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Assignment 1

① suppose two functions $f(x)$ and $g(x)$ are defined on some subset of the real numbers:

$$f(x) = O(g(x))$$

if and only if there exists a positive real c and a positive integer n_0, such that for all $n \geq n_0$, $f(n) \leq c \cdot g(n)$

Important Item:

- change of base rule:

$$\log_b(a) = \frac{\log_x(a)}{\log_x(b)}$$

proof:

$$\log_a x = y$$

$$a^y = x$$

$$\log_b a^y = \log_b x \quad \leftarrow \text{divid both sides by } \log_b$$

$$y \log_{\cancel{b}} a = \log_b x \quad \leftarrow \text{move the } y \text{ out front}$$

$$\frac{y \log_{\cancel{b}} a}{\cancel{\log_b a}} = \frac{\log_b x}{\log_b a} \quad \leftarrow \text{divide both sides by } \log_b a$$

$$y = \frac{\log_b x}{\log_b a}$$

continued →

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Assignment 1

continued

1

We say two set are equal if they have the same elements.

$$f(x) = \{\log_2(10)\} \quad , \quad g(x) = \{\log_{10}(10)\}$$
$$\rightarrow f(x) = \{3.32\} \quad , \quad g(x) = \{\frac{\log_2(10)}{\log_2(2)}\}$$

Therefore:

$$f(x)\{3.32\} = g(x)\{3.32\}$$

and the two sets are equal. So, $f(x) = O(g(x))$ and shows that the base of the logarithm doesn't matter.

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Assignment 2

② $T(n) \begin{cases} 1 & , n=1 \\ 2T(n/3)+n & , n>1 \end{cases}$

$$\begin{aligned} T(n) &= T(n/3) + T(n/3) + n \\ &= 2T(n/3) + n \\ &= 2(2T(n/3^2) + n/3) + n \\ &= 2^2 T(n/3^2) + n + n \\ &= 2^2(2T(n/3^3) + n/3^2) + 2n \\ &= 2^3 T(n/3^3) + 3n \\ &\vdots \end{aligned}$$

let :

$n/3^k = 1$, $n = 3^k$, $k = \log n$

now :

$$\begin{aligned} T(n) &= 1n + n \log n \\ &= O(n \log n) \end{aligned}$$

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Assignment 1

3.

1	7	1	5	3	-1	9

↑ - pointer

1	7	1	5	3	-1	9

1	1	7	5	3	-1	9

1	1	5	7	3	-1	9

1	1	3	5	7	-1	9

-1	1	1	3	5	7	9

-1	1	1	3	5	7	9

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Assignment 1

⑤ The median-of-three takes the middle element of the sorted left-most, right-most and middle of the array.

Example:

	1/3	2/3
<p	Good Pivot	>p

We would at most pick two partitions of size 2/3. By picking the median of three elements we are reducing the probability of picking a bad pivot. By sorting the three elements we increase are chance of picking a good pivot (increases the probability).

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Assignment 1

sources:

- Khanacademy - change of base rule

- Khanacademy - change of base rule proof

- Data structures + Algorithm Analysis in C++
  4th edition, Mark Allen Weiss