The expressions for A, B, and C stated in [JSR] are only applicable in the limit $N \pm 1 = N$ therefore the expressions for σ_a, σ_v, and γ are not optimal for the intermediate N that DFXRM utilizes. Here we derivation of σ_a, σ_v, and γ without the assumption of large N:

$$\frac{\mu}{R}\sum_{n=1}^{N} y_n^2 = \frac{(\alpha+\gamma y)^2}{2\sigma_a^2} + \frac{y^2}{2\sigma_v^2} \tag{1}$$

$$y_n = (d\alpha + y)T_1 + f\phi\alpha T_2 = \alpha T_3 + yT_1 \tag{2}$$

Assuming small ϕ so that $1+\phi^2 = 1$ the parameters are:

$$\begin{aligned}
T_1 &= \cos(n\phi) + \frac{\phi}{2}\sin(n\phi) = \cos\left(\left(n-\frac{1}{2}\right)\phi\right) && (3)\\
T_2 &= \sin(n\phi) - \frac{\phi}{2}\cos(n\phi) = \sin\left(\left(n-\frac{1}{2}\right)\phi\right) && (4)\\
T_3 &= dT_1 + f\phi T_2 && (5)
\end{aligned}$$

$$y_n^2 = \alpha^2 T_3^2 + y^2 T_1^2 + 2\alpha y T_1 T_3 \tag{6}$$

$$\begin{aligned}
\sum_{n=1}^{N} y_n^2 &= \alpha^2 \sum_{n=1}^{N} T_3^2 + y^2 \sum_{n=1}^{N} T_1^2 + 2\alpha y \sum_{n=1}^{N} T_1 T_3 && (7)\\
&= \alpha^2 A + 2\alpha y B + y^2 C && (8)\\
&= A\left(\alpha + y\frac{B}{A}\right)^2 + \left(C - \frac{B^2}{A}\right) y^2 && (9)
\end{aligned}$$

Defining the trigonometric functions S_1 and S_2:

$$\begin{aligned}
S_1 &= \frac{\sin((N+1)\phi)\cos((N-1)\phi))}{\sin\phi} - 1 && (10)\\
&\approx \frac{1}{\phi}\sin(N\phi)\cos(N\phi) = \frac{1}{2\phi}\sin(2N\phi) = N\mathrm{sinc}(2N\phi) && (11)\\
S_2 &= \frac{\sin{(N+1)\phi}\sin{(N-1)\phi)}}{\sin\phi} && (12)\\
&\approx \frac{1}{\phi}\sin^2(N\phi) = \frac{1}{2\phi}\left(1 - \cos(2N\phi)\right) && (13)
\end{aligned}$$

$$\sum_{n=1}^{N} T_1^2 = \sum_{n=1}^{N} \cos^2\left(\left(n-\frac{1}{2}\right)\phi\right) = \frac{1}{2}\left(N+S_1\right) \tag{14}$$

$$\sum_{n=1}^{N} T_2^2 = \sum_{n=1}^{N} \sin^2\left(\left(n-\frac{1}{2}\right)\phi\right) = \frac{1}{2}\left(N-S_1\right) \tag{15}$$

$$\sum_{n=1}^{N} T_1 T_2 = \sum_{n=1}^{N} \cos\left(\left(n-\frac{1}{2}\right)\phi\right)\sin\left(\left(n-\frac{1}{2}\right)\phi\right) = \frac{1}{2}S_2 \tag{16}$$

$$\begin{aligned}
\sum_{n=1}^{N} T_3^2 &= \sum_{n=1}^{N} (dT_1 + f\phi T_2)^2 && (17)\\
&= d^2 \sum_{n=1}^{N} T_1^2 + (f\phi)^2 \sum_{n=1}^{N} T_2^2 + 2df\phi \sum_{n=1}^{N} T_1 T_2 && (18)\\
&= \frac{1}{2}\left(N(d^2+(f\phi)^2) + (d^2-(f\phi)^2)S_1 + 2df\phi S_2\right) && (19)
\end{aligned}$$

$$\begin{aligned}
\sum_{n=1}^{N} T_1 T_3 &= \sum_{n=1}^{N} (dT_1 + f\phi T_2)T_1 = d\sum_{n=1}^{N} T_1^2 + f\phi \sum_{n=1}^{N} T_1 T_2 && (20)\\
&= \frac{1}{2}\left(d\left(N+S_1\right) + f\phi S_2\right) && (21)
\end{aligned}$$

The coefficients are:

$$A = \sum_{n=1}^{N} T_3^2 = \frac{1}{2}\left(N(d^2+(f\phi)^2) + (d^2-(f\phi)^2)S_1 + 2df\phi S_2\right) \tag{22}$$

$$B = \sum_{n=1}^{N} T_1 T_3 = \frac{1}{2}\left(d\left(N+S_1\right) + f\phi S_2\right) \tag{23}$$

$$C = \sum_{n=1}^{N} T_1^2 = \frac{1}{2}\left(N+S_1\right) \tag{24}$$

Furthermore, for the vignetting we derive $AC - B^2$:

$$\begin{aligned}
AC - B^2 &= \sum_{n=1}^{N} T_1^2 \sum_{n=1}^{N} T_3^2 - \left(\sum_{n=1}^{N} T_1 T_3\right)^2 && (25)\\
&= \sum_{n=1}^{N} T_1^2 \sum_{n=1}^{N} (dT_1 + f\phi T_2)^2 - \left(\sum_{n=1}^{N} (dT_1 + f\phi T_2)T_1\right)^2 && (26)\\
&= (f\phi)^2 \left(\sum_{n=1}^{N} T_1^2 \sum_{n=1}^{N} T_2^2 - \left(\sum_{n=1}^{N} T_1 T_2\right)^2\right) && (27)\\
&= \left(\frac{f\phi}{2}\right)^2 \left(N^2 - S_1^2 - S_2^2\right) && (28)
\end{aligned}$$

This expression is independent of the distance to the sample, d. Now we can derive expressions for σ_D,σ_a, σ_v, and γ, with $D = \frac{d}{f\phi}$:

$$\frac{1}{2\sigma_D^2} = \frac{\mu}{R} C \Rightarrow \tag{29}$$

$$\sigma_D = \sqrt{\frac{R}{\mu}} \sqrt{\frac{1}{2C}} = \sqrt{\frac{R}{\mu}} \left[N + S_1\right]^{-\frac{1}{2}} \tag{30}$$

$$\frac{1}{2\sigma_a^2} = \frac{\mu}{R} A \Rightarrow \tag{31}$$

$$\sigma_a = \sqrt{\frac{R}{\mu}} \sqrt{\frac{1}{2A}} \tag{32}$$

$$= \sqrt{\frac{R}{\mu}} \frac{1}{f\phi} \left[N(1 + D^2) - (1 - D^2) S_1 + 2DS_2\right]^{-\frac{1}{2}} \tag{33}$$

$$\frac{1}{2\sigma_v^2} = \frac{\mu}{R} \frac{AC - B^2}{A} \Rightarrow \tag{34}$$

$$\sigma_v = \sqrt{\frac{R}{2\mu}} \sqrt{\frac{A}{AC - B^2}} \tag{35}$$

$$= \frac{1}{2} \sqrt{\frac{R}{\mu}} \left[\frac{N\left(1 + D^2\right) - \left(1 - D^2\right) S_1 + 2DS_2}{(N^2 - S_1^2 - S_2^2)}\right]^{\frac{1}{2}} \tag{36}$$

$$\gamma = \frac{B}{A} = \frac{d\,(N + S_1) + f\phi S_2}{N(d^2 + (f\phi)^2) + (d^2 - (f\phi)^2) S_1 + 2df\phi S_2} \tag{37}$$

$$= \frac{1}{f\phi} \frac{D\,(N + S_1) + S_2}{N(1 + D^2) - (1 - D^2) S_1 + 2DS_2} \tag{38}$$

Furthermore, from the definition of σ_v in terms of A, B, and C we have:

$$\frac{1}{2\sigma_v^2} = \frac{\mu}{R} \frac{AC - B^2}{A} = \frac{\mu}{R} \left(C - \left(\frac{B}{A}\right)^2 A\right) \tag{39}$$

$$= \frac{1}{2\sigma_D^2} - \gamma^2 \frac{1}{2\sigma_a^2} \tag{40}$$