Q5 #5

$a_0 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ $a_1 = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 3 \end{bmatrix}$

$a_2 = 2a_0 + a_1 = \begin{bmatrix} -2 \\ 2 \\ -2 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ -3 \\ 5 \end{bmatrix}$

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[11]: a = np.array([ [-1, 1, -1, 1], [-1, 3, -1, 3]]) #, [1, 3, 5, 7] ])
      b = np.array([a[0], a[1], 2*a[0] + a[1]])
      q = gram_schmidt(b)
      print(q)

      Vectors are linearly dependent.
      GS algorithm terminates at iteration  3
      [array([-0.5,  0.5, -0.5,  0.5]), array([0.5, 0.5, 0.5, 0.5])]
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Verify

$q_0 = \frac{a_0}{\|a_0\|} = \frac{a_0}{\sqrt{(-1)^2+(1)^2+(-1)^2+(1)^2}} = \frac{a_0}{\sqrt{4}} = \frac{a_0}{2}$

$q_0 = \begin{bmatrix} -0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$ $q_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$

BASES

$\tilde{q}_0 = a_0 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

$\tilde{q}_1 = a_1 - (q_0^T a_1) q_0$

$\begin{bmatrix} -0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} \times \begin{bmatrix} -1 \\ 3 \\ -1 \\ 3 \end{bmatrix} = 0.5+1.5+0.5+1.5 = 4$

$= \begin{bmatrix} -1 \\ 3 \\ -1 \\ 3 \end{bmatrix} - 4 \cdot \begin{bmatrix} -0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -2 \\ 2 \end{bmatrix}$

$\tilde{q}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$q_1 = \frac{\tilde{q}_1}{\|\tilde{q}_1\|} = \frac{\tilde{q}_1}{\sqrt{1^2+1^2+1^2+1^2}} = \frac{\tilde{q}_1}{\sqrt{4}} = \frac{\tilde{q}_1}{2}$

$\tilde{q}_2 = a_2 - (q_0^T a_2) q_0 - (q_1^T a_2) q_1$

$\begin{bmatrix} -0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} \times \begin{bmatrix} -3 \\ 5 \\ -3 \\ 5 \end{bmatrix} = 1.5+2.5+1.5+2.5 = 8$

$\begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \times \begin{bmatrix} -3 \\ 5 \\ -3 \\ 5 \end{bmatrix} = -1.5+2.5-1.5+2.5 = 2$

$= \begin{bmatrix} -3 \\ 5 \\ -3 \\ 5 \end{bmatrix} - 8 \cdot \begin{bmatrix} -0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix} - 2 \cdot \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$

$= \begin{bmatrix} -3 \\ 5 \\ -3 \\ 5 \end{bmatrix} - \begin{bmatrix} -4 \\ 4 \\ -4 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$\tilde{q}_2 = \vec{0}$, STOP, return $[q_0, q_1]$