Fictionalism and Counterpossibles

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Counterfactualist Fictionalism

(1) The number of stones at Stonehenge is 83.
(2) The number of stones at Stonehenge is 43.

Do the truth conditions of these sentences require the existence of numbers?

Platonists: Yes.
Nominalists: No.

Fictionalism: The truth conditions of mathematical statements can be specified by *supposing* or *hypothetically entertaining* the existence of abstract objects, or by speaking *as if* abstract objects exist. [3, 1].

Counterfactualist Fictionalism: The truth conditions of mathematical statements are parasitic on the truth conditions of counterfactual conditionals. Where X is some Platonist commitment needed for specifying the truth conditions of sentence S, and $>$ is the counterfactual conditional connective:

$$[\![S]\!] \text{ is true iff } [\![X > S]\!] \text{ is true} \tag{CF}$$

CF then accounts for the truth of (1) and falsity of (2) as follows:

$$[\![(1)]\!] \text{ is true iff } [\![\text{Numbers exist} > (1)]\!] \text{ is true}$$
$$[\![(2)]\!] \text{ is true iff } [\![\text{Numbers exist} > (2)]\!] \text{ is true}$$

Counterfactualist Fictionalism Trivialised

For (CF) to be an analysis, we need to specify when $\varphi > \psi$ is true in compositional terms. Let W be the logical space and let $f : \mathscr{P}(W) \times W \longrightarrow \mathscr{P}(W)$. Intuitively, $f(\cdot,\cdot)$ maps a proposition p and world w to a set of closest p-worlds to w. $\varphi > \psi$ is true at w if and only if ψ holds in all the closest worlds to w where φ holds. [5, 2].

$$[\![\varphi > \psi]\!]^{w} \text{ is true iff } \forall w' \in f([\![\varphi]\!], w), [\![\psi]\!]^{w'} \text{ is true} \tag{*}$$

Counterfactuals with impossible antecedents are commonly called *counterpossibles*. If φ in $\varphi > \psi$ is such that $[\![\varphi]\!] = \emptyset$, f($[\![\varphi]\!], w) = \emptyset$ for any w. But then $\varphi > \psi$ is vacuously true. This can be captured model-theoretically, where ψ is any sentence:

$$\neg\Diamond\varphi \models \varphi > \psi \tag{Vac.}$$

Nominalism maintains that, of necessity, there are no abstract objects. Where C and E are predicates '… is concrete' and '… exists' (of type $\langle e \rangle$):

$$\Box\forall x_e \Box(E_{\langle e\rangle}x \rightarrow C_{\langle e\rangle}x) \tag{Nom.}$$

As numbers are paradigmatic examples of non-concrete objects, (Nom.) entails that necessarily, numbers do not exist, or $[\![\text{Numbers exist}]\!] = \bot$, or that $[\![\neg\Diamond(\text{Numbers exist})]\!] = \top$.

From this it follows, for any ψ, that

$$[\![\text{Numbers exist} > \psi]\!] \text{ is true}$$

But then $[\![\text{Numbers exist} > (1)]\!]$ and $[\![\text{Numbers exist} > (2)]\!]$ are both true, making both (1) and (2) true. This is undesirable, as (CF) was supposed to deliver the verdict that (1) is true and (2) is false.

Moreover, as (*) entails that any ψ follows from an impossible supposition, (CF), (*) and (Nom.) together predict the truth of all sentences that involve apparent quantification over abstract objects.

(CF), (*), and (Nom.) jointly **TRIVIALISE** Fictionalism.

Avoiding Triviality

Can we avoid trivialising Fictionalism? The options:

1) Reject (Nom.)
   - (Nom.) is problematic as it entails that $[\![\neg\Diamond(A)]\!] = \top$, for any sentence A that attributes existence to abstract objects. One way to avoid this entailment is to claim that Nominalism is a *contingent* thesis. However, if metaphysical theses like Nominalism are meant to hold of necessity, this is not a promising route.
2) Reject (*) [1]
   - (*) is problematic as it entails (Vac.). *Contra* (Vac.), some counterpossibles are intuitively non-vacuously true or false, and accommodating such intuitions is an independent motivation for revising (*) [4]. However, avoiding (Vac.) plausibly involves revisionary semantic accounts (e.g. ones involving *im*possible worlds), and this route should be taken only as a last resort [6, 7].
3) Reject (CF)
   - (CF) is problematic in conjunction with (*) and (Nom.) as it overgenerates. However, (CF) is arguably the simplest way of fleshing out Fictionalism, and one that makes appeal to an independently motivated account of entertaining contrary-to-fact hypotheses. Even if we reject (CF), a proposal that conserves the core of its motivations is preferable.

An Alternative Counterfactualist Fictionalism: Numbers

The main idea behind (CF) is that the existence of abstract objects is counterfactually supposed. This approach overgenerates, but there is still something to the proposal. We need an alternative view of what is being supposed to capture the truth of (1) and the falsity of (2).

What do we do when we counterfactually entertain the existence of numbers *qua* nominalists? First, we suppose that a kind of object exists. Second, we suppose that a certain arithmetical structure holds between these abstract objects.

What accounts for the truth of (1) and the falsity of (2) is not so much the existence of abstract objects, but the overarching arithmetical structure. For a suppositional account to yield adequate predictions, the subject matter (in the sense of [8]) of the counterfactual supposition should reflect the structure-claim, and not the existence-claim.

We need a formally precise way to capture the structural portion of the fictionalist supposition.

Let $\mathscr{N}$ be a set of predicates:

- $\lambda X^0.\neg\exists x\, X^0 x$
- $\lambda X^1.\exists x\, X^1 x$
- $\dots$
- $\lambda X^n.\exists x_1,\dots,\exists x_n\ (\bigwedge_{i=1}^n X^n x_i \wedge \bigwedge_{1\leq i<j\leq n} x_i \neq x_j)$

Let $\mathscr{A}$ be a set of surrogate objects $a_1,\dots,a_n$, and $\mathscr{C}$ a set of concrete objects $c_1,\dots,c_n$, where both sets are countably infinite.

Let $\mathscr{M}$ be a set of bijective maps $m_i : \mathscr{A} \longrightarrow \mathscr{N}$, let $m^* \in \mathscr{M}$ be such that $m^*(a_n) = X^n$, and M be the predicate '... is the actual mapping'. We assume that to for every $m_i \in \mathscr{M}$ there is a $w \in W$ such that $[\![Mm_i]\!]^w$ is true.

Let $\text{EXT}_w(F)$ for any F of type $\langle e\rangle$ be $\{c_i \in \mathscr{C} \mid [\![F(c)]\!]^w \text{ is true}\}$.

Let $[\![\#(F,a_i)]\!]^w$ be true iff $\mid \text{EXT}_w(F) \mid = \mid \text{EXT}_w(m_i(a_i)) \mid$ (where m_i is such that $[\![Mm_i]\!]^w$ is true).

The alternative proposal. Where S is some sentence of the form 'The number of Fs is i':

$$[\![S]\!]^w \text{ is true iff } [\![M(m^*) > \#(F,a_i)]\!]^w \text{ is true} \qquad \text{(CF')}$$

To see how the proposal works, consider (1) again:

(1) The number of stones at Stonehenge is 83.

(1) is of the form 'The number of Fs is i', so its truth can be evaluated via (CF'). Let F be the predicate '… is a stone at Stonehenge'.

$$
\begin{aligned}
&[\![\text{The number of stones at the Stonehenge is 83}]\!]^{w} \text{ is true}\\
&\text{iff } [\![M(m^{*}) > \#(F, a_{83})]\!]^{w} \text{ is true}\\
&\text{iff } \forall w' \in f(w), [\![\#(F, a_{83})]\!]^{w'} \text{ is true}\\
&\text{iff } \forall w' \in f(w), \mid \mathrm{EXT}_{w}(F) \mid = \mid \mathrm{EXT}_{w}(m^{*}(a_{83})) \mid\\
&\text{iff } \forall w' \in f(w), \mid \mathrm{EXT}_{w}(F) \mid = \mid \mathrm{EXT}_{w}(X^{83}) \mid\\
&\text{iff } 83 = 83 \text{ iff } \top
\end{aligned}
$$

However, as can be checked, assuming the same mapping m^{*} delivers the verdict that (2) is false.

References

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