

A Semantics for Weak, Question-Sensitive Belief

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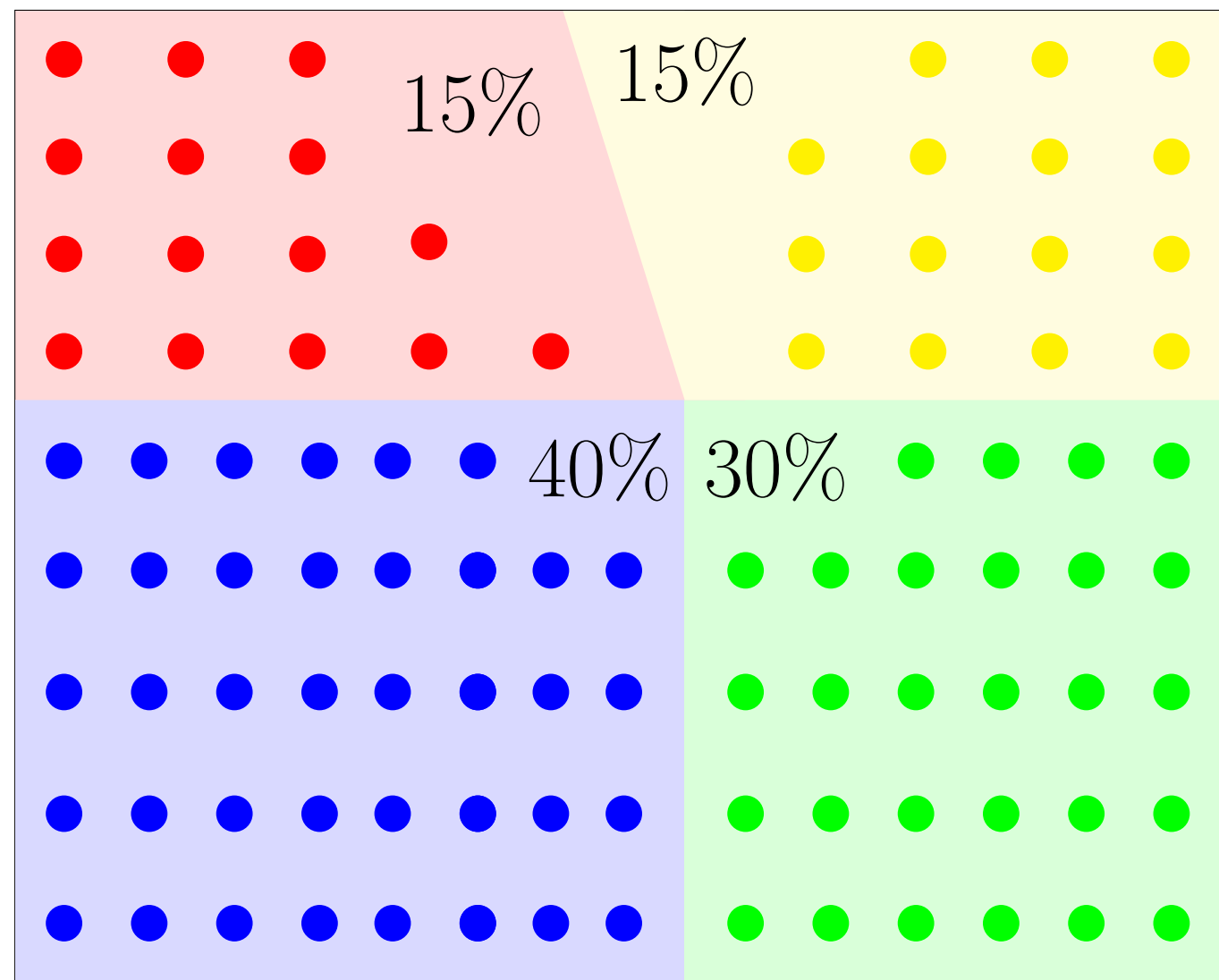
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Belief is:

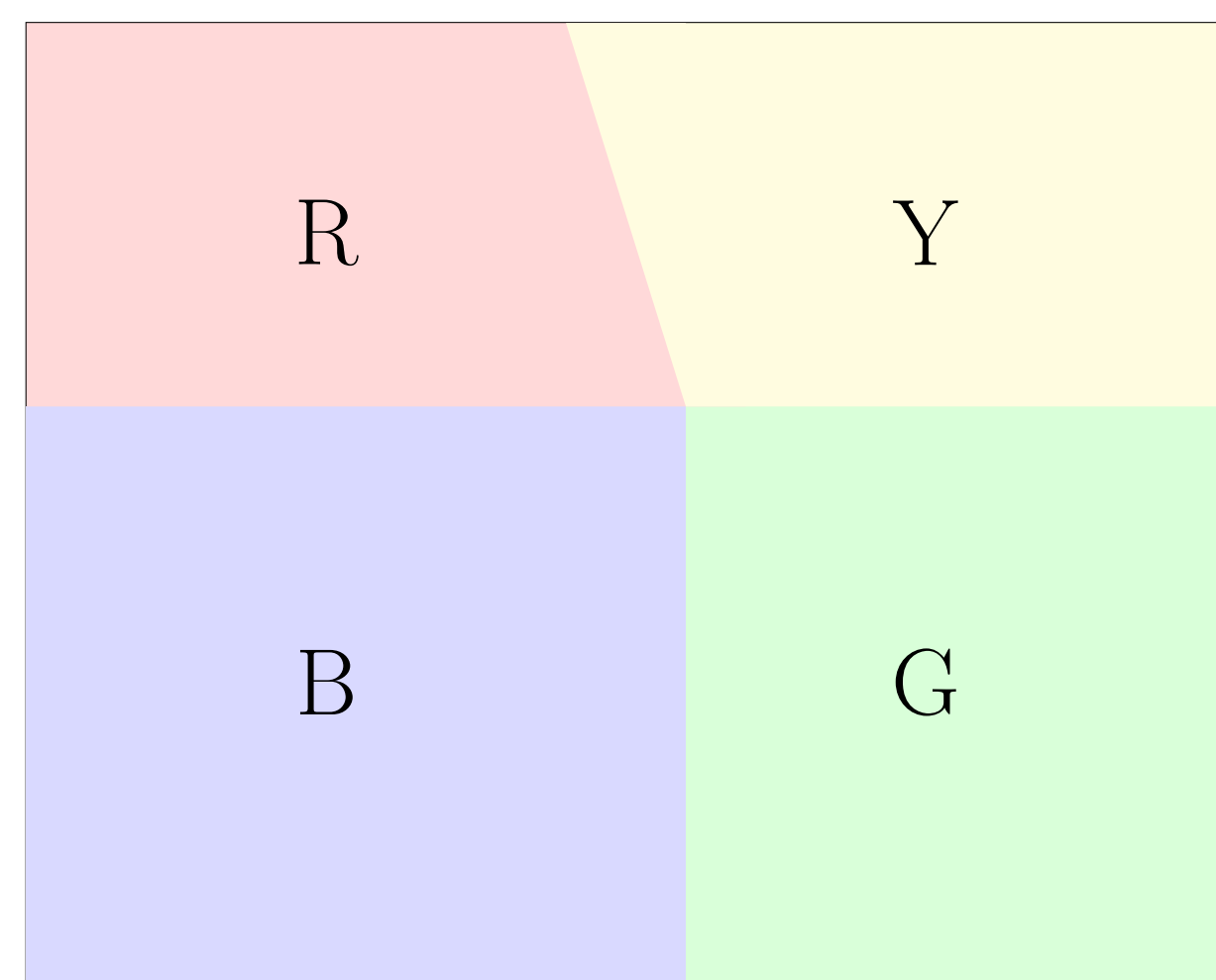
- **weak**
- **question-sensitive**
- **a matter of guessing**

Some examples:

- (1) **The Urn** An urn contains a 100 marbles: 40 blue, 30 green, 15 red, 15 yellow. A marble is randomly chosen from the urn.



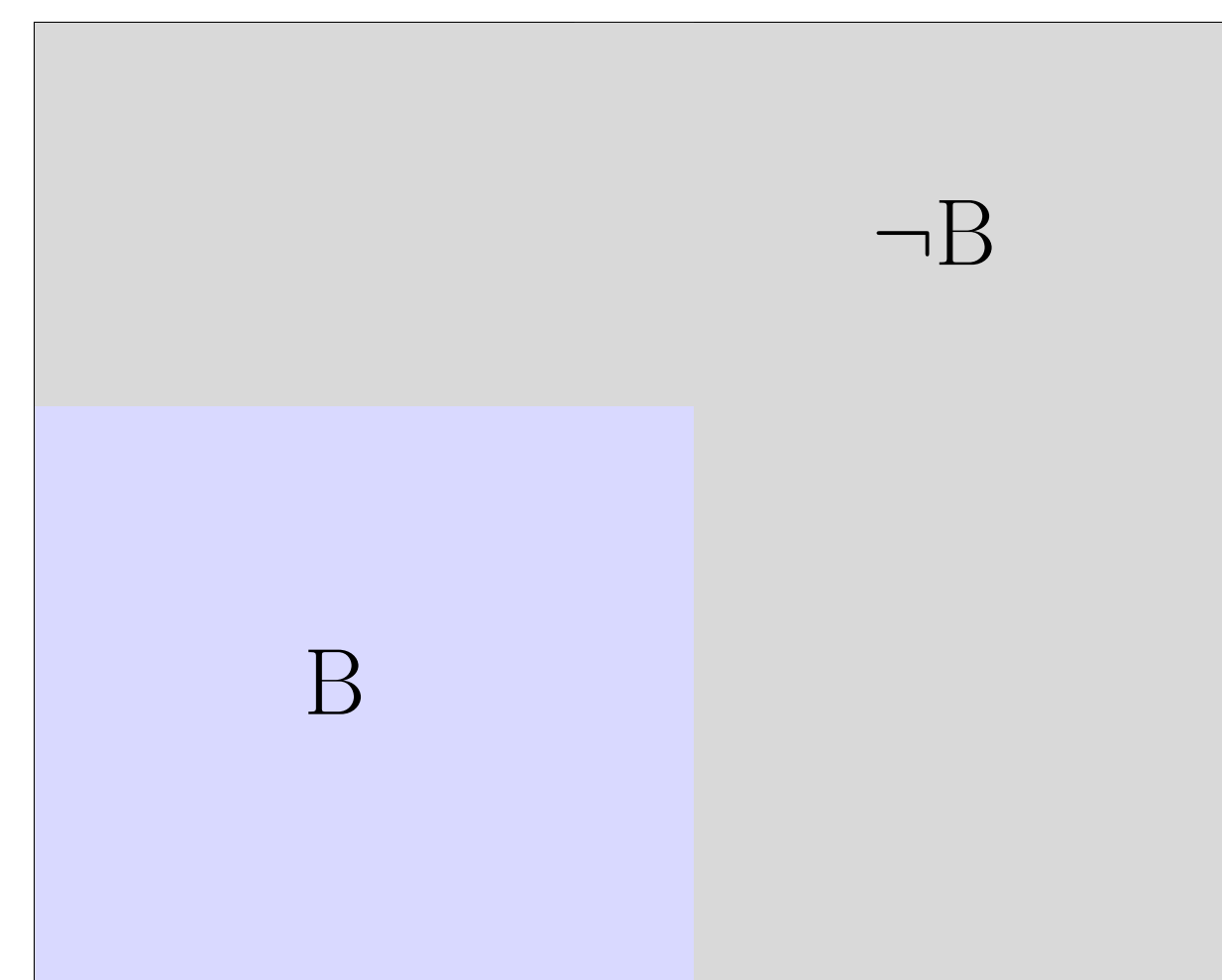
Q: What is the colour of the chosen marble?



✓ (1) I {believe}{think} that the chosen marble is blue.

✗ (2) I {believe}{think} that the chosen marble is non-blue.

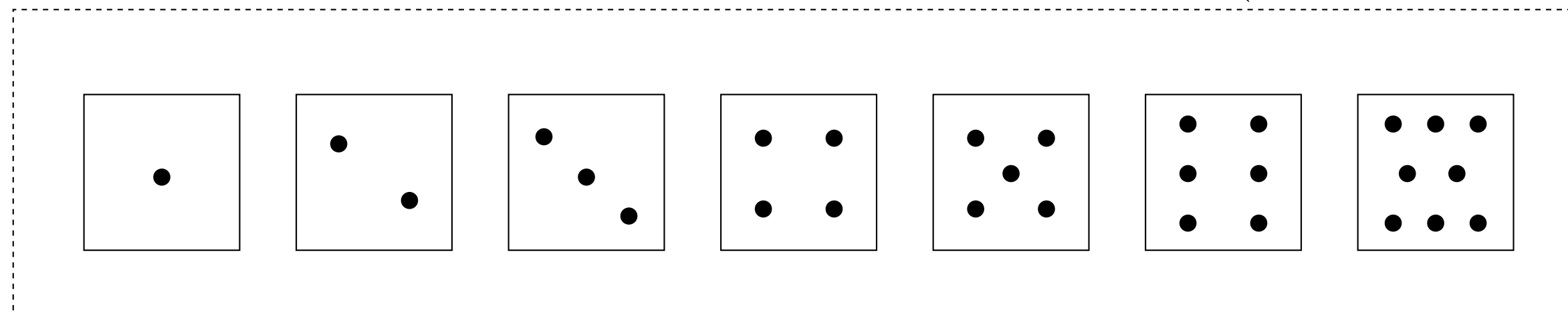
Q': Is the chosen marble blue or non-blue?



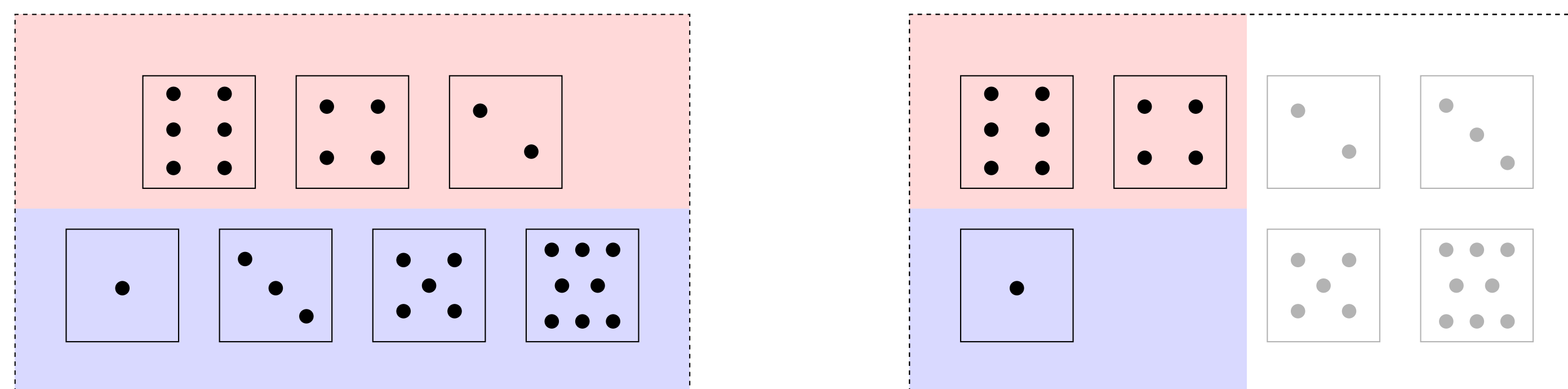
✗ (1) I {believe}{think} that the chosen marble is blue.

✓ (2) I {believe}{think} that the chosen marble is non-blue.

- (2) **Seven-sided die** A fair, seven-sided die is rolled (outcome unknown).



Q'': Did the die land on **even** or **odd**?



✓ (3) I {believe}{think} that the die landed on odd.

✓ (4) I {believe}{think} that if the die landed on composite, it landed on even.

✓ (4') If the die landed on composite, I {believe}{think} it landed on even.

✗ (5) I {believe}{think} that if the die landed on composite, it landed on odd.

✗ (5') If the die landed on composite, I {believe}{think} it landed on odd.

Desiderata:

- The semantics should capture belief-ascriptions with both categorical and conditional contents.
- Belief-ascriptions should be sensitive to QUDs and contextually salient 'chances'.
- Beliefs in conditionals and conditional beliefs should be semantically equivalent.

Belief is a **modal** sensitive to

information states + **probabilistic** and **inquisitive** structure

$$s \in S = \mathcal{P}(W) \quad Pr : \mathcal{P}(s) \longrightarrow [0, 1] \text{ with:} \quad Q = \{X_i\}_{i \in I} \text{ with:}$$

- $Pr(s) = 1$
- $\forall s_1, s_2 \subseteq s : s_1 \neq s_2 \rightarrow Pr(s_1 \cup s_2) = Pr(s_1) + Pr(s_2)$
- $\bigcup_{i \in I} X_i = s$
- $\forall i, j \in I : X_i \cap X_j = \emptyset$
- $\forall i \in I : X_i \neq \emptyset$

Main component: probabilistic-inquisitive information state, $i = \langle s, Pr, Q \rangle$

Beliefs with **categorical** contents:

$$\llbracket B\phi \rrbracket^{w,i} \text{ is true iff } \underbrace{s \cap \llbracket \phi \rrbracket \in Q}_{\text{Condition 1}} \text{ and } \underbrace{Pr(s \cap \llbracket \phi \rrbracket) > \max_{s \cap \llbracket \phi \rrbracket \neq X \in Q} Pr(X)}_{\text{Condition 2}}$$

Condition 1 $\sim \phi$ is a **complete answer** to Q;

Condition 2 $\sim \phi$ **probabilistically dominates** other answers to Q;

Path Semantics for indicative conditionals.

- A **PATH** is a sequence of worlds without repetitions;
- The **SET OF PATHS** on a state $s = \{w_1, \dots, w_n\}$ is $\text{PATH}(s) = \{\langle w'_1, \dots, w'_n \rangle \mid \{w'_1, \dots, w'_n\} = s, w'_i \neq w'_j \text{ for all } i \neq j\}$;
- p' is a **PERMUTATION** of p ($p' * p$) iff $p' \in \text{PATH}(s)$;
- A **PATH** p 's **STATE** $\mathbf{S}(p)$ is $\{p'_1 \mid p' * p, p'_1 \neq p_1\}$;

Update:

- A **PATH UPDATE** of p with q , $p + q$, is the largest member of the set $\{p' \leq p \mid \forall p'' \text{ if } p'' * p', \text{ then } p''_1 \in \llbracket q \rrbracket\}$;
- An **INFORMATION STATE UPDATE** of s with q , s_q , is $s \cap \llbracket q \rrbracket$;
- A **QUESTION UPDATE** of Q with q , Q_q , is $\{X_i \cap \llbracket q \rrbracket \mid X_i \in Q \text{ and } X_i \cap \llbracket q \rrbracket \neq \emptyset\}$;
- A **PROBABILITY DISTRIBUTION UPDATE** of Pr with q , Pr_q , is $Pr(\cdot \mid q)$;

$$\llbracket \phi \rightarrow \psi \rrbracket^{p_1, p} \text{ is true iff } \llbracket \psi \rrbracket^{p_1, p+q} \text{ is true iff } \llbracket \psi \rrbracket^{(p+q)_1} \text{ is true.}$$

$$\llbracket B\phi \rrbracket^{p_1, p, i} \text{ is true iff } s \cap \llbracket \phi \rrbracket \in Q \text{ and } Pr(s \cap \llbracket \phi \rrbracket) > \max_{s \cap \llbracket \phi \rrbracket \neq X \in Q} Pr(X)$$

Conditional beliefs:

$$\llbracket \phi \rightarrow B\psi \rrbracket^{p_1, p, i} \text{ is true iff } \llbracket B\psi \rrbracket^{p_1, p+[\phi], i_{[\phi]}} \text{ is true}$$

Beliefs in conditionals:

$$\llbracket B(\phi \rightarrow \psi) \rrbracket^{p_1, p, i} \text{ is true iff } s \cap \llbracket \psi \rrbracket \in Q \text{ and } C(\phi \rightarrow \psi) > \max_{X_i \in \mathcal{A}_i} C(\phi \rightarrow \chi_i)$$

Some predictions (see Appendix):

Fact 1: (1) is true and (2) is false w.r.t. Q, and vice versa w.r.t. Q'.

Fact 2: (4) and (4') are true, and (5) and (5') are false w.r.t. Q''.

Fact 3: $\llbracket B(A \rightarrow C) \rrbracket^{p_1, p, i}$ is true iff $\llbracket A \rightarrow B(C) \rrbracket^{p_1, p, i}$ is true.

Further work:

- Properties of belief: e.g. **closure**;
- The interaction of belief and knowledge;
- Information states with no fixed questions; questions as dynamic effects;