Meta Analysis

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1 Frameworks

- Fixed effect: The studies are assumed to investigate the same population, use the same variable and outcome definitions, etc.
- Random effects: The studies are assumed to be heterogeneous

2 Models

We are considering the case where there are k independent studies done, resulting in k 2 x 2 tables as shown below. The parameter of interest in these cases is often the odds ratio, defined as $OR = \frac{a/b}{c/d} = \frac{ad}{bc}$. We are comparing different methods of pooling the odds ratios.

a	b
c	d

Table 1: 2 x 2 table

Following are several common models for pooling the odds ratios.

2.1 Inverse Variance

$$\hat{OR}_{pooled} = \frac{\sum_{i=1}^{k} \mathrm{Var}(\frac{a_i d_i}{b_i c_i})^{-1} \frac{a_i d_i}{b_i c_i}}{\sum_{i=1}^{k} \mathrm{Var}(\frac{a_i d_i}{b_i c_i})^{-1}} \tag{1}$$

- Assumes fixed effect framework.

2.2 DerSimonian-Laird

$$\hat{OR}_{pooled} = \tag{2}$$

- Assumes random effects framework.
- Adaption of inverse variance for random effects framework.

- Most common for medical and psychological research
- Prone to producing false positives, especially when the number of studies is small

2.3 Mantel-Haenszel

$$\hat{OR}_{pooled} = \frac{\sum_{i=1}^{k} \frac{a_i d_i}{n_i}}{\sum_{i=1}^{k} \frac{b_i c_i}{n_i}}, \text{where } n_i = a_i + b_i + c_i + d_i \tag{3}$$

- Assumes fixed effect framework
- Better at binary outcomes than inverse variance.

2.4 Peto

$$\hat{OR}_{pooled} = \exp(\frac{\sum_{i=1}^{k}(O_i - E_i)}{\sum_{i=1}^{k} V_i}) \tag{4}$$

O, E, and V are defined such that $O = a$, $E = \frac{(a+b)(a+c)}{n}$, and $V = \frac{(a+b)(c+d)(a+c)(b+d)}{n^2(n-1)}$ (where $n = a + b + c + d$).

- Assumes fixed effect framework
- Only used for odds ratios
- Works best when odds ratios are close to one
- Performs well for rare events
- Can cause bias when samples are very unbalanced
- Corrections for zero cell counts are not necessary