Efficient Computation of L_1 Low-Rank Matrix Approximations in the Presence of Missing Data

Anders Eriksson and Anton van den Hengel
School of Computer Science
University of Adelaide, Australia
{anders.eriksson, anton.vandenhengel}@adelaide.edu.au

Errata & Clarifications

Below are some corrections to our paper. Not all is exactly errata, some are suggestions to clarify the text.

Section 2.1

- The line following equation (4) should read:
'where $G(U) = W(I_n \otimes U)$.'

- The line following equation (6) should read:
'and $F(V) = W(V^T \otimes I_m)$.'

- Equation (7) should read

$$\min_U ||Wy - W\text{vec}(UV^*(U))||_2^2 = ||Wy - \phi(U)||_2^2. \quad (7)$$

There are also some additional places in the text that should read $W\text{vec}(UV^*(U))$ instead of $WUV^*(U)$, or similar, equations (32), (40) and just before equation (37) to name some.

Section 4

The part of this section between and including equations (37) to (40) is better formulated as

$$\frac{\partial G}{\partial U} = (I_{nr} \otimes W)(I_n \otimes T_{r,n} \otimes I_m)(\text{vec}(I_n) \otimes I_{mr}) \quad (37)$$

$$\frac{\partial B}{\partial G} = \frac{\partial (AQ)}{\partial G} = \left(Q^T \otimes I_{2mn}\right)\frac{\partial A}{\partial G} = $$
$$= \left(Q^T \otimes I_{2mn}\right)\left[\begin{array}{cc} \frac{\partial}{\partial G}\left(\begin{smallmatrix} -G & G \\ G & -G\end{smallmatrix}\right) & 0 \end{array}\right] =$$
$$= \left(Q^T \otimes I_{2mn}\right)\left[\begin{array}{cc} \frac{\partial\left(\left[\begin{smallmatrix} -1 & 1 \\ 1 & -1\end{smallmatrix}\right] \otimes G\right)}{\partial G} & 0 \end{array}\right] \quad (38)$$

$$\frac{\partial z^*}{\partial B} = Q\frac{\partial z_B^*}{\partial B} = -Q\left((z_B^*)^T \otimes B^{-1}\right) \quad (39)$$

with

$$\frac{\partial\left(\left[\begin{smallmatrix} -1 & 1 \\ -1 & 1\end{smallmatrix}\right] \otimes G\right)}{\partial G} = (I_2 \otimes T_{nr,2} \otimes I_{nm})(\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1\end{bmatrix} \otimes I_{nm,nr}).$$

Here $T_{m,n}$ denotes the $mn \times mn$ matrix for which $T_{m,n}\text{vec}(C) = \text{vec}(C^T)$ for any $C \in \mathbb{R}^{m\times n}$, and $Q \in \mathbb{R}^{(2nr+3nm)\times mn}$ is obtained by removing the columns corresponding to the non-basic variables of z^* from the identity matrix $I_{(2nr+3nm)}$. With $z = \left[\begin{smallmatrix} v^+ & v^- & t & s\end{smallmatrix}\right]^T$.

Combining the above expressions gives us

$$\frac{\partial z^*}{\partial U} = \begin{bmatrix} \frac{\partial v^{*+}}{\partial U} \\ \frac{\partial v^{*-}}{\partial U} \\ \frac{\partial t^*}{\partial U} \\ \frac{\partial s^*}{\partial U}\end{bmatrix} = \frac{\partial z^*}{\partial B}\frac{\partial B}{\partial G}\frac{\partial G}{\partial U}$$

Since $v^* = v^{*+} - v^{*-}$ we obtain the partial derivatives of v^* as

$$\frac{\partial v^*}{\partial U} = \frac{\partial v^{*+}}{\partial U} - \frac{\partial v^{*-}}{\partial U},$$

and the Jacobian of $\phi_1(U)$ is given by

$$J(U) = F(V) + G(U)\frac{\partial v^*}{\partial U}. \quad (40)$$