1 Foreword

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Disclaimer : This notes is written only for my own memorization purpose after I have studied online lecture notes and blogs.

2 K-Means Clustering

Simple Description : Identify clusters by finding the centroid of data points

Algorithm :

1. Initialize μ_1,μ_2.... μ_k randomly (k is hyper-parameter)

2. Repeated until converge :

(i) $c^{(i)} := \arg\min_j ||x^{(i)} - \mu_j||^2, j \in [1 : k]$, (i.e. $c^{(i)}$ denote which μ the $x^{(i)}$ is linked to. Link each data point to nearest μ_j. If $x^{(i)}$ is nearest to μ_s, then $c^{(i)} = j$. Thus, k partitions are created.)

(ii) $\mu_j := \frac{\sum_{i=1}^m 1\{c^{(i)}=j\}x^{(i)}}{\sum_{i=1}^m 1\{c^{(i)}=j\}}$, (i.e. For each data point in each partition from (i) , find the new centroid and assign to μ_k

Proof of convergence of the algorithm : consider

$J(c, \mu) = \sum_{i=1}^m ||x^{(i)} - \mu_{c^{(i)}}||^2$
Observation : J must be monotonically decreasing. It is because for step (i) It is adjusting $c^(i)$ to reduce J, for step (ii) we are adjusting μ_j to reduce J

3 Linear Regression(MSE approach)

4 Linear Regression(MLE approach)

5 Logistic Regression

6 Logistic Regression(MLE approach)

7 Softmax Regression(Multi-Class Logistic)

8 Loss function in Classification(Binary) Problem - General treatment

General Hypothesis : $h_\theta(x) = x^T\theta$

Adjustment for binary classification :
$\mathrm{sign}(h_\theta(x)) = \mathrm{sign}(\theta^T x) = \mathrm{sign}(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t = 0 \\ -1 & \text{if } t < 0 \end{cases}$

Measure of confidence : $h_\theta(x) = x^T\theta$ gives larger value, more confident

Margin ($yx^T\theta$) : (i) if $h_\theta(x)$ classify correctly, margin is positive, otherwise negative.
(ii) Therefore our objective is to maximize the margin (we want both correct classification and be confident)

Consider the following loss function :

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \phi\left(y^{(i)}\theta^T x^{(i)}\right)$$

We want penalize wrong classfication and encourage correct one , we design ϕ as $\phi(z) \to 0$ as $z \to \infty$, while $\phi(z) \to \infty$ as $z \to -\infty$ where $z = yx^T\theta$, and examples are :

logistic loss : $\phi_{\text{logistic}}(z) = \log(1 + e^{-z})$,used in logistic regression

hinge loss : $\phi_{\text{hinge}}(z) = [1 - z]_+ = \max 1 - z, 0$, used in SVM

Exponential loss $\phi_{\exp}(z) = e^{-z}$, used in boosting