Trees

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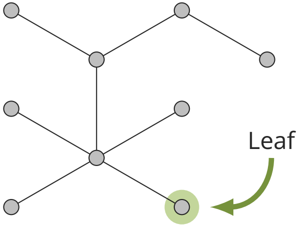
Tree

A *tree* is a connected graph without cycles.

Nodes which have only one edge are called *leaves*.



Observations

- Each tree with at least two nodes has at least two leaves.
- If v is a leaf of a tree T, then $T - v$ is a still a tree.

Trees

Theorem

For an undirected simple graph G following are equivalent.

(i) G is a tree (i. e., connected and without cycles).

(ii) G is connected and has $n - 1$ edges.

(iii) G is cycle-free and has $n - 1$ edges.

(iv) G edge-maximal cycle-free, i. e., adding any edge creates a cycle.

(v) There is exactly one simple path between each pair of vertices.

(vi) G edge-minimal connected, i. e., removing any edge disconnects G.

(vii) If $n > 1$, then G has a leave v and $G - v$ is a tree.

Statement (vii) implies that (1) every tree can be constructed by starting with a single vertex and adding leaves to the existing graph, and (2) every graph constructed this way is a tree.