MATRICES

Introduction

MATRIX

- An ordered set of numbers listed in rectangular form.
- A rectangular array of numbers, symbols or expressions arranged in rows and columns.

Example: 2x3 Matrix (2 rows and 3 columns)

$$\begin{bmatrix} 2 & 9 & -5 \\ 7 & -2 & 8 \end{bmatrix}$$

Element/Entry - an item in a matrix

A_{22} - refers to item @ row 2 and column 2 = -2

MATRIX

- Matrices of same size can be added or subtracted element by element.

- The rule for matrix multiplication is more complicated; two matrices can be multiplied only when the number of columns in the first equals to the number of rows in the second.

APPLICATIONS OF MATRICES

- To represent linear transformation, that is, generalization of linear function f(x) - **major application**

- In Physics, matrices are used to study electrical circuits, optics and quantum mechanics.

- In Computer Graphics, matrices are used to project a 3-dimensional onto a 2-dimensional screen, and to create a realistic screening motion.

- In Calculus, matrices generalize classical analytical motions such as derivatives and exponential to higher dimensions.

MATRIX

- Together with determinants, matrix is a useful tool in Linear Algebra
- **Linear Algebra**
    - The study of linear set of equations and their transformation properties.
    - A way to solve and manipulate systems of linear equations.

LINEAR EQUATIONS

Examples:

$2x + y = 0$

$x + 3y = 8$

The two equations form a system of linear equations. It is called ***LINEAR*** because none of the variables is raised to a power.

KINDS OF MATRICES

1. Square Matrix
2. Diagonal Matrix
3. Zero Matrix
4. Identity Matrix
5. Scalar Matrix
6. Same kind Matrices
7. Transposed Matrix of a Matrix
8. Opposite Matrix of a Matrix
9. Symmetric Matrices
10. Skew-Symmetric Matrices

SQUARE MATRIX

- A matrix with equal number of rows and columns.
- 1x1 matrix is equal to an ordinary number.
- 2x2 matrix

$$\begin{bmatrix} 4 & 2 \\ 3 & 9 \end{bmatrix}$$

- In a square matrix, the elements A_{ii}, with i = 1,2,3, … are called *diagonal elements.*

DIAGONAL MATRIX

- A square matrix wherein only diagonal elements are non-zeroes.

- Example:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

ZERO MATRIX

- A matrix wherein all elements are zeroes.

- Example:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

ROW MATRIX

- A matrix with one row.
- Example:

$$\begin{bmatrix} 2 & 7 & -9 & 10 \end{bmatrix}$$

COLUMN MATRIX

- A matrix with one column.
- Example:

$$\begin{bmatrix} 2 \\ 7 \\ -9 \\ 10 \end{bmatrix}$$

IDENTITY MATRIX

- A diagonal matrix wherein all diagonal elements are 1s.
- Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SCALAR MATRIX

- A diagonal matrix wherein all diagonal elements are with the same scalar value.
- Example:

$$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

- Thus, identity matrix is also a scalar matrix.

SAME KIND MATRICES

- Matrices that are of the same dimension.

- Example1: 2x2

$$\begin{bmatrix} 4 & 2 \\ 3 & 9 \end{bmatrix} \quad \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix}$$

- Example 2: 3x3

$$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & -6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} -1 & 6 & 5 \\ 4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$$

TRANSPOSED MATRIX OF A MATRIX

- The *nxm* matrix B is the transposed matrix of *mxn* matrix A, where $\mathbf{A}_{ij} = \mathbf{B}_{ji}$

- Example:

Matrix A   Matrix B

$$\begin{bmatrix} 1 & 2 \\ -4 & 5 \\ 7 & 8 \end{bmatrix} \qquad \begin{bmatrix} 1 & -4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$$

$A_{21} = -4$ $B_{12} = -4$

$A_{32} = 8$ $B_{23} = 8$

OPPOSITE MATRIX OF A MATRIX

- The *nxm* matrix B is the opposite matrix of *mxn* matrix A, where **A*ij* = -B*ij***

- Example:

Matrix A Matrix B

$$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 & -3 \\ 4 & -5 & -4 \\ -7 & -8 & -9 \end{bmatrix}$$

$A_{21} = -4$ $B_{21} = 4$

$A_{32} = 8$ $B_{32} = -8$

SYMMETRIC MATRIX

- A matrix which is equal to its transpose, $\mathbf{A}_{ij} = \mathbf{B}_{ij}$
- Example:

Matrix A

Matrix B = *Transposed Matrix A*

$$\begin{bmatrix} 7 & 1 & 5 \\ 1 & 3 & 0 \\ 5 & 0 & 7 \end{bmatrix} \qquad \begin{bmatrix} 7 & 1 & 5 \\ 1 & 3 & 0 \\ 5 & 0 & 7 \end{bmatrix} \qquad \mathrm{B} = \mathrm{A}$$

$A_{21} = 1$ $B_{21} = 1$

$A_{32} = 0$ $B_{32} = 0$

SKEW-SYMMETRIC MATRIX

- A matrix which is equal to the opposite of its transpose, **A_{ij} = -B_{ij}**

- Example:

Matrix A   Matrix B = *Transposed Matrix A*

$$\begin{bmatrix} 0 & 1 & -5 \\ -1 & 0 & 0 \\ 5 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 0 \\ -5 & 0 & 0 \end{bmatrix} \quad \mathrm{B} = \text{-}\mathrm{A}$$

$A_{12} = 1$ $B_{12} = -1$

$A_{31} = 5$ $B_{31} = -5$

Exercise1:

Describe the following matrices, give all possible descriptions:

1. $\begin{bmatrix} 7 & 1 & 5 \\ 1 & 3 & 0 \\ 5 & 0 & 7 \end{bmatrix}$ ___A___

2. $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ ___ABE___

CHOICES

A. Square Matrix

B. Diagonal Matrix

C. 0- Matrix

D. Identity Matrix

E. Scalar Matrix

Exercise 2:

Describe the following matrices, give all possible descriptions:

1. $\begin{bmatrix} 7 & 1 & 5 \\ 1 & 3 & 0 \\ 5 & 0 & 7 \end{bmatrix} \begin{bmatrix} 7 & 1 & 5 \\ 1 & 3 & 0 \\ 5 & 0 & 7 \end{bmatrix}$ ___ABD___

2. $\begin{bmatrix} 3 & -8 & 0 \\ 0 & -3 & 0 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} -3 & 8 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & -3 \end{bmatrix}$ ___AC___

CHOICES

A. Same kind Matrices

B. Transposed Matrix of a Matrix

C. Opposite Matrix of a Matrix

D. Symmetric Matrices

E. Skew-Symmetric Matrices