Cálculo dos Autovalores e Autovetores p/A Não-simétrica

1. Dimensão da matriz

```
NN = 5
```

```
5
```

2. Matriz de entrada

```
A = {{3., 7., 8, 9, 12}, {5, -7, 4, -7, 8},
  {1, 1, -1, 1, -1}, {4, 3, 2, 1, 7}, {9, 3, 2, 5, 4}}
```

```
{{3., 7., 8, 9, 12}, {5, -7, 4, -7, 8},
 {1, 1, -1, 1, -1}, {4, 3, 2, 1, 7}, {9, 3, 2, 5, 4}}
```

3. Decomposição de Hessemberg (Método de Householder)

p: é o acúmulo das matrizes de Householder

h: é a matriz de Hessemberg resultante do método de Householder

```
{p, h} = HessenbergDecomposition[A]
```

```
{{{1., 0., 0., 0., 0.}, {0., -0.450835, -0.508605, -0.733385, -0.0146591},
  {0., -0.090167, -0.16282, 0.148933, 0.971174},
  {0., -0.360668, 0.842248, -0.365662, 0.163795},
  {0., -0.811503, -0.0736829, 0.553405, -0.172562}},
 {{3., -16.8612, 1.83324, -0.592338, 7.07019},
  {-11.0905, 8.5935, 0.435104, -4.91868, -2.55286},
  {0., -5.31272, 0.289487, -4.70995, -0.0668302},
  {0., 0., 4.40812, -11.0209, -0.358208}, {0., 0., 0., -1.41106, -0.862041}}}
```

- Mostrar que A = p h p^T

```
MatrixForm[A]
```

$$\begin{pmatrix} 3. & 7. & 8 & 9 & 12 \\ 5 & -7 & 4 & -7 & 8 \\ 1 & 1 & -1 & 1 & -1 \\ 4 & 3 & 2 & 1 & 7 \\ 9 & 3 & 2 & 5 & 4 \end{pmatrix}$$

```
MatrixForm[p.h.Transpose[p]]
```

$$\begin{pmatrix} 3. & 7. & 8. & 9. & 12. \\ 5. & -7. & 4. & -7. & 8. \\ 1. & 1. & -1. & 1. & -1. \\ 4. & 3. & 2. & 1. & 7. \\ 9. & 3. & 2. & 5. & 4. \end{pmatrix}$$

- Mostrar que h = p^T A p

```
MatrixForm[h]
```

$$\begin{pmatrix} 3. & -16.8612 & 1.83324 & -0.592338 & 7.07019 \\ -11.0905 & 8.5935 & 0.435104 & -4.91868 & -2.55286 \\ 0. & -5.31272 & 0.289487 & -4.70995 & -0.0668302 \\ 0. & 0. & 4.40812 & -11.0209 & -0.358208 \\ 0. & 0. & 0. & -1.41106 & -0.862041 \end{pmatrix}$$

```
MatrixForm[Transpose[p].A.p]
```

$$\begin{pmatrix} 3. & -16.8612 & 1.83324 & -0.592338 & 7.07019 \\ -11.0905 & 8.5935 & 0.435104 & -4.91868 & -2.55286 \\ -2.22045\times 10^{-16} & -5.31272 & 0.289487 & -4.70995 & -0.0668302 \\ 0. & 1.77636\times 10^{-15} & 4.40812 & -11.0209 & -0.358208 \\ -2.22045\times 10^{-16} & 7.77156\times 10^{-16} & 1.04083\times 10^{-16} & -1.41106 & -0.862041 \end{pmatrix}$$

4. Decomposição QR da matriz A

```
{QT, R} = QRDecomposition[A]
```

```
{{{0.261116, 0.435194, 0.0870388, 0.348155, 0.783349},
  {0.60593, -0.754906, 0.0759203, 0.209139, 0.116029},
  {0.724617, 0.465944, -0.24829, -0.0863952, -0.434411},
  {0.161463, -0.0628997, -0.00415312, -0.906683, 0.384555},
  {0.116303, 0.140223, 0.961767, -0.0742358, -0.190539}},
 {{11.4891, 2.26301, 6.00568, 3.65563, 12.0984},
  {0., 10.5773, 2.40223, 11.6029, 3.08408}, {0., 0., 6.86739, 0.753202, 10.3288},
  {0., 0., 0., 2.9054, -3.37005}, {0., 0., 0., 0., 0.273848}}}
```

```
Q = Transpose[QT]
```

```
{{0.261116, 0.60593, 0.724617, 0.161463, 0.116303},
 {0.435194, -0.754906, 0.465944, -0.0628997, 0.140223},
 {0.0870388, 0.0759203, -0.24829, -0.00415312, 0.961767},
 {0.348155, 0.209139, -0.0863952, -0.906683, -0.0742358},
 {0.783349, 0.116029, -0.434411, 0.384555, -0.190539}}
```

```
MatrixForm[R]
```

$$\begin{pmatrix} 11.4891 & 2.26301 & 6.00568 & 3.65563 & 12.0984 \\ 0. & 10.5773 & 2.40223 & 11.6029 & 3.08408 \\ 0. & 0. & 6.86739 & 0.753202 & 10.3288 \\ 0. & 0. & 0. & 2.9054 & -3.37005 \\ 0. & 0. & 0. & 0. & 0.273848 \end{pmatrix}$$

```
MatrixForm[Q]
```

$$\begin{pmatrix} 0.261116 & 0.60593 & 0.724617 & 0.161463 & 0.116303 \\ 0.435194 & -0.754906 & 0.465944 & -0.0628997 & 0.140223 \\ 0.0870388 & 0.0759203 & -0.24829 & -0.00415312 & 0.961767 \\ 0.348155 & 0.209139 & -0.0863952 & -0.906683 & -0.0742358 \\ 0.783349 & 0.116029 & -0.434411 & 0.384555 & -0.190539 \end{pmatrix}$$

- Mostrar que A = QR

```
MatrixForm[Q.R]
```

$$\begin{pmatrix} 3. & 7. & 8. & 9. & 12. \\ 5. & -7. & 4. & -7. & 8. \\ 1. & 1. & -1. & 1. & -1. \\ 4. & 3. & 2. & 1. & 7. \\ 9. & 3. & 2. & 5. & 4. \end{pmatrix}$$

5. Aplicação do método QR para autovalores e autovetores direto na matriz A

```
Ai = A
```

```
{{3., 7., 8, 9, 12}, {5, -7, 4, -7, 8},
 {1, 1, -1, 1, -1}, {4, 3, 2, 1, 7}, {9, 3, 2, 5, 4}}
```

```
Qa = IdentityMatrix[NN]
```

```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}
```

```
For[i = 1, i < 1000, i++,
 Clear[R, QT, Q];
 {QT, R} = QRDecomposition[Ai];
 Q = Transpose[QT];
 Ai = R.Q;
 Qa = Qa.Q
]
```

- A matriz após 1000 iterações do método QR é Bloco-Triangular superior

```
MatrixForm[Ai]
```

$$\begin{pmatrix} 19.9655 & 3.64311 & 5.98959 & -2.19175 & 6.48571 \\ 0. & -8.41303 & 4.50606 & 9.86926 & 0.68819 \\ 0. & -1.24727 & -8.01444 & -0.433066 & -2.34584 \\ 0. & 0. & 0. & -3.40433 & 1.52222 \\ 0. & 0. & 0. & 0. & -0.133739 \end{pmatrix}$$

- A matriz Qa = Q1 Q2 ... Q1000 é usada para recuperar os autovetores de A

```
MatrixForm[Qa]
```

$$\begin{pmatrix} 0.702104 & -0.33334 & -0.481958 & 0.390638 & 0.105134 \\ 0.200224 & 0.925222 & -0.29223 & 0.0336559 & 0.131694 \\ 0.0349374 & -0.0700178 & 0.225343 & -0.104371 & 0.965507 \\ 0.390111 & -0.0796418 & -0.128473 & -0.904039 & -0.087633 \\ 0.559958 & 0.14698 & 0.784241 & 0.134501 & -0.178101 \end{pmatrix}$$

- Mostrar que a matriz Qa é ortogonal: I = Qa. Qa^T

```
MatrixForm[Qa.Transpose[Qa]]
```

$$\begin{pmatrix} 1. & -2.498\times10^{-15} & 1.3739\times10^{-15} & -2.75301\times10^{-15} & 7.38298\times10^{-15} \\ -2.498\times10^{-15} & 1. & -1.16573\times10^{-15} & -8.93383\times10^{-16} & 2.498\times10^{-16} \\ 1.3739\times10^{-15} & -1.16573\times10^{-15} & 1. & 5.41234\times10^{-16} & -2.7478\times10^{-15} \\ -2.75301\times10^{-15} & -8.93383\times10^{-16} & 5.41234\times10^{-16} & 1. & 7.68482\times10^{-16} \\ 7.38298\times10^{-15} & 2.498\times10^{-16} & -2.7478\times10^{-15} & 7.68482\times10^{-16} & 1. \end{pmatrix}$$

- Mostrar que, de fato, Qa acumula as transformações de similaridade

```
MatrixForm[Transpose[Qa].A.Qa]
```

$$\begin{pmatrix} 19.9655 & 3.64311 & 5.98959 & -2.19175 & 6.48571 \\ -3.08642\times10^{-14} & -8.41303 & 4.50606 & 9.86926 & 0.68819 \\ -3.86358\times10^{-14} & -1.24727 & -8.01444 & -0.433066 & -2.34584 \\ -3.31402\times10^{-14} & -1.39055\times10^{-14} & -1.28786\times10^{-14} & -3.40433 & 1.52222 \\ -8.67882\times10^{-15} & -1.12584\times10^{-15} & -2.08861\times10^{-15} & 1.30104\times10^{-16} & -0.133739 \end{pmatrix}$$

- Achar os blocos da diagonal da matriz Bloco-triangular superior

- Inicializar a lista que armazenará os blocos (no máximo NN blocos de dimensão 1x1)

```
BL = Table[0, {NN}]
{0, 0, 0, 0, 0}
```

- Inicializar a lista com a informação dos tamanhos de blocos que forem sendo identificados

```
dBL = Table[1, {NN}]
{1, 1, 1, 1, 1}
```

- Número de blocos inicialmente

```
nb = 0
0
```

- Percorrer as colunas, identificar se há dente abaixo do elemento da diagonal e copiar os blocos

```
For[j = 1, j < NN, j++,
 If[Abs[Ai[[j + 1, j]]] > (10^(-8)),
  ++nb;
  dBL[[nb]] = 2;
  BL[[nb]] = IdentityMatrix[2];
  BL[[nb]] = {{Ai[[j, j]], Ai[[j, j + 1]]}, {Ai[[j + 1, j]], Ai[[j + 1, j + 1]]}};
  ++j
  ,
  ++nb;
  BL[[nb]] = Ai[[j, j]];
  If[j == (NN - 1), ++nb; BL[[nb]] = Ai[[NN, NN]]]
 ]
]
```

- Número de blocos identificados e lista de blocos

```
Print[nb]
```

4

```
Print[BL]
```

{19.9655, {{-8.41303, 4.50606}, {-1.24727, -8.01444}}, -3.40433, -0.133739, 0}

- Achar os autovalores de cada bloco

```
Lamb = Table[0, {NN}]
```

{0, 0, 0, 0, 0}

```
For[j = 1; k = 0, j ≤ nb, j++,
 If[dBL[[j]] == 1,
  ++k;
  Lamb[[k]] = BL[[j]]
  ,
  ++k;
  Lamb[[k]] = Extract[Eigenvalues[BL[[j]]], 1];
  ++k;
  Lamb[[k]] = Extract[Eigenvalues[BL[[j]]], 2]
 ]
]
```

```
Print[Lamb]
```

{19.9655, -8.21374 + 2.36232 i, -8.21374 - 2.36232 i, -3.40433, -0.133739}

- Achar os autovetores da matriz Bloco-triangular superior

```
Psi = Transpose[Eigenvectors[Ai]]
```

{{1., -0.112295 - 0.106838 i, -0.112295 + 0.106838 i, -0.00711204, -0.183677},
 {0., 0.874295 + 0. i, 0.874295 + 0. i, 0.804436, 0.335835},
 {0., 0.0386684 + 0.458354 i, 0.0386684 - 0.458354 i, -0.267464, -0.308364},
 {0., 0. + 0. i, 0. + 0. i, 0.530373, 0.367468}, {0., 0. + 0. i, 0. + 0. i, 0., 0.78953}}

```
MatrixForm[Psi]
```

$$\begin{pmatrix} 1. & -0.112295-0.106838\,i & -0.112295+0.106838\,i & -0.00711204 & -0.183677 \\ 0. & 0.874295+0.\,i & 0.874295+0.\,i & 0.804436 & 0.335835 \\ 0. & 0.0386684+0.458354\,i & 0.0386684-0.458354\,i & -0.267464 & -0.308364 \\ 0. & 0.+0.\,i & 0.+0.\,i & 0.530373 & 0.367468 \\ 0. & 0.+0.\,i & 0.+0.\,i & 0. & 0.78953 \end{pmatrix}$$

- Recuperar os autovetores da matriz A

```
Phi = Qa.Psi
```

```
{{0.702104 + 0. i, -0.388916 - 0.295919 i,
  -0.388916 + 0.295919 i, 0.0629461 + 0. i, 0.134264 + 0. i},
 {0.200224 + 0. i, 0.775133 - 0.155336 i, 0.775133 + 0.155336 i, 0.838869 + 0. i,
  0.480402 + 0. i}, {0.0349374 + 0. i, -0.0564259 + 0.0995542 i,
  -0.0564259 - 0.0995542 i, -0.1722 + 0. i, 0.624524 + 0. i},
 {0.390111 + 0. i, -0.118406 - 0.100565 i, -0.118406 + 0.100565 i,
  -0.511957 + 0. i, -0.460178 + 0. i}, {0.559958 + 0. i, 0.0959489 + 0.299634 i,
  0.0959489 - 0.299634 i, -0.0241665 + 0. i, -0.386513 + 0. i}}
```

```
MatrixForm[Phi]
```

$$\begin{pmatrix} 0.702104+0.\,i & -0.388916-0.295919\,i & -0.388916+0.295919\,i & 0.0629461+0.\,i \\ 0.200224+0.\,i & 0.775133-0.155336\,i & 0.775133+0.155336\,i & 0.838869+0.\,i \\ 0.0349374+0.\,i & -0.0564259+0.0995542\,i & -0.0564259-0.0995542\,i & -0.1722+0.\,i \\ 0.390111+0.\,i & -0.118406-0.100565\,i & -0.118406+0.100565\,i & -0.511957+0.\,i \\ 0.559958+0.\,i & 0.0959489+0.299634\,i & 0.0959489-0.299634\,i & -0.0241665+0.\,i \end{pmatrix}$$

6. Mostrar que os autovetores encontrados são realmente autovetores

- Criar a matriz diagonal de autovalores

```
LAMB = IdentityMatrix[NN]
```

```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}
```

```
For[i = 1, i ≤ NN, i++, LAMB[[i, i]] = Lamb[[i]]]
```

```
LAMB
```

```
{{19.9655, 0, 0, 0, 0}, {0, -8.21374 + 2.36232 i, 0, 0, 0},
 {0, 0, -8.21374 - 2.36232 i, 0, 0}, {0, 0, 0, -3.40433, 0}, {0, 0, 0, 0, -0.133739}}
```

- Sbendo que [A][Psi] = [Psi][LAMB], então [A][Psi][LAMB]^-1 = [Psi]

```
MatrixForm[Ai.Psi.Inverse[LAMB]]
```

$$\begin{pmatrix} 1.+0.\,i & -0.112295-0.106838\,i & -0.112295+0.106838\,i & -0.00711204+0.\,i & - \\ 0.+0.\,i & 0.874295+5.55112\times10^{-17}\,i & 0.874295-5.55112\times10^{-17}\,i & 0.804436+0.\,i & 0 \\ 0.+0.\,i & 0.0386684+0.458354\,i & 0.0386684-0.458354\,i & -0.267464+0.\,i & - \\ 0.+0.\,i & 0.+0.\,i & 0.+0.\,i & 0.530373+0.\,i & 0 \\ 0.+0.\,i & 0.+0.\,i & 0.+0.\,i & 0.+0.\,i & \end{pmatrix}$$

```
MatrixForm[Psi]
```

$$\begin{pmatrix} 1. & -0.112295-0.106838\,i & -0.112295+0.106838\,i & -0.00711204 & -0.183677 \\ 0. & 0.874295+0.\,i & 0.874295+0.\,i & 0.804436 & 0.335835 \\ 0. & 0.0386684+0.458354\,i & 0.0386684-0.458354\,i & -0.267464 & -0.308364 \\ 0. & 0.+0.\,i & 0.+0.\,i & 0.530373 & 0.367468 \\ 0. & 0.+0.\,i & 0.+0.\,i & 0. & 0.78953 \end{pmatrix}$$

- Sbendo que [A][Phi] = [Phi][LAMB], então [A][Phi][LAMB]^-1 = [Phi]

```
MatrixForm[A.Phi.Inverse[LAMB]]
```

$$\begin{pmatrix} 0.702104+0.\,i & -0.388916-0.295919\,i & -0.388916+0.295919\,i & 0.0629461+0.\,i \\ 0.200224+0.\,i & 0.775133-0.155336\,i & 0.775133+0.155336\,i & 0.838869+0.\,i \\ 0.0349374+0.\,i & -0.0564259+0.0995542\,i & -0.0564259-0.0995542\,i & -0.1722+0.\,i \\ 0.390111+0.\,i & -0.118406-0.100565\,i & -0.118406+0.100565\,i & -0.511957+0.\,i \\ 0.559958+0.\,i & 0.0959489+0.299634\,i & 0.0959489-0.299634\,i & -0.0241665+0.\,i \end{pmatrix}$$

```
MatrixForm[Phi]
```

$$\begin{pmatrix} 0.702104+0.\,i & -0.388916-0.295919\,i & -0.388916+0.295919\,i & 0.0629461+0.\,i \\ 0.200224+0.\,i & 0.775133-0.155336\,i & 0.775133+0.155336\,i & 0.838869+0.\,i \\ 0.0349374+0.\,i & -0.0564259+0.0995542\,i & -0.0564259-0.0995542\,i & -0.1722+0.\,i \\ 0.390111+0.\,i & -0.118406-0.100565\,i & -0.118406+0.100565\,i & -0.511957+0.\,i \\ 0.559958+0.\,i & 0.0959489+0.299634\,i & 0.0959489-0.299634\,i & -0.0241665+0.\,i \end{pmatrix}$$