Efficiently Learning Fourier Sparse Set Functions

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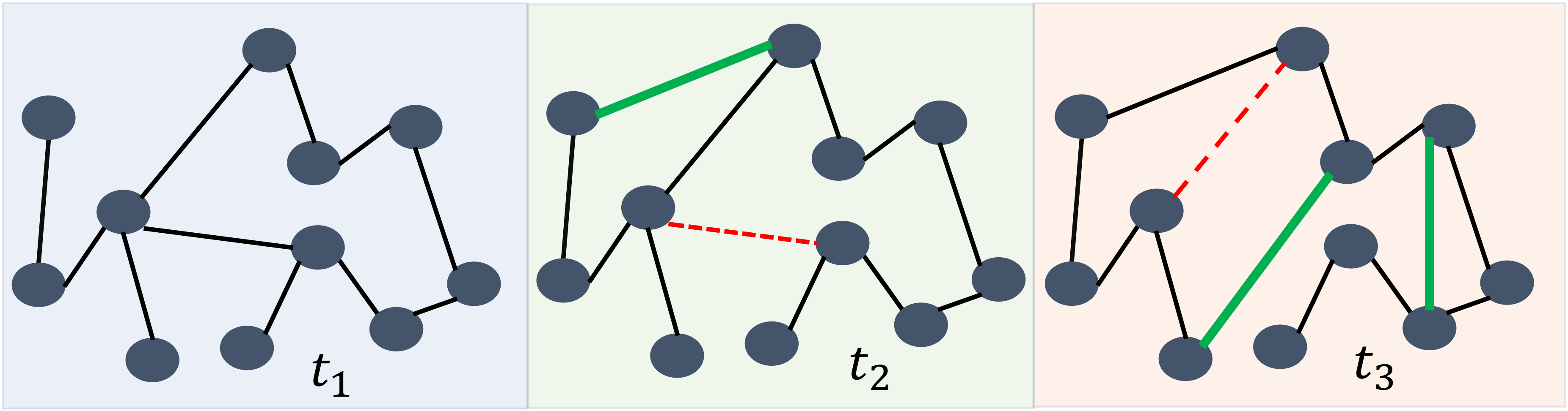
* The first two authors contributed equally

Motivation – sketching graphs



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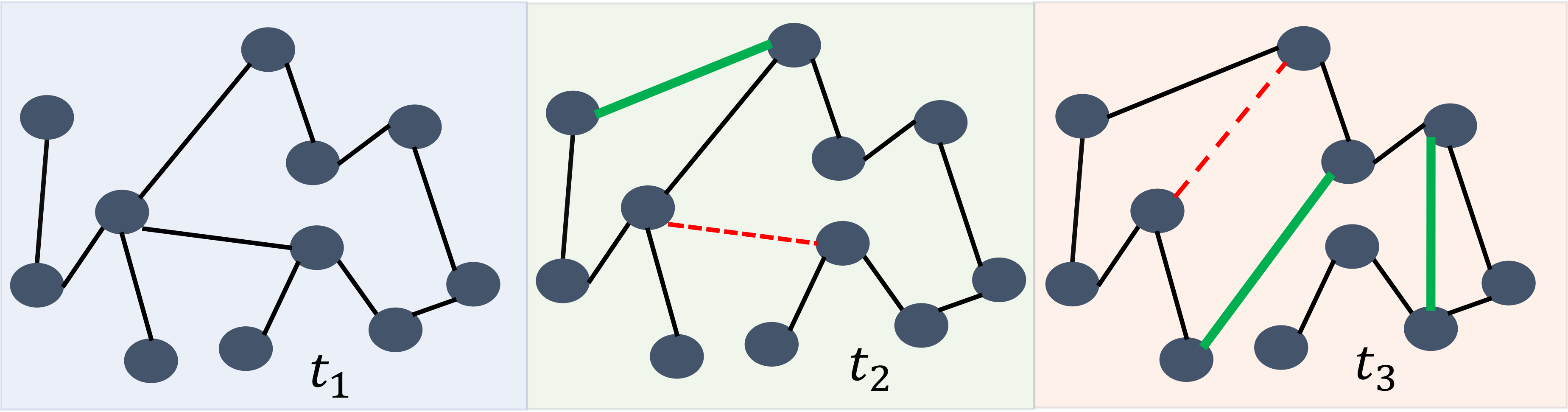
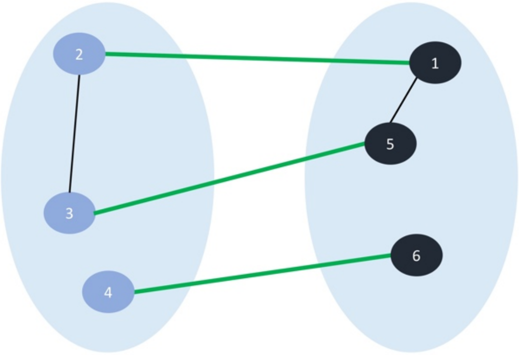
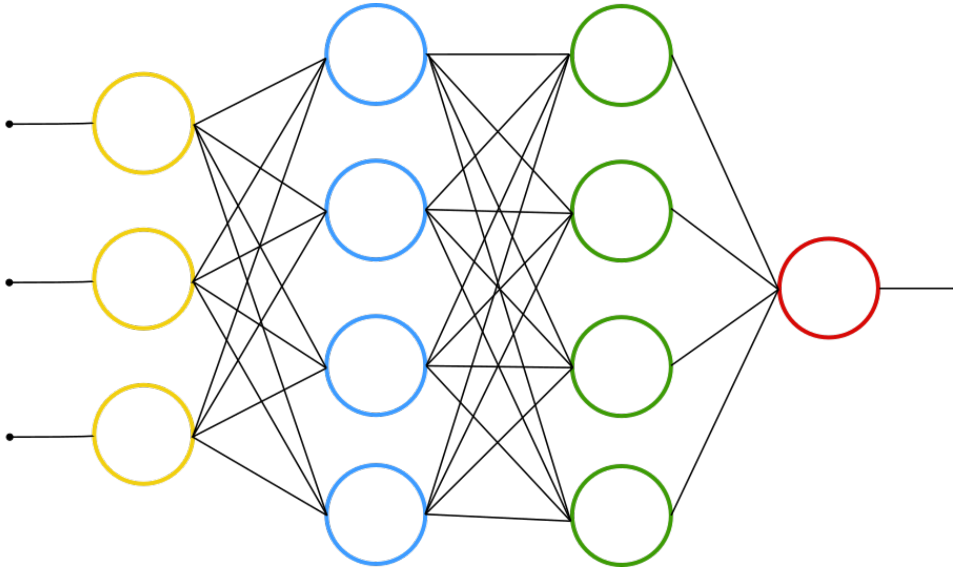




$A = \{2, 3, 4\}$ $\quad$ $V \setminus A = \{1, 5, 6\}$

$$\underbrace{F(A)}_{\text{Size of the cut between A and V \ A}} = 3$$

Motivation – hyperparameter optimization



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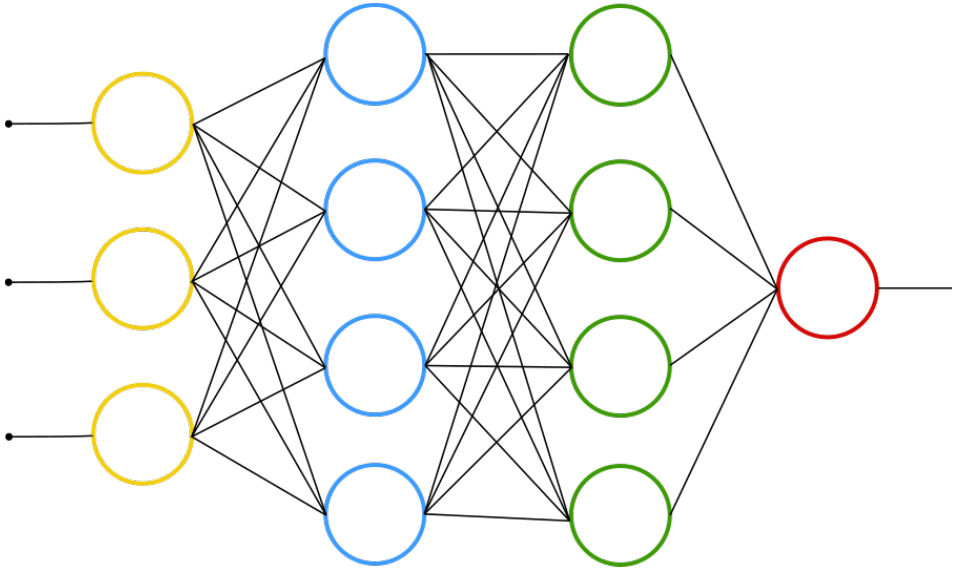

$F(x_1\, x_2\, x_3\, x_4\, x_5)$ = Validation error using hyperparameters **x**

$$x_1 = \begin{cases} 0 \; if \; optimizer \; is \; ADAM \\ 1 \; if \; optimizer \; is \; SGD \end{cases}$$

$$x_2 = \begin{cases} 0 \; if \; filtersize = 3x3 \\ 1 \; if \; filtersize = 5x5 \end{cases}$$

…

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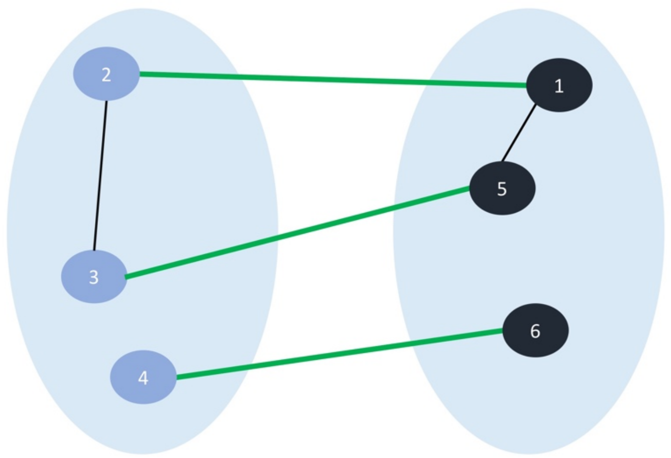

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$k = |E| + 1 = 6$ $d = 2$

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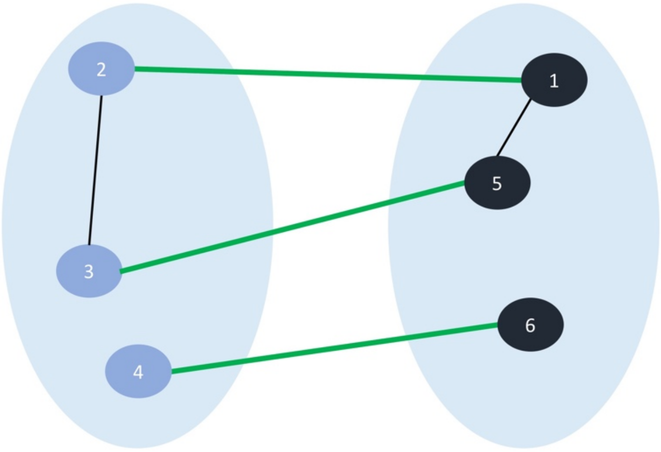

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$$F(x_1\, x_2\, x_3\, x_4\, x_5) =$$

$$F_1(x_1) + F_2(\underbrace{x_3\, x_4}_{d=2}) + F_3(x_2\, x_5)$$

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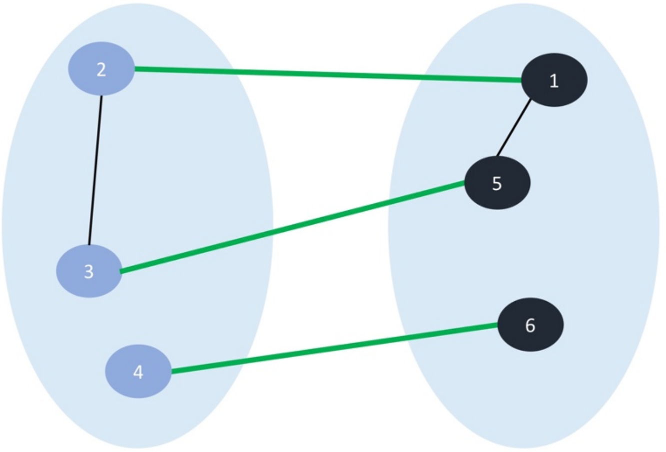

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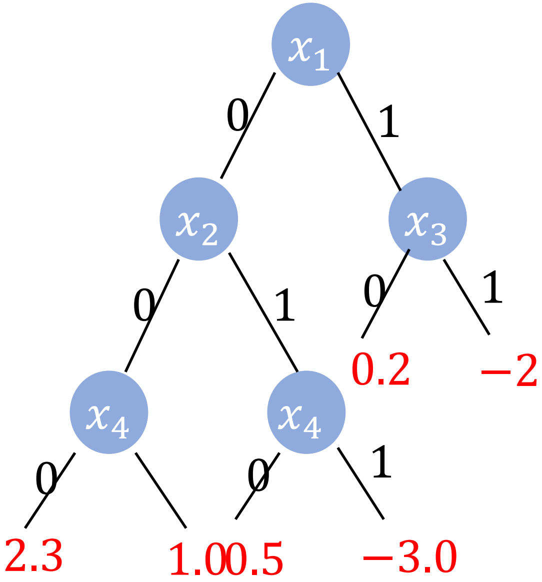

$d = 3$

Motivation – Learning set functions

Approximate Fourier transform of

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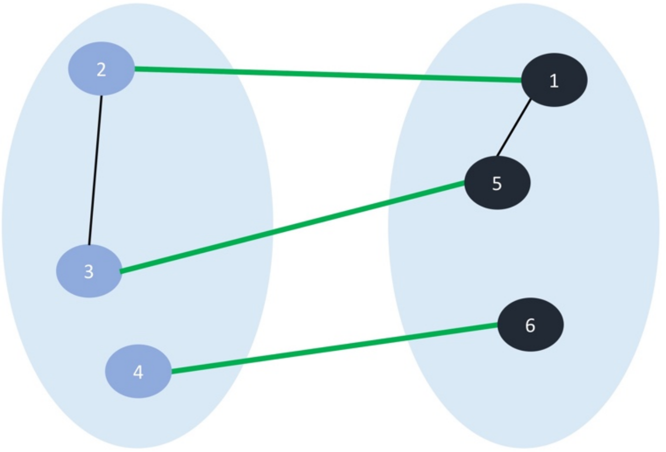

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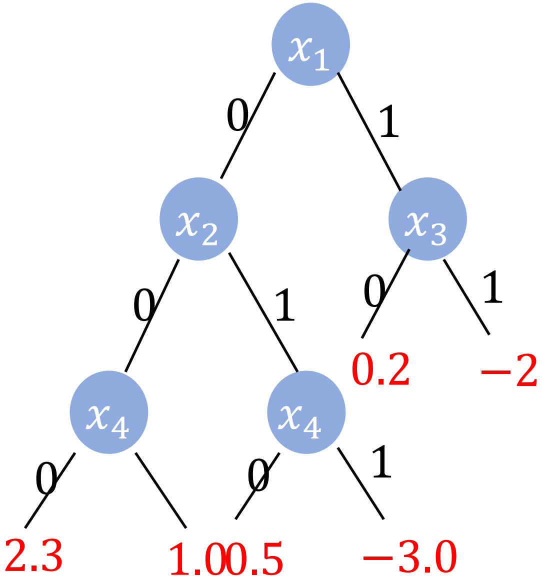

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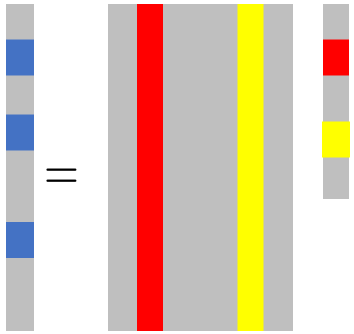
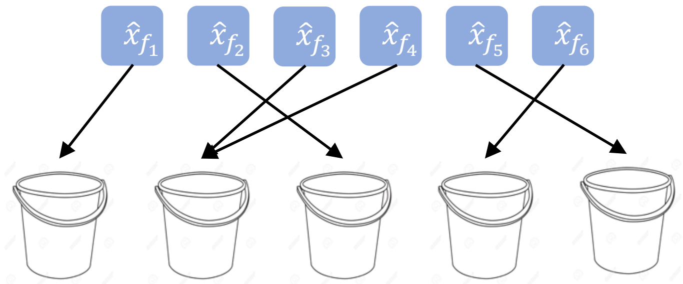
Previous work and our contributions

	Compressive sensing	Sparse FFT
Runtime		
Sampling complexity		
Assumptions		
Robustness		





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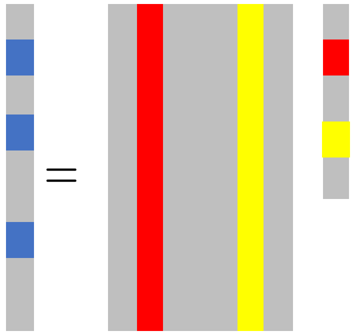

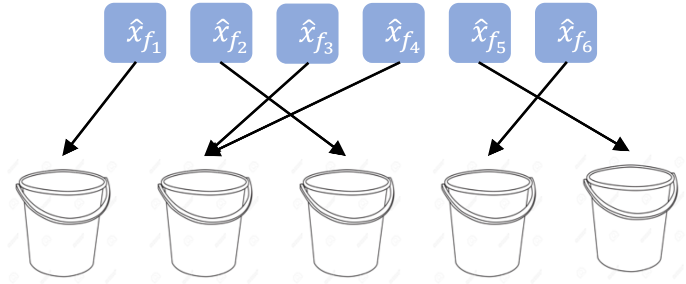

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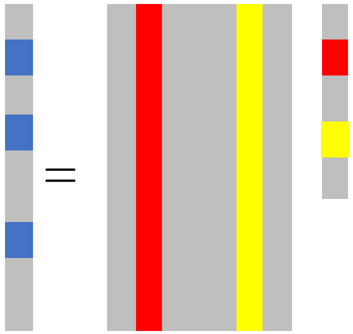

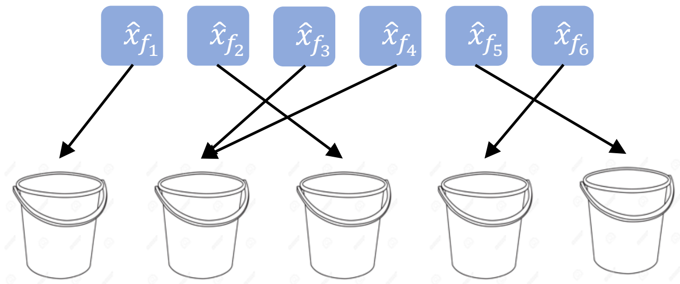

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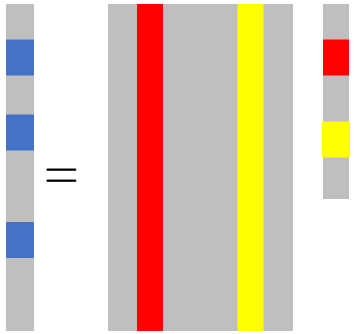
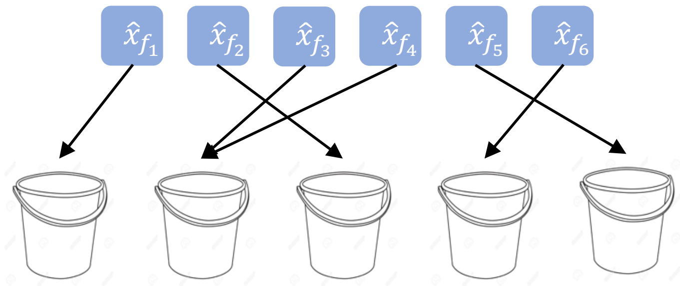
Assumptions	None	Randomness of support
Robustness	Worst case noise	Gaussian noise + …





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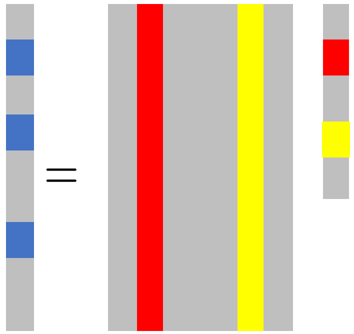
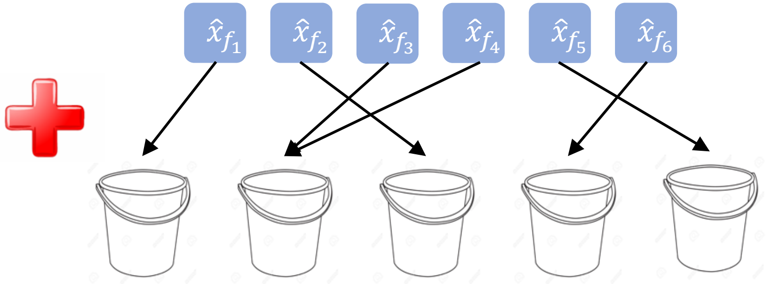
Compressive sensing over finite fields





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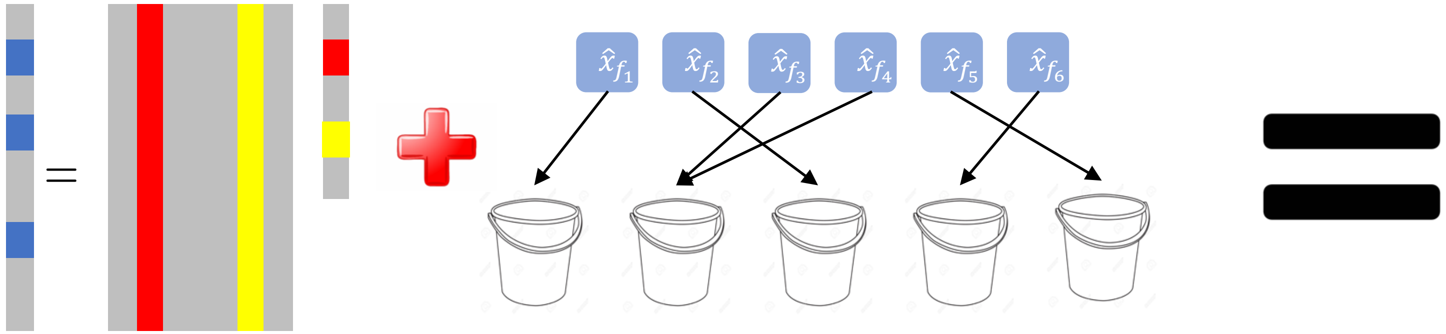
Sampling complexity	$\tilde{O}(kd)$	$\tilde{O}(kn)$	$\tilde{O}(kd)$
Assumptions	None	Randomness of support	None
Robustness	Worst case noise	Gaussian noise + …	Worst case noise

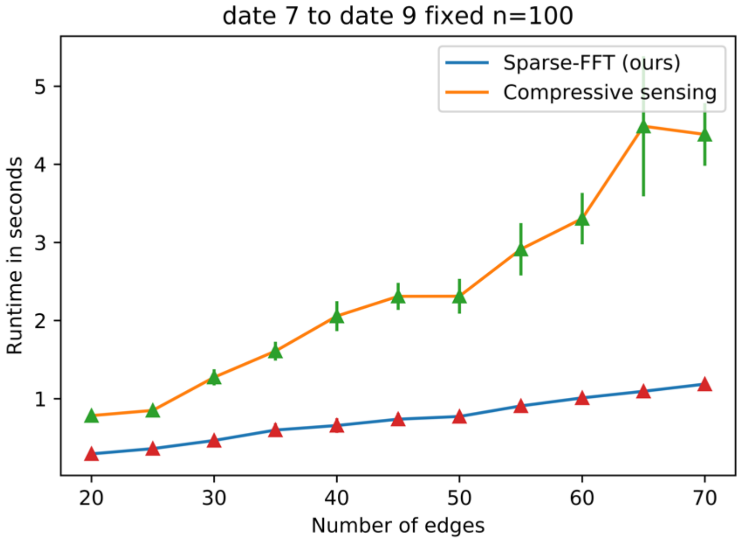
Compressive sensing over finite fields

New hashing schemes



Best of both worlds!

Please visit our poster for experimental results, more applications, and details of our algorithms



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