COSC 4200
Computability & Complexity

Name: Jacob Tuttle

Homework 06

April 23, 2020

1. (a) Let f be a reduction and M be the recognizer for A_{TM}. We describe the recognizer for N for B.

   N: On input w:

   i. compute $f(w)$
   ii. Run M on input $f(w)$. If M accepts, accept $f(w)$. If M rejects, reject $f(w)$.

   Therefore, $B \leq_M A_{TM}$ because if $w \in B$, $f(w) \in A_{TM}$ and if $w \notin B$, $f(w) \notin A_{TM}$.

   (b) By the definition of Turing Recognizable, C is turing recognizable. Since B is decidable, it is also recognizable. Let f be the mapping reduction by which $B \leq_M C$ where M is the recognizer for C. We can then use the recognizer described above to show how we can reduce B to C

2. Since $\{ax|x \in A\} \subseteq A$ and $\{by|y \in B\} \subseteq A$, C is a subset of A. Therefore, since A reduces to D, C reduces to D as well.

3. The following turing machine F computes a reduction f.

   F: On input $< M, w >$:

   (a) Construct the following machine M' where on input x:

   i. run M on x
   ii. if M accepts, accept
   iii. if M rejects, M' enters an infinite loop

   (b) Output $< M', w >$

   Just as in the lecture slides, we have shown a way that $HALT_{TM} \leq_M A_{TM}$ since $< M, w > \in HALT_{TM} \iff < M', w > \in A_{TM}$ since their behavior will be the same.

4. Let f be the reduction that will accept an input from ALL_{CFG} that is two CFGs if and onlly if, for some decider M both languages are decided or not decided in the same way. In this way, they are checked to be equivalent and ALL_{CFG} is reduced to EQ_{CFG}.

5. Let M be the Turing Machine that decides ALL_{TM}. Run our diagonalize turing machine, D, with M as input. D will accept M when M rejects and reject when it accepts. Now, run D on itself. It will now accept when M accepts and reject when M rejects, this contradicts the previous execution and raises a contradiction to ALL_{TM} being decidable.

6. Since B is made up of a part of D, if D is undecidable, the undecidable part could be the portion that is used to construct B, meaning that B would also be undecideable, and therefore not turing recognizable. If D is decidable, that means that, since B is a part of D, that B will be decidable and therefore have a Turing Machine that decides it, and therefore will be recognized by a turing machine, making it recognizable.