

DFS and BFS – non-recursive versions

DFS(G, v_0)

Input: graph G and starting vertex v_0

Output: labels $L(v)$ for each vertex v that give the order in which the search visits the vertices. (Each vertex label is initially zero, with a zero label designating an unvisited vertex.)

- $T \leftarrow \{v_0\}$ (T is maintained as a stack)
 - For all vertices v , $L(v) \leftarrow 0$.
 - $Count \leftarrow 1$.
 - $L(v_0) \leftarrow Count$
 - While $T \neq \emptyset$ do
 - $v \leftarrow$ top of stack T
 - If there is a vertex w adjacent to v with $L(w) = 0$ then do
 - * $Count \leftarrow Count + 1$
 - * $L(w) \leftarrow Count$
 - * Push w onto stack T
 - Else pop v from stack T
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BFS(G, v_0)

Input: graph G and starting vertex v_0

Output: labels $L(v)$ for each vertex v that give the length of the shortest path from that vertex to v_0 .

- $T \leftarrow \{v_0\}$ (T is maintained as a queue)
- For all vertices v , $L(v) \leftarrow 0$.
- $L(v_0) \leftarrow 0$
- While $T \neq \emptyset$ do
 - $v \leftarrow$ first item on queue T
 - Remove v from T
 - For all w adjacent to v with $L(w) = 0$ and $w \neq v_0$ do
 - * add w to queue T
 - * $L(w) \leftarrow L(v) + 1$

Efficiency analysis of DFS:

- We'll measure input size by the number n of vertices in G .
- The basic operation will be the label check implicit in the line “If there is a vertex w adjacent to v with $L(w) = 0$ then ...”
- There are at most $n - 1$ of these label checks each time through the “While ...” loop.
- But the “While ...” loop is executed at most $2n$ times: each time either adds a vertex to the stack or takes one off, and each vertex is added once and taken off once.
- This makes the total number of basic steps no more than $n \cdot 2n$, so the algorithm has efficiency in $O(n^2)$.

Efficiency analysis of BFS:

- Again, we'll measure input size by the number n of vertices in G .
- The basic operation is again a label check, this time implicit in the line “For all w adjacent to v with $L(w) = 0$ do ...”
- There are at most $n - 1$ label checks each time through the “While ...” loop.
- But the “While ...” loop is executed at most n times, since each time one vertex is removed from the queue.
- This makes the total number of basic steps no more than n^2 , so the algorithm has efficiency in $O(n^2)$.

Note: Can you see how the label checks would be handled differently depending on if G is input as an adjacency matrix or as adjacency lists? Is one better than the other?