Advanced Time Series Analysis.
Non-Linear Time Series Analysis

EXERCISE 1, lecture.

Its sometimes possible to approximate the conditional mean $E(X_t|X_{t-j}), j = 1, 2, \ldots$ for at SETAR(2;2,2) by using the piecewise linearity of the process. A condition under which the approximation might work is given by

$$\mathrm{Var}(E(X_t|X_{t-i})) \approx \rho_i^2 \mathrm{Var}(X_t), i = \pm 1.$$

A: Give an interpretation of what this condition means!

A second order SETAR model with two regimes for the Canadian Lynx data from 1821-1919, given in Tong (1990), is

$$\begin{aligned} X_t &= (0.62 + 1.25X_{t-1} - 0.43X_{t-2} + \varepsilon_t)I_{t-2}(3.25) + \\ &+ (2.25 + 1.52X_{t-1} - 1.24X_{t-2} + \varepsilon_t^{'})(1 - I_{t-2}(3.25)), \end{aligned}$$

where $I_t(y)$ is an indicator variable defined by

$$I_t(y) = \begin{cases} 1 & \text{if } X_t \le y \\ 0 & \text{otherwise} \end{cases}$$

and $\{\varepsilon_t\}$, $\{\varepsilon_t^{'}\}$ are two independent sequences of independent random variables from the distributions $\mathcal{N}(0, 0.0381)$ and $\mathcal{N}(0, 0.0626)$ respectively.

From data one can note firstly that the condition mentioned above is met, and secondly that by fitting a linear AR(2) model to data the following conditional mean is obtained:

$$E(X_t|X_{t\pm 1} = x) = 0.6 + 0.8x.$$

B: Use the nonlinear SETAR(2;2,2) model to write down the conditional mean $E(X_t|X_{t-1} = x)$.

C: Note that the variance of the two noise sources are small. Hence assume we have either a large or a small value of x and compute an approximation of the conditional mean for these two cases. If we agree on having a continuous conditional mean with respect to x, give an approximation of the conditional mean for all values of x. Compare with what you got in B above.

D: This same procedure could be followed to compute, iteratively, the conditional means $E(X_t|X_{t-i} = x, i = 2, 3, \ldots)$, however with decreasing precision. Do that for at least $i = 2$.