1 Foreword

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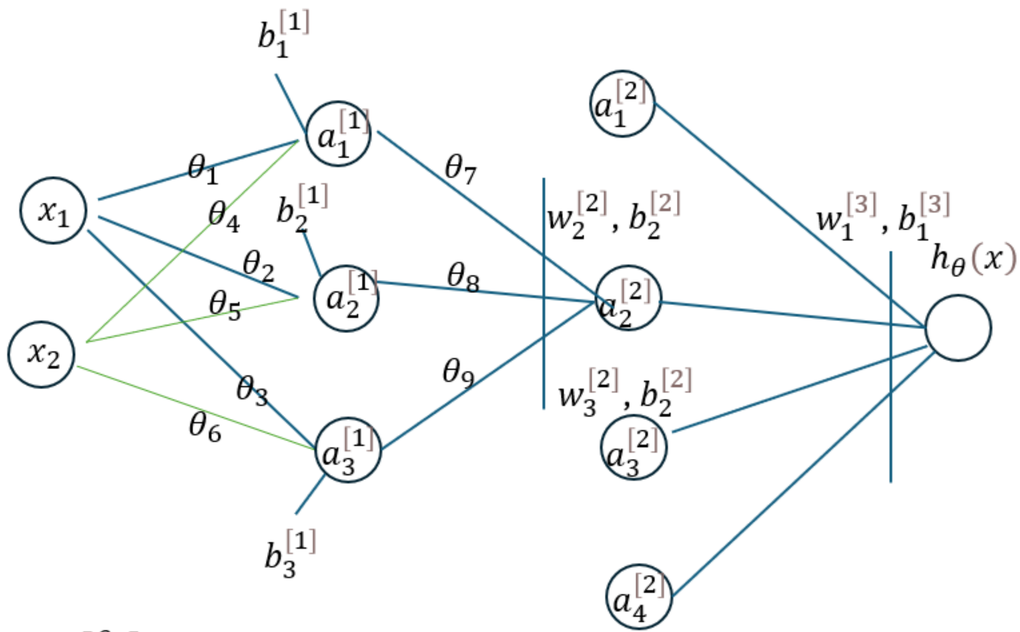
Disclaimer : The notes are written only for my understanding and memorization purpose after I have self-studied those online lecture notes.

2 Main Idea

The old trick : (i) Forward feed (ii) Compute Loss (NN output vs training data) (iii) Backpropagation with Gradient Descent to tune parameters
2. Use activation functions to avoid the model to be a pure linear model, which is useless (just ax+b)
3. Examples of activation functions : Signmoid , (Leaky) ReLU, tanh etc.

3 ANN Structure



$$w_2^{[1]} = \begin{bmatrix} \theta_2 \\ \theta_5 \end{bmatrix} \quad w_3^{[1]} = \begin{bmatrix} \theta_3 \\ \theta_6 \end{bmatrix} \quad w_2^{[2]} = \begin{bmatrix} \theta_7 \\ \theta_8 \\ \theta_9 \end{bmatrix}$$

w's and b's are parameters : Totally ther are Hidden Layer 1 (layer 1) (2x3 + 3) + Hidden Layer 1 (layer 2) (3x4 + 4) + Output Layer (layer 3)(4x1 +1) number of parameters

4 Forward Feed (see above NN diagram)

$$a_1 = \text{ReLU}(\theta_1 x_1 + \theta_4 x_2 + b_1^{[1]})$$
$$a_2 = \text{ReLU}(\theta_2 x_1 + \theta_5 x_2 + b_2^{[1]})$$

Let i be i^{th} layer and j be j^{th} neuron (count vertifically) this layer of NN
Rewriting the notation : $w_j^{[i]}$ as **VECTOR** of input θ's for layer i and j^{th} neuron.
With the above, Layer 1 of NN can be expressed as:

For all $j \in [1, \ldots, m]$: (m=3, count vertically, is the number of number of neuron layer 1)

$$z_j = \mathbf{w}_j^{[1]\top}\mathbf{x} + b_j^{[1]} \quad \text{where} \quad \mathbf{w}_j^{[1]} \in R^d,\ b_j^{[1]} \in R$$

(d = 2 , count vertically, is the number of number of previous layer , layer 0)

$$a_j = \mathrm{ReLU}(z_j),$$

$$\mathbf{a} = [a_1, \ldots, a_m]^\top \in R^m$$

For Layer 3 :

$$\bar{h}_\theta(\mathbf{x}) = \mathbf{w}^{[3]\top}\mathbf{a} + b^{[3]} \quad \text{where} \quad \mathbf{w}^{[3]} \in R^n,\ b^{[3]} \in R$$

(n = 4, count vertically, is the number of number of neuron layer, layer 2)

5 Vectorization of Forward Feeding

Just by stacking the parameters . for layer 1 :

$$W^{[1]} = \begin{bmatrix} -w_1^{[1]T}- \\ -w_2^{[1]T}- \\ \vdots \\ -w_m^{[1]T}- \end{bmatrix} \in R^{m \times d}$$

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} \in \mathbb{R}^{m \times 1} = \begin{bmatrix} -w_1^{[1]T}- \\ -w_2^{[1]T}- \\ \vdots \\ -w_m^{[1]T}- \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ \vdots \\ b_m^{[1]} \end{bmatrix}$$

Hence we get:

$$\begin{aligned} z^{[1]} &= W^{[1]}x + b^{[1]} \\ a^{[1]} &= ReLU(z^{[1]}) \\ z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\ a^{[2]} &= ReLU(z^{[2]}) \\ h &= W^{[3]}a^{[2]} + b^{[3]} \end{aligned}$$

In General :

$$\begin{aligned} a^{[1]} &= \mathrm{ReLU}(W^{[1]}x + b^{[1]}) \\ a^{[2]} &= \mathrm{ReLU}(W^{[2]}a^{[1]} + b^{[2]}) \\ &\vdots \\ a^{[r-1]} &= \mathrm{ReLU}(W^{[r-1]}a^{[r-2]} + b^{[r-1]}) \\ h(x) &= W^{[r]}a^{[r-1]} + b^{[r]} \end{aligned}$$

Also, for loss function J

$$J = (1/2)(h - o)^2$$

Some common loss function for J : Common : (1) Mean Sequared Error (2) Cross Entropy Loss

6 Vector/Matrix Calculus: Some important equations

1. Gradient, where f is $R^n toR$:

$$\nabla f(x_1, x_2, \ldots, x_n) = \begin{bmatrix} \dfrac{\partial f}{\partial x_1}(x_1, x_2, \ldots, x_n) \\ \dfrac{\partial f}{\partial x_2}(x_1, x_2, \ldots, x_n) \\ \vdots \\ \dfrac{\partial f}{\partial x_n}(x_1, x_2, \ldots, x_n) \end{bmatrix}$$

Example : $f(x_1, x_2) = 3x_1^2 x_2$, then $\nabla f(x_1, x_2) = [6x_1x_2, 3x_1]^T$

2. Jacobian of f, Note that there are m DIFFERENT functions

$$\begin{aligned} \mathbf{f}(\mathbf{x}) &= \mathbf{f}(x_1, x_2, \ldots, x_n) \\ &= (f_1(x_1, x_2, \ldots, x_n), , \ldots, f_m(x_1, x_2, \ldots, x_n)) \\ &= (f_1(\mathbf{x}), , \ldots, f_m(\mathbf{x})) \end{aligned}$$

$$\mathbb{J} = \begin{bmatrix} \dfrac{\partial \mathbf{f}(\mathbf{x})}{\partial x_1} & \cdots & \dfrac{\partial \mathbf{f}(\mathbf{x})}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla^T f_1(\mathbf{x}) \\ \vdots \\ \nabla^T f_m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1(x_1, x_2 ...., x_n)}{\partial x_1} & \cdots & \dfrac{\partial f_1(x_1, x_2 ...., x_n)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m((x_1, x_2 ...., x_n)}{\partial x_1} & \cdots & \dfrac{\partial f_m((x_1, x_2 ...., x_n)}{\partial x_n} \end{bmatrix}$$

3. (Law of total derivatives) for $f(g_1, g_2 ...., g_k)$, if **EACH** of $g_1, g_2 ...., g_k$ actually depends on $x_1, x_2 .... x_n$, then

$$\frac{\partial f_m((g_1, g_2 ...., g_n)}{\partial x_1} = \frac{\partial f}{\partial g_1}\frac{\partial g_1}{\partial x_1} + \frac{\partial f}{\partial g_2}\frac{\partial g_2}{\partial x_1} + ... + \frac{\partial f}{\partial u_n}\frac{\partial g_n}{\partial x_1}$$

$$\frac{\partial f_m((g_1, g_2 ...., g_n)}{\partial x_2} = \frac{\partial f}{\partial g_1}\frac{\partial g_1}{\partial x_2} + \frac{\partial f}{\partial g_2}\frac{\partial g_2}{\partial x_2} + ... + \frac{\partial f}{\partial u_n}\frac{\partial g_n}{\partial x_2}$$

4. By 3 , Jacobian again, but this time :

$$y_1 = f_1(g_1(x_1, ..., x_n), g_2(x_1, ..., x_n), ..., g_k(x_1, ..., x_n))$$
$$y_2 = f_1(g_1(x_1, ..., x_n), g_2(x_1, ..., x_n), ..., g_k(x_1, ..., x_n))$$

....

$$y_m = f_m(g_1(x_1, ..., x_n), g_2(x_1, ..., x_n), ..., g_k(x_1, ..., x_n))$$

$$\mathbf{y} = (y_1, y_2..., y_m), \mathbf{x} = (x_1, ..., x_n)$$

$$J = \frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \nabla^T f_1(\mathbf{x}) \\ \nabla^T f_2(\mathbf{x}) \\ \vdots \\ \nabla^T f_m(\mathbf{x}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^k \frac{\partial f_1}{\partial g_i} \frac{\partial g_i}{\partial x_1} & \sum_{i=1}^k \frac{\partial f_1}{\partial g_i} \frac{\partial g_i}{\partial x_2} & \cdots & \sum_{i=1}^k \frac{\partial f_1}{\partial g_i} \frac{\partial g_i}{\partial x_2} \\ \sum_{i=1}^k \frac{\partial f_2}{\partial g_i} \frac{\partial g_i}{\partial x_1} & \sum_{i=1}^k \frac{\partial f_2}{\partial g_i} \frac{\partial g_i}{\partial x_2} & \cdots & \sum_{i=1}^k \frac{\partial f_2}{\partial g_i} \frac{\partial g_i}{\partial x_2} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{i=1}^k \frac{\partial f_m}{\partial g_i} \frac{\partial g_i}{\partial x_1} & \sum_{i=1}^k \frac{\partial f_m}{\partial g_i} \frac{\partial g_i}{\partial x_2} & \cdots & \sum_{i=1}^k \frac{\partial f_m}{\partial g_i} \frac{\partial g_i}{\partial x_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial f_1}{\partial g_1} & \frac{\partial f_1}{\partial g_2} & \cdots & \frac{\partial f_1}{\partial g_k} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_m}{\partial g_1} & \frac{\partial f_m}{\partial g_2} & \cdots & \frac{\partial f_m}{\partial g_k} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial g_k}{\partial x_1} & \frac{\partial g_k}{\partial x_2} & \cdots & \frac{\partial g_k}{\partial x_n} \end{bmatrix}$$

5. Jacobian of elementwise operation fuunction are diagonal. Therefore $\frac{\partial a^{[i]}}{\partial z^{[i]}}$ is a diagonal matrix

7 Neural Network breakdown

$$z_1^{[1]} = \begin{bmatrix} w_{1,1}^{[1]}, w_{1,2}^{[1]} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b_1^{[1]}$$

$$z_2^{[1]} = \begin{bmatrix} w_{2,1}^{[1]}, w_{2,2}^{[1]} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b_2^{[1]}$$

$$z_3^{[1]} = \begin{bmatrix} w_{3,1}^{[1]}, w_{3,2}^{[1]} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b_3^{[1]}$$

$$a_1^{[1]} = ReLU(z_1^{[1]}), a_2^{[1]} = ReLU(z_2^{[1]}), a_3^{[1]} = ReLU(z_3^{[1]})$$

$$a^{[1]} = ReLU\left[\begin{bmatrix} w^{[1]}_{1,1}, w^{[1]}_{1,2} \\ w^{[1]}_{2,1}, w^{[1]}_{2,2} \\ w^{[1]}_{3,1}, w^{[1]}_{3,2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b^{[1]}_1 \\ b^{[1]}_2 \\ b^{[1]}_3 \end{bmatrix}\right]$$

$$z^{[2]}_1 = \begin{bmatrix} w^{[2]}_{1,1}, w^{[2]}_{1,2}, w^{[2]}_{1,3} \end{bmatrix}\begin{bmatrix} a^{[1]}_1 \\ a^{[1]}_2 \\ a^{[1]}_3 \end{bmatrix} + b^{[2]}_1$$

$$z^{[2]}_2 = \begin{bmatrix} w^{[2]}_{2,1}, w^{[2]}_{2,2}, w^{[2]}_{2,3} \end{bmatrix}\begin{bmatrix} a^{[1]}_1 \\ a^{[1]}_2 \\ a^{[1]}_3 \end{bmatrix} + b^{[2]}_2$$

$$z^{[2]}_3 = \begin{bmatrix} w^{[2]}_{3,1}, w^{[2]}_{3,2}, w^{[2]}_{3,3} \end{bmatrix}\begin{bmatrix} a^{[1]}_1 \\ a^{[1]}_2 \\ a^{[1]}_3 \end{bmatrix} + b^{[2]}_3$$

$$z^{[2]}_4 = \begin{bmatrix} w^{[2]}_{4,1}, w^{[2]}_{4,2}, w^{[2]}_{4,3} \end{bmatrix}\begin{bmatrix} a^{[1]}_1 \\ a^{[1]}_2 \\ a^{[1]}_3 \end{bmatrix} + b^{[2]}_4$$

$$a^{[2]}_1 = ReLU(z^{[2]}_1), a^{[2]}_2 = ReLU(z^{[2]}_2), a^{[2]}_3 = ReLU(z^{[2]}_3), a^{[2]}_4 = ReLU(z^{[2]}_4)$$

$$a^{[2]} = ReLU\left[\begin{bmatrix} w^{[2]}_{1,1}, w^{[2]}_{1,2}, w^{[2]}_{1,3} \\ w^{[2]}_{2,1}, w^{[2]}_{2,2}, w^{[2]}_{2,3} \\ w^{[2]}_{3,1}, w^{[2]}_{3,2}, w^{[2]}_{3,3} \\ w^{[2]}_{4,1}, w^{[2]}_{4,2}, w^{[2]}_{4,3} \end{bmatrix}\begin{bmatrix} a^{[1]}_1 \\ a^{[1]}_2 \\ a^{[1]}_3 \end{bmatrix} + \begin{bmatrix} b^{[1]}_1 \\ b^{[1]}_2 \\ b^{[1]}_3 \\ b^{[1]}_4 \end{bmatrix}\right]$$

$$z^{[3]}_1 = \begin{bmatrix} w^{[2]}_{4,1}, w^{[2]}_{4,2}, w^{[2]}_{4,3}, w^{[2]}_{4,4} \end{bmatrix}\begin{bmatrix} a^{[1]}_1 \\ a^{[1]}_2 \\ a^{[1]}_3 \\ a^{[1]}_4 \end{bmatrix} + b^{[3]}_1$$

$$o = a^{[3]}_1 = ReLU(z^{[3]}_1)$$

$$a^{[3]} = ReLU\left[\begin{bmatrix} w^{[3]}_{1,1}, w^{[3]}_{1,2}, w^{[3]}_{1,3}, w^{[3]}_{1,3} \end{bmatrix}\begin{bmatrix} a^{[2]}_1 \\ a^{[2]}_2 \\ a^{[2]}_3 \\ a^{[2]}_4 \end{bmatrix} + \begin{bmatrix} b^{[1]}_1 \\ b^{[1]}_2 \\ b^{[1]}_3 \\ b^{[1]}_4 \end{bmatrix}\right]$$

8 Vectorized Backpropagation. (Refer to Section 7)

Our objective is to make use of matrix calculus, gradient descent to tune W and b to minimize J by SGD or mini-batch GD

Algo 1 SGD : 1: Hyperparameter: learning rate α, number of total iteration n_{iter}.
2: Initialize θ randomly.
3: for i = 1 to niter do
4: Sample j uniformly from 1,...,n, and update θ by

$$\theta := \theta - \alpha \nabla_\theta J^{(j)}(\theta)$$

By Chain Rule of matrix calculus

$$\frac{\partial L}{\partial W_{ij}^{[2]}} = (\frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}})(\frac{\partial z^{[3]}}{\partial a^{[2]}})(\frac{\partial a^{[2]}}{\partial z^{[2]}})(\frac{\partial z^{[2]}}{\partial W_{ij}^{[2]}})$$

$$= [(a^{[3]} - y)W[3] \bigodot g'(z^{[2]})]_i a_j^{[1]} e_i$$

$$= [(a^{[3]} - y)W[3] \bigodot g'(z^{[2]})]_i a_j^{[1]T}$$

stacking up again :

$$\frac{\partial L}{\partial W^{[2]}} = [(a^{[3]} - y)W[3] \bigodot g(z^{[2]})]a^{[1]T}$$

But How ? Let's do it one-by-one:

(i) Assume using sigmoid, by differentiation rule :

$$\frac{\partial L}{\partial z^{[3]}} = \frac{\partial[-ylog(o) - (1-y)(log(1-o))]}{\partial z^{[3]}} = a^{[3]} - y,$$

dimension : ϵR^{1x1}

(ii) for $\frac{\partial z^{[3]}}{\partial a^{[2]}}$

$$\frac{\partial z^{[3]}}{\partial a^{[2]}} = \frac{\partial(w_{1,1}^{[3]} a_1^{[2]} + w_{1,2}^{[3]} a_2^{[2]} + w_{1,3}^{[3]} a_3^{[2]} + w_{1,4}^{[3]} a_4^{[2]})}{\partial(a_1^{[2]}, a_2^{[2]}, a_3^{[2]}, a_4^{[2]})}$$

$$= (w_{1,1}^{[3]}, w_{1,2}^{[3]}, w_{1,3}^{[3]}, w_{1,4}^{[3]}) = W^{[3]}$$

dimension : ϵR^{1x4}

(ii) for $\frac{\partial a^{[2]}}{\partial z^{[2]}}$

$$\frac{\partial a^{[2]}}{\partial z^{[2]}} = \frac{\partial(a_1^{[2]}, a_2^{[2]}, a_3^{[2]}, a_4^{[2]})}{\partial(z_1^{[2]}, z_2^{[2]}, z_3^{[2]}, z_4^{[2]})}$$

$$= \begin{bmatrix} \frac{\partial g(z_1^{[2]})}{\partial z_1^{[2]}} & \frac{\partial g(z_1^{[2]})}{\partial z_2^{[2]}} & \frac{\partial g(z_1^{[2]})}{\partial z_3^{[2]}} & \frac{\partial g(z_1^{[2]})}{\partial z_4^{[2]}} \\ \frac{\partial g(z_2^{[2]})}{\partial z_1^{[2]}} & \frac{\partial g(z_2^{[2]})}{\partial z_2^{[2]}} & \frac{\partial g(z_2^{[2]})}{\partial z_3^{[2]}} & \frac{\partial g(z_2^{[2]})}{\partial z_4^{[2]}} \\ \frac{\partial g(z_3^{[2]})}{\partial z_1^{[2]}} & \frac{\partial g(z_3^{[2]})}{\partial z_2^{[2]}} & \frac{\partial g(z_3^{[2]})}{\partial z_3^{[2]}} & \frac{\partial g(z_3^{[2]})}{\partial z_4^{[2]}} \\ \frac{\partial g(z_4^{[2]})}{\partial z_1^{[2]}} & \frac{\partial g(z_4^{[2]})}{\partial z_2^{[2]}} & \frac{\partial g(z_4^{[2]})}{\partial z_3^{[2]}} & \frac{\partial g(z_4^{[2]})}{\partial z_4^{[2]}} \end{bmatrix}$$

$$= \begin{bmatrix} g'(z_1^{[2]}) & 0 & 0 & 0 \\ 0 & g'(z_2^{[2]}) & 0 & 0 \\ 0 & 0 & g'(z_3^{[2]}) & 0 \\ 0 & 0 & 0 & g'(z_4^{[2]}) \end{bmatrix} = diag(g'(z^{[2]}))$$

dimension : ϵR^{4x4}

(iii) for $\frac{\partial z^{[2]}}{\partial W_{i,j}^{[2]}}$

$$\frac{\partial z^{[2]}}{\partial W_{i,j}^{[2]}} = \frac{\partial \begin{bmatrix} w_{1,1}^{[2]}a_1^{[1]} + w_{1,2}^{[2]}a_2^{[1]} + w_{1,3}^{[2]}a_3^{[1]} + w_{1,4}^{[2]}a_4^{[1]} + b_1^{[1]} \\ w_{2,1}^{[2]}a_1^{[1]} + w_{2,2}^{[2]}a_2^{[1]} + w_{2,3}^{[2]}a_3^{[1]} + w_{2,4}^{[2]}a_4^{[1]} + b_2^{[1]} \\ w_{3,1}^{[2]}a_1^{[1]} + w_{3,2}^{[2]}a_2^{[1]} + w_{3,3}^{[2]}a_3^{[1]} + w_{3,4}^{[2]}a_4^{[1]} + b_3^{[1]} \\ w_{4,1}^{[2]}a_1^{[1]} + w_{4,2}^{[2]}a_2^{[1]} + w_{4,3}^{[2]}a_3^{[1]} + w_{4,4}^{[2]}a_4^{[1]} + b_4^{[1]} \end{bmatrix}}{\partial W_{i,j}^{[2]}}$$

$$= a_j^{[i]} e_i$$

for example , if i = 2 , j = 3 : then it is $a_3(0,1,0,0)^T = (0, a_3, 0, 0)^T$
for example , if i = 4 , j = 4 : then it is $a_4(0,0,0,1)^T = (0, 0, 0, a_4)^T$
dimension : ϵR^{4x1}

Recall :

$$\frac{\partial L}{\partial W_{ij}^{[2]}} = (\frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}})(\frac{\partial z^{[3]}}{\partial a^{[2]}})(\frac{\partial a^{[2]}}{\partial z^{[2]}})(\frac{\partial z^{[2]}}{\partial W_{ij}^{[2]}})$$

$$= (a^{[3]} - y)(w_{1,1}^{[3]}, w_{1,2}^{[3]}, w_{1,3}^{[3]}, w_{1,4}^{[3]}) \begin{bmatrix} g'(z_1^{[2]}) & 0 & 0 & 0 \\ 0 & g'(z_2^{[2]}) & 0 & 0 \\ 0 & 0 & g'(z_3^{[2]}) & 0 \\ 0 & 0 & 0 & g'(z_4^{[2]}) \end{bmatrix} a_j^{[1]} e_i$$

$$= (a^{[3]} - y)(w^{[3]}_{1,1}g'(z^{[2]}_1), w^{[3]}_{1,2}g'(z^{[2]}_2), w^{[3]}_{1,3}g'(z^{[2]}_3), w^{[3]}_{1,4}g'(z^{[2]}_4))a^{[1]}_j e_i$$

Therefore :

$$\frac{\partial L}{\partial W^{[2]}_{ij}} = (a^{[3]} - y)(w^{[3]}_{1,1}g'(z^{[2]}_1), w^{[3]}_{1,2}g'(z^{[2]}_2), w^{[3]}_{1,3}g'(z^{[2]}_3), w^{[3]}_{1,4}g'(z^{[2]}_4))e_i a^{[1]}_j$$

Stacking it up vertically :

$$\frac{\partial L}{\partial W^{[2]}} = (a^{[3]} - y) \begin{bmatrix} (w^{[3]}_{1,1}g'(z^{[2]}_1)a^{[1]}_1 & w^{[3]}_{1,1}g'(z^{[2]}_2)a^{[1]}_2 & w^{[3]}_{1,1}g'(z^{[2]}_3)a^{[1]}_3 & w^{[3]}_{1,1}g'(z^{[2]}_4)a^{[1]}_4 \\ (w^{[3]}_{1,2}g'(z^{[2]}_1)a^{[1]}_1 & w^{[3]}_{1,2}g'(z^{[2]}_2)a^{[1]}_2 & w^{[3]}_{1,2}g'(z^{[2]}_3)a^{[1]}_3 & w^{[3]}_{1,2}g'(z^{[2]}_4)a^{[1]}_4 \\ (w^{[3]}_{1,3}g'(z^{[2]}_1)a^{[1]}_1 & w^{[3]}_{1,3}g'(z^{[2]}_2)a^{[1]}_2 & w^{[3]}_{1,3}g'(z^{[2]}_3)a^{[1]}_3 & w^{[3]}_{1,3}g'(z^{[2]}_4)a^{[1]}_4 \\ (w^{[3]}_{1,4}g'(z^{[2]}_1)a^{[1]}_1 & w^{[3]}_{1,4}g'(z^{[2]}_2)a^{[1]}_2 & w^{[3]}_{1,4}g'(z^{[2]}_3)a^{[1]}_3 & w^{[3]}_{1,4}g'(z^{[2]}_4)a^{[1]}_4 \end{bmatrix}$$

$$= (a^{[3]} - y) \begin{bmatrix} (w^{[3]}_{1,1} \\ (w^{[3]}_{1,2} \\ (w^{[3]}_{1,3} \\ (w^{[3]}_{1,4} \end{bmatrix} \bigodot \begin{bmatrix} g'(z^{[2]}_1) \\ g'(z^{[2]}_2) \\ g'(z^{[2]}_3) \\ g'(z^{[2]}_4) \end{bmatrix} \begin{bmatrix} a^{[1]}_1 & a^{[1]}_2 & a^{[1]}_3 & a^{[1]}_4 \end{bmatrix}$$

Therefore

$$\frac{\partial L}{\partial W^{[2]}} = [(a^{[3]} - y)W[3] \bigodot g(z^{[2]})]a^{[1]T}$$

We see here the step of GD is based on next layer's value, therefore we have update back propagated to previous layer.

9 Testing

$$\sum_{i=1}^{k} \frac{\partial f_i}{\partial x_i} dx_i$$