WENSLO (Weights by ENvelope and SLOpe) ALWAS (Aczel-Alsina Weighted Assessment) Method

Index

Summary

- **WENSLO**: objective method to calculate weights of criteria based on the *envelope/slope* ratio, using linear normalization, accumulation of values (concept inspired by Grey theory), calculation of the envelope (accumulated Euclidean distance) and slope. Final weight: additive envelope/slope ratio normalization.
- **WENSLO**: Normalizes → accumulates → calculates slope/envelope → obtains QJ → normalizes on w_j.
- **ALWAS**: The aggregation/ranking method based on nonlinear norms Aczel–Alsina — defines two strategies (weighted average and weighted geometric) and integrates both through a function with stabilization parameters (φ, θ) and parameter ξ of the Aczel–Alsina family. The output is a score of 0–1 per alternative, used to sort.
- **ALWAS**: standardizes → calculates $R^{(1)}$ and $R^{(2)}$ via Aczel–Alsina functions → integrates with **Initial matrix (ℜ):** Build the matrix "m x n" where "m" are the alternatives and "n" are the criteria. ζ_{ij} values represent the performance of alternative "i" in criterion "j"(19) → obtains scores and ranking; parameters ξ, φ, θ control behavior/sensitivity.

WENSLO

1. **Initial matrix (ℜ):** Build the matrix "m x n" where "m" are the alternatives and "n" are the criteria. ζ_{ij} values represent the performance of alternative "i" in criterion "j".

$$\Re(A,C) = [\zeta_{ij}]_{m\times n} = \begin{bmatrix} A/C & C_1 & C_2 & \dots & C_j \\ \text{target} & \text{maxmin} & \text{maxmin} & \dots & \text{maxmin} \\ A_1 & \zeta_{11} & \zeta_{12} & \dots & \zeta_{1j} \\ A_2 & \zeta_{21} & \zeta_{22} & \dots & \zeta_{2j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_i & \zeta_{i1} & \zeta_{i2} & \dots & \zeta_{mn} \end{bmatrix}$$

2. **Linear normalization**: Transform the initial matrix into a normalized matrix "Z" using linear normalization by sum (Eq. 2). The result is "z_{ij}" values between 0 and 1, making the criteria dimensionless and comparable.

$$z_{ij} = \frac{\zeta_{ij}}{\sum_{i=1}^{m} \zeta_{ij}}$$

Produces matrix $Z=[z_{ij}]$ with $0< z_{ij} <1$

3. **Class Range** Δz_j (Sturges rule): Use Sturges' Rule (Eq. 5) to calculate the amplitude of the interval for each criterion "j". This step is fundamental to "graduate" the axis of alternatives, treating the sequence of normalized values of a criterion as a time series where the "distance" between two consecutive alternatives is fixed and equal to Δz_j.

$$\Delta z_j = \frac{\max_i z_{ij} - \min_i z_{ij}}{1 + 3.322 \log(m)}$$

— used to graduate the axis and treat alternatives as "points in time".

4. **Slope (Criterion Inclination) tan φj**: Calculate the tangent of the angle of inclination of the hypotenuse of the right triangle formed by the artificial accumulation of the data (Eq. 7). This slope represents the "ideal" or average growth rate of the criterion.

$$\tan\varphi_j = \frac{\sum_{i=1}^{m} z_{ij}}{(m-1)\cdot \Delta z_j} \quad \forall j \in [1,2,\dots,n]\,.$$

5. **Criterion envelope E_j**: Calculate the total length of the poly line (zig-zag) that connects the points of the ordered normalized values of each criterion (Eq. 8). This envelope captures the volatility and actual behavior of the criterion through alternatives.

$$E_j = \sum_{i=1}^{m-1} \sqrt{(z_{i+1,j} - z_{i,j})^2 + \Delta z_j^2}$$

$$\forall j \in [1,2,\dots,n]$$

6. **Envelope/slope ratio** (q_j): Calculate the ratio of the envelope (real volatility) to the slope (average trend) for each criterion (Eq. 9). A criterion

with a high envelope (high volatility) and low slope (low upward trend) will have a high ratio, indicating greater importance in the decision process.

$$q_j = \frac{E_j}{\tan \varphi_j}$$

$$\forall j \in [1, 2, \ldots, n]$$

7. **Calculation of the Final Weights of the Criteria (w_j):** Normalize the ratios q_j to obtain the final weights (Eq. 10).

$$w_j = \frac{q_j}{\sum_{j=1}^{n} q_j}$$

$$\forall j \in [1, 2, \ldots, n]$$

Interpretations: *High envelope* + *low slope* → *heavier weight*.

Important notes (WENSLO)

- **Process Validation (Accumulation):** The method is based on Grey Systems Theory. Validation is done by comparing the actual accumulation of the normalized data with an artificial linear accumulation (the hypotenuse). The high correlation (r ≈ 1) and the low mean square error (MSE ≈ 0) between the two sequences (Table XI) confirm that the hypotenuse is a valid representation for calculating the slope (tan φj).
- The accumulation of z_{ij} (cumulative *sequence*) is used to reduce volatility and facilitate slope calculation — see eq. (11) In the article and concept of artificial accumulation (hypotenuse) vs. real.
- Validation of the artificial approximation was performed with **MSE** and **correlation coefficient**, e.g., for criterion C_{12} MSE=0.0013 and r=0.9981; mean r≈0.9747, mean MSE≈0.0126 — indicates good adherence and validity of the procedure.

ALWAS

1. **Home matrix** ζ_{ij}: Use the original decision matrix $\mathfrak{R} = [\zeta_{ij}]$ (the same as in step 1 of WENSLO).
2. **Standardization**: transforms ζ_{ij} in standardized values using the article rule (eq. (16)), with different treatment for criteria such as benefit (B) and cost (C) and preserving the proportion of the original values (Eq. 16). This step is different from WENSLO's linear normalization.

$$\mathfrak{R}^s = [\hat{\zeta}_{ij}]_{m \times n}$$

$$\hat{\zeta}_{ij} = \begin{cases} \hat{\zeta}_{ij} = \frac{\zeta_{ij}}{\zeta'_j}, \text{ if } j \in \mathrm{B} \\ \hat{\zeta}_{ij} = -\frac{\zeta_{ij}}{\zeta'_j} + \max\limits_{1 \le i \le m} \left(\frac{\zeta_{ij}}{\zeta'_j}\right) + \min\limits_{1 \le i \le m} \left(\frac{\zeta_{ij}}{\zeta'_j}\right), \text{ if } j \in \mathrm{C} \end{cases}$$

3. **Define Weighted Aczel–Alsina Strategies**: Apply two nonlinear aggregation operators based on the triangular Aczel-Alsina norms, which introduce a flexibility parameterξ (ξ ≥ 1) to simulate different attitudes of the decision-maker (e.g., risk aversion).
   - **Aczel-Alsina Weighted Average Strategy: $Ri^{(1)}$ξ (Aczel–Alsina weighted averaging)** — eq. (17).
   - **Aczel-Alsina Weighted Geometric Strategy $R^{(2)}$ξ (geometric)** — eq. (18).

where $f(\hat{\zeta}_{ij}) = \hat{\zeta}_{ij} / \sum_i \hat{\zeta}_{ij}$ and the weights w_j are those obtained by WENSLO.

$$\mathbb{R}_i^{(1)\xi} = \sum_{j=1}^{n} \hat{\zeta}_{ij} \left(1 - e^{-\left(\sum\limits_{j=1}^{n} w_j \left(-\ln\left(1 - f(\hat{\zeta}_{ij})\right)\right)^{\xi} \right)^{1/\xi}} \right)$$

$$\mathbb{R}_i^{(2)\xi} = \sum_{j=1}^{n} \hat{\zeta}_{ij} \cdot e^{-\left(\sum\limits_{j=1}^{n} w_j \left(-\ln\left(f(\hat{\zeta}_{ij})\right)\right)^{\xi} \right)^{1/\xi}}$$

4. **Integration of strategies (𝕊i)**: Combine the two previous strategies $R_{\xi}^{(1)}$ e $R_{\xi}^{(2)}$ in a single integrated evaluation measure using a nonlinear function that incorporates two stabilization parameters, φ (which balances the importance between the two strategies) and θ (Eq. 19). The final ranking is obtained by ordering the alternatives in descending order by the values of 𝕊i.

$$\Im_i = \frac{\mathbb{R}_i^{(1)\xi} + \mathbb{R}_i^{(2)\xi}}{1 + \left\{ \phi \left(\frac{1 - f\left(\mathbb{R}_i^{(1)\xi}\right)}{f\left(\mathbb{R}_i^{(1)\xi}\right)} \right)^{\theta} + (1 - \phi) \left(\frac{1 - f\left(\mathbb{R}_i^{(2)\xi}\right)}{f\left(\mathbb{R}_i^{(2)\xi}\right)} \right)^{\theta} \right\}^{1/\theta}} \tag{19}$$

where $\theta > 0$, $\phi \geq 0$, $f(\mathbb{R}_i^{(1)\xi}) = \mathbb{R}_i^{(1)\xi} / \mathbb{R}_i^{(1)\xi} + \mathbb{R}_i^{(2)\xi}$, and $f(\mathbb{R}_i^{(2)\xi}) = \mathbb{R}_i^{(2)\xi} / \mathbb{R}_i^{(1)\xi} + \mathbb{R}_i^{(2)\xi}$.

Sensitivity Analysis - Parameters and sensitivity (ALWAS)

- **ξ (xi)** controls "nonlinearity". The authors recommend low values (e.g., ξ = 1) for good separation; In the experimentation of the article, large variations (ξ ≥ 36) leveled scores and reversed orders (e.g., change between A1 and A6 to high ξ). It is recommended to explore $1 \le \xi \le 5$ for useful breakdown.

- **φ e θ**: φ regulates the relative weight between the two strategies; φ=0.5 and θ=1 were used as the initial solution. Changes in φ (below ~0.46) caused small trades between some positions (e.g. A4 vs A5). θ affects stability, but not major rearrangements in the study.

Validation and comparison (in the article)

- WENSLO: Comparison with other methods of objective weighting (**Entropy, CRITIC, SIDev**). WENSLO showed a high Spearman correlation (>96%) with all of them, confirming the credibility of the weights.
- ALWAS: was compared with **MABAC, TOPSIS, WASPAS, TODIM** — high degree of correlation; small differences appeared in TOPSIS for intermediate positions.

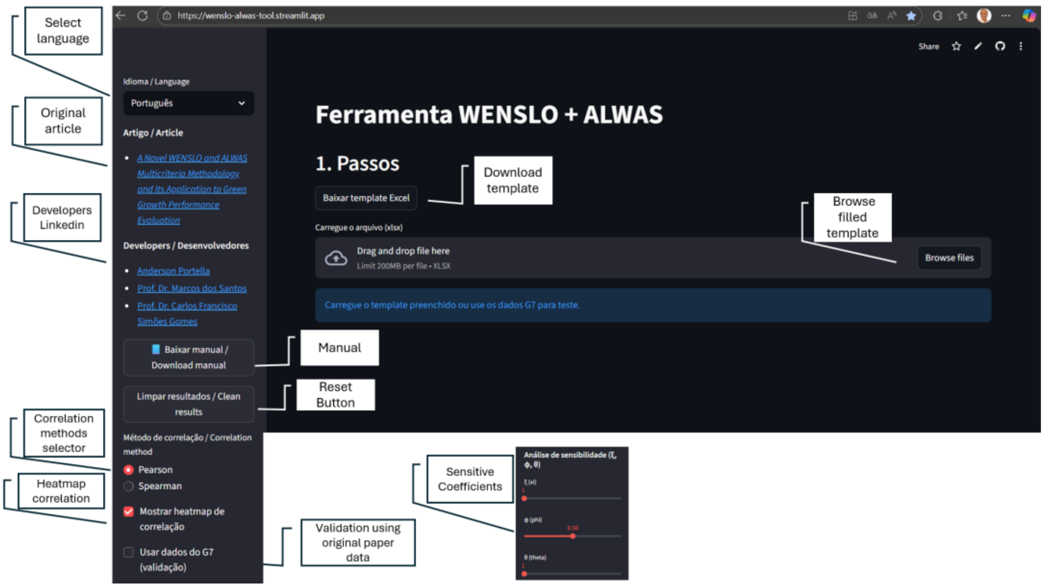
Limitations pointed out in the article

- The method **does not incorporate uncertainty** (noise/intervals/missing values) authors suggest extension with uncertainty theory (fuzzy, intervals, etc.).

Computational Tool

1 – Main Screen

Web site: [WENSLO + ALWAS Tool · Streamlit](https://wenslo-alwas-tool.streamlit.app) <https:// wenslo-alwas-tool.streamlit.app>



2 – Uploaded Data

Calcullating button

Dados carregados (confira se estão corretos)

	C11	C12	C13	C14	C15	C21	C22	C23	C31	C32	C33	C41	C42	C43	C44	C45	C46	C51	C52	C53
A1	523.19	5727.67	7.57	17.26	67.9	77.64	5.78	10.51	16.78	84	85.7	12.27	0.44	3.84	11.26	1.17	8.8	144.43	66.12	1.5
A2	258.23	12337.01	3.23	11.81	23.81	52.32	43.65	0.74	17.4	79	81	12.54	0.54	1.84	12.14	2.38	45.57	126.03	61.59	1.85
A3	585.26	14372.84	3.33	16.38	43.56	46.33	45.54	1.24	28.3	97	97.13	13.33	0.57	3.22	19.91	1.71	45.35	124.68	64.36	1.6
A4	280.37	15433.84	2.32	19.42	41.51	38.89	53.44	1.01	57.65	96	94	9.63	0.41	2.94	19.53	3.09	5.87	101.03	63.71	1.31
A5	1024.07	12787.94	3.18	6.77	19.04	67.15	24.72	1.48	24.06	81	79.7	9.87	0.43	2.57	13.73	1.24	8.99	111.43	59.15	1.37
A6	306.32	17447.05	2.33	13.91	43.13	63.11	29.7	1.42	8.9	98	96	11.33	0.49	1.78	12.1	2.07	41.46	134.46	63.67	1.75
A7	4285.89	9461.82	6.17	8.5	19.74	73.04	20.37	2.37	38.86	98	75.4	8.84	0.38	0.34	10.91	2.8	19.11	147.51	65	1.78

Calcular

3 – Normalized Matrix

Matriz de Decisão Normalizada

	Criterion	Alternative	z_ij	class_index_int	class_index	class_lower	class_upper	pos_in_class
0	C11	A2	0.0356	0	0.0000	0.0356	0.1812	0.0000
1	C11	A4	0.0386	0	0.0209	0.0356	0.1812	0.0209
2	C11	A6	0.0422	0	0.0455	0.0356	0.1812	0.0455
3	C11	A1	0.0720	0	0.2505	0.0356	0.1812	0.2505
4	C11	A3	0.0806	0	0.3091	0.0356	0.1812	0.3091
5	C11	A5	0.1410	0	0.7240	0.0356	0.1812	0.7240
6	C11	A7	0.5901	3	3.8074	0.4725	0.6181	0.8074
7	C12	A1	0.0654	0	0.0000	0.0654	0.1006	0.0000
8	C12	A7	0.1081	1	1.2132	0.1006	0.1357	0.2132
9	C12	A2	0.1409	2	2.1473	0.1357	0.1709	0.1473

4 – Summary of graduation by criteria

Resumo da graduação por critério

Criterion	Nº de classes	z (mín)	z (máx)
C32	3	0.1248	0.1548
C33	3	0.1238	0.1595
C41	3	0.1136	0.1713
C42	4	0.1166	0.1748
C43	4	0.0206	0.2323
C44	3	0.1096	0.1999
C45	4	0.0809	0.2137
C46	3	0.0335	0.2602
C51	4	0.1136	0.1658
C52	4	0.1333	0.1491

5 – Weights (WENSLO)

Pesos (WENSLO)

	Weight	q	E	tan_phi	Rank
C11	0.3012	1.2016	1.3751	1.1444	1
C12	0.0196	0.0781	0.3704	4.7416	11
C13	0.0392	0.1565	0.5322	3.4001	8
C14	0.0248	0.0988	0.4661	4.7179	10
C15	0.0468	0.1868	0.6275	3.3597	6
C21	0.0093	0.0373	0.2555	6.8530	12
C22	0.0483	0.1926	0.5724	2.9718	5
C23	0.2541	1.0140	1.2362	1.2191	2
C31	0.0784	0.3128	0.7816	2.4986	4
C32	0.0013	0.0052	0.1096	21.1412	19

6 – Normalized Matrix Z

Matriz normalizada Z / Normalized matrix Z

	C11	C12	C13	C14	C15	C21	C22	C23	C31	C32
A1	0.0720	0.0654	0.2691	0.1835	0.2625	0.1855	0.0259	0.5599	0.0874	0.1327
A2	0.0356	0.1409	0.1148	0.1256	0.0920	0.1250	0.1956	0.0394	0.0906	0.1248
A3	0.0806	0.1641	0.1184	0.1742	0.1684	0.1107	0.2040	0.0661	0.1474	0.1532
A4	0.0386	0.1762	0.0825	0.2065	0.1605	0.0929	0.2394	0.0538	0.3003	0.1517
A5	0.1410	0.1460	0.1130	0.0720	0.0736	0.1605	0.1108	0.0788	0.1253	0.1280
A6	0.0422	0.1992	0.0828	0.1479	0.1667	0.1508	0.1331	0.0757	0.0464	0.1548
A7	0.5901	0.1081	0.2193	0.0904	0.0763	0.1745	0.0913	0.1263	0.2024	0.1548

	C33	C41	C42	C43	C44	C45	C46	C51	C52	C53
A1	0.1407	0.1577	0.1350	0.2323	0.1131	0.0809	0.0502	0.1624	0.1491	0.1344
A2	0.1330	0.1612	0.1656	0.1113	0.1219	0.1646	0.2602	0.1417	0.1388	0.1658
A3	0.1595	0.1713	0.1748	0.1948	0.1999	0.1183	0.2589	0.1402	0.1451	0.1434
A4	0.1544	0.1238	0.1258	0.1779	0.1961	0.2137	0.0335	0.1136	0.1436	0.1174
A5	0.1309	0.1268	0.1319	0.1555	0.1379	0.0858	0.0513	0.1253	0.1333	0.1228
A6	0.1577	0.1456	0.1503	0.1077	0.1215	0.1432	0.2367	0.1512	0.1435	0.1568
A7	0.1238	0.1136	0.1166	0.0206	0.1096	0.1936	0.1091	0.1658	0.1465	0.1595

7 – Δz (Sturges)

Δz (Sturges) per criterion

	Delta_z
C11	0.1456
C12	0.0352
C13	0.0490
C14	0.0353
C15	0.0496
C21	0.0243
C22	0.0561
C23	0.1367
C31	0.0667
C32	0.0079

8 – Envelope E and tan_phi

Envelope E and tan_phi

	E	tan_phi
C11	1.3751	1.1444
C12	0.3704	4.7416
C13	0.5322	3.4001
C14	0.4661	4.7179
C15	0.6275	3.3597
C21	0.2555	6.8530
C22	0.5724	2.9718
C23	1.2362	1.2191
C31	0.7816	2.4986
C32	0.1096	21.1412

9 – ALWAS Matrix

ALWAS: standardized home matrix ($\hat{\zeta}$)

	C11	C12	C13	C14	C15	C21	C22	C23	C31	C32
A1	0.9382	0.3283	1.0000	0.8888	1.0000	1.0000	0.1082	1.0000	0.8633	0.8571
A2	1.0000	0.7071	0.4267	0.6081	0.3507	0.6739	0.8168	0.0704	0.8526	0.8061
A3	0.9237	0.8238	0.4399	0.8435	0.6415	0.5967	0.8522	0.1180	0.6635	0.9898
A4	0.9948	0.8846	0.3065	1.0000	0.6113	0.5009	1.0000	0.0961	0.1544	0.9796
A5	0.8213	0.7330	0.4201	0.3486	0.2804	0.8649	0.4626	0.1408	0.7370	0.8265
A6	0.9888	1.0000	0.3078	0.7163	0.6352	0.8129	0.5558	0.1351	1.0000	1.0000
A7	0.0603	0.5423	0.8151	0.4377	0.2907	0.9408	0.3812	0.2255	0.4803	1.0000

	C33	C41	C42	C43	C44	C45	C46	C51	C52	C53
A1	0.8823	0.9205	0.7719	1.0000	0.5655	0.3786	0.1931	0.9791	1.0000	0.8108
A2	0.8339	0.9407	0.9474	0.4792	0.6097	0.7702	1.0000	0.8544	0.9315	1.0000
A3	1.0000	1.0000	1.0000	0.8385	1.0000	0.5534	0.9952	0.8452	0.9734	0.8649
A4	0.9678	0.7224	0.7193	0.7656	0.9809	1.0000	0.1288	0.6849	0.9636	0.7081
A5	0.8205	0.7404	0.7544	0.6693	0.6896	0.4013	0.1973	0.7554	0.8946	0.7405
A6	0.9884	0.8500	0.8596	0.4635	0.6077	0.6699	0.9098	0.9115	0.9629	0.9459
A7	0.7763	0.6632	0.6667	0.0885	0.5480	0.9061	0.4194	1.0000	0.9831	0.9622

10 – Ranking (ALWAS)

Ranking (ALWAS)

	R1	R2	S	Rank
A1	0.8261	0.7188	0.7687	1
A6	0.6617	0.5083	0.5750	2
A3	0.6528	0.4977	0.5648	3
A2	0.6430	0.4286	0.5143	4
A4	0.5709	0.3660	0.4461	5
A5	0.5031	0.3954	0.4428	6
A7	0.2908	0.1985	0.2359	7

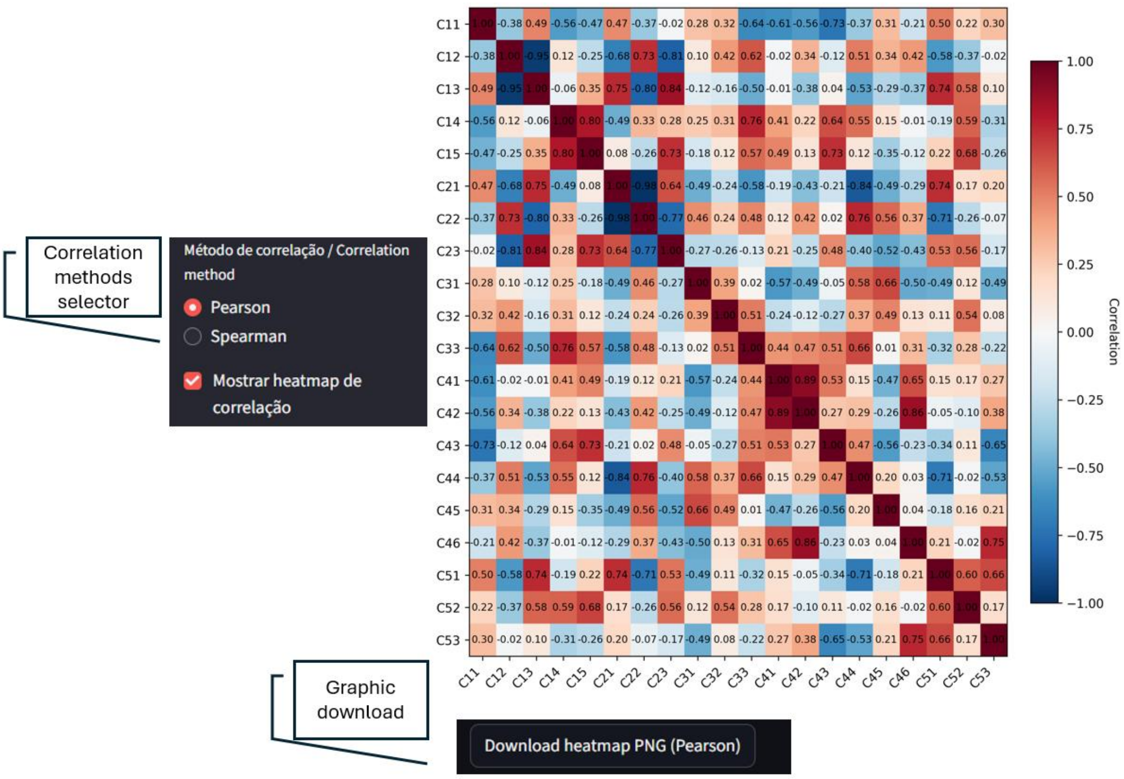
11 – Accumulation validation

Validação da acumulação (MSE / Correlação)

	MSE	Correlation
C14	0.0182	0.9930
C15	0.0272	0.9954
C21	0.0090	0.9950
C22	0.0081	0.9886
C23	0.0969	0.9870
C31	0.0046	0.9862
C32	0.0052	0.9996
C33	0.0076	0.9996
C41	0.0134	0.9981
C42	0.0105	0.9982

Baixar todos os resultados (Excel)

12 – **Heatmap — Correlation between criteria (Pearson)**



13 – Real vs Artificial Accumulation per criterion