Hamiltonian Learning with Unitary Dynamics

Andi Gu[1] Gerwin Koolstra[1]

[1]Quantum Nanoelectronics Laboratory, University of California, Berkeley

Hamiltonian Learning

Quantum systems are characterized by an operator known as the Hamiltonian H, which corresponds to the total energy of the system.

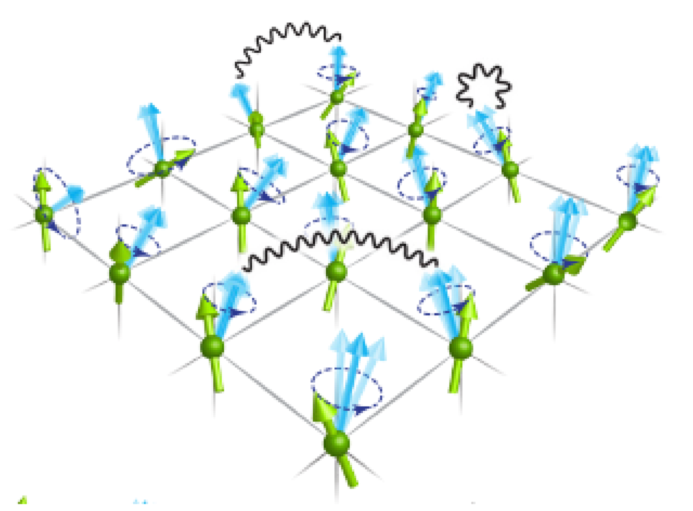

Figure 1. Interacting many body quantum system

We aim to learn the Hamiltonian of a black-box quantum system using very **few resources**, making as **few assumptions** as possible about the nature of the system.

Problem Formulation

We expect our system is governed by a Hamiltonian that can be written as a linear combination of terms that act on no more than k sites:

$$H = \sum_{\ell=1}^{K} c_\ell P_\ell, \quad P_\ell \in \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}^{\otimes k}$$

Our goal is to use the **unitary dynamics** of this system to infer the coefficients $\{c_\ell\}$ of the Hamiltonian. We can control:

- The initial state of the system ψ_0
- The time length it is evolved for t
- Measurement of some observable O after time evolution

With this, we aim to infer H given access to

$$\langle \psi_0 | e^{iHt} O e^{-iHt} | \psi_0 \rangle$$

Methods

The Heisenberg Picture

The following operator identity ('Taylor expansion') tells us:

$$O(t) \equiv \left\langle e^{iHt} O e^{-iHt} \right\rangle = \langle O \rangle + \boldsymbol{it} \, \langle [\boldsymbol{H}, \boldsymbol{O}] \rangle + \frac{(it)^2}{2} \langle [H, [H, O]] \rangle + \mathcal{O}\left(t^3\right) \tag{1}$$

By setting O and ψ_0 correctly, $\langle [H, O] \rangle$ can directly tell us one of the $\{c_\ell\}$. Our entire goal, then, is to accurately infer $\langle [H, O] \rangle$ given access only to noisy estimates of $O(t)$.

Intuition

Technically, we can access $\langle [H, O] \rangle$ in a straightforward manner, since $O'(0) = i \langle [H, O] \rangle$. But, measurements of $O(t)$ are **noisy**, so derivative estimates will be very noisy. We can do better!

Polynomial Regression

The key observation is: $O(t)$ is a polynomial in t, with the coefficients being $\langle [H, \ldots, [H, O]] \rangle$. Therefore, we measure $O(t)$ for appropriately chosen evolution times $\{t_i\}$, and fit a polynomial to $O(t)$.

Theoretical Results

The error of the polynomial regression method scales **polynomially** in the locality k and **slightly worse than noise-limited** in the number of measurements required $\mathcal{O}\left(N^{-(2+\alpha)}\right)$, where $\alpha > 0$. The origin of α is that we can only fit for finitely many polynomial coefficients in Equation (1), so the remaining coefficients induce a bias in our estimator.

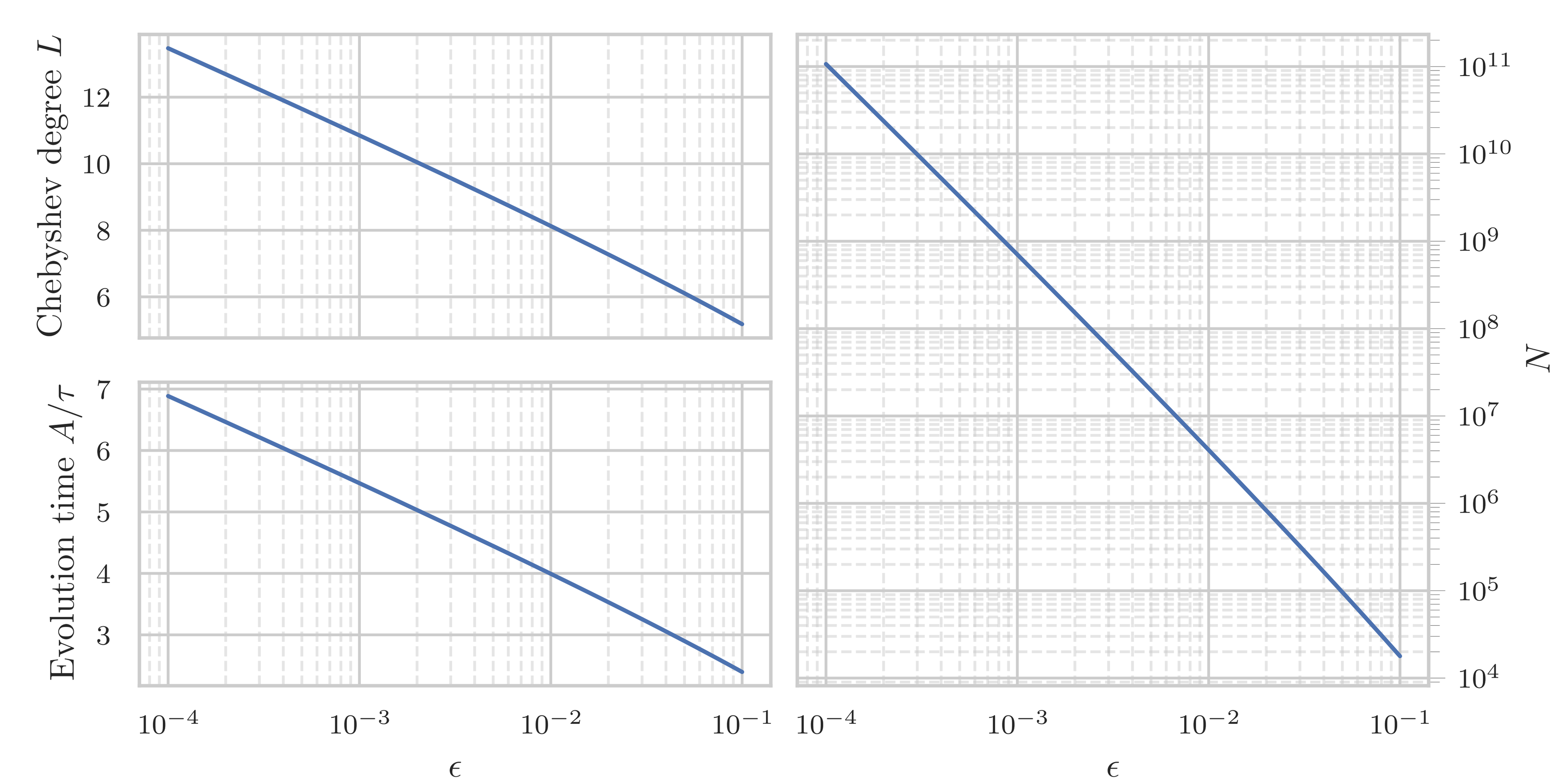

Figure 2. Error bound

Numerical Results

We test our method on a well-understood system: the transverse field ising model:

$$H = \sum_{i=1}^{n-1} J_i \sigma_z^{(i)} \otimes \sigma_z^{(i+1)} + \sum_{i=1}^{n} B_i \sigma_x^{(i)} \tag{2}$$

We find that our theoretical results are in places a very good estimate of the error, and in others, a dramatic overestimate.

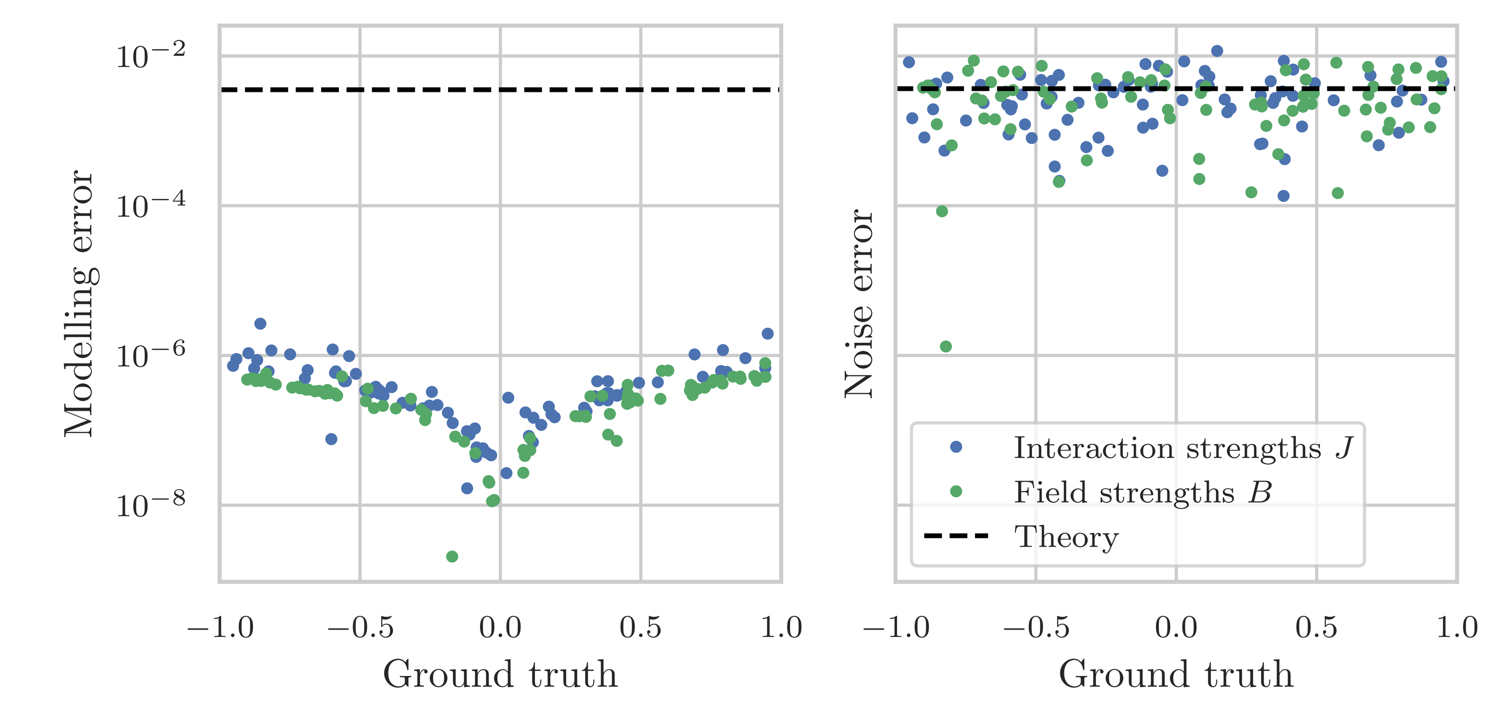

Figure 3. Coefficient recovery errors

Insights

The polynomial regression method for Hamiltonian learning offers an insight into the nature of the problem.

- Careful choices of initial state ψ_0 and observable O suffice to make Hamiltonian learning efficient
- Using classical resources, we need to rely on small time evolution: long time evolution introduces too much chaos

Further Work

Our theoretical error estimates are in some places tight, and in others, dramatic overestimates. A crucial next step is to investigate why this bound is so loose, and how to improve it.