Exercise 3

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1 General Homography Transformation Matrix

An homography ($\tilde{H} \in \mathbb{P}^{n\times n}$) on homogeneous coordinates is defined as the composition of a general Affine transformation $\mathbf{A} \in \mathbb{R}^{(n-1)\times(n-1)}$ (Rotation, Scaling, Reflection, Shearing), a translation $\mathbf{t} \in \mathbb{R}^{(n-1)\times 1}$ and a projection $\mathbf{u} \in \mathbb{R}^{1\times(n-1)}$. Finally, the matrix is defined on terms of a scalar scaling factor $\alpha \in \mathbb{R}$ that accounts for the homogeneous coordinates transform. Written as a n-dimensional matrix (1), the homography has n^2-1 degrees of freedom (DoF), dissected as it follows: the affine transformation accounts for $(n-1)^2$ DoF, the translation transform has $n-1$ DoF and the projection presents $n-1$ DoF.

It is necessary to establish that the scalar term α does not increase the total number of DoF of the transformation, as it describes an scaling transform that only takes effect when a coordinate on projective space is projected back into Cartesian coordinates.

$$\tilde{H} = \begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{u}^T & \alpha \end{bmatrix} \tag{1}$$

2 Proof that an Homogeneous transform preserves lines

Given a point x on projective space $\mathbb{P}^2$, a line is defined using a coefficient vector $L \in \mathbb{R}^{3\times 1}$, as shown on (2). It is possible to proof that an homography transform, described through a projective matrix H preserves lines by showing that the line coeffcients are transformed by the inverse homography matrix B. This implies that the resulting point x_h after applying the homography belongs to the line defined by the new set of coefficients, therefore, the Homography preserves lines.

$$\begin{aligned}
L^T x &= 0 \qquad && L, x \in \mathbb{P}^{2\times 1} && (2) \\
\underbrace{L^T H^{-1}}_{B} Hx &= 0 \qquad && H \in \mathbb{P}^{2\times 2} \\
B \underbrace{Hx}_{x_h} &= 0 \qquad && B \in \mathbb{P}^{1\times 2} \\
\Rightarrow \boxed{Bx_h = 0} & \qquad && x_h \in \mathbb{P}^{2\times 1} \quad \square && (3)
\end{aligned}$$

References

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