Nama : Andi Suci Khairunnisa

NIM : D121241085

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(A.) Tentukan F_x dan F_y

1. $F(x,y) = x^3 \cos(x+y) + y \sin 2xy$

=>

- Terhadap x

$u_1 = x^3$ $u_1' = 3x^2$ $u_2 = y$ $u_2' = 0$

$v_1 = \cos(x+y)$ $v_1' = -\sin(x+y)$ $v_2 = \sin 2xy$ $v_2' = 2y \cos 2xy$

$\therefore F_x(x,y) = 3x^2 \cos(x+y) - x^3 \sin(x+y) + 2y^2 \cos 2xy$

- Terhadap y

$u_1 = x^3$ $u_1' = 0$ $u_2 = y$ $u_2' = 1$

$v_1 = \cos(x+y)$ $v_1' = -\sin(x+y)$ $v_2 = \sin 2xy$ $v_2' = 2x \cos 2xy$

$\therefore F_y(x,y) = -x^3 \sin(x+y) + \sin 2xy + 2xy \cos(2xy)$

2. $F(x,y) = \int_x^y e^{\cos t}\, dt$

=>

- terhadap x

$F_x(x,y) = -e^{\cos(x)}$

- terhadap y

$F_y(x,y) = e^{\cos(y)}$

3. $F(x,y) = x^3 \cos(x+y) + y \sin(2xy)$

=> <Soal mirip no. 1>

- terhadap x

$F_x(x,y) = 3x^2 \cos(x+y) - x^3 \sin(x+y) + 2y^2 \cos 2xy$

- terhadap y

$F(x,y) = -x^3 \sin(x+y) + \sin 2xy + 2xy \cos(2xy)$

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B. Tentukan F_x, F_y, dan F_z

1. $F(x,y,z) = xy + y^2 z + 3xz$

$\Rightarrow$

- $F_x(x,y,z) = y + 3z$
- $F_y(x,y,z) = x + 2yz$
- $F_z(x,y,z) = y^2 + 3x$

2. $F(x,y,z) = x\cos(y-z) + 2xy$

$\Rightarrow$

- $F_x(x,y,z) = \cos(y-z) + 2y$
- $F_y(x,y,z) = -x\sin(y-z) + 2x$
- $F_z(x,y,z) = -x\sin(y-z)$

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(B) Tentukan f_{xx}, f_{yy}, f_{xy}, f_{yx}

1. $F(x,y) = x\cos(xy) + xy\,e^{x+y}$

=>

$*\ f_x(x,y) = \cos(xy) - xy\sin(xy) + ye^{x+y} + xye^{x+y}$

- $f_{xx} = -y\sin(xy) - y\sin(xy) - xy^2\cos(xy) + ye^{x+y} + ye^{x+y} + xye^{x+y} \cdot 1$
  $= -2y\sin(xy) - xy^2\cos(xy) + 2ye^{x+y} + xye^{x+y}$
- $f_{xy} = -x\sin(xy) - x\sin(xy) - x^2y\cos(xy) + e^{x+y} + ye^{x+y} + xe^{x+y} + xye^{x+y}$
  $= -2x\sin(xy) - x^2y\cos(xy) + e^{x+y}(1+y+x+xy)$

$*\ f_y(x,y) = -x^2\sin(xy) + xe^{x+y} + xye^{x+y}$

- $f_{yx} = f_{xy}$
  $= -2x\sin(xy) - x^2y\cos(xy) + e^{x+y}(1+y+x+xy)$
- $f_{yy} = -x^2\cos(xy)\cdot x + xe^{x+y} + xe^{x+y} + xye^{x+y}$
  $= -x^3\cos(xy) + 2xe^{x+y} + xye^{x+y}$

2. $F(x,y) = \ln(x^2+2xy+y^3)$

=>

$*\ F_x(x,y) = \dfrac{2x+2y}{x^2+2xy+y^3}$

- $F_{xx} = \dfrac{2(x^2+2xy+y^3) - (2x+2y)^2}{(x^2+2xy+y^3)^2}$
- $F_{xy} = \dfrac{2(x^2+2xy+y^3) - (2x+2y)(2x+3y^2)}{(x^2+2xy+y^3)^2}$

$*\ F_y(x,y) = \dfrac{2x+3y^2}{x^2+2xy+y^3}$

- $F_{yx} = F_{xy} = \dfrac{2(x^2+2xy+y^3) - (2x+2y)(2x+3y^2)}{(x^2+2xy+y^3)^2}$
- $F_{yy} = \dfrac{6y(x^2+2xy+y^3) - (2x+3y^2)^2}{(x^2+2xy+y^3)^2}$

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3. $F(x,y) = \tan^{-1}\left(\frac{y^2}{x}\right)$

$\Rightarrow$

$* \; F_x(x,y) = \frac{1}{1+\left(\frac{y^2}{x}\right)^2} \cdot \left(\frac{d}{dx} \cdot \frac{y^2}{x}\right)$

$= \frac{1}{1+\frac{y^4}{x^2}} \cdot \left(-\frac{y^2}{x^2}\right) = \frac{-y^2}{x^2+y^4}$

- $F_{xx} = \frac{2xy^2}{(x^2+y^4)^2}$

- $F_{xy} = \frac{-2y\,(x^2+y^4)+4y^5}{(x^2+y^4)^2}$

$* \; F_y(x,y) = \frac{1}{1+\left(\frac{y^2}{x}\right)^2} \cdot \left(\frac{d}{dx} \; \frac{y^2}{x}\right)$

$= \frac{1}{1+\left(\frac{y^2}{x}\right)^2} \cdot \frac{2y}{x} = \frac{2xy}{x^2+y^4}$

- $F_{yx} = F_{xy} = \frac{-2y\,(x^2+y^4)+4y^5}{(x^2+y^4)^2}$

- $F_{yy} = \frac{2x(x^2+y^4)-(2xy)(4y^3)}{(x^2+y^4)^2}$

$= \frac{2x\,(x^2+y^4)-8xy^4}{(x^2+y^4)^2}$

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4. $F(x,y) = \ln(x^2 + 2xy + y^2)$

$\Rightarrow$

* $F_x(x,y) = \dfrac{2x + 2y}{x^2 + 2xy + y^2}$

  - $F_{xx} = \dfrac{2(x^2+2xy+y^2) - (2x+2y)^2}{(x^2+2xy+y^2)^2}$

  - $F_{xy} = \dfrac{2(x^2+2xy+y^2) - (2x+2y)^2}{(x^2+2xy+y^2)^2}$

* $F_y(x,y) = \dfrac{2x+2y}{x^2+2xy+y^2}$

  - $F_{yx} = F_{xy} = \dfrac{2(x^2+2xy+y^2) - (2x+2y)^2}{(x^2+2xy+y^2)^2}$

  - $F_{yy} = \dfrac{2(x^2+2xy+y^2) - (2x+2y)^2}{(x^2+2xy+y^2)^2}$

5. $F(x,y) = \dfrac{2x-y}{xy}$

$\Rightarrow$

* $F_x(x,y) = \dfrac{2xy - (2xy - y^2)}{(xy)^2} = \dfrac{y^2}{(xy)^2} = \dfrac{1}{x^2}$

  - $F_{xx} = \dfrac{-2}{x^3}$

  - $F_{xy} = 0$

* $F_y(x,y) = \dfrac{-xy - (2x^2 - xy)}{(xy)^2} = \dfrac{-2x^2}{x^2y^2} = \dfrac{-2}{y^2}$

  - $F_{yy} = \dfrac{4}{y^3}$

  - $F_{yx} = 0$

(c) Tentukan $\bar{\nabla} f$ dari

1. $f(x,y) = \dfrac{x^2 y}{x+y}$

$\Rightarrow$

$$f_x(x,y) = \frac{2xy(x+y) - x^2 y}{(x+y)^2}$$

$$f_y(x,y) = \frac{x^2(x+y) - x^2 y}{(x+y)^2}$$

$$\therefore \bar{\nabla} f(x,y) = \left(\frac{2xy(x+y) - x^2 y}{(x+y)^2} , \frac{x^2(x+y) - x^2 y}{(x+y)^2} \right)$$

2. $f(x,y) = \ln \sqrt{x^2 + y^2}$

$\Rightarrow$

$$f_x(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x$$

$$= \frac{2x}{2 \cdot (x^2+y^2)^{1/2 + 1/2}}$$

$$= \frac{2x}{2(x^2+y^2)} = \frac{x}{x^2+y^2}$$

$$f_y(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y$$

$$= \frac{2y}{2(x^2+y^2)}$$

$$= \frac{y}{x^2+y^2}$$

$$\therefore \bar{\nabla} F(x,y) = \left(\frac{x}{x^2+y^2} , \frac{y}{x^2+y^2} \right)$$

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(D) Tentukan ∇F dititik yang diberikan

1. $F(x,y) = x^2y - xy^2$ di $P(-2,3)$

$\Rightarrow$

$F_x(-2,3) = 2xy - y^2 = 2(-2)(3) - (3)^2$
$= -21$

$F_y(-2,3) = x^2 - 2xy = (-2)^2 - 2(-2)(3)$
$= 16$

$\therefore \nabla F(-2,3) = (-21, 16)$

2. $F(x,y) = \ln(x^3 - xy^2 + 4y^3)$ di $P(-3,3)$

$\Rightarrow$

$F_x(-3,3) = \dfrac{3x^2 - y^2}{x^3 - xy^2 + 4y^3} = \dfrac{3(-3)^2 - (3)^2}{(-3)^3 - (-3)(3)^2 + 4(3)^3} = \dfrac{18}{108}$

$F_y(-3,3) = \dfrac{-2xy + 12y^2}{x^3 - xy^2 + 4y^3} = \dfrac{-2(-3)(3) + 12(3)^2}{108} = \dfrac{126}{108}$

$\therefore \nabla F(-3,3) = \left(\dfrac{18}{108}, \dfrac{126}{108}\right)$
$= \left(\dfrac{1}{6}, \dfrac{7}{6}\right)$

3. $f(x,y) = \dfrac{x^2}{y}$ di $P(2,-1)$

$\Rightarrow$

$F_x(2,-1) = \dfrac{2x}{y} = \dfrac{2(2)}{-1} = -4$

$F_y(2,-1) = -\dfrac{x^2}{y^2} = -\dfrac{(2)^2}{(-1)^2} = -4$

$\therefore \nabla F(2,-1) = (-4, -4)$

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3. $f(x,y) = \sin^3(x^2y)$

$\Rightarrow$

$f_x(x,y) = 3\sin^2(x^2y) \cdot \cos(x^2y) \cdot 2xy$

$= 6xy\sin^2(x^2y) \cdot \cos(x^2y)$

$f_y(x,y) = 3\sin^2(x^2y)\cos(x^2y) \cdot x^2$

$= 3x^2\sin^2(x^2y) \cdot \cos(x^2y)$

$\therefore \bar{\nabla} f(x,y) = \left(6xy\sin^2(x^2y)\cos(x^2y), 3x^2\sin^2(x^2y)\cos(x^2y)\right)$

4. $f(x,y) = xy\ln(x+y)$

$\Rightarrow$

$f_x(x,y) = y\ln(x+y) + \frac{xy}{x+y}$

$f_y(x,y) = x\ln(x+y) + \frac{xy}{x+y}$

$\therefore \bar{\nabla} f(x,y) = \left(y\ln(x+y) + \frac{xy}{x+y}, x\ln(x+y) + \frac{xy}{x+y}\right)$

5. $f(x,y,z) = x^2ye^{x-z}$

$\Rightarrow$

$f_x(x,y,z) = 2xye^{x-z} + x^2ye^{x-z}$

$f_y(x,y,z) = x^2e^{x-z}$

$f_z(x,y,z) = -x^2ye^{x-z}$

$\therefore \bar{\nabla} f(x,y,z) = \left(2xye^{x-z} + x^2ye^{x-z}, x^2e^{x-z}, -x^2ye^{x-z}\right)$

6. $f(x,y,z) = xe^{-2y}\sec z$

$\Rightarrow$

$f_x(x,y,z) = e^{-2y}\sec z$

$f_y(x,y,z) = -2xe^{-2y}\sec z$

$f_z(x,y,z) = xe^{-2y} \cdot \sec z\tan z$

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(E). Tentukan turunan berarah fungsi F pada titik P yang diberikan dalam vektor a.

a. $F(x,y) = y^2 \ln x$, $P(1,4)$, $a = -3i + 3j$

$\Rightarrow$

- $\bar{\nabla} F(x,y) = \frac{y^2}{x} i + 2y \ln x \, j$

$\bar{\nabla} F(1,4) = \left(\frac{4^2}{1}\right) i + 2(4) \ln 1 \, j$

$= 16 i + 0 \cdot j$

- $|a| = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}$

$u = \frac{a}{|a|} = \left(\frac{-3}{3\sqrt{2}} i + \frac{3}{3\sqrt{2}} j \right) = \frac{-\sqrt{2}}{2} i + \frac{\sqrt{2}}{2} j$

- $D_u F(1,4) = \bar{\nabla} F(1,4) \cdot u$

$= (16i + 0j) \cdot \left(\frac{-\sqrt{2}}{2} i + \frac{\sqrt{2}}{2} j \right)$

$= \frac{-16\sqrt{2} + 0 \cdot \sqrt{2}}{2}$

$= \frac{-16\sqrt{2}}{2} = -8\sqrt{2}$

b. $F(x,y) = xe^y - ye^x$, $P(0,0)$, $a = 5i - 2j$

$\Rightarrow$

- $\bar{\nabla} F(x,y) = (e^y - ye^x) i + (xe^y - e^x) j$

$\bar{\nabla} F(0,0) = (e^0 - 0.e^0) i + (0.e^y - e^0) j$

$= i + (-j)$

- $|a| = \sqrt{5^2 + (-2)^2} = \sqrt{29}$

$u = \frac{a}{|a|} = \left(\frac{5}{\sqrt{29}} i + \frac{-2}{\sqrt{29}} j \right)$

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- $D_u F\ (0,0) = \bar{\nabla} F(0,0) \cdot u$

$$= i + (-j) \cdot \left(\frac{5}{\sqrt{29}} i + \frac{-2}{\sqrt{29}} j \right)$$

$$= \frac{5 + (-2)(-1)}{\sqrt{29}}$$

$$= \frac{7}{\sqrt{29}}$$

c. $F(x,y) = e^{-xy}$, di $P(1,-1)$, $a = 1-i + \sqrt{3}j)$

=>

- $\bar{\nabla} F(x,y) = (-ye^{-xy})i + (-xe^{-xy})j$

$$\bar{\nabla} F(1,-1) = (-(-1)e^{-1(-1)})i + (-1e^{-1(-1)})j$$

$$= (e^1)i + (-e^1)j$$

$$= ei + (-e)j$$

- $|a| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$$u = \frac{a}{|a|} = \left(\frac{-1}{2} i + \frac{\sqrt{3}}{2} j \right)$$

- $D_u f\ (1,-1) = \bar{\nabla} F(-1,1) \cdot u$

$$= (ei + (-e)j) \cdot \left(\frac{-1}{2} i + \frac{\sqrt{3}}{2} j \right)$$

$$= \frac{-e - \sqrt{3}e}{2}$$

$$= \frac{-e(1+\sqrt{3})}{2}$$