Andi Suci Khairunnisa --> D121241085 Date

- Gambarkan kurva ketinggian $z = k$ dari

1. $F(x,y) = x^2/y$ , $k = -4, -1, 0, 1, 4$

$\Rightarrow$

$F(x,y) = z = k$

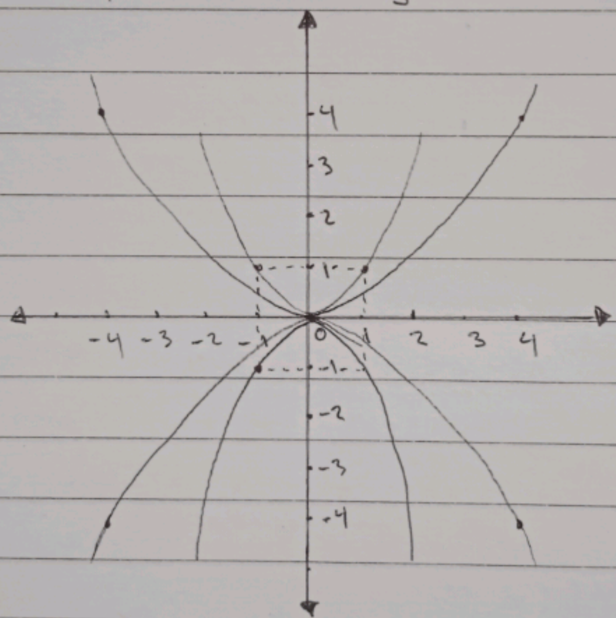
u/ $k = -4$ $\Rightarrow$ $\frac{x^2}{y} = -4$ $\Rightarrow$ $y = \frac{x^2}{-4}$

u/ $k = -1$ $\Rightarrow$ $\frac{x^2}{y} = -1$ $\Rightarrow$ $y = x^2/-1$

u/ $k = 0$ $\Rightarrow$ $\frac{x^2}{y} = 0$ $\Rightarrow$ $y = 0$

u/ $k = 1$ $\Rightarrow$ $\frac{x^2}{y} = 1$ $\Rightarrow$ $y = x^2$

u/ $k = 4$ $\Rightarrow$ $\frac{x^2}{y} = 4$ $\Rightarrow$ $y = x^2/4$




2. $F(x,y) = x^2 + y^2$ , $k = 0, 1, 4, 9$

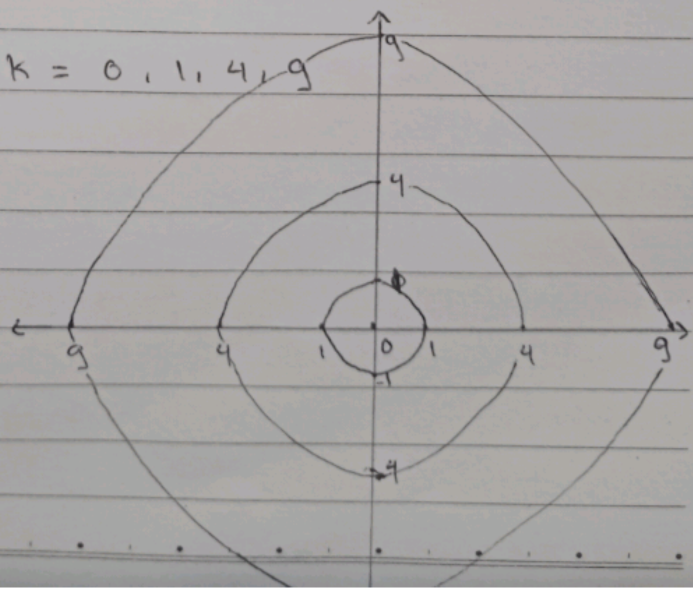
$\Rightarrow$ $F(x,y) = z = k$

u/ $k = 0$ $\Rightarrow$ $x^2 + y^2 = 0$

u/ $k = 1$ $\Rightarrow$ $x^2 + y^2 = 1$

u/ $k = 4$ $\Rightarrow$ $x^2 + y^2 = 4$

u/ $k = 9$ $\Rightarrow$ $x^2 + y^2 = 9$

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3. $F(x,y) = xy$, $k = -4, -1, 0, 1, 4$

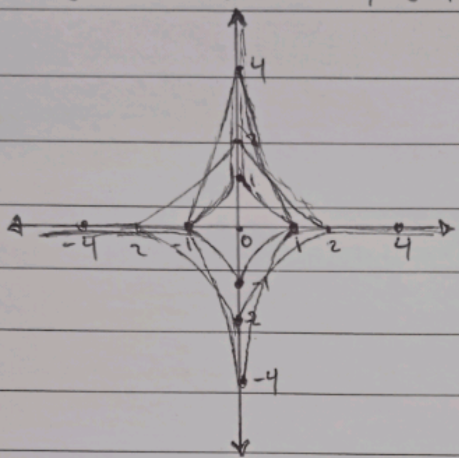
$\Rightarrow$

w/ $k = -4 \Rightarrow (1,-4), (-1,4), (2,-2), (-2,2)$

w/ $k = -1 \Rightarrow (1,-1), (-1,1), (\frac{1}{2},-2), (-\frac{1}{2}, 2)$

w/ $k = 1 \Rightarrow (1,1), (-1,-1), (-\frac{1}{2},-2), (\frac{1}{2},2)$

w/ $k = 4 \Rightarrow (-1,-4), (1,4), (2,2), (-2,-2)$




4. $f(x,y) = y^2 - x^2$, $k = 1,2,3,4$

$\Rightarrow$

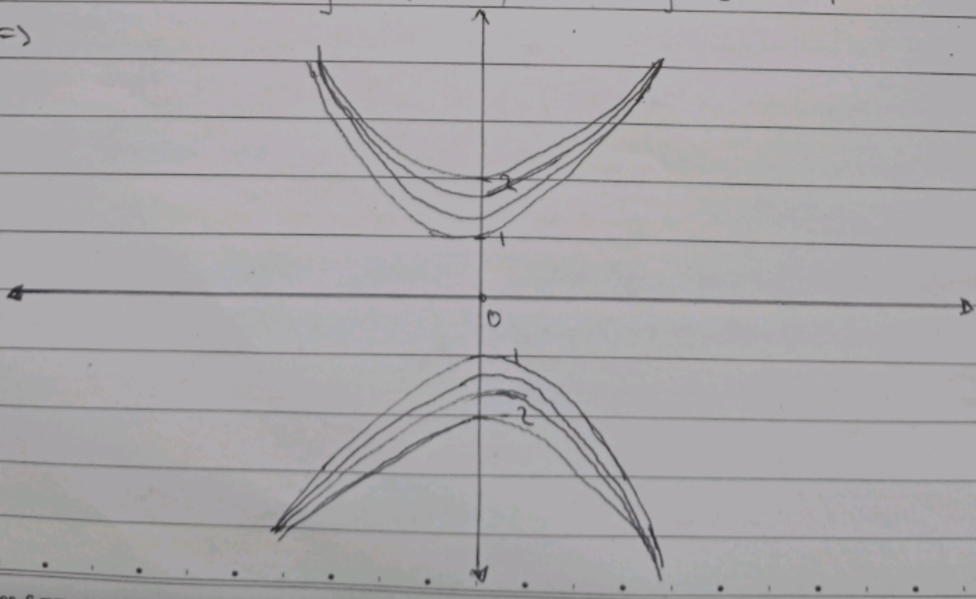
w/ $k = 1 \Rightarrow y^2 - x^2 = 1 \Rightarrow y^2 = x^2 + 1$

w/ $k = 2 \Rightarrow y^2 - x^2 = 2 \Rightarrow y^2 = x^2 + 2$

w/ $k = 3 \Rightarrow y^2 - x^2 = 3 \Rightarrow y^2 = x^2 + 3$

w/ $k = 4 \Rightarrow y^2 - x^2 = 4 \Rightarrow y^2 = x^2 + 4$

$\Rightarrow$



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- Tentukan nilai $\delta > 0$ u/ setiap $\varepsilon > 0$ sehingga

1. $\lim_{(x,y) \to (3,2)} (3x - 4y) = 1$

$\Rightarrow$

$\forall \varepsilon > 0 \quad \exists \delta > 0 \quad \ni |3x - 4y - 1| < \varepsilon$

dimana $0 < \sqrt{(x-3)^2 + (y-2)^2}$

maka :

$|3x - 4y - 1| = |3x - 9 - 4y + 8|$

$\leq |3(x-3) + 4(y-2)|$

karena :

$|x-3| < \sqrt{(x-3)^2 + (y-2)^2}$ dan $|y-2| < \sqrt{(x-3)^2 + (y-2)^2}$

maka

$|3x - 4y - 1| = |3(x-3) - 4(y-2)| \leq 3|x-3| + 4|y-2|$

$< 3\sqrt{(x-3)^2 + (y-2)^2} + 4\sqrt{(x-3)^2 + (y-2)^2}$

$< 3\delta + 4\delta = 7\delta$

Sehingga

$7\delta < \varepsilon$ ambil $\delta = \frac{\varepsilon}{7}$

2. $\lim_{(x,y) \to (2,4)} (5x - 3y) = -2$

$\Rightarrow$

$\forall \varepsilon > 0 \ \exists \ \delta > 0 \quad \ni |5x - 3y + 2| < \varepsilon$

dimana $0 < \sqrt{(x-2)^2 + (y-4)^2}$

maka :

$|5x - 3y + 2| = |5x - 10 - 3y + 12| \leq |5(x-2) - 3(y-4)|$

karena

$|x-2| < \sqrt{(x-2)^2 + (y-4)^2}$ dan $|y-4| < \sqrt{(x-2)^2 + (y-4)^2}$

maka : $|5(x-2) - 3(y-4)| \leq 5|x-2| + 3|y-4|$

$< 5\sqrt{(x-2)^2 + (y-4)^2} + 3\sqrt{(x-2)^2 + (y-4)^2}$

$< 5\delta + 3\delta = 8\delta$

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Sehingga $8\delta < \varepsilon$ ambil $\delta < \frac{\varepsilon}{8}$

3. Buktikan $\lim_{(x,y) \to (1,2)} (3x + 8y) = 19$

$\Rightarrow$

$\forall \delta > 0 \ \exists \ \varepsilon > 0 \ \ni \ |3x + 8y - 19| < \varepsilon$

dimana $0 < \sqrt{(x-1)^2 + (y-2)^2} < \delta$

maka :

$|3x + 8y - 19| = |3x - 3 + 8y - 16| \leq |3(x-1) + 8(y-2)|$

karena

$|x-1| < \sqrt{(x-1)^2 + (y-2)^2}$ dan $|y-2| < \sqrt{(x-1)^2 + (y-2)^2}$

maka :

$$|3(x-1) + 8(y-2)| \leq 3|x-1| + 8|y-2|$$
$$< 3\sqrt{(x-1)^2 + (y-2)^2} + 8\sqrt{(x-1)^2 + (y-2)^2}$$
$$< 3\delta + 8\delta = 11\delta$$

sehingga :

$11\delta < \varepsilon$ ambil $\delta < \frac{\varepsilon}{11}$

Diperoleh :

$$|3x + 8y - 19| \leq 3|x-1| + 8|y-2|$$
$$< 3\delta + 8\delta$$
$$< 11\delta$$
$$< 11\left(\frac{\varepsilon}{11}\right) = \varepsilon$$

Terbukti bahwa

$\lim_{(x,y) \to (1,2)} (3x + 8y) = 19$

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- Selidiki apakah fungsi di bawah limitnya ada, jika iya carilah nilai limit untuk $(x,y) \to (0,0)$

1. $f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$

   - pendekatan sepanjang sumbu x $(y=0)$

     $f(x,0) = \frac{x^2 - 0}{x^2 + 0} = 1$

   - pendekatan sepanjang sumbu y $(x=0)$

     $f(0,y) = \frac{0 - y^2}{0 + y^2} = \frac{-y^2}{y^2} = -1$

   hasil limit dari 2 arah berbeda, limit tidak ada

2. $f(x,y) = \frac{x^2}{x^2 - y}$

   - Jika $x = 0$

     $f(0,y) = \frac{0}{0 - y} = 0$

   - Jika $y = 0$

     $f(x,0) = \frac{0}{0 - y} = 0$

   - Jika $y = x^2$

     $f(x,x^2) = \frac{x^2}{x^2 - x^2} = \frac{x^2}{0} = \sim$

   karena ada jalur limit tidak terdefinisi, maka limit tidak ada