Chapter 5

Portfolio Construction Notes

When designing optimisation, plan your comparison between the mean-variance minimisation (Markowitz portfolio) and at least one alternative index of satisfaction, which could be Max Sharpe Ratio, Tracking Error, Min VaR kinds of optimisation. The second dimension to comparison can vary: comparison among varied levels risk aversion for a simple Min Var, comparison between naive sample covariance and robust shrunk covariance for Max Sharpe Ratio. These comparisons ask for themselves and more can be designed.

5.1 Portfolio Choice and Data

The main principle is to come up with a set of assets which promises an optimal diversification (in accepted sense, or according to quantitative criterion). There is analytical and computational work to be done at the stage of portfolio choice. The extent of the work is up to you: you can apply commonly accepted stylised facts (e.g, equity vs. bonds), compute correlation matrices, or go in depth and analyse the rolling beta for each asset *wrt* the market or specific factor (factor tilts). While you are not limited to Options A,B,C below it will be helpful to indicate in your project report which is the closest option to the approach you took.

Option A. Identify 15-20 assets from a large market and check their correlation over consistent past period (that gives a large sample but not interrupted by a major crisis/regime change). Narrow down your choice to < 10 'optimal bets'.

Scenario: a set of large caps from S&P500 is strongly exposed to the market factor without benefit either diversification or concentrated, event-based bet (in other words it is a concentrated bet on large cap, without shoring of large-cap under-performers).

Option B. Identify a large liquid ETF or index for a specialised portfolio that focuses on an industry, emerging market, fixed income assets. Those assets will be correlated but that was your purpose – to get exposure but stay diversified within that market segment by using 5-10 names.

Scenario: for the Option B portfolio, you need to implant 1-3 exogenous, most uncorrelated assets, such as short commodity or long VIX. This is not exactly a hedging exercise but helps to understand what would happen to your portfolio if other markets experience turbulence or even regime change.

Option C. Similarly to a specialised portfolio Option B. you will choose a segment of market but based on asset factor tilts (betas against factor returns). The factors (momentum, small cap, betting against beta) themselves are constructed by going long top X stocks/ short bottom X stocks. Such long/short factor portfolios are regularly rebalanced, which constitutes time diversification. Returns P&L from a factor portfolio is supposed to grow (not mean-revert).

Scenario: factor portfolio are not amenable to imposition of views in Black-Litterman style, in fact, long top X/short bottom X is already a relative view. Other kinds of optimisation that do not take expected returns into computation: the ERC (*aka* naive risk parity), Risk Budget, the MDP.

You can compute Risk Contributions on top of any kind of optimisation mean-variance, Max SR – the contributions just won't be equal. It possible to create a version of each option mirroring into Risk Budgeting portfolio, particularly when then the investor have no implicit discretion to enter concentrated bets/Kelly Criterion type allocation.

- It is possible to make a project out of factor replication and/or *factor construction* – you can look into the methodology of how Fama-French factors computed by themselves and replicated by others. You will need the numerical metric for going long/short, rebalancing scheme, and produce factor time series. However, pure factor portfolios are domain of academic study that investigates an underlying structure of the market.

- *Replication of a large index* (e.g., country) solves the portfolio choice problems 'in one go' – you know which data you need, and you have benchmark weights for your BL prior. It follows you will be considering 50+, 250+ assets and the issue of diversification require explicit treatment – re-invent the index by computing Most Diversified Portfolio (MDP) optimisation, or Risk-Budgeted Portfolio optimisation.[1] It is also logical to implement *Tracking Error* kind of optimisation for this set up.

- Tactical Allocation is a common trendy approach where asset choice is driven by 1-2 macro variables (economic data plus beliefs of fund managers).[2] That gives a fruitful opportunity to formalise Tactical Allocation in terms of Black-Litterman views. Also, GTA advisors prefer to use regression on economic variables to make the prediction. That can be further developed and 'automated' by constructing ML pipelines from preferred features (to economic data) and pre-configured classifiers.

[1] Index-following can be improved by introducing *optimal bets maximisation* which seeks fewer assets that deliver risk-return profile similar to the larger set of assets.
The advantage of this all-market portfolio investigation and choice is that you depend less on the estimates of individual asset returns.

[2] `markovprocesses.com/blog/2017/02/parsing-the-dynamics-of-global-tactical-asset-allocation-gtaa-funds/`

Equilibrium returns $\boldsymbol{\pi}$ for the Black-Litterman prior reflect the allocation by 'all global investors' on average. That is an idea, but it is difficult to have a good information. So any reasonable approximation is suitable. The common choice is market weights from a global-enough benchmark. For example, the market caps of S&P 500 vs Barclays Capital U.S. Aggregate Bond Index give a good idea about allocation into equity vs bonds for the US and global investors.

- ETFs typically have benchmarks, against which they measure performance or even compute a tracking error. AUM values can somewhat differ reflecting funds in the process of of investment/divestment.
- Forming a portfolio from several large-cap ETFs and use: fund market cap/sum of market caps to produce 'benchmark allocation'. However, please let the index providers do the job for you and search for a benchmark, a market index that covers your portfolio, form which you can borrow weights.

5.2 Advanced Topics in Portfolio Science

5.2.1 Index of Satisfaction

We obtain optimal allocations by solving variance minimisation

$$\text{Portfolio Mean} - \lambda \times \text{Portfolio Variance}$$

But that is only one method to define **a trade-off between two moments:** expected return $\mathbb{E}[X_w]$ and its standard deviation $\text{SD}[X_w]$.

We can construct other trade-offs, the most common is

$$SR_w = \frac{\mathbb{E}[X_w]}{\text{SD}[X_w]}$$

BL used the Sharpe Ratio 0.5 for the broad market index. Hedge funds aim for strategies to have the SR > 1.

5.2.2 Tracking Error

Portfolio relative **tracking error** is difference in return (over each period) as compared to the benchmark, usually an MSCI index for the relevant market. The error can also be estimated as a quadratic mean over the period.

The alternative name for the tracking error is **annualized tracking risk**. So it is measured as the standard deviation of **active returns**

$$a_i = r_i - b_i$$

which is the difference between the portfolio *monthly* returns r_i and benchmark monthly returns b_i, measured over the past 120 months.

$$\mathrm{TR}_j(nY) = \sqrt{\frac{12}{12n-1}\sum_{i=1}^{12n}(a_{i,j} - \langle a_{i,j}\rangle)^2}$$

where $\langle a_{i,j}\rangle$ is the *inner product* (scalar product) represented by *SUMPRODUCT()* function in Excel and $j = 12n, \ldots, N$. The expression sets up sampling of annualised tracking risk from the set of observations for 120 months.

Optimal allocations can be obtained to minimise the tracking error.

5.2.3 Quantitative and Optimal Diversification

There are three papers by Attilio Meucci et al. to help you get quantitative grasp: Managing Diversification (2010), Review of Linear Factor Models (2010) and Factors on Demand (2010).

Effective Number of Bets – the reference technical is Risk Budgeting and Diversification Based on Optimized Uncorrelated Factors `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2276632`, also download Matlab code at `https://uk.mathworks.com/matlabcentral/fileexchange/43245-portfolio-diversi-cation-based-on-optimized-uncorrelated-factors`.

Topics below generalise Black-Litterman approach – or go beyond it towards the roots and more fundamental techniques. The computation of posterior (returns and standard error) within the BL is a simplified case of more general entropy pooling.

- **Entropy Pooling** with Flexible Views – replaces mean-variance optimisation. Particularly important is correlation stress-testing.
- **Flexible Probabilities** (particularly, crisp conditioning) is a technique to apply filters on historical data. We do apply EMWA/GARCH filter to prioritise recent input in estimation of volatility and similar schemes can be applied to returns themselves.

5.2.4 For Analysis and Discussion

Compare your allocations to market benchmark weights – check if being over/underweight in a particular asset is a desired result, eg, impact of the view. Being over/underweight compared to benchmark is also called 'Active Risk'. Example of comparison and analysis is Table 6 from the Guide to Black-Litterman Model by Thomas Idzorek.

Check for the common pitfalls such as 'corner solutions' result of mean-variance optimisation. Discuss concentrated positions, particularly in high volatility assets and are you shorting low volatility assets (betting against bet)? Identify common themes that come across different kinds of optimisation, and that perhaps were resolved by devising Risk Budget portfolio.

Of separate interest is impact of covariance matrix on optimisation, and your work on covariance shrinkage – please see updated Workshop presentation.