Homework #2

COSC 4200 Computability and Complexity
Spring 2020

Due: 11:00am, Thursday, February 20

1. (10) Let A and B be languages. The *difference* of A and B is the language

$$A - B = \{x \in \Sigma^* \mid x \in A \text{ and } x \notin B\}.$$

The *symmetric difference* of A and B is the language

$$A \Delta B = \{x \in \Sigma^* \mid x \text{ is in exactly one of } A \text{ or } B\} = (A - B) \cup (B - A).$$

Prove the following statements:

(a) If A and B are regular, then $A - B$ is also regular.
(b) If A and B are regular, then $A \Delta B$ is also regular.

2. (30) Designing NFAs:

(a) Give a 3-state NFA for $\{1x0 \mid x \in \{0,1\}^*\}$.
(b) Give a 4-state NFA for $\{x \in \{0,1\}^* \mid x \text{ ends with 101 or 11}\}$.
(c) Give a 3-state NFA for $\{0^i 1^j 0^k \mid i, j, k \geq 0\}$.
(d) Give a 6-state NFA for $\{x \in \{0,1\}^* \mid x \text{ contains 01010}\}$.
(e) Give an NFA for $\{wxw^R \mid x \in \{0,1\}^*, w \in \{0,1\}^2\}$ that is simpler than your DFA from Homework #1.
(f) Given an NFA for

$$\{w \in \{a,b\}^* \mid \text{the first symbol of } w \text{ is different from the last symbol of } w\}$$

with as few states as possible.

Explain how each NFA works.

3. (10) Give a DFA for the language

$$\{w \in \{a,b\}^* \mid w \text{ contains } aba \text{ but does not contain } bb\}.$$

4. (10) Use the subset construction to convert your NFA from 2(d) to a DFA.

5. (10) Describe the languages of these regular expressions (write an English description that is as simple as possible):

   (a) $((0 \cup 1)(0 \cup 1))^*$

   (b) $(0 \cup 1 \cup \epsilon)(0 \cup 1 \cup \epsilon)(0 \cup 1 \cup \epsilon)$

   (c) $(0 \cup 1)^*111(0 \cup 1)^*000(0 \cup 1)^*$

   (d) $(0^*100)^*0^*$

   (e) $(0^*1^*)^*$

6. (10) Give regular expressions for these languages:

   (a) $\{w \in \{a, b\}^* \mid w$ contains at least three b's$\}$

   (b) $\{w \in \{a, b\}^* \mid$ every third symbol of w is $b\}$

   (c) $\{w \in \{a, b\}^* \mid w$ contains at most two a's$\}$

   (d) $\{a^n \mid n$ is a multiple of 2 or a multiple of 3$\}$

   (e) $\{w \in \{a, b\}^* \mid w$ does not contain exactly two a's$\}$

7. (10) Convert these regular expressions to NFAs:

   (a) $(0 \cup 11)^*(1 \cup 01)$

   (b) $((00)^*11 \cup 10)^*$

8. (10) Give a regular expression for $\{x \in \{0, 1\}^* \mid \text{bin}(x)$ is a multiple of 5$\}$. (Start with a DFA and convert it to a regular expression.)