

Advanced Time Series Analysis: Computer Exercise 3

Anders Launer Bæk (s160159)

08 November 2017

Sparring partners:

- Anja Liljedahl Christensen (s162876)
- Marie Mørk (s112770)

Part 1: Simulation and discretization of diffusion processes

Equation 2a and 2b from the description have been discretized and they are showed in eqn. 1.

$$\begin{aligned} Y_{n+1}^1 &= Y_n^1 + \theta_3 \left(Y_n^1 + Y_n^2 - \frac{1}{3} (Y_n^1)^3 + \theta_4 \right) \Delta + \sigma \Delta W_{n+1}^1 \\ Y_{n+1}^2 &= Y_n^2 - \frac{1}{\theta_3} (Y_n^1 + \theta_2 Y_n^2 - \theta_1) \Delta \end{aligned} \quad (1)$$

The initial parameters for this diffusion process are given in eqn. 2.

$$\begin{aligned} \theta_{1,2,3,4} &= [0.7, 0.8, 3, -0.34] \\ \Delta &= 0.0019531 \\ \sigma &= 0 \\ T &= 100 \\ t &= 1 : \Delta : T \\ \Delta W_{n+1}^1 &\sim \mathcal{N}(0, \Delta) \end{aligned} \quad (2)$$

Question 1a

It is possible to change the process by changes the value of σ . An increase in σ will provide a bigger variation in the Wiener process. Below there have been plotted realizations of Y_k^1 and Y_k^2 wrt. time and a phase plot of Y_k^1 and Y_k^2 . The following function `model_func()` has been used to plot

```
# function ----
model_func <- function(sigma, delta, t, Theta, init_values) {
  # initialize data.frame and initial values
  data <- data.frame(T = t, Y_1 = NA, Y_2 = NA)
  data$Y_1[1] <- init_values[1]
  data$Y_2[1] <- init_values[2]
  # simulate wiener process
  set.seed(22)
  data$W <- rnorm(nrow(data), mean = 0, sd = delta)
  # run the simulation loop
  for (k in 1:(nrow(data) - 1)) {
    # Y_k^1
    data$Y_1[k + 1] <- data$Y_1[k] + Theta[3] * (data$Y_1[k] + data$Y_2[k] -
      1/3 * data$Y_1[k]^3 + Theta[4]) * delta + sigma * data$W[k + 1]
    # Y_k^2
    data$Y_2[k + 1] <- data$Y_2[k] - Theta[1] * (data$Y_1[k] + data$Y_2[k] -
      1/3 * data$Y_2[k]^3 + Theta[2]) * delta + sigma * data$W[k + 1]
  }
}
```

```

    data$Y_2[k + 1] <- data$Y_2[k] - 1/Theta[3] * (data$Y_1[k] + Theta[2] *
      data$Y_2[k] - Theta[1]) * delta
  }
# realizations
re_plot <- ggplot2::ggplot(data) + ggplot2::geom_point(ggplot2::aes(x = T,
  y = Y_1, color = "Y_k^1"), alpha = 1/2) + ggplot2::geom_point(ggplot2::aes(x = T,
  y = Y_2, color = "Y_k^2"), alpha = 1/2) + ggplot2::labs(x = "t", y = "Y_k^*(t)",
  color = "") + theme_TS()
# phase
ph_plot <- ggplot2::ggplot(data) + ggplot2::geom_point(ggplot2::aes(x = Y_1,
  y = Y_2, color = "Phase"), alpha = 1/2) + ggplot2::labs(x = "Y_k^1(t)",
  y = "Y_k^2(t)", color = "") + theme_TS()
return(list(sim = data, re_plot = re_plot, ph_plot = ph_plot))
}

```

$\sigma = 0.00$

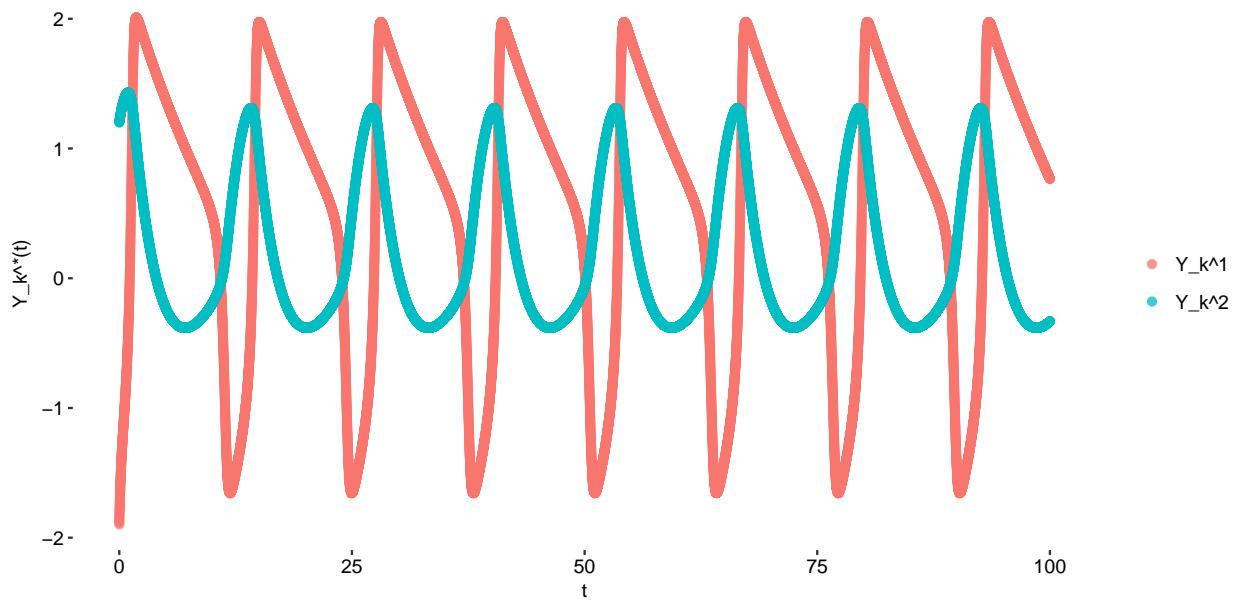


Figure 1: Plot of the simulation realizations with $\sigma = 0.0$.

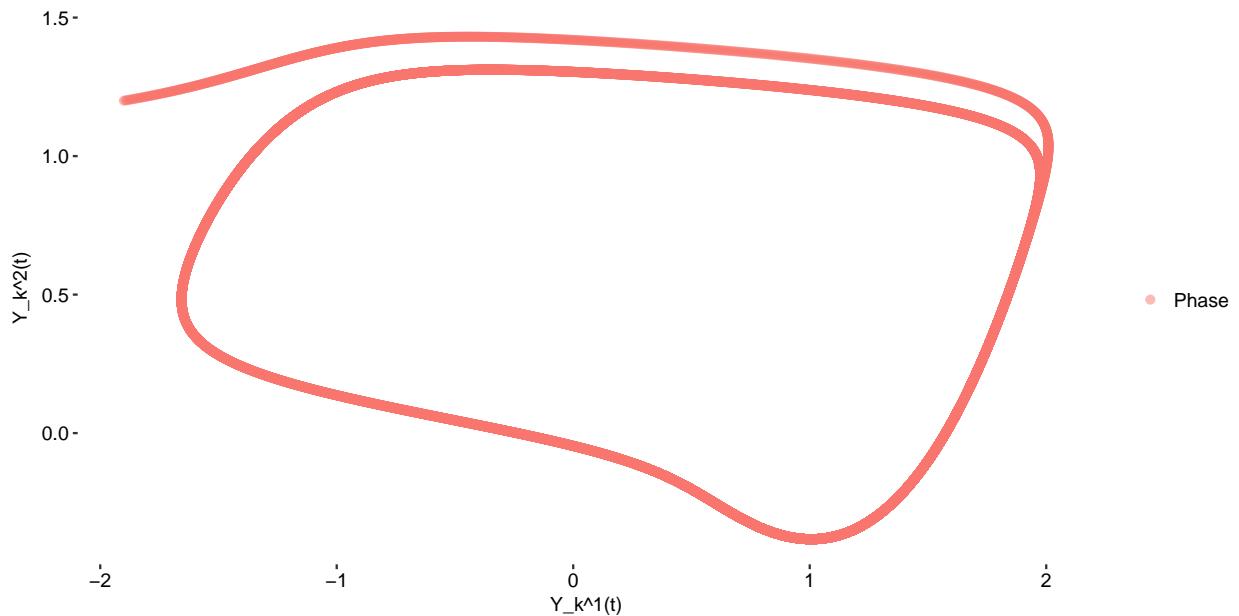


Figure 2: Phase plot of the simulation with $\sigma = 0.0$.

$\sigma = 0.10$

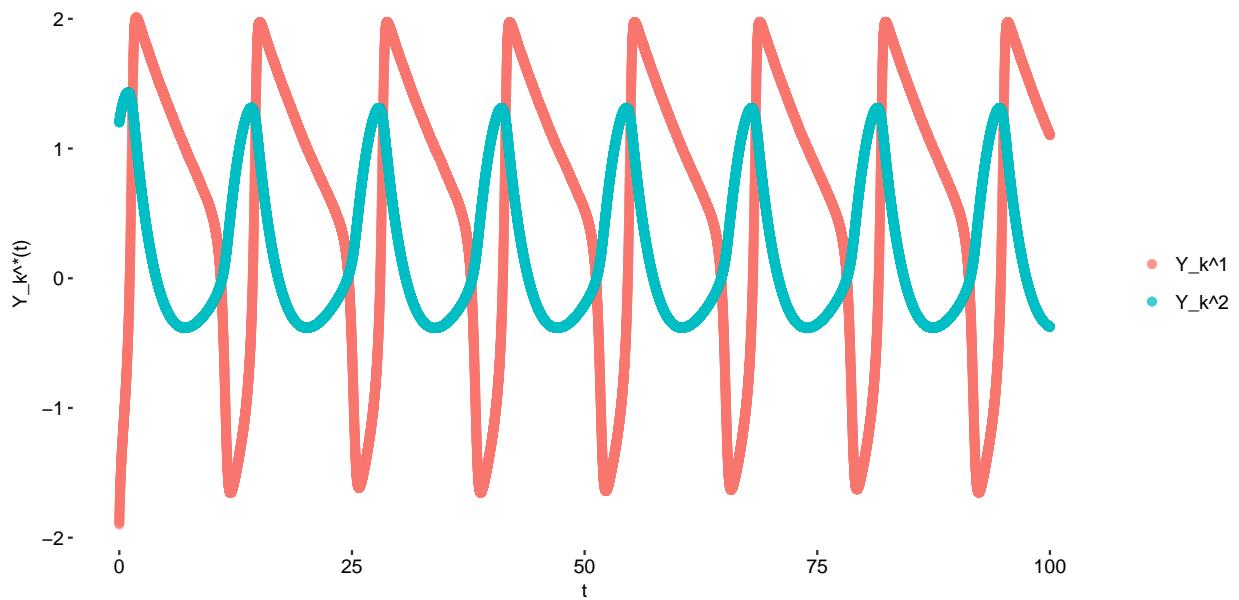


Figure 3: Plot of the simulation realizations with $\sigma = 0.10$.

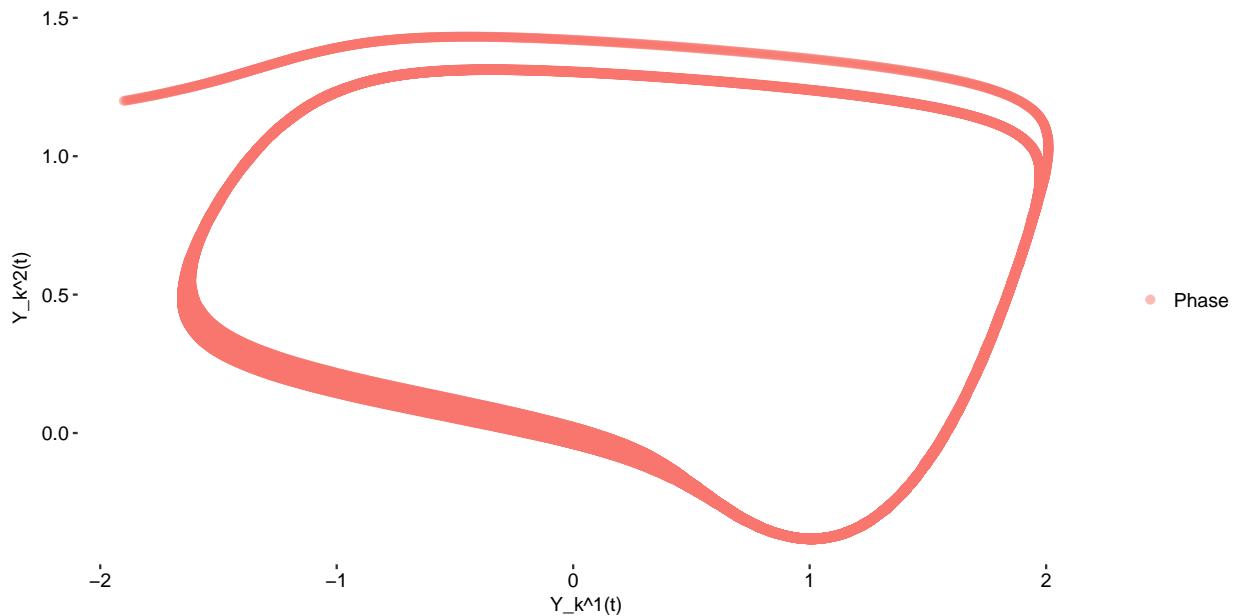


Figure 4: Phase plot of the simulation with $\sigma = 0.10$.

$\sigma = 0.20$

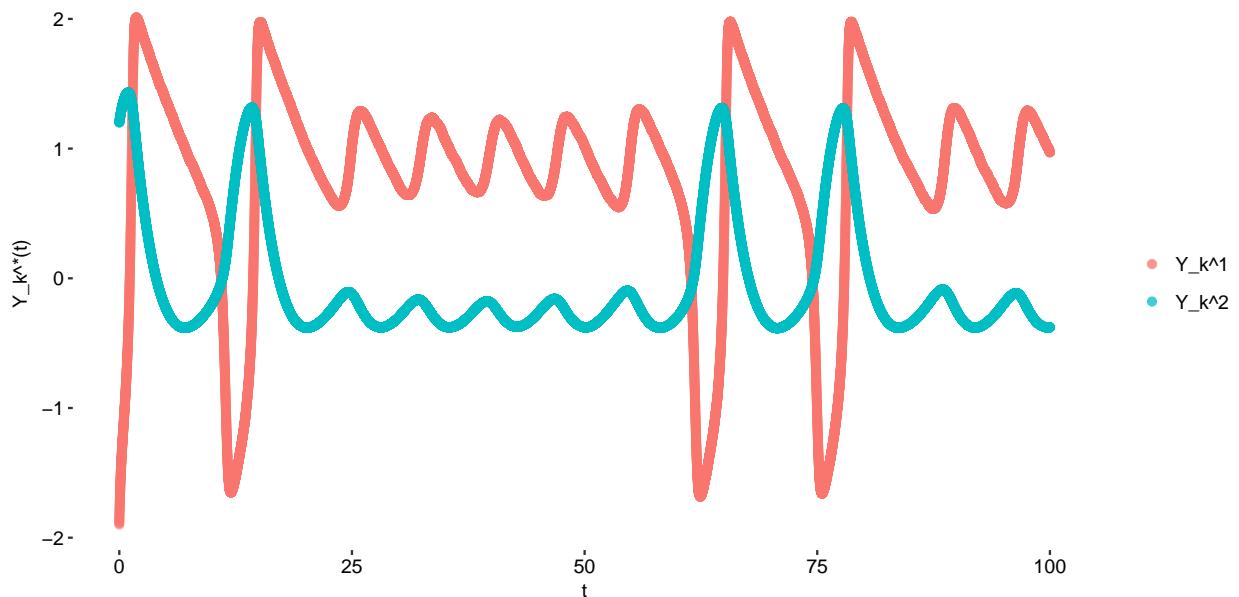


Figure 5: Plot of the simulation realizations with $\sigma = 0.20$.

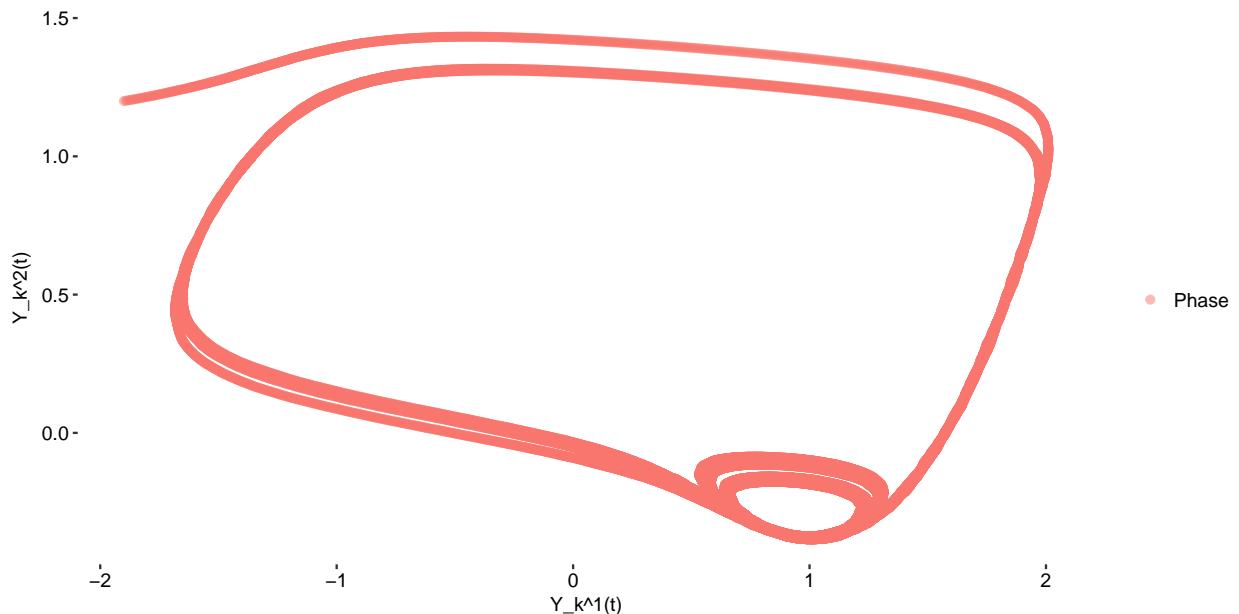


Figure 6: Phase plot of the simulation with $\sigma = 0.20$.

$\sigma = 0.30$

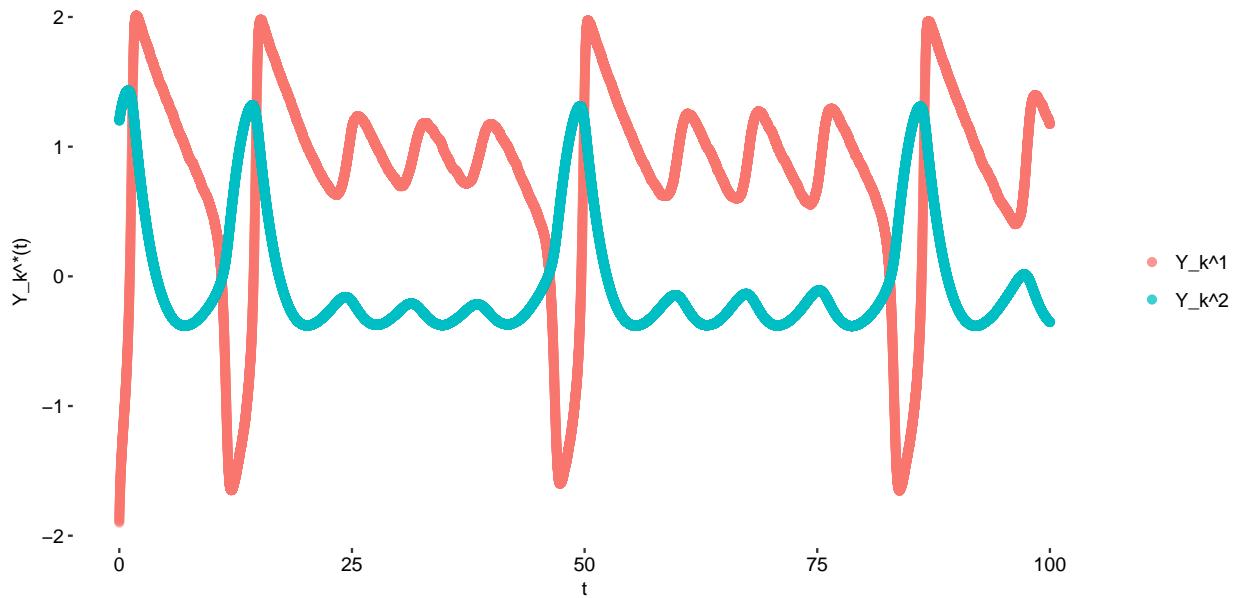


Figure 7: Plot of the simulation realizations with sigma = 0.30.

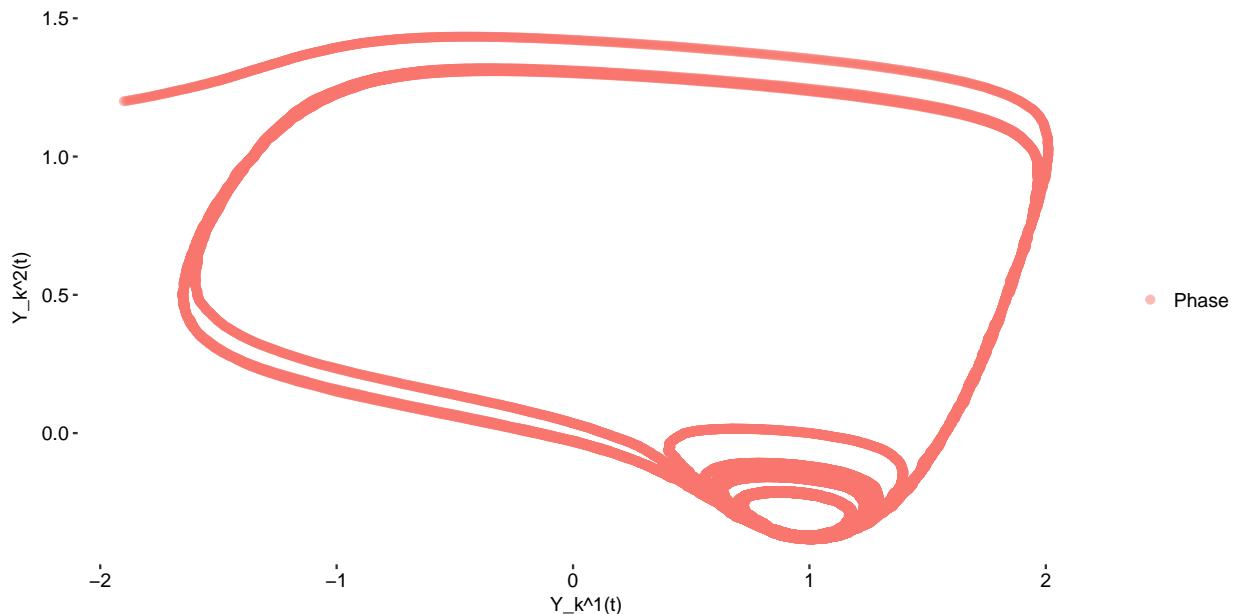


Figure 8: Phase plot of the simulation with sigma = 0.30.

$$\sigma = 0.40$$

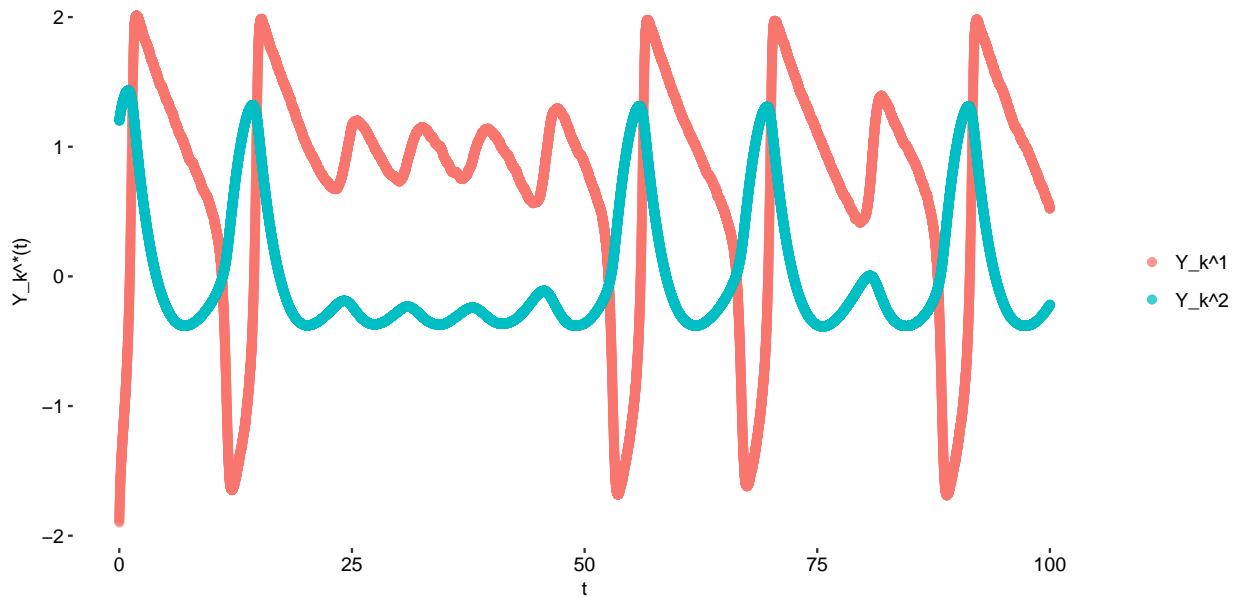


Figure 9: Plot of the simulation realizations with sigma = 0.40.

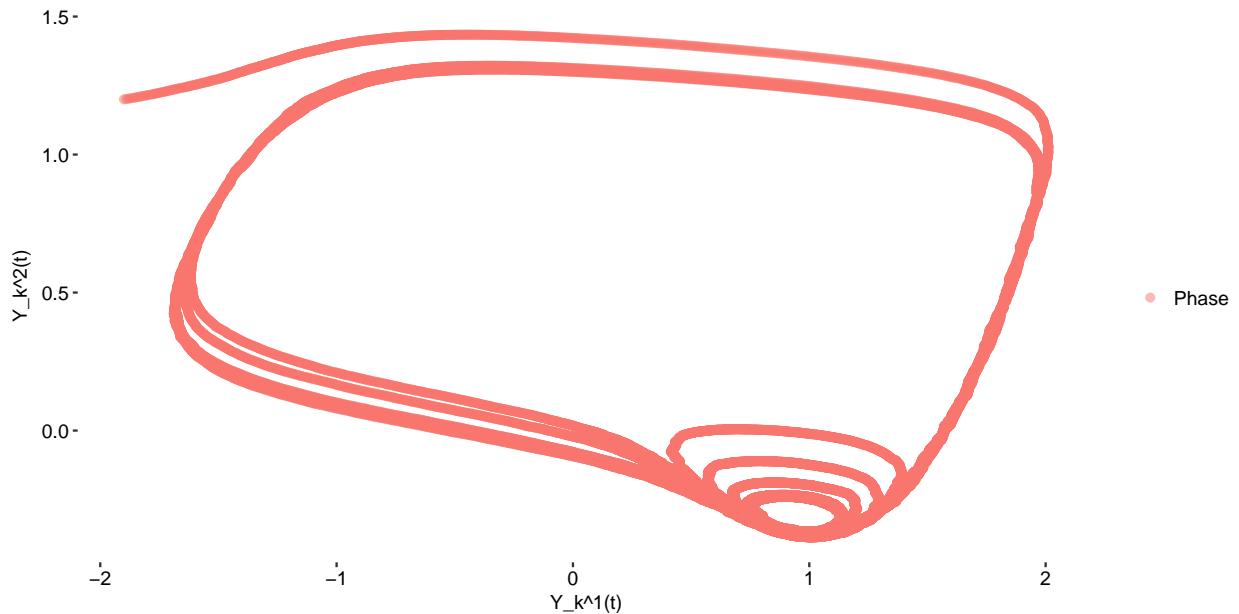


Figure 10: Phase plot of the simulation with sigma = 0.40.

Comment on the effect of adding noise to the equations

-

Question 1b

$$\sigma = 0.10$$

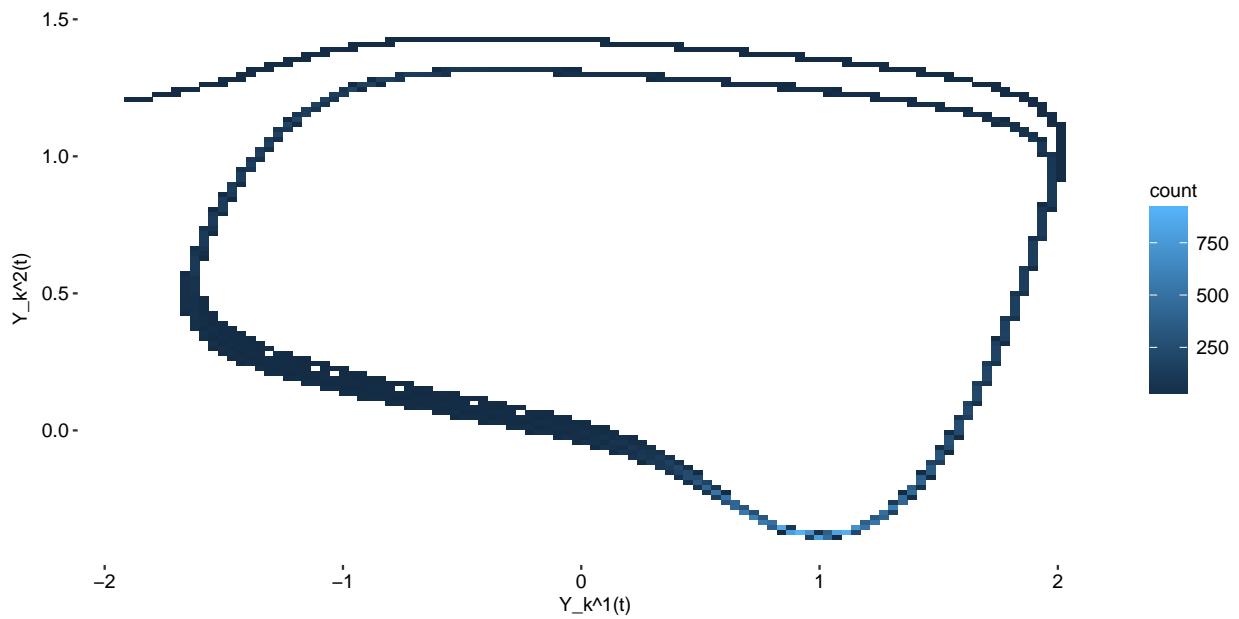


Figure 11: Phase plot of the simulation with $\sigma = 0.10$.

$$\sigma = 0.20$$

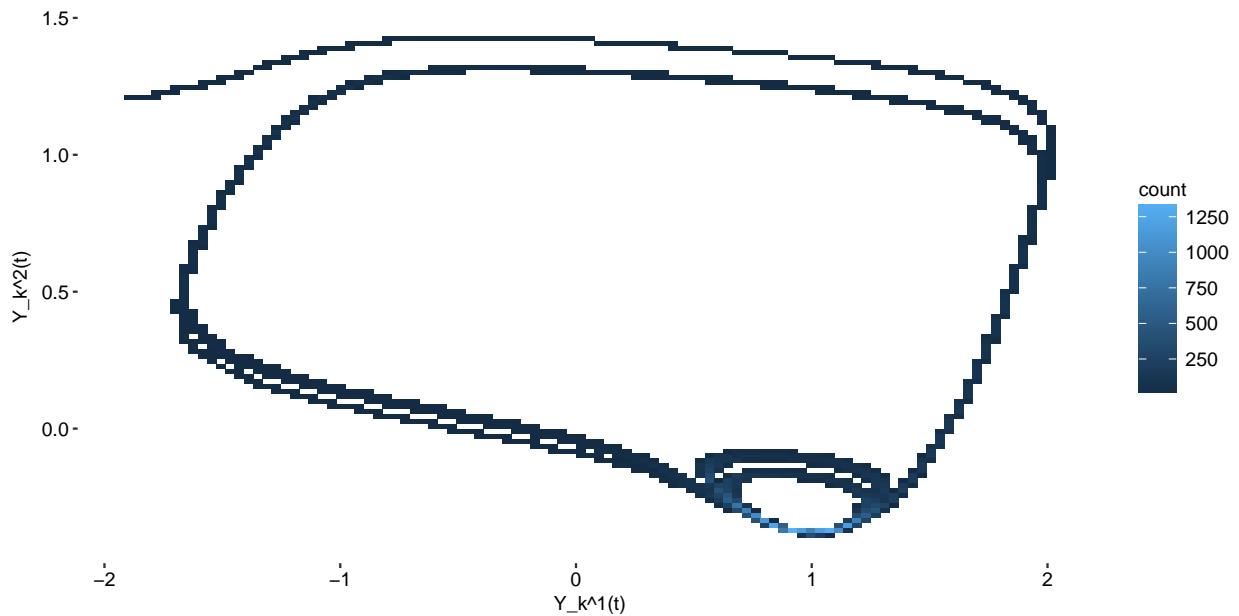


Figure 12: Phase plot of the simulation with $\sigma = 0.20$.

$$\sigma = 0.30$$

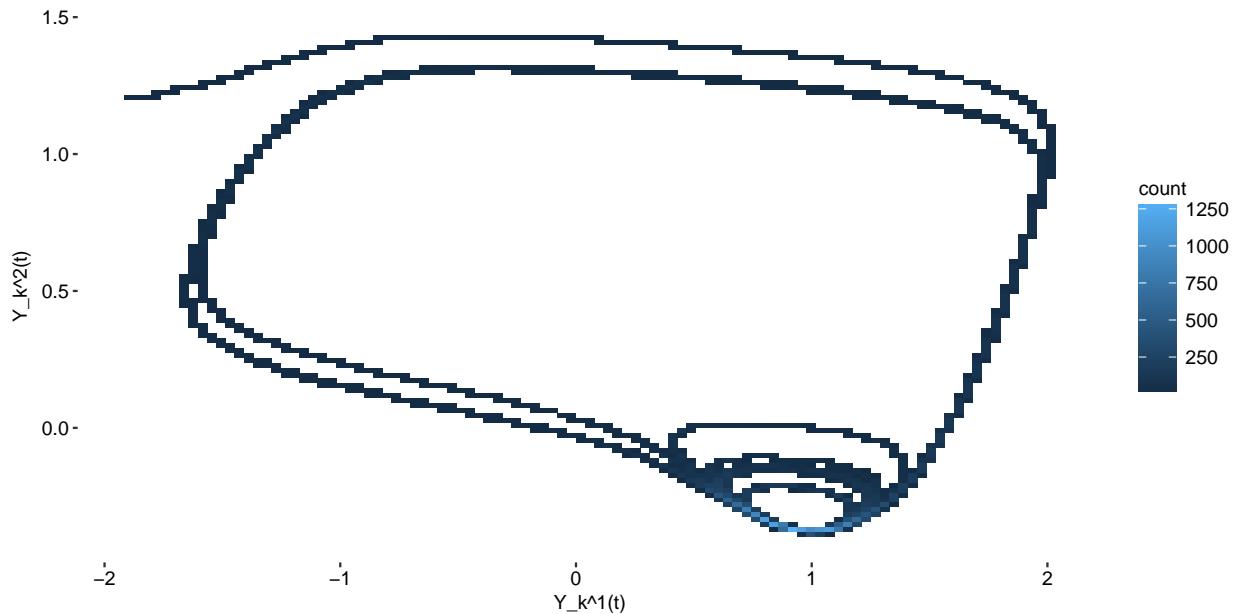


Figure 13: Phase plot of the simulation with $\sigma = 0.30$.

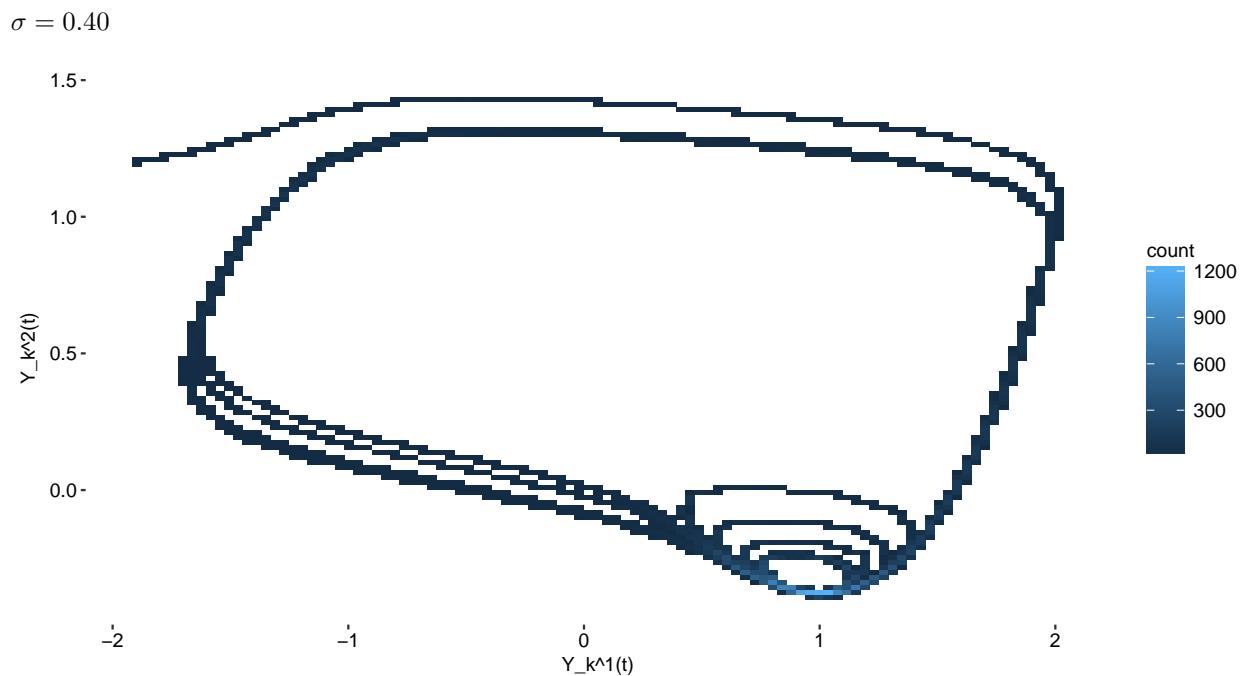


Figure 14: Phase plot of the simulation with $\sigma = 0.40$.

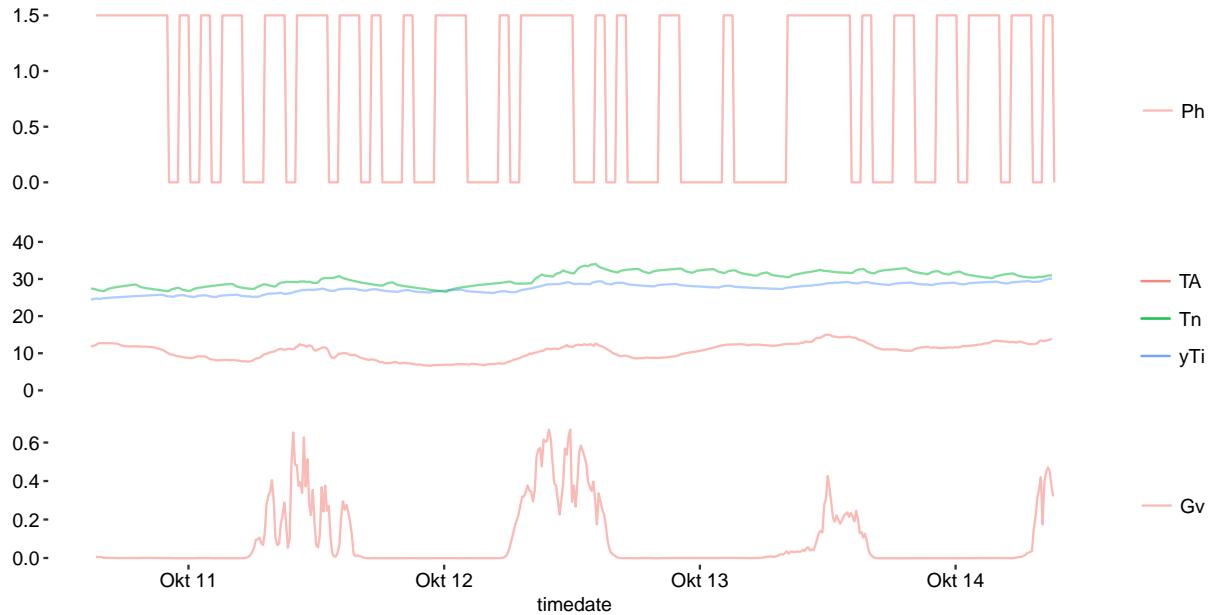
Which extra information does the plot contain, compared to the standard phase-plot?

-

Part 2:

Data

describe the data #### Question 2a ##### 1. Step



2. Step

figure ??

adasd

```
## Iteration 1, F(x) = 7.2459137029635326e+02, max|g(x)| = 1.7514e+03
## Parameter:
## [1] 15 1 -1 1 5
##
## Iteration 2, F(x) = 6.2396610979858667e+02, max|g(x)| = 1.2840e+03
## Parameter:
## [1] 15.0055938 1.0040652 -1.4052689 0.2833903 5.0056845
##
## Iteration 3, F(x) = 4.6757277294075141e+02, max|g(x)| = 1.9130e+03
## Parameter:
## [1] 15.03370679 1.03854411 -3.12851664 0.03052911 5.05261286
##
## Iteration 4, F(x) = 3.8352658897433201e+01, max|g(x)| = 5.5567e+03
## Parameter:
## [1] 15.561654 2.773932 -6.463466 -1.328303 6.443921
##
## Iteration 5, F(x) = -1.4776076256063934e+02, max|g(x)| = 1.0677e+04
## Parameter:
## [1] 24.592643 6.647852 -10.627572 -2.219071 6.749663
##
## Iteration 6, F(x) = -4.1709371269261379e+02, max|g(x)| = 5.1805e+03
## Parameter:
```

```

## [1] 24.632125 6.168585 -11.079956 -1.942853 7.215577
##
## Iteration 7, F(x) = -5.3027644843280086e+02, max|g(x)| = 1.3893e+03
## Parameter:
## [1] 24.503086 8.430776 -11.926992 -1.673834 6.230826
##
## Iteration 8, F(x) = -5.7250672706296621e+02, max|g(x)| = 2.7391e+02
## Parameter:
## [1] 24.599315 6.409254 -13.009370 -1.622418 8.655600
##
## Iteration 9, F(x) = -5.8069085829373751e+02, max|g(x)| = 2.9637e+01
## Parameter:
## [1] 24.530338 7.082640 -13.496469 -1.606170 8.758678
##
## Iteration 10, F(x) = -5.8944737943456437e+02, max|g(x)| = 9.4155e+01
## Parameter:
## [1] 24.419679 8.205528 -14.524126 -1.611656 9.092194
##
## Iteration 11, F(x) = -5.9428677853718682e+02, max|g(x)| = 6.4375e+01
## Parameter:
## [1] 24.455159 9.083578 -15.497850 -1.624357 9.363861
##
## Iteration 12, F(x) = -5.9723583155863457e+02, max|g(x)| = 4.0540e+01
## Parameter:
## [1] 24.495715 10.117732 -16.702422 -1.642178 9.602276
##
## Iteration 13, F(x) = -5.9823017249292946e+02, max|g(x)| = 8.0069e+01
## Parameter:
## [1] 24.509883 10.875842 -17.645262 -1.648693 9.728360
##
## Iteration 14, F(x) = -5.9874281340884636e+02, max|g(x)| = 6.5288e+01
## Parameter:
## [1] 24.512553 11.547964 -18.561331 -1.647928 9.815021
##
## Iteration 15, F(x) = -5.9899174877821110e+02, max|g(x)| = 2.7880e+01
## Parameter:
## [1] 24.508318 12.081048 -19.387256 -1.644040 9.871715
##
## Iteration 16, F(x) = -5.9908656612919901e+02, max|g(x)| = 1.2325e+00
## Parameter:
## [1] 24.503563 12.408487 -20.006021 -1.641127 9.904731
##
## Iteration 17, F(x) = -5.9913422835660060e+02, max|g(x)| = 1.0356e+01
## Parameter:
## [1] 24.501103 12.573551 -20.456810 -1.639875 9.925261
##
## Iteration 18, F(x) = -5.9919088060711931e+02, max|g(x)| = 1.5321e+01
## Parameter:
## [1] 24.499917 12.652631 -20.964408 -1.639415 9.945283
##
## Iteration 19, F(x) = -5.9928124478589700e+02, max|g(x)| = 1.3985e+01
## Parameter:
## [1] 24.499986 12.571554 -21.627066 -1.639766 9.965942
##

```

```

## Iteration 20, F(x) = -5.9941779944312668e+02, max|g(x)| = 5.7495e+00
## Parameter:
## [1] 24.501365 12.135642 -22.486726 -1.641050 9.983638
##
## Iteration 21, F(x) = -5.9951111428892750e+02, max|g(x)| = 4.0023e+00
## Parameter:
## [1] 24.502935 11.329133 -23.099141 -1.642489 9.991796
##
## Iteration 22, F(x) = -5.9953615349418362e+02, max|g(x)| = 3.3558e+00
## Parameter:
## [1] 24.502790 11.096761 -22.776358 -1.642435 9.990544
##
## Iteration 23, F(x) = -5.9955192999954727e+02, max|g(x)| = 6.9410e-02
## Parameter:
## [1] 24.501963 10.907724 -22.088829 -1.642019 9.985844
##
## Iteration 24, F(x) = -5.9955195978908444e+02, max|g(x)| = 2.1753e-02
## Parameter:
## [1] 24.501871 10.927251 -22.091681 -1.642008 9.985780
##
## Iteration 25, F(x) = -5.9955196121716824e+02, max|g(x)| = 1.4114e-02
## Parameter:
## [1] 24.501852 10.924038 -22.092395 -1.642012 9.985802
##
## Iteration 26, F(x) = -5.9955196123396809e+02, max|g(x)| = 9.2784e-03
## Parameter:
## [1] 24.501842 10.923998 -22.092305 -1.642011 9.985801
##
## Iteration 27, F(x) = -5.9955196123661347e+02, max|g(x)| = 2.8515e-03
## Parameter:
## [1] 24.501836 10.923983 -22.092456 -1.642011 9.985803

## Coefficients:
##             Estimate Std. Error   t value Pr(>|t|)    dF/dPar dPen/dPar
## Ti0    2.4502e+01 7.5142e-02 3.2607e+02 0.0000e+00 -3.4124e-03  0.2468
## Ci     1.0924e+01 2.0632e+00 5.2948e+00 1.7518e-07 -4.0554e-05  0.0003
## e11   -2.2092e+01 4.1092e+00 -5.3763e+00 1.1438e-07 -1.6859e-04  0.0001
## p11   -1.6420e+00 2.6167e-02 -6.2751e+01 0.0000e+00  5.6728e-04  0.0000
## Ria    9.9858e+00 3.7836e-02  2.6392e+02 0.0000e+00  6.0841e-03 49.5438
##
## Correlation of coefficients:
##      Ti0   Ci   e11   p11
## Ci  -0.11
## e11  0.24 -0.28
## p11 -0.12  0.05 -0.07
## Ria -0.22  0.03 -0.97  0.05
##
## [1] "loglikelihood = 599.62755058139"

## Coefficients:
##             Estimate Std. Error   t value Pr(>|t|)    dF/dPar dPen/dPar
## Ti0    2.4502e+01 7.5142e-02 3.2607e+02 0.0000e+00 -3.4124e-03  0.2468
## Ci     1.0924e+01 2.0632e+00 5.2948e+00 1.7518e-07 -4.0554e-05  0.0003
## e11   -2.2092e+01 4.1092e+00 -5.3763e+00 1.1438e-07 -1.6859e-04  0.0001
## p11   -1.6420e+00 2.6167e-02 -6.2751e+01 0.0000e+00  5.6728e-04  0.0000

```

```

## Ria 9.9858e+00 3.7836e-02 2.6392e+02 0.0000e+00 6.0841e-03 49.5438
##
## Correlation of coefficients:
##   Ti0 Ci e11 p11
## Ci -0.11
## e11 0.24 -0.28
## p11 -0.12 0.05 -0.07
## Ria -0.22 0.03 -0.97 0.05

```

3. Step

```

## Coefficients:
##             Estimate Std. Error    t value Pr(>|t|)    dF/dPar dPen/dPar
## Ti0  2.4502e+01 7.5142e-02 3.2607e+02 0.0000e+00 -3.4124e-03 0.2468
## Ci   1.0924e+01 2.0632e+00 5.2948e+00 1.7518e-07 -4.0554e-05 0.0003
## e11 -2.2092e+01 4.1092e+00 -5.3763e+00 1.1438e-07 -1.6859e-04 0.0001
## p11 -1.6420e+00 2.6167e-02 -6.2751e+01 0.0000e+00 5.6728e-04 0.0000
## Ria  9.9858e+00 3.7836e-02 2.6392e+02 0.0000e+00 6.0841e-03 49.5438
##
## Correlation of coefficients:
##   Ti0 Ci e11 p11
## Ci -0.11
## e11 0.24 -0.28
## p11 -0.12 0.05 -0.07
## Ria -0.22 0.03 -0.97 0.05
##
## [1] "loglikelihood = 599.62755058139"

```

The optimization procedure works out without any problems, but if we look at the extended summary of the fit then dPen/dPar is rather large for one of the parameters. What does this mean and how to fix the associated problem?

Issue: Ria and Ti0 are binding

see the manuel and fix the fix

```

## Iteration 1, F(x) = 7.2459120055951109e+02, max|g(x)| = 1.7514e+03
## Parameter:
## [1] 15 1 -1 1 5
##
## Iteration 2, F(x) = 6.2292737397961503e+02, max|g(x)| = 1.2840e+03
## Parameter:
## [1] 15.0114181 1.0040652 -1.4052689 0.2833903 5.0169917
##
## Iteration 3, F(x) = 4.5984022065109758e+02, max|g(x)| = 1.8610e+03
## Parameter:
## [1] 15.06777684 1.03768481 -3.09476332 0.01842093 5.15642223
##
## Iteration 4, F(x) = -1.5096176898793544e+01, max|g(x)| = 7.8542e+03
## Parameter:
## [1] 16.121330 2.488508 -7.770612 -1.872988 11.429733
##
## Iteration 5, F(x) = -1.1598527969248778e+02, max|g(x)| = 1.0951e+04
## Parameter:
## [1] 26.335387 2.275225 -9.520657 -2.426135 18.625047
##

```

```

## Iteration 6, F(x) = -2.8353632516365241e+02, max|g(x)| = 8.2080e+03
## Parameter:
## [1] 25.876900 2.037587 -9.568760 -2.286478 19.992857
##
## Iteration 7, F(x) = -4.2080780066587295e+02, max|g(x)| = 4.6432e+03
## Parameter:
## [1] 26.365974 2.975544 -9.471621 -1.991601 13.160585
##
## Iteration 8, F(x) = -5.3233060589609465e+02, max|g(x)| = 3.2393e+03
## Parameter:
## [1] 23.249246 3.127150 -10.455286 -1.981509 17.011781
##
## Iteration 9, F(x) = -6.1631086748721566e+02, max|g(x)| = 2.8776e+02
## Parameter:
## [1] 25.204009 3.300772 -10.876595 -1.714468 17.819689
##
## Iteration 10, F(x) = -6.4433527260240101e+02, max|g(x)| = 2.0034e+02
## Parameter:
## [1] 24.44459 3.27501 -11.12494 -1.70690 19.06705
##
## Iteration 11, F(x) = -6.4724018244599131e+02, max|g(x)| = 1.2146e+02
## Parameter:
## [1] 24.545814 3.280909 -11.254751 -1.721663 20.048766
##
## Iteration 12, F(x) = -6.5119246102799298e+02, max|g(x)| = 9.8024e+01
## Parameter:
## [1] 24.507764 3.281753 -11.568705 -1.754794 22.441113
##
## Iteration 13, F(x) = -6.5250451156412475e+02, max|g(x)| = 1.2003e+02
## Parameter:
## [1] 24.502561 3.300104 -11.872156 -1.759433 24.470511
##
## Iteration 14, F(x) = -6.5292975839757992e+02, max|g(x)| = 4.3141e+01
## Parameter:
## [1] 24.501935 3.337134 -12.098481 -1.750837 25.704667
##
## Iteration 15, F(x) = -6.5303966093733584e+02, max|g(x)| = 7.6182e+00
## Parameter:
## [1] 24.501804 3.380044 -12.248602 -1.744910 26.345081
##
## Iteration 16, F(x) = -6.5307648045673830e+02, max|g(x)| = 2.0850e+01
## Parameter:
## [1] 24.501804 3.417663 -12.339886 -1.743356 26.608829
##
## Iteration 17, F(x) = -6.5310419303752167e+02, max|g(x)| = 1.8139e+01
## Parameter:
## [1] 24.501802 3.462506 -12.428298 -1.743665 26.726679
##
## Iteration 18, F(x) = -6.5311636217567604e+02, max|g(x)| = 7.6040e+00
## Parameter:
## [1] 24.501807 3.487359 -12.471385 -1.744901 26.654049
##
## Iteration 19, F(x) = -6.5312267756360427e+02, max|g(x)| = 2.6900e+00
## Parameter:

```

```

## [1] 24.501817 3.495316 -12.491824 -1.745857 26.525038
##
## Iteration 20, F(x) = -6.5312971002517031e+02, max|g(x)| = 6.5284e+00
## Parameter:
## [1] 24.501834 3.496597 -12.522070 -1.746577 26.379599
##
## Iteration 21, F(x) = -6.5314143092708866e+02, max|g(x)| = 1.1567e+01
## Parameter:
## [1] 24.501859 3.494226 -12.590581 -1.747177 26.220576
##
## Iteration 22, F(x) = -6.5318158831693097e+02, max|g(x)| = 2.5165e+01
## Parameter:
## [1] 24.501962 3.475086 -13.064703 -1.748760 25.683961
##
## Iteration 23, F(x) = -6.5328838763382862e+02, max|g(x)| = 2.2786e+00
## Parameter:
## [1] 24.501935 3.459592 -14.151112 -1.745582 26.332155
##
## Iteration 24, F(x) = -6.5330207866969158e+02, max|g(x)| = 2.7940e+00
## Parameter:
## [1] 24.501895 3.453973 -14.620172 -1.745525 26.311375
##
## Iteration 25, F(x) = -6.5331473320616999e+02, max|g(x)| = 1.1389e+00
## Parameter:
## [1] 24.501904 3.453482 -15.406232 -1.745722 26.261512
##
## Iteration 26, F(x) = -6.5331986918967220e+02, max|g(x)| = 1.0703e-01
## Parameter:
## [1] 24.501901 3.455864 -16.077076 -1.745843 26.253132
##
## Iteration 27, F(x) = -6.5332256937974194e+02, max|g(x)| = 2.4613e-01
## Parameter:
## [1] 24.501901 3.457158 -16.788518 -1.745883 26.257542
##
## Iteration 28, F(x) = -6.5332386904328825e+02, max|g(x)| = 1.9434e-01
## Parameter:
## [1] 24.501900 3.457122 -17.480957 -1.745877 26.260348
##
## Iteration 29, F(x) = -6.5332452856610780e+02, max|g(x)| = 5.5234e-02
## Parameter:
## [1] 24.501900 3.456770 -18.188509 -1.745861 26.260476
##
## Iteration 30, F(x) = -6.5332484597946882e+02, max|g(x)| = 2.3419e-02
## Parameter:
## [1] 24.501900 3.456603 -18.878587 -1.745852 26.259976
##
## Iteration 31, F(x) = -6.5332499722347563e+02, max|g(x)| = 3.1135e-02
## Parameter:
## [1] 24.501900 3.456611 -19.528695 -1.745851 26.259787
##
## Iteration 32, F(x) = -6.5332506930696104e+02, max|g(x)| = 1.1976e-02
## Parameter:
## [1] 24.501900 3.456654 -20.115694 -1.745853 26.259812
##

```

```

## Iteration 33, F(x) = -6.5332512217597878e+02, max|g(x)| = 6.0124e-03
## Parameter:
## [1] 24.501900 3.456684 -21.067023 -1.745855 26.259888
##
## Iteration 34, F(x) = -6.5332513709258080e+02, max|g(x)| = 6.1679e-03
## Parameter:
## [1] 24.501900 3.456679 -21.793061 -1.745855 26.259902
##
## Iteration 35, F(x) = -6.5332514101220249e+02, max|g(x)| = 1.9117e-03
## Parameter:
## [1] 24.501900 3.456670 -22.247794 -1.745855 26.259889
##
## Iteration 36, F(x) = -6.5332514188191021e+02, max|g(x)| = 3.4971e-05
## Parameter:
## [1] 24.501900 3.456668 -22.441382 -1.745855 26.259884
##
## Iteration 37, F(x) = -6.5332514204409972e+02, max|g(x)| = 1.4766e-04
## Parameter:
## [1] 24.501900 3.456669 -22.491122 -1.745855 26.259885

## Coefficients:
##             Estimate Std. Error     t value   Pr(>|t|)      dF/dPar dPen/dPar
## Ti0    2.4502e+01 7.3830e-02  3.3187e+02 0.0000e+00  0.0000e+00     8e-04
## Ci     3.4567e+00 2.4954e-01  1.3852e+01 0.0000e+00 -4.3892e-06     0e+00
## e11   -2.2491e+01 2.3055e+02 -9.7556e-02 9.2232e-01 -2.0623e-05     1e-04
## p11   -1.7459e+00 2.6627e-02 -6.5568e+01 0.0000e+00 -5.4510e-06     0e+00
## Ria   2.6260e+01 2.4301e+00  1.0806e+01 0.0000e+00 -1.4811e-05     0e+00
##
## Correlation of coefficients:
##      Ti0   Ci   e11   p11
## Ci   0.00
## e11  0.00  0.00
## p11 -0.01  0.04  0.00
## Ria  0.00  0.25  0.00  0.05
##
## [1] "loglikelihood = 653.325639755608"

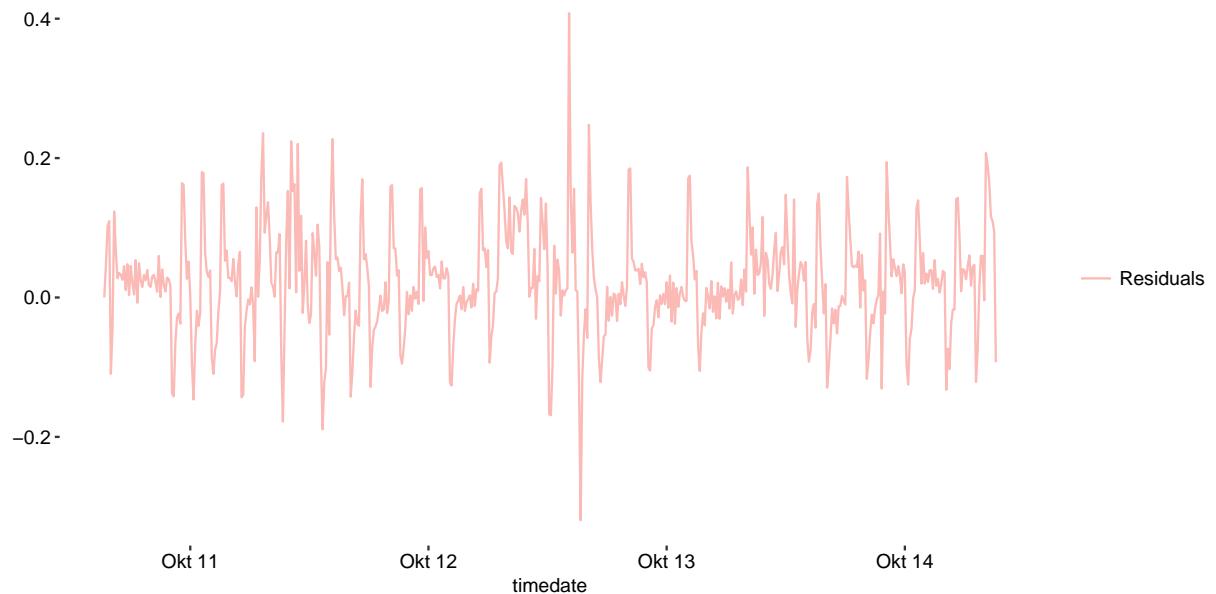
## Coefficients:
##             Estimate Std. Error     t value   Pr(>|t|)      dF/dPar dPen/dPar
## Ti0    2.4502e+01 7.3830e-02  3.3187e+02 0.0000e+00  0.0000e+00     8e-04
## Ci     3.4567e+00 2.4954e-01  1.3852e+01 0.0000e+00 -4.3892e-06     0e+00
## e11   -2.2491e+01 2.3055e+02 -9.7556e-02 9.2232e-01 -2.0623e-05     1e-04
## p11   -1.7459e+00 2.6627e-02 -6.5568e+01 0.0000e+00 -5.4510e-06     0e+00
## Ria   2.6260e+01 2.4301e+00  1.0806e+01 0.0000e+00 -1.4811e-05     0e+00
##
## Correlation of coefficients:
##      Ti0   Ci   e11   p11
## Ci   0.00
## e11  0.00  0.00
## p11 -0.01  0.04  0.00
## Ria  0.00  0.25  0.00  0.05

```

4. Step

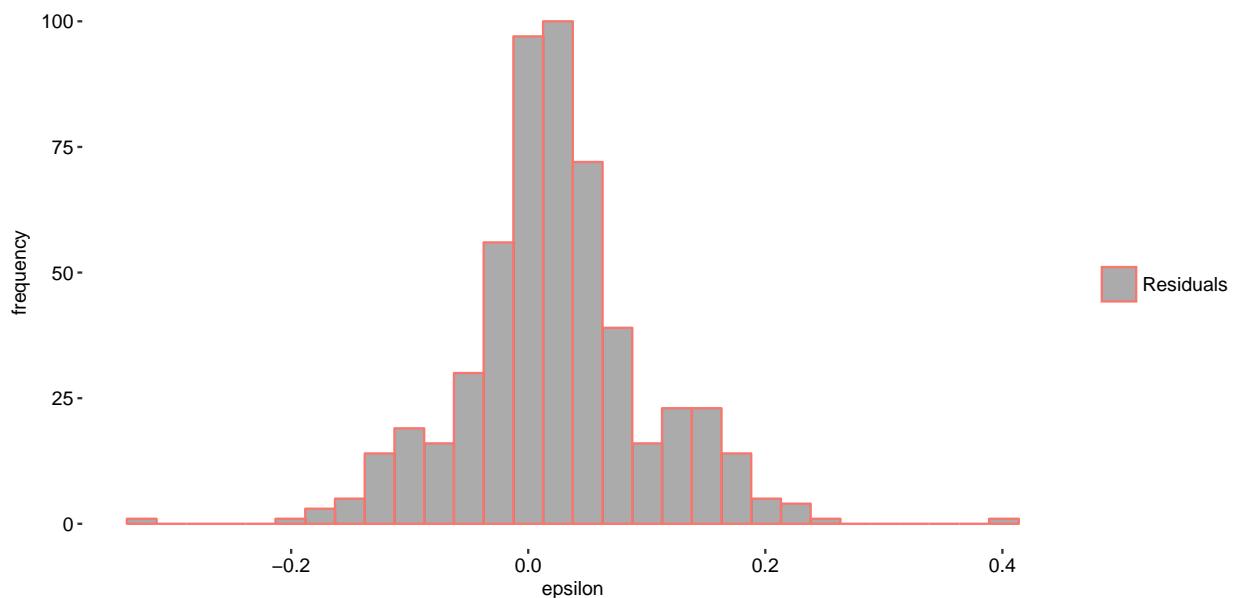
One-step ahead residuals

Time Series of the residuals

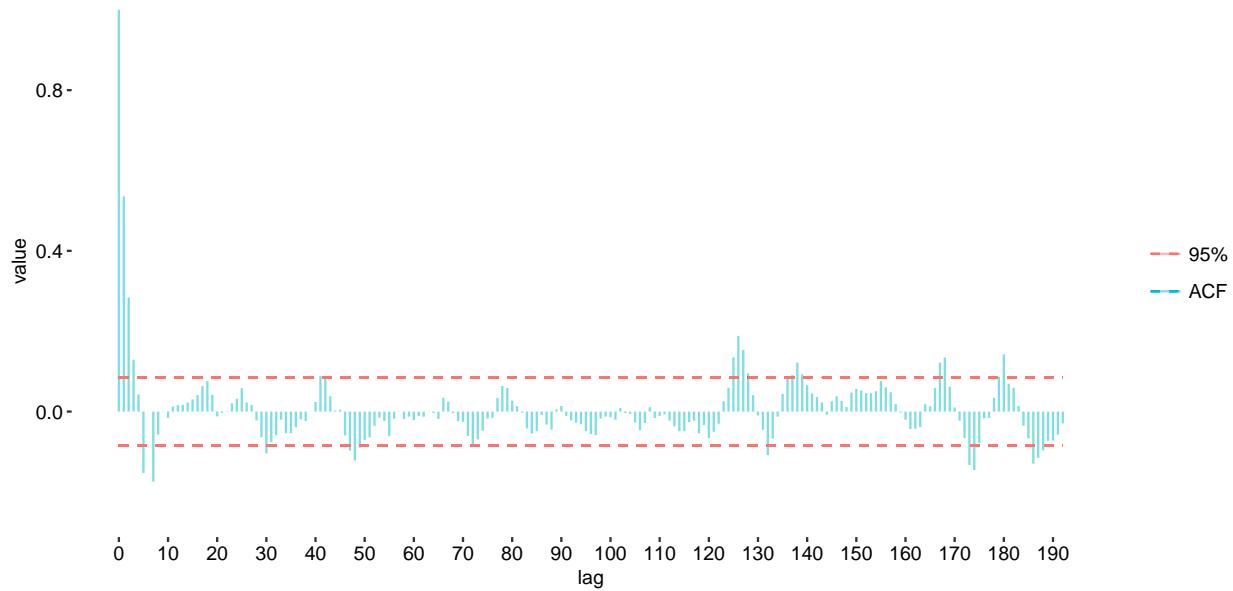


Distribution of the residuals

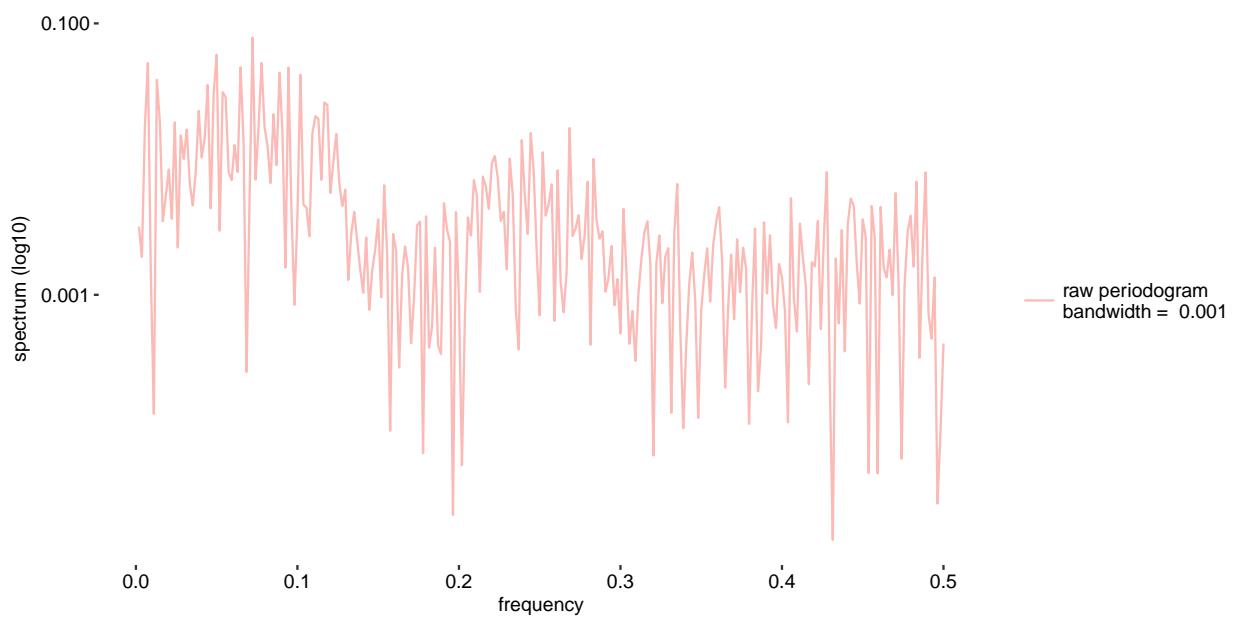
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



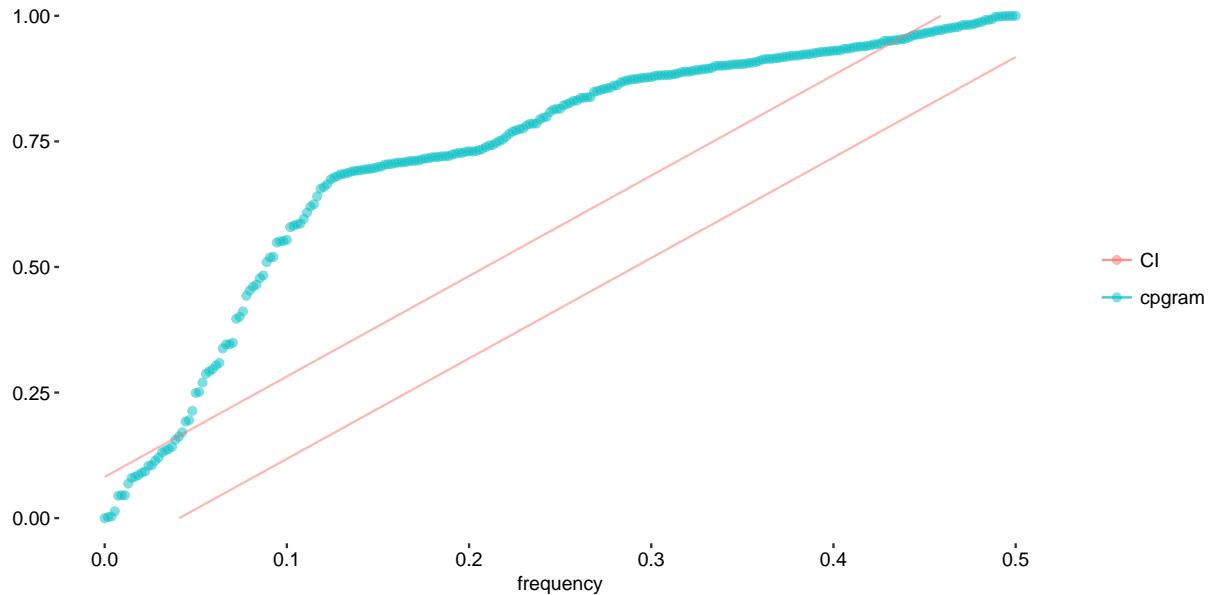
ACF of the residuals



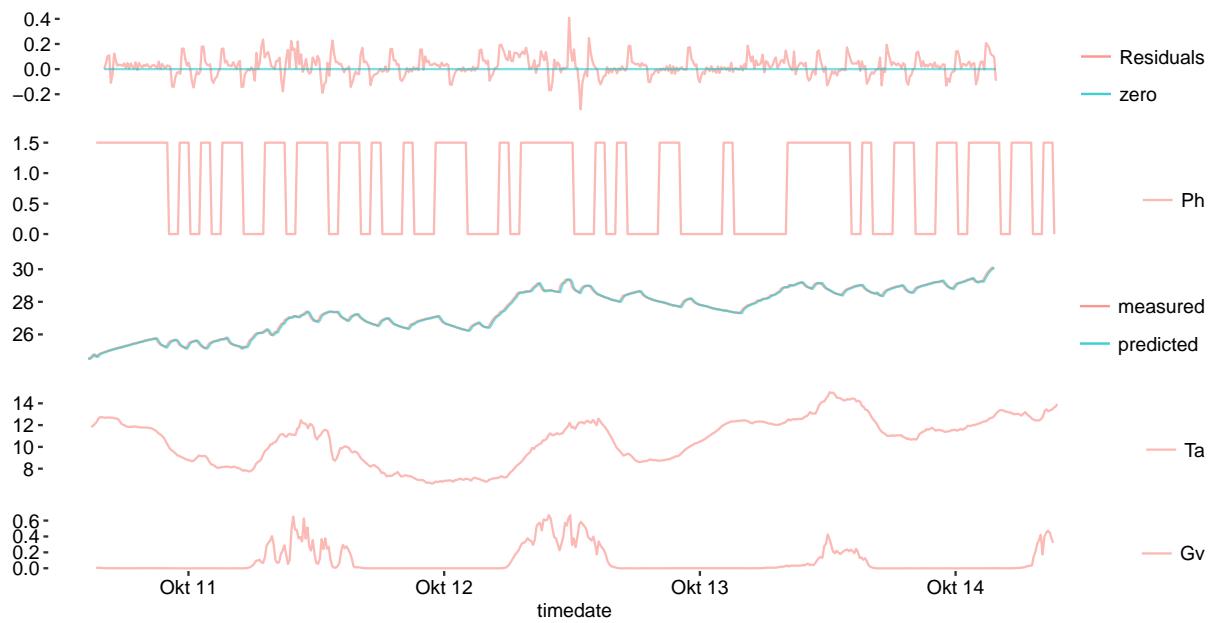
Periodogram of the residuals



Commulated Periodogram of the residuals



combined



5. Step

figure ?? new

```
## Iteration 1, F(x) = 6.4608887398147544e+02, max|g(x)| = 2.3175e+03
## Parameter:
## [1] 25 25 1 1 -1 1 1 20 1
##
## Iteration 2, F(x) = -3.4784719690753536e+02, max|g(x)| = 1.6463e+03
## Parameter:
## [1] 24.9856844 24.9971932 0.9961833 0.9963113 -4.4343835 -7.6433722
```

```

## [7] 0.4238932 19.9890260 0.9920505
##
## Iteration 3, F(x) = -6.0653972410656183e+02, max|g(x)| = 7.1992e+02
## Parameter:
## [1] 24.7216589 24.9833569 1.0074303 0.9940519 -6.6495622 -8.6640149
## [7] -0.2652974 19.9897815 0.9963850
##
## Iteration 4, F(x) = -6.2313487291594060e+02, max|g(x)| = 3.9072e+02
## Parameter:
## [1] 24.5054767 24.9724137 1.0124681 0.9944953 -6.8399063 -8.7222290
## [7] -0.1890326 19.9948077 0.9927579
##
## Iteration 5, F(x) = -6.2810305620379302e+02, max|g(x)| = 3.8369e+02
## Parameter:
## [1] 24.58579759 24.92977514 1.03226764 0.99535760 -7.03899006 -8.72543915
## [7] -0.04874208 20.00580380 0.98267927
##
## Iteration 6, F(x) = -6.8546990989261519e+02, max|g(x)| = 6.1187e+02
## Parameter:
## [1] 24.50750074 24.21510715 1.58440432 0.99648382 -7.33572762 -9.05864590
## [7] -0.08190329 20.18056368 0.82881061
##
## Iteration 7, F(x) = -6.9572306498979970e+02, max|g(x)| = 6.1111e+02
## Parameter:
## [1] 24.5329805 26.4104903 1.6788813 0.9188280 -7.3355674 -9.0336807
## [7] -0.2331343 21.2036502 0.5893744
##
## Iteration 8, F(x) = -7.1175662186181410e+02, max|g(x)| = 4.1900e+02
## Parameter:
## [1] 24.4899919 24.8441911 1.6688834 0.9446035 -7.1730940 -8.9797799
## [7] -0.3536015 22.6432743 0.4815130
##
## Iteration 9, F(x) = -7.2904542893414998e+02, max|g(x)| = 2.7234e+02
## Parameter:
## [1] 24.4529348 24.2782304 1.7958681 1.6965337 -7.0508292 -8.9408977
## [7] -0.4636590 25.6524324 0.3823342
##
## Iteration 10, F(x) = -7.3341988683910699e+02, max|g(x)| = 2.8180e+02
## Parameter:
## [1] 24.4453906 23.5994744 1.7607145 2.1640560 -7.0611765 -8.9448603
## [7] -0.7358482 26.7856886 0.2928776
##
## Iteration 11, F(x) = -7.4952124764012046e+02, max|g(x)| = 2.5597e+02
## Parameter:
## [1] 24.4482919 24.3017015 1.8509429 3.8823339 -7.2146855 -8.9837990
## [7] -1.1561343 30.2825879 0.1741817
##
## Iteration 12, F(x) = -7.5541703687490315e+02, max|g(x)| = 1.4689e+02
## Parameter:
## [1] 24.5038126 24.7587818 1.8133025 4.5349573 -7.4813456 -9.0080467
## [7] -1.0943489 30.1611267 0.1738719
##
## Iteration 13, F(x) = -7.6088207814958514e+02, max|g(x)| = 4.0066e+02
## Parameter:

```

```

## [1] 24.5357031 24.3447184 1.6230609 6.4024428 -7.5590265 -8.9821989
## [7] -1.2741877 31.6060848 0.1606238
##
## Iteration 14, F(x) = -7.6468693995403862e+02, max|g(x)| = 1.7072e+02
## Parameter:
## [1] 24.495655 24.410844 1.471847 8.396971 -7.442158 -8.934701 -1.442693
## [8] 32.673569 0.159123
##
## Iteration 15, F(x) = -7.6896357668288067e+02, max|g(x)| = 3.2292e+02
## Parameter:
## [1] 24.4485845 24.4131437 1.3074688 9.2766287 -7.2500004 -8.8755220
## [7] -1.4486402 33.7561039 0.1985014
##
## Iteration 16, F(x) = -7.7009297501964022e+02, max|g(x)| = 6.5063e+01
## Parameter:
## [1] 24.4857903 24.4436348 1.1164962 12.2410116 -7.0303550 -8.7871944
## [7] -1.6017379 38.3412507 0.2252434
##
## Iteration 17, F(x) = -7.7079245432870869e+02, max|g(x)| = 1.6656e+01
## Parameter:
## [1] 24.4934289 24.4011589 1.2098831 11.0052661 -7.1438181 -8.8330699
## [7] -1.5371212 38.5354553 0.2112259
##
## Iteration 18, F(x) = -7.7085075926574825e+02, max|g(x)| = 5.2164e+00
## Parameter:
## [1] 24.4926434 24.4068522 1.2038701 11.1697670 -7.1300833 -8.8323634
## [7] -1.5415436 40.1155470 0.2147736
##
## Iteration 19, F(x) = -7.7088868712935914e+02, max|g(x)| = 1.3051e+01
## Parameter:
## [1] 24.4912778 24.3981820 1.1974934 11.4641133 -7.1186189 -8.8314719
## [7] -1.5513783 42.9645746 0.2166812
##
## Iteration 20, F(x) = -7.7092299589405161e+02, max|g(x)| = 1.8323e+01
## Parameter:
## [1] 24.4928459 24.4036408 1.2021684 11.6619567 -7.1141569 -8.8331208
## [7] -1.5526987 47.1370328 0.2168775
##
## Iteration 21, F(x) = -7.7096454857098206e+02, max|g(x)| = 1.7034e+01
## Parameter:
## [1] 24.4911864 24.3860809 1.2056805 11.9551767 -7.1193753 -8.8355224
## [7] -1.5519199 54.0459611 0.2161362
##
## Iteration 22, F(x) = -7.7098582798842472e+02, max|g(x)| = 8.3931e+00
## Parameter:
## [1] 24.491795 24.383119 1.210885 12.194720 -7.125532 -8.835946 -1.548097
## [8] 60.172197 0.214309
##
## Iteration 23, F(x) = -7.7099129964094914e+02, max|g(x)| = 2.4410e+00
## Parameter:
## [1] 24.4917511 24.3807649 1.2133318 12.4088428 -7.1282245 -8.8360148
## [7] -1.5458740 64.1282464 0.2136979
##
## Iteration 24, F(x) = -7.7099433309940684e+02, max|g(x)| = 2.9846e-01

```

```

## Parameter:
## [1] 24.491733 24.379575 1.215609 12.652393 -7.129401 -8.836243 -1.544997
## [8] 67.329235 0.213423
##
## Iteration 25, F(x) = -7.7099696423473711e+02, max|g(x)| = 6.9509e-01
## Parameter:
## [1] 24.4916840 24.3788141 1.2188539 12.9573033 -7.1299253 -8.8368034
## [7] -1.5441822 70.2889580 0.2133623
##
## Iteration 26, F(x) = -7.7099836348024792e+02, max|g(x)| = 3.8977e-01
## Parameter:
## [1] 24.4916602 24.3790587 1.2210886 13.1520750 -7.1301510 -8.8372589
## [7] -1.5438757 70.9780298 0.2134223
##
## Iteration 27, F(x) = -7.7099926405891290e+02, max|g(x)| = 7.0220e-01
## Parameter:
## [1] 24.4916340 24.3798586 1.2222687 13.2306677 -7.1300704 -8.8373638
## [7] -1.5436279 69.7482068 0.2135111
##
## Iteration 28, F(x) = -7.7099973620444359e+02, max|g(x)| = 8.4209e-01
## Parameter:
## [1] 24.4916356 24.3809220 1.2224119 13.2139635 -7.1302840 -8.8371868
## [7] -1.5434091 67.5586006 0.2135313
##
## Iteration 29, F(x) = -7.7099980379109138e+02, max|g(x)| = 3.3649e-01
## Parameter:
## [1] 24.4916354 24.3811678 1.2221640 13.1780942 -7.1304324 -8.8370069
## [7] -1.5432267 66.7914198 0.2134877
##
## Iteration 30, F(x) = -7.7099981138251485e+02, max|g(x)| = 6.6862e-02
## Parameter:
## [1] 24.4916414 24.3811700 1.2220443 13.1663245 -7.1305994 -8.8369569
## [7] -1.5431613 66.7274053 0.2134516
##
## Iteration 31, F(x) = -7.7099981157662182e+02, max|g(x)| = 9.8989e-03
## Parameter:
## [1] 24.491640 24.381144 1.222045 13.166278 -7.130594 -8.836946 -1.543143
## [8] 66.754314 0.213447
##
## Iteration 32, F(x) = -7.7099981158602736e+02, max|g(x)| = 2.4626e-03
## Parameter:
## [1] 24.491641 24.381147 1.222046 13.166741 -7.130607 -8.836948 -1.543144
## [8] 66.758555 0.213446
##
## Iteration 33, F(x) = -7.7099981158677906e+02, max|g(x)| = 4.5237e-04
## Parameter:
## [1] 24.4916405 24.3811462 1.2220476 13.1669154 -7.1306054 -8.8369473
## [7] -1.5431437 66.7591522 0.2134461

## Coefficients:
##             Estimate Std. Error     t value   Pr(>|t|)      dF/dPar dPen/dPar
## Ti0    2.4492e+01 3.7617e-02  6.5109e+02  0.0000e+00  4.4632e-03   4e-04
## Tm0    2.4381e+01 1.2695e-01  1.9205e+02  0.0000e+00  5.9287e-04   4e-04
## Ci     1.2220e+00 1.0998e-01  1.1111e+01  0.0000e+00 -3.5434e-05   0e+00

```

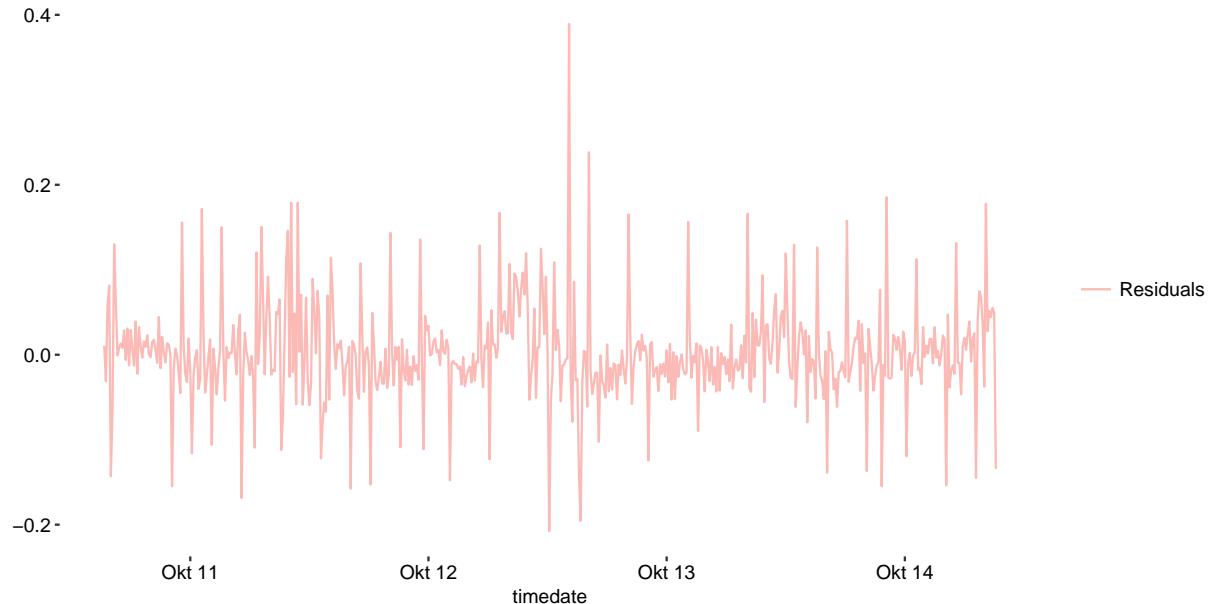
```

## Cm  1.3167e+01  6.5997e+00  1.9951e+00  4.6544e-02 -7.6129e-06    0e+00
## e11 -7.1306e+00  1.2886e-01 -5.5338e+01  0.0000e+00 -1.7809e-04    0e+00
## p11 -8.8369e+00  1.9800e-01 -4.4630e+01  0.0000e+00  4.9749e-05    0e+00
## p22 -1.5431e+00  8.5533e-02 -1.8042e+01  0.0000e+00 -1.2287e-04    0e+00
## Ria  6.6759e+01  9.2717e+01  7.2003e-01  4.7182e-01  8.4109e-07    0e+00
## Rim  2.1345e-01  1.8749e-02  1.1385e+01  0.0000e+00 -4.3269e-05    0e+00
##
## Correlation of coefficients:
##   Ti0   Tm0   Ci    Cm   e11   p11   p22   Ria
##   Tm0  0.24
##   Ci   0.00  0.07
##   Cm   0.00 -0.13  0.46
##   e11 -0.02 -0.02 -0.36 -0.12
##   p11  0.02  0.01 -0.14  0.02  0.17
##   p22  0.00  0.07  0.66  0.00 -0.37 -0.08
##   Ria  0.01 -0.38  0.23  0.53 -0.11 -0.06 -0.03
##   Rim -0.02 -0.04 -0.27 -0.18  0.51  0.02 -0.03 -0.17
##
## [1] "loglikelihood = 771.000411040433"

```

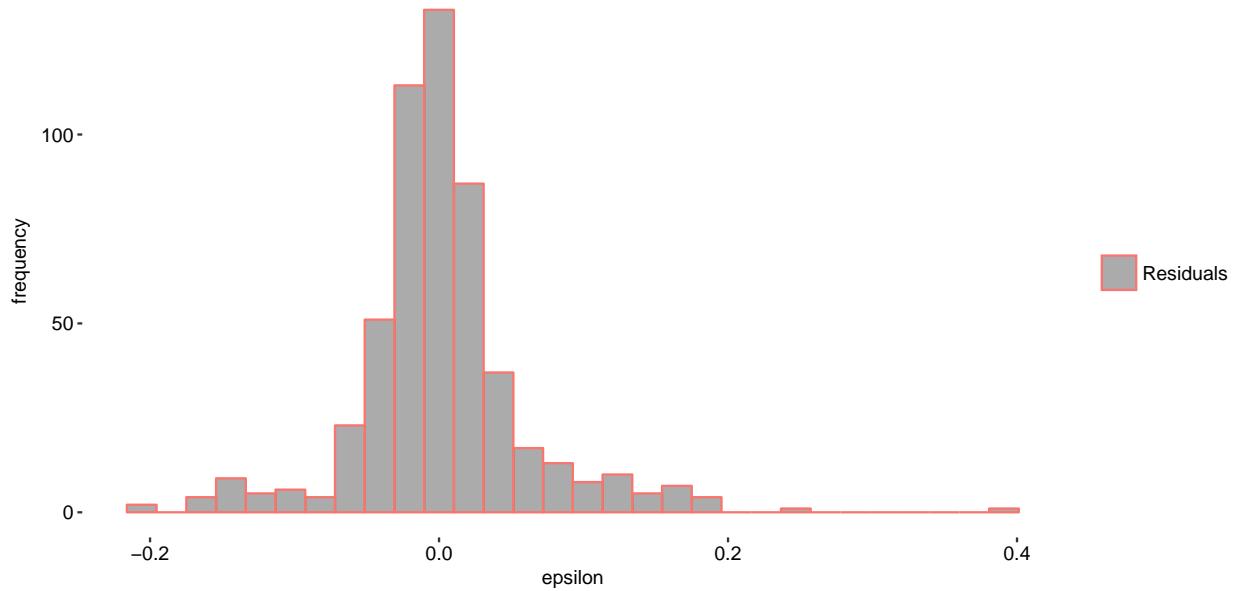
6. Step

Time Series of the residuals

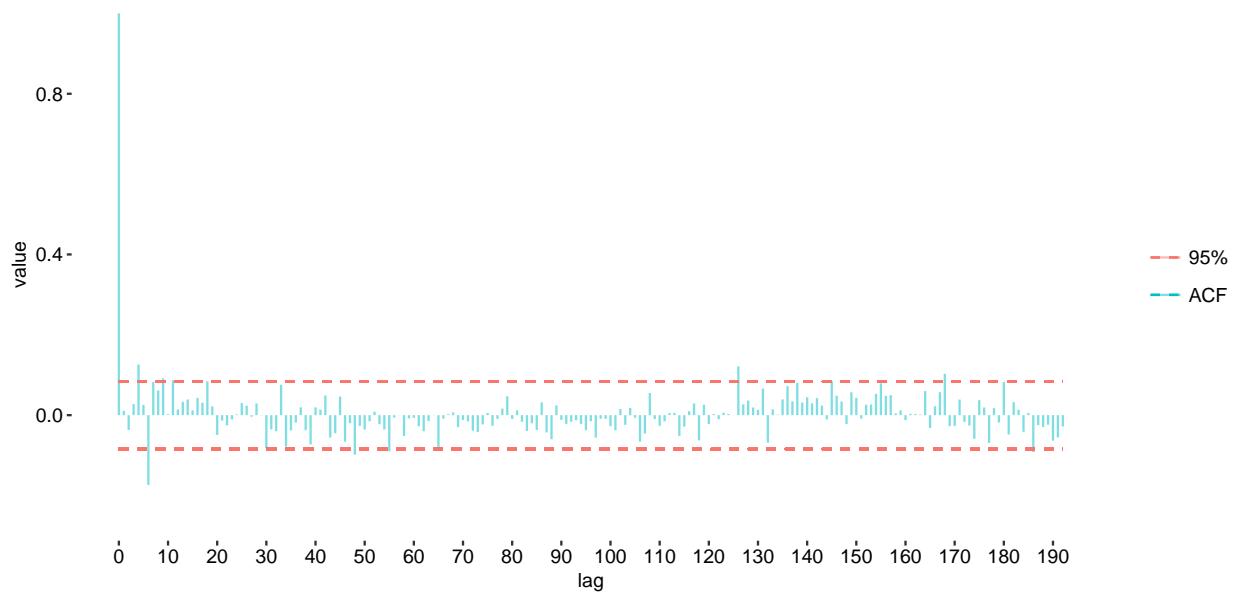


Distribution of the residuals

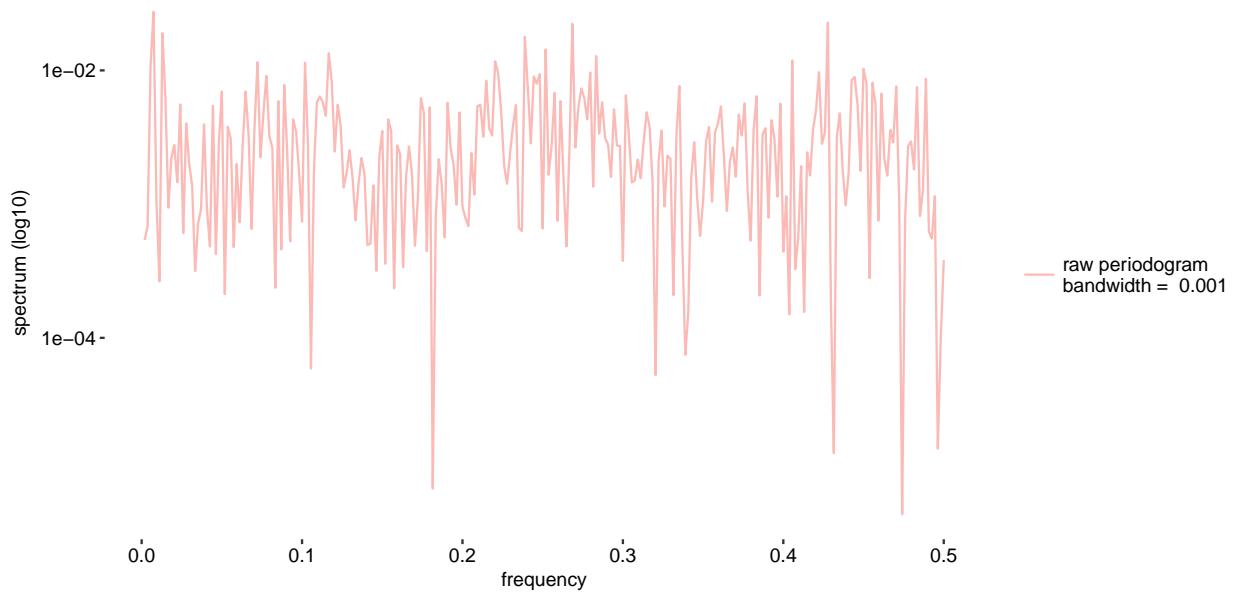
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



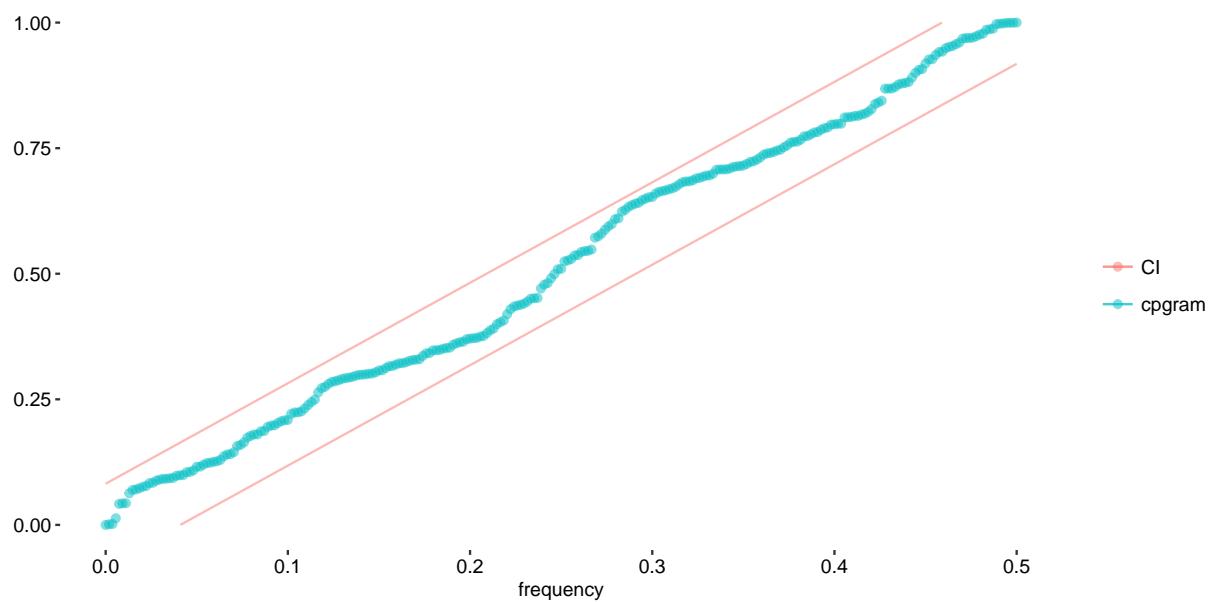
ACF of the residuals



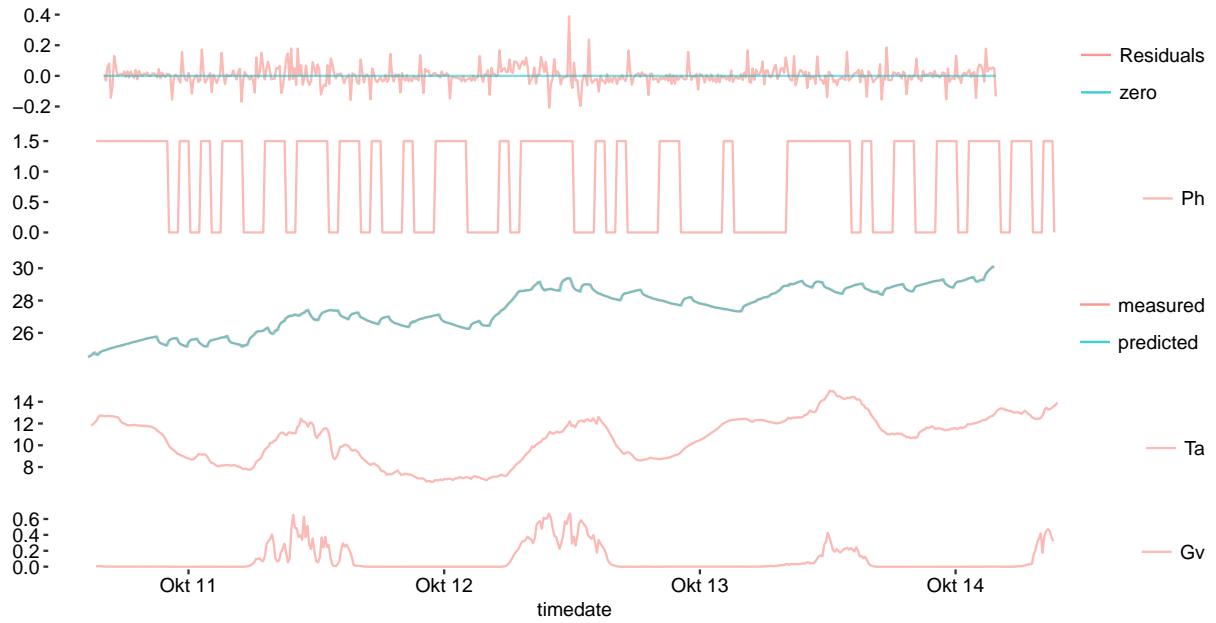
Periodogram of the residuals



Commulated Periodogram of the residuals



combined



7. Step

The P value from 0

Consider the following

- Discuss the white-noise properties of the (one-step ahead) residuals for model T_i .
 -
 -
- What useful information can be obtained from the time series plots of the residuals and the inputs for model T_i ?
 -
 -
- Discuss the white-noise properties of the one-step ahead residuals for model $T_i T_m$.
 -
 -
- What useful information can be obtained from the time series plots of the residuals and inputs for model $T_i T_m$?
 -
 -
- Based on the likelihood-ratio test is model $T_i T_m$ then to be preferred over model T_i ?
 -
 -

Question 2b

Describe the good way to implement it...!

what is Aw, Gv and p.. must

```
## A two-state model implemented in functions/TiTm.R
model.TiTm_2b <- ctsm()
## Add a system equation and thereby also a state
model.TiTm_2b$addSystem(dTi ~ (1/(Ci * Rim) * (Tm - Ti) + 1/(Ci * Ria) * (Ta -
    Ti) + (p * Aw)/Ci * Gv + 1/Ci * Ph) * dt + exp(p11) * dw1)
model.TiTm_2b$addSystem(dTm ~ (1/(Cm * Rim) * (Ti - Tm) + ((1 - p) * Aw)/Cm *
    Gv) * dt + exp(p22) * dw2)
## Set the names of the inputs
model.TiTm_2b$addInput(Ta, Ph, Gv)
## Set the observation equation: Ti is the state, yTi is the measured output
model.TiTm_2b$addObs(yTi ~ Ti)
## Set the variance of the measurement error
model.TiTm_2b$setVariance(yTi ~ exp(e11))
## Set the initial value (for the optimization) of the value of the state at
## the starting time point
model.TiTm_2b$setParameter(Ti = c(init = 25, lb = 0, ub = 40))
model.TiTm_2b$setParameter(Tm = c(init = 25, lb = 0, ub = 40))
## Set the initial value for the optimization
model.TiTm_2b$setParameter(Ci = c(init = 1, lb = 1e-05, ub = 20))
model.TiTm_2b$setParameter(Cm = c(init = 1, lb = 1e-04, ub = 100))
model.TiTm_2b$setParameter(Ria = c(init = 20, lb = 1, ub = 1e+05))
model.TiTm_2b$setParameter(Rim = c(init = 1, lb = 1e-04, ub = 100))
model.TiTm_2b$setParameter(p11 = c(init = 1, lb = -30, ub = 10))
model.TiTm_2b$setParameter(p22 = c(init = 1, lb = -30, ub = 10))
model.TiTm_2b$setParameter(e11 = c(init = -1, lb = -50, ub = 10))

model.TiTm_2b$setParameter(p = c(init = 0.5, lb = 0, ub = 1))
model.TiTm_2b$setParameter(Aw = 7.5 + 4.8)
# model.TiTm_2b$setParameter( Gv = c(init=mean(data$Gv) ,lb=min(data$Gv)
# ,ub=max(data$Gv)) )

## Run the parameter optimization
fit.TiTm_2b <- model.TiTm_2b$estimate(data)

## Iteration 1, F(x) = 6.5737311467757854e+02, max|g(x)| = 2.2321e+03
## Parameter:
## [1] 25.0 25.0 1.0 1.0 -1.0 0.5 1.0 1.0 20.0 1.0
##
## Iteration 2, F(x) = -2.6799800773998277e+02, max|g(x)| = 1.4469e+03
## Parameter:
## [1] 24.9851043 24.9969781 1.0017803 1.0015918 -4.5727684 0.4999159
## [7] -7.6433722 0.6282581 19.9477880 0.9919895
##
## Iteration 3, F(x) = -4.5099876281386730e+02, max|g(x)| = 8.4174e+02
## Parameter:
## [1] 24.6445249 24.9875946 1.0249996 1.0131155 -7.1459708 0.4992945
## [7] -8.3683874 0.3902562 19.8925631 0.9993662
##
## Iteration 4, F(x) = -4.6144558528042711e+02, max|g(x)| = 5.2433e+02
```

```

## Parameter:
## [1] 24.3816985 24.9835463 1.0440201 1.0231354 -7.4991673 0.4984472
## [7] -8.4862741 0.6704139 19.8540520 0.9986748
##
## Iteration 5, F(x) = -4.7110948639406320e+02, max|g(x)| = 6.2801e+02
## Parameter:
## [1] 24.5842772 24.9488365 1.0896187 1.0389780 -7.6790912 0.4966263
## [7] -8.5371148 0.8127337 19.7855053 1.0014007
##
## Iteration 6, F(x) = -5.9684996945608918e+02, max|g(x)| = 6.5525e+02
## Parameter:
## [1] 24.5829911 24.6400904 1.8974627 1.2449630 -8.2267143 0.4732934
## [7] -9.2262475 0.8196597 18.9893980 1.0591120
##
## Iteration 7, F(x) = -6.3125007306024577e+02, max|g(x)| = 1.0437e+03
## Parameter:
## [1] 24.2832750 22.2212715 3.1499355 0.8230772 -8.3606467 0.4045752
## [7] -9.3472769 0.5797982 18.2321603 0.4611138
##
## Iteration 8, F(x) = -6.7235087298001747e+02, max|g(x)| = 6.0760e+02
## Parameter:
## [1] 24.5454930 27.9464775 3.8472956 1.0235519 -8.0444564 0.4199308
## [7] -9.4441834 0.5382240 14.6524586 0.3882532
##
## Iteration 9, F(x) = -6.8704552410069255e+02, max|g(x)| = 5.3246e+02
## Parameter:
## [1] 24.3921629 23.3037296 4.6027048 1.7406562 -7.1191616 0.5146687
## [7] -9.7483650 0.6863713 8.8740854 0.5470782
##
## Iteration 10, F(x) = -7.0453560135557223e+02, max|g(x)| = 1.1787e+02
## Parameter:
## [1] 24.5072038 24.9517367 5.2830345 1.4329142 -7.5985842 0.5140933
## [7] -9.7968156 0.7613344 10.0319739 0.4069486
##
## Iteration 11, F(x) = -7.1388529350388296e+02, max|g(x)| = 1.2767e+02
## Parameter:
## [1] 24.5060133 25.5808417 6.5491348 1.4709754 -7.9421247 0.5525314
## [7] -9.9051975 0.5769602 10.2870580 0.2201182
##
## Iteration 12, F(x) = -7.1797931942634034e+02, max|g(x)| = 5.7083e+01
## Parameter:
## [1] 24.5064187 24.6749245 7.8641304 1.8009158 -8.2004929
## [6] 0.6000210 -10.0591504 0.4283133 13.6910309 0.1312887
##
## Iteration 13, F(x) = -7.1985412514973791e+02, max|g(x)| = 6.8525e+01
## Parameter:
## [1] 24.4972606 24.6725609 8.0779100 2.2240414 -8.1940337
## [6] 0.6036894 -10.1462890 0.3554150 14.5673687 0.1223168
##
## Iteration 14, F(x) = -7.2270026061276951e+02, max|g(x)| = 6.1860e+01
## Parameter:
## [1] 24.499900747 24.762550462 8.704287297 3.814133717 -8.158449963
## [6] 0.615444311 -10.326127484 0.001490912 15.371629645 0.084886097
##

```

```

## Iteration 15, F(x) = -7.2346705486002736e+02, max|g(x)| = 1.9047e+02
## Parameter:
## [1] 24.4964725 24.4689980 8.9044462 5.4553128 -8.1246838
## [6] 0.6199581 -10.4157639 -0.2489301 15.9426904 0.0669720
##
## Iteration 16, F(x) = -7.2390678088682694e+02, max|g(x)| = 2.3043e+02
## Parameter:
## [1] 24.50018070 24.68403207 8.96864501 6.47111493 -8.08052974
## [6] 0.62177434 -10.44938285 -0.36118135 16.23209106 0.06042984
##
## Iteration 17, F(x) = -7.2563128916615631e+02, max|g(x)| = 5.1071e+01
## Parameter:
## [1] 24.5000478 24.7562931 8.5273068 8.2153587 -7.9456929
## [6] 0.6137147 -10.4443051 -0.4260946 16.1183445 0.0596948
##
## Iteration 18, F(x) = -7.2734923231236530e+02, max|g(x)| = 2.0088e+02
## Parameter:
## [1] 24.48232645 24.51617744 7.54190827 15.74471742 -7.75803328
## [6] 0.58369542 -10.46958422 -0.84190653 16.65618871 0.04841499
##
## Iteration 19, F(x) = -7.5378423781551692e+02, max|g(x)| = 7.5964e+02
## Parameter:
## [1] 24.404507 24.184887 2.039775 23.748542 -7.448178 0.302929 -9.767884
## [8] -1.101215 12.883628 0.133505
##
## Iteration 20, F(x) = -7.5504118982828163e+02, max|g(x)| = 9.6349e+02
## Parameter:
## [1] 24.3878251 24.1366717 1.3871517 27.6948763 -7.3488545 0.2439464
## [7] -9.6044342 -1.2124352 12.2022593 0.1666558
##
## Iteration 21, F(x) = -7.6298367121856654e+02, max|g(x)| = 7.7519e+02
## Parameter:
## [1] 24.4164254 24.7312149 1.2770589 39.8727784 -7.4336259 0.2148174
## [7] -9.6880404 -1.4945233 14.0104299 0.1321983
##
## Iteration 22, F(x) = -7.6710882832189077e+02, max|g(x)| = 2.2326e+02
## Parameter:
## [1] 24.4292858 24.3359519 1.2900114 47.9815104 -7.3858469 0.2064533
## [7] -9.7801111 -1.6481054 14.9051746 0.1203399
##
## Iteration 23, F(x) = -7.7378943599406909e+02, max|g(x)| = 1.8828e+02
## Parameter:
## [1] 24.4570577 24.3501711 1.3631301 45.7930283 -7.3195327 0.2068924
## [7] -9.8058446 -1.5794970 14.9795618 0.1331114
##
## Iteration 24, F(x) = -7.8429922934321485e+02, max|g(x)| = 3.4242e+02
## Parameter:
## [1] 24.4975431 24.4484263 1.2165611 48.8852378 -7.1315689 0.1788128
## [7] -9.8180883 -1.5924547 15.6230833 0.1771799
##
## Iteration 25, F(x) = -7.8712433902991972e+02, max|g(x)| = 2.7716e+02
## Parameter:
## [1] 24.5281218 24.5401544 1.2066091 54.4948075 -6.9520379 0.1624146
## [7] -9.9767745 -1.6214779 18.6273679 0.2583454

```

```

## 
## Iteration 26, F(x) = -7.9000478538632615e+02, max|g(x)| = 1.7999e+02
## Parameter:
## [1] 24.5065217 24.5265461 1.2579010 52.5466701 -7.0569655 0.1772568
## [7] -9.9481987 -1.6233198 18.3489399 0.2204863
##
## Iteration 27, F(x) = -7.9040633815925082e+02, max|g(x)| = 2.3765e+02
## Parameter:
## [1] 24.4852477 24.5099961 1.1237718 54.2497405 -6.9687402 0.1810719
## [7] -9.8623116 -1.6949240 18.4036860 0.2201982
##
## Iteration 28, F(x) = -7.9107797111407763e+02, max|g(x)| = 1.8495e+02
## Parameter:
## [1] 24.4854183 24.5074603 1.1513264 54.3231831 -6.9720594 0.1852309
## [7] -9.8793617 -1.7031922 19.0246699 0.2196442
##
## Iteration 29, F(x) = -7.9188794568090191e+02, max|g(x)| = 7.1065e+01
## Parameter:
## [1] 24.4819511 24.4722288 1.1696403 54.1429170 -7.0107545 0.1881851
## [7] -9.8977258 -1.7345483 21.0777230 0.2226346
##
## Iteration 30, F(x) = -7.9217006514480647e+02, max|g(x)| = 4.1849e+01
## Parameter:
## [1] 24.495270 24.487191 1.182064 53.695438 -6.930867 0.196732 -9.873093
## [8] -1.786737 23.799206 0.225007
##
## Iteration 31, F(x) = -7.9318670295018251e+02, max|g(x)| = 9.6043e+01
## Parameter:
## [1] 24.4753911 24.3842141 1.1262116 47.4783590 -6.9938260 0.1894929
## [7] -9.7626893 -1.7954396 27.2962162 0.2297252
##
## Iteration 32, F(x) = -7.9879381463168602e+02, max|g(x)| = 2.0402e+02
## Parameter:
## [1] 24.4824807 24.3331118 1.0396213 22.3484857 -7.0076589 0.1819647
## [7] -9.2518198 -1.8567611 50.8855622 0.2312904
##
## Iteration 33, F(x) = -8.0001404440850456e+02, max|g(x)| = 3.5313e+01
## Parameter:
## [1] 24.4882818 24.3780468 1.0250980 19.4150702 -6.8800960 0.1903757
## [7] -9.1301779 -1.9087141 55.4413824 0.2269009
##
## Iteration 34, F(x) = -8.0043229800799452e+02, max|g(x)| = 3.9610e+01
## Parameter:
## [1] 24.4908125 24.4028683 1.0308925 21.4004332 -6.8502667 0.1919325
## [7] -9.1742055 -1.9123503 51.6976559 0.2260131
##
## Iteration 35, F(x) = -8.0059086450490645e+02, max|g(x)| = 2.9387e+01
## Parameter:
## [1] 24.4885272 24.3836394 1.0229566 21.8002469 -6.8510068 0.1893114
## [7] -9.1869499 -1.9228526 50.1936572 0.2300495
##
## Iteration 36, F(x) = -8.0071342313997093e+02, max|g(x)| = 1.3581e+01
## Parameter:
## [1] 24.4895297 24.3917821 1.0135299 21.9088580 -6.8576520 0.1861157

```

```

## [7] -9.1932072 -1.9351239 46.0871154 0.2315623
##
## Iteration 37, F(x) = -8.0115003056787361e+02, max|g(x)| = 5.4698e+01
## Parameter:
## [1] 24.4857814 24.4070434 0.9924293 21.6883035 -6.8552055 0.1803089
## [7] -9.2181588 -1.9692711 27.3349266 0.2371572
##
## Iteration 38, F(x) = -8.0144778969111144e+02, max|g(x)| = 5.9557e+01
## Parameter:
## [1] 24.4880228 24.4556847 0.9817228 20.4085715 -6.8524571 0.1788662
## [7] -9.2058997 -1.9794814 18.9923597 0.2374521
##
## Iteration 39, F(x) = -8.0184143501851122e+02, max|g(x)| = 3.4017e+01
## Parameter:
## [1] 24.4906176 24.4826524 0.9847911 19.3027413 -6.8448568 0.1822386
## [7] -9.1727494 -1.9726816 19.4263483 0.2343969
##
## Iteration 40, F(x) = -8.0192466243419290e+02, max|g(x)| = 4.8999e+00
## Parameter:
## [1] 24.4880658 24.4779329 0.9853106 18.3439028 -6.8267060 0.1856447
## [7] -9.1425132 -1.9712150 19.1265036 0.2324245
##
## Iteration 41, F(x) = -8.0192969859296306e+02, max|g(x)| = 1.7471e+00
## Parameter:
## [1] 24.4885997 24.4753758 0.9866571 18.6258630 -6.8287686 0.1855076
## [7] -9.1491634 -1.9678242 19.6735930 0.2326015
##
## Iteration 42, F(x) = -8.0193032413894264e+02, max|g(x)| = 4.5846e-01
## Parameter:
## [1] 24.4888844 24.4776712 0.9863945 18.5901403 -6.8296341 0.1854225
## [7] -9.1490346 -1.9679752 19.4178348 0.2325080
##
## Iteration 43, F(x) = -8.0193035958583164e+02, max|g(x)| = 6.1756e-02
## Parameter:
## [1] 24.4888333 24.4772867 0.9864327 18.5801954 -6.8293211 0.1854903
## [7] -9.1486114 -1.9679960 19.4479963 0.2325012
##
## Iteration 44, F(x) = -8.0193035985673384e+02, max|g(x)| = 7.7772e-03
## Parameter:
## [1] 24.4888408 24.4772869 0.9864407 18.5818581 -6.8293424 0.1854872
## [7] -9.1486616 -1.9679763 19.4482358 0.2325011
##
## Iteration 45, F(x) = -8.0193035985825122e+02, max|g(x)| = 9.9352e-04
## Parameter:
## [1] 24.4888418 24.4772893 0.9864402 18.5817912 -6.8293415 0.1854879
## [7] -9.1486595 -1.9679751 19.4480344 0.2325012
##
## Iteration 46, F(x) = -8.0193035985836616e+02, max|g(x)| = 3.3132e-04
## Parameter:
## [1] 24.4888419 24.4772887 0.9864405 18.5817905 -6.8293423 0.1854880
## [7] -9.1486592 -1.9679751 19.4480563 0.2325009

## Coefficients:
## Estimate Std. Error t value Pr(>|t|) dF/dPar dPen/dPar

```

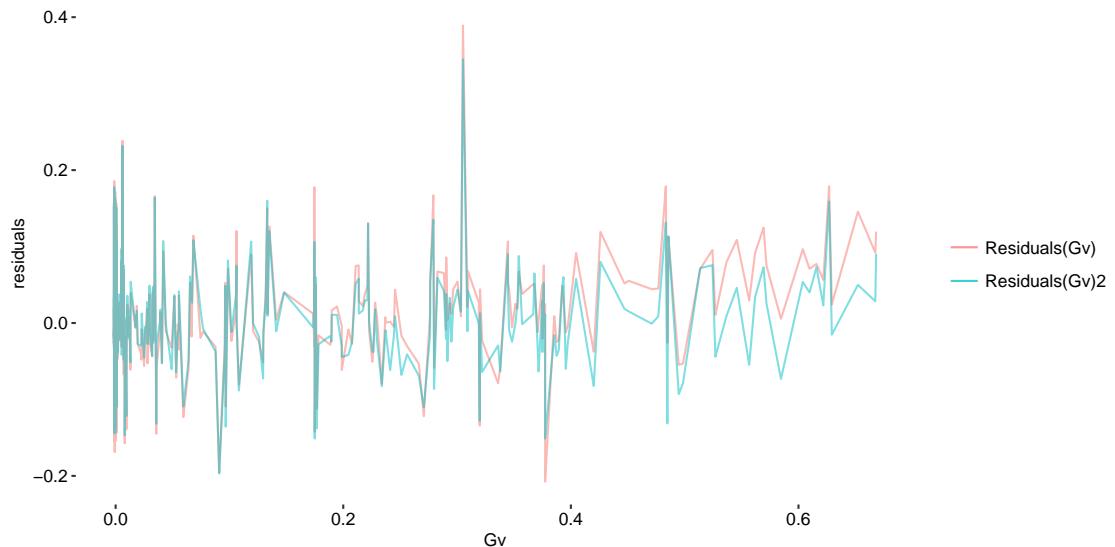
```

## Ti0 2.4489e+01 3.6668e-02 6.6786e+02 0.0000e+00 4.3000e-05 4e-04
## Tm0 2.4477e+01 9.8163e-02 2.4935e+02 0.0000e+00 -2.9409e-05 4e-04
## Aw 1.2300e+01 NA NA NA NA NA
## Ci 9.8644e-01 6.8555e-02 1.4389e+01 0.0000e+00 -1.1716e-06 0e+00
## Cm 1.8582e+01 2.6874e+00 6.9145e+00 1.3832e-11 -2.9918e-06 0e+00
## e11 -6.8293e+00 1.0673e-01 -6.3988e+01 0.0000e+00 -1.5967e-05 0e+00
## p 1.8549e-01 2.9369e-02 6.3157e+00 5.7724e-10 -1.4033e-06 0e+00
## p11 -9.1487e+00 2.2310e-01 -4.1007e+01 0.0000e+00 4.9089e-05 0e+00
## p22 -1.9680e+00 8.6330e-02 -2.2796e+01 0.0000e+00 -1.6512e-06 0e+00
## Ria 1.9448e+01 6.6528e+00 2.9233e+00 3.6122e-03 4.7629e-07 0e+00
## Rim 2.3250e-01 1.3792e-02 1.6857e+01 0.0000e+00 -2.2276e-06 0e+00
##
## Correlation of coefficients:
##      Ti0   Tm0   Ci    Cm   e11     p    p11    p22   Ria
## Tm0  0.14
## Ci   0.02 -0.05
## Cm   -0.03 -0.21  0.25
## e11 -0.01  0.00 -0.37 -0.09
## p    -0.07  0.00  0.14  0.07  0.00
## p11  0.04  0.01 -0.31 -0.32  0.17  0.04
## p22  0.05 -0.03  0.64  0.12 -0.47 -0.07 -0.20
## Ria -0.01 -0.53  0.11  0.40 -0.02  0.04 -0.04  0.03
## Rim -0.02 -0.02 -0.18  0.02  0.44 -0.19 -0.06 -0.24 -0.02
##
## [1] "loglikelihood = 801.930993840359"

```

Consider

- Findings
 - Plot of the resi



- Changes in squared residuals:

- * Question 2a: 1.8746528
- * Question 2b: 1.6630286

- Likelihood
 - Question 2a: 771.000411
 - Question 2b: 801.9309938
- Likelihood ratio test
The P value from $3.6859404 \times 10^{-14}$
- Conclusion:

Question 2c

Describe how i changed input

```
## A two-state model implemented in functions/TiTm.R
model.TiTm_2c <- ctsm()
## Add a system equation and thereby also a state
model.TiTm_2c$addSystem(dTi ~ (1/(Ci * Rim) * (Tm - Ti) + 1/(Ci * Ria) * (Ta -
  Ti) + (p * Aw)/Ci * Gv + 1/Ci * Ph) * dt + exp(p11) * dw1)
model.TiTm_2c$addSystem(dTm ~ (1/(Cm * Rim) * (Ti - Tm) + ((1 - p) * Aw)/Cm *
  Gv) * dt + exp(p22) * dw2)
## Set the names of the inputs
model.TiTm_2c$addInput(Ta, Ph, Gv)
## Set the observation equation: Ti is the state, yTi is the measured output
model.TiTm_2c$addObs(yTi ~ Ti)
## Set the variance of the measurement error
model.TiTm_2c$setVariance(yTi ~ exp(e11))
## Set the initial value (for the optimization) of the value of the state at
## the starting time point
model.TiTm_2c$setParameter(Ti = c(init = 25, lb = 0, ub = 40))
model.TiTm_2c$setParameter(Tm = c(init = 25, lb = 0, ub = 40))
## Set the initial value for the optimization
model.TiTm_2c$setParameter(Ci = c(init = 1, lb = 1e-05, ub = 20))
model.TiTm_2c$setParameter(Cm = c(init = 1, lb = 1e-04, ub = 100))
model.TiTm_2c$setParameter(Ria = c(init = 20, lb = 1, ub = 1e+05))
model.TiTm_2c$setParameter(Rim = c(init = 1, lb = 1e-04, ub = 100))
model.TiTm_2c$setParameter(p11 = c(init = 1, lb = -30, ub = 10))
model.TiTm_2c$setParameter(p22 = c(init = 1, lb = -30, ub = 10))
model.TiTm_2c$setParameter(e11 = c(init = -1, lb = -50, ub = 10))
model.TiTm_2c$setParameter(p = c(init = 0.5, lb = 0, ub = 1))
model.TiTm_2c$setParameter(Aw = 7.5 + 4.8)

## Run the parameter optimization
fit.TiTm_2c <- model.TiTm_2c$estimate(data)

## Iteration 1, F(x) = 6.5619928275591690e+02, max|g(x)| = 2.2295e+03
## Parameter:
## [1] 25.0 25.0 1.0 1.0 -1.0 0.5 1.0 1.0 20.0 1.0
##
## Iteration 2, F(x) = -1.8100069563243019e+02, max|g(x)| = 1.1281e+03
## Parameter:
## [1] 25.0691387 25.0044847 0.9990285 0.9999422 -4.5554271 0.4999277
## [7] -7.6433722 0.5442943 19.9526549 0.9924216
##
```

```

## Iteration 3, F(x) = -2.7336236936776930e+02, max|g(x)| = 8.0427e+02
## Parameter:
## [1] 26.0095073 25.0146005 1.0036698 1.0185718 -8.4134543 0.4987631
## [7] -8.5069667 1.1623974 19.8453068 0.9848241
##
## Iteration 4, F(x) = -3.0230739365643910e+02, max|g(x)| = 1.0875e+03
## Parameter:
## [1] 26.5735962 24.9928280 1.0204859 1.0217688 -9.1506308 0.4979862
## [7] -8.5911022 0.8179555 19.8108747 0.9881627
##
## Iteration 5, F(x) = -3.3267759855140366e+02, max|g(x)| = 1.3080e+03
## Parameter:
## [1] 27.4301456 24.9457189 1.0419439 1.0242267 -9.3987980 0.4965636
## [7] -8.7711074 0.7691135 19.7706282 0.9822545
##
## Iteration 6, F(x) = -3.7281449318413479e+02, max|g(x)| = 1.4733e+03
## Parameter:
## [1] 27.4697416 25.2302671 1.4715934 1.0566193 -9.7241254 0.4732379
## [7] -8.9199576 0.9110544 19.3573783 0.9004944
##
## Iteration 7, F(x) = -3.9949359997640374e+02, max|g(x)| = 1.6031e+03
## Parameter:
## [1] 27.0920698 23.6523572 1.7712291 0.8674584 -10.2259631
## [6] 0.4298931 -8.7877051 0.5226880 17.8601634 0.4492875
##
## Iteration 8, F(x) = -4.3185835846026879e+02, max|g(x)| = 5.4572e+02
## Parameter:
## [1] 27.0227787 25.1160212 1.8855066 1.0741743 -9.8604292 0.4264471
## [7] -8.8494780 0.6033083 15.5543992 0.4248760
##
## Iteration 9, F(x) = -4.5443140926547051e+02, max|g(x)| = 2.9237e+02
## Parameter:
## [1] 27.38964634 29.17082156 2.04947145 1.97650647 -9.53941681
## [6] 0.39457615 -8.99484988 0.05106789 11.68663095 0.19914755
##
## Iteration 10, F(x) = -4.6601745995171507e+02, max|g(x)| = 2.9375e+02
## Parameter:
## [1] 27.3084616 25.8962133 2.3276338 3.9081579 -9.2410021 0.3630993
## [7] -9.1653396 -0.1211586 10.0368410 0.1412566
##
## Iteration 11, F(x) = -4.8263246152875706e+02, max|g(x)| = 2.4433e+02
## Parameter:
## [1] 27.4061555 27.4776264 2.1946623 4.5585483 -9.0248267 0.3207644
## [7] -9.1720206 -0.2194409 12.8943807 0.1440310
##
## Iteration 12, F(x) = -4.8596250911084115e+02, max|g(x)| = 1.1756e+02
## Parameter:
## [1] 27.3992388 27.7193989 2.0707066 6.0664605 -8.8000400 0.2831990
## [7] -9.1912728 -0.4214307 13.4445574 0.1309105
##
## Iteration 13, F(x) = -5.1316686689109952e+02, max|g(x)| = 2.6774e+02
## Parameter:
## [1] 27.3511006 26.9549780 1.0705157 16.6115899 -7.0130622 0.0624036
## [7] -9.2015110 -0.8554846 23.5931954 0.2326541

```

```

## 
## Iteration 14, F(x) = -5.2028604732526378e+02, max|g(x)| = 2.8601e+02
## Parameter:
## [1] 27.34485646 26.85322736 0.70722273 20.56803322 -6.31373820
## [6] 0.02450055 -9.17225983 -0.88509169 32.02954133 0.41091696
##
## Iteration 15, F(x) = -5.2408564758263367e+02, max|g(x)| = 2.4115e+02
## Parameter:
## [1] 27.34718514 26.82891479 0.76617497 18.10461247 -6.67541778
## [6] 0.02873763 -9.18865156 -0.80888546 30.86760125 0.40249132
##
## Iteration 16, F(x) = -5.2529203344884525e+02, max|g(x)| = 1.7774e+02
## Parameter:
## [1] 27.39020872 27.03656402 0.83521593 17.53616092 -6.73610344
## [6] 0.03498074 -9.19818063 -0.78958329 29.50779716 0.36938637
##
## Iteration 17, F(x) = -5.2615776088522978e+02, max|g(x)| = 7.3063e+01
## Parameter:
## [1] 27.37528214 26.94461639 0.83924785 18.33040190 -6.72304865
## [6] 0.03752659 -9.19605176 -0.82018705 29.72656108 0.37309980
##
## Iteration 18, F(x) = -5.2720967123092782e+02, max|g(x)| = 6.5296e+01
## Parameter:
## [1] 27.39357069 26.97696665 0.80787157 20.14951098 -6.64184746
## [6] 0.04252954 -9.18746050 -0.88462618 31.38440752 0.40290015
##
## Iteration 19, F(x) = -5.2855990308632613e+02, max|g(x)| = 1.1826e+02
## Parameter:
## [1] 27.36691652 26.85963445 0.79821304 20.74901591 -6.52121962
## [6] 0.06250898 -9.13405633 -0.91156900 31.69569182 0.43680116
##
## Iteration 20, F(x) = -5.3070843173038622e+02, max|g(x)| = 1.5431e+02
## Parameter:
## [1] 27.3466553 26.7655615 0.7647491 21.5890649 -6.3489614 0.1243613
## [7] -9.0367086 -0.9775802 32.1970836 0.4887754
##
## Iteration 21, F(x) = -5.3154684673509928e+02, max|g(x)| = 1.2952e+02
## Parameter:
## [1] 27.3442698 26.7642329 0.7342869 21.8938468 -6.2862954 0.1586115
## [7] -8.9942175 -1.0210135 32.4232327 0.5015581
##
## Iteration 22, F(x) = -5.3285083876844374e+02, max|g(x)| = 8.0271e+01
## Parameter:
## [1] 27.3522344 26.8110566 0.7150378 21.6422726 -6.2952985 0.1544421
## [7] -8.9897445 -1.0337568 32.3568568 0.4893522
##
## Iteration 23, F(x) = -5.3369543959149132e+02, max|g(x)| = 2.6460e+01
## Parameter:
## [1] 27.3785807 26.9076062 0.7230632 19.8222773 -6.4076343 0.1659250
## [7] -8.9801488 -1.0158327 31.1934663 0.4473303
##
## Iteration 24, F(x) = -5.3377903873539094e+02, max|g(x)| = 2.4881e+01
## Parameter:
## [1] 27.3848053 26.8851917 0.7066542 19.8387367 -6.3772832 0.1539834

```

```

## [7] -8.9802843 -1.0132665 31.9009863 0.4584579
##
## Iteration 25, F(x) = -5.3380564673032814e+02, max|g(x)| = 1.1592e+01
## Parameter:
## [1] 27.3798449 26.8837029 0.7111910 19.7688110 -6.3786204 0.1574465
## [7] -8.9768379 -1.0144635 31.8181688 0.4555041
##
## Iteration 26, F(x) = -5.3382217958073284e+02, max|g(x)| = 7.8612e+00
## Parameter:
## [1] 27.3782800 26.8871089 0.7115268 19.3194911 -6.3774617 0.1616905
## [7] -8.9623957 -1.0169198 32.5421284 0.4566099
##
## Iteration 27, F(x) = -5.3388101251355761e+02, max|g(x)| = 9.6038e+00
## Parameter:
## [1] 27.3763338 26.8476376 0.7088580 17.5827094 -6.3658306 0.1588480
## [7] -8.9163040 -1.0221624 35.8030809 0.4561572
##
## Iteration 28, F(x) = -5.3389196598139324e+02, max|g(x)| = 2.6371e+00
## Parameter:
## [1] 27.3795893 26.8542277 0.7037982 17.6053134 -6.3623673 0.1583181
## [7] -8.9144389 -1.0272720 36.0413167 0.4586142
##
## Iteration 29, F(x) = -5.3389229451157212e+02, max|g(x)| = 1.0650e+00
## Parameter:
## [1] 27.3799228 26.8546538 0.7034465 17.5811857 -6.3628044 0.1582310
## [7] -8.9137877 -1.0281517 36.0292172 0.4587612
##
## Iteration 30, F(x) = -5.3389258719926033e+02, max|g(x)| = 7.4966e-01
## Parameter:
## [1] 27.3800936 26.8555844 0.7031901 17.5621673 -6.3627613 0.1581777
## [7] -8.9134721 -1.0288413 35.8840022 0.4588715
##
## Iteration 31, F(x) = -5.3389384306664795e+02, max|g(x)| = 5.5838e+00
## Parameter:
## [1] 27.3807268 26.8636498 0.7019129 17.3717295 -6.3622981 0.1578989
## [7] -8.9112797 -1.0316952 34.4705183 0.4595178
##
## Iteration 32, F(x) = -5.3389789830879738e+02, max|g(x)| = 5.9562e-01
## Parameter:
## [1] 27.3799827 26.8897344 0.7010270 16.8084157 -6.3582120 0.1577851
## [7] -8.9077548 -1.0292628 30.2373303 0.4604137
##
## Iteration 33, F(x) = -5.3389807772919130e+02, max|g(x)| = 2.3575e-01
## Parameter:
## [1] 27.3798844 26.8848375 0.7013827 16.8908385 -6.3590422 0.1578431
## [7] -8.9076974 -1.0289175 31.1661020 0.4601811
##
## Iteration 34, F(x) = -5.3389810887544854e+02, max|g(x)| = 3.6561e-02
## Parameter:
## [1] 27.3798988 26.8858391 0.7012686 16.8767244 -6.3590183 0.1578282
## [7] -8.9078503 -1.0290427 30.9387336 0.4602390
##
## Iteration 35, F(x) = -5.3389810911909149e+02, max|g(x)| = 1.0327e-02
## Parameter:

```

```

## [1] 27.3799125 26.8860808 0.7012632 16.8720359 -6.3589701 0.1578254
## [7] -8.9078022 -1.0290387 30.9122498 0.4602441
##
## Iteration 36, F(x) = -5.3389810913339079e+02, max|g(x)| = 5.8126e-04
## Parameter:
## [1] 27.3799088 26.8860450 0.7012622 16.8726290 -6.3589776 0.1578260
## [7] -8.9078079 -1.0290400 30.9154068 0.4602440
##
## Iteration 37, F(x) = -5.3389810913344513e+02, max|g(x)| = 2.2877e-04
## Parameter:
## [1] 27.3799091 26.8860452 0.7012623 16.8726711 -6.3589779 0.1578260
## [7] -8.9078076 -1.0290400 30.9155399 0.4602439

## Coefficients:
##          Estimate Std. Error   t value Pr(>|t|)    dF/dPar dPen/dPar
## Ti0  2.7380e+01 5.4465e-02 5.0270e+02 0.0000e+00 -2.3262e-05 7e-04
## Tm0  2.6886e+01 3.5651e-01 7.5415e+01 0.0000e+00 6.2791e-05 6e-04
## Aw   1.2300e+01           NA           NA           NA           NA       NA
## Ci   7.0126e-01 5.4524e-02 1.2862e+01 0.0000e+00 1.1018e-05 0e+00
## Cm   1.6873e+01 7.2707e+00 2.3206e+00 2.0685e-02 -2.5728e-06 0e+00
## e11  -6.3590e+00 1.5539e-01 -4.0922e+01 0.0000e+00 -1.5480e-05 0e+00
## p    1.5783e-01 3.1869e-02 4.9523e+00 9.9102e-07 6.4493e-06 0e+00
## p11 -8.9078e+00 2.8365e-01 -3.1404e+01 0.0000e+00 2.0439e-04 0e+00
## p22 -1.0290e+00 9.2965e-02 -1.1069e+01 0.0000e+00 1.1677e-05 0e+00
## Ria  3.0916e+01 3.9330e+01 7.8605e-01 4.3219e-01 1.5932e-06 0e+00
## Rim  4.6024e-01 3.9437e-02 1.1670e+01 0.0000e+00 4.2493e-06 0e+00
##
## Correlation of coefficients:
##      Ti0   Tm0   Ci    Cm   e11     p    p11    p22   Ria
## Tm0  0.17
## Ci   0.00 -0.12
## Cm  -0.07 -0.54  0.29
## e11 -0.02  0.03 -0.65 -0.15
## p   -0.06 -0.12 -0.03  0.02  0.12
## p11  0.19  0.08 -0.20 -0.26  0.14 -0.01
## p22 -0.01  0.07  0.76  0.07 -0.55 -0.14 -0.24
## Ria -0.02 -0.83  0.19  0.64 -0.10  0.10  0.05 -0.06
## Rim  0.00  0.10 -0.57 -0.22  0.62 -0.08  0.11 -0.32 -0.23
##
## [1] "loglikelihood = 533.898842680385"

```

Findings

- Log like
 - Question 2b: 801.9309938
 - Question 2c: 533.8988427
- Cap
 - $C_i + C_m$
 - Question 2b: 19.5682341
 - Question 2c: 17.5739334

Question 2d

Not considered due to time.