

Advanced Time Series Analysis: Computer Exercise 1

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Part 1

There is generated $n = 1000$ noise samples from a $x \sim \mathcal{N}(0, 1)$ which is used in all simulation in part one.

The equations below are the used parameters though out this exercise. Let us call the eq. 1 and eq. 2 parameter set one (par_1), eq. 3 and eq. 4 parameter set two (par_2).

$$a_0 = [2.0, -1.0] \tag{1}$$

$$a_1 = [0.6, -0.4] \tag{2}$$

$$a_{02} = [3.0, -2.0] \tag{3}$$

$$a_{12} = [-0.6, 0.4] \tag{4}$$

SETAR(2,1,1)

The Self-Exciting Threshold AR (SETAR) model is given by eq. 5.

$$X_t = a_0^{(J_t)} + \sum_{i=1}^{k(J_t)} a_i^{(J_t)} X_{t-i} + \epsilon^{(J_t)} \tag{5}$$

where J_t are regime processes. The complete model are defined in eq. 6.

$$X_t = \begin{cases} a_{0,1} + a_{1,1}X_{t-1} + \epsilon_t & \text{for } X_{t-1} \leq 0 \\ a_{0,2} + a_{1,2}X_{t-1} + \epsilon_t & \text{for } X_{t-1} > 0 \end{cases} \tag{6}$$

The model (eq. 6) have been simulated with two different set of parameters (eq. 1 - eq. 4) and their processes are plotted in fig. 1.

Fig. 1 shows the plot of the SETAR(2,1,1) model with the two different parameter sets.

- For both model it is possible to differentiate between the regimes and their transitions.
- It is also possible to see the inverse properties of the two model defined by the slope.
- Both models are using different offsets where the transition are most separated in the model which is using par_2 .

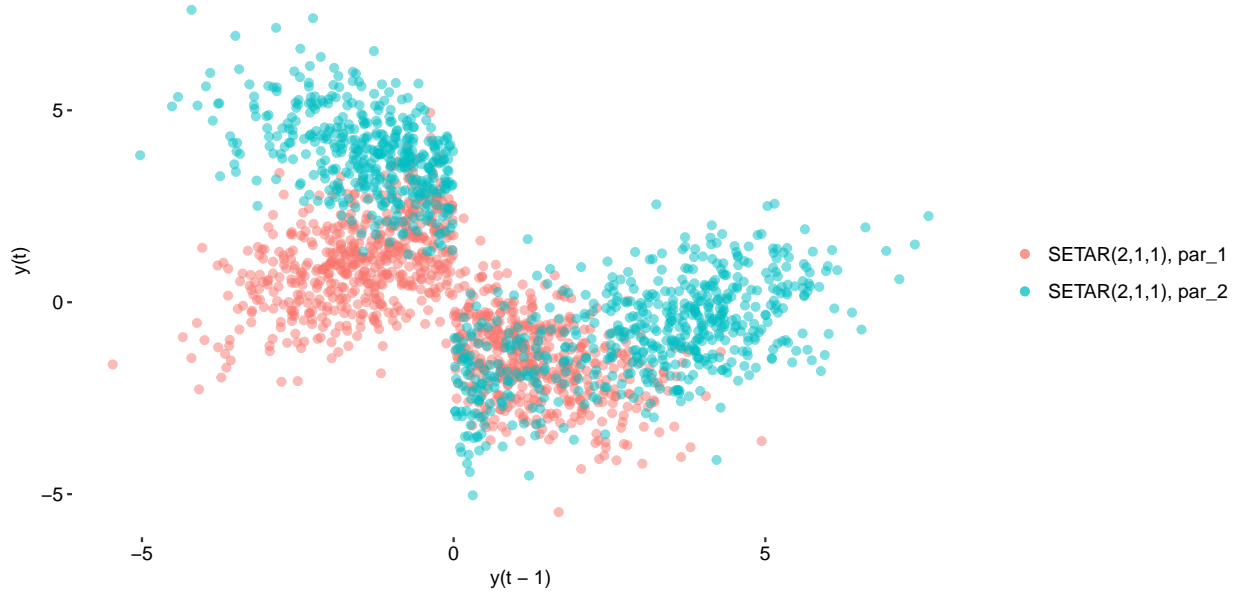


Figure 1: Two simulated SETAR(2,1,1) models using par_1 and par_2 .

IGAR(2,1)

The IGAR model are given by the same equation as the SETAR model (eq. 5) but using an external parameter to switch between regimes.

The complete simulated IGAR model is given in eq. 7.

$$X_t = \begin{cases} a_{0,1} + a_{1,1}X_{t-1} + \epsilon_t & \text{for } p \leq 0.5 \\ a_{0,2} + a_{1,2}X_{t-1} + \epsilon_t & \text{for } p > 0.5 \end{cases} \quad (7)$$

Fig. 2 shows the plot of the IGAR(2,1) model with the two different parameter sets.

- Different from the SETAR model the IGAR model using a given external parameter to switch between regimes. In this case there have been sampled a random number between $[0, 1]$.
- The shift threshold is 0.5 (eq. 7) which supports the distribution of the data points in fig. 2. The data point are more less equally distributed in both regimes for both IGAR models.

MMAR(2,1)

The simulated MMAR model has same properties as the IGAR model in eq. 7. The main difference are the properties of the transition parameter p . The transition parameters between regimes are given by the transition matrix P in eq. 8.

$$P = \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix} \in R_1 \quad (8)$$

Fig. 3 shows the plot of the MMAR(2,1) model with the two different parameter sets.

- P is useful for setting different thresholds for shifting between regimes.

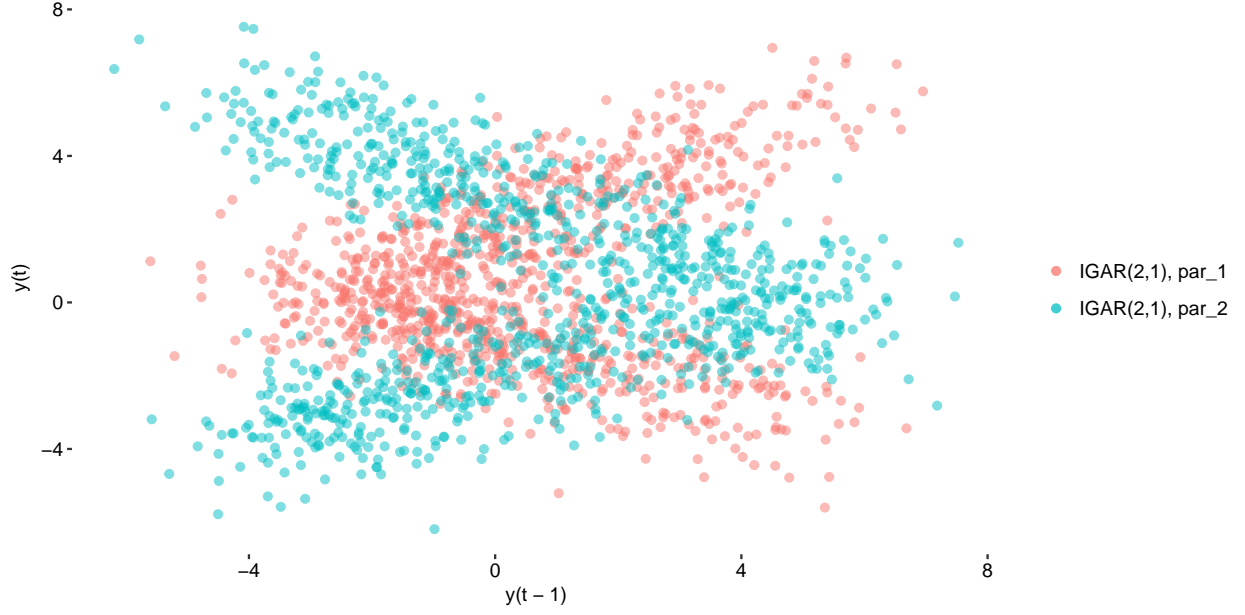


Figure 2: Two simulated IGAR(2,1) models using par_1 and par_2 .

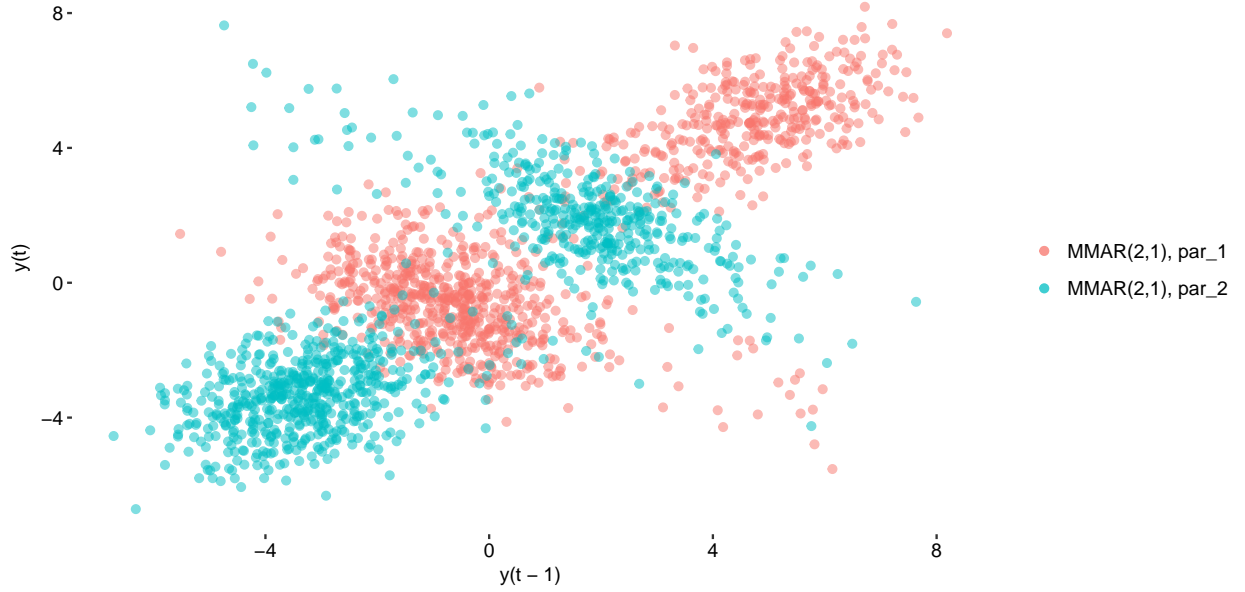


Figure 3: Two simulated MMAR(2,1) models using par_1 and par_2 .

- The diagonal in P is, in this case, determine external parameter for being this the current regime. The off-diagonal are external parameter for shifting to a new regime.

Common for above models

- The main difference between the three models are the properties for shifting to a new regime. The shift in the SETAR model depends on the previous value of the model and some predefined regimes and the

IGAR model and the MMAR model shifts with respect to an external parameter.

- The main difference between the IGAR model and the MMAR model is that it is possible to determine different thresholds for shifting between given regimes. Maybe there are some physical properties which make the transition from a given stage to another given stage.

Part 2

Using the same SETAR model with par_1 from part 1, eq. 6.

Compute the theoretical mean

The theoretical mean, eq. 9

$$M(x) = E \{X_{t+1} | X_t = x\} \quad (9)$$

By the fact that the noise are Gaussian distributed, $\epsilon_t = 0$ it is possible to rewrite the SETAR(2,1,1) model (eq. 6) to the theoretical mean in eq. 10

$$M_t = \begin{cases} a_{0,1} + a_{1,1}X_{t-1} & \text{for } X_{t-1} \leq 0 \\ a_{0,2} + a_{1,2}X_{t-1} & \text{for } X_{t-1} > 0 \end{cases} \quad (10)$$

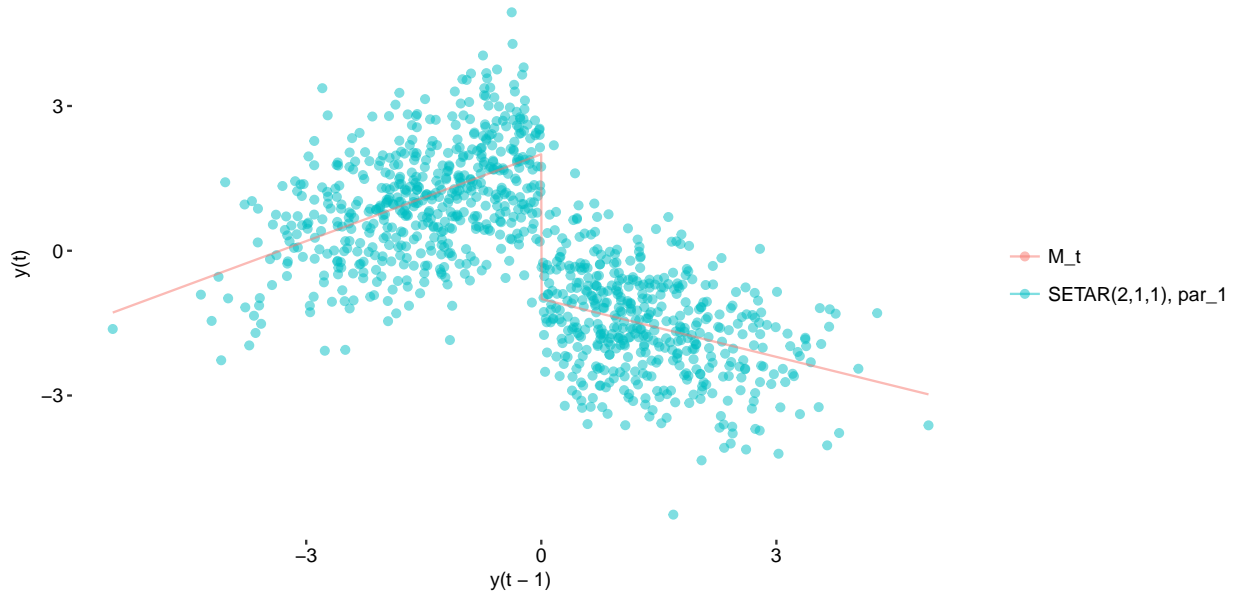


Figure 4: SETAR(2,1,1) with theoretical mean.

Fig. 4 shows the plot of the theoretical mean with two. I do not have any comments to this plot.

Estimate the mean

I have chosen to use the native function `lm()` have been used to estimate the mean of the SETAR(2,1,1) model with two selected bandwidths, $bw_n = (0.2, 0.7)$. `lm()` is set to use a local second order polynomial regression.

The `lm()` uses the bandwidth restricts number of samples to include to each fit $idx = as.integer(bw_n * n)$. Besides the bandwidth each fit also uses weights from the Epanechnikov kernel (function from sample code).

Fig. 5 shows the plot of the estimated mean with two different bandwidths.

- The conceptual interpretation of the bandwidth is a measure for how many samples which should be used in the local fit of the second order polynomial.

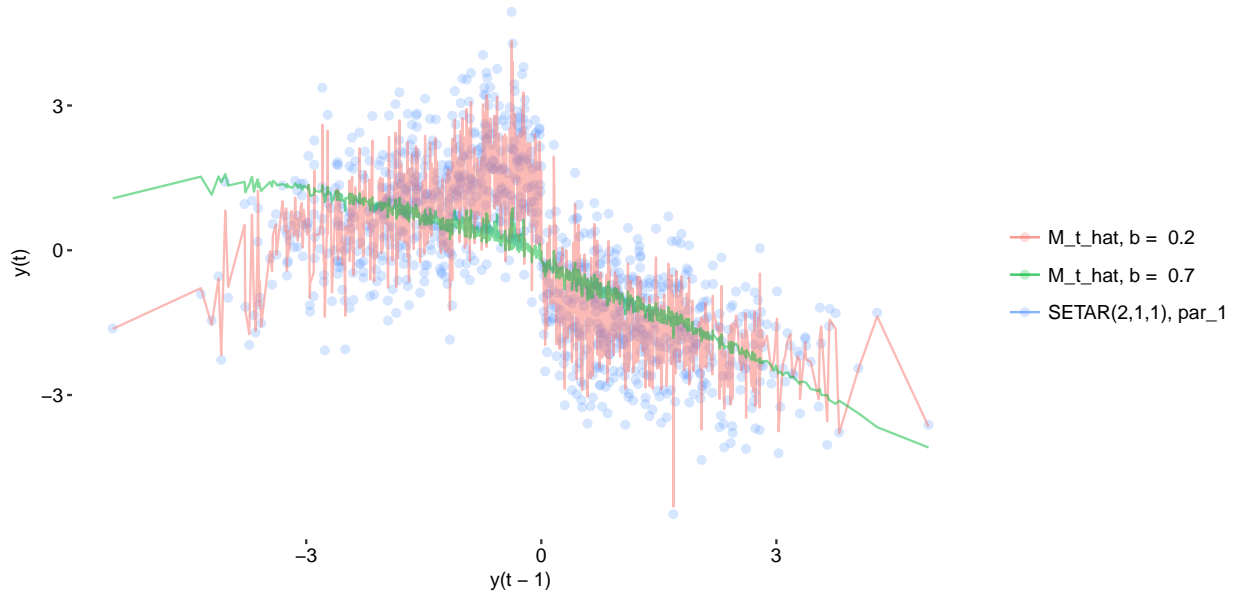


Figure 5: Plots of the estimated mean with different bandwidths.

- A higher bandwidth will decrease the noise in the signal and vice versa. The high bandwidth makes estimated mean is less flexibel to regime shifts for the SETAR(2,1,1) model.
- A lower bandwidth is more suitable to estimate the mean value of the SETAR(2,1,1) model, due to the charaterica of its regimes.
- It is also possible to obtain better estimation in the boundaries by using the lower bandwidth.

Part 3

The `cumulativeMeans.R` script have been used to calculate the cumulative conditional means.

The theoretical cumulative conditional means are determined by eq. 11.

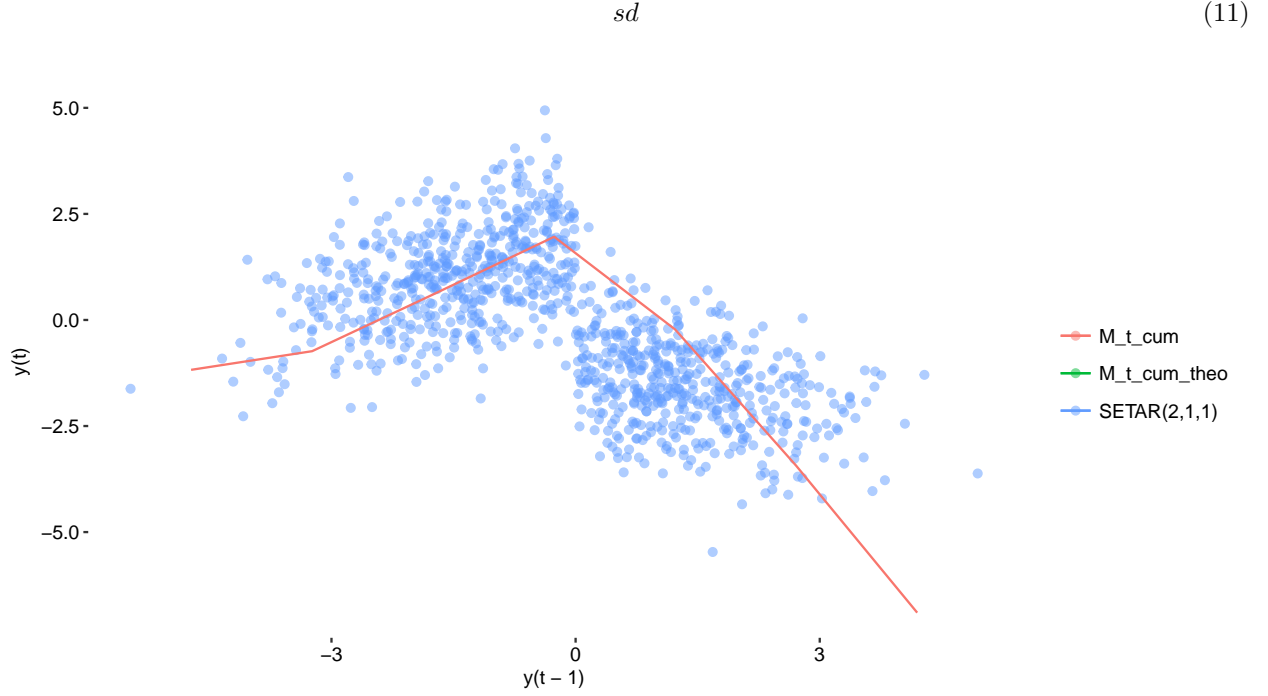


Figure 6: Plots of the theoretical cumulative conditional means and estimated cumulative conditional means.

Fig. 6 shows the plot of the theoretical cumulative conditional means and the estimated cumulative conditional means.

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Part 5

Native function `acf()`

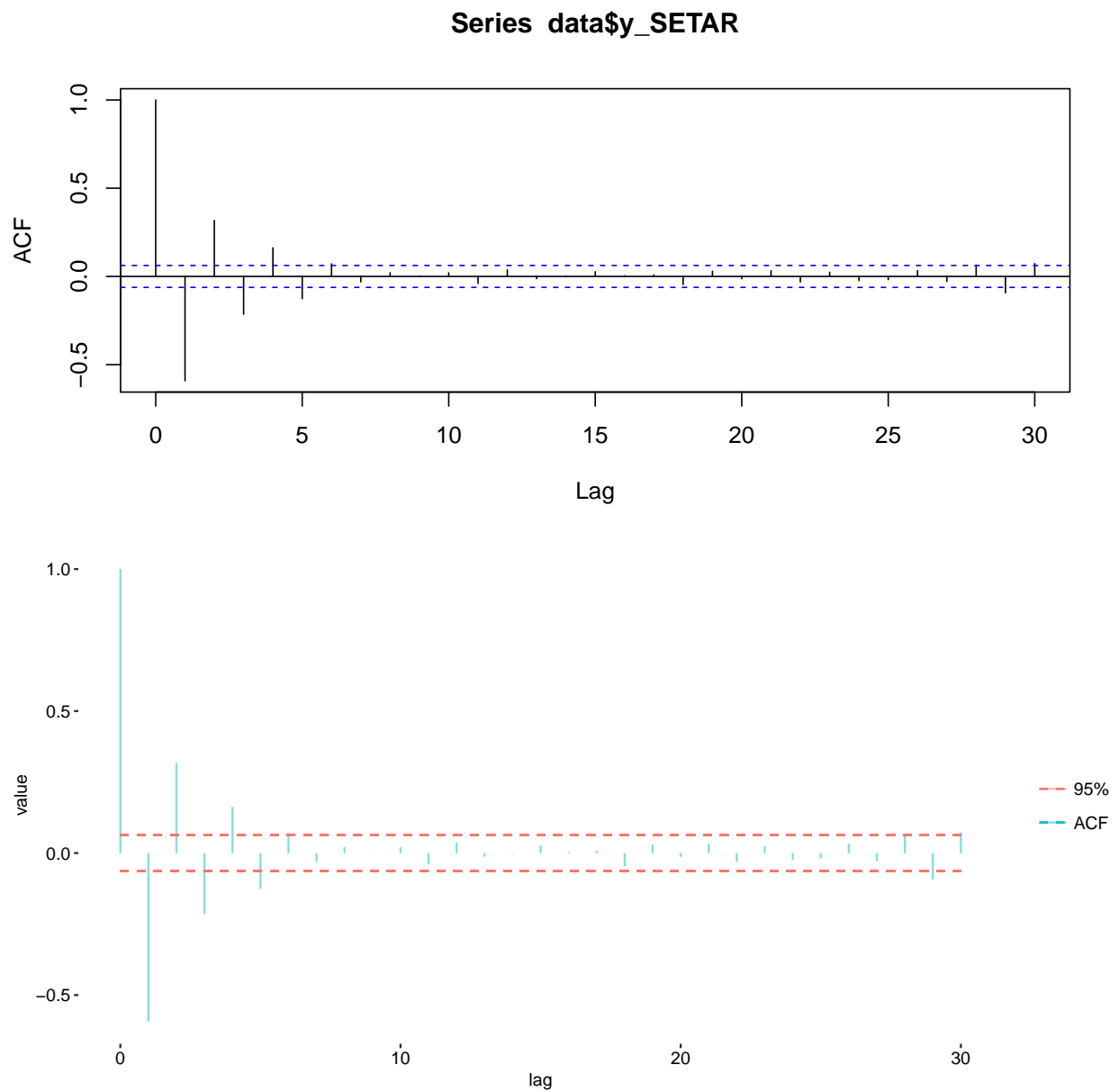


Figure 7:

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## Calculating ldf no. 1 of 1
## Fitting for bandwidth 1 of 12
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## Fitting for bandwidth 3 of 12
## Fitting for bandwidth 4 of 12
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## Fitting for bandwidth 5 of 12
## Fitting for bandwidth 6 of 12
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## Fitting for bandwidth 8 of 12
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## Fitting for bandwidth 10 of 12
## Fitting for bandwidth 11 of 12
## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 1 of 30
## Fitting for bandwidth 1 of 12
## Fitting for bandwidth 2 of 12
## Fitting for bandwidth 3 of 12
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## Fitting for bandwidth 7 of 12
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## Calculating bootstrap no. 2 of 30
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## Fitting for bandwidth 3 of 12
## Fitting for bandwidth 4 of 12
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## Fitting for bandwidth 11 of 12
## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 3 of 30
## Fitting for bandwidth 1 of 12
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## Fitting for bandwidth 2 of 12
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## Fitting for bandwidth 11 of 12
## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 4 of 30
## Fitting for bandwidth 1 of 12
## Fitting for bandwidth 2 of 12
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## Calculating bootstrap no. 5 of 30
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## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 6 of 30
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## Fitting for bandwidth 11 of 12
## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 7 of 30
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## Fitting for bandwidth 11 of 12
## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 8 of 30
## Fitting for bandwidth 1 of 12
## Fitting for bandwidth 2 of 12
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## Fitting for bandwidth 9 of 12
## Fitting for bandwidth 10 of 12
## Fitting for bandwidth 11 of 12
## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 9 of 30
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## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 10 of 30
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## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 11 of 30
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## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 12 of 30
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## Calculating bootstrap no. 13 of 30
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## Calculating bootstrap no. 14 of 30
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## Calculating bootstrap no. 15 of 30
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## Calculating bootstrap no. 16 of 30
## Fitting for bandwidth 1 of 12
## Fitting for bandwidth 2 of 12
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## Calculating bootstrap no. 19 of 30
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## Calculating bootstrap no. 20 of 30
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## Calculating bootstrap no. 21 of 30
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## Calculating bootstrap no. 22 of 30
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## Fitting for bandwidth 12 of 12
## Calculating bootstrap no. 28 of 30
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## Calculating bootstrap no. 30 of 30
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## Fitting for bandwidth 11 of 12
## Fitting for bandwidth 12 of 12
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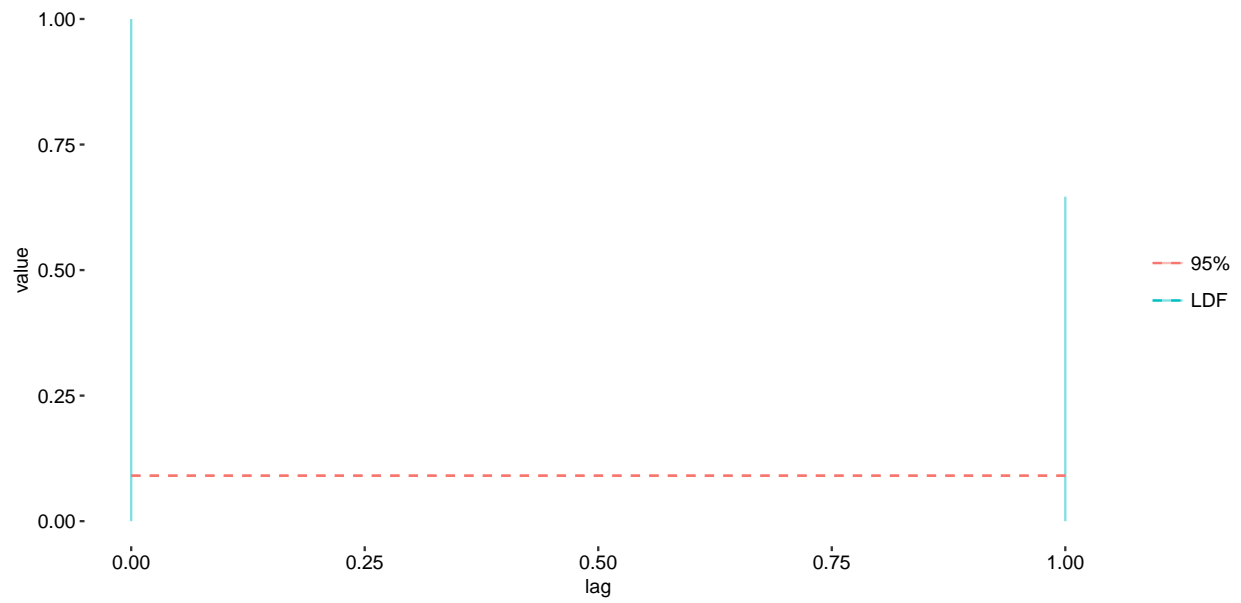


Figure 8: