Haselgrove, J., 1957, "Oblique ray paths in the ionosphere." Proceed of the Physical Society B 70.7: 653.

$$\frac{dx}{dt} = \frac{1}{\mu^2}\left(\mu\cos\chi - \frac{\partial\mu}{\partial\chi}\sin\chi\right)$$

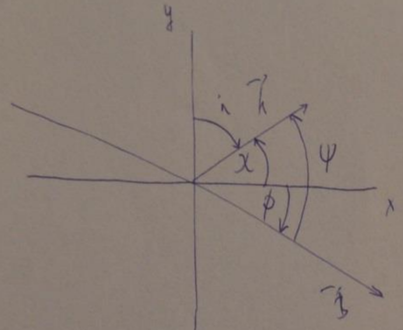
$$\frac{dy}{dt} = \frac{1}{\mu^2}\left(\mu\sin\chi - \frac{\partial\mu}{\partial\chi}\cos\chi\right)$$

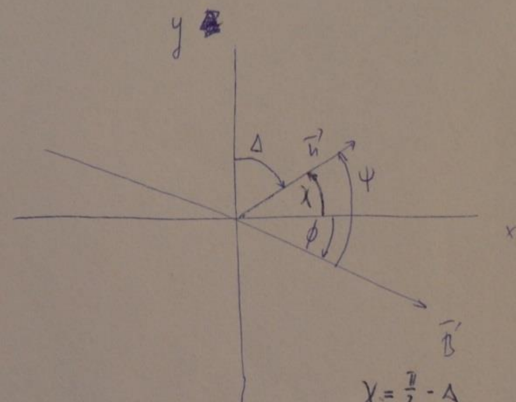
$$\frac{d\chi}{dt} = \frac{1}{\mu^2}\left(\frac{\partial\mu}{\partial y}\cos\chi - \frac{\partial\mu}{\partial x}\sin\chi\right)$$

$$\frac{dx}{dt} = \frac{1}{n^2}\left(n\sin\Delta + \frac{\partial n}{\partial\psi}\cos\Delta\right)$$

$$\frac{dy}{dt} = \frac{1}{n^2}\left(n\cos\Delta - \frac{\partial n}{\partial\psi}\sin\Delta\right)$$

$$\frac{d\Delta}{dt} = \frac{1}{n^2}\left(\frac{\partial n}{\partial y}\sin\Delta - \frac{\partial n}{\partial x}\cos\Delta\right)$$




$$\psi = -\phi + \chi$$
$$\chi = \psi + \phi$$

$$\chi = \frac{\pi}{2} - \Delta$$
$$\psi = \chi + (-\phi) = \frac{\pi}{2} - \Delta - \phi$$

---

$$\chi = 90^\circ - \Delta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

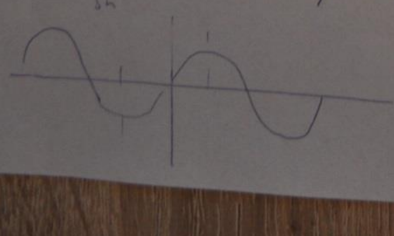
$$\cos\chi = \cos(90^\circ - \Delta) = \cos 90^\circ\cos(-\Delta) - \sin 90^\circ\sin(-\Delta) = -\sin(-\Delta) = \sin(\Delta)$$

$$\sin\chi = \sin(90^\circ - \Delta) = \sin 90^\circ\cos(-\Delta) + \cancel{\cos(90^\circ)\sin(-\Delta)} = \cos(-\Delta) = \cos(\Delta)$$

$$\frac{dr}{dt} = \frac{1}{\mu^2}\left(\mu\cos\chi + \frac{\partial\mu}{\partial\chi}\sin\chi\right)$$

$$\frac{d\theta}{dt} = \frac{1}{r\mu^2}\left(\mu\sin\chi - \frac{\partial\mu}{\partial\chi}\cos\chi\right)$$

$$\frac{d\chi}{dt} = \frac{1}{r\mu^2}\left(\frac{\partial\mu}{\partial\theta}\cos\chi - \left[r\frac{\partial\mu}{\partial r} + \mu\right]\sin\chi\right)$$




$$\Delta = 10^\circ$$
$$\chi = 80^\circ$$

$$\frac{dx}{dt} = \frac{1}{\mu^2}\left(\mu\cos\chi - \frac{\partial\mu}{\partial\chi}\sin\chi\right)$$

$$\frac{dy}{dt} = \frac{1}{\mu^2}\left(\mu\sin\chi - \frac{\partial\mu}{\partial\chi}\cos\chi\right)$$

$$\frac{d\chi}{dt} = \frac{1}{\mu^2}\left(\frac{\partial\mu}{\partial y}\cos\chi - \frac{\partial\mu}{\partial x}\sin\chi\right)$$

$$\frac{dP'}{dt} = \frac{1}{\mu}\mu_G \quad \Big| \quad \frac{dP}{dt} = \frac{\mu_g}{\mu} = 1 + \frac{\omega}{\mu}\frac{\partial\mu}{\partial\omega}$$

$$\mu_G = \mu - 2X\frac{\partial\mu}{\partial X} - Y\frac{\partial\mu}{\partial Y} \qquad X = Ne^2/\varepsilon_0 \ldots \qquad Y = f_H/f \,; \ldots$$

$$\chi = \phi + \psi$$

$\chi$ angle between horizontal & wave n…

$\phi$ dip angle

$\psi$ angle between wave normal & mag…

$X = X_m\left(1 - \left(1 - h/h_t\right)^2\right)$ • electron density … height $h$ given b…

$h_t =$ … h

---

- magnetic field & frequency defined by $X_m$, $Y$, & $\phi$

  $X_m = f_c^2/f$ ; $f_c$ is critical frequency

figure 1

- initial conditions $X_m$, $Y$, $\phi$, $i$ at $t = 0\,s$

  $i$ is incident angle

  ! $\frac{\partial\mu}{\partial X}, \frac{\partial\mu}{\partial\chi}, \frac{\partial\mu}{\partial\ldots}$ numerically !

$X_m = 2,$ ~~…~~

mag. field strength $Y = \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4$

$\phi = 0, 22.5°, 45°, 60°, 90°$

$i = 60°$

$X =$

$Y =$

$P' =$

$X = (\frac{\ldots}{2}$

---

- good accuracy check $\mu\cos\chi$ should remain constant according to Snell

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$$\frac{\partial\mu}{\partial\chi} = \frac{\partial\mu}{\partial(\phi+\psi)} = \frac{\partial\mu}{\partial\psi} \quad \phi = \text{const.} \;?\; \text{check}$$

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$$\cancel{\frac{dx}{dt} = \frac{1}{\mu^2}\left(\mu\cos\chi - \frac{\partial\mu}{\partial\psi}\sin\chi\right)}$$

$$\cancel{\frac{dy}{dt} = \frac{1}{\mu^2}\left(\mu\sin\chi - \frac{\partial\mu}{\partial\psi}\cos\chi\right)}$$

$$\cancel{\frac{d\chi}{dt} = \frac{1}{\mu^2}\left(\frac{\partial\mu}{\partial y}\cos\chi - \frac{\partial\mu}{\partial x}\sin\chi\right)}$$

$$\cancel{X_p^2 = 1 - \frac{X}{1 - \frac{Y_T^2}{2(1-X)}}}$$

Davies.

$$n_p^2 = 1 - \frac{X}{1 - \frac{Y_T^2}{2(1-X)} \pm \left[\frac{Y_T^4}{4(1-X)^2} + Y_L^2\right]^{1/2}}$$

$$X = \frac{\omega_p^2}{\omega^2} \qquad Y_L = -\frac{\omega_g}{\omega}\cos\psi \qquad Y_T = -\frac{\omega_g}{\omega}\sin\psi$$

$$\omega = 2\pi f \qquad \omega_p^2 = \frac{N_e q^2}{m_e\varepsilon_0} \qquad \omega_g = \frac{Bq}{m_e}$$


$$X = X_m\left(1 - \left(1 - h/h_t\right)\right) \qquad Y_L = -Y\cos\psi \qquad Y_T = -Y\sin\ldots$$

$$\psi = \chi - \phi$$