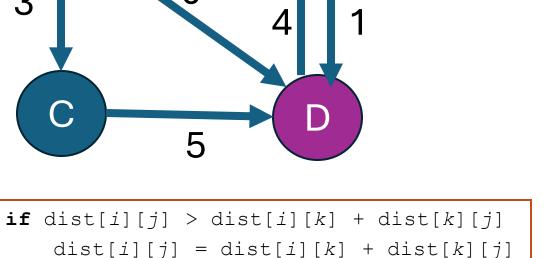
Α

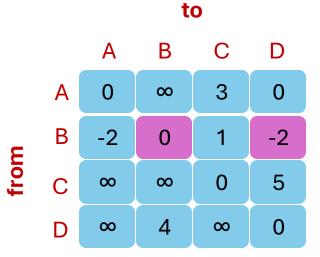
В

В

D







C

Α

Α

D

Α

C

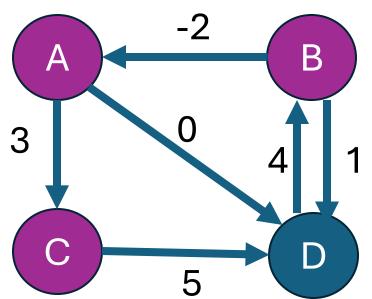
В

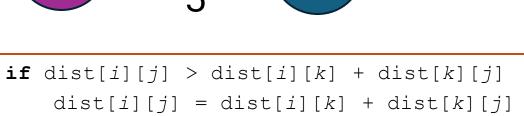
Α

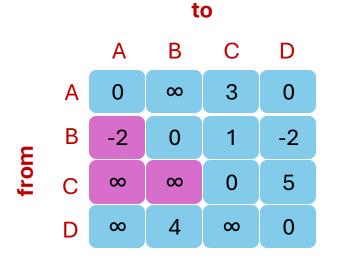
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

```
CA = \infty > CB = \infty + BA = -2

if dist[3][1] > dist[3][2] + dist[2][1]

dist[3][1] = dist[3][2] + dist[2][1]
```

C

Α

Α

D

Α

С

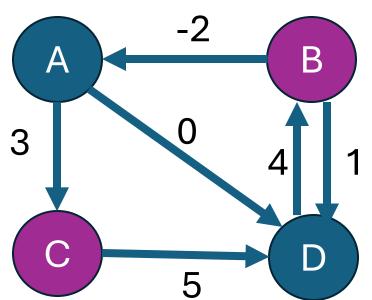
В

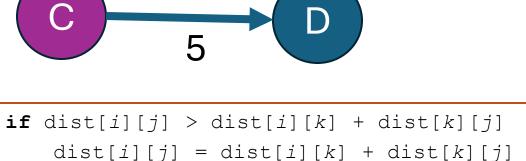
Α

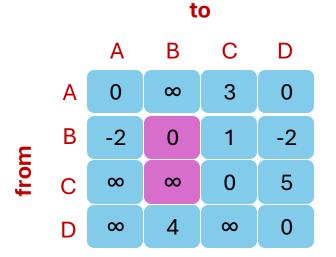
В

В

D







```
CB = \infty > CB = \infty + BB = 0

if dist[3][2] > dist[3][2] + dist[2][2]

dist[3][2] = dist[3][2] + dist[2][2]
```

C

Α

Α

D

Α

С

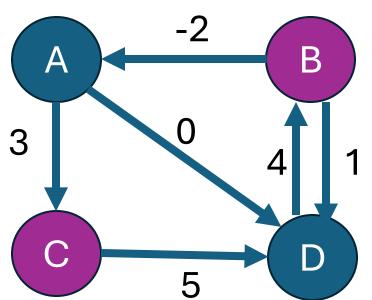
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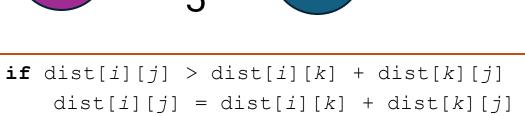
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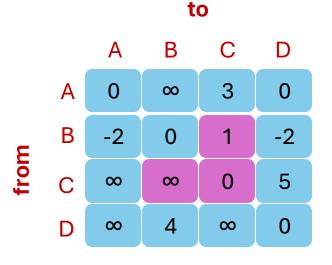
В

В

D







$$CC = 0$$
 > $CB = \infty$ + $BC = 1$
if dist[3][3] > dist[3][2] + dist[2][3]
dist[3][3] = dist[3][2] + dist[2][3]

C

Α

Α

D

C

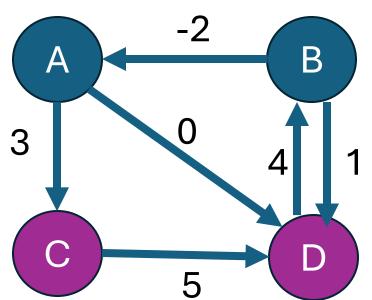
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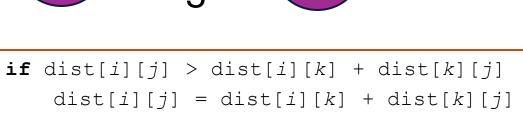
Α

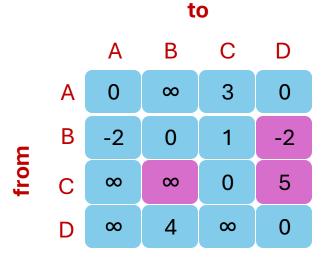
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

CD=5 > CB=
$$\infty$$
 + BD=-2
if dist[3][4] > dist[3][2] + dist[2][4]
dist[3][4] = dist[3][2] + dist[2][4]

C

Α

Α

D

C

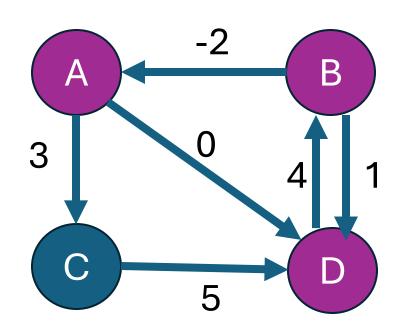
В

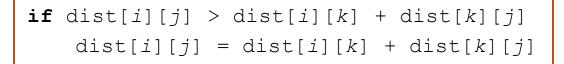
Α

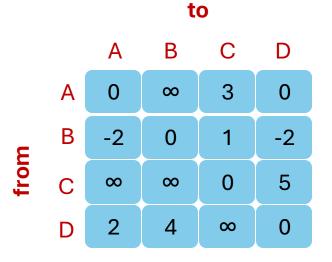
В

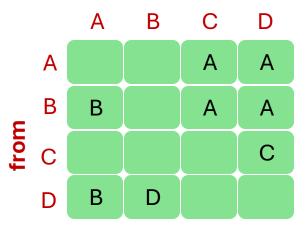
В

D



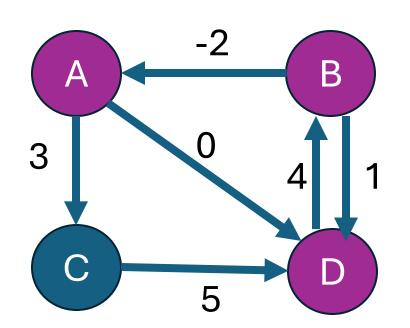


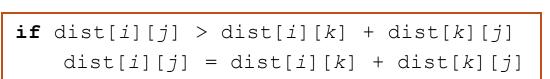


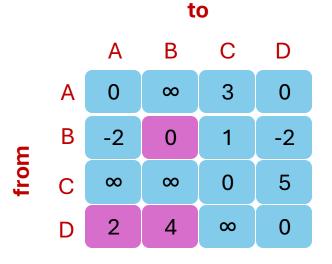


YES!!!!!!

$$DA = \infty$$
 > $DB = 4$ + $BA = -2$
if dist[4][1] > dist[4][2] + dist[2][1]
dist[4][1] = dist[4][2] + dist[2][1]







```
DA = ∞ > DB = 1 + BA=-2

if dist[4][1] > dist[4][2] + dist[2][1]

dist[4][1] = dist[4][2] + dist[2][1]
```

C

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Α

С

В

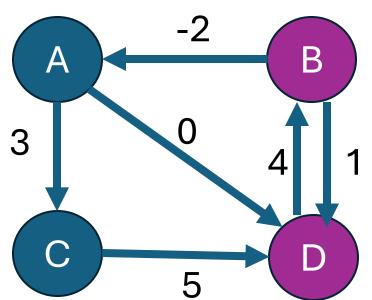
Α

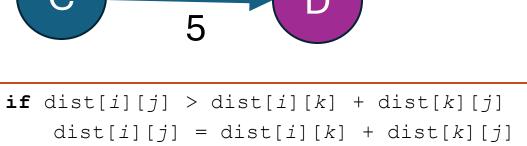
В

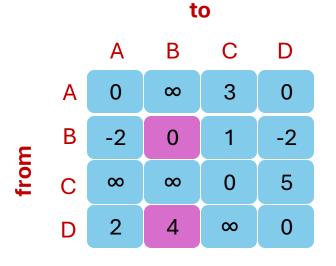
В

В

D







C

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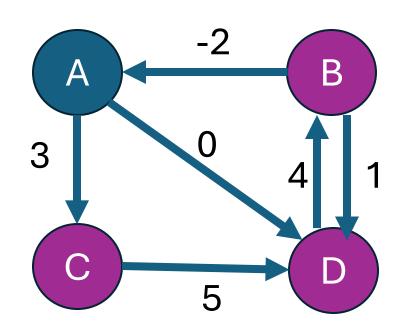
Α

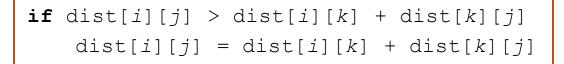
В

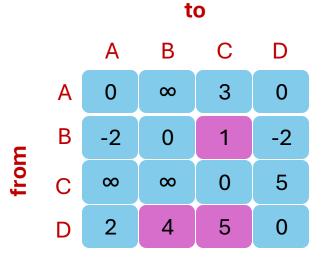
В

В

D







YES!!!!!!

to

C

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В

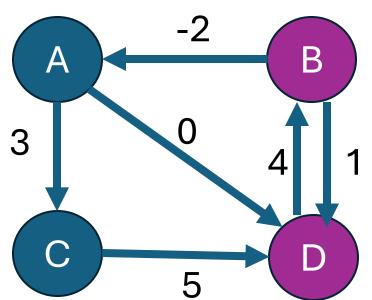
Α

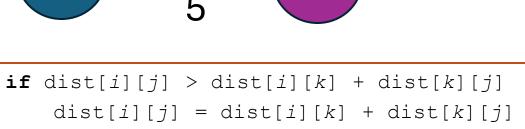
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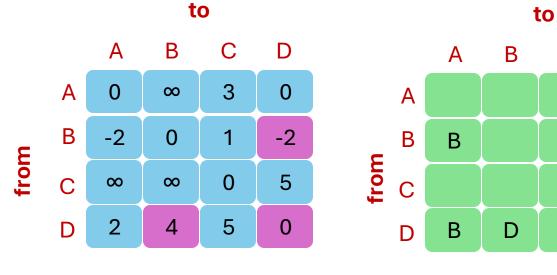
В

В

D







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Α

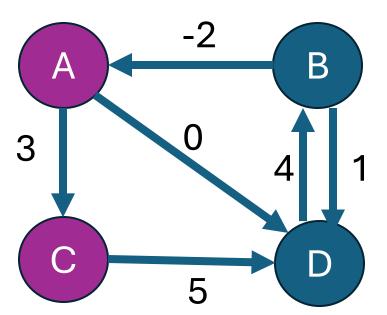
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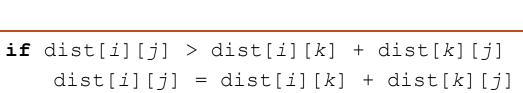
D

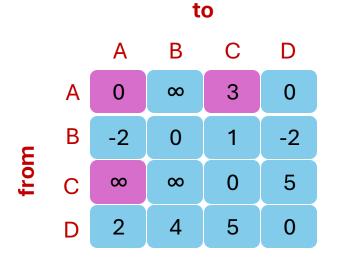
Α

С

$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4







C

Α

Α

В

D

Α

С

В

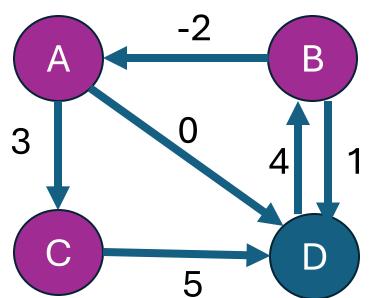
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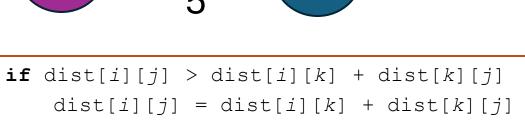
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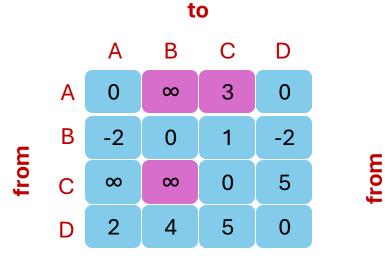
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

C

Α

Α

В

D

Α

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В

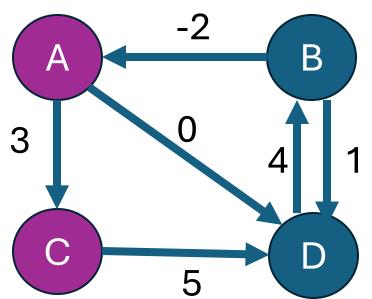
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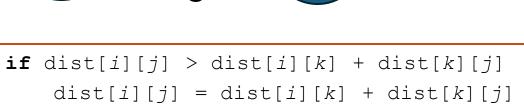
В

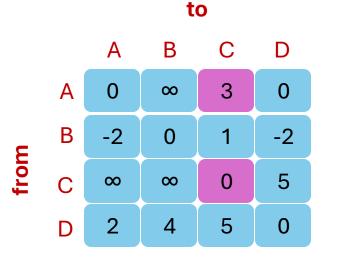
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

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В

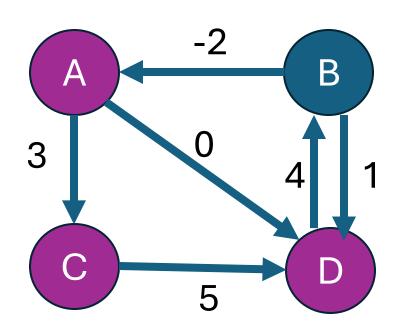
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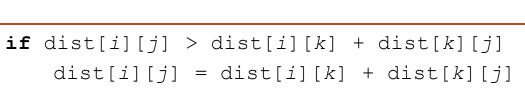
В

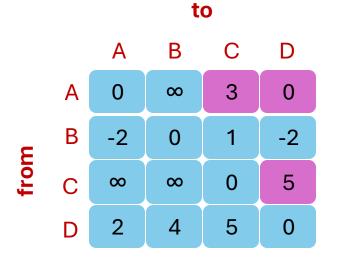
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

C

Α

Α

В

D

Α

С

В

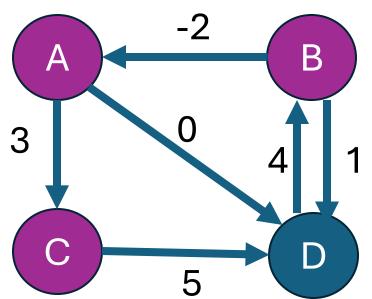
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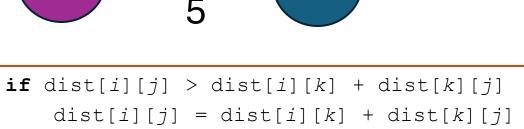
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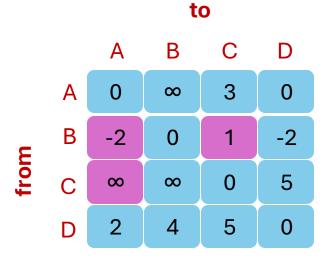
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

Α

Α

В

D

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В

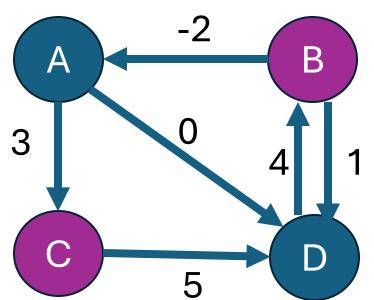
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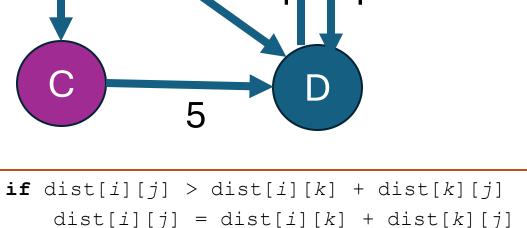
В

В

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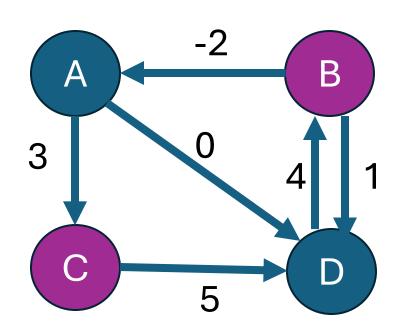
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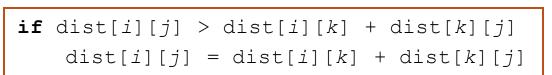
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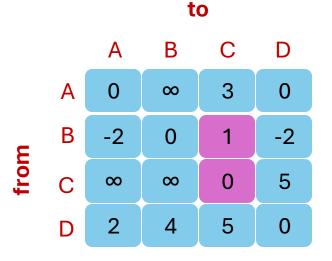
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

Α

Α

В

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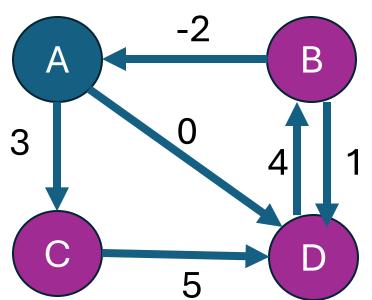
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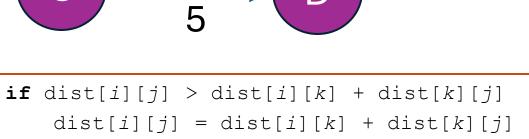
В

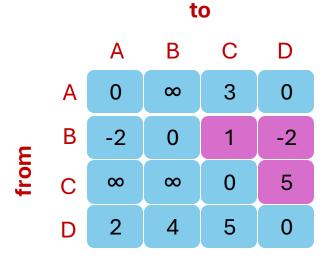
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

Α

Α

В

D

Α

С

В

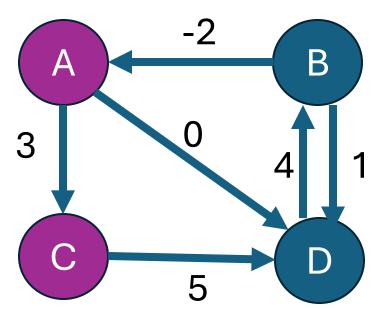
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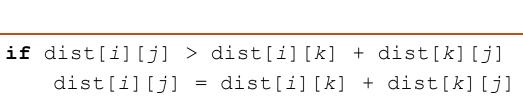
В

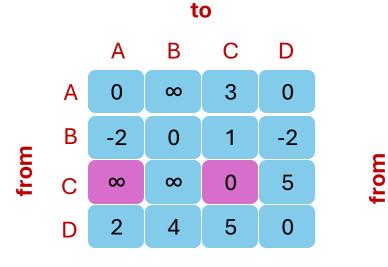
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

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В

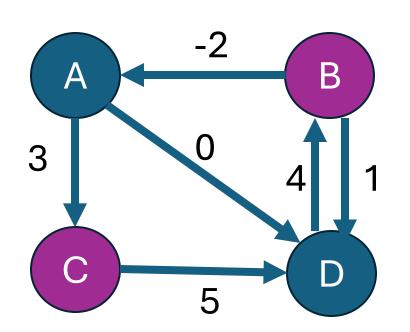
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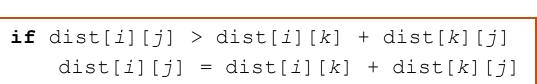
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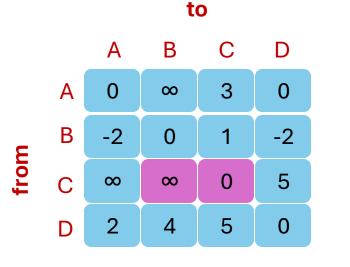
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

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В

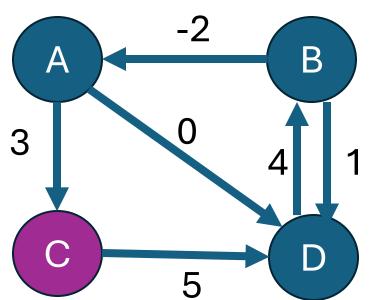
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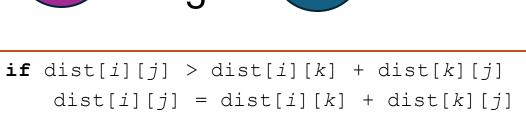
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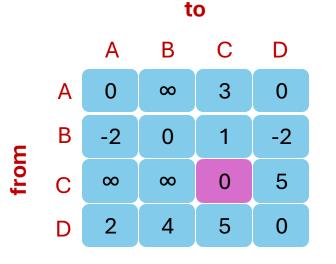
В

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$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

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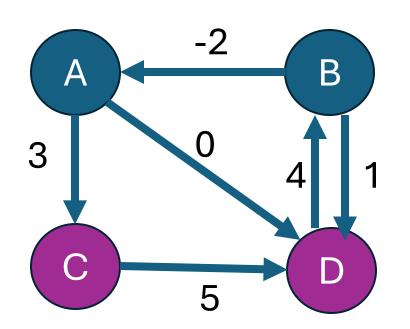
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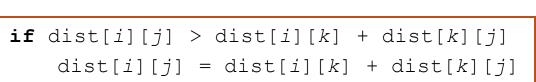
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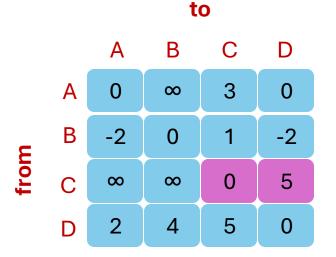
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В

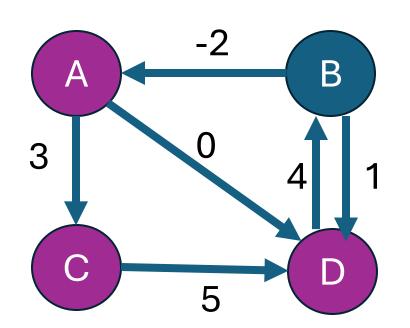
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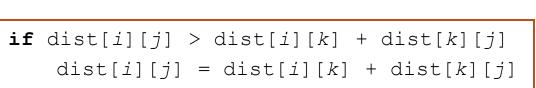


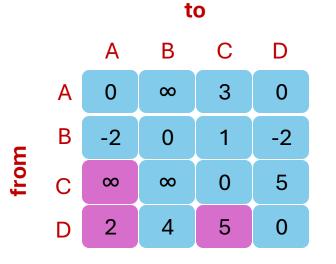




$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

C

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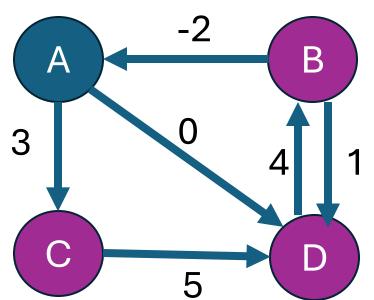
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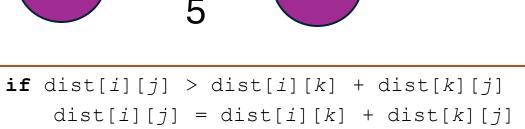
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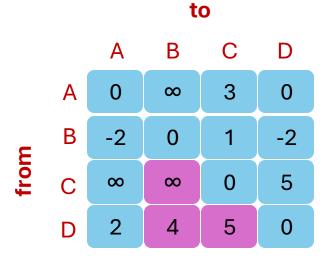
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$$k = 1$$
 2 3 4
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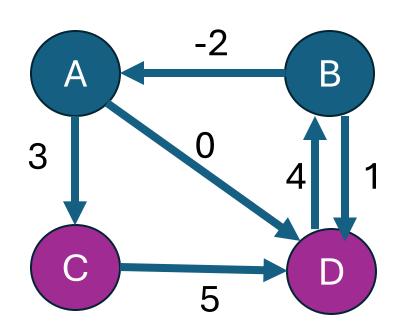
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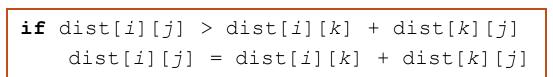
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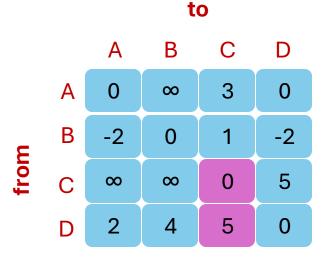
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$$k = 1$$
 2 3 4
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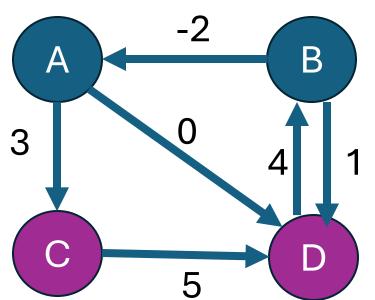
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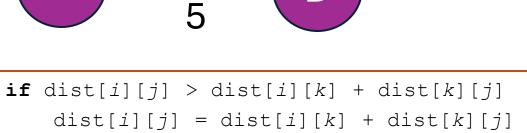
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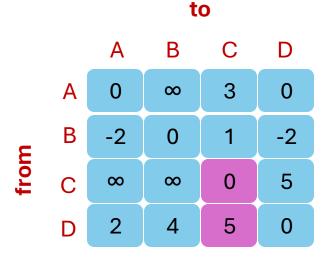
В

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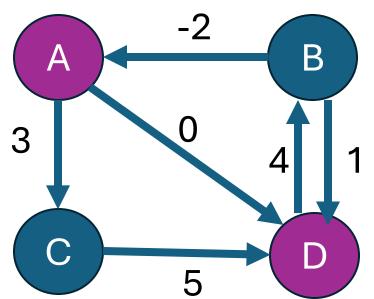
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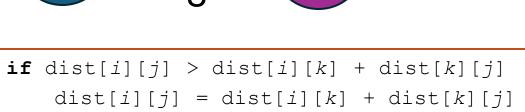
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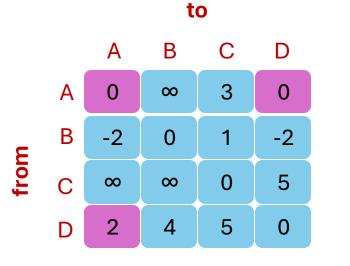
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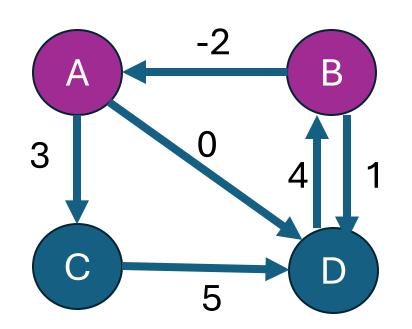
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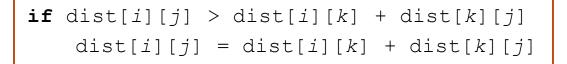
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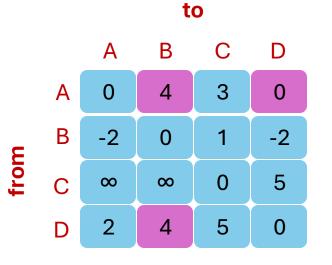
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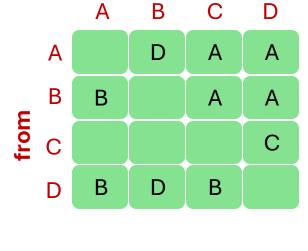
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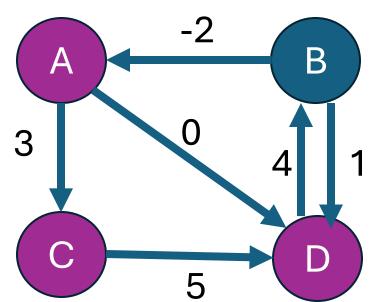


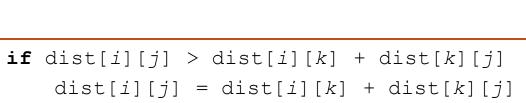


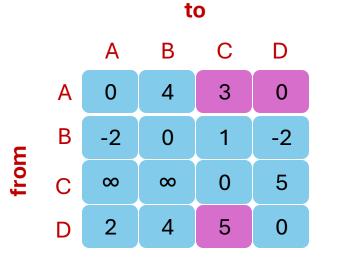


$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $j = 1$ 2 3 4

YES!!!!!!







Α

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Α

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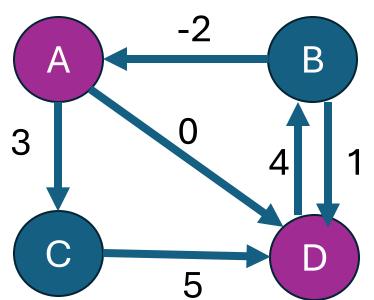
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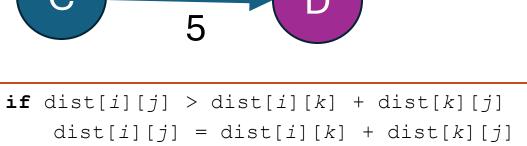
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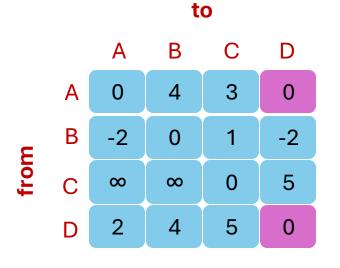
В

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$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

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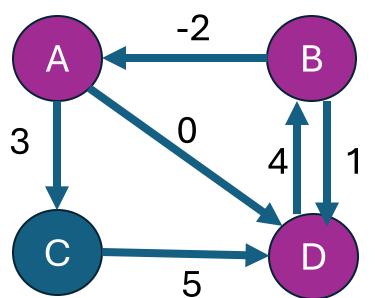
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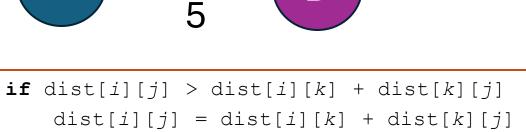
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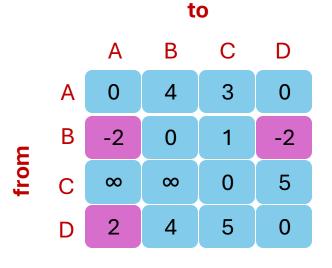
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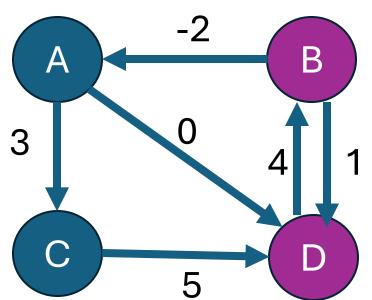
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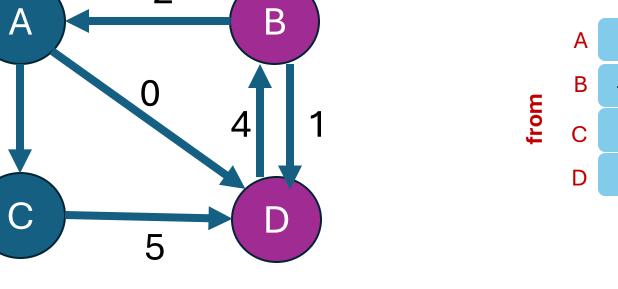
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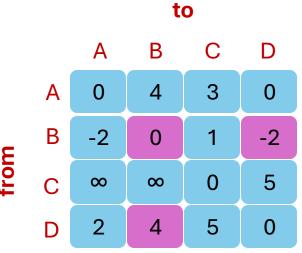
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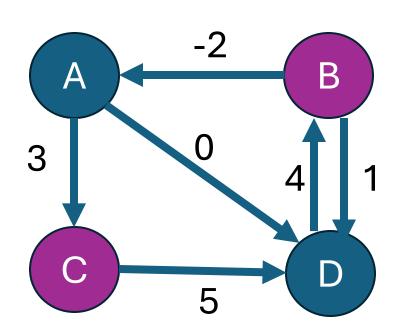
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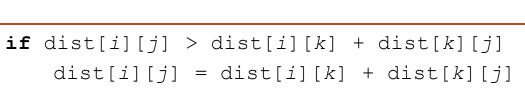
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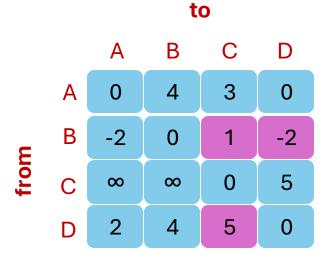
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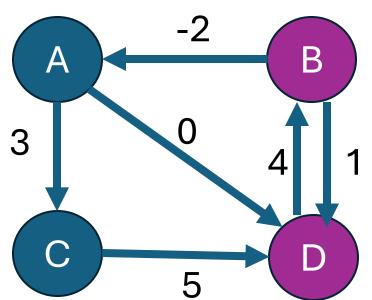
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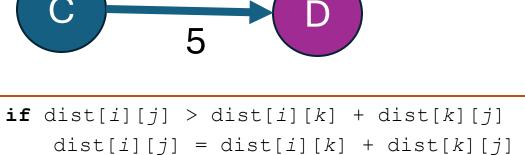


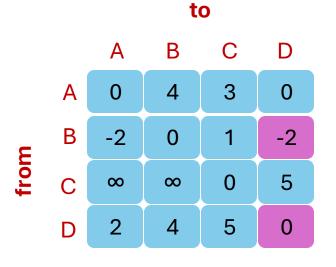




$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4







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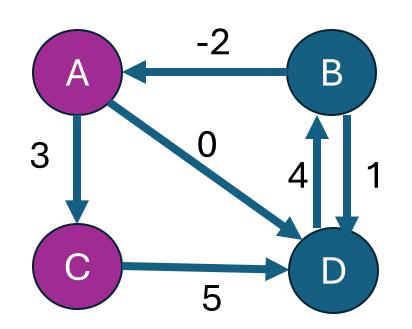
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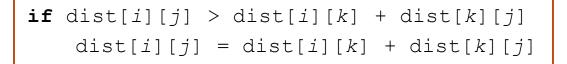
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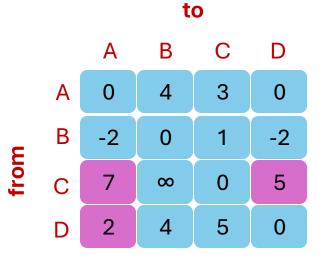
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

YES!!!!!!

to

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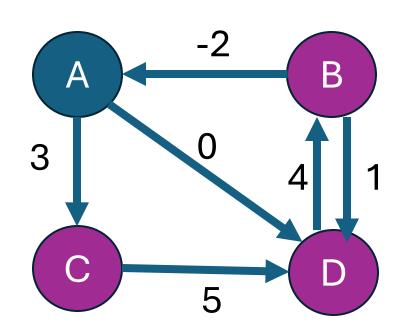
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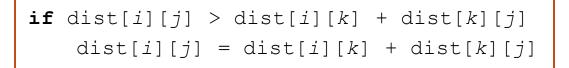
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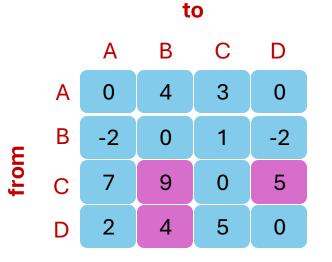
В

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YES!!!!!!

to

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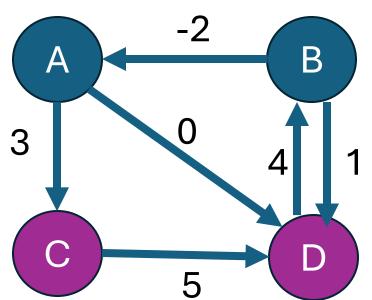
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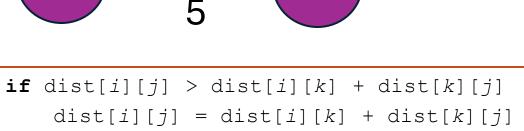
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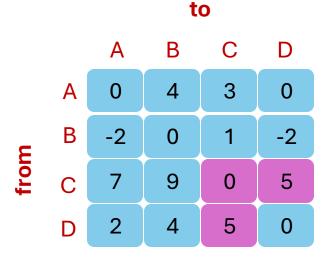
В

В

D







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

Α

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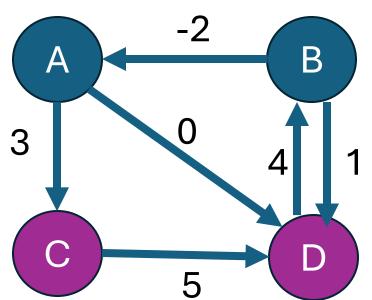
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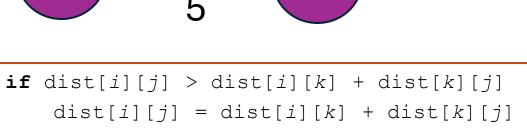
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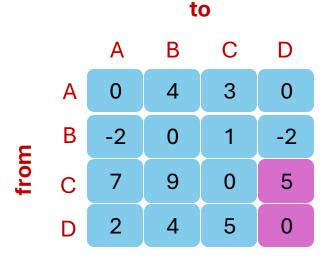
В

В

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$$k = 1 \quad 2 \quad 3 \quad 4$$
 $i = 1 \quad 2 \quad 3 \quad 4$
 $i = 1 \quad 2 \quad 3 \quad 4$

Α

В

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Α

В

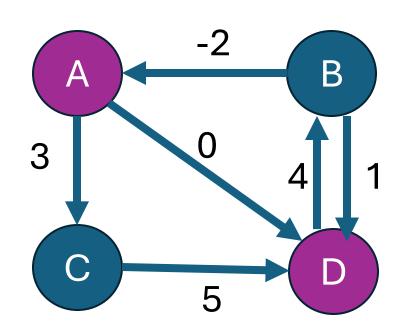
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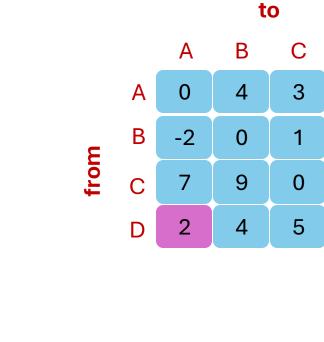
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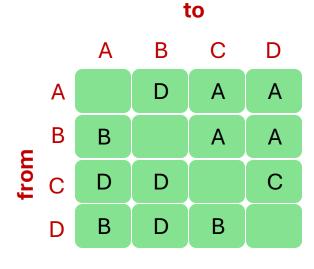
В

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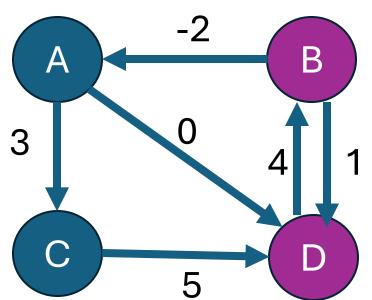


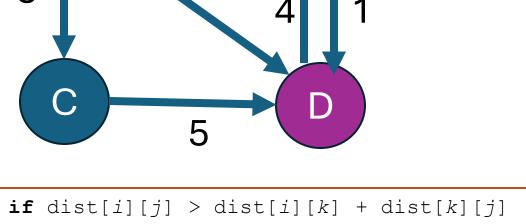




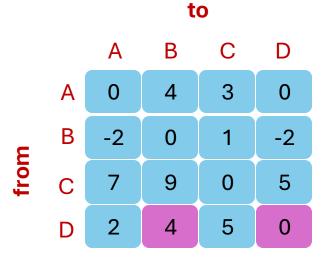
$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

-2



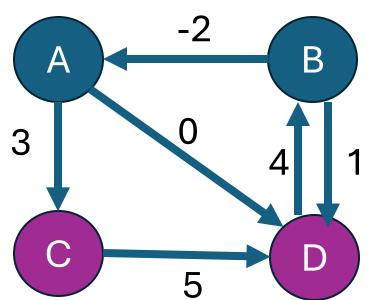


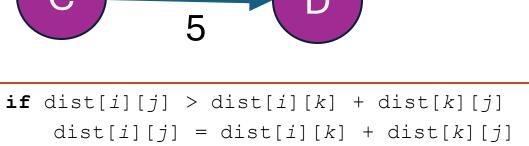
dist[i][j] = dist[i][k] + dist[k][j]

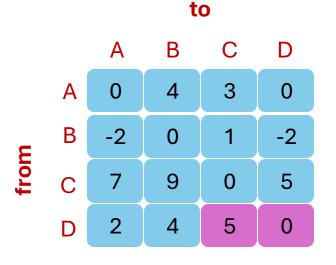


to

$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4







$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

```
DC=5 > DD=0 + DC=5

if dist[4][3] > dist[4][4] + dist[4][3]

dist[4][3] = dist[4][4] + dist[4][3]
```

Α

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Α

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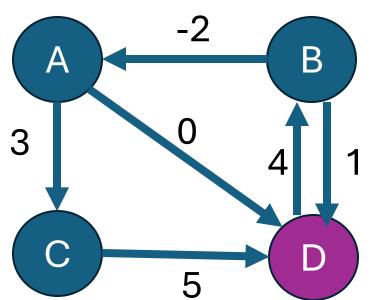
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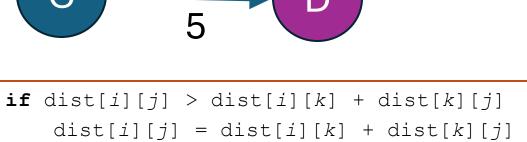
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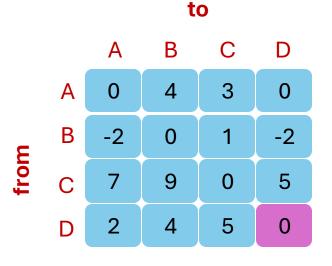
В

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$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

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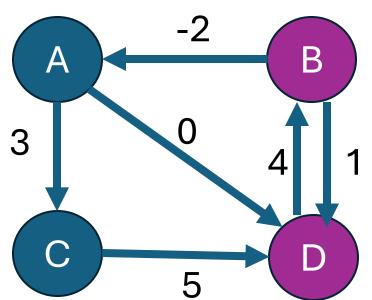
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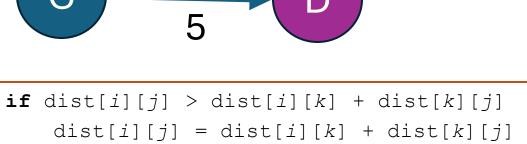
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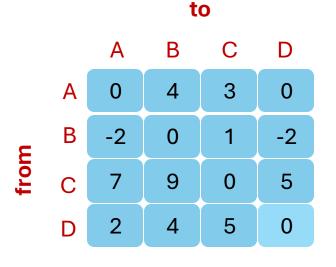
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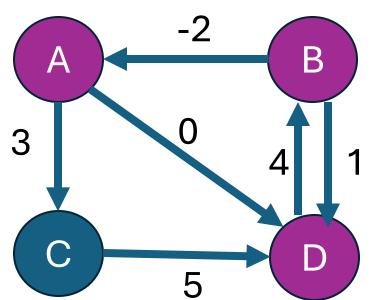
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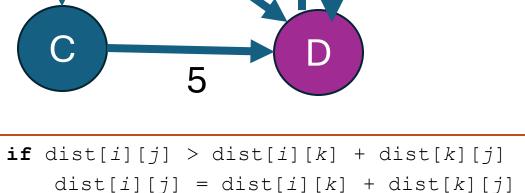
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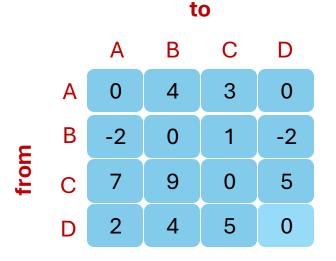
В

В

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$$k = 1$$
 2 3 4
 $i = 1$ 2 3 4
 $i = 1$ 2 3 4

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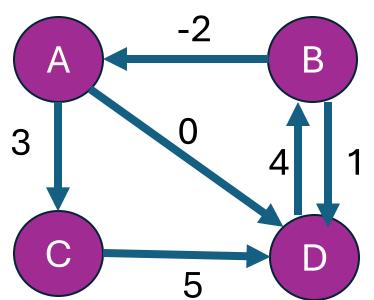
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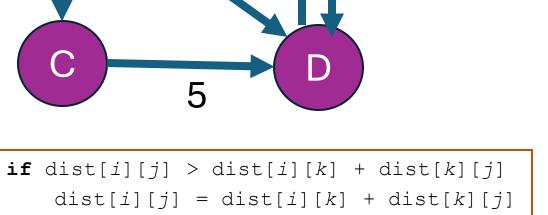
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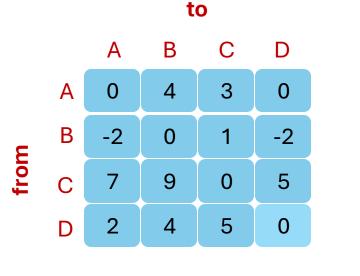
В

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D

```
Johnson(G)
create G' where G'.V = G.V + \{s\}
G'.E = G.E + ((s, u) \text{ for } u \text{ in } G.V),
and weight(s, u) = 0 for u in G.V
if Bellman-Ford(s) == False
     return "The input graph has a negative weight cycle"
else:
     for vertex v in G'.V:
          h(v) = distance(s, v) computed by Bellman-Ford
     for edge (u, v) in G'.E:
          weight (u, v) = weight(u, v) + h(u) - h(v)
D = new matrix of distances initialized to infinity
for vertex u in G.V:
     run Dijkstra(G, weight', u) to compute distance'(u, v) for all v in G.V
     for each vertex v in G.V:
               D (u, v) = distance'(u, v) + h(v) - h(u) return D
```

Johnson(G)

```
create G' where G'.V = G.V + \{s\}
G'.E = G.E + ((s, u) for u in G.V),
and weight(s, u) = 0 for u in G.V
```

. Adding a new vertex, ss, to the graph and connecting it to all other vertices with a zero weight edge

```
if Bellman-Ford(s) == False
    return "The input graph has a negative weight cycle"
else:
    for vertex v in G`.V:
        h(v) = distance(s, v) computed by Bellman-Ford
    for edge (u, v) in G`.E:
        weight`(u, v) = weight(u, v) + h(u) - h(v)
```

D = new matrix of distances initialized to infinity for vertex u in G.V: run Dijkstra(G, weight`, u) to compute distance`(u, v) for all v in G.V

for each vertex v in G.V:

$$D_{u}(u, v) = distance'(u, v) + h(v) - h(u) return D$$

```
Johnson(G)
create G' where G'.V = G.V + \{s\}
G'.E = G.E + ((s, u) for u in G.V),
and weight(s, u) = 0 for u in G.V
```

```
if Bellman-Ford(s) == False
    return "The input graph has a negative weight cycle"
else:
    for vertex v in G`.V:
        h(v) = distance(s, v) computed by Bellman-Ford
    for edge (u, v) in G`.E:
        weight`(u, v) = weight(u, v) + h(u) - h(v)
```

D = new matrix of distances initialized to infinity for vertex u in G.V:

run Dijkstra(G, weight', u) to compute distance'(u, v) for all v in G.V for each vertex v in G.V:

$$D_{u}(u, v) = distance'(u, v) + h(v) - h(u) return D$$

Reweighting is a process by which edge weight is changed to satisfy two properties.

- 1. For all pairs of vertices u, v in the graph, if a certain path is the shortest path between those vertices before reweighting, it must also be the shortest path between those vertices after reweighting.
- 2. For all edges, (u, v), in the graph, weight(u, v) must be non-negative.

```
Johnson(G)
create G' where G'.V = G.V + {s}
G'.E = G.E + ((s, u) for u in G.V),
and weight(s, u) = 0 for u in G.V

if Bellman-Ford(s) == False
return "The input graph has a negative weight cycle"
else:
for vertex v in G'.V:
h(v) = distance(s, v) computed by Bellman-Ford
for edge (u, v) in G'.E:
weight'(u, v) = weight(u, v) + h(u) - h(v)
```

Finally, Dijkstra's algorithm is run on all vertices to find the shortest path. This is possible because the weights have been transformed into non-negative weights.

D = new matrix of distances initialized to infinity
for vertex u in G.V:
run Dijkstra(G, weight`, u) to compute distance`(u, v) for all v in G.V
for each vertex v in G.V: $D_{(u, v)} = distance`(u, v) + h(v) - h(u) return D$

```
Johnson(G)
create G' where G'.V = G.V + \{s\}
                                                                            O(V)
G'.E = G.E + ((s, u) \text{ for } u \text{ in } G.V),
and weight(s, u) = 0 for u in G.V
if Bellman-Ford(s) == False
     return "The input graph has a negative weight cycle"
else:
     for vertex v in G'.V:
          h(v) = distance(s, v) computed by Bellman-Ford
     for edge (u, v) in G'.E:
          weight (u, v) = weight(u, v) + h(u) - h(v)
D = new matrix of distances initialized to infinity
for vertex u in G.V:
     run Dijkstra(G, weight', u) to compute distance'(u, v) for all v in G.V
     for each vertex v in G.V:
               D (u, v) = distance'(u, v) + h(v) - h(u) return D
```

```
Johnson(G)
create G' where G'.V = G.V + \{s\}
                                                                            O(V)
G'.E = G.E + ((s, u) \text{ for } u \text{ in } G.V),
and weight(s, u) = 0 for u in G.V
if Bellman-Ford(s) = False
     return "The input graph has a negative weight cycle"
                                                                            O(VE)
else:
     for vertex v in G'.V:
          h(v) = distance(s, v) computed by Bellman-Ford
     for edge (u, v) in G'.E:
          weight (u, v) = weight(u, v) + h(u) - h(v)
D = new matrix of distances initialized to infinity
for vertex u in G.V:
     run Dijkstra(G, weight', u) to compute distance'(u, v) for all v in G.V
     for each vertex v in G.V:
               D (u, v) = distance'(u, v) + h(v) - h(u) return D
```

	Average
Johnson's algorithm	$O(V^2log(V) + VE)$

Johnson(G)

```
create G' where G'.V = G.V + \{s\}
G'.E = G.E + ((s, u) \text{ for } u \text{ in } G.V),
and weight(s, u) = 0 for u in G.V
```

O(V)

```
if Bellman-Ford(s) == False
return "The input graph has a negative weight cycle"
else:
```

O(VE)

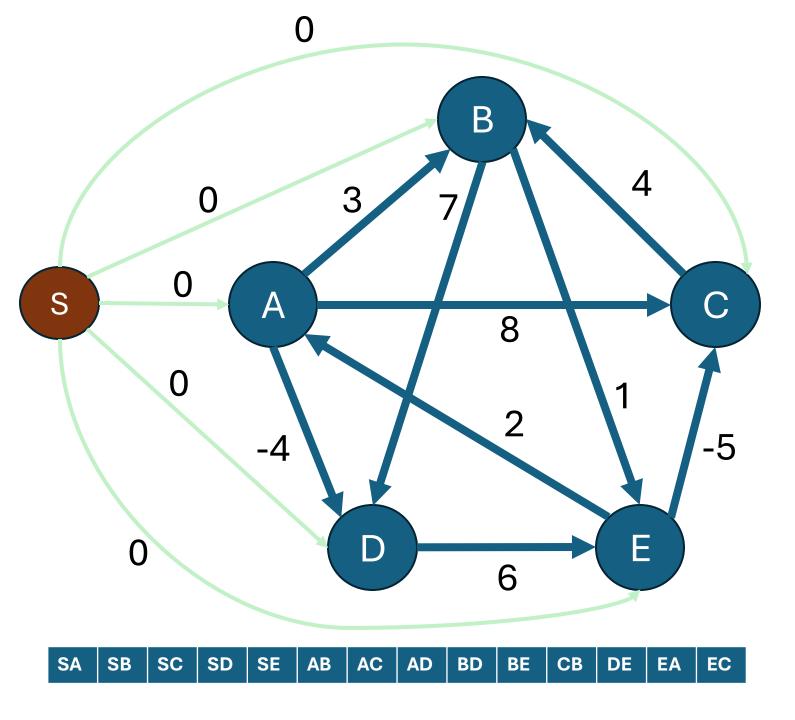
```
for vertex v in G'.V:
 h(v) = distance(s, v) computed by Bellman-Ford
for edge (u, v) in G'.E:
```

weight (u, v) = weight(u, v) + h(u) - h(v)

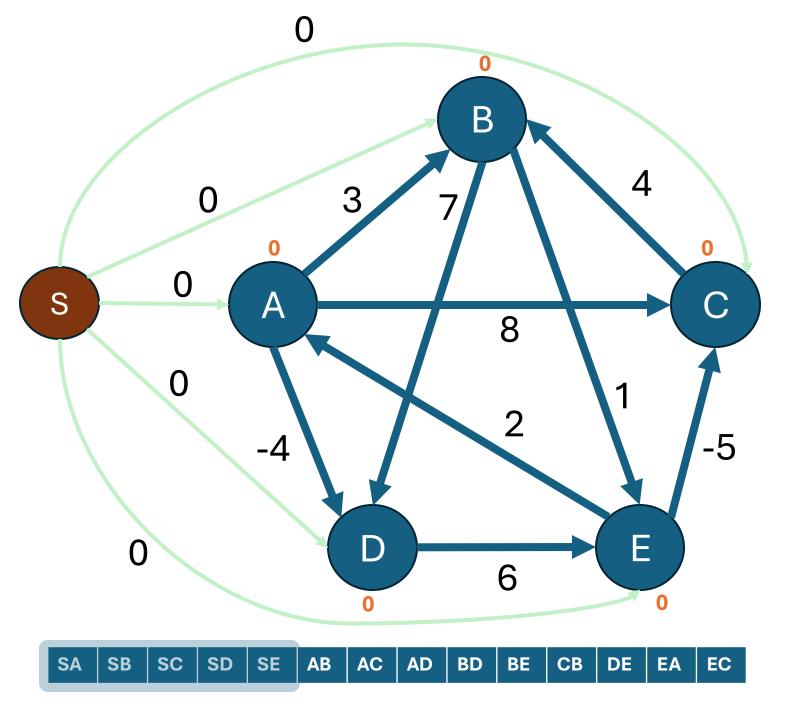
```
D = new matrix of distances initialized to infinity for vertex u in G.V:
```

run Dijkstra(G, weight', u) to compute distance'(u, v) for all v in G.V for each vertex v in G.V:

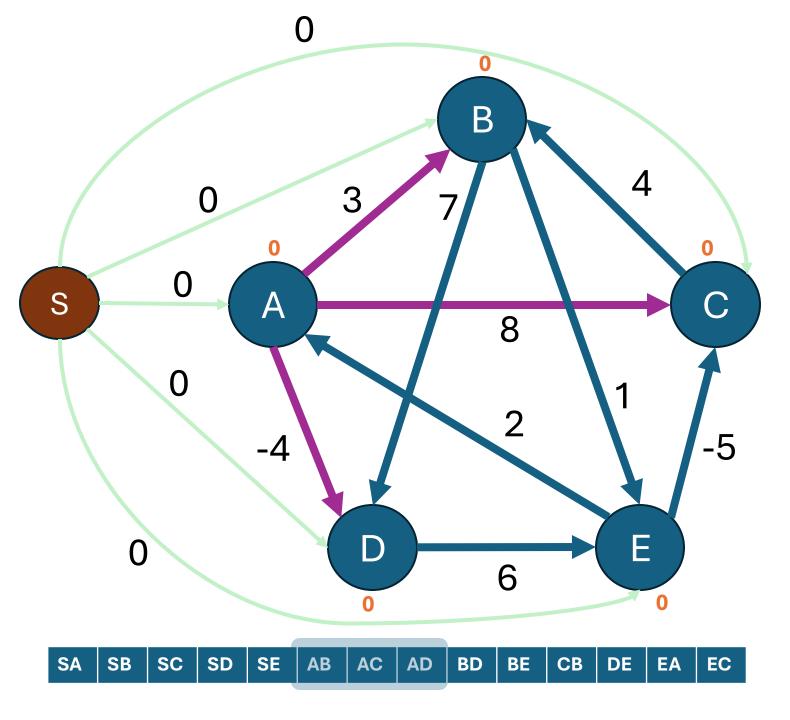
 $D_{u}(u, v) = distance'(u, v) + h(v) - h(u) return D$



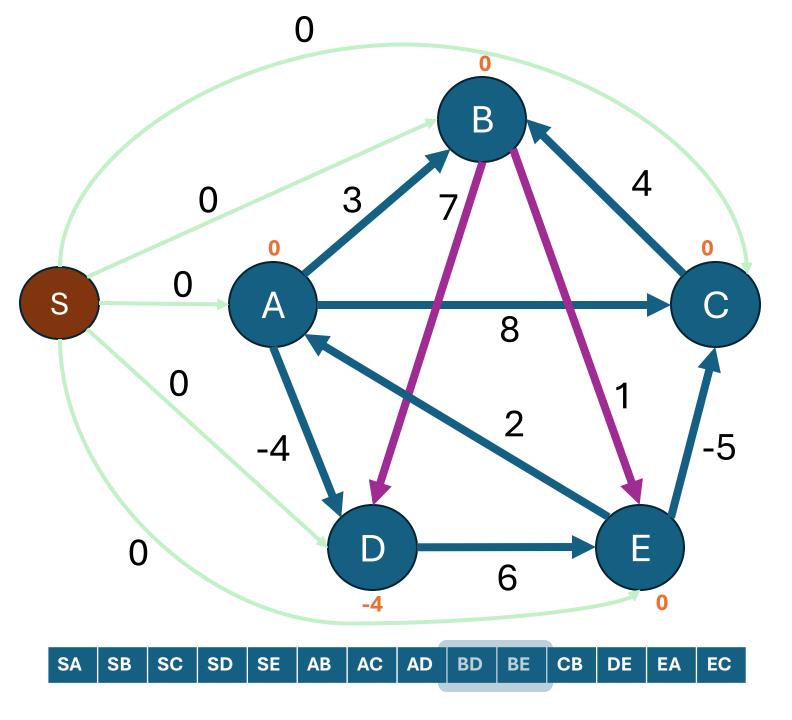
	Cost	Prev
S	0	-
Α	0	S
В	0	S
С	0	S
D	0	S
Е	0	S



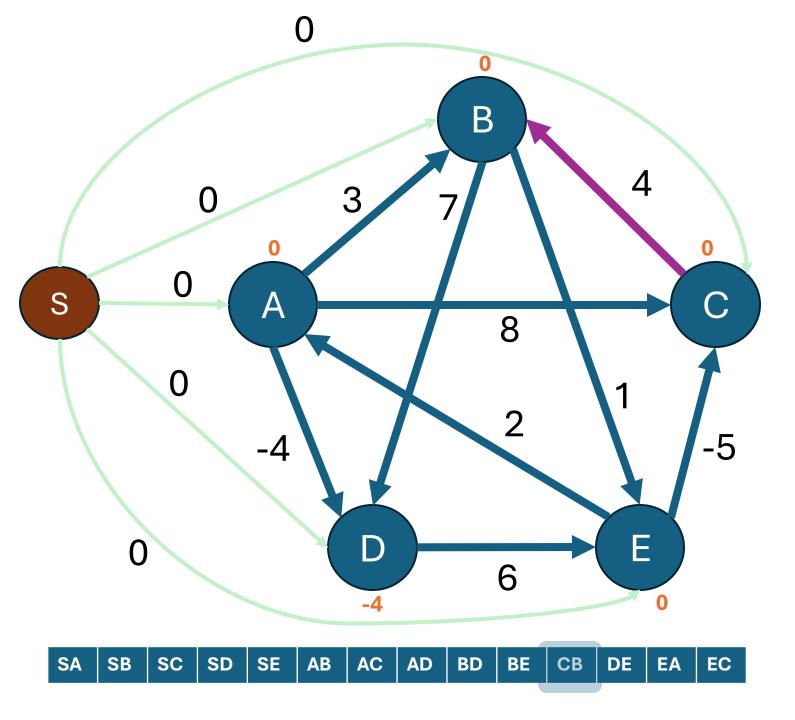
	Cost	Prev
S	0	-
Α	0	S
В	0	S
С	0	S
D	0	S
Е	0	S



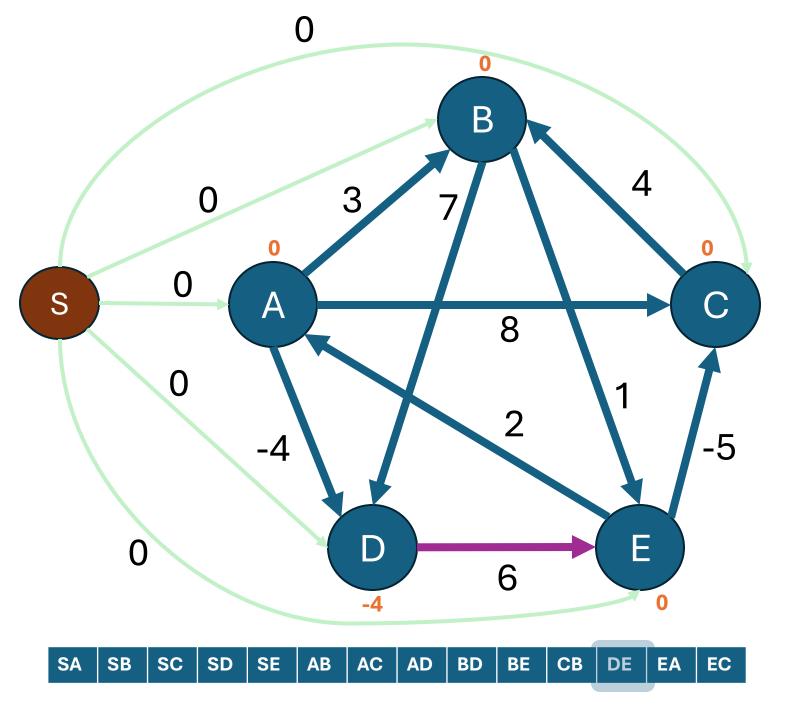
	Cost	Prev
S	0	-
Α	0	S
В	0	S
С	0	S
D	-4	Α
Е	0	S



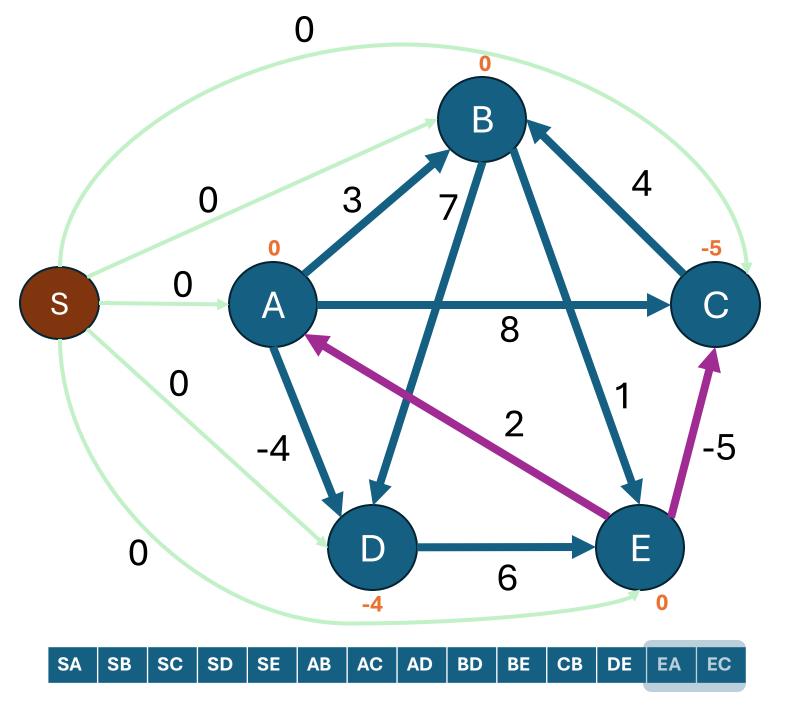
	Cost	Prev
S	0	-
Α	0	S
В	0	S
С	0	S
D	-4	А
Е	0	S



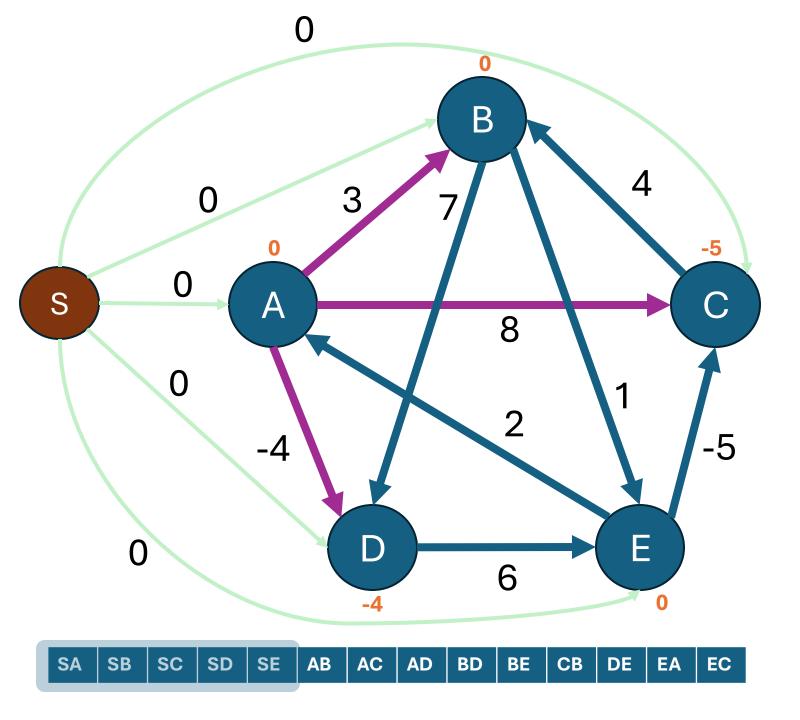
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Α	0	S
В	0	S
С	0	S
D	-4	Α
Е	0	S



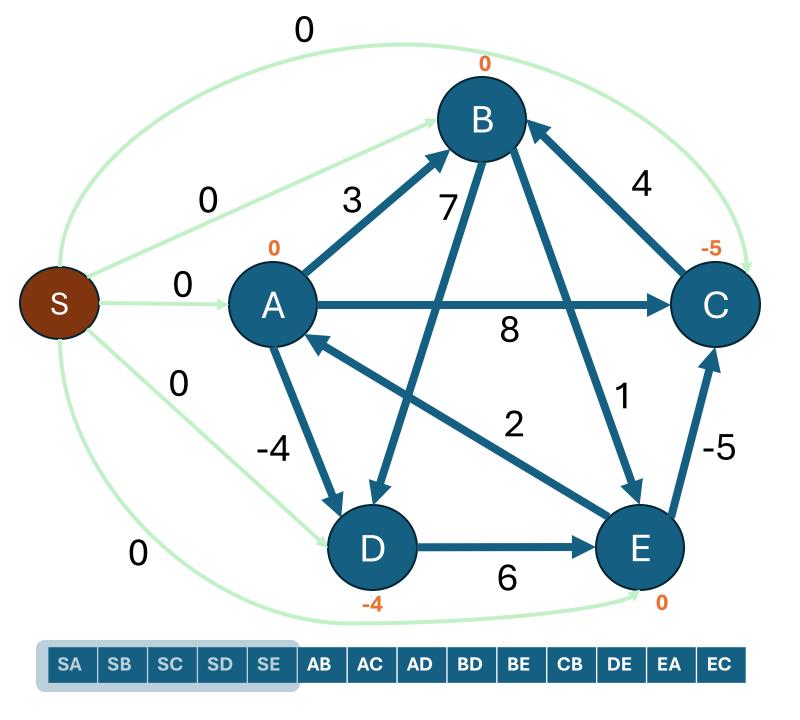
	Cost	Prev
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В	0	S
С	0	S
D	-4	Α
Е	0	S



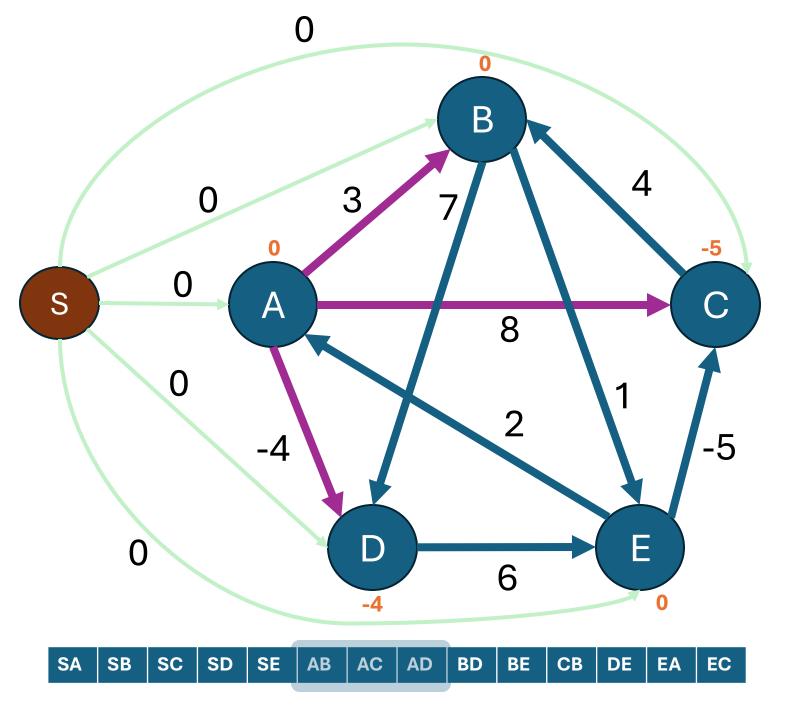
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В	0	S
С	-5	Е
D	-4	Α
Е	0	S



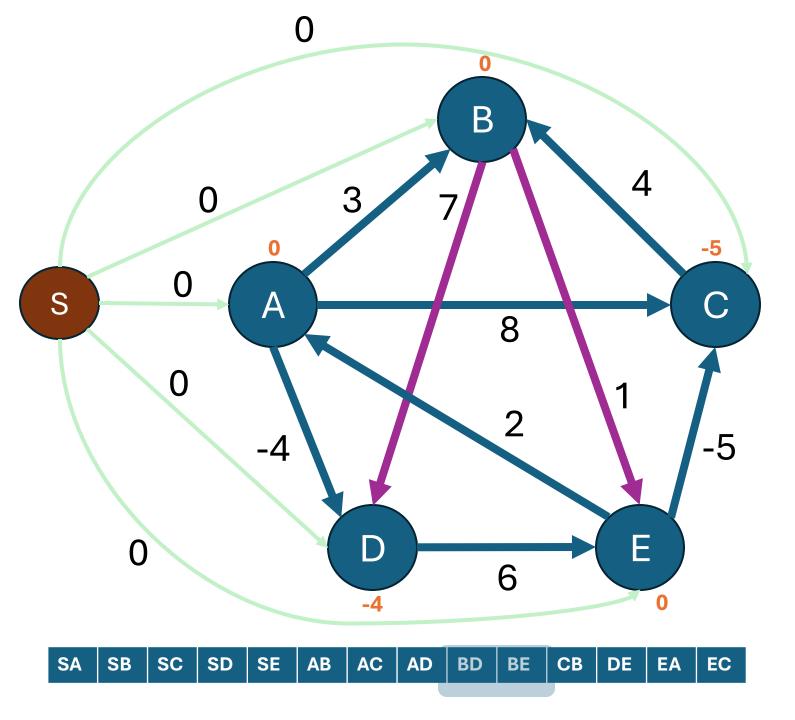
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В	0	S
С	-5	Е
D	-4	Α
Е	0	S



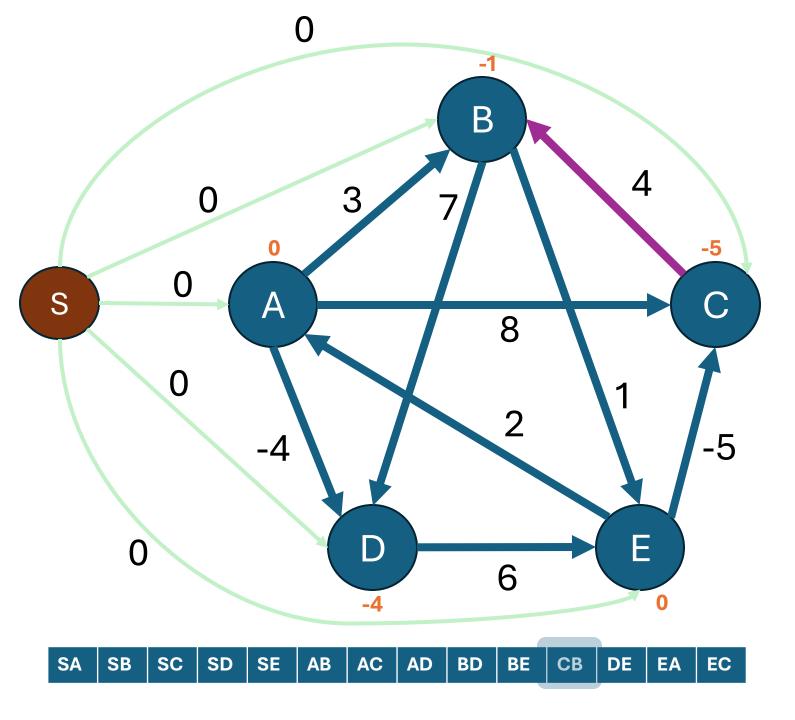
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В	0	S
С	-5	Е
D	-4	Α
Е	0	S



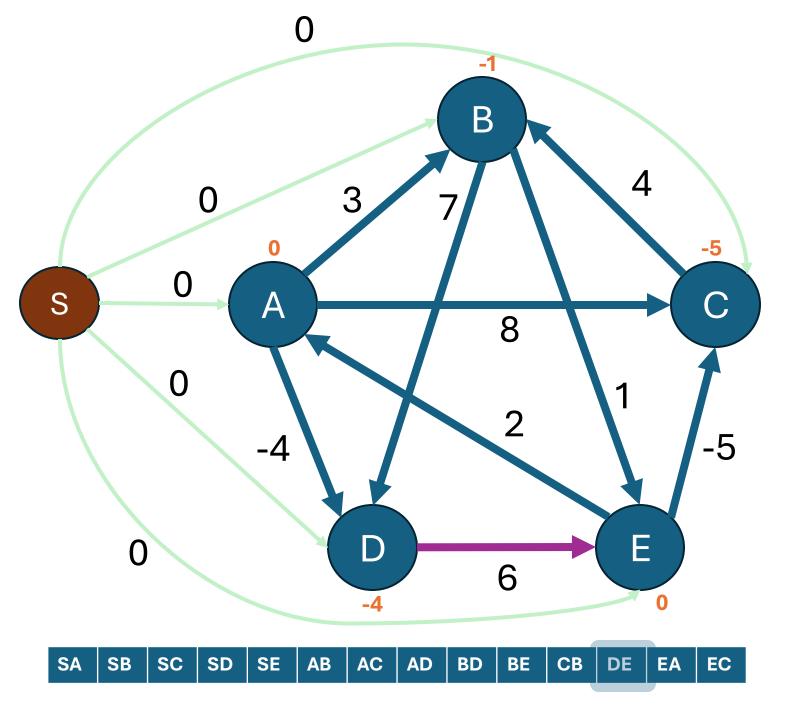
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Α	0	S
В	0	S
С	-5	Е
D	-4	Α
Е	0	S



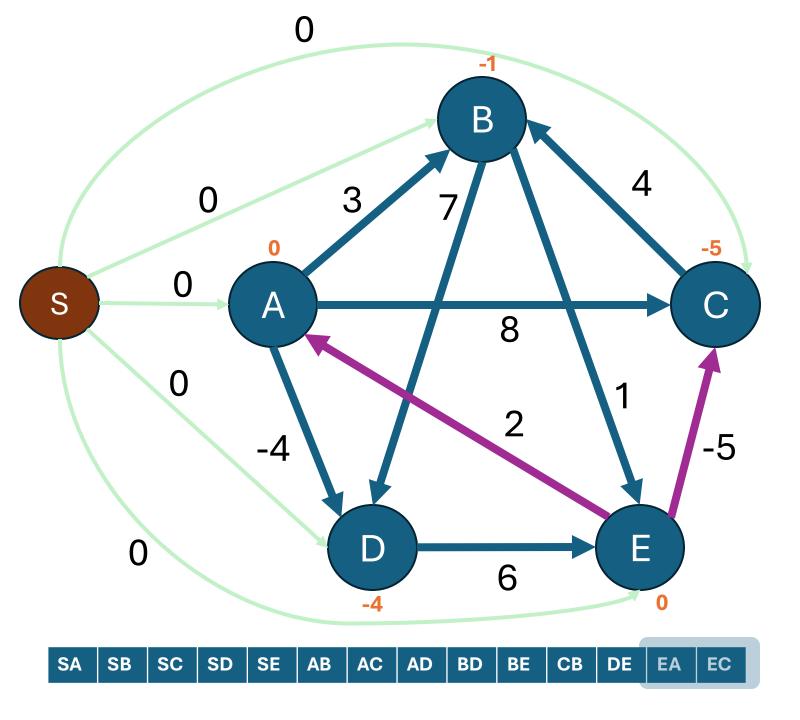
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В	0	S
С	-5	Е
D	-4	Α
Е	0	S



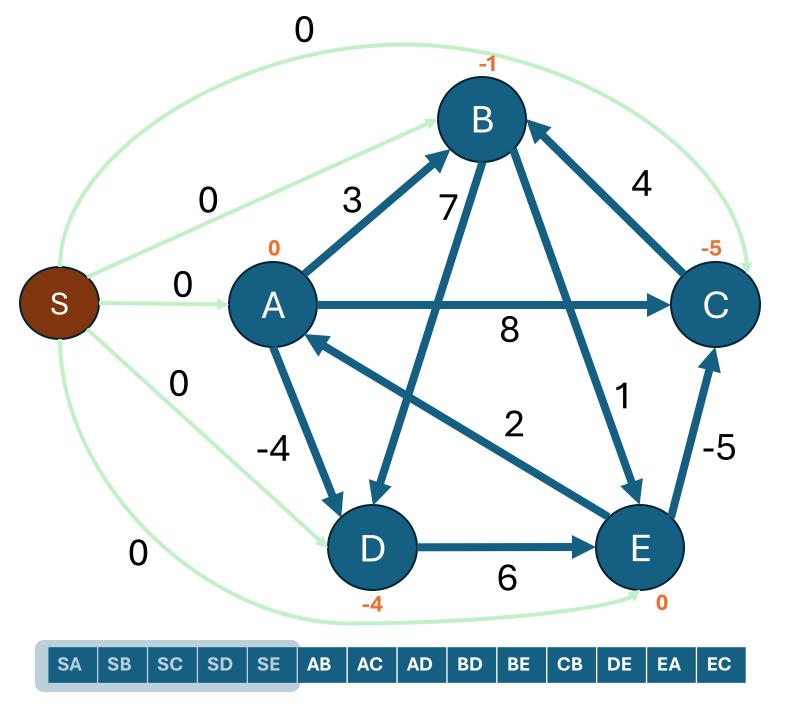
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



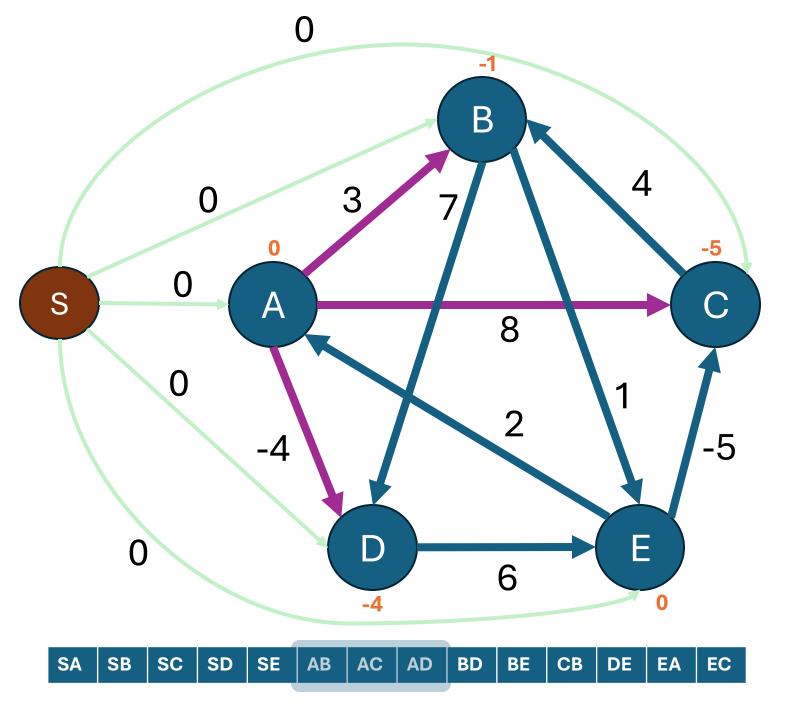
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В	-1	С
С	-5	Е
D	-4	А
Е	0	S



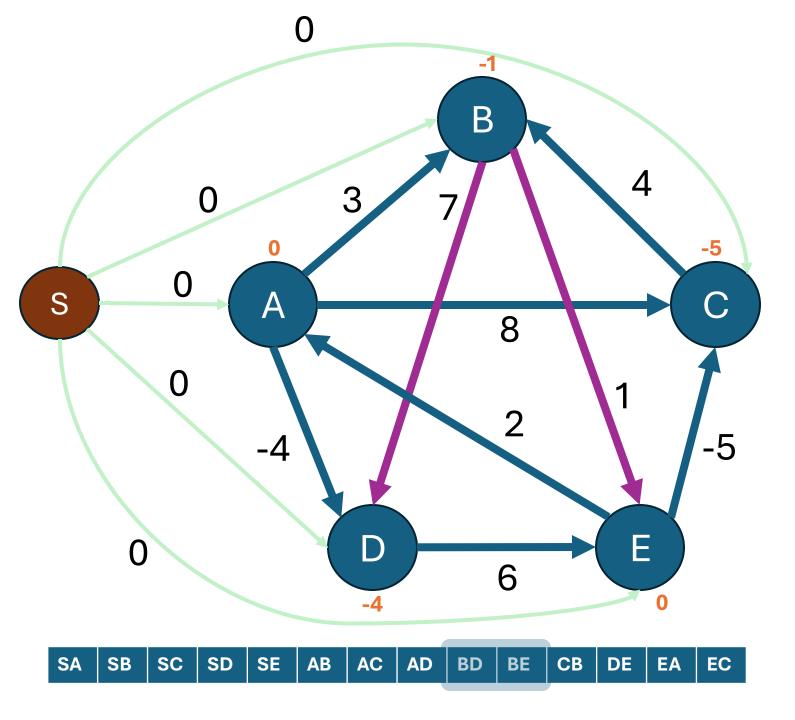
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Α	0	S
В	-1	С
С	-5	Е
D	-4	А
Е	0	S



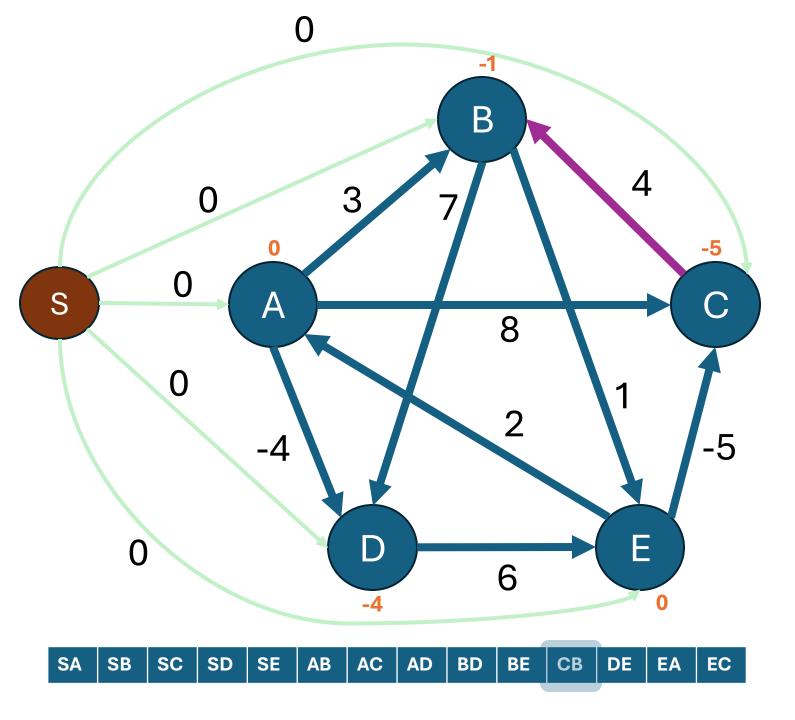
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



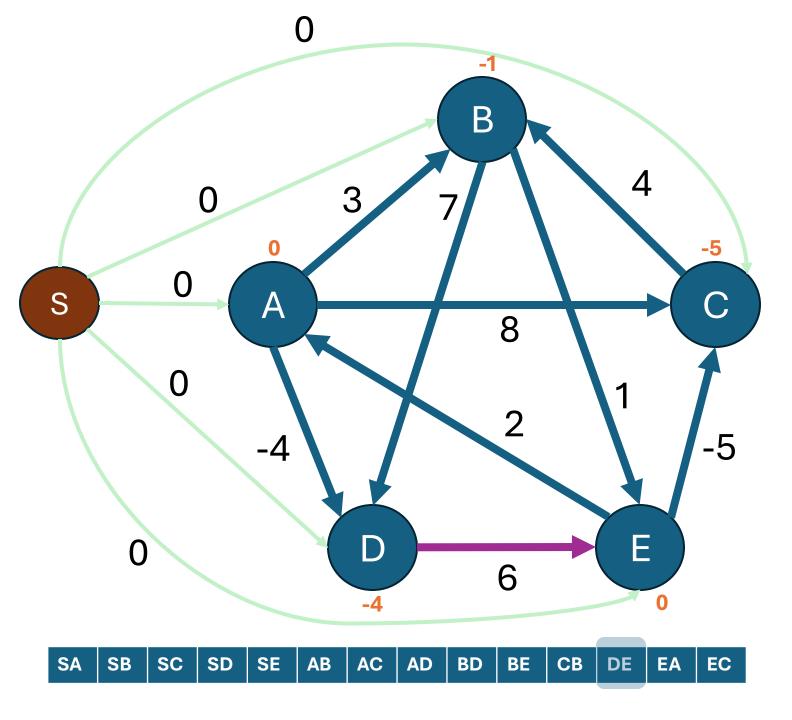
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



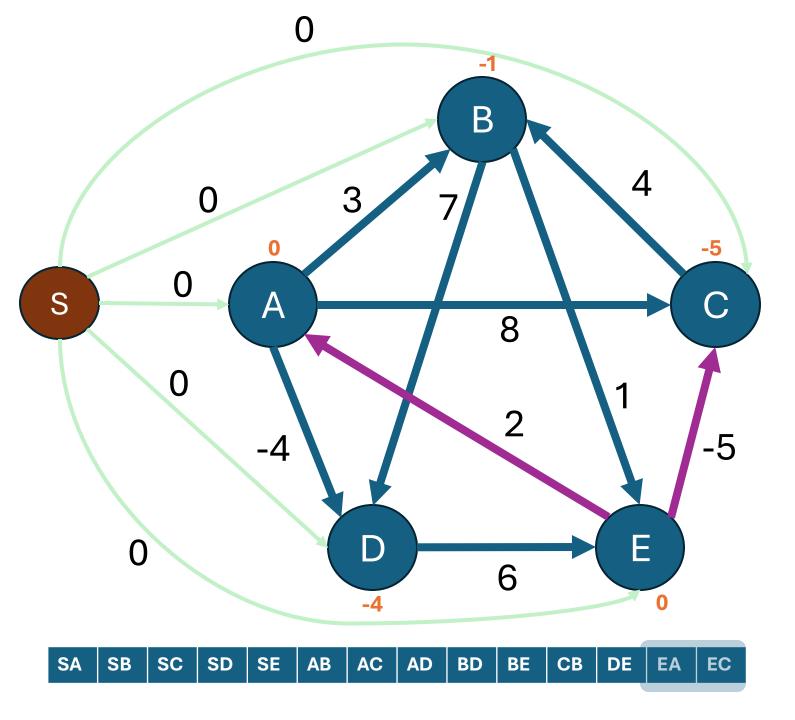
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



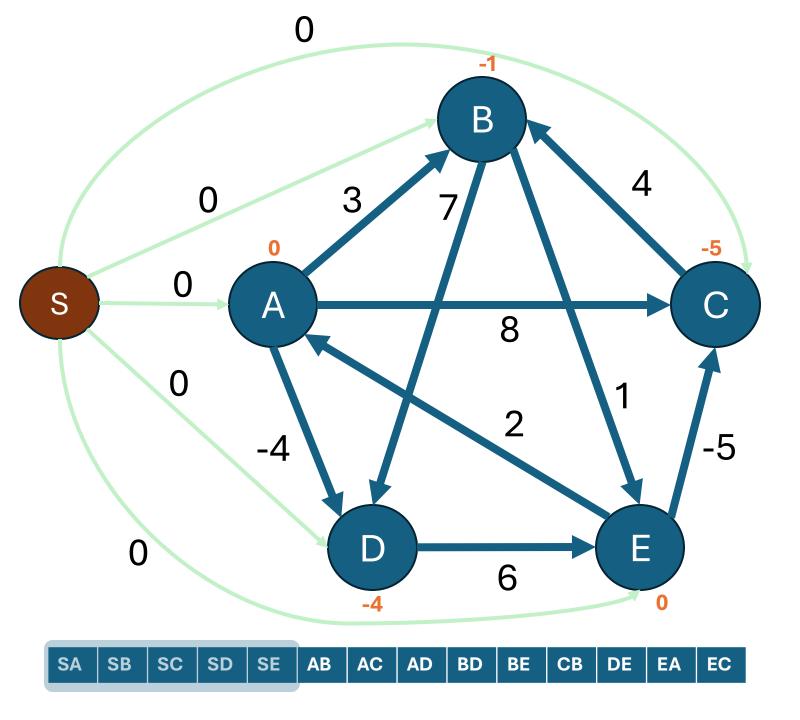
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С	-5	Е
D	-4	А
Е	0	S



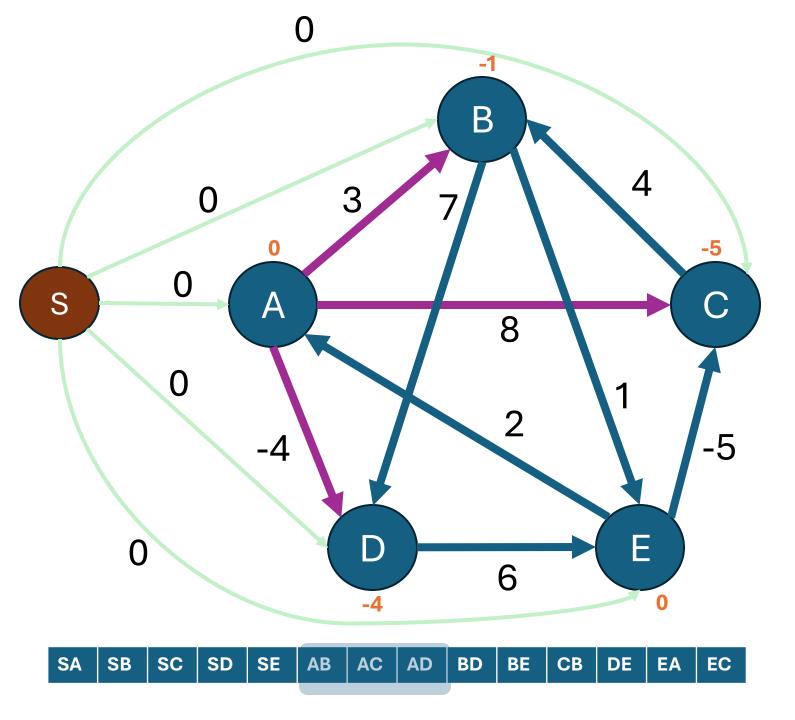
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



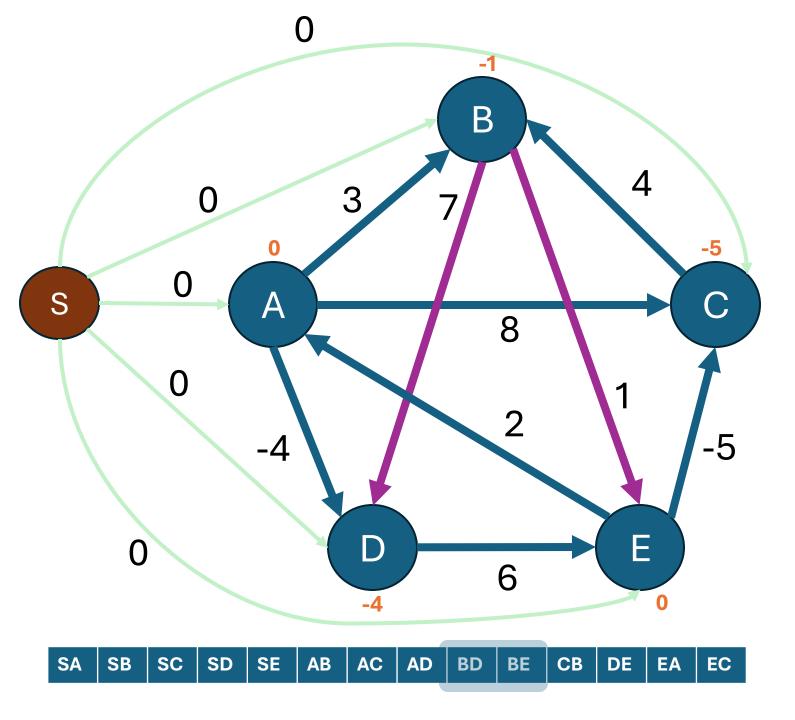
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В	-1	С
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Е	0	S



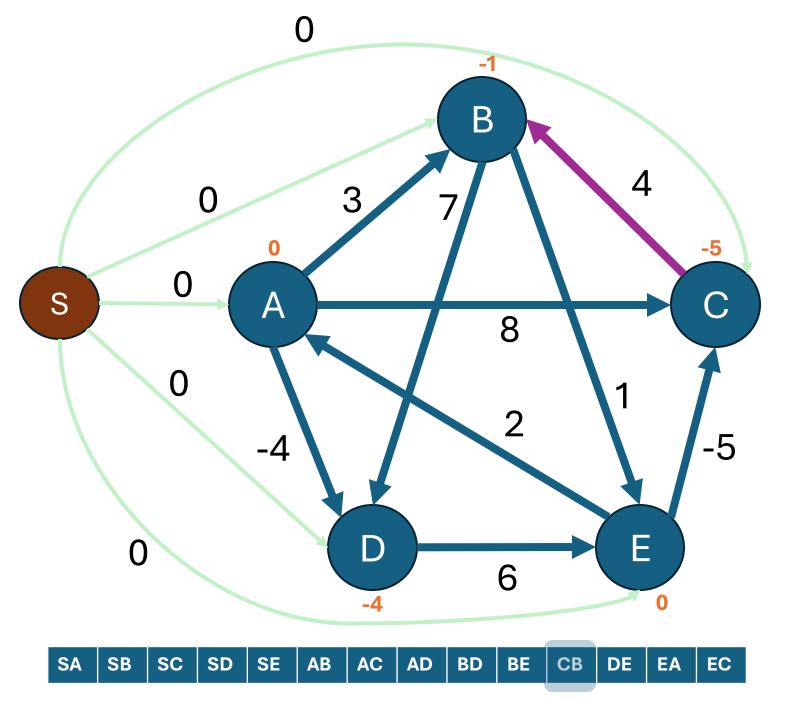
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В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



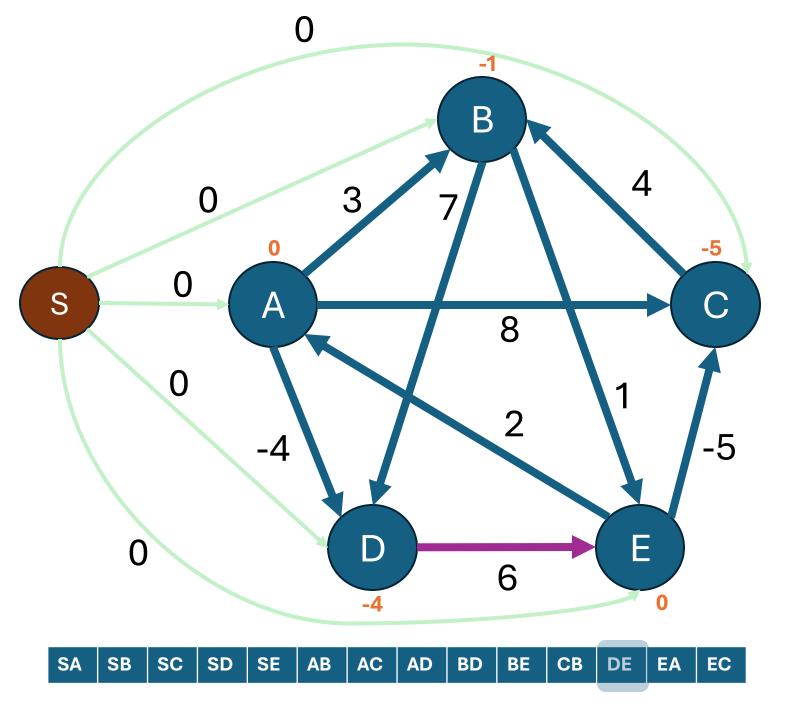
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



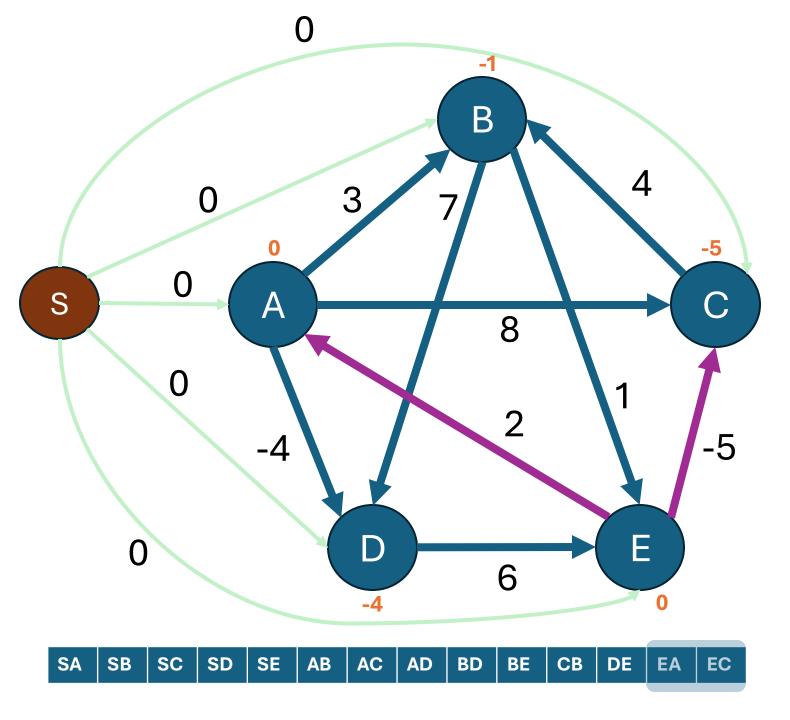
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



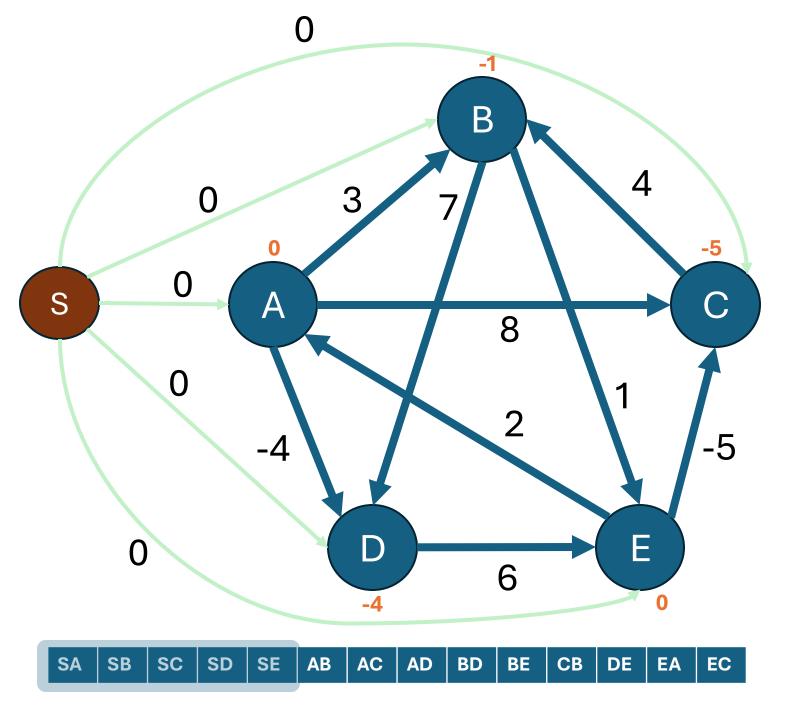
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Α	0	S
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С	-5	Е
D	-4	Α
Е	0	S



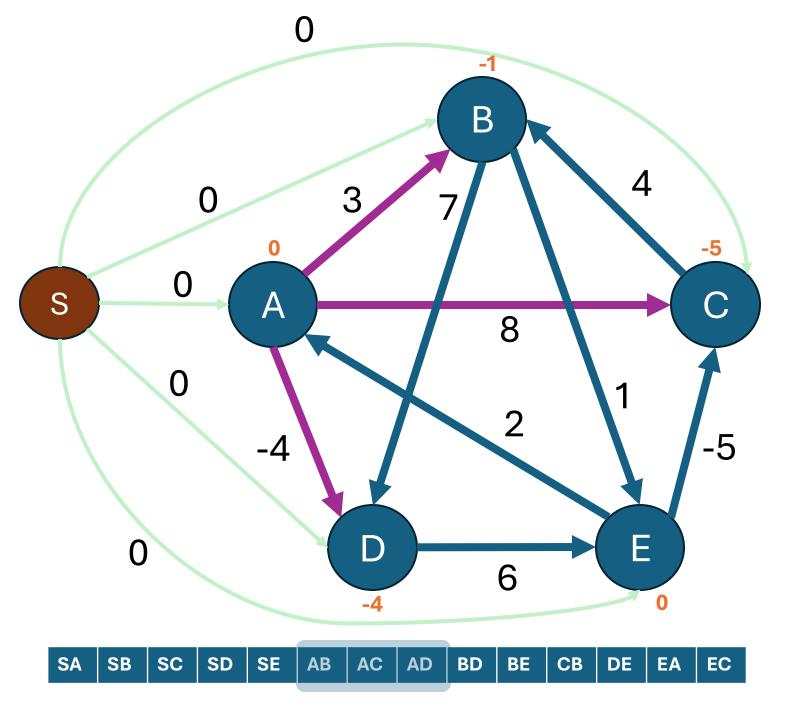
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



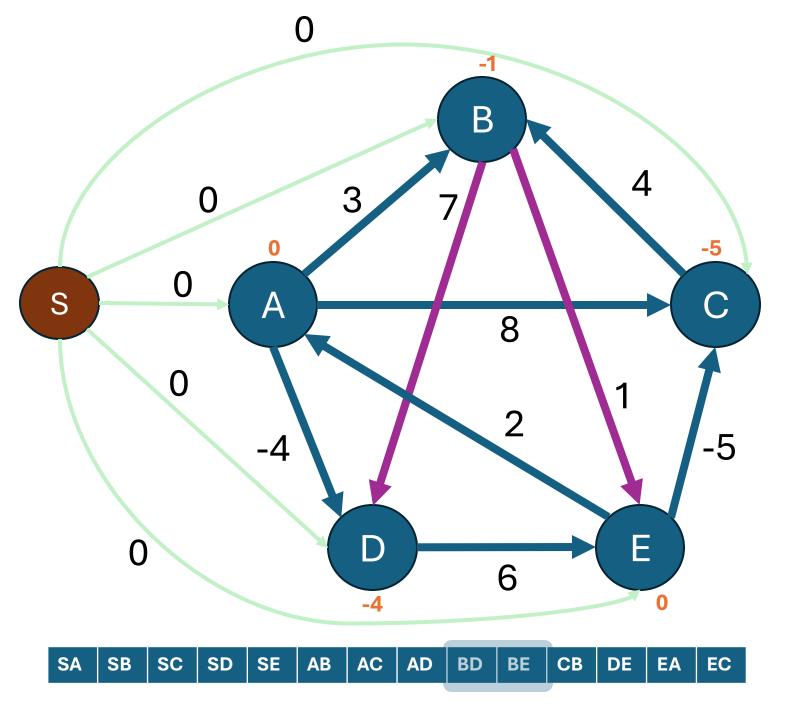
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В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



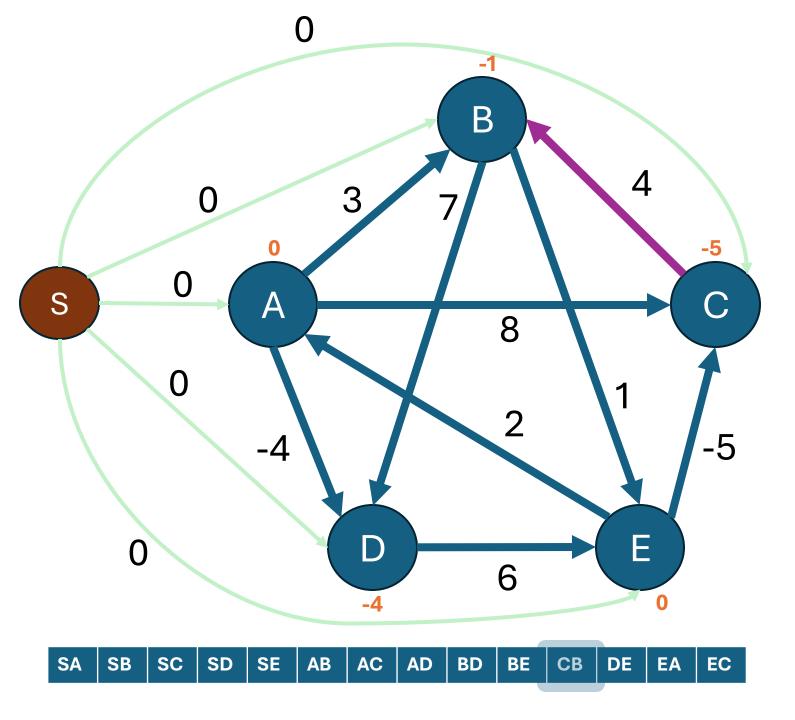
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



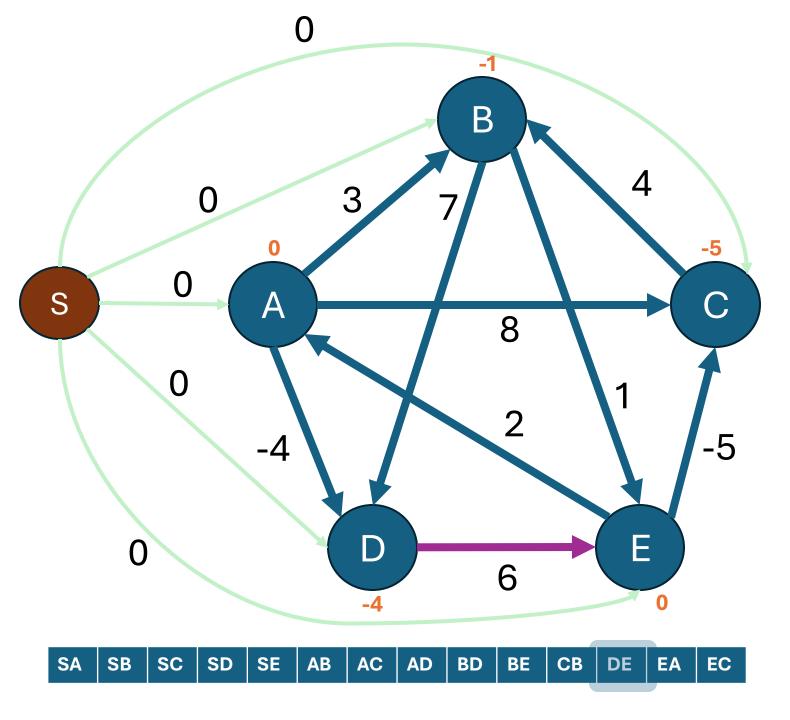
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Α	0	S
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С	-5	Е
D	-4	Α
Е	0	S



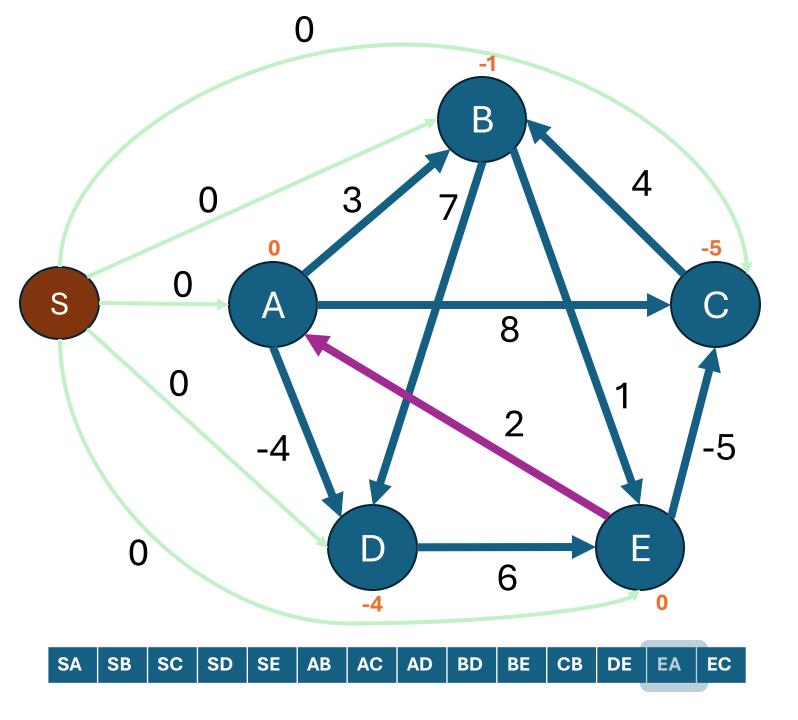
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



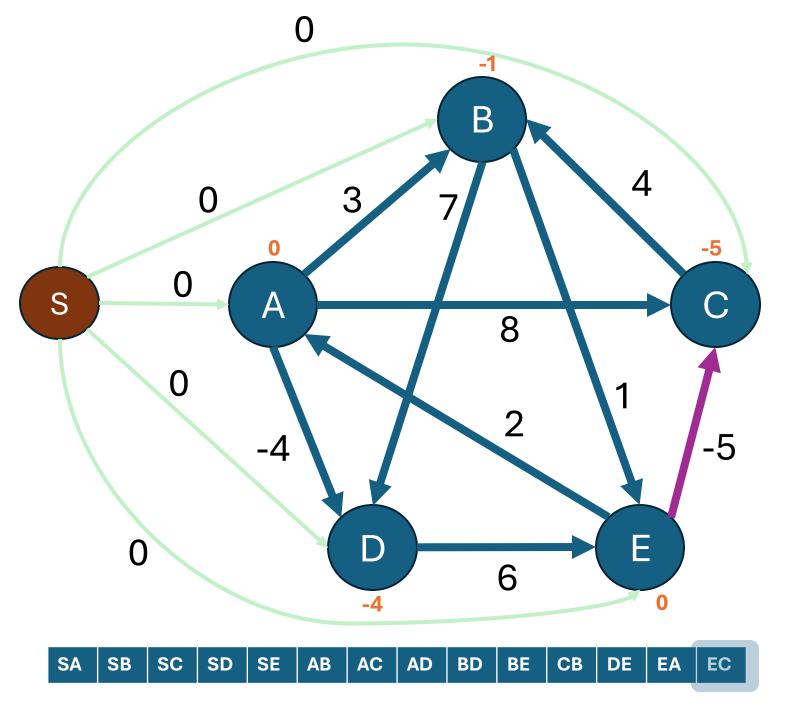
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



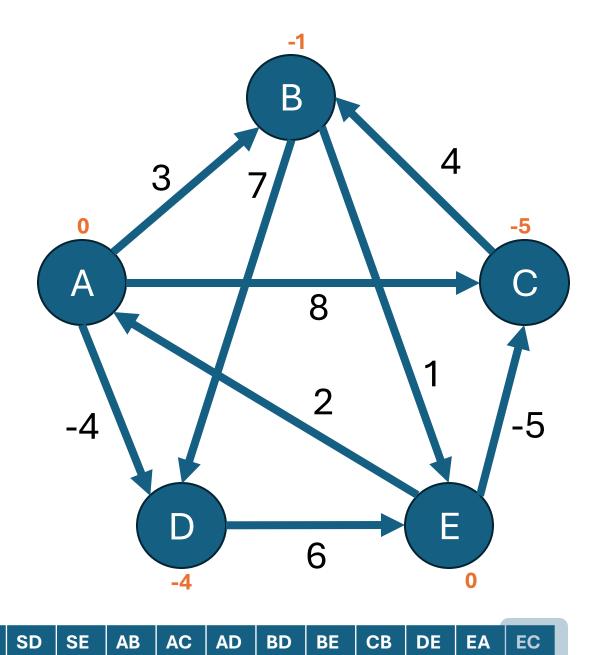
	Cost	Prev
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



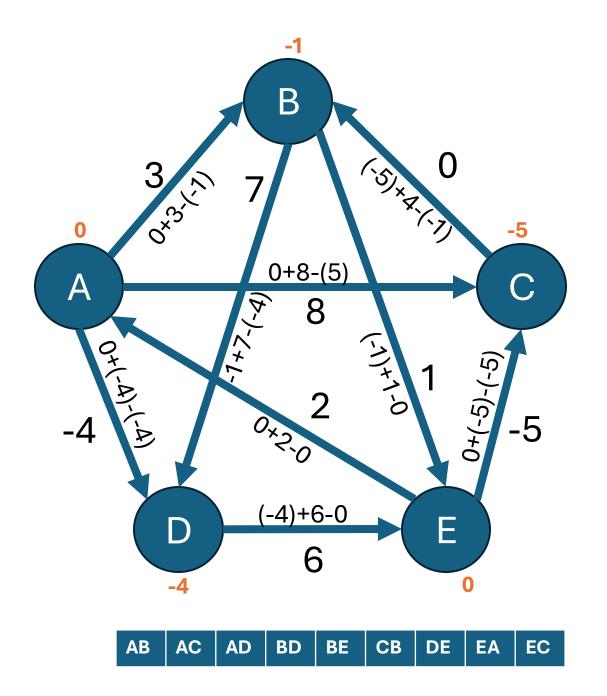
	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S

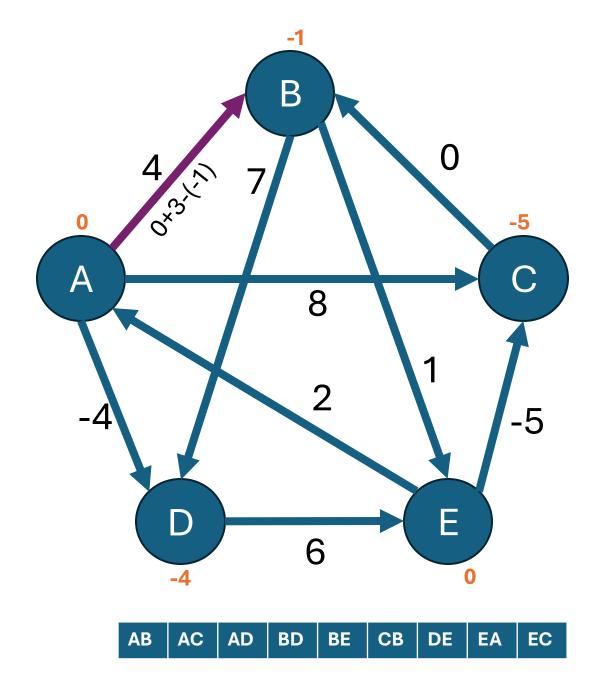


	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S

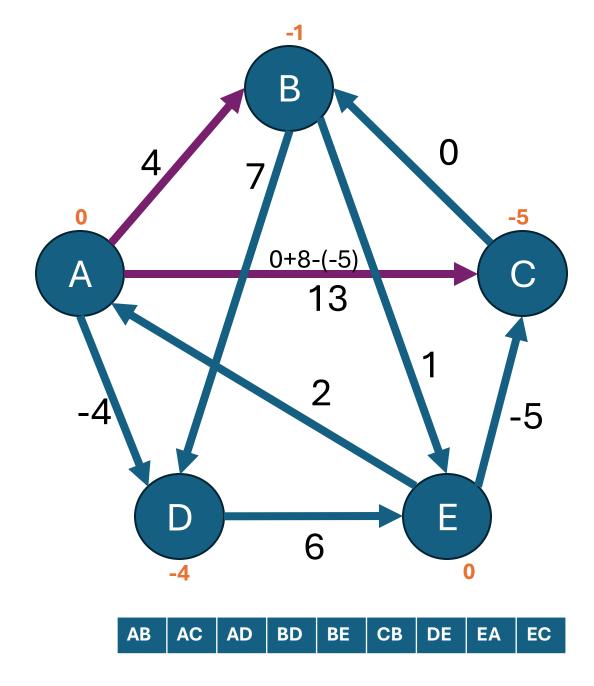


	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S

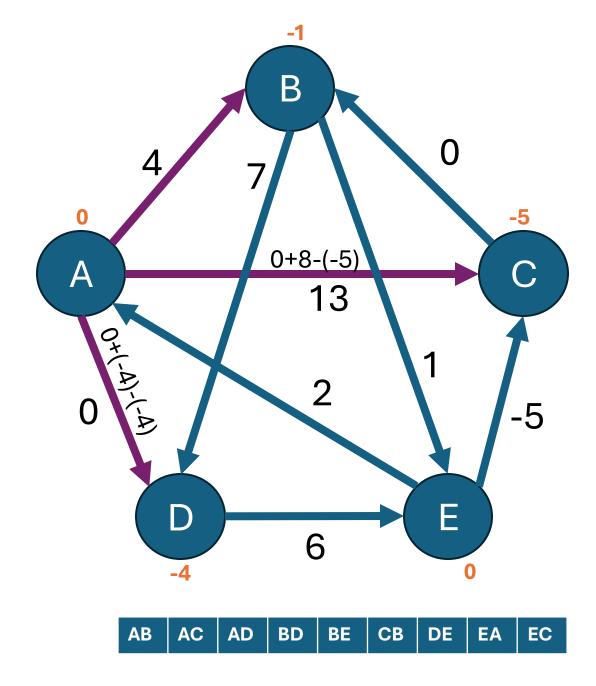
$$W_{new}(u,v) = W(u,v) + h(u) - h(v) >= 0$$



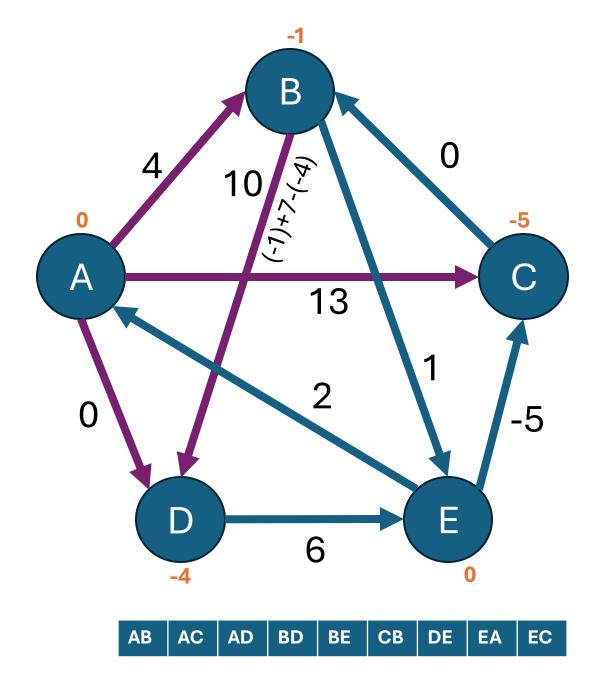
	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



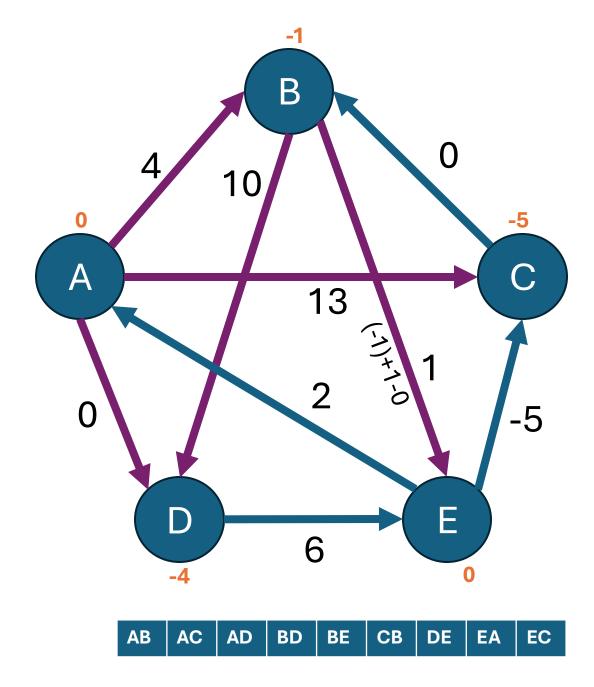
	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



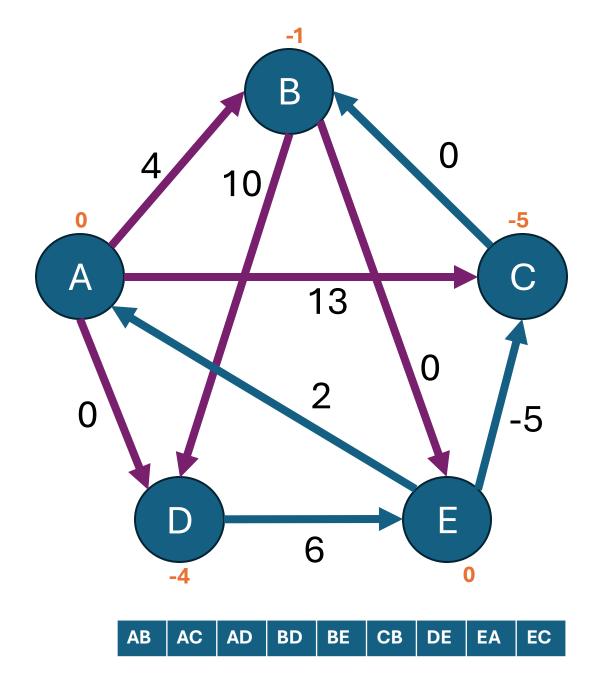
	Cost	Prev
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



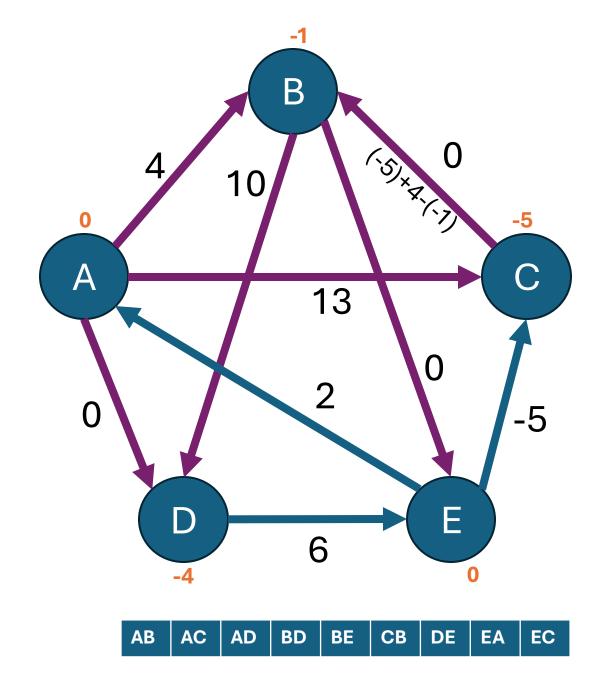
	Cost	Prev
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



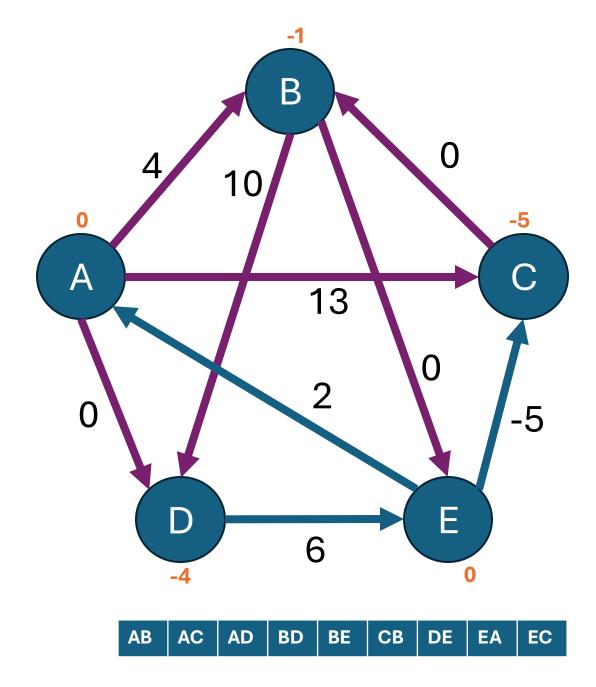
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



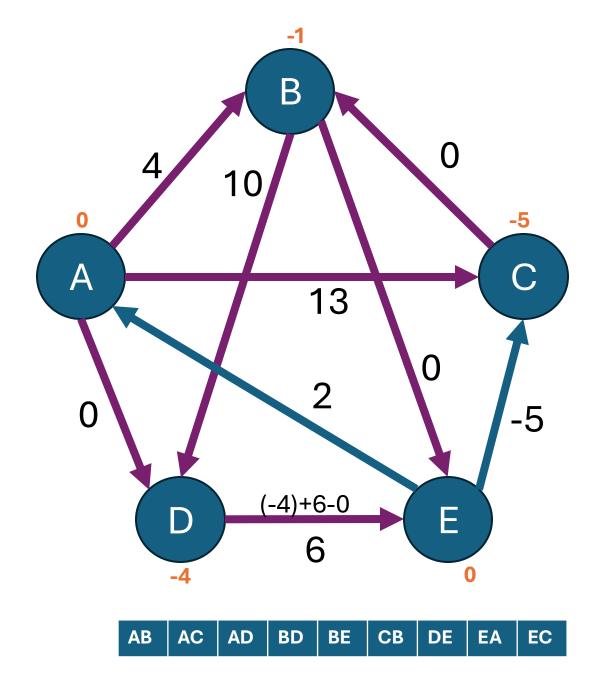
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



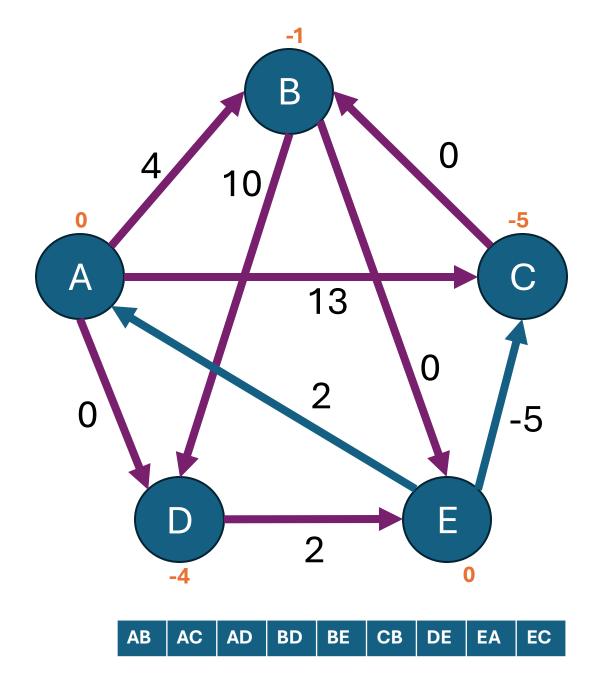
	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	А
Е	0	S



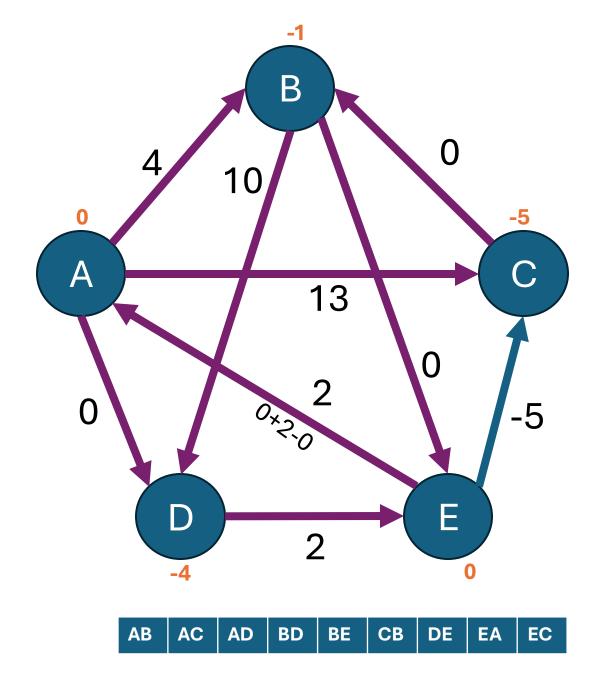
	Cost	Prev
S	0	-
Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



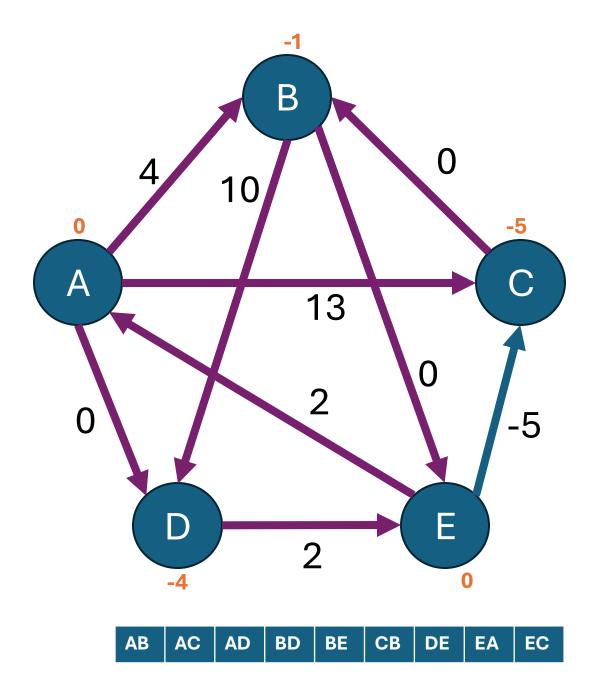
	Cost	Prev
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Α	0	S
В	-1	С
С	-5	Е
D	-4	Α
Е	0	S



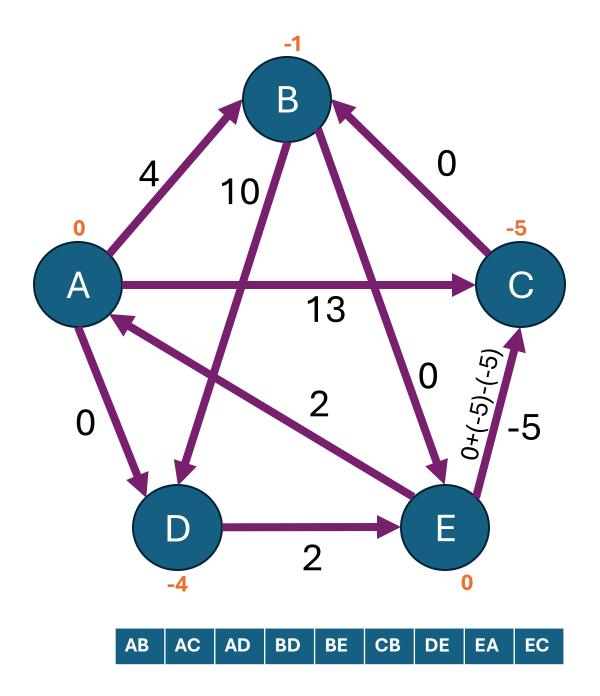
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С	-5	Е
D	-4	Α
Е	0	S



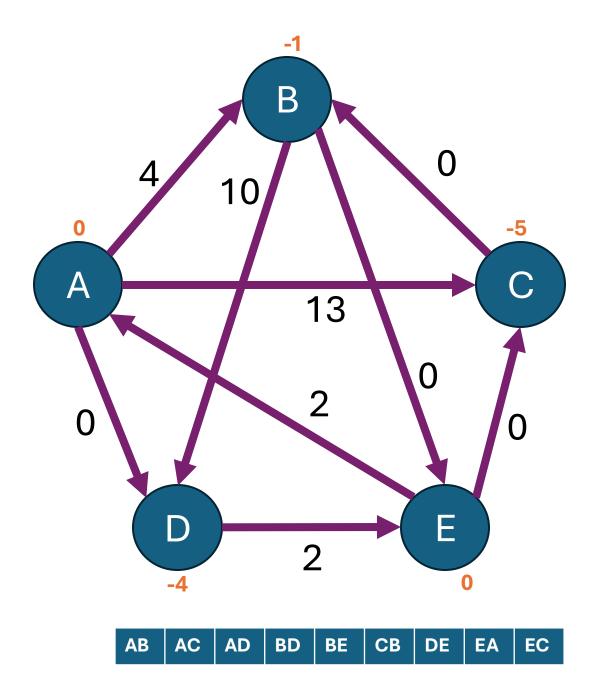
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С	-5	Е
D	-4	Α
Е	0	S



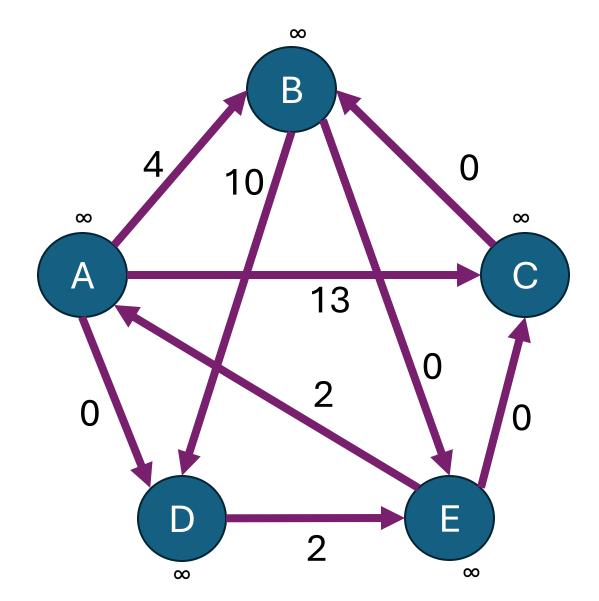
	Cost	Prev
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С	-5	Е
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Е	0	S



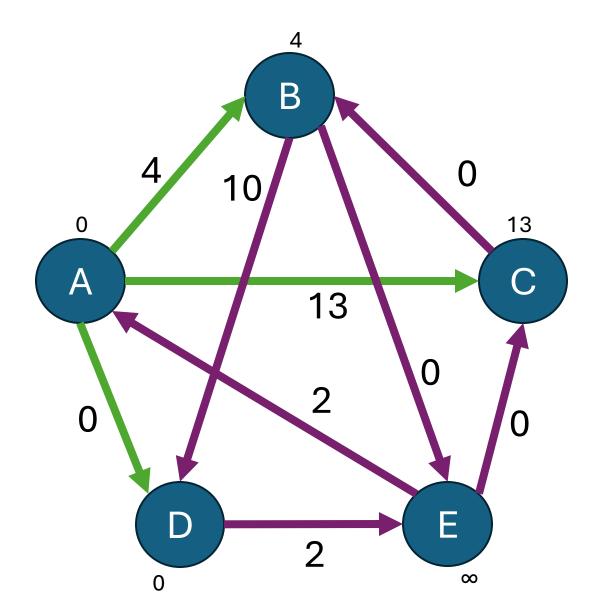
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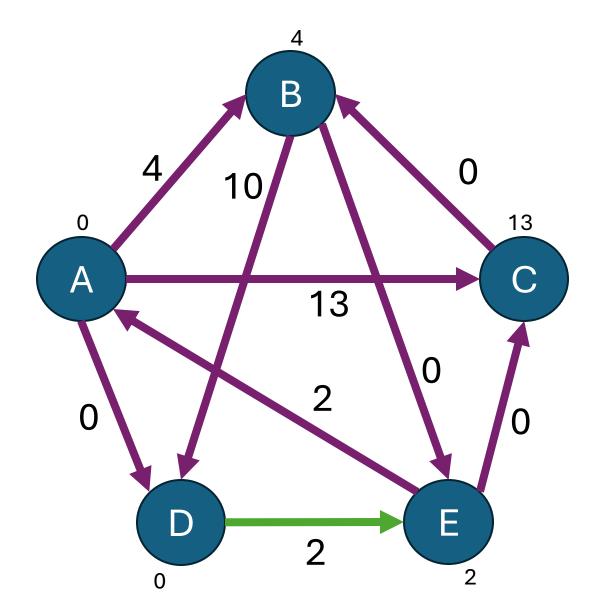
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Α	0	S
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С	-5	Е
D	-4	Α
Е	0	S



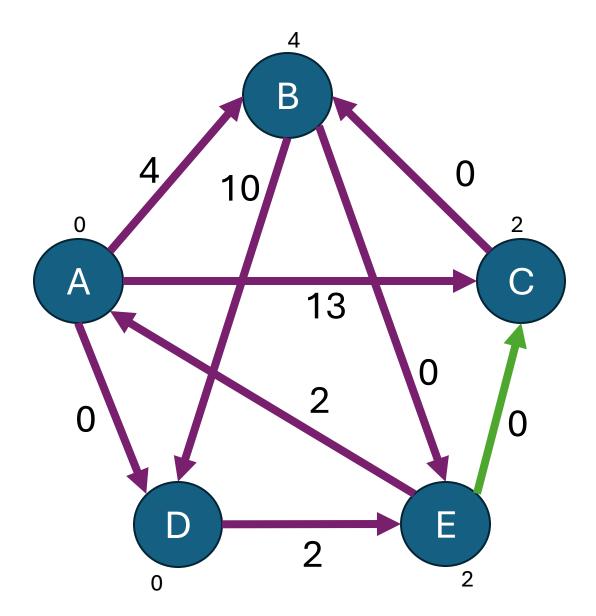
	Α	В	С	D	Е
Α					



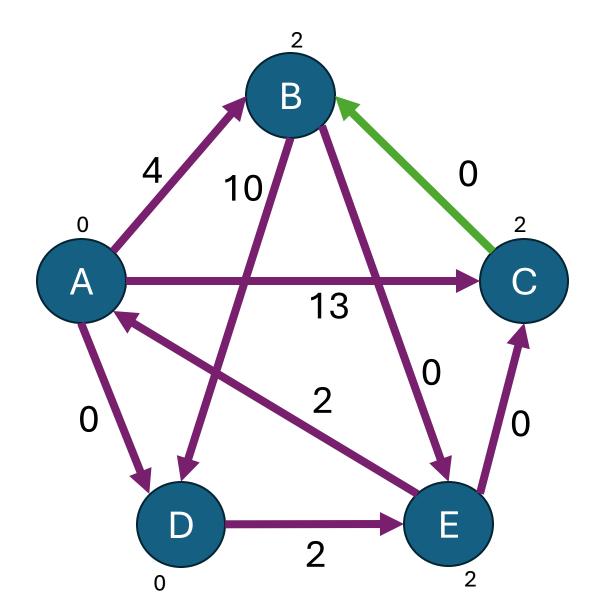
	Α	В	С	D	Е
Α	0	4	13	0	
		Α	Α	Α	



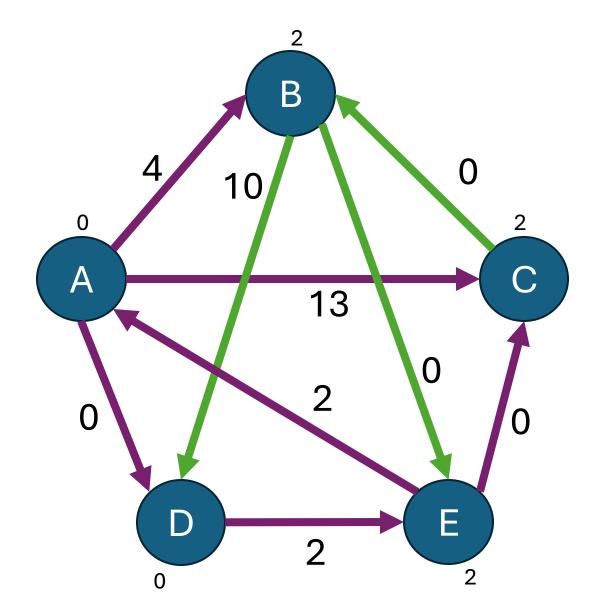
	Α	В	С	D	Е
A	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D



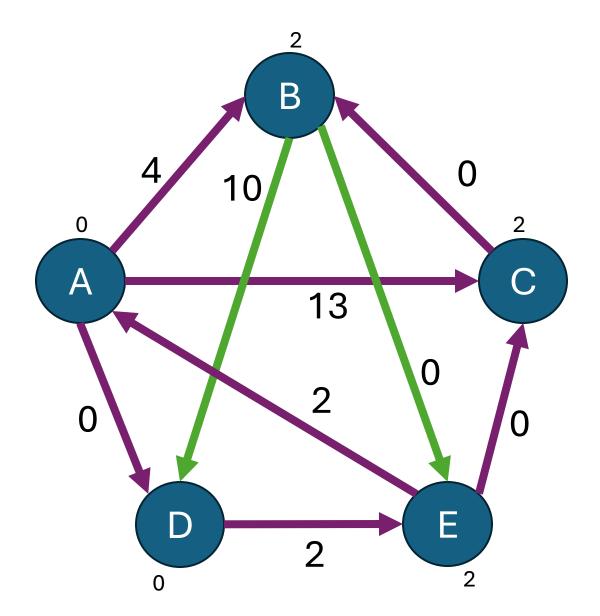
	Α	В	С	D	Е
A	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D



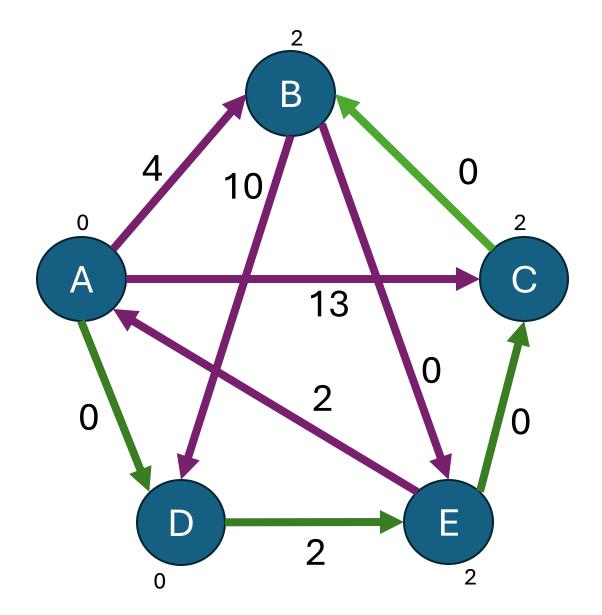
	Α	В	С	D	Е
A	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D



	Α	В	С	D	Е
A	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D

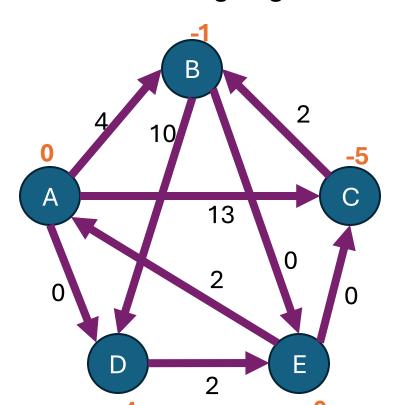


	Α	В	С	D	Е
A	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D

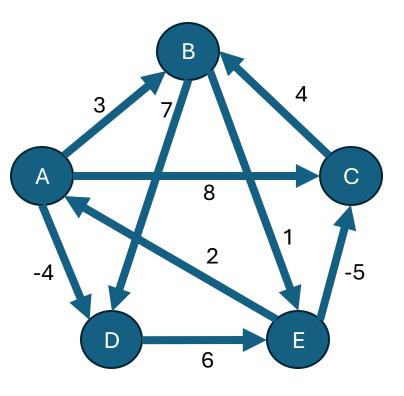


	Α	В	С	D	Е
A	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D

Let us reweight the edges to retrieve the original weights



Original Graph



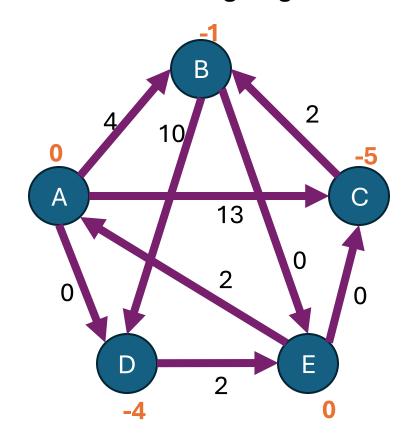
	A	В	С	D	E
Α	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D

$$d(u,v)=d'(u,v)-h(u)+h(v)$$

$$d(a,b) = 2-0+(-1) = 1$$

$$d(a,d) = 0-0+(-4) = -4$$

$$d(a,c) = 2-0+(-5) = -3$$



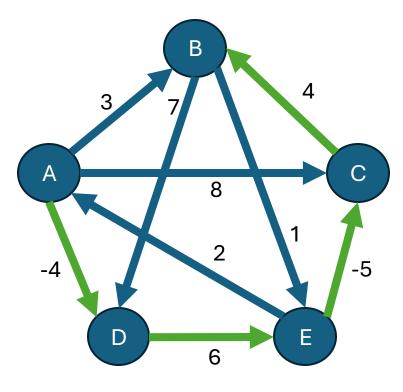
$$d(u,v)=d'(u,v)-h(u)+h(v)$$

$$d(a,b) = 2-0+(-1) = 1$$

$$d(a,d) = 0-0+(-4) = -4$$

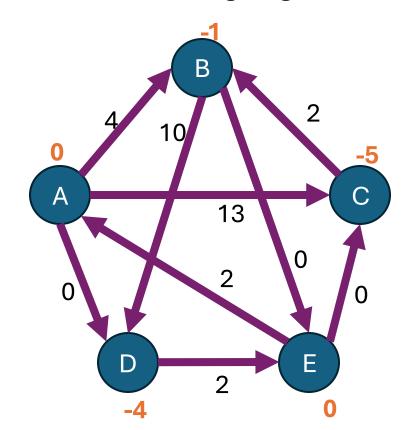
$$d(a,c) = 2-0+(-5) = -3$$

Original Graph



Total is 1!

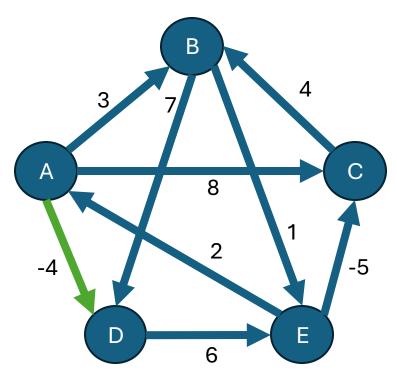
	Α	В	С	D	E
Α	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D



$$d(u,v)=d'(u,v)-h(u)+h(v)$$

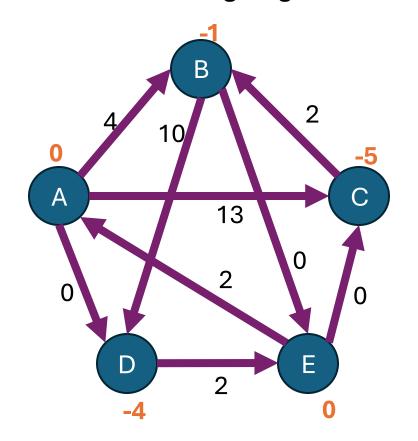
 $d(a,b) = 2-0+(-1) = 1$
 $d(a,d) = 0-0+(-4) = -4$
 $d(a,c) = 2-0+(-5) = -3$

Original Graph



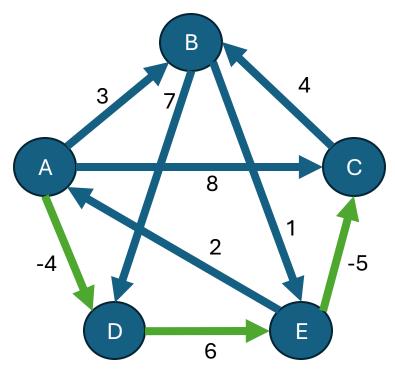
Total is -4!

	Α	В	С	D	Е
Α	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
E	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D



d(u,v)=d'(u,v)-h(u)+h(v) d(a,b) = 2-0+(-1) = 1 d(a,d) = 0-0+(-4) = -4d(a,c) = 2-0+(-5) = -3

Original Graph

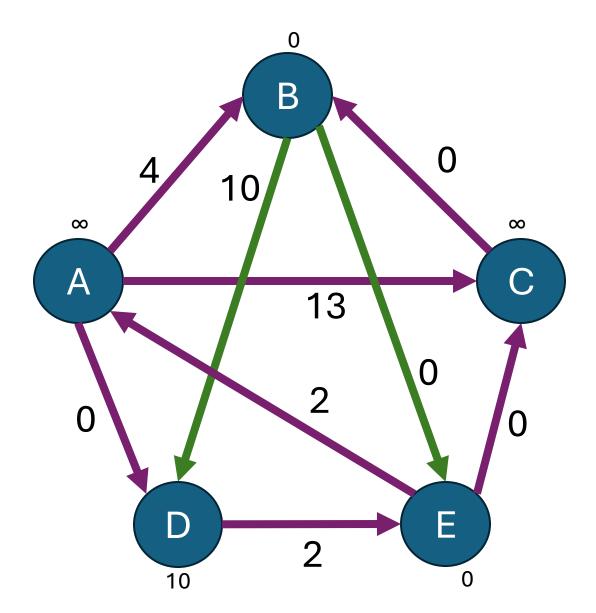


Total is -3!

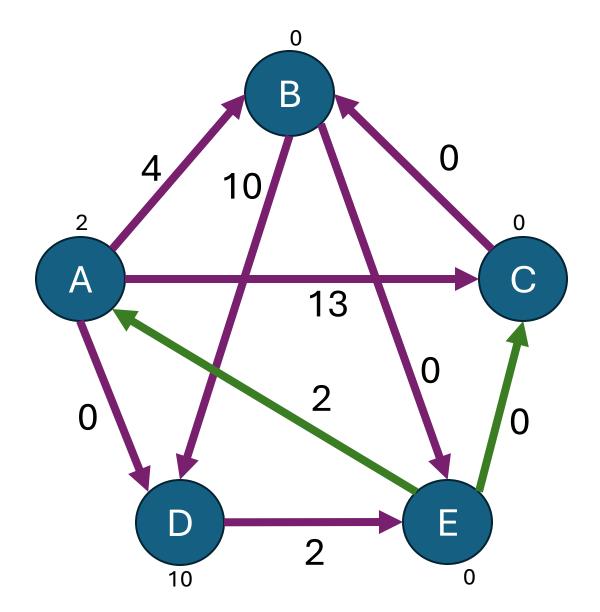
	Α	В	С	D	E
A	0	4 A	13 A	0 A	
D	0	4 A	13 A	0 A	2 D
Е	0	4 A	2 E	0 A	2 D
С	0	2 C	2 E	0 A	2 D
В	0	2 C	2 E	0 A	2 D

Perform Dijkstra on every other vertex in the graph

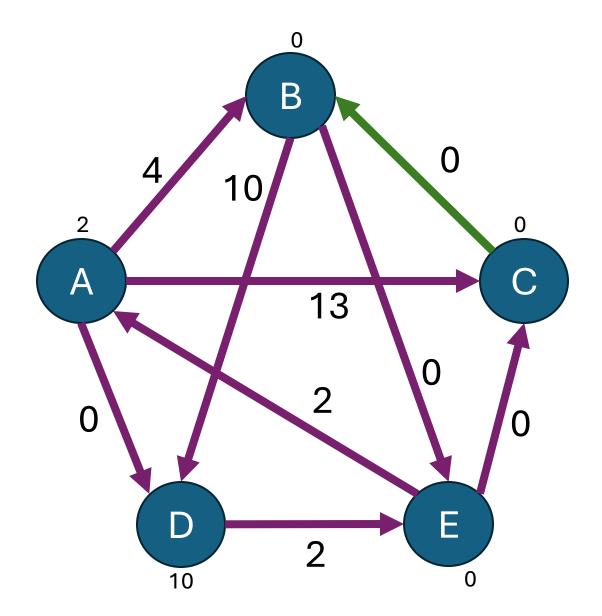
Perform Dijkstra on vertex B



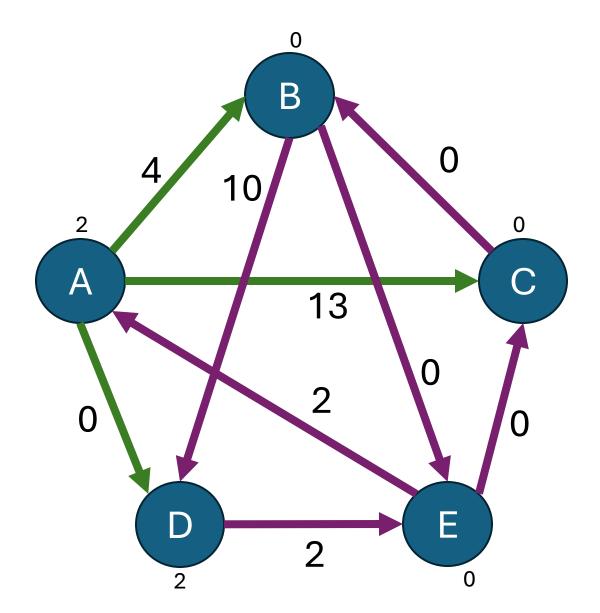
	В	Α	С	D	Е
В	0			10	0
				В	В



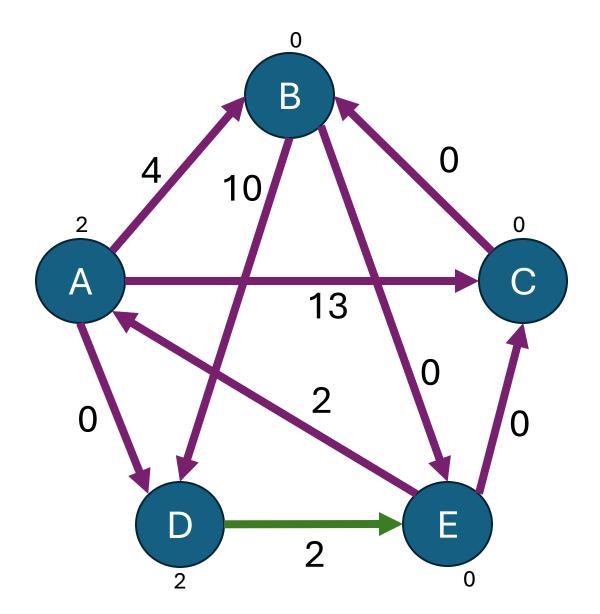
	В	Α	С	D	Е
В	0			10 B	0 B
Е	0	2 E	0 E	10 B	0 B



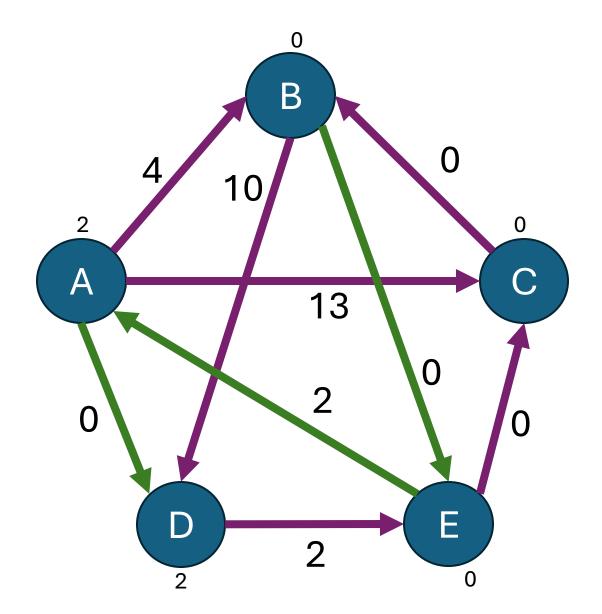
	В	Α	С	D	Е
В	0			10 B	0 B
Е	0	2 E	0 E	10 B	0 B
С	0	2 E	0 E	10 B	0 B



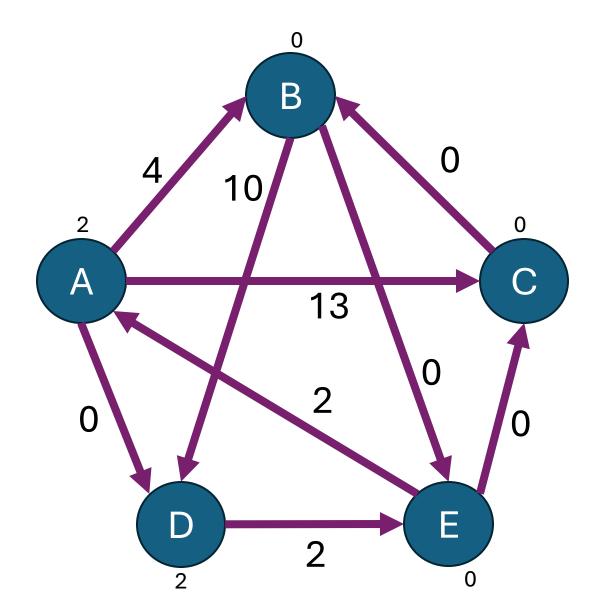
	В	Α	С	D	Е
В	0			10 B	0 B
Е	0	2 E	0 E	10 B	0 B
С	0	2 E	0 E	10 B	0 B
A	0	2 E	0 E	2 A	0 B



	В	Α	С	D	Е
В	0			10 B	0 B
Е	0	2 E	0 E	10 B	0 B
С	0	2 E	0 E	10 B	0 B
A	0	2 E	0 E	2 A	0 B
D	0	2 E	0 E	2 A	0 B

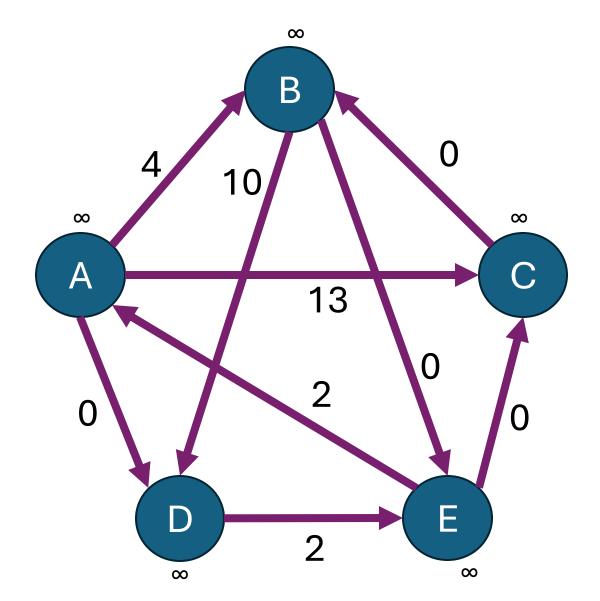


	В	Α	С	D	Е
В	0			10 B	0 B
E	0	2 E	0 E	10 B	0 B
С	0	2 E	0 E	10 B	0 B
A	0	2 E	0 E	2 A	0 B
D	0	2 E	0 E	2 A	0 B

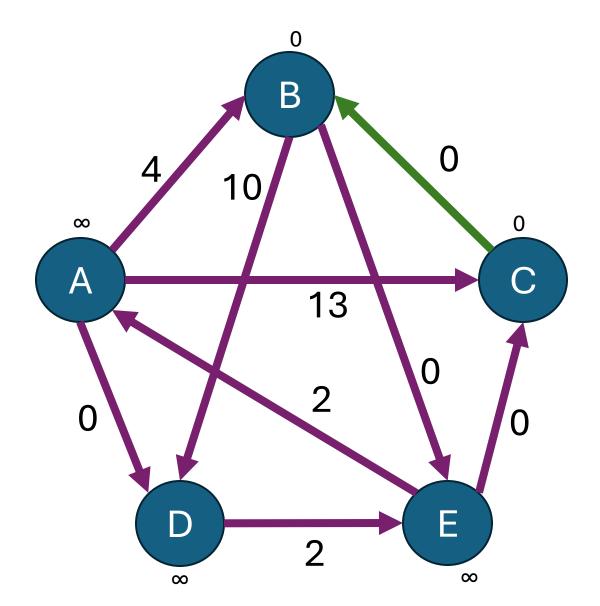


	В	Α	С	D	Е
В	0			10 B	0 B
E	0	2 E	0 E	10 B	0 B
С	0	2 E	0 E	10 B	0 B
A	0	2 E	0 E	2 A	0 B
D	0	2 E	0 E	2 A	0 B

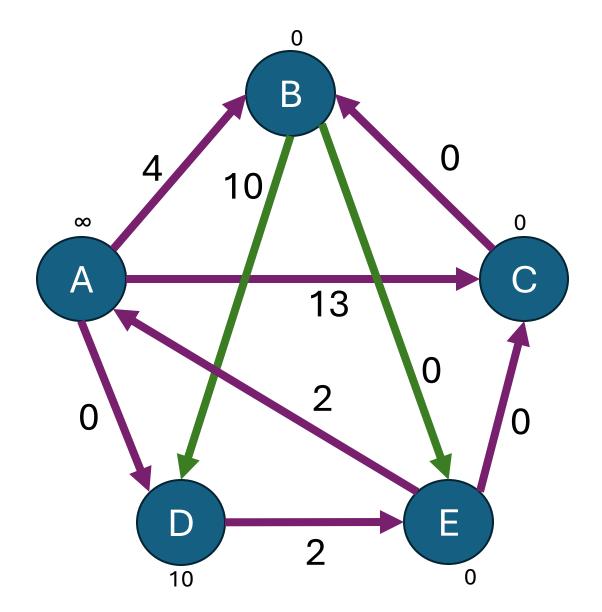
Perform Dijkstra on vertex C



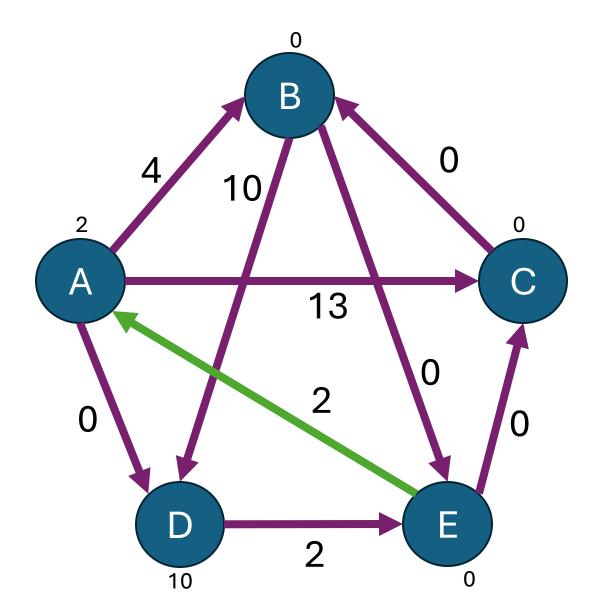
	С	A	В	D	E
С					



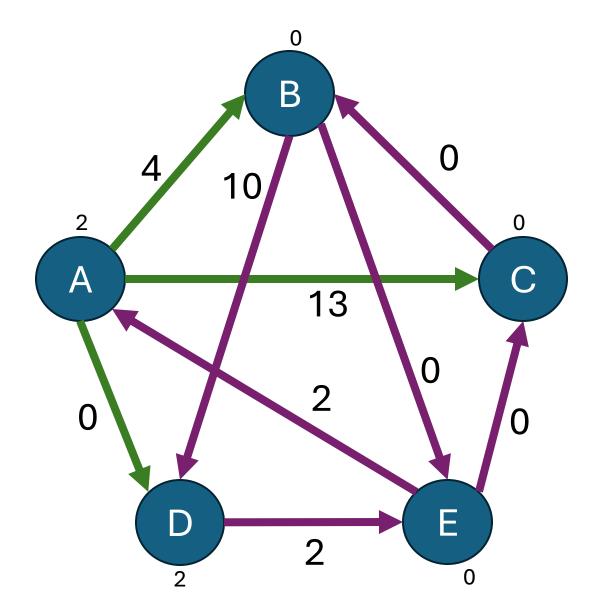
	С	Α	В	D	Е
С	0		0 C		



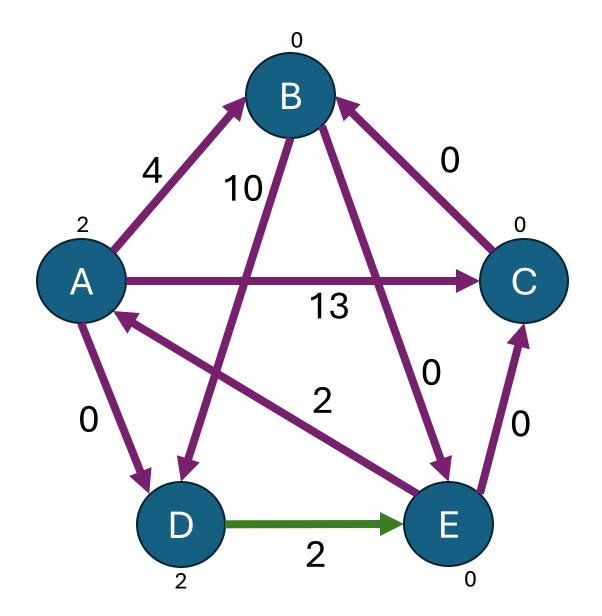
	С	Α	В	D	Е
С	0		0 C		
В	0		0 C	10 B	0 B



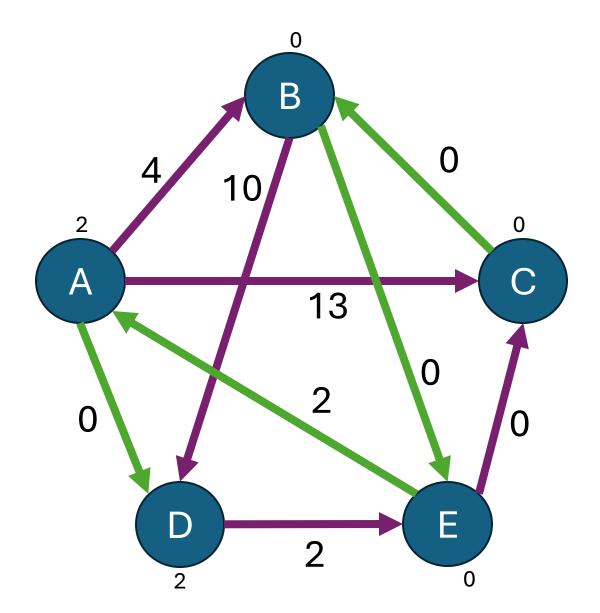
	С	Α	В	D	Е
С	0		0 C		
В	0		0 C	10 B	0 B
Е	0	2 E	0 C	10 B	0 B



	С	Α	В	D	Е
С	0		0 C		
В	0		0 C	10 B	0 B
Е	0	2 E	0 C	10 B	0 B
Α	0	2 E	0 C	2 A	0 B



	С	Α	В	D	Е
С	0		0 C		
В	0		0 C	10 B	0 B
Е	0	2 E	0 C	10 B	0 B
A	0	2 E	0 C	2 A	0 B
D	0	2 E	0 C	2 A	0 B



	С	Α	В	D	Е
С	0		0 C		
В	0		0 C	10 B	0 B
Е	0	2 E	0 C	10 B	0 B
A	0	2 E	0 C	2 A	0 B
D	0	2 E	0 C	2 A	0 B