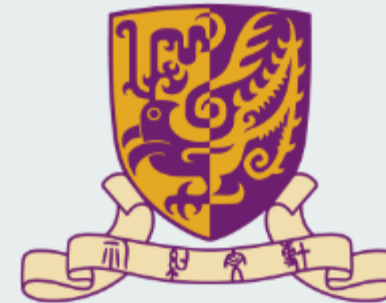
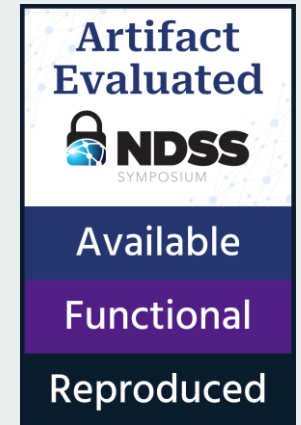




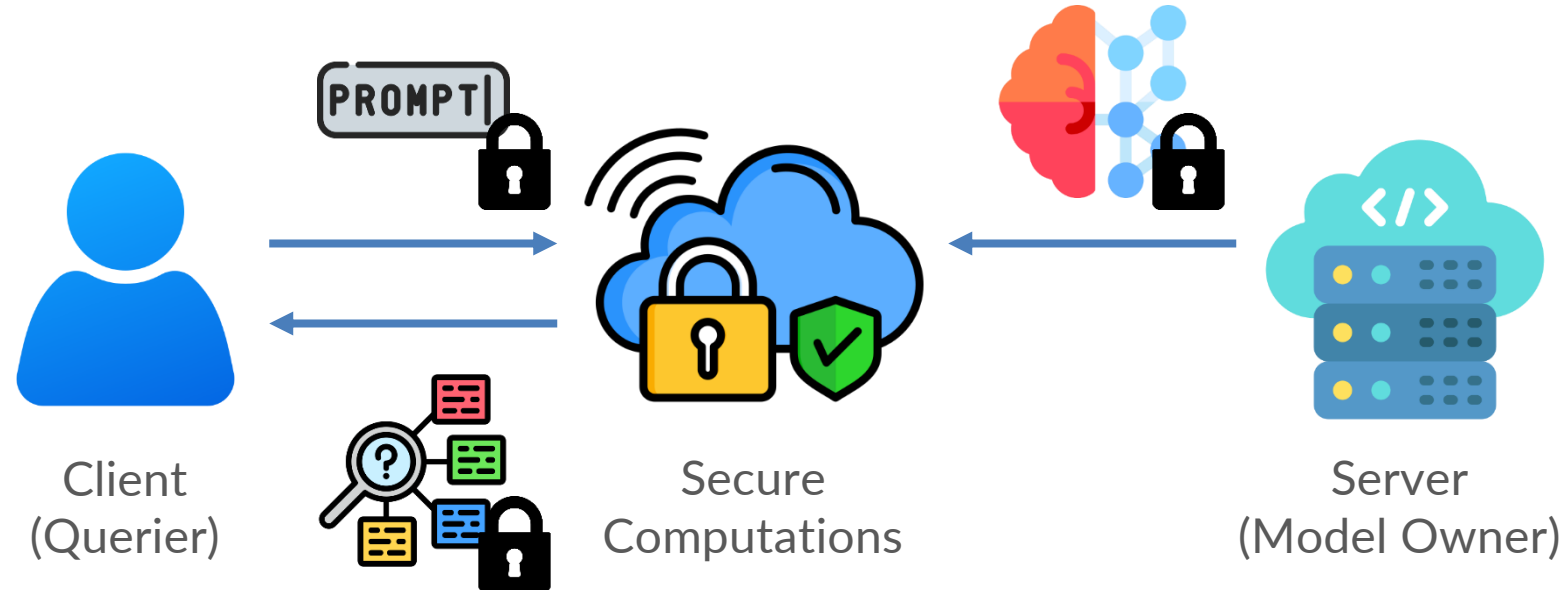
SHAFT: Secure, Handy, Accurate, and Fast Transformer Inference

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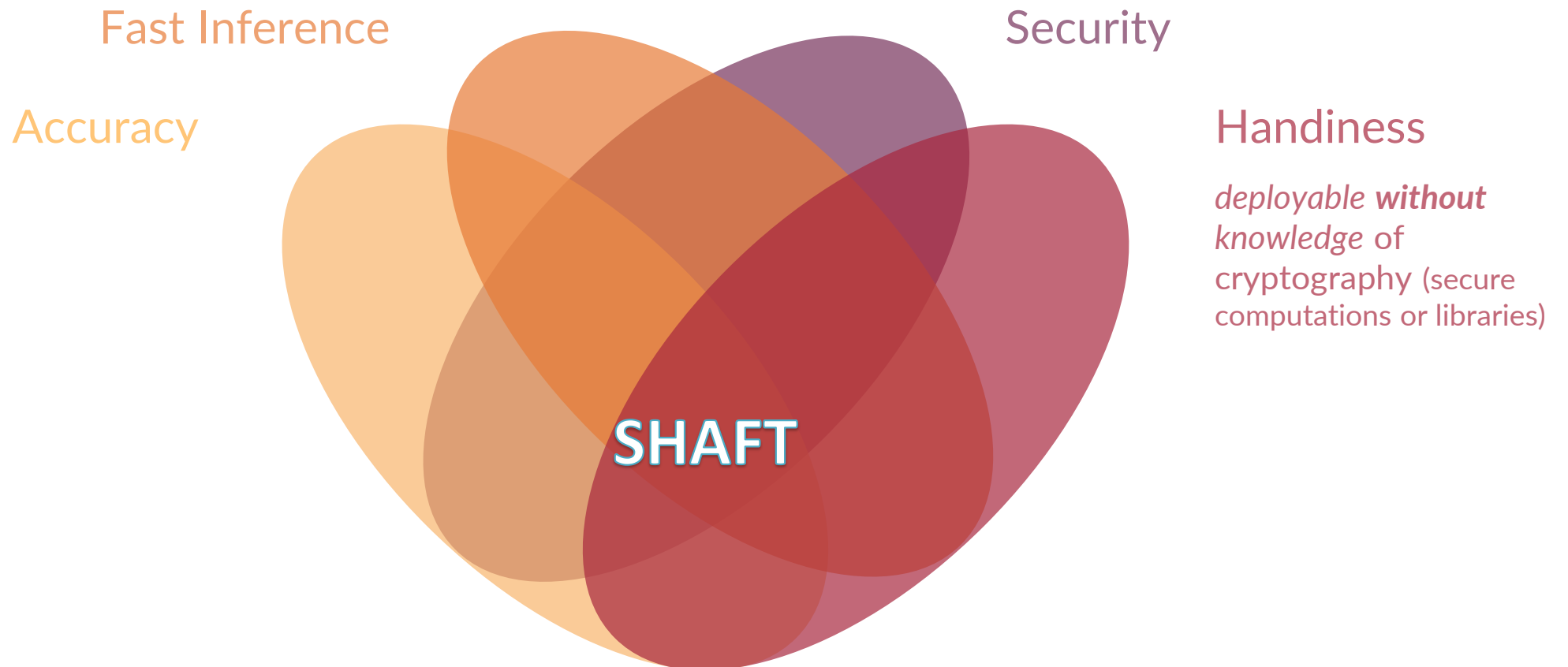


Background

- **Transformer**: the current “standard” architecture for large language models (LLMs)
- **Privacy concerns** arise due to the increasing adoption of LLMs (e.g., ChatGPT)
- **Private inference**: performs model inference without leaking the *model* or *query*

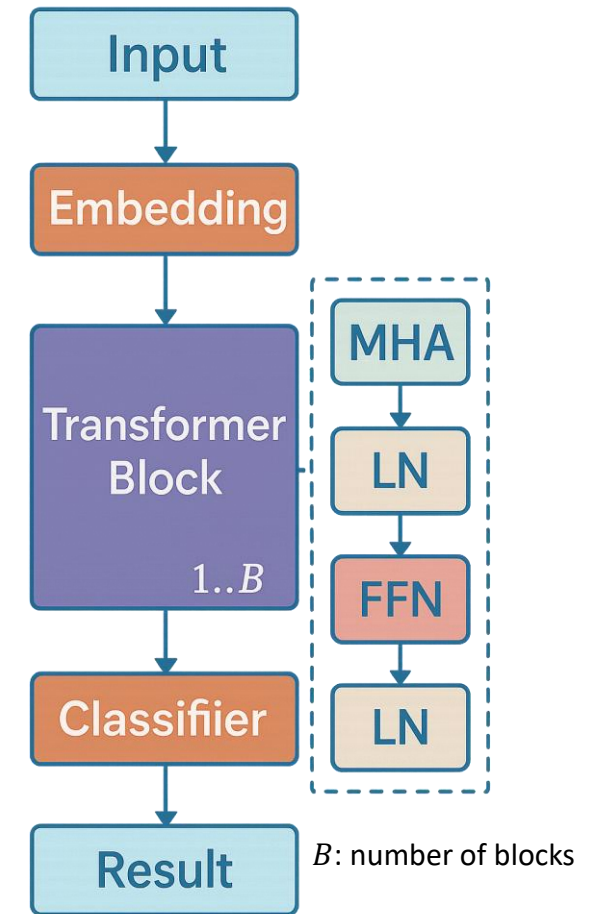


Desirable Properties for Private **Transformer** Inference



Challenges in Private Transformer Inference

- Embedding
 - One-hot vector conversion
- Multi-head attention (MHA)
 - **Softmax**
- Feed-forward network (FFN)
 - **Gaussian error linear unit (GELU)**
- Layer normalization (LN)
 - Inverse square root
- More challenging than (convolutional) neural networks
 - **Expensive secure softmax** in **every** MHA layer
 - **Sophisticated secure GELU** activation in **every** FFN layer



Note: other **linear** operations/layers within the above can be easily realized.

Private Transformer Inference Frameworks

Framework	Core Techniques		Accuracy	Efficiency
MPCFormer (ICLR '23)	Rough Approximations			✓✗
SIGMA (PETS '24)	Lookup Tables from FSS		✓	✓✗
BumbleBee (NDSS '25)	RLWE-based Homomorphic Multiplication		✓	✓✗
SHAFT (Ours)	ODE	Fourier Series	✓	✓

- Newer works applied recent paradigm or improved secure linear protocols
 - SIGMA uses function secret sharing (FSS) to reduce running time
 - BumbleBee optimizes homomorphic matrix multiplication to save communication
- Observation: non-linear function approximations remain underexplored

D. Li et al. "MPCFormer: Fast, performant and private transformer inference with MPC." *ICLR* 2023.

K. Gupta et al. "SIGMA: Secure GPT inference with function secret sharing." *PETS* 2024.

W. Lu et al. "BumbleBee: Secure two-party inference framework for large transformers." *NDSS* 2025.

Private Transformer Inference Frameworks

Framework	Security	Handiness	Accuracy	Efficiency
MPCFormer (ICLR '23)	✓✗			✓✗
SIGMA (PETS '24)	✓		✓	✓✗
BumbleBee (NDSS '25)	✓		✓	✓✗
SHAFT (Ours)	✓	✓	✓	✓

- SHAFT outperforms two recent works in **efficiency**
 - vs. SIGMA: 1.8-**2.5× faster** and **reduces communication** by 62-70%
 - vs. BumbleBee: 2.6-**3.7× faster** on LAN
- SHAFT achieves **comparable accuracy to plaintext inference**

OS: Ubuntu 20.04.

Models: BERT-base, BERT-large, GPT-2, ViT-base.

CPU: Intel Xeon Gold 5318Y, GPU: two NVIDIA A40, RAM: 256 GB.

Datasets: QNLI, CoLA, SST-2 from the GLUE benchmark.

Private Transformer Inference Frameworks

Framework	Security	Handiness	Accuracy	Efficiency
MPCFormer (ICLR '23)	✓✗			✓✗
SIGMA (PETS '24)	✓		✓	✓✗
BumbleBee (NDSS '25)	✓		✓	✓✗
SHAFT (Ours)	✓	✓	✓	✓

- For **handy** deployment of secure inference, we offer an **open-source** framework
 - **PyTorch-like APIs** smoothly integrate with the **Hugging Face** transformer library





Importing Hugging Face Transformers

- Existing works like CrypTen (NeurIPS '21) allow importing **simple** models
 - but **lack support** for transformer-specific layers (e.g., GELU)
- We implement **code conversion** of these layers

```
1 from transformers import AutoModelForSequenceClassification
2 import crypten as ct
3 # (standard) data loading and preprocessing omitted
4 model = AutoModelForSequenceClassification.from_pretrained("user/bert-base-cased-qnli")
5 ct.init()
6 model_ss = ct.nn.from_pytorch(model, dummy_data).encrypt().cuda()
7 data_ss = ct.cryptensor(data).cuda()
8 output_ss = model_ss(data_ss)
9 output = output_ss.get_plain_text()
```



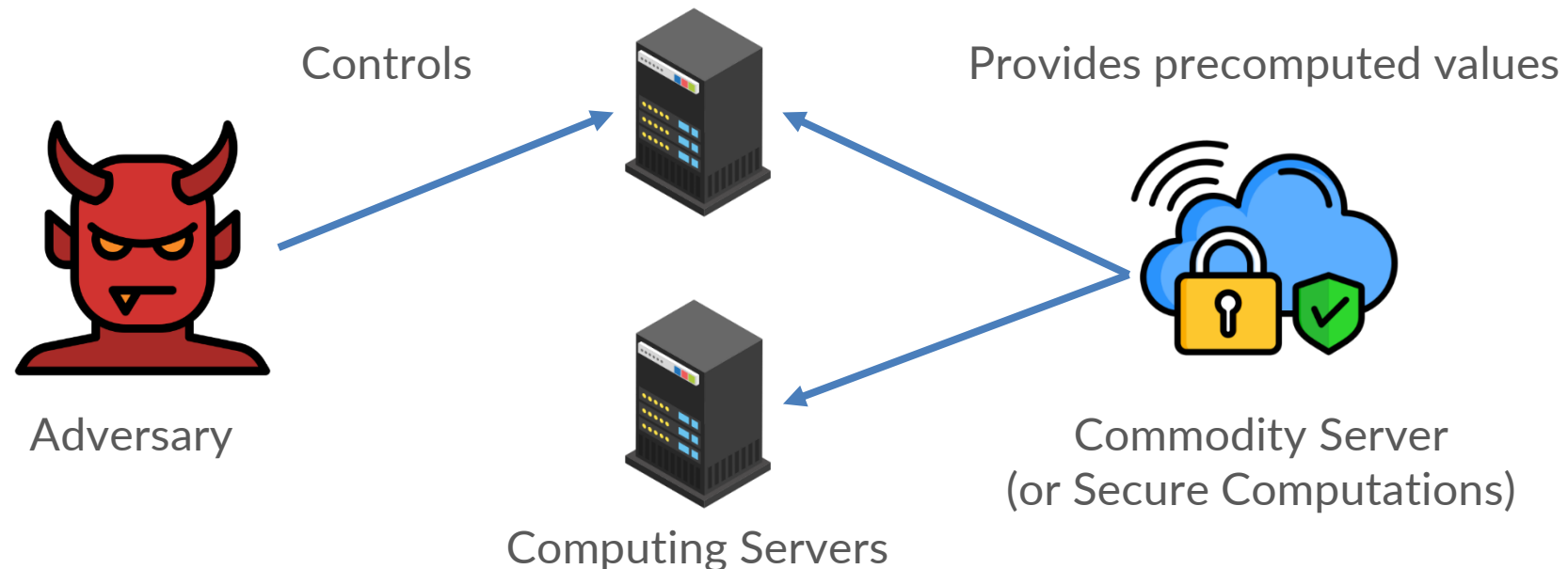
Our Technical Novelties

1. First **constant-round** private **softmax** protocol for transformers
 - Prior works need **logarithmic** rounds (in input length m) for **numerical stability**
 - We guarantee the same in **constant** rounds by uniquely combining **ordinary differential equation (ODE)** and **input clipping**
2. A **precise** and **efficient** private **GELU** protocol
 - We design a **GELU characterization** for **Fourier series** approximation
 - **Reduce** round complexity from **two** (S&P '24) to **one**

hundreds of thousands of softmax & GELU in transformers

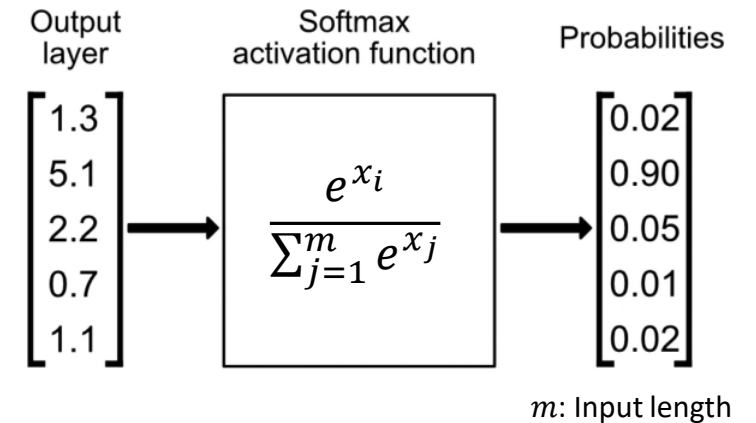
Security Model: 2-Party Outsourced Setting w/ Precomputation

- Well-established 2-party setting, e.g., NDSS '09
- Model and query are **secret-shared** to two servers
- Adversary: **semi-honest**, controls **one** of the two servers
- **Commodity server** can be replaced by **2-party computation** between servers



Private Softmax with Private Max()

- $\text{Softmax}(\vec{x})_i = e^{x_i} / \sum_j e^{x_j}$
 - Converts values in a vector into probabilities
- Basic idea: evaluates a sequence of e^x and $1/x$
 - Problem: e^x and $1/x$ **overflow easily**
- Typical solution: computes $\text{Softmax}(\vec{x} - \max(\vec{x}))$



<https://medium.com/towards-data-science/softmax-activation-function-explained-a7e1bc3ad60>



- **Avoids overflows** without affecting correctness



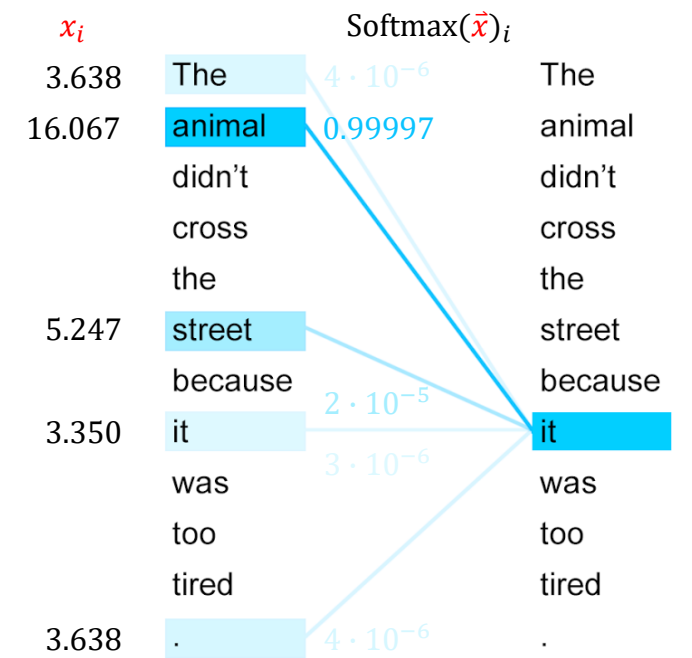
- Secure evaluation of maximum requires **logarithmic** rounds

Numerically-Unstable Private Softmax with ODE (ACSAC '23)

- Let t be the number of iterations, m be the input length
- **ODE approximation** of $\text{Softmax}(\vec{x})$:
 - Initial guess: $\vec{y}_0 = \vec{1}/m$
 - Iterative updates: $\vec{y}_i = \vec{y}_{i-1} + \frac{1}{t} (\vec{x} - \langle \vec{x}, \vec{y}_{i-1} \rangle \vec{1}) * \vec{y}_{i-1}$
 - Inner product
 - All-one vector
 - Entry-wise product
- Total $2t$ rounds (2 per iteration)
- Needs **large** t (e.g., 128) for **unbounded** \vec{x} in transformers
 - Correctness **requires** $\max(\vec{x}) - \min(\vec{x}) \leq t$

Our Private Softmax with Input-Clipped ODE

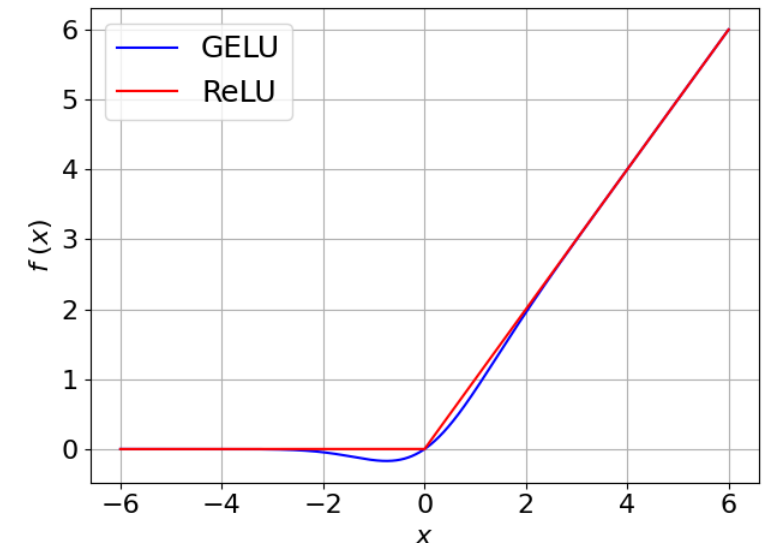
- Key idea: **clips** input to a **pre-defined** range $[a, b]$
 - $t = b - a$ ensures correctness, even with **small** t
 - **Constant round!**
- Why clipping matters in $\text{Softmax}(\vec{x})_i = e^{x_i} / \sum_j e^{x_j}$?
 - Attention layers map **word relevance** to probabilities
 - Words usually relate to *only a few* others in a sentence
 - **Most** x_j s are **small**, but a **few** large x_j s **dominate** the sum
 - Clipping large x_j aggressively can cause **significant** errors
- How to select a and b ?
 - Sets a **larger positive** b to minimize errors from *large* x_j s
 - Chooses a **slightly negative** a to include most *small* x_j s
 - $t = 16, a = -4, b = 12$ in all our experiments



<https://research.google/blog/transformer-a-novel-neural-network-architecture-for-language-understanding>

Private GELU with (Piecewise) Polynomial

- $\text{GELU}(x) = 0.5x \left(1 + \text{Erf}(x/\sqrt{2})\right)$, $\text{Erf}(x) = (2/\sqrt{\pi}) \int_0^x e^{-u^2} du$
- Standard approach:
 - Idea: $\text{GELU}(x)$ is close to $\text{ReLU}(x) = \max(x, 0)$ when $|x|$ is relatively large
 - Approximates $\text{GELU}(x)$ for x near 0 with polynomial(s)
 - Sets $\text{GELU}(x) = \text{ReLU}(x)$ for larger $|x|$
- State-of-the-art: a **degree-4** polynomial (S&P '24)
 - Secure evaluation requires **two** rounds
 - **Substantial** overheads for transformers with **hundreds of thousands** of GELU



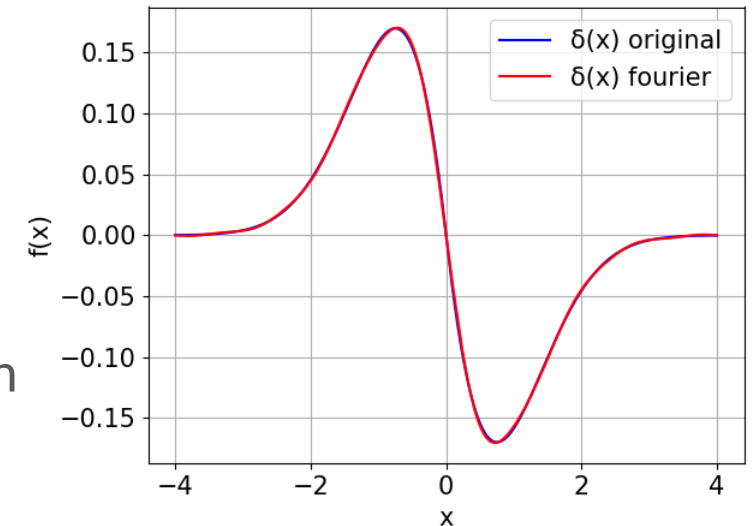


Private GELU with Fourier Series (FS)

- **Fourier Series (FS)**: provide precise approximations of functions
 - with a **sinusoidal** shape for a **bounded** input range
- Securely evaluating an FS takes only **one** round (ACSAC '23)
- Problem: GELU is **not** sinusoidal (*i.e.*, **direct** approximation with FS **fails**)
- Simple solution: approximates $\text{Erf}(x)$ with FS (ACL Findings '24):
 - Recall: $\text{GELU}(x) = 0.5x \left(1 + \text{Erf}(x/\sqrt{2}) \right)$
 - Requires an **additional** round to **securely multiply** the result by x
 - Increases approximation **error** (when $|x| > 2$)

Our Private GELU with Fourier Series (FS)

- Goal: **designs a suitable function** for FS approximation
- Our formulation: $\delta(x) = \text{sgn}(x)(\text{GELU}(x) - \text{ReLU}(x))$
 - Modified from **non-sinusoidal** $\text{GELU}(x) - \text{ReLU}(x)$ for table lookup (PETS '24)
 - A **sinusoidal** function for x near 0
 - **Ideal** for accurate FS approximation
- **FS approximation** of $\delta(x)$ for $|x| < 4$:
 - $\delta(x) \approx \sum_{n=1}^8 \beta_n \sin\left(\frac{n\pi x}{4}\right), \beta_n = \frac{1}{4} \int_{-4}^4 \delta(u) \sin\left(\frac{n\pi u}{4}\right) du$
 - Coefficients β_n precomputable via numerical integration
- Enforces $\delta(x) \approx 0$ for $|x| \geq 4$
- GELU characterization: $\text{GELU}(x) = \text{ReLU}(x) + \delta(|x|)$
 - **No extra round** needed: $|x| = 2\text{ReLU}(x) - x$





Other Contributions

1. **First private embedding** protocol “**natively**” taking **indices** as inputs
 - Prior works **assume** inputs are **one-hot vectors**, requiring **extra conversions** by clients
 - Our approach is inspired by **pre-computed one-hot pairs** from Grotto (CCS '23)
 - (unlike Grotto for *spline evaluation*)
2. Extension of our GELU characterization to **other activations**
 - *E.g.*, **sigmoid linear unit/SiLU**, used in the Meta AI's LLaMA model
3. Optimizations for **smaller bitwidth**
 - **Reduces communication** in **mixed-bitwidth** frameworks

Final Remarks

- We propose **secure**, **accurate**, and **fast** protocols for **softmax** and **GELU**
- Code: github.com/andeskyl/SHAFT
 - Interoperable with Hugging Face for **handy transformer** deployment
- Future directions:
 - Private transformer ***fine-tuning/training*** (GPU-TEE co-design?)
 - Security against ***malicious*** adversaries (replicated/authenticated sharing?)
- Contact: {kyl022, sherman}@ie.cuhk.edu.hk

