02427 Advanced Time Series Analysis

Computer exercise 3

The topics of this exercise is an investigation of some properties of simulated solutions to stochastic differential equations (SDE), and estimation of parameters in SDEs.

Part 1: Simulation and discretization of diffusion processes

The Bonhoeffer-Van der Pol equations give a two-dimensional simplification of a famous four-dimensional system of ordinary differential equations proposed by Hodgkin and Huxley, in order to describe the firing of a single neuron. The equations are

$$\frac{dx_t^1}{dt} = \theta_3 \left(x_t^1 + x_t^2 - \frac{1}{3}(x_t^1)^3 + \theta_4 \right) \tag{1a}$$

$$\frac{dx_t^2}{dt} = -\frac{1}{\theta_3}(x_t^1 + \theta_2 x_t^2 - \theta_1) \tag{1b}$$

where x_t^1 is the negative potential over the membrane, x_t^2 is the permeability of the membrane, and $\theta_1, \theta_2, \theta_3$ and θ_4 denote the physical parameters. When $(\theta_1, \theta_2, \theta_3, \theta_4) = (0.7, 0.8, 3.0, -0.34)$ the system has a limit cycle, which describes the periodic slow charging and fast decharging that have been observed through experiments.

The effects of imperfections in the membrane and firing of the surrounding neurons can be simulated by incorporating additive noise with a small standard deviation in Eq. (1a). This leads to a two-dimensional system of Itô stochastic diffential equation

$$dX_t^1 = \theta_3 \left(X_t^1 + X_t^2 - \frac{1}{3}(X_t^1)^3 + \theta_4 \right) dt + \sigma dW_t \tag{2a}$$

$$dX_t^2 = -\frac{1}{\theta_3}\left(X_t^1 + \theta_2 X_t^2 - \theta_1\right) dt \tag{2b}$$

where W_t is a standard Wiener process and $\sigma > 0$ is the incremental standard deviation of the noise.

In order to be able to simulate Eq. (2a), it is necessary to discretize it, i.e. to consider finite steps in the Wiener process ΔW_n instead of the infinitesimal dW_t. It can be shown that the so called Euler-Marayama approximation to Eq. (2a) is given by the two-dimensional stochastic difference equation

$$\begin{aligned} Y_{n+1}^1 &= Y_n^1 + \theta_3 \left(Y_n^1 + Y_n^2 - \frac{1}{3}(Y_n^1)^3 + \theta_4 \right) \Delta + \sigma \Delta W_{n+1}^1 \\ Y_{n+1}^2 &= Y_n^2 - \frac{1}{\theta_3}\left(Y_n^1 + \theta_2 Y_n^2 - \theta_1\right)\Delta \end{aligned} \tag{3}$$

where Δ is a suitably chosen small time interval, $\Delta W_{n+1} \in N(0, \Delta)$ and $(Y_n^1, Y_n^2), n = 1, ..., N$ is a discrete approximation to (X_t^1, X_t^2) in the time interval $0 \le t \le T$.

Question 1a

Let $\Delta = 2^{-9}$, $\theta_1 = 0.7$, $\theta_2 = 0.8$, $\theta_3 = 3.0$, $\theta_4 = -0.34$ og $\sigma = 0$. Simulate Eq. (3) in the time interval $0 \le t \le T = 100$ with initial conditions $Y_0^1 = -1.9$ and $Y_0^2 = 1.2$.

Plot the realizations of Y_k^1 and Y_k^2 and make a phaseplot of (Y_k^1, Y_k^2). Repeat for $\sigma = 0.10, 0.20, 0.30$ and 0.40. Comment on the effect of adding noise to the equations.

Question 1b

The Euler-Marayama method is an example of a *weak method*, implying that it only gives an approximation of (functionals of) the moments of X_t^1, X_t^2, whereas a strong method approximates the whole distribution. However, a weak method can give some essential visual information about the performance of the stochastic dynamic system in question. A histogram might for example indicate the form and support of the density function of the asymptotically stable, stationary solution.

Let $\sigma = 0.10$ and simulate Eq. (2a) using the approximation Eq. (3) with the same parameter values as given above (you may reuse the results from question 1 if you like). Partition the phase plane in $100\times$ 100 equal cells. Count the number of trajectories that passes through each cell, and make a three-dimensional plot (the units on the axes have no essential meaning in this case).

Which extra information does the plot contain, compared to the standard phase-plot?

Repeat for $\sigma = 0.20, 0.30$ and 0.40 (again using the results from question 1 if you like).

Hints

You can download a Matlab and R script for making the 2D histogram plot.

Part 2: Models for the heat dynamics of a high performance test building
This part deals with modeling of the heat dynamics of a high performance test building with grey-box models based on SDEs. Data from an experiment carried out at DTU is provided. The task is to find a model which describes the heat dynamics of the building and provides a description thermal performance parameters.

CTSM-R

In order to carry out this part of the exercise it is needed to install R[1] and CTSM-R, see the Userguide (`ctsmr.pdf` in the `.zip` file). Furthermore it is recommend to install an editor for R, especially RStudio is a good choice.

High performance test building

The high performance test building is a single-storey building with a pitched roof, see the photo in Figure 1. The loft floor is used as office and the ground floor forms a test area

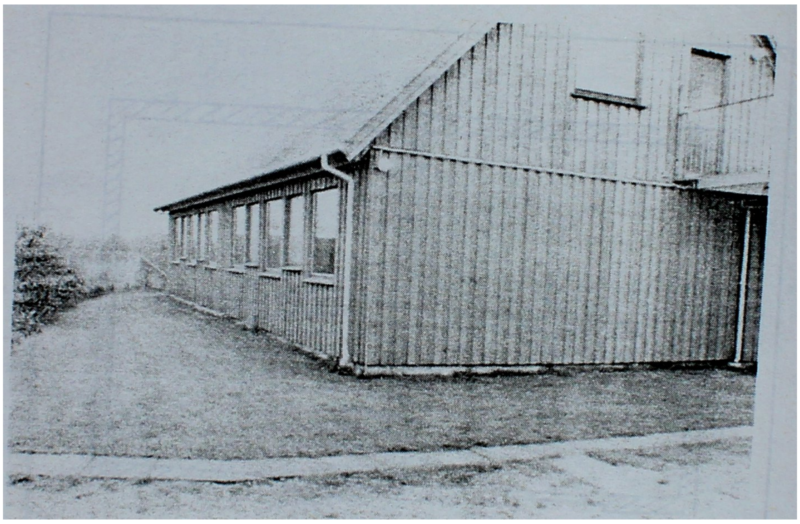

Figure 1: The test building.

of 120 m^2. The loft floor is used as an office space, which is accessed from an outdoor stair. Heat loss through the ceiling between the ground and loft floor can be considered adiabatic (there no significant heat flow to the loft) since the temperature is kept at 20 °C by a separate heater in the loft rooms. The test area is divided into two equally spaced rooms: each room is 60 m^2 as illustrated in Figure 2. The room height is 2.4 meters. The building envelope is extremely air tight and very well insulated. The windows are

[1] `www.r-project.org`

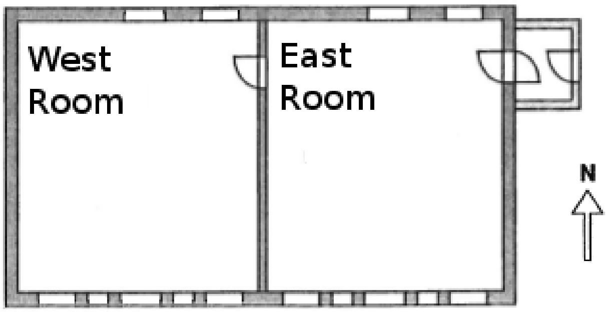

Figure 2: Sketch of the ground plan of the test building.

positioned identically in both rooms, each room having 7.5 m^2 facing south and 4.8 m^2 facing north. The wall separating the two rooms is insulated with 95 mm mineral wool and in each room three electrical heaters are placed that can provide in total 1.5 kW.

The West room has a thermally light floor and the East room has a concrete floor with a thickness of 75 mm.

Furthermore the East Room is filled with concrete flagstones (specific heat capacity of 0.879 kJ/(kg K), i.e 0.879/3600 kWh/(kg K)) in order to increase the heat capacity of the room. The concrete flagstones are each 500x500x50 mm. They are placed on racks to provide heat transport by convection on both sides, as seen on the photo in Figure 3. Some further description of the building is found in Appendix A.

Data

In the experiment the heat input was controlled with a PRBS sequence with $n = 6$ and $\lambda = 1$h, see [Godfrey(1980)] for more details. Time series of the recorded signals are found in the attached file `data.csv`. The columns are:

- `timedate` The time of the sample in UTC.
- `TiE` The indoor air temperature of the East room (°C).
- `TiW` The indoor air temperature of the West room (°C).
- `Ta` The ambient temperature (°C).
- `Gv` The solar radiation on a vertical surface facing south (kW/m^2).
- `PhE` The power of the heater in the East room (kW).
- `PhW` The power of the heater in the West room (kW).

The sample period is 10 minutes and the samples are instantaneous values.

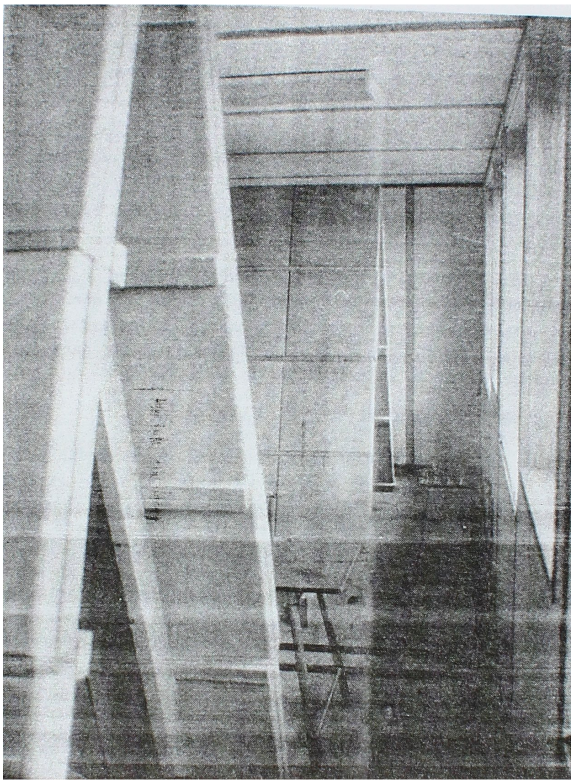

Figure 3: Concrete flagstones on racks in the East Room.

Question 2a

First it is noted that a description of all the parameters used in the models below is given in Appendix B. Some tips on creating RC-diagrams in LaTeXdocuments are given in Appendix C.

Use the script in `fitmodelQ2a.R` to get started. Run the script step-by-step and get to know how the models are fitted and analyzed, and read the following points alongside

1. Data is plotted as time series.
2. The most simple feasible model *Ti* is fitted using CTSM-R. It is a one-state model with the system equation

$$dT_{\mathrm{i}} = \Big(\frac{1}{R_{\mathrm{ia}} C_{\mathrm{i}}} (T_{\mathrm{a}} - T_{\mathrm{i}}) + \frac{1}{C_{\mathrm{i}}} \Phi_{\mathrm{h}} \Big) dt + \sigma_{\mathrm{i}} d\omega_{\mathrm{i}} \tag{4}$$

   and the measurement equation

$$(Y_{t_k} = T_{\mathrm{i},t_k} + e_{t_k} \tag{5}$$

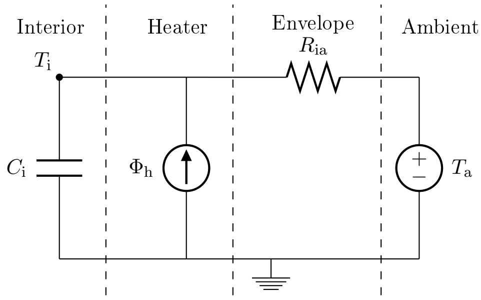

Figure 4: RC-network of the most simple model.

The RC-diagram in Figure 4 illustrates the deterministic part of the system equation.

3. The summary of the fit and estimated parameters is run. Note that the optimization of the loglikelihood did not end well, please consider Section 3.6.1 "Summary of estimated parameters" in the CTSM-R Userguide (`ctsmr.pdf`) to see the important steps in interpreting the summary of the result and fix the problem.

4. The fit of model *Ti* is analyzed with time series plots, and the ACF, periodogram and cumulated periodogram of the residuals.

5. A two-state model *TiTm* is fitted using CTSM-R. It has the system equations

$$dT_{\mathrm{i}} = \left(\frac{1}{R_{\mathrm{im}}C_{\mathrm{i}}}(T_{\mathrm{m}} - T_{\mathrm{i}}) + \frac{1}{R_{\mathrm{ia}}C_{\mathrm{i}}}(T_{\mathrm{a}} - T_{\mathrm{i}}) + \frac{1}{C_{\mathrm{i}}}\Phi_{\mathrm{h}}\right)dt + \sigma_{\mathrm{i}}d\omega_{\mathrm{i}} \tag{6}$$

$$dT_{\mathrm{m}} = \left(\frac{1}{R_{\mathrm{im}}C_{\mathrm{m}}}(T_{\mathrm{i}} - T_{\mathrm{m}})\right)dt + \sigma_{\mathrm{m}}d\omega_{\mathrm{m}} \tag{7}$$

and the measurement equation is still

$$Y_{t_k} = T_{\mathrm{i},t_k} + e_{t_k} \tag{8}$$

The RC-diagram in Figure 5 illustrates the deterministic part of the system equation.

6. The fit of model *TiTm* is analyzed.

7. The model *Ti* and model *TiTm* are compared with a loglikelihood-ratio test.

Consider the following in the report

- Discuss the white-noise properties of the (one-step ahead) residuals for model *Ti*.
- What useful information can be obtained from the time series plots of the residuals and the inputs for model *Ti*?

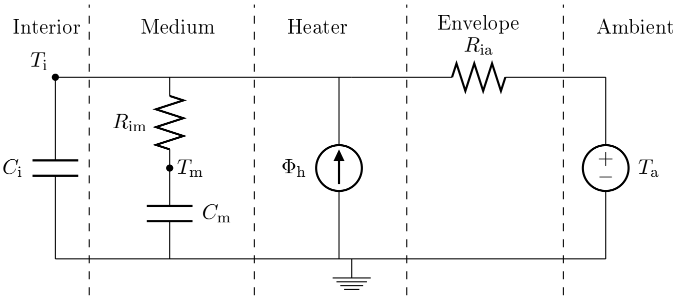

Figure 5: RC-network of the most simple model.

- Discuss the white-noise properties of the one-step ahead residuals for model *TiTm*.
- What useful information can be obtained from the time series plots of the residuals and inputs for model *TiTm*?
- Based on the likelihood-ratio test is model *TiTm* then to be preferred over model *Ti*?

Question 2b

Next task is to extend the model to include the solar radiation `Gv`. Find a good way to include it in the model by

- either include it in different ways, for example let it enter into T_{m} with

$$dT_{\mathrm{m}} = \left(\frac{1}{R_{\mathrm{im}}C_{\mathrm{m}}}(T_{\mathrm{i}} - T_{\mathrm{m}}) + \frac{A_{\mathrm{w}}}{C_{\mathrm{m}}}G_{\mathrm{v}}\right)dt + \sigma_{\mathrm{m}}d\omega_{\mathrm{m}} \tag{9}$$

  in one model and into T_{i} in another. Compare the performance of the models with their likelihoods.

- or make a linear interpolation with

$$\begin{aligned} dT_{\mathrm{i}} &= \left(\frac{1}{R_{\mathrm{im}}C_{\mathrm{i}}}(T_{\mathrm{m}} - T_{\mathrm{i}}) + \frac{1}{R_{\mathrm{ia}}C_{\mathrm{i}}}(T_{\mathrm{a}} - T_{\mathrm{i}}) + \frac{pA_{\mathrm{w}}}{C_{\mathrm{i}}}G_{\mathrm{v}} + \frac{1}{C_{\mathrm{i}}}\Phi_{\mathrm{h}}\right)dt + \sigma_{\mathrm{i}}d\omega_{\mathrm{i}} \qquad (10) \\ dT_{\mathrm{m}} &= \left(\frac{1}{R_{\mathrm{im}}C_{\mathrm{m}}}(T_{\mathrm{i}} - T_{\mathrm{m}}) + \frac{(1-p)A_{\mathrm{w}}}{C_{\mathrm{m}}}G_{\mathrm{v}}\right)dt + \sigma_{\mathrm{m}}d\omega_{\mathrm{m}} \end{aligned}$$

In the report consider:

- The findings regarding the extension with the solar radiation.
- Considering a likelihood-ratio test should the solar radiation be included in the model?

Question 2c

Identify a suitable model for the West room. An easy way to do this is simply to rename the inputs in order to use the same implementation of the models, i.e. same functions for fitting and validating. In the report:

- Describe how the model was identified and validated.
- Consider the estimated heat capacities of the rooms and compare this with the amount of concrete in the each room. Argue if the estimated values seems reasonable or not?

Question 2d (Optional)

It is both possible to fit a model for each separate room or one model including both rooms. The latter should have states for each room and two outputs, which are simply specified by adding two observation equations, for example by

```
model$addSystem(dTiE ~ (...)*dt + exp(p11E)*dw1)
model$addSystem(dTiW ~ (...)*dt + exp(p11W)*dw2)
...
## Set the observation equation: Ti is the state, yTi is the measured output
model$addObs(yTiE ~ TiE)
model$addObs(yTiW ~ TiW)
## Set the variance of the measurement error
model$setVariance(yTiE ~ exp(e11E))
model$setVariance(yTiW ~ exp(e11W))
```

where `TiE` is the East Room indoor temperature state, `TiW` is the West Room indoor temperature state, `yTiE` is the observed temperature in the East Room, and `yTiW` is the observed temperature in the West Room. See the function in `r/functions/analyzeFitEW.R` for details on getting the one-step predictions from a multi-output model.

Appendix

A Some more description of the building

The building is extremely air tight (an air change rate around 0.005 ACH, measured by means of tracer gas). The outer walls and ceiling of the ground floor are light sandwich constructions, based on a Masonite beam insulated with 300 mm mineral wool. All windows are triple glazed in wooden frames. The floors are composed of 22 mm plywood, 300 mm mineral wool, 22 mm plywood. On top of this, the East Room has 75 mm concrete, while the West Room has 50 mm polystyrene and 22 mm chipboard.

Sensors

Each room is equipped with a thermocouple (type T) air temperature sensor, which are positioned in the northern part of the rooms and shielded for solar radiation. The ambient temperature is measured next to the building in a height of 1.5 meters. The solar radiation is measured as the vertical total radiation the south facing wall. The radiation sensor type is unknown.

B Parts and parameters in the models

The individual model parts are indicated on the figures. The model parts are:

Interior In the full model the interior is considered to be the indoor air (again remember that, since the models are lumped models, the building part represented by "Interior" is mostly different for each model) and it is modelled as a heat capacity connected to other parts by thermal resistances.

Medium A thermal medium inside the building is the interior walls and furniture, which is modelled with a heat capacity and a thermal resistance to the interior.

Heater The heaters are modelled by a heat capacity and a thermal resistance to the interior.

Solar The heat input from solar radiation is modelled by the global irradiance multiplied with the effective window area.

Envelope The building envelope is modelled with a heat capacity and thermal resistances to both the interior and the ambient. A thermal resistance directly coupled to the ambient is also included.

Ambient The ambient is represented by the observed ambient air temperature.

The full model includes five state variables, that each represents the temperature in a part of the building, and they are:

T_{i} The temperature of the indoor air. This is used as the output Y_{t_k} in the measurement equation, Eq. (5).

T_{m} The temperature of an interior thermal medium, i.e. interior walls and furniture.

The parameters of the model represent different thermal properties of the building. This includes thermal resistances:

R_{im} between the interior and the interior thermal medium,

R_{ia} between the interior and the ambient,

R_{in} between the interior and the other room.

The effective heat capacities of different parts of the building are represented by:

C_{i} for the indoor air,

C_{m} for the interior walls and furniture, i.e. in this case including the concrete flags.

Finally, more coefficients can be included in extended models. For example an estimate of an effective area in which the energy from solar radiation enters the building:

A_{w} The effective window area of the building.

C Creating RC-diagrams in LaTeXwith Circuitikz

The RC-diagrams in the document is created with CircuiTikz (`www.ctan.org/pkg/circuitikz`), which is a package for Tikz (`www.texample.net/tikz`). The Tikz code for the diagrams are in the files: `figures/Ti.tikz`, `figures/TiTm.tikz` and `figures/TiTmAw.tikz`. First, the CircuiTikz package must be installed and then the LaTeXcode for inclusion is: in the header (i.e. before `\begin{document}`)

```
%% For tikz graphics
\usepackage{tikz}
%% Required by circuitikz
\usepackage{xstring}
%% For drawing electrical networks
\usepackage[american]{circuitikz}
%% Controlling the size of the resistors
\ctikzset{bipoles/resistor/width=.7}
\ctikzset{bipoles/resistor/height=.36}
```

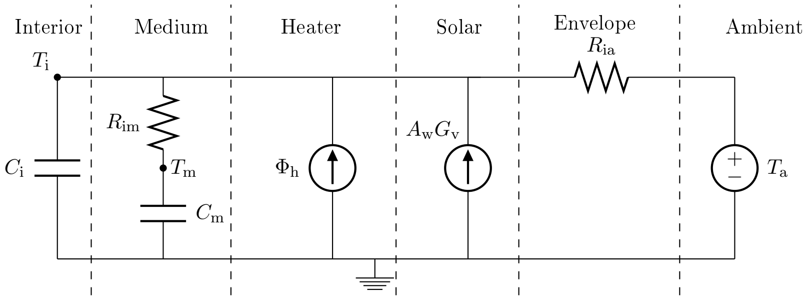

Figure 6: RC-network of a model example with the solar radiation entering directly into indoor air state.

```
% Variables
\newcommand{\n}[1]{_{\mathrm{#1}}}
\newcommand{\Aw}{A\n{w}}
\newcommand{\Ci}{C\n{i}}
\newcommand{\Cm}{C\n{m}}
\newcommand{\Gv}{G\n{v}}
\newcommand{\Ph}{\Phi\n{h}}
\newcommand{\Ria}{R\n{ia}}
\newcommand{\Rim}{R\n{im}}
\newcommand{\Rin}{R\n{in}}
\newcommand{\Ta}{T\n{a}}
\newcommand{\Ti}{T\n{i}}
\newcommand{\Tm}{T\n{m}}
```

and then in a figure

```
\begin{figure}[tb]
  \newcommand{\scaleTikz}{0.8}
  \centering
  \input{figures/TiTmAw.tikz}
  \caption{RC-network of a model example with the solar radiation entering directly into ir
  \label{fig:modelTiTmAw}
\end{figure}
```

This will create the diagram in Figure 6.

References

[Godfrey(1980)] K.R. Godfrey. Correlation methods. *Automatica*, 16(5):527–534, 1980. ISSN 00051098.