Exercise 3

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1 General Homography Transformation Matrix

An homography ($\tilde{H} \in \mathbb{P}^{n \times n}$) on homogeneous coordinates is defined as the composition of a general Affine transformation $\mathbf{A} \in \mathbb{R}^{(n-1)\times(n-1)}$ (Rotation, Scaling, Reflection, Shearing), a translation $\mathbf{t} \in \mathbb{R}^{(n-1)\times 1}$ and a projection $\mathbf{u} \in \mathbb{R}^{1\times(n-1)}$. Finally, the matrix is defined on terms of a scalar scaling factor $\alpha \in \mathbb{R}$ that accounts for the homogeneous coordinates transform. Written as a n-dimensional matrix (1), the homography has n^2-1 degrees of freedom (DoF), dissected as it follows: the affine transformation accounts for $(n-1)^2$ DoF, the translation transform has $n-1$ DoF and the projection presents $n-1$ DoF.

It is necessary to establish that the scalar term α does not increase the total number of DoF of the transformation, as it describes an scaling transform that only takes effect when a coordinate on projective space is projected back into Cartesian coordinates.

$$\tilde{H} = \begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{u}^T & \alpha \end{bmatrix} \tag{1}$$

2 Proof that an Homogeneous transform preserves lines

Given a point x on projective space $\mathbb{P}^2$, a line is defined using a coefficient vector $L \in \mathbb{R}^{3\times 1}$, as shown on (2). It is possible to proof that an homography transform, described through a projective matrix H preserves lines by showing that the line coeffcients are transformed by the inverse homography matrix B. This implies that the resulting point x_h after applying the homography belongs to the line defined by the new set of coefficients, therefore, the Homography preserves lines.

$$L^T x = 0 \qquad L, x \in \mathbb{P}^{2\times 1} \tag{2}$$

$$\underbrace{L^T H^{-1}}_{B} H x = 0 \qquad H \in \mathbb{P}^{2\times 2}$$

$$B \underbrace{H x}_{x_h} = 0 \qquad B \in \mathbb{P}^{1\times 2}$$

$$\Rightarrow \boxed{B x_h = 0} \qquad x_h \in \mathbb{P}^{2\times 1} \quad \square \tag{3}$$

3   Geometrical meaning of translation vector values

By applying the extrinsic parameter matrix $[R|t]$ to the camera pose coordinate system origin $(0, 0, 0, 1)^T$, is possible to obtain the corrected position of the camera with respect to the world coordinate system. *i.e.,* The translation vector t. On this case, the displacement vector establishes the real position of the camera with respect to the world and the movement should the camera have to take in order to be perpendicular to the image plane.

It is possible to infer from the negative values on the translation vector that the coordinate system is defined around the chessboard center. This implies that the chessboard corners are defined in terms of offsets from the center. *i.e.,* Left coordinates are negative and Right coordinates are positive. This can happen if the homography computation was done taking into account offset coordinates from the board center instead of top-left centered coordinates.

References

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[3]   Richard Szeliski. *Computer vision: algorithms and applications*. Springer Science & Business Media, 2010.