

Poisson Process

Introduction

The Poisson process is a fundamental concept in probability theory and statistics. This process is named after the French mathematician Siméon Denis Poisson, who developed it in the 19th century.

The Poisson process is widely used in science to model and mathematically describe randomly and independently occurring events.

One of its most common applications is in modeling event rates, such as the number of calls received by a call center in an hour or the number of traffic accidents in a given time interval.

The Poisson process is particularly useful in situations where events are rare and independent, making it difficult to use other, more complex statistical methods.

Why the Poisson Process is important

The Poisson process is important because it provides a rigorous mathematical framework for understanding random and rare events. This process helps estimate the probability of future events, which is critical in many practical decisions.

Its simplicity and versatility make it an indispensable tool for analysts and researchers studying data relating to random events.

Formula

The formula associated with the Poisson process is as follows:

$$P(N(t) = n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

Where:

- $P(N(t) = n)$ represents the probability that there will be exactly n events in the time period t .
- n is the number of events you want to count.
- t is the length of the time interval considered.
- λ represents the average number of events that occur in a unit time interval (often called "**rate**").
- e is Euler's constant, which is approximately equal to 2.71828.

The formula is based on the idea that events occur randomly and independently over time.

The formula tells us how likely it is that n events will occur in a given time interval t when the average of those events is given by λt .

The presence of $e^{-\lambda t}$ in the denominator takes into account the fact that, as n increases, the probability of having exactly n events decrease rapidly.

The Poisson process and the Poisson distribution

“Poisson process” and “Poisson distribution” are two closely related concepts.

Both are used to model random or rare events, but they apply to different contexts and provide different information.

Poisson process:

The Poisson process is a model that describes the time sequence of random events that occur independently and at a constant rate.

In other words, the Poisson process focuses on describing the arrival of events in time or space. This process is characterized by a parameter λ which represents the average arrival rate of events in an interval of time or space.

For example, it could be used to model the number of phone calls in a call center in an hour.

Poisson distribution:

The Poisson distribution, on the other hand, is a discrete probability distribution that describes the number of events that occur in a time interval, given an average event rate λ .

The Poisson distribution is used to calculate the probability of observing a specific number of events in a given time interval in a Poisson process.

So the Poisson process is a model that describes the arrival sequence of events over time, while the Poisson distribution is a probability distribution used to calculate the probabilities relating to the number of events in a time interval given an average rate.

Applications in Cybersecurity

The Poisson process and Poisson distribution can find applications in the field of cybersecurity in different scenarios.

Cyber attack detection:

Attempts to gain unauthorized access to a system or intrusion attempts can be modeled using a Poisson process. For example, if you track the number of unauthorized access attempts on a computer system over time, you can use the Poisson distribution to evaluate how likely a certain number of attempts are to occur over a given period.

Software vulnerability analysis:

In the field of cybersecurity, it is important to evaluate and mitigate software vulnerabilities. Vulnerability analysis often involves counting errors or vulnerabilities found in an application. The Poisson distribution can be used to model the number of vulnerabilities detected in a certain period, which can help set security objectives and plan resources for remediation activities.

Detection of suspicious activity in networks:

In network monitoring, Poisson process-based techniques can be used to identify suspicious activity. For example, data flow in a network can be modeled as a Poisson process, and detection of significant deviations from this model can indicate unauthorized activity or security threats.

Analysis of network traffic and event logs:

Monitoring network traffic and analyzing event logs are important cybersecurity tasks. Poisson distributions can be used to evaluate the frequency of certain events, such as the number of login attempts or incoming data traffic, to identify anomalies or suspicious behavior.

The Poisson process and Poisson distribution can be valuable tools for modeling and analyzing rare and random events in the context of cybersecurity, allowing organizations to detect threats, plan security strategies, and make informed decisions to protect their systems and data.

Simulation

The following simulation was calculated using the program written for homework 5.

In this case I used the following values for the variables:

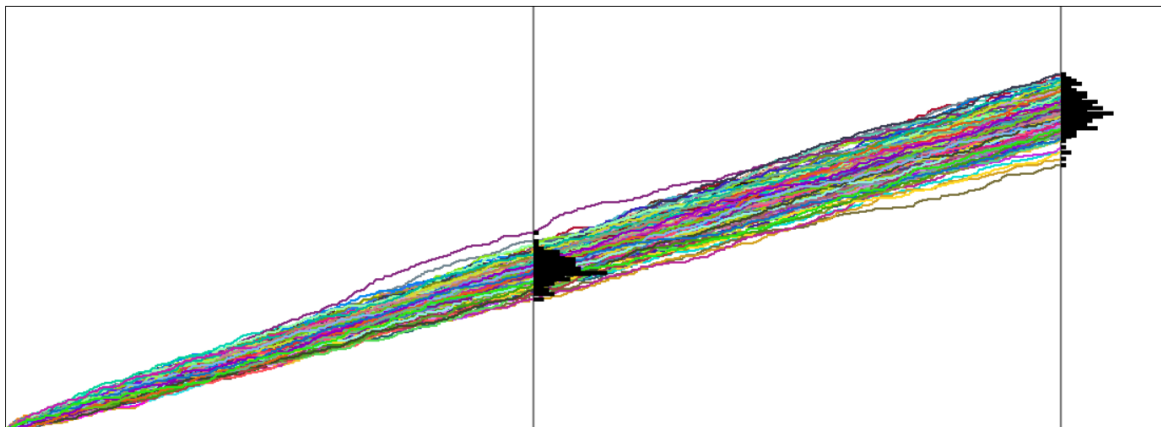
- The simulation was carried out on 200 different systems
- the simulation time is 1000 days
- divided into 1400 intervals
- each interval is therefore long $\frac{1000}{1400}$
- in each interval there are on average 400 penetrations

Numero di Sistemi:

T:

N:

λ :



References:

Poisson process - *Wikipedia*

https://en.wikipedia.org/wiki/Poisson_point_process

Poisson distribution - *Wikipedia*

https://en.wikipedia.org/wiki/Poisson_distribution

Poisson process vs Poisson distribution - *Builton*

<https://builton.com/data-science/poisson-process>