Date

I. Aturan Rantai

1). Tentukan $\frac{\partial F}{\partial r}$ dan $\frac{\partial F}{\partial \theta}$ dari fungsi-fungsi berikut.

a. $F(x,y) = 3x^2 - 5y^2$, $x = r\sin 2\theta$; $y = r\cos\theta$

=>

* $F(x,y) = 3x^2 - 5y^2$ → $F_x = 6x$, $F_y = -10y$

* $x = r\sin(2\theta)$
  - $\frac{dx}{dr} = \sin(2\theta)$
  - $\frac{dx}{d\theta} = 2r\cos(2\theta)$

* $y = r\cos\theta$
  - $\frac{dy}{dr} = \cos\theta$
  - $\frac{dy}{d\theta} = -r\sin\theta$

- $\frac{dF}{dr} = F_r = F_x \frac{dx}{dr} + F_y \cdot \frac{dy}{dr}$
  $= 6x \cdot \sin(2\theta) + (-10y) \cdot \cos\theta$
  $= 6x\sin 2\theta - 10y\cos\theta$

- $\frac{dF}{d\theta} = F_\theta = F_x \cdot \frac{dx}{d\theta} + F_y \cdot \frac{dy}{d\theta}$
  $= 6x \cdot 2r\cos(2\theta) + (-10y)(-r\sin\theta)$
  $= 12rx\cos(2\theta) + 10ry\sin\theta$

∴ $F_r(x,y) = 6r\sin 2\theta \cdot \sin 2\theta - 10r\cos\theta \cdot \cos\theta$
$= 6r\sin^2 2\theta - 10r\cos^2\theta$

$F_\theta(x,y) = 12r \cdot r\sin 2\theta\cos 2\theta + 10r \cdot r\cos\theta\sin\theta$
$= 12r^2\sin 2\theta\cos 2\theta + 10r^2\cos\theta\sin\theta$

b. $F(x,y) = 4x^2 + 7y^2$, $x = \sin(r-\theta)$, $y = \cos(\theta - r)$

=>

* $F(x,y) = 4x^2 + 7y^2$ → $F_x = 8x$, $F_y = 14y$

* $x = \sin(r-\theta)$
  - $\frac{dx}{dr} = \cos(r-\theta)$
  - $\frac{dx}{d\theta} = -\cos(r-\theta)$

* $y = \cos(\theta - r)$
  - $\frac{dy}{dr} = \sin(\theta - r)$
  - $\frac{dy}{d\theta} = -\sin(\theta - r)$

Date

- $F_r = F_x \cdot \frac{dx}{dr} + F_y \cdot \frac{dx}{dr}$

$= 8x \cdot \cos(r-\theta) + 14y\sin(\theta - r)$

$= 8\sin(r-\theta)\cos(r-\theta) + 14\cos(\theta-r)\sin(\theta-r)$

- $F_\theta = F_x \cdot \frac{dx}{d\theta} + F_y \cdot \frac{dy}{d\theta}$

$= 8x \cdot (-\cos(r-\theta)) + 14y(-\sin(\theta-r))$

$= -8\sin(r-\theta)\cos(r-\theta) - 14\cos(\theta-r)\sin(\theta-r)$

2). Bila $F(x,y) = x^3y$; $x^5 + y = t$; $x^2+y^2 = t^2$ tentukan $\frac{\partial F}{\partial t}$!

=>

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial t}$$

- $\frac{\partial F}{\partial x} = F_x = 3x^2y$ • $\frac{\partial F}{\partial y} = F_y = x^3$

- $x^5 + y = t$ turunan terhadap t

: $5x^4\frac{\partial x}{\partial t} + \frac{\partial y}{\partial t} = 1 \approx \frac{dy}{dt} = 1 - 5x^4\frac{dx}{dt}$

- $x^2 + y^2 = t^2$ turunan terhadap t

: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2t \approx x\frac{dx}{dt} + y\frac{dy}{dt} = t$

misal $\frac{dy}{dt} = Y$ dan $\frac{dx}{dt} = X$

maka: $Y = 1 - 5x^4 X$

=> $xX + yY = t$

$xX + y.(1-5x^4X) = t$

$xX + y - 5x^4yX = t$

$xX - 5x^4yX = t - y$

$X(x-5x^4y) = t-y$

$$X = \frac{t-y}{x-5x^4y} = \frac{dx}{dt}$$

$$Y = 1 - 5x^4\cdot\frac{t-y}{x-5x^4y} = \frac{dy}{dt}$$

Date

Sehingga

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$= 3x^2y\left(\frac{t-y}{x-5x^4y}\right) + x^3\left(1-5x^4\left(\frac{t-y}{x-5x^4y}\right)\right)$$

3). Tentukan nilai dF/dt dititik $t = \pi/2$, jika $F(x,y) = e^{xy^2}$, $x = t\cos t$, $y = 1/t$

=>

$$xy = t\cos t \cdot \frac{1}{t} = \cos t$$

Shg:

$$F(t) = e^{(\cos t)^2}$$

$$\frac{dF}{dt} F(t) = e^{(\cos t)^2} \cdot 2\cos t \cdot -\sin t$$

$$= -2e^{\cos^2 t} \cos t \sin t$$

$$F_t\left(\frac{\pi}{2}\right) = -2e^{\cos^2\frac{\pi}{2}} \cdot \cos\frac{\pi}{2} \cdot \sin\frac{\pi}{2}$$

$$= -2e^0 \cdot 0 \cdot 1$$

$$= 0$$

4. Jika $F(x,y) = \sqrt{x^2+y^2}$ tunjukkan bahwa $xF_x + yF_y = F$!

=>

$$F_x = \frac{x}{\sqrt{x^2+y^2}} \qquad F_y = \frac{y}{\sqrt{x^2+y^2}}$$

$$xF_x + y.F_y = x.\frac{x}{\sqrt{x^2+y^2}} + y.\frac{y}{\sqrt{x^2+y^2}} = \frac{x^2+y^2}{\sqrt{x^2+y^2}} =$$

$$= \frac{x^2+y^2}{\sqrt{x^2+y^2}} \cdot \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$$

No
Date

$$= \frac{x^2+y^2 \cdot \sqrt{x^2+y^2}}{x^2+y^2} = \sqrt{x^2+y^2} = f$$

5. Jika $z = \ln\sqrt{x^2+y^2}$, tunjukkan $x z_x + y z_x = 1$

$\Rightarrow$

$$z_x = \frac{1}{(x^2+y^2)^{1/2}} \cdot \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x$$

$$= \frac{\cancel{2}x}{\cancel{2}(x^2+y^2)} = \frac{x}{(x^2+y^2)}$$

$$z_y = \frac{1}{(x^2+y^2)^{1/2}} \cdot \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2y$$

$$= \frac{2y}{2(x^2+y^2)} = \frac{y}{(x^2+y^2)}$$

$\Rightarrow x z_x + y z_x = 1$

$$= x \cdot \frac{x}{x^2+y^2} + y \cdot \frac{y}{x^2+y^2} = \frac{x^2+y^2}{x^2+y^2}$$

$$= 1$$

No
Date

II. Turunan Fungsi Implisit

1. Misalkan $z = z(x,y)$, tentukan $\frac{\partial z}{\partial x}$ dan $\frac{\partial z}{\partial y}$ dari fungsi implisit berikut.

   a. $ye^x + 15x - 17z = 0$

   b. $z \sin x + z \cos y + xyz = 0$

   c. $x^2 \cos yz - y^2 \sin xz = 2$

Jawab.

a. $ye^x + 15x - 17z = 0$

$\Rightarrow \frac{d}{dx} ye^x + 15x - 17z = \frac{d}{dx} 0$

$= ye^x + 15 - 17\frac{dz}{dx} = 0$

$= \quad \frac{dz}{dx} = \frac{ye^x + 15}{17}$

$\Rightarrow \frac{d}{dy} ye^x + 15x - 17z = \frac{d}{dy} 0$

$= e^x - 17\frac{dz}{dy} = 0$

$\frac{dz}{dy} = e^x / 17$

b. $z \sin x + z \cos y + xyz = 0$

$\Rightarrow \frac{d}{dx} z \sin x + z \cos y + xyz = \frac{d}{dx} 0$

$= \frac{dz}{dx} \sin x + z \cos x + \frac{dz}{dx} \cos y + yz + xy\frac{dz}{dx} = 0$

$= \frac{dz}{dx} (\sin x + \cos y + xy) + z \cos x + yz = 0$

$$\frac{dz}{dx} = -\frac{z \cos x + yz}{\sin x + \cos y + xy}$$

$\Rightarrow \frac{d}{dy} z \sin x + z \cos y + xyz = \frac{d}{dy} 0$

$= \frac{dz}{dy} \sin x + \frac{dz}{dy} \cos y - \sin y . z + xz + xy\frac{dz}{dy} = 0$

$= \frac{dz}{dy} (\sin x + \cos y + xy) - z \sin y + xz = 0$

$$\frac{dz}{dy} = \frac{z \sin y - xz}{\sin x + \cos y + xy}$$

JOYKO 30 Lines, 6 mm

c. $x^2 \cos(yz) - y^2 \sin(xz) = 2$

$\Rightarrow \frac{d}{dx} x^2 \cos(yz) - y^2 \sin(xz) = \frac{d}{dx} 2$

$= 2x \cos(yz) - x^2 y \sin(yz) \frac{dz}{dx} - y^2 \cos(xz)(z + x \frac{dz}{dx}) = 0$

$= \frac{dz}{dx}(-x^2 y \sin(yz) - xy^2 \cos(xz)) + 2x \cos(yz) - y^2 \cos(xz) z = 0$

$= \frac{dz}{dx} = \frac{2x \cos(yz) - y^2 \cos(xz) z}{x^2 y \sin(yz) + xy^2 \cos(xz)}$

$\Rightarrow \frac{d}{dy} x^2 \cos(yz) - y^2 \sin(xz) = \frac{d}{dy} 2$

$= -x^2 \sin(yz)(z + y \frac{dz}{dy}) - 2y \sin(xz) - y^2 \cos(xz)(x \frac{dz}{dy}) = 0$

$= \frac{dz}{dy}(-x^2 y \sin(yz) - xy^2 \cos(xz)) - 2y \sin(xz) - x^2 z \sin(yz) = 0$

$= \frac{dz}{dy} = \frac{-2y \sin(xz) - x^2 z \sin(yz)}{x^2 y \sin(yz) + xy^2 \cos(xz)}$

Date

7. Tentukan $\frac{dz}{dx}$ dan $\frac{dz}{dy}$ dari fungsi implisit berikut :

a. $ye^{-x} + 5x - 17 = 0$

b. $x \cdot \sin y + y \cos x = 0$

c. $x^2 \cos y - y^2 \sin x = 0$

Jawab.

a. $ye^{-x} + 5x - 17 = 0$

- $\frac{dy}{dx}(ye^{-x} + 5x - 17) = 0$

$$e^{-x}\frac{dy}{dx} - ye^{-x} + 5 = 0$$

$$\frac{dy}{dx} = \frac{ye^{-x} + 5}{e^{-x}}$$

- $\frac{dx}{dy}(ye^{-x} + 5x - 17) = 0$

$$e^{-x} - ye^{-x}\frac{dx}{dy} + 5\frac{dx}{dy} = 0$$

$$\frac{dx}{dy}(5 - ye^{-x}) = -e^{-x}$$

$$\frac{dx}{dy} = \frac{-e^{-x}}{-(ye^{-x} + 5)} = \frac{e^{-x}}{ye^{-x} - 5}$$

b. $x \sin y + y \cos x = 0$

- $\frac{dy}{dx}(\sin y + y \cos x) = 0$

$$\sin y + x \cos y \frac{dy}{dx} + \frac{dy}{dx}\cos x - y \sin x = 0$$

$$\frac{dy}{dx}(x \cos y + \cos x) + \sin y - y \sin x = 0$$

$$\frac{dy}{dx} = \frac{y \sin x - \sin y}{x \cos y + \cos x}$$

No
Date

- $\frac{dx}{dy}(x \sin y + y \cos x) = 0$

$\frac{dx}{dy} \sin y + x \cos y + \cos x - y \sin x \frac{dx}{dy} = 0$

$\frac{dx}{dy}(\sin y - y \sin x) = -\cos x \quad \cos y$

$\frac{dx}{dy} = \frac{-(\cos x + \cos y)}{-(y \sin x - \sin y)}$

$= \frac{\cos x + \cos y}{y \sin x - \sin y}$

3. $x^2 \cos y - y^2 \sin x = 0$

- $\frac{dy}{dx}(x^2 \cos y - y^2 \sin x) = 0$

$2x \cos y - x^2 \sin y \frac{dy}{dx} - 2y \frac{dy}{dx} - y^2 \cos x = 0$

$\frac{dy}{dx}(-x^2 \sin y - 2y) = y^2 \cos x - 2x \cos y$

$\frac{dy}{dx} = \frac{-(2x \cos y - y^2 \cos x)}{-(x^2 \sin y + 2y)}$

$= \frac{2x \cos y - y^2 \cos x}{x^2 \sin y + 2y}$

- $\frac{dx}{dy}(x^2 \cos y - y^2 \sin x) = 0$

$2x \frac{dx}{dy} \cos y - x^2 \sin y - 2y \sin x - y^2 \cos x \frac{dx}{dy} = 0$

$\frac{dx}{dy}(2x \cos y - y^2 \cos x) = x^2 \sin y + 2y \sin x$

$\frac{dx}{dy} = \frac{x^2 \sin y + 2y \sin x}{2x \cos y - y^2 \cos x}$