# A Semantics for Weak, Question-Sensitive Belief

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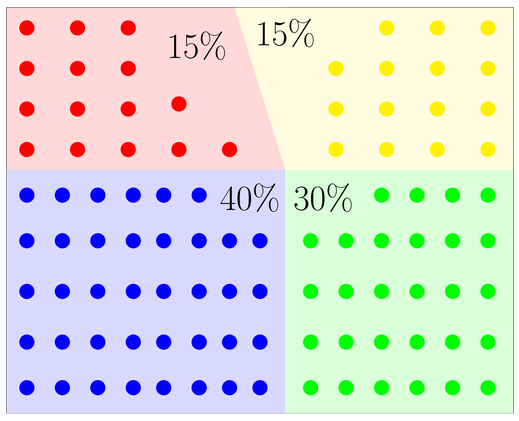
Belief is:

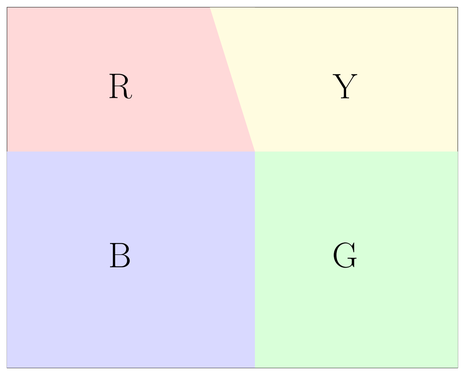
- **weak**
- **question-sensitive**
- **a matter of guessing**

Some examples:

(1) **The Urn** An urn contains a 100 marbles: 40 blue, 30 green, 15 red, 15 yellow. A marble is randomly chosen from the urn.



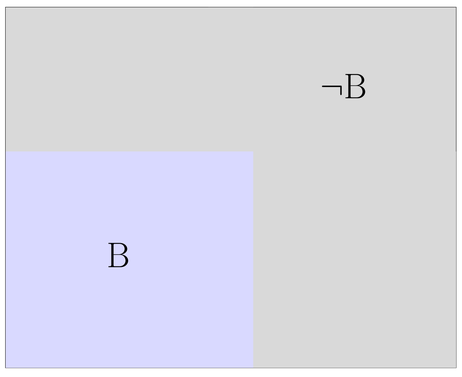

**Q**: What is the colour of the chosen marble?




✓ (1) I {believe}{think} that the chosen marble is blue.

✗ (2) I {believe}{think} that the chosen marble is non-blue.

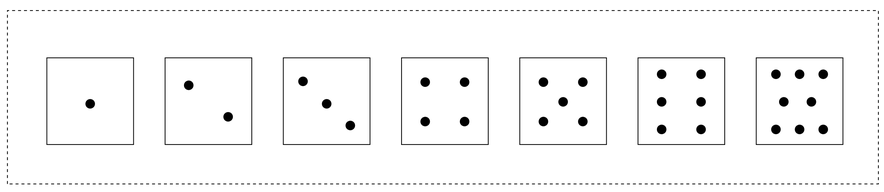
**Q'**: Is the chosen marble blue or non-blue?



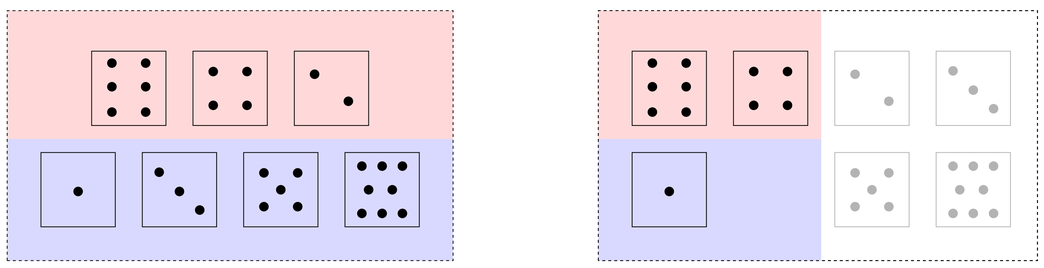

✗ (1) I {believe}{think} that the chosen marble is blue.

✓ (2) I {believe}{think} that the chosen marble is non-blue.

(2) **Seven-sided die** A fair, seven-sided die is rolled (outcome unknown).



**Q"**: Did the die land on **even** or **odd**?



✓ (3) I {believe}{think} that the die landed on odd.

✓ (4) I {believe}{think} that if the die landed on a composite, it landed on an even.

✗ (4') If the die landed on a composite, I {believe}{think} that it landed on an even.

**Desiderata**:

- The semantics should capture belief-ascriptions with both categorical and conditional contents.
- Belief-ascriptions should be sensitive to QUDs and contextually salient 'chances'.
- Beliefs in conditionals and conditional beliefs should be semantically equivalent.

Belief is a **modal** sensitive to
**information states** + **probabilistic** and **inquisitive** structure

$s \in S = \mathscr{P}(W)$

$Pr : \mathscr{P}(s) \longrightarrow [0,1]$ with:

- $Pr(s) = 1$
- $\forall s_1, s_2 \subseteq s : s_1 \neq s_2 \rightarrow Pr(s_1 \cup s_2) = Pr(s_1) + Pr(s_2)$

$Q = \{X_i\}_{i \in I}$ with:

- $\bigcup_{i \in I} X_i = s$
- $\forall i, j \in I : X_i \cap X_j = \emptyset$
- $\forall i \in I : X_i \neq \emptyset$

Main component: probabilistic-inquisitive information state, $i = \langle s, Pr, Q \rangle$

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Beliefs with **categorical** contents:

$$[\![B\phi]\!]^{w,i} \text{ is true iff } \underbrace{s \cap [\![\phi]\!] \in Q}_{\text{Condition 1}} \text{ and } \underbrace{Pr(s \cap [\![\phi]\!]) > \max_{s \cap [\![\phi]\!] \neq X \in Q} Pr(X)}_{\text{Condition 2}}$$

Condition 1 $\sim \phi$ is a **complete answer** to Q;

Condition 2 $\sim \phi$ **probabilistically dominates** other answers to Q;

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**Path Semantics** for indicative conditionals.

- A PATH is a sequence of worlds without repetitions;
- The SET OF PATHS on a state $s = \{w_1, \ldots, w_n\}$ is $\text{PATH}(s) = \{\langle w'_1, \ldots, w'_n \rangle \mid \{w'_1, \ldots, w'_n\} = s, w'_1 \neq w'_n \text{ for all } i \neq j\}$;
- $p'$ is a PERMUTATION of $p$ ($p' * p$) iff $p' \in \text{PATH}(s)$;
- A PATH $p$'S STATE $\mathbf{S}(p)$ is $\{p'_1 \mid p' * p, p'_1 \neq p_1\}$;

**Update**:

- A PATH UPDATE of $p$ with $q$, $p + q$, is the largest member of the set $\{p' \leq p \mid \forall p''$ if $p'' * p'$, then $p''_1 \in [\![q]\!]\}$;
- An INFORMATION STATE UPDATE of $s$ with $q$, $s_q$, is $s \cap [\![q]\!]$;
- A QUESTION UPDATE of $Q$ with $q$, $Q_q$, is $\{X_i \cap [\![q]\!] \mid X_i \in Q \text{ and } X_i \cap [\![q]\!] \neq \emptyset\}$;
- A PROBABILITY DISTRIBUTION UPDATE of $Pr$ with $q$, $Pr_q$, is $Pr(\cdot \mid q)$;

$\|\phi \rightarrow \psi\|^{p_1,p}$ is true iff $\|\psi\|^{p_1,p+q}$ is true iff $[\![\psi]\!]^{(p+q)_1}$ is true.

$\|B\phi\|^{p_1,p,i}$ is true iff $s \cap [\![\phi]\!] \in Q$ and $Pr(s \cap [\![\phi]\!]) > \max_{s \cap [\![\phi]\!] \neq X \in Q} Pr(X)$

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## Conditional beliefs:

$\|\phi \rightarrow B\psi\|^{p_1,p,i}$ is true iff $\|B\psi\|^{p_1,p+q,i_q}$ is true

## Beliefs in conditionals:

$\|B(\phi \rightarrow \psi)\|^{p_1 p,i}$ is true iff $s \cap [\![\psi]\!] \in Q$ and $C(\phi \rightarrow \psi) > \max_{X_i \in \mathcal{A}^i_r} C(\phi \rightarrow \chi_i)$

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## Some predictions (see Appendix):

**Fact 1:** (1) is true and (2) is false w.r.t. Q, and vice versa w.r.t. Q'.

**Fact 2:** (4) and (4') are true, and (5) and (5') are false w.r.t. Q".

**Fact 3:** $\|B(A \rightarrow C)\|^{p_1,p,i}$ is true iff $\|A \rightarrow B(C)\|^{p_1,p,i}$ is true.

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## Further work:

- Properties of belief: e.g. **closure**;
- The interaction of belief and knowledge;
- Information states with no fixed questions; questions as dynamic effects;