Random variate generation

A random variate refers to a single value obtained from a random variable.

In probability theory and statistics, a random variable is a variable whose possible values are the result of a random phenomenon.

When a specific value is obtained from this random variable through a process such as sampling or simulation, that particular value is called random variates.

Random variate generation is the process of generating values that follow a specific probability distribution such as the normal distribution, uniform distribution or the exponential distribution.

Random variate generation plays a crucial role in various domains, allowing the emulation of complex systems, statistical analysis, and computational modeling by producing values that follow a specific probability distribution.

Algorithms for random variates generation

Generating random variables involves using various techniques to produce values that emulate specific probability distributions.

Inverse Transform sampling

Inverse transformation sampling is a fundamental method used to generate random variations from a given probability distribution function (PDF) by exploiting the inverse of its cumulative distribution function (CDF). This method is particularly useful when the CDF of the distribution is known and can be inverted analytically or numerically.

The core principle of Inverse Transform sampling is based on the probability integral transform, which states that if X is a continuous random variable with a continuous cumulative distribution function $F_X(x)$, then $F_X(X)$ has a uniform distribution on the interval **[0,1]**. This forms the basis for generating random variates from other distributions.

The process involves three steps:

- **Computing the inverse CDF**
  - Given a probability distribution function $f(x)$ and its cumulative distribution function $F(x)$ the first step is to find the inverse of $F(x)$, denoted as $F^{-1}(y)$ where y is a uniform random variate in the range [0,1].

- **Generating Uniform Random Variates:**
  - Generate U, a random number uniformly distributed on the interval [0,1].

- **Inverse Transformation:**
  - Apply the inverse CDF function $F^{-1}(U)$ to the generated uniform variate U to obtain the corresponding random variate from the desired distribution. This transformed value will follow the specified distribution.

Rejection sampling

Rejection sampling is a versatile method used to generate random variates from a target probability distribution, particularly when it's hard or impossible to directly sample from the distribution. This technique is valuable for distributions without an easily invertible cumulative distribution function (CDF) or for non-standard distributions.

The concept behind rejection sampling involves generating random samples from a simpler "proposal" distribution, which entirely encloses the target distribution.

- **Selection of an Envelope Distribution:**
  - Choose a simpler and known distribution (usually a majorizing distribution) that completely envelops the target distribution. The envelope distribution should be easy to sample from and should cover the entire range of the target distribution.

- **Sampling from the Envelope Distribution:**
  - Generate random variates from the envelope distribution.

- **Acceptance-Rejection Criterion:**
  - For each generated sample from the envelope distribution, accept or reject it based on whether it falls beneath the target distribution curve.

- **Generation of Random Variates:**
  - Retain the accepted samples from the envelope distribution that also satisfy the acceptance criteria to approximate samples from the target distribution.

The acceptance-rejection criterion is determined by comparing the ratio of the target distribution's probability density function (PDF) to the envelope distribution's PDF at the sampled point with a uniformly generated random number. If the random number is less than this ratio, the sample is accepted; otherwise, it's rejected.

Box Muller transform

The Box-Muller transformation is a method used to generate random samples from a standard normal distribution (mean = 0, variance = 1) using uniformly distributed random numbers. It provides a straightforward and efficient way to produce pairs of independent, normally distributed random variables.

The process involves the following steps:

- **Generation of uniform random numbers:**
  - Generate two independent random numbers U_1 and U_2 that are uniformly distributed in the range [0, 1].

- **Transformation formula:**
  - Using the Box-Muller transformation, these uniform random numbers (U_1 and U_2) are converted into two independent standard normal random variables (Z_1 and Z_2)
  - $Z_1 = \sqrt{-2\,ln(U_1)} \; * \; cos(2\pi\, U_2)$
  - $Z_2 = \sqrt{-2\,ln(U_1)} \; * \; sin(2\pi\, U_2)$

- **Obtaining normal variates:**
  - The resulting Z_1 and Z_2 pairs are independent and identically distributed as standard normal random variables.

The Box-Muller transformation generates normally distributed random variates by exploiting the properties of the standard normal distribution and trigonometric functions based on uniformly distributed random numbers.

Each method offers unique strengths and limitations, and the choice of method depends on the characteristics of the target distribution, computational efficiency, and the specific requirements of the application.

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