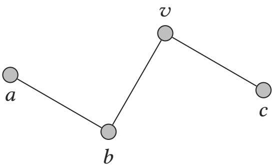
Terminology

Adjacency

Adjacency

Two vertices u and v are *adjacent* if there is an edge connecting them. This is sometimes written as $u \sim v$.

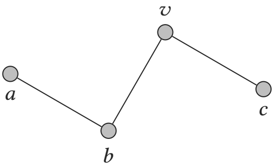


v is adjacent to b and c but not to a.

Neighbourhood

Neighbourhood

The *open neighbourhood* $N(v) = \{ u \in V \mid u \neq v, u \sim v\}$ of a vertex v is the set of vertices adjacent to v (not including v). The *closed neighbourhood* $N[v] = N(v) \cup \{v\}$ includes v.



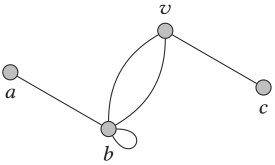
$N(v) = \{b, c\}$ $\qquad$ $N[v] = \{v, b, c\}$

Degree

Degree

The *degree* $\deg(v)$ of a vertex v is the number of incident edges.

Note that the degree is not necessarily equal to the cardinality of neighbours.

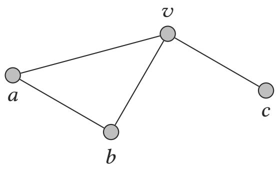


$\deg(v) = 3$ $\qquad$ $\deg(a) = 1$ $\qquad$ $\deg(b) = 5$ $\qquad$ $\deg(c) = 1$

Minimum and Maximum Degree

Minimum and Maximum Degree

For a graph $G = (V, E)$, $\delta(G)$ denotes the *ninimum* and $\Delta(G)$ denotes the *maximum degree* of G, i. e., $\delta(G) := \min\{ \deg(v) \mid v \in V \}$ and $\Delta(G) := \max\{ \deg(v) \mid v \in V \}$.

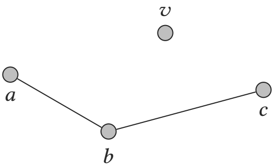


$$\delta(G) = 1 \qquad \Delta(G) = 3$$

Isolated Vertex

Isolated Vertex

A vertex v is called *isolated*, if it has no neighbours, i. e., $N(v) = \emptyset$.

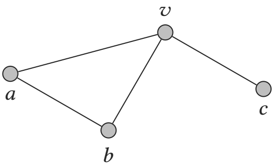


The vertex v is an isolated vertex.

Universal Vertex

Universal Vertex

A vertex v is called *universal*, if it adjacent to all other vertices in the graph, i. e., $N[v] = V$.

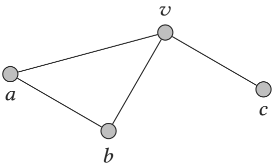


The vertex v is a universal vertex.

Pendant Vertex

Pendant Vertex

A vertex v is called *pendant*, if it adjacent to exactly one other vertex, i. e., $\left|N(v)\right| = 1$.

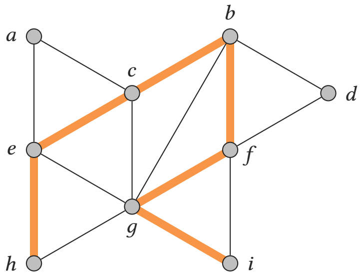


The vertex c is a pendant vertex.

Path

Path

A set $P = \{v_0, v_1, \ldots, v_k\}$ of distinct vertices is called *path* (of *length* k) if v_i is adjacent to v_{i+1} for all i with $0 \leq i < k$.

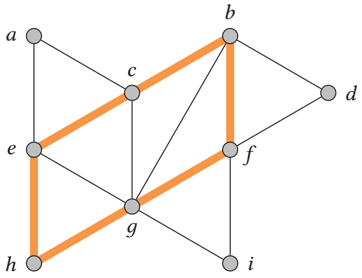


$P = \{h, e, c, b, f, g, i\}$ is a path of length 6.

Cycle

Cycle

A path $P = \{v_0, v_1, \ldots, v_k\}$ is called *cycle* (of *length* $k+1$) if v_0 is adjacent to v_k.

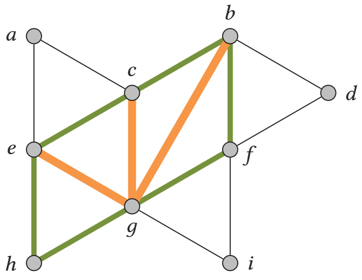


$\{h, e, c, b, f, g\}$ is a cycle of length 6.

Chord

Chord

A *chord* in a path (or cycle) is an edge connecting two non-consecutive vertices of the path (or cycle).

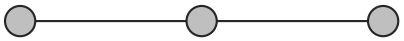


The edges bg, cg, and eg are chords of the cycle $\{h, e, c, b, f, g\}$.

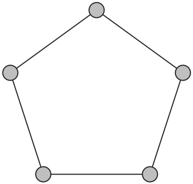
Induced Path / Cycle

Induced Path / Cycle

A path (or cycle) is called *induced* if it has no chords. For each $k \geq 3$, an induced path of k vertices is called P_k and an induced cycle of length k is called C_k.



P_3

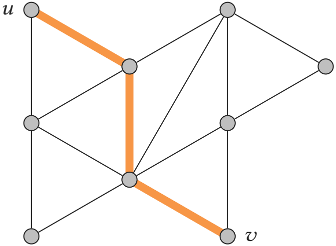


C_5

Distance

Distance

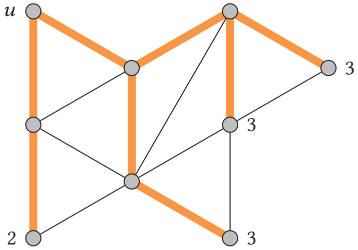
The *distance* $d(u, v)$ of two vertices u and v is the length of the shortest path from u to v.



$$d(u, v) = 3$$

Eccentricity

The *eccentricity* $\mathrm{ecc}(v)$ of a vertex v is its maximal distance to any vertex, i. e., $\mathrm{ecc}(v) = \max_{u \in V} d(u, v)$.

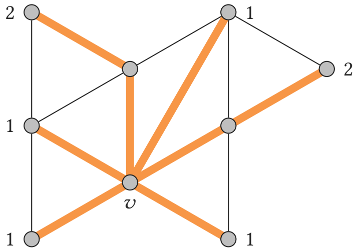


$$\mathrm{ecc}(u) = 3$$

Eccentricity

Eccentricity

The *eccentricity* $\mathrm{ecc}(v)$ of a vertex v is its maximal distance to any vertex, i. e., $\mathrm{ecc}(v) = \max_{u \in V} d(u, v)$.



$$\mathrm{ecc}(v) = 2$$

Radius and Diameter

Diameter

The *diameter* $\operatorname{diam}(G)$ of a graph G is the maximal eccentricity of all vertices in G, i. e., $\operatorname{diam}(G) = \max_{v \in V} \operatorname{ecc}(v)$.

Radius

The *radius* $\operatorname{rad}(G)$ of a graph G is the minimal eccentricity of all vertices in G, i. e., $\operatorname{rad}(G) = \min_{v \in V} \operatorname{ecc}(v)$.

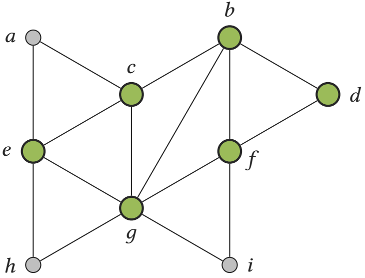
Lemma

For each graph G, $\operatorname{rad}(G) \leq \operatorname{diam}(G) \leq 2 \operatorname{rad}(G)$

Interval

The *interval* $I(u,v)$ of two vertices u and v is the set of vertices which are on a shortest path from u to v. Formally,

$$I(u,v) = \{\, w \mid d(u,v) = d(u,w) + d(w,v) \,\}.$$

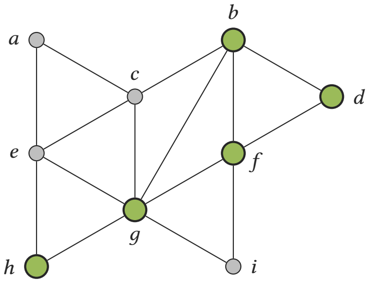


$$I(e,d) = \{b, c, d, e, f, g\}$$

Interval

The *interval* $I(u, v)$ of two vertices u and v is the set of vertices which are on a shortest path from u to v. Formally,

$$I(u, v) = \{\, w \mid d(u, v) = d(u, w) + d(w, v) \,\}.$$

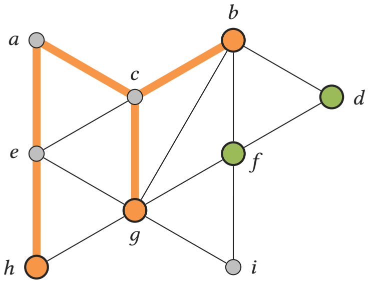


$$I(h, d) = \{b, d, f, g, h\}$$

Projection

For a vertex v and a vertex set S, the *projection* $\mathrm{Pr}(v, S)$ is the set of vertices in S with minimal distance to v. Formally,

$$\mathrm{Pr}(v, S) = \{\, u \in S \mid d(u, v) = d(v, S) \,\}.$$

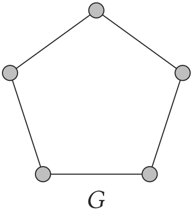


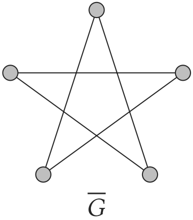
$$\mathrm{Pr}(a, I(h, d)) = \{b, g, h\}$$

Complement

Complement

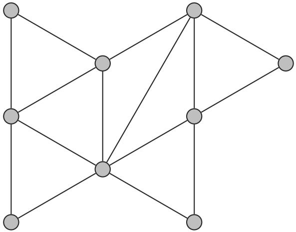
The *complement* $\overline{G} = (V, \overline{E})$ of a graph $G = (V, E)$ is the graph with the edges not contained in G, i. e., $\overline{E} = \{ uv \mid uv \notin E \}$.



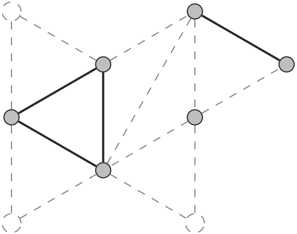


Subgraph

A graph $G' = (V', E')$ is a *subgraph* of a graph $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$.



G

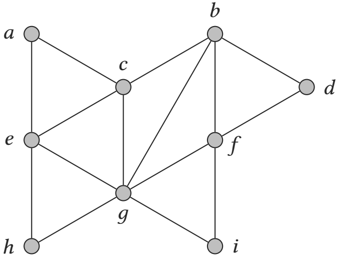


G'

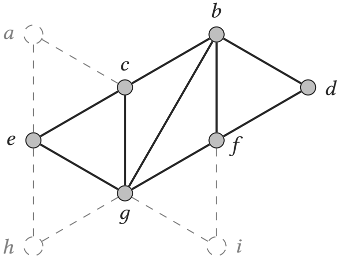
Note that $u, v \in V \cap V'$ and $uv \in E$ does *not* imply $uv \in E'$.

Induced Subgraph

For a graph $G = (V, E)$ and a set $U \subseteq V$, the *induced subgraph* $G[U]$ of G is defined as $G[U] = (U, E')$ with $E' = \{\, uv \mid u, v \in U; uv \in E \,\}$



G

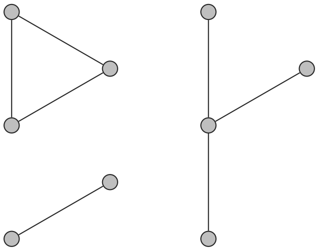


$G\big[\{b, c, d, e, f, g\}\big]$

Connected Component

Connected Component

A *connected component* of an (undirected) graph is a maximal subgraph in which any two vertices can be connected by a path.

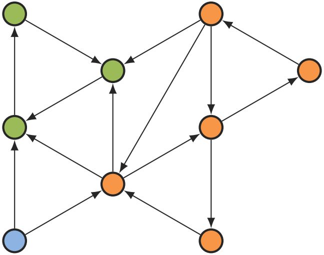


A graph with three connected components.

Strongly Connected Component

Strongly Connected Component

A directed graph is *strongly connected* if every vertex is reachable from every other vertex. A *strongly connected component* is a maximal subgraph which is strongly connected.

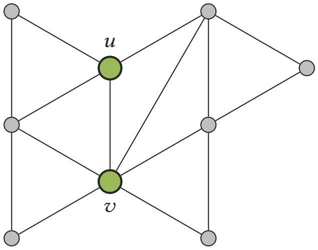


A graph with three strongly connected components.

Separator

Separator

A vertex set S is called *seperator* of a graph G if removing S from G increases the number of connected components.

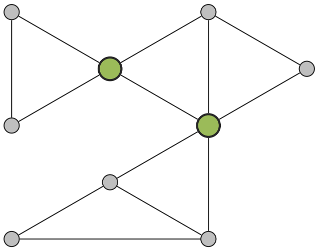


The set $S = \{u, v\}$ is a separator for the given graph.

Articulation Point

Articulation Point

A vertex v is an *articulation point* (also called *cut vertex*) if $\{v\}$ is a separator, i. e., removing it increases the number of connected components.

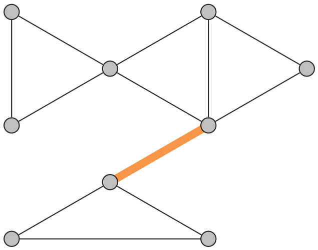


A graph with two articulation points.

Bridge

Bridge

An edge is called *bridge* if removing it from the graph (while keeping the vertices) increases the number of connected components.

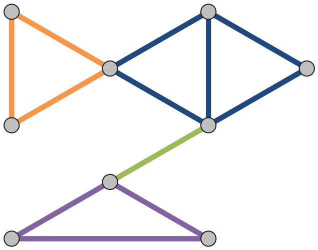


A graph with a bridge.

Block

Block

A *block* (also called *2-connected component*) is a maximal subgraph without articulation points.



A graph with four blocks.