DYNAMIC PROGRAMMING PART II

Lecture 5.1, CMSC 142

Today's Topics

- Longest Common Subsequence

Longest Common Subsequence

- X is a subsequence of Y if X is obtained from dropping or removing 0 or more elements of Y, preserving the relative order of elements
- Whereas in a substring the elements needs to be contiguous, it need not be for subsequences
- Example: "ad" is a subsequence of "abcde", by dropping b,c,e

Common subsequence

- Given two subsequences A and B, C is a common subsequence of A and B if the subsequence C appears in both A and B

Longest Common Subsequence

Input:

- Sequences A = $\{a_1, a_2, \ldots, a_m\}$ and B = $\{b_1, b_2, \ldots, b_n\}$

Output

- Sequence C = $\{c_1, c_2, \ldots, c_p\}$ where C is a subsequence of both A and B (common subsequence) and it is the longest possible common subsequence (common subsequence with the longest length)

Example

- A = "abcde", B = "bceadq"
- Common subsequences: "bcd", "ad", "bce", "be", "bd", "bc“
- LCS: "bcd","bce"

LCS

- Goal: Find a longest common subsequence(A,B)
- Note: 'a' longest common subsequence, not 'the' LCS, since there might be more than one

Motivation

- length of LCS(A,B) = similarity between A and B
- The longer the length of LCS, the more similar A and B are
- Usage: DNA sequences, misspelled word, autocomplete

Activity

- List (with Ranking) of TV Shows You Like:
- Heroes, Gossip Girl, Awkward, How to Get Away with Murder, Friends, Big Bang Theory, HIMYM, Sherlock, Game of Thrones, American Horror Story, Agents of SHIELD, Arrow, Once Upon A Time, The Walking Dead, Grey's Anatomy, The Flash

Question

How can we solve this problem?

NOOBgrammer says…

Use **Brute-force** approach.

Brute-force Algorithm

- Generate all possible subsequences of A (length m)
- Check which are also subsequences of B (length n)
- Retain the longest subsequence (only one)

Question

- Will it return the correct answer?

- NO

Question

- Will it be fast? NO

Analysis

- All possible subsequences of A = $2^n - 1$
- Example: "1234" has 15 subsequences

Analysis

- Checking if subsequence of A is also subsequence of B = O(n) time per subsequence
- Worst-Case RT = $O(n2^m)$ [exponential time]

Can we do better?

PROgrammer says..

Use DP approach!

Optimal Substructure

- First, we need to characterize a longest common subsequence
- The LCS problem has an optimal substructure property: optimal solutions to the subproblems contribute to the optimal solution of the given problem

Idea

Example:

A = "ABCD“

B = "BCDEF“

LCS = "BCD"

Example:

A = "ABCD“

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Idea

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Example:

- A = "ABC“
- B = "BCDE“

- LCS = "BC"

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- B = "BCD“

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Idea

Example:

- A = "AB“
- B = "BCDE“
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Example:

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Example

String 1: BDCABA

String 2: ABCBDAB

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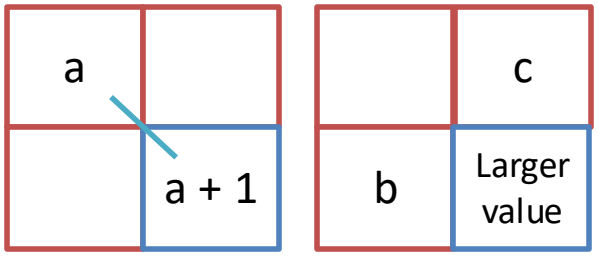
B							
C							
B							
D							
A							
B							



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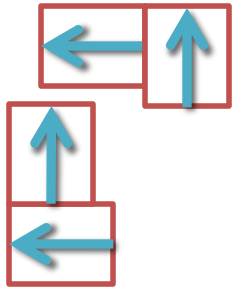

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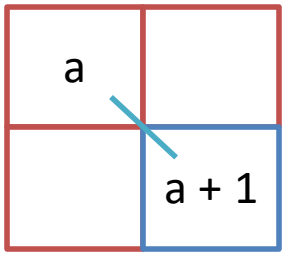

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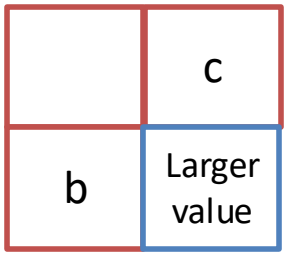

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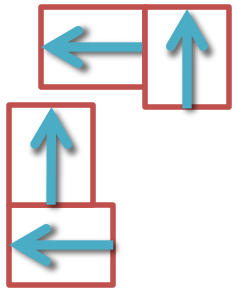

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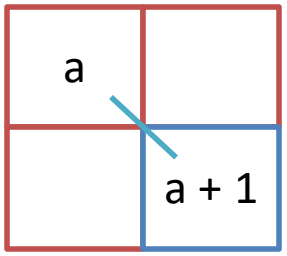

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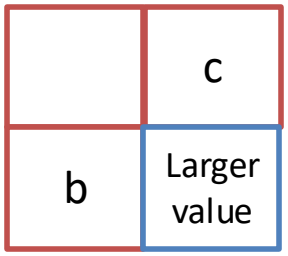

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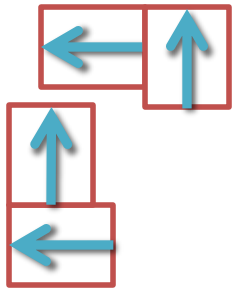

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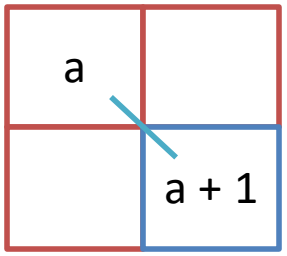

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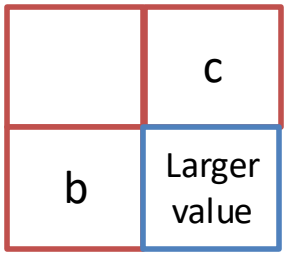

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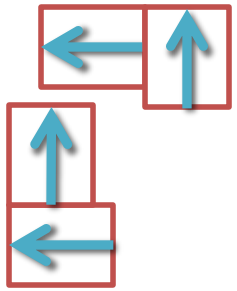

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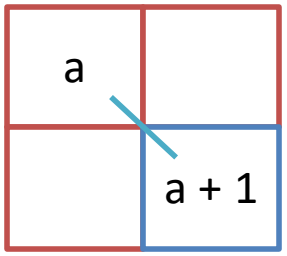

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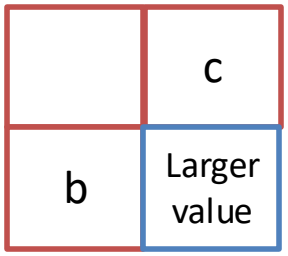

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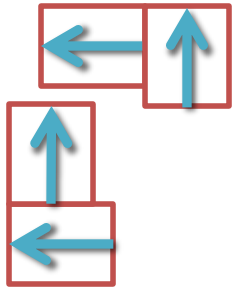

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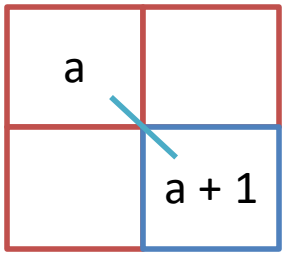

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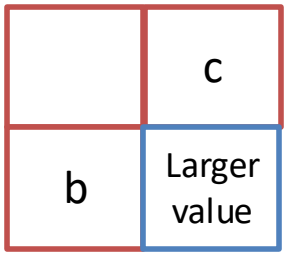

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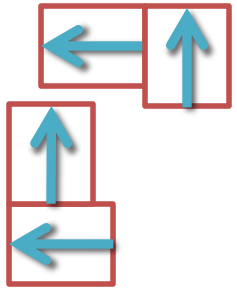

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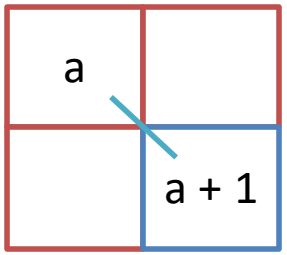

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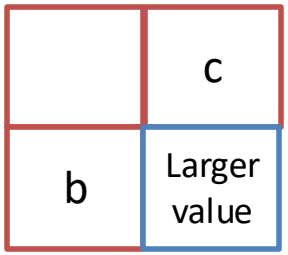

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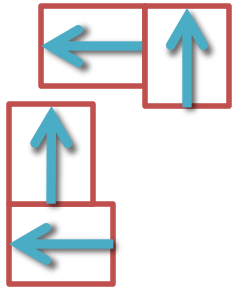

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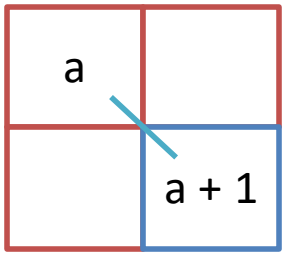

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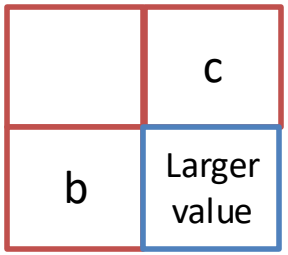

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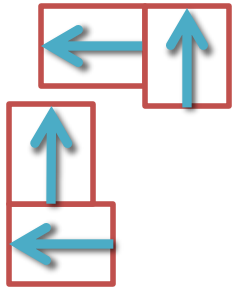

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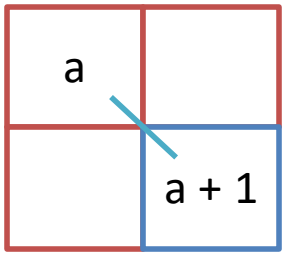

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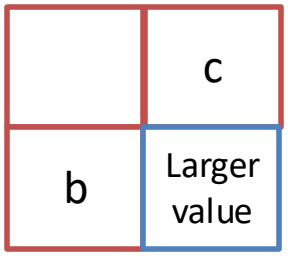

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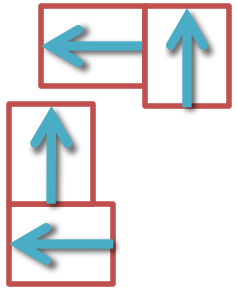

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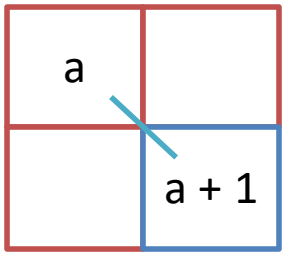

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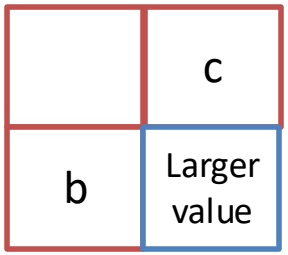

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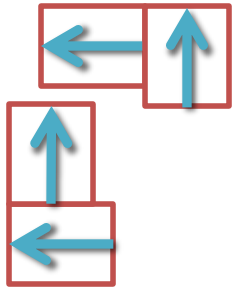

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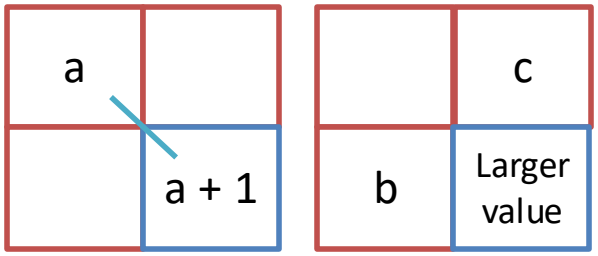

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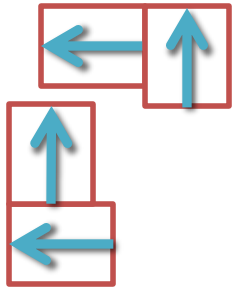

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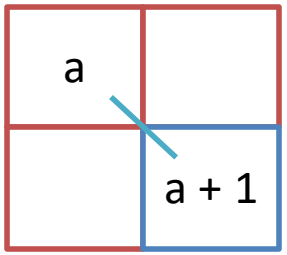

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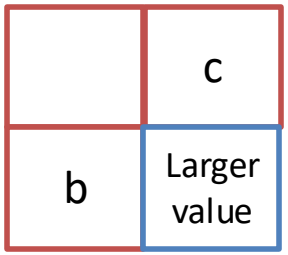

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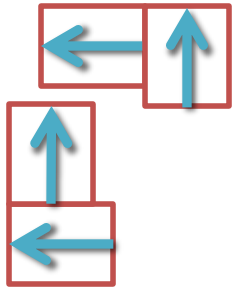

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A	0	↑0	↑0	↑0	↖1		
B	0						
C	0						
B	0						
D	0						
A	0						
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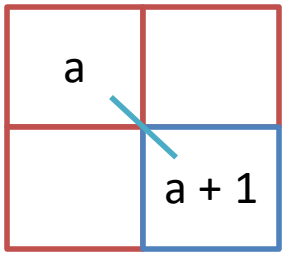

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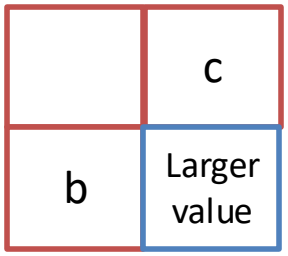

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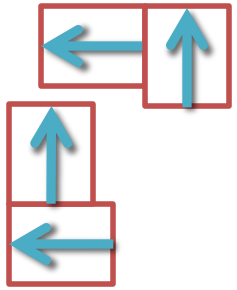

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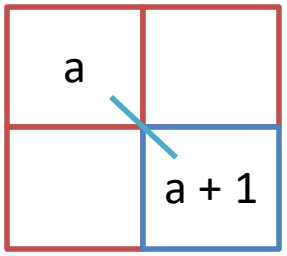

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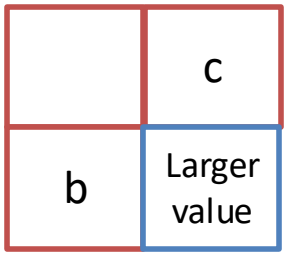

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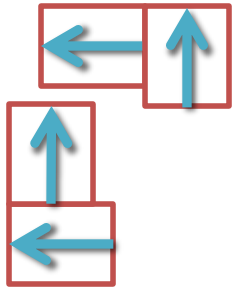

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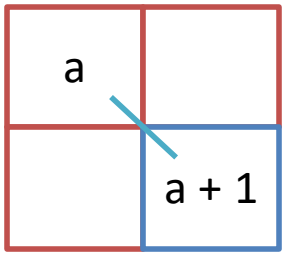

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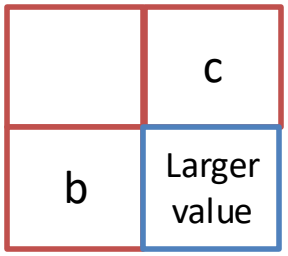

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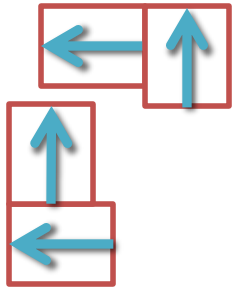

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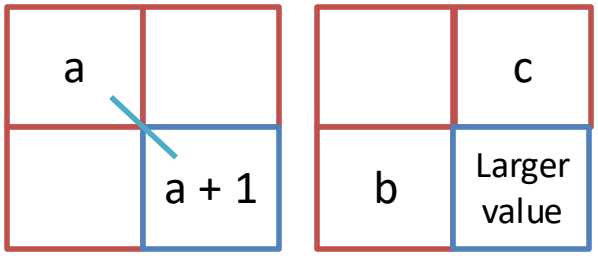

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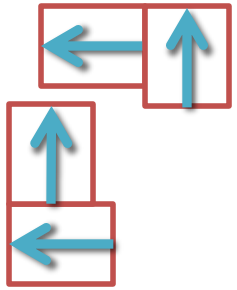

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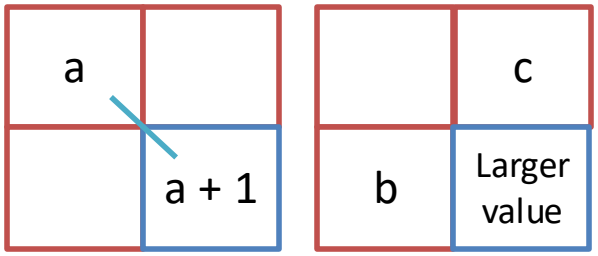

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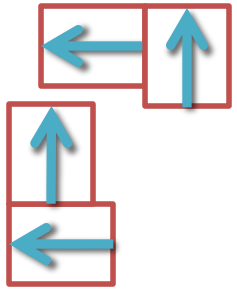

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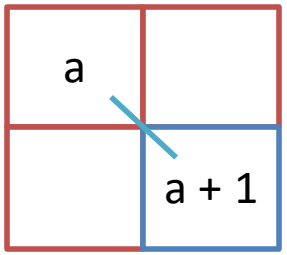

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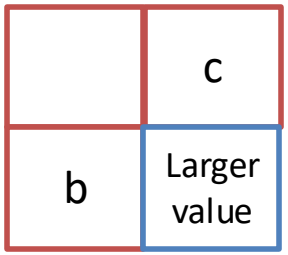

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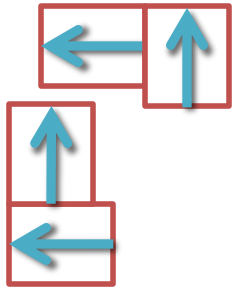

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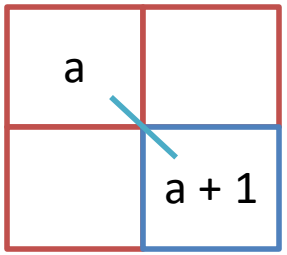

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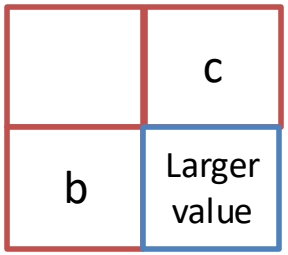

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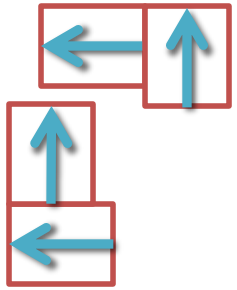

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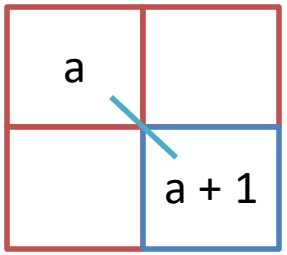

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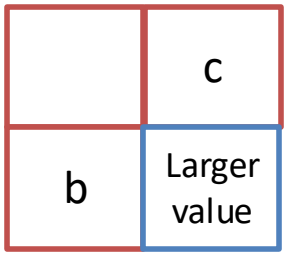

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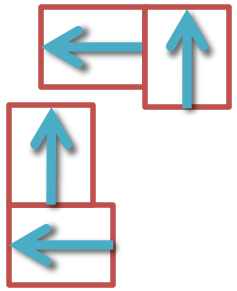

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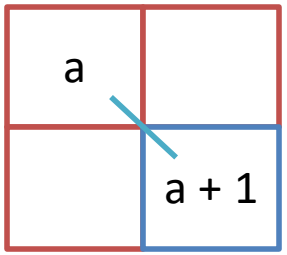

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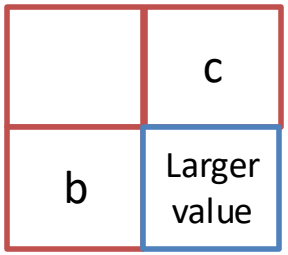

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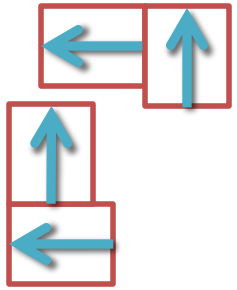

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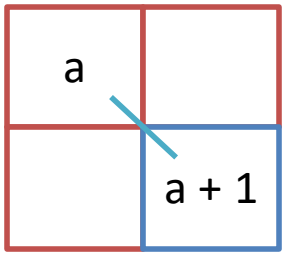

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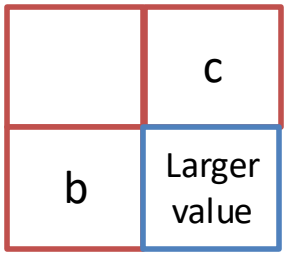

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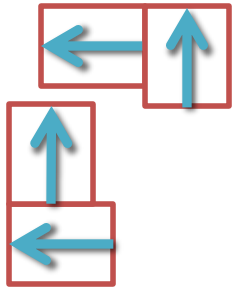

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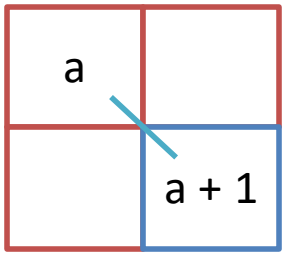

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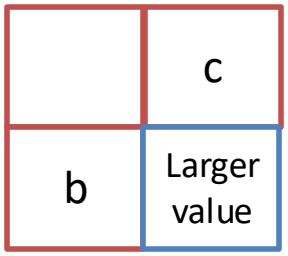

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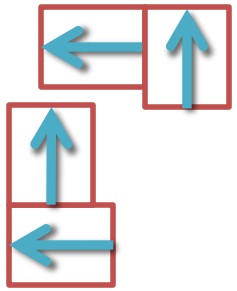

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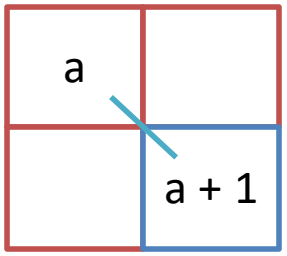

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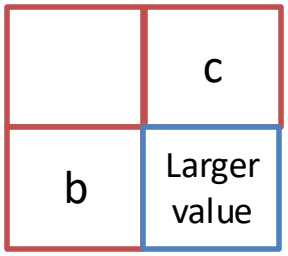

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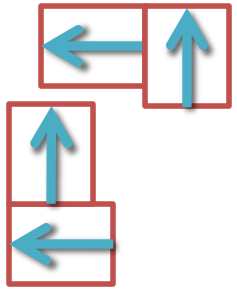

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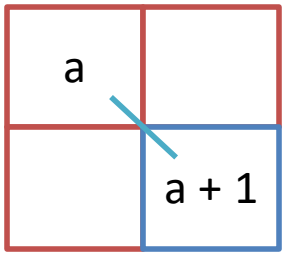

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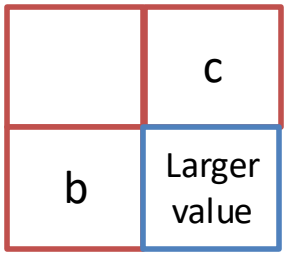

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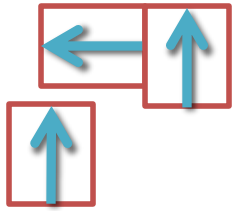

String 1: BDCABA

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		B	D	C	A	B	A
	0	0	0	0	0	0	0
A	0	↑0	↑0	↑0	↖1	←1	↖1
B	0	↖1	←1	←1	↑1	↖2	
C	0						
B	0						
D	0						
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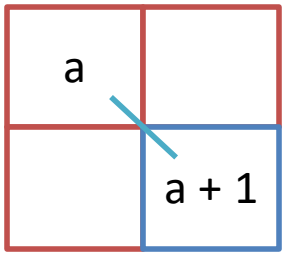

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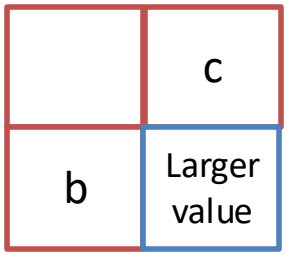

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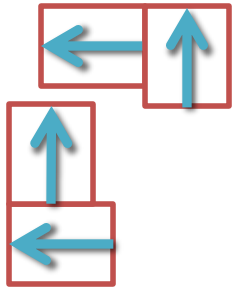

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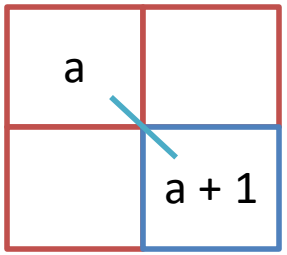

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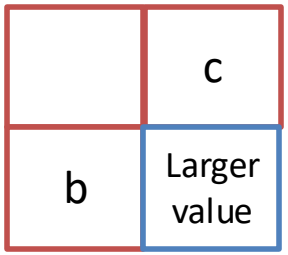

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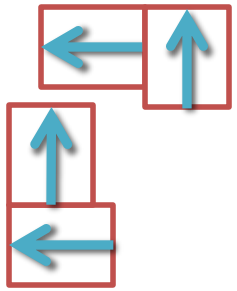

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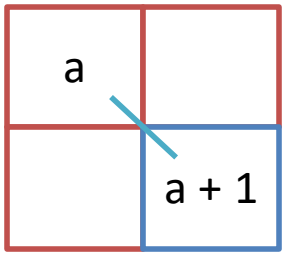

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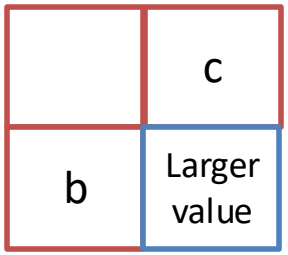

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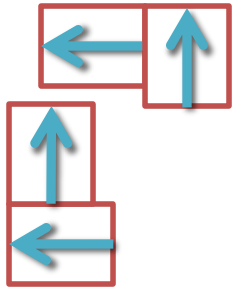

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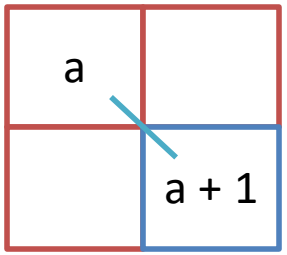

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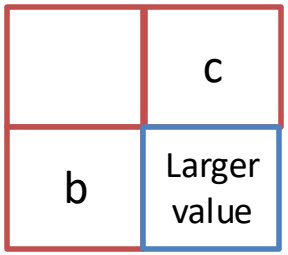

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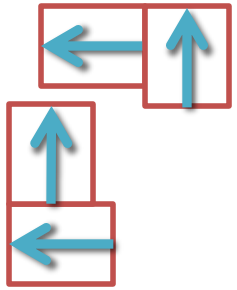

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C	0	↑1					
B	0						
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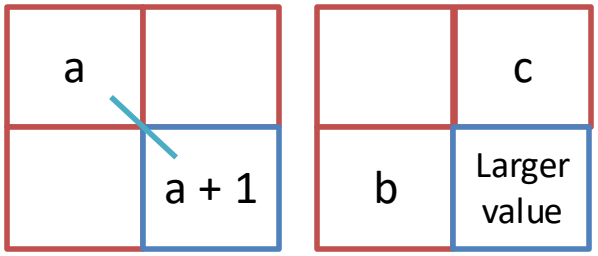

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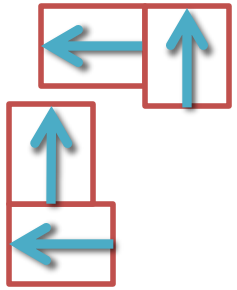

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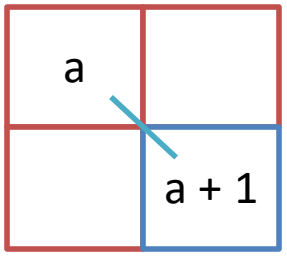

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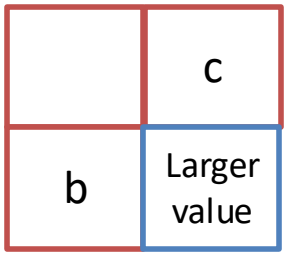

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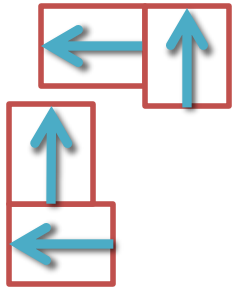

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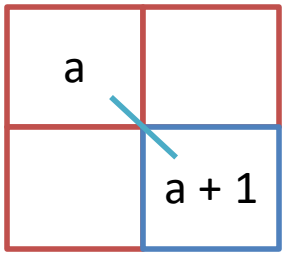

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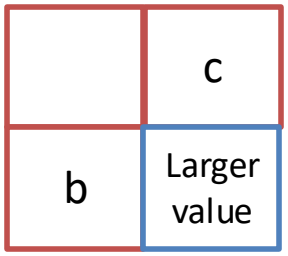

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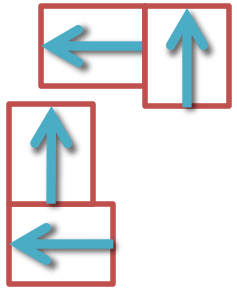

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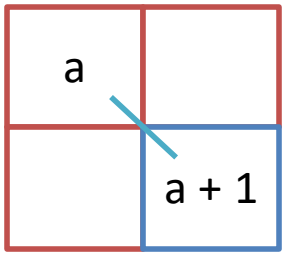

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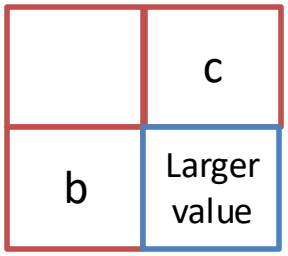

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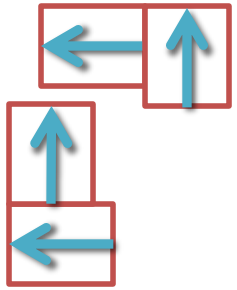

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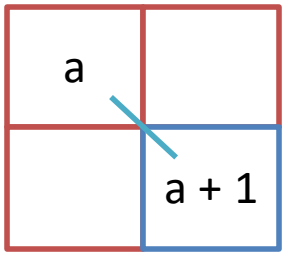

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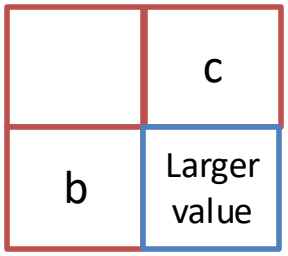

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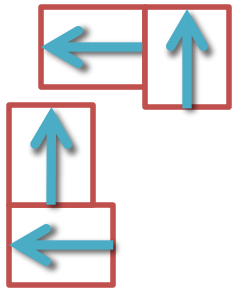

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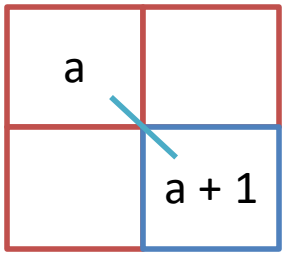

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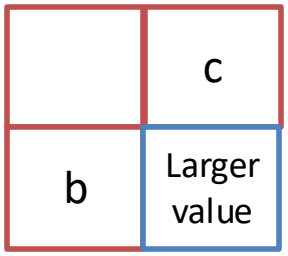

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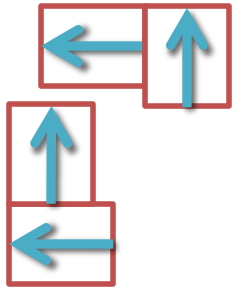

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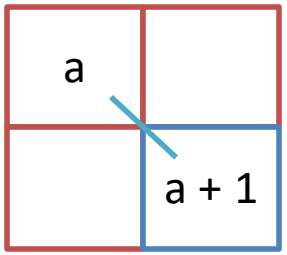

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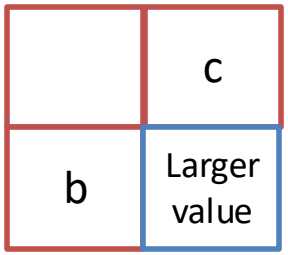

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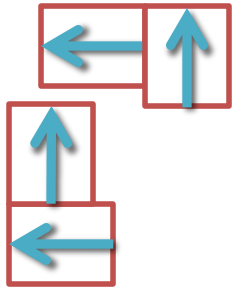

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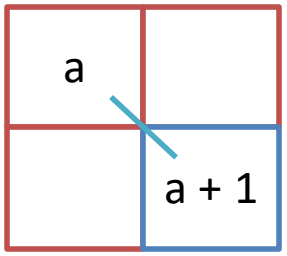
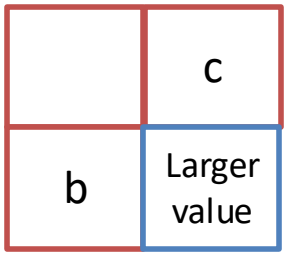


match!



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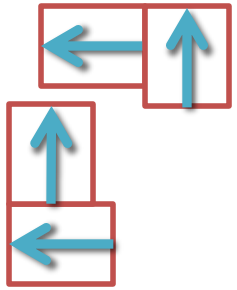

String 1: BDCABA

String 2: ABCBDAB

What's the LCS of the two strings?

		B	D	C	A	B	A
	0	0	0	0	0	0	0
A	0	↑0	↑0	↑0	↖1	←1	↖1
B	0	↖1	←1	←1	↑1	↖2	←2
C	0	↑1	↑1	↖2	←2	↑2	
B	0						
D	0						
A	0						
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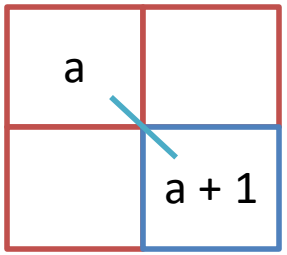

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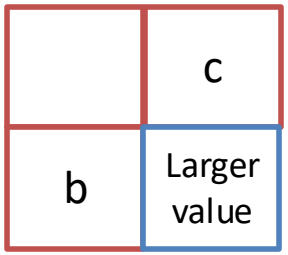

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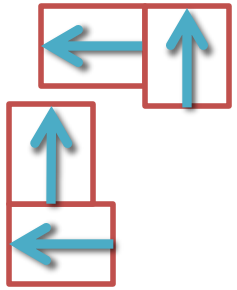

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C	0	↑1	↑1	↖2	←2	↑2	
B	0						
D	0						
A	0						
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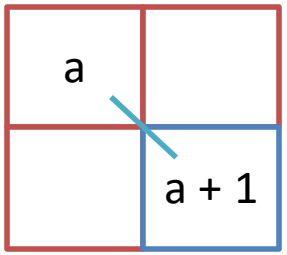

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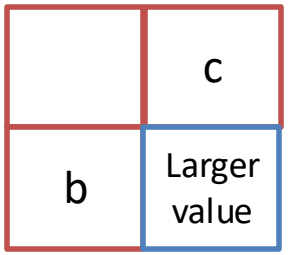

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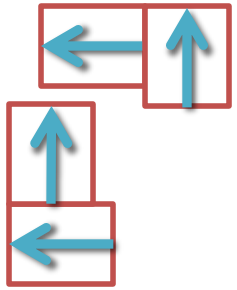

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C	0	↑1	↑1	↖2	←2	↑2	↑2
B	0						
D	0						
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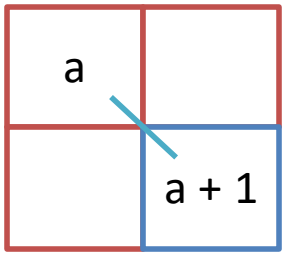

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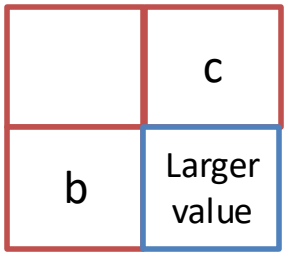

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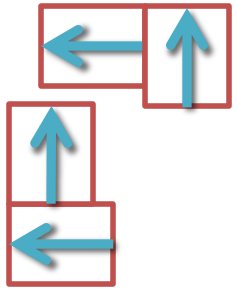

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B	0						
D	0						
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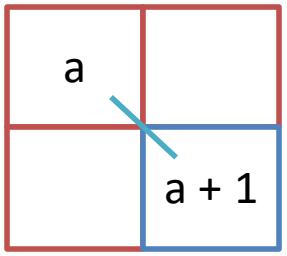

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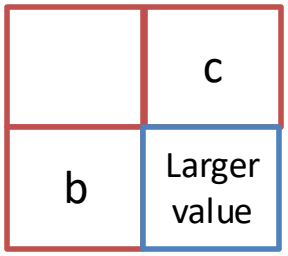

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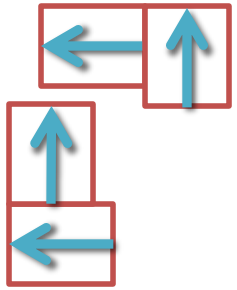

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B	0	↖1					
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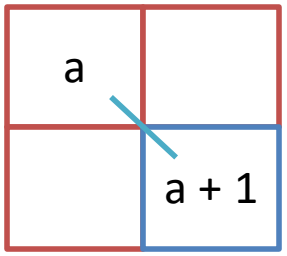
A	0						
B	0						



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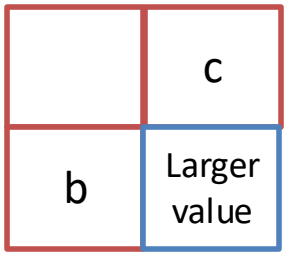

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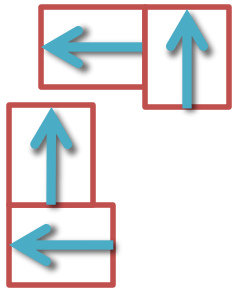

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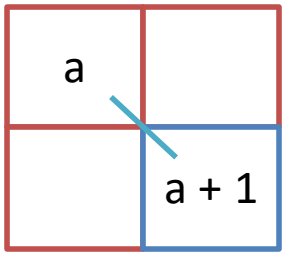

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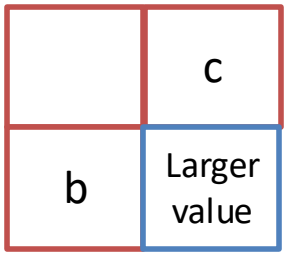

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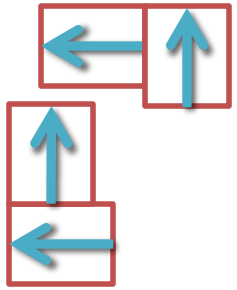

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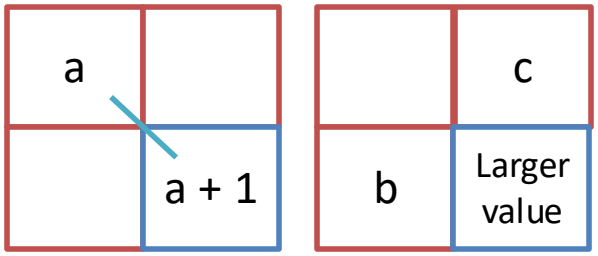

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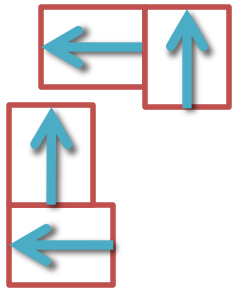

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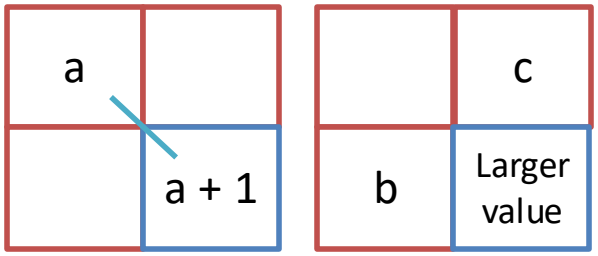

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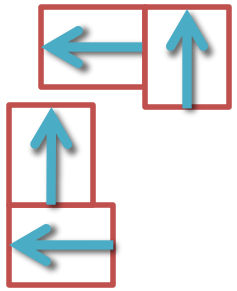

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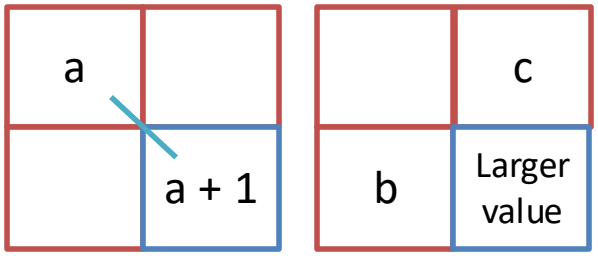

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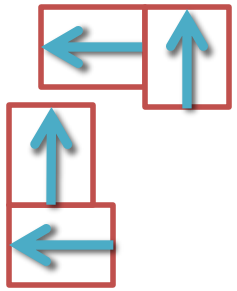

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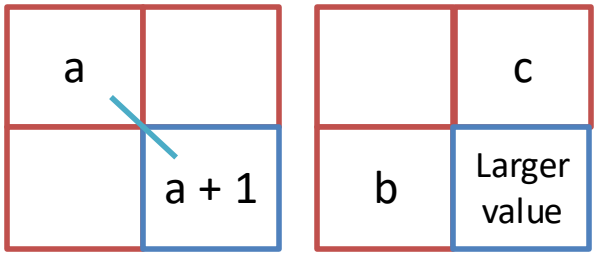

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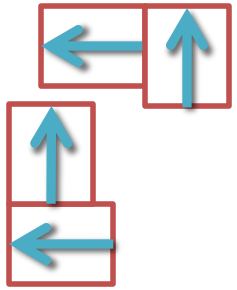

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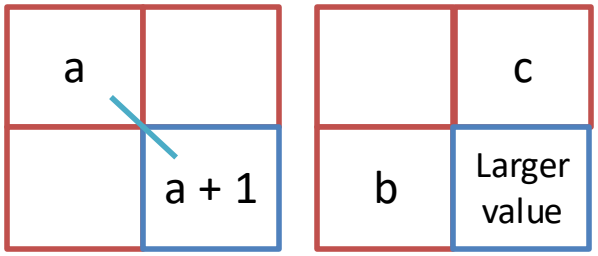

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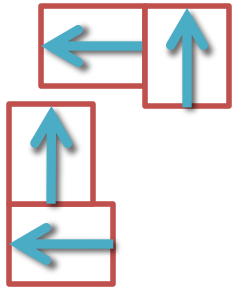

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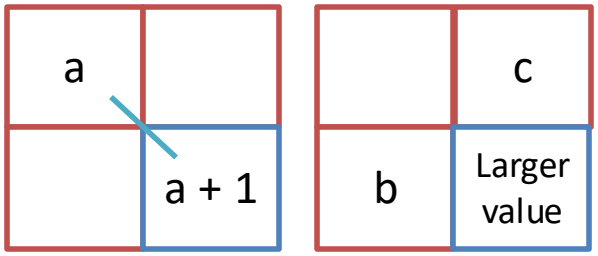

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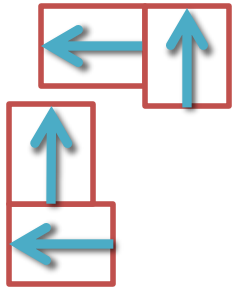

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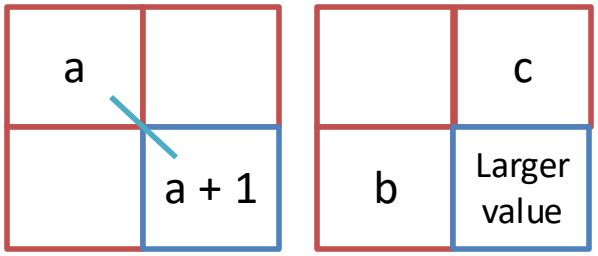

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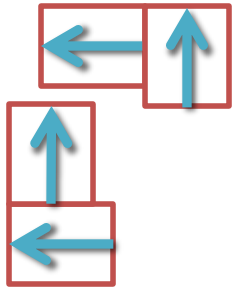

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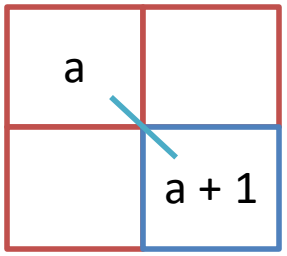

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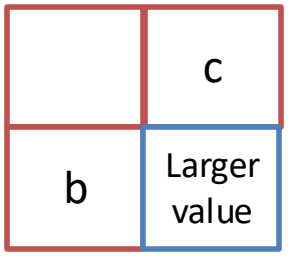

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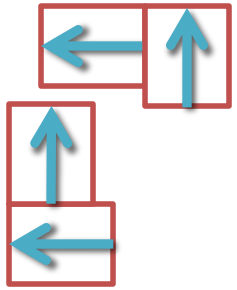

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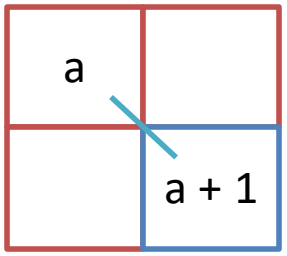

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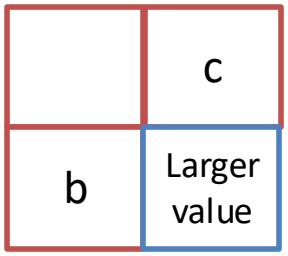

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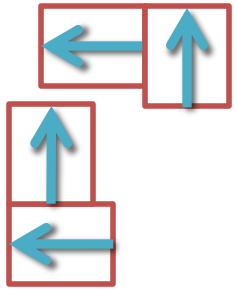

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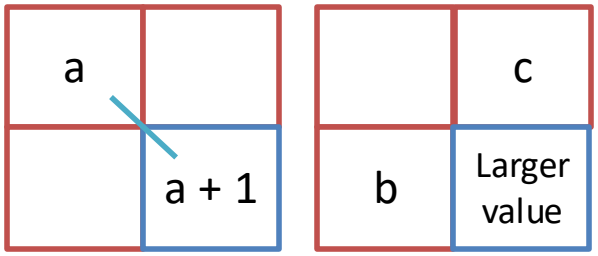

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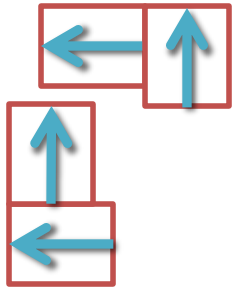

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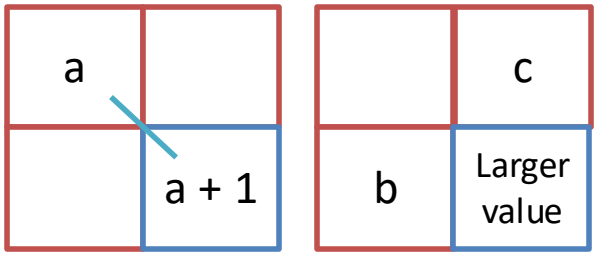

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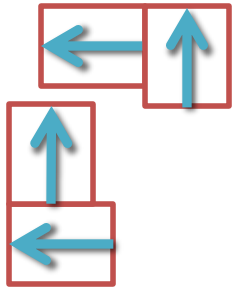

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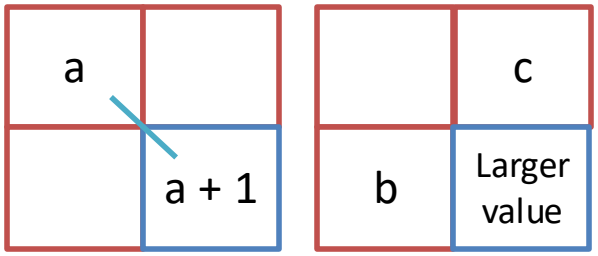

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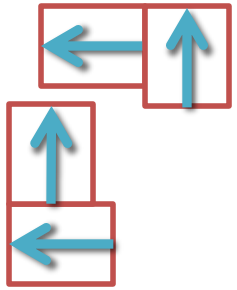

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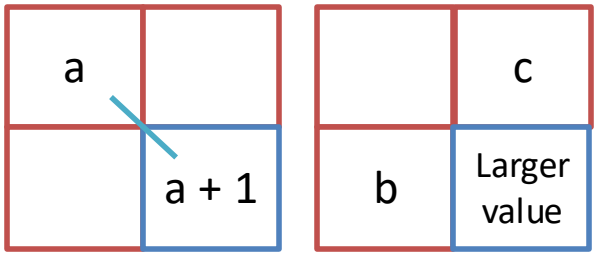

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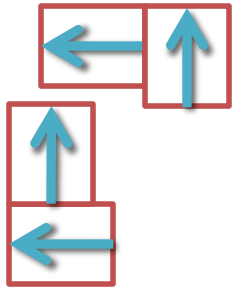

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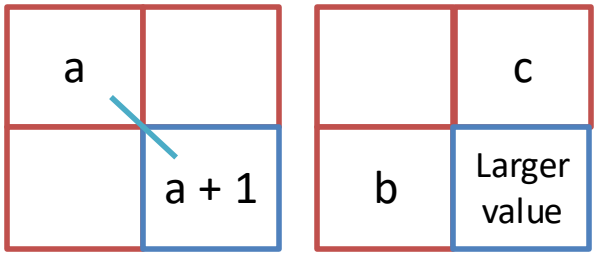

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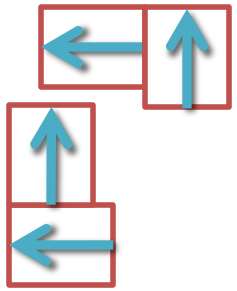

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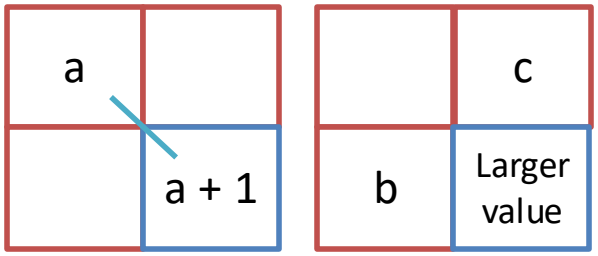

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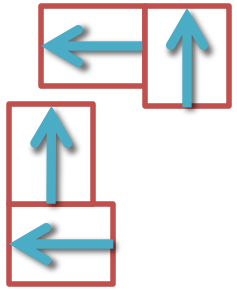

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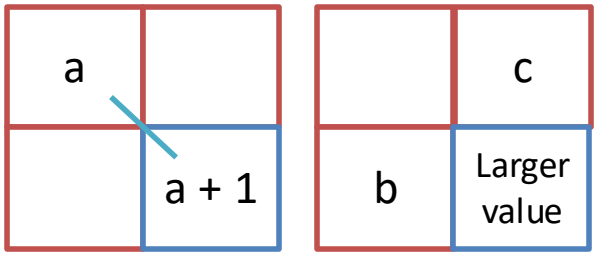

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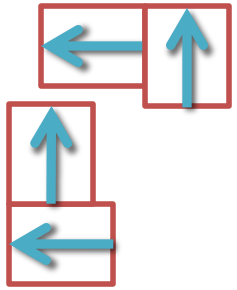

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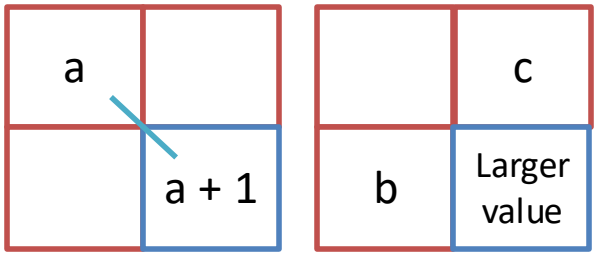

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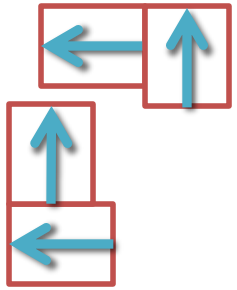

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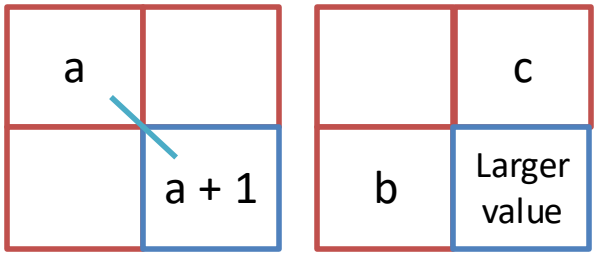

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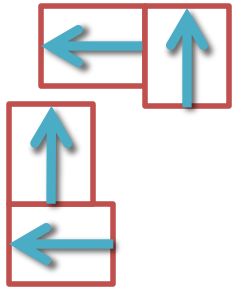

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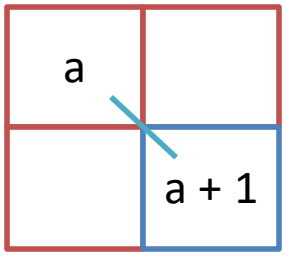

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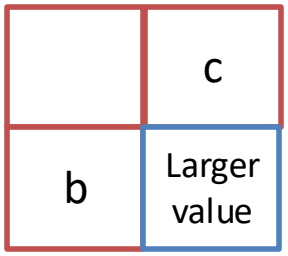

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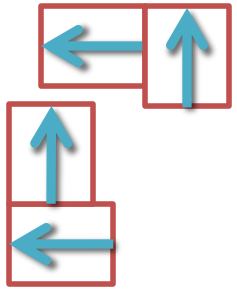

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A	0	↑0	↑0	↑0	↖1	←1	↖1
B	0	↖1	←1	←1	↑1	↖2	←2
C	0	↑1	↑1	↖2	←2	↑2	↑2
B	0	↖1	↑1	↑2	↑2	↖3	←3
D	0	↑1	↖2	↑2	↑2	↑3	
A	0						
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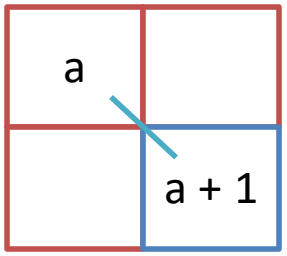

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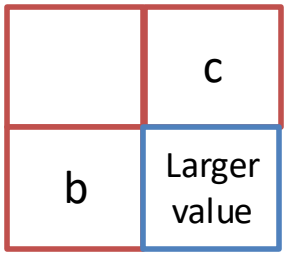

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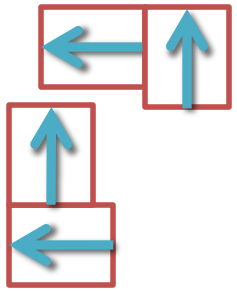

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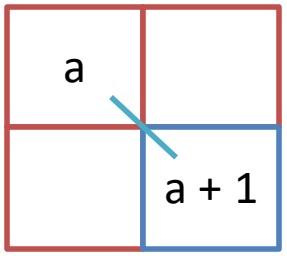

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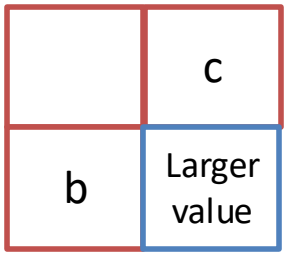

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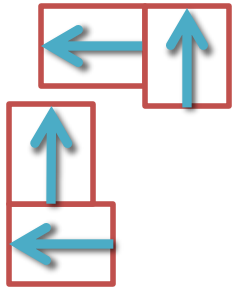

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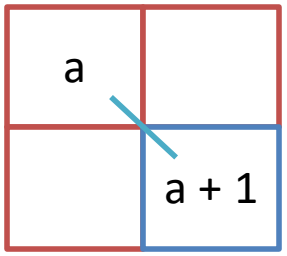

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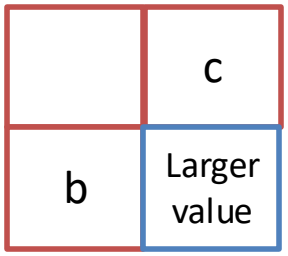

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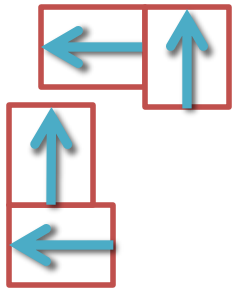

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B	0	↖1	↑1	↑2	↑2	↖3	←3
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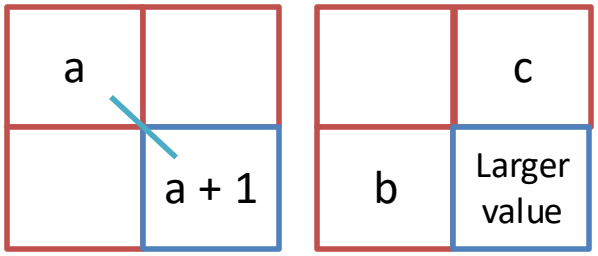

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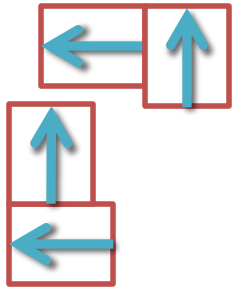

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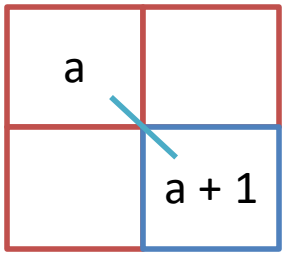

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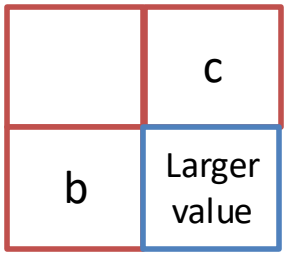

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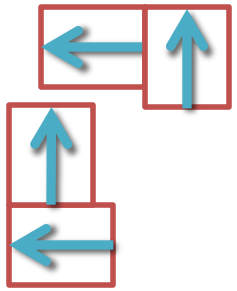

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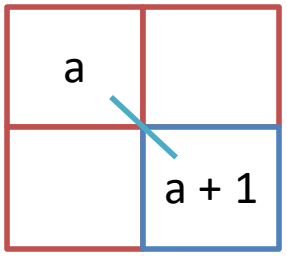

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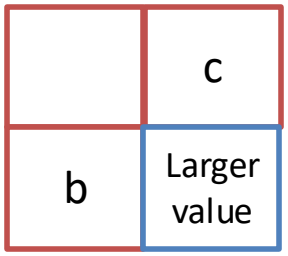

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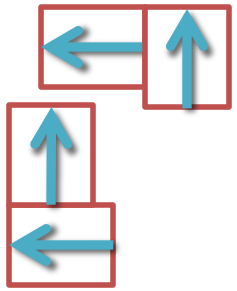

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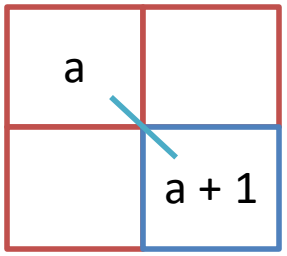

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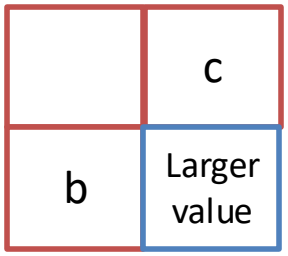

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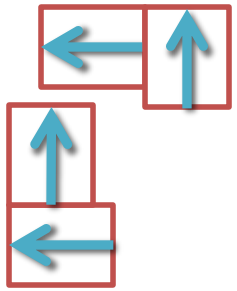

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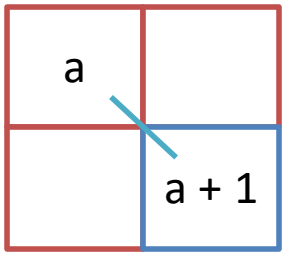

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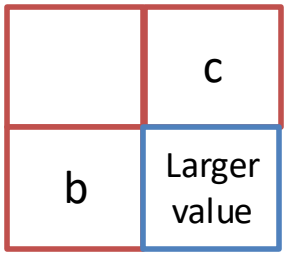

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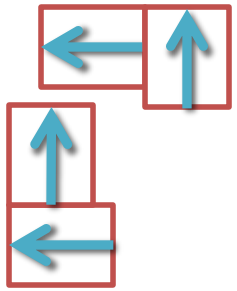

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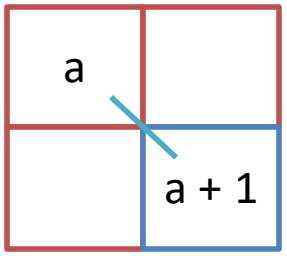

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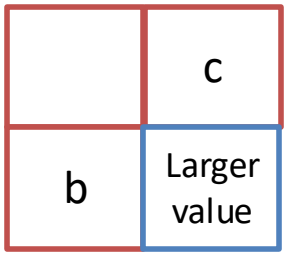

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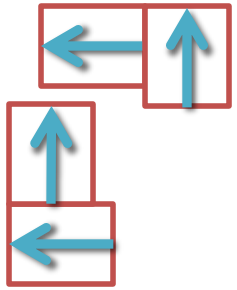

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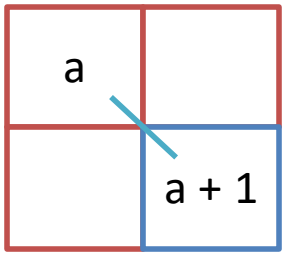

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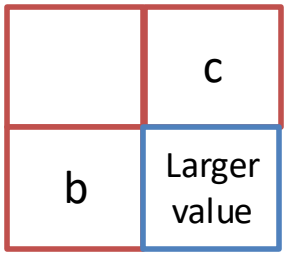

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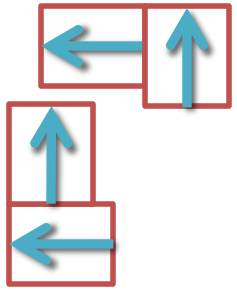

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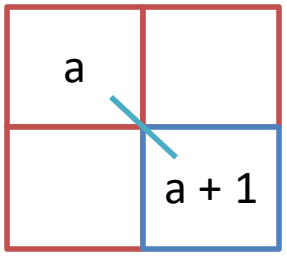

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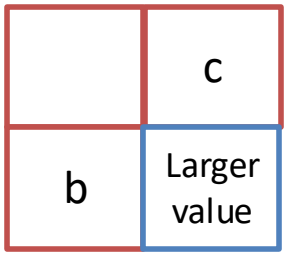

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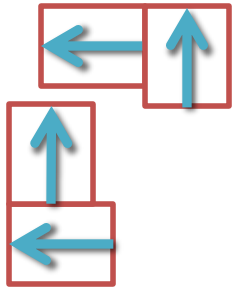

String 1: BDCABA

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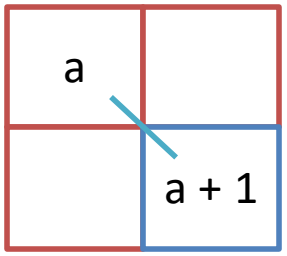
What's the LCS of the two strings?

		B	D	C	A	B	A
	0	0	0	0	0	0	0
A	0	↑0	↑0	↑0	↖1	←1	↖1
B	0	↖1	←1	←1	↑1	↖2	←2
C	0	↑1	↑1	↖2	←2	↑2	↑2
B	0	↖1	↑1	↑2	↑2	↖3	←3
D	0	↑1	↖2	↑2	↑2	↑3	↑3
A	0	↑1	↑2	↑2	↖3	↑3	
B	0						



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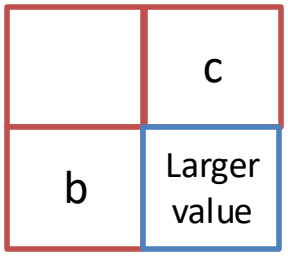

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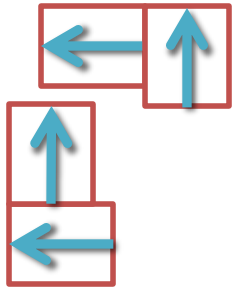

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B	0	↖1	↑1	↑2	↑2	↖3	←3
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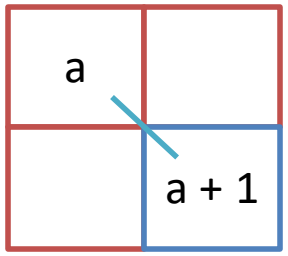

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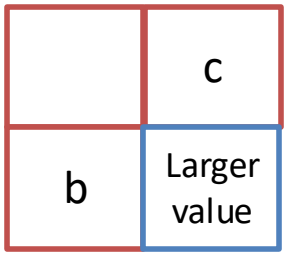

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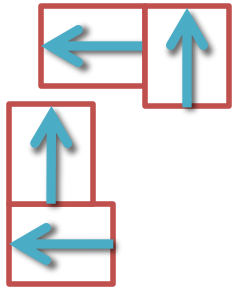

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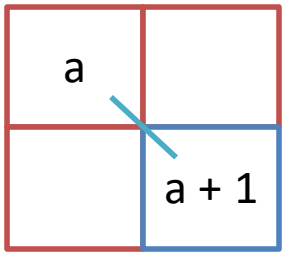
D	0	↑1	↖2	↑2	↑2	↑3	↑3
A	0	↑1	↑2	↑2	↖3	↑3	↖4
B	0						



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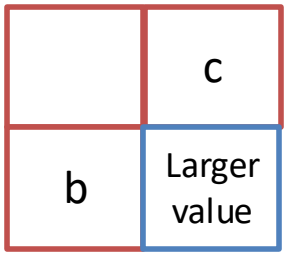

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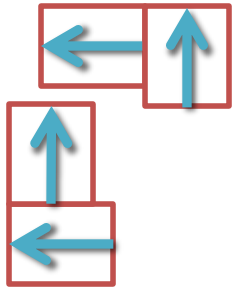

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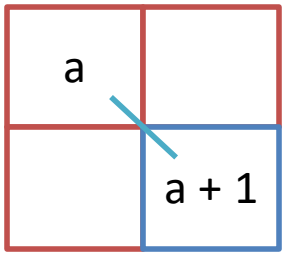
B	0	↖1	↑1	↑2	↑2	↖3	←3
D	0	↑1	↖2	↑2	↑2	↑3	↑3
A	0	↑1	↑2	↑2	↖3	↑3	↖4
B	0						



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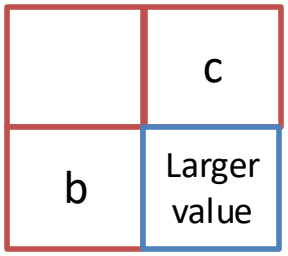

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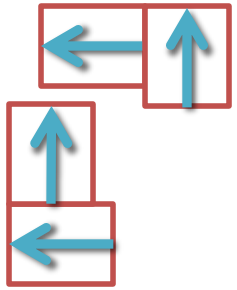

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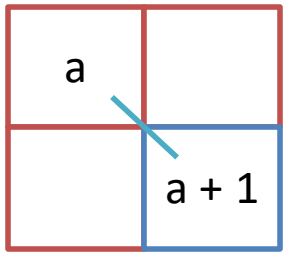

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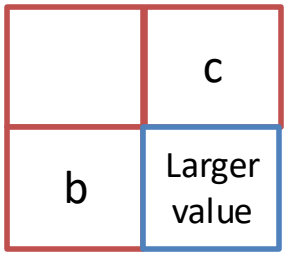

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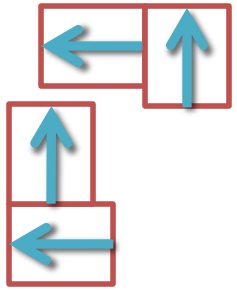

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D	0	↑1	↖2	↑2	↑2	↑3	↑3
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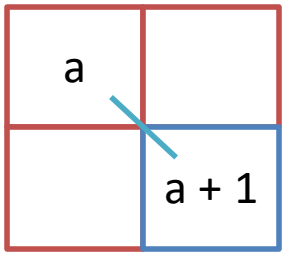

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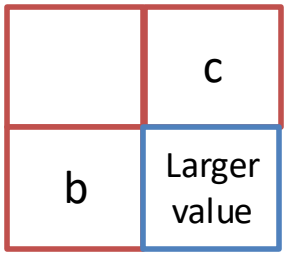

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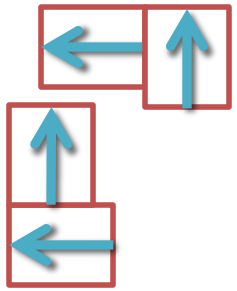

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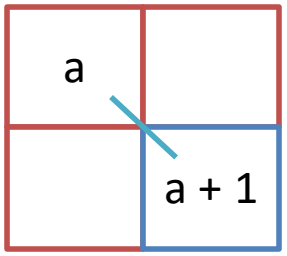

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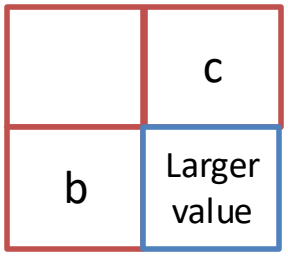

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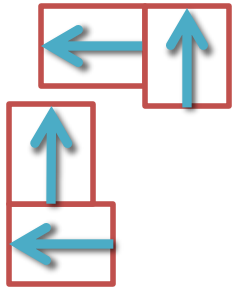

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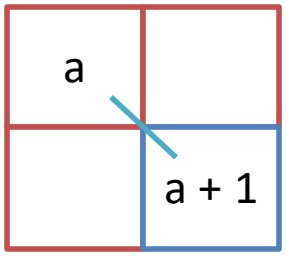

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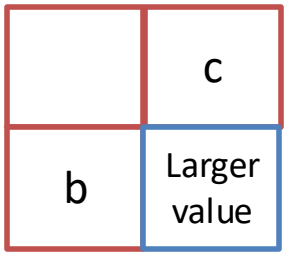

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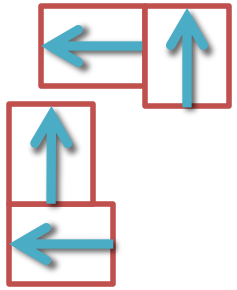

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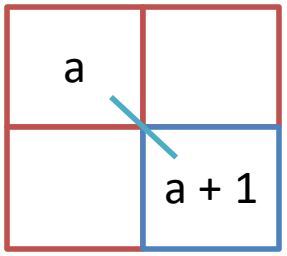

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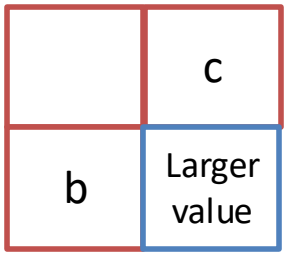

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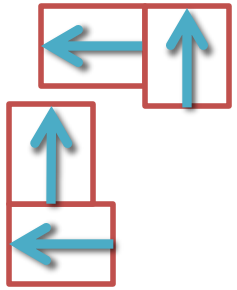

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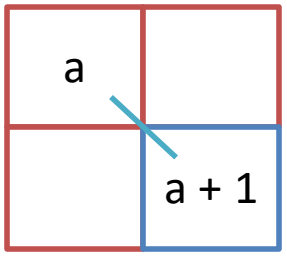

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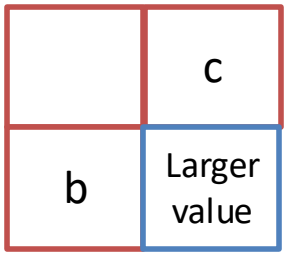

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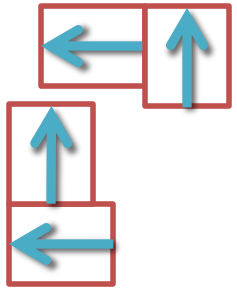

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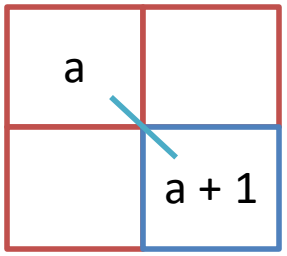
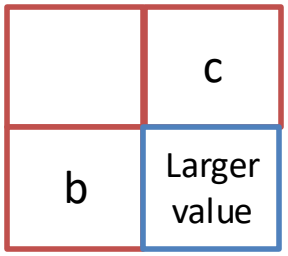
		B	D	C	A	B	A
	0	0	0	0	0	0	0
A	0	↑0	↑0	↑0	↖1	←1	↖1
B	0	↖1	←1	←1	↑1	↖2	←2
C	0	↑1	↑1	↖2	←2	↑2	↑2
B	0	↖1	↑1	↑2	↑2	↖3	←3
D	0	↑1	↖2	↑2	↑2	↑3	↑3
A	0	↑1	↑2	↑2	↖3	↑3	↖4
B	0	↖1	↑2	↑2	↑3		



match!



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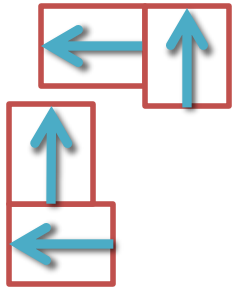

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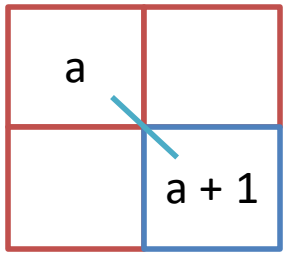

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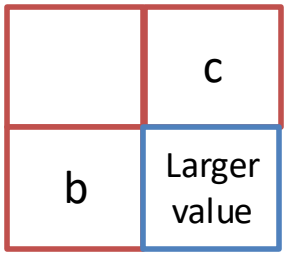

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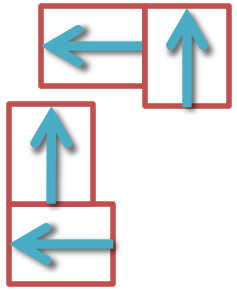

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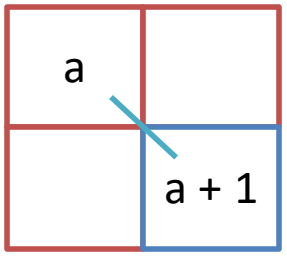

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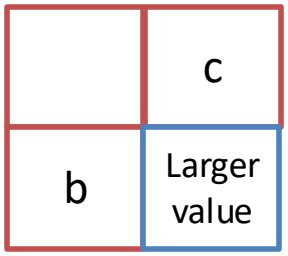

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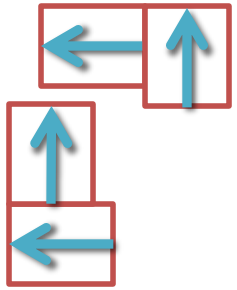

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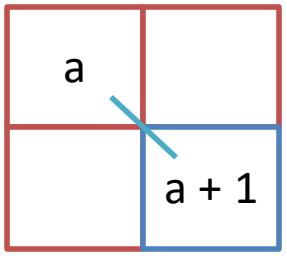

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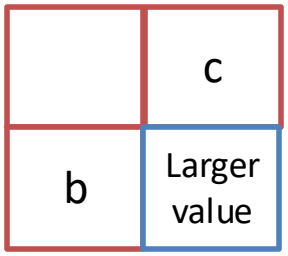

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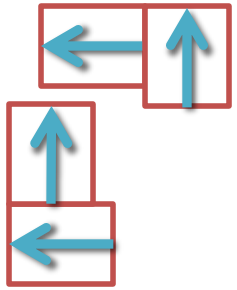

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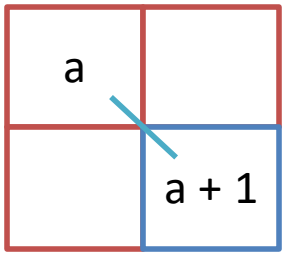

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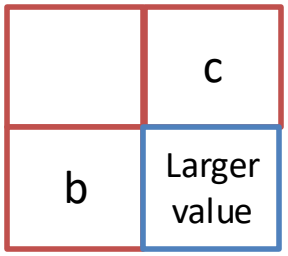

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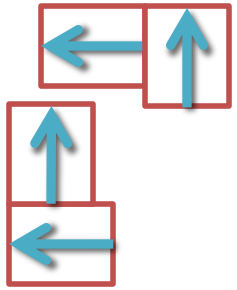

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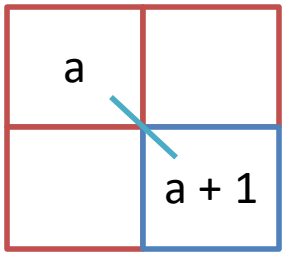

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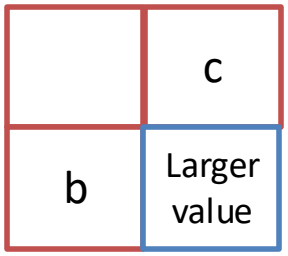

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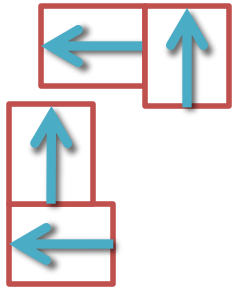

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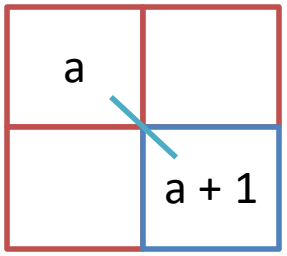

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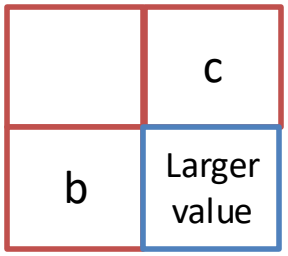

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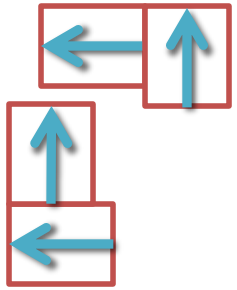

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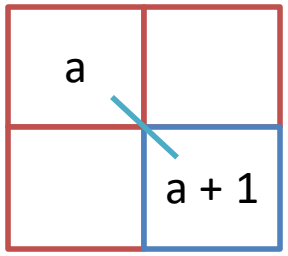

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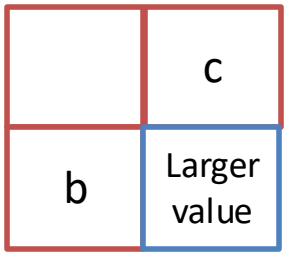

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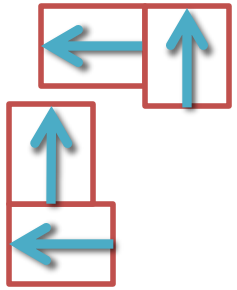

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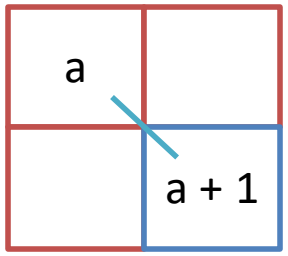

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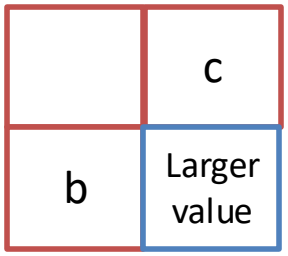

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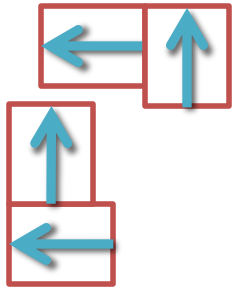

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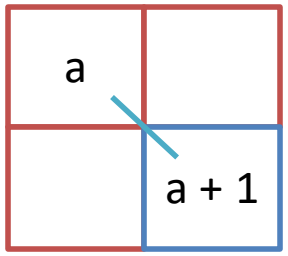

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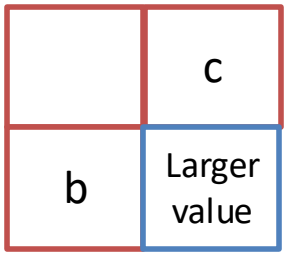

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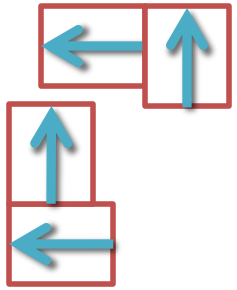

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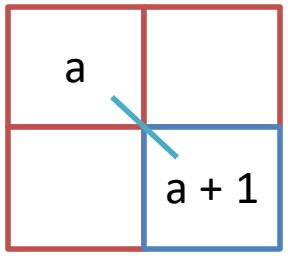
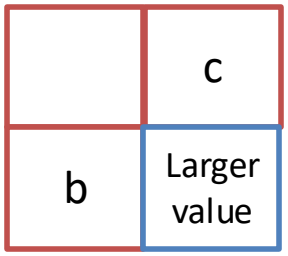


match!



no match!

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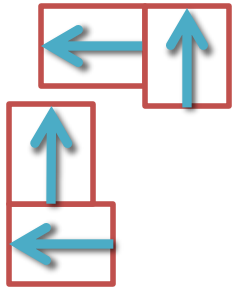
Example



String 1: BDCABA

String 2: ABCBDAB

What's the LCS of the two strings?

		B	D	C	A	B	A
	0	0	0	0	0	0	0
A	0	↑0	↑0	↑0	↖1	←1	↖1
B	0	↖1	←1	←1	↑1	↖2	←2
C	0	↑1	↑1	↖2	←2	↑2	↑2
B	0	↖1	↑1	↑2	↑2	↖3	←3
D	0	↑1	↖2	↑2	↑2	↑3	↑3
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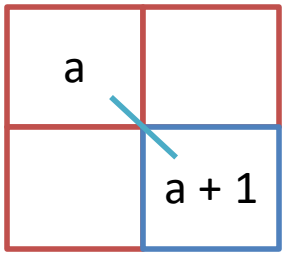

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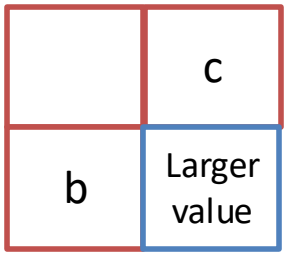

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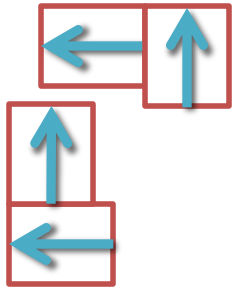

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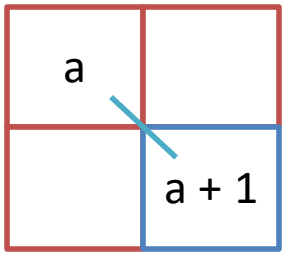

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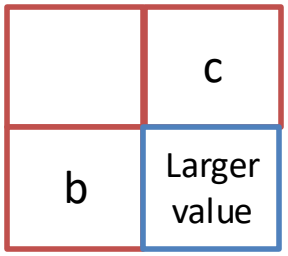

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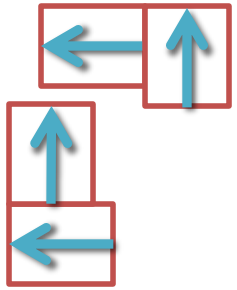

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D	0	↑1	↖2	↑2	↑2	↑3	↑3
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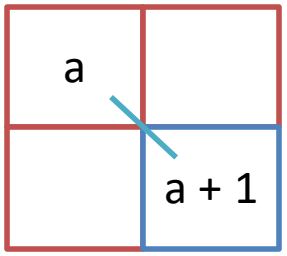

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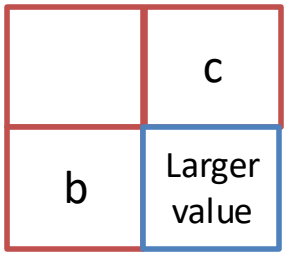

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Optimal Substructure

- The subproblems correspond to pairs of "prefixes" of the two input sequences

- LCS of two sequences contains within it an LCS of prefixes of 2 sequences

Prefix

Given a sequence A = {a_1 ,a_2 ,...,a_n }, we define the ith prefix of A, for i = 0 to n, as A_i = {a_1 ,a_2 ,...,a_i }

- Example: A = "ABCBDAB“
- A_4 = "ABCB“
- A_0 = ""

Theorem

- Let A = $\{a_1, a_2, \ldots, a_n\}$ and B = $\{b_1, b_2, \ldots, b_m\}$ be subsequences, and let C = $\{c_1, c_2, \ldots, c_k\}$ be any LCS of A and B

Step 2

- Recursively Defining Value of Optimal Solution
- Theorem implies that there are either one or two subproblems to examine when finding LCS of A and B

Subproblems

If $a_n = b_m$, we must find LCS of A_{n-1} and B_{m-1} :

- appending $a_n = b_m$ to this LCS yields an LCS of A and B

If $a_n \neq b_m$, we must solve two subproblems :

- finding an LCS of A_{n-1} and B and finding an LCS of A and B_{m-1} .

- Whichever of the two is longer is an LCS of A and B

Subproblems

- Because these three cases exhaust all possibilities, we know that one of the optimal subproblem solutions must be used within an LCS of A and B

- Can you already see the overlapping subproblems?

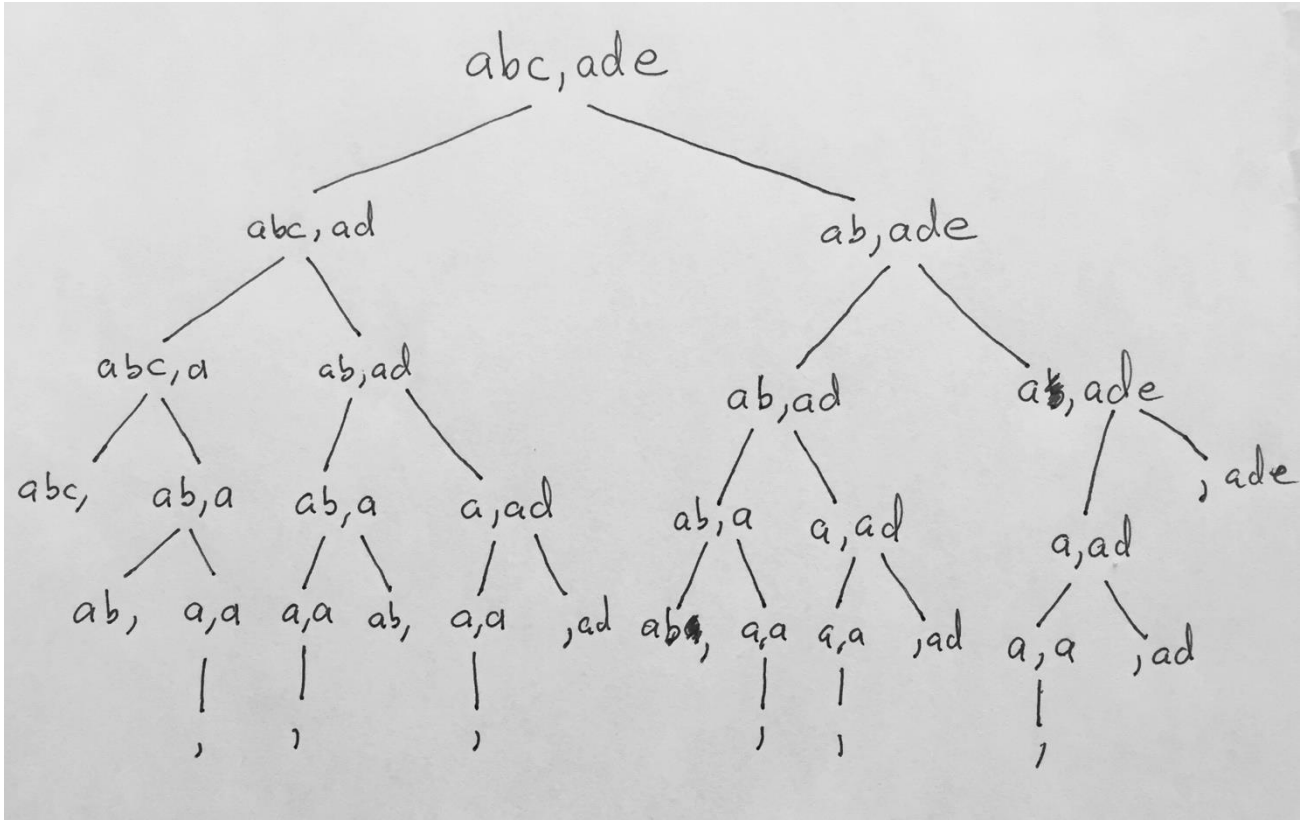
Overlapping subproblems

To find LCS of A and B, we may need to find the LCS of (A_{n-1} ,B) and (A,B_{m-1})

- But each of this subproblems has the subsubproblem of finding the LCS of A_{n-1} ,B_{m-1}

- Many other subproblems share subsubproblems

Overlapping Subproblems



- https://www.usna.edu/Users/cs/wcbrown/courses/S18SI335/lec/l20/lec.html

Recurrence relation

c[i,j]= length of an LCS of the sequences A_i and B_j

$$c_{i,j} = \begin{cases} 0 & \textit{if } i=0 \text{ or } j=0 \\ C_{i-1,\,j-1} + 1 & \textit{if } a_i = b_j \\ \text{Max}(C_{i,j-1},\, C_{i-1,\,j}) & \textit{if } a_i \neq b_j \end{cases}$$

Value of the optimal solution

- It also maintains the table d[1..m,1..n] to simplify the construction of an optimal solution
- Intuitively, d[i,j] points to the table entry corresponding to the optimal subproblem solution chosen when computing c[i,j]

Value of optimal solution

- Compute the length of LCS by defining a procedure LCS_LENGTH that takes two sequences A and B as inputs

- It stores the $c_{[i,j]}$ values in a table $c_{[0..m,0..n]}$ (m x n table) whose entries are computed in row-major order (first row, LTR, second row, LTR, so on)

LCS_LENGTH

```
LCS_LENGTH (A,B)
1       m = length(A), n = length(B)
2       for i = 1 to m
3               c[i,0] = 0
4       for j = 0 to n
5               c[0,j] = 0
6       for i = 1 to m
7               for j = 1 to n
8                       if a_i == b_j
9                               c[i,j] = c[i-1, j-1] + 1
10                              d[i,j] = "↖"
11                      else
12                              if c[i-1,j] ≥ c[i,j-1]
13                                      c[i,j] = c[i-1,j]
14                                      d[i,j] = "↑"
15                              else
16                                      c[i,j] = c[i,j-1]
17                                      d[i,j] = "←"
18      return c and d
```

Analysis

- The LCS_LENGTH procedure runs in O(mn) time
- Filling up a m x n table = mn time O(1) time to compute each entry

Constructing LCS

- Step 4 is constructing the optimal solution from the computed info
- The d table returned by LCS_LENGTH can be used to quickly construct an LCS of A and B

Constructing LCS

- Begin at d[m,n] and trace through the table following the directions
- Using this method, we will encounter the LCS in reverse order

Print LCS

- We can use a recursive procedure to print out the LCS of A and B in the proper, forward order
- Initial Invocation: PRINT_LCS(d,A,length(A),length(B))

PRINT_LCS

```
PRINT_LCS (d, A, i, j)
1       if i == 0 or j == 0
2               return
3       if d[i,j] == "↖"
4               PRINT_LCS (d, A, i-1, j-1)
5               print a_i
6       else if d[i,j] == "↑"
7               PRINT_LCS (d, A, i-1, j)
8       else if d[i,j] == "←"
9               PRINT_LCS (d, A, i, j-1)
```

Analysis

- Procedure takes O(m+n) time
- At least one of i and j is decremented in each stage of the recursion

End of Lecture

Practice Exercise

Item	Value/Benefit	Weight (kg)
A	3	5
B	7	4
C	5	6
D	9	3
E	4	2
F	3	7

X = "HELLOWORLD" Y = "HOLDTHELINE"

Example of Multiple LCS
Given Strings:
- **X = "ACDBE"**
- **Y = "ABCDE"**

The **LCS length = 4**, and we have **two possible LCS**:
1."ACDE" (Following a diagonal path choosing "A → C → D → E")
2."ABDE" (Following another diagonal path choosing "A → B → D → E")

X \ Y	Ø	A	B	C	D	E
Ø	0	0	0	0	0	0
A	0	1	1	1	1	1
C	0	1	1	2	2	2
D	0	1	1	2	3	3
B	0	1	2	2	3	3
E	0	1	2	2	3	4