Incomparable, All Dimensions Considered

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Abstract

We present a new argument for the incompleteness of individual goodness orderings. Building on the idea that 'good' is a multidimensional concept, we model the multidimensionality of individual goodness orderings in a framework akin to that of social choice, where dimensions of goodness play the role of individuals, and the individual goodness ordering the role of the social goodness ordering. Drawing on an analogous result, we show that, given two Pareto principles and a permutation-theoretic constraint, the existence of infinitely many dimensions of goodness entails incompleteness. We argue that the background principles and infinitary multidimensionalism are plausible, and thereby motivate incompleteness. Along the way, we explore the metaphysics of dimensions of goodness, discuss the costs of accepting completeness, and offer a principled account of when outcomes are incomparable. We conclude that, despite recent arguments to the contrary, incompleteness is theoretically well-motivated and still has a leg to stand on.

Keywords: axiology; multidimensionality; dimensions of goodness incomparability; incompleteness;

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The question of whether all outcomes are comparable in terms of goodness for an individual – i.e., whether individual goodness orderings over outcomes are complete – is central in axiological theorising.[1] The view that some outcomes are incomparable enjoys pretheoretic appeal and has been defended, most notably, via the Small Improvements Argument, first formulated by Chang (1997), foreshadowed by De Sousa (1974), and endorsed by Hare (2010) *inter alia*. Moreover, since subjective preferences involve comparisons between outcomes, incomparability appears pervasive at least in the preferences of actual agents (see von Neumann & Morgenstern, 1944; Aumann, 1962).[2] However, compelling arguments against incomparability, appealing to considerations as different as the logic of comparatives (Dorr, Nebel, & Zuehl, 2023, 2021) and choice under uncertainty (Tarsney, Lederman, & Spears, in press; Lederman, MS; Gustafsson, 2025), have recently been proposed. These arguments suggest that comparability is theoretically well-motivated, and arguably undermine the intuitive case for incomparability.

This article offers a new argument for incomparability. We start with the idea that 'good' is a multidimensional concept, or that the goodness of an outcome depends on how it fares along different dimensions of goodness (Hedden & Muñoz, 2024). Following Hedden and Nebel (2024), we model the multidimensionality of 'good' in a social-choice-like framework, where the relevant dimensions of goodness play the role of individuals, and the individual goodness ordering that of the social goodness ordering. Drawing on an analogous result first suggested by Askell (2018) and streamlined by Goodman and Lederman (MS), we show that, given two plausible Pareto principles and a permutation-theoretic constraint, the existence of infinitely many dimensions of goodness entails incompleteness. In other words, we show that, given some background principles, some two outcomes are incomparable for an individual provided there are infinitely many dimensions of goodness. We argue, moreover, that the requisite background principles and infinitary multidimensionalism are independently plausible, and suggest that accepting incompleteness on the basis of this argument is well-motivated.

While the immediate upshot of this article is the fact that incompleteness follows from a seemingly independent package of commitments, and that this offers a plausible new way of motivating incompleteness, several subsidiary issues are treated along the way. (1) In arguing for the premises of our argument, we begin discussing some open questions on the multidimensionality of goodness (brought to attention by Muñoz (MSa)), such as what dimensions of goodness are, how many such dimensions there are, as well as whether one

[1] In what follows, we take incompleteness (a property of orderings) and incomparability (a relation holding between outcomes) to be interchangeable.

[2] Also see Lederman (MS) and references therein.

may permissibly accord some more importance over others. (2) As the argument shows that completeness has unacknowledged, non-trivial axiological consequences, we discuss the theoretical costs incurred by proponents of completeness, and suggest this significantly alters the dialectic in the current literature. (3) Finally, we note that the argument not only helps the case of incomparability in avoiding appeal to intuition, but also suggests a natural account of why some outcomes are incomparable in a multidimensional setting.

We proceed as follows. Section 2 outlines the background for formally capturing multidimensionalism about goodness and develops the main result. Section 3 defends the plausibility of the Pareto and permutation-theoretic principles, as well as that of infinitary multidimensionalism. Section 4 situates the argument against the Small Improvements Argument and sketches a natural account of incomparability available in a multidimensional setting. Section 5 summarises the main argument and findings.

2

This section outlines the result that, given two Pareto principles and a permutation-theoretic constraint, the existence of infinitely many dimensions of goodness entails incompleteness.[3] Section 2.1 motivates a formal framework for capturing (infinitary) multidimensionalism; Section 2.2 introduces the premises and presents the main result.

2.1 *The Framework*

The goodness of states of affairs for a given individual is taken to involve comparisons between states of affairs, so that states of affairs are ordered with respect to how good they are for the individual. We call such orderings (individual) **goodness orderings** and, for simplicity of exposition, speak of **outcomes** instead of states of affairs. This article adopts the view that 'good' is a multidimensional concept, or that there are different ways for an outcome to be good, all of which contribute in some way to the outcome's overall goodness. If 'good' is multidimensional, the goodness of an outcome depends on how the outcome fares along different **dimensions of goodness**. More specifically, an individual goodness ordering over outcomes is taken to depend on the relevant dimensions of goodness in the sense that, once it is specified how each outcome compares to others along each dimension of goodness, the overall goodness ordering is forthcoming.

[3] The formal treatment of this section, especially that of Sub-section 2.2, is heavily indebted to Goodman and Lederman (MS).

Individual goodness orderings are ethically significant, for they capture what is better or worse for an individual (i.e., the individual's 'preferences').[4] Dimensions of goodness are ethically significant, for they are what the goodness orderings depend on. A chief aim of axiological theorising that takes these components as central is that of specifying *what* dimensions of goodness are, *how many* such dimensions exist, and *how* goodness orderings supervene on dimensions. Here's what needs to be said for our purposes. This sub-section offers a way of *representing* dimensions of goodness, but stays neutral on the ontological score.[5] The next sub-section introduces three plausible constraints on the supervenience score, remaining neutral on others. As for the cardinality question, one can either be a multidimensionalist (i.e., pluralist), holding that there are at least two dimensions of goodness, or a unidimensionalist (i.e., monist), holding that there is exactly one dimension of goodness.[6] As a sub-species of multidimensionalism, we single out *infinitary* multidimensionalism, on which there are infinitely many distinct dimensions of goodness.[7]

On the picture sketched above, the overall goodness comparison over outcomes depends on how the outcomes compare along different dimensions of goodness. Drawing on Hedden and Nebel (2024), we model this formally in a framework akin to that employed in the literature on social choice. We let $\mathcal{O}$ be a non-empty set of outcomes, representing all (evaluatively relevant) ways the world can be, and $\mathcal{D}$ be a non-empty set of dimensions, representing the relevant dimensions of goodness (i.e., ways in which an outcome can be good). A member $\langle d, o\rangle$ of the set of dimension-outcome pairs $\mathcal{D}\times\mathcal{O}$ represents the goodness of outcome o with respect to dimension d. Comparisons between members of $\mathcal{D}\times\mathcal{O}$ are taken as a basis for comparisons between outcomes.

We assume inter-dimensional comparability, so that it makes sense to compare how good o is along d with how good o' is along d'. We let $\succeq$ be a reflexive, transitive and complete ordering over $\mathcal{D}\times\mathcal{O}$.[8] Intuitively, $\langle d, o\rangle \succeq \langle d', o'\rangle$ captures the claim that outcome o is at least as good along d as o' is along d'. We adopt some standard abbreviations: $\langle d, o\rangle \succ \langle d', o'\rangle$ (read: o is better along d than o' is along d') abbreviates $\langle d, o\rangle \succeq \langle d', o'\rangle$ & $\langle d', o'\rangle \not\succeq \langle d, o\rangle$ and $\langle d, o\rangle \sim \langle d', o'\rangle$ (read: o is equally good along d as o' is along d') abbreviates

[4] Talk of preferences is intended to be neutral between 'subjective' and 'objective' readings of what is good for an individual. On a rough gloss: the 'subjective' reading tracks what some individual considers good for her, while an 'objective' reading tracks what is good from some 'impartial' point of view. The distinction is plausibly vague, but this should not affect our discussion.

[5] We revisit the question of what dimensions are in Section 3.4, when considering two ways in which our result may be interpreted.

[6] A paradigmatic example of unidimensionalism is the utilitarian position, on which only the outcome's utility matters for the outcome's goodness. While unidimensionalism is the standard position in the literature, some notable multidimensionalists, on our reading, include Nagel (2012), A. K. Sen (1985), Walzer (1983), *inter alia*.

[7] As our result hinges on infinitary multidimensionalism, we take it on board for the time being and revisit it in Section 3.4.

[8] Although these assumptions are not philosophically innocent, they are dialectically appropriate for our purposes (as discussed in Section 3.1).

$\langle d, o\rangle \succeq \langle d', o'\rangle$ & $\langle d', o'\rangle \succeq \langle d, o\rangle$. That orderings over $\mathcal{D} \times \mathcal{O}$ result in overall orderings over $\mathcal{O}$ amounts to there being a function f such that $f(\succeq) = \succcurlyeq$, where $\succcurlyeq$ orders $\mathcal{O}$ (with $\succ$ and $\approx$ the analogs of $\succ$ and $\sim$).

To make our discussion more perspicuous, we introduce the following simplification. Let $\rho(\cdot) : \mathcal{D} \times \mathcal{O} \longrightarrow \mathbb{N}$ be a ranking function such that $\rho(\langle d, o\rangle) = \rho(\langle d', o'\rangle)$ iff $\langle d, o\rangle \sim \langle d', o'\rangle$ and $\rho(\langle d, o\rangle) > \rho(\langle d', o'\rangle)$ iff $\langle d, o\rangle \succ \langle d', o'\rangle$. Each rank $i \in \mathbb{N}$ may be taken to represent a distinct 'level' of goodness of an outcome with respect to a given dimension. To each outcome o there corresponds a function $o(\cdot) : \mathcal{D} \longrightarrow \mathbb{N}$ such that $o(d) = \rho(\langle d, o\rangle)$. For a sequence of dimensions $(d_1, \ldots, d_n)$ containing all $d \in \mathcal{D}$, o determines a sequence $(o(d_1), \ldots, o(d_n))$. The sequence in turn represents a given outcome's distribution of goodness along multiple dimensions. The overall goodness ordering $\succeq$ over $\mathcal{O}$ may then, without confusion, be taken to be an ordering over the outcomes' distributions of goodness (qua sequences of natural numbers).

As noted above, our formal machinery is parallel to that employed in the literature on social choice (A. Sen, 1984; Adler, 2019). This framework is natural since goodness is treated as a multidimensional concept[9] and the properties of multidimensional concepts are fruitfully captured via the machinery of social choice.[10] In social choice, we assume a non-empty set of outcomes $\mathcal{O}$ and a non-empty set of individuals $\mathcal{I}$, representing all individuals under consideration. A member $\langle i, o\rangle$ of the set of individual-outcome pairs $\mathcal{I} \times \mathcal{O}$ represents the welfare of individual i in outcome o. We assume interpersonal comparability, so that it makes sense to compare the welfare of individual i in o to that of individual i' in o'. We let $\succeq$ be an ordering over $\mathcal{I} \times \mathcal{O}$, with $\langle i, o\rangle \succeq \langle i', o'\rangle$ capturing the claim that i in o has at least well off as i' in o' (with $\sim$ and $\succ$ as above). A central question in social choice is that of determining an ordering over $\mathcal{O}$ based on the ordering over $\mathcal{I} \times \mathcal{O}$, representing an *overall* goodness ordering over outcomes (i.e., an ordering over outcomes that takes individual preferences into account). An answer to this question specifes a so-called social welfare functional f such that $f(\succeq) = \succcurlyeq$, where $\succcurlyeq$ orders $\mathcal{O}$.

2.2 *The Result*

The previous section introduced a framework that formally captures multidimensionalism about goodness. The multidimensionality consists in the goodness ordering over outcomes $\succcurlyeq$ being a function of a more fine-grained ordering $\succeq$ that takes dimensions of goodness

[9]Note that the framework itself does *not* settle whether goodness is a multidimensional concept, since $\mathcal{D}$ can be a singleton.

[10]*Cf.* Hedden and Nebel (2024), who note that 'each underlying dimension of a multidimensional concept [is] akin to an individual whose preferences or utilities correspond to that dimension's ranking of alternatives' (p. 266).

into account. This section presents the result that infinitary multidimensionalism about goodness, paired with three principles about how the goodness ordering supervenes on the fine-grained ordering, entails incompleteness. More precisely stated, the result is: provided (i) two Pareto and one permutation-theoretic constraint on $f(\cdot)$ hold, and (ii) the cardinality of $\mathcal{D}$ is countably infinite, $\geqslant$ is an incomplete ordering (i.e., there exist o, o' such that neither $o \geqslant o'$ nor $o' \geqslant o$ holds).

We first introduce three constraints on how the overall goodness ordering over outcomes supervenes on the goodness ordering over dimension-outcome pairs. The principles, like the result under consideration, are close analogs of constraints on social welfare functionals in the setting of social choice. We now only note the motivation for the principles, reserving further defence for Sections 3.2 and 3.3.

The first constraint is Dimensioned Pareto Equivalence (**DPE**):

Dimensioned Pareto Equivalence.
$\forall o, o' \in \mathcal{O},\ \forall d \in \mathcal{D} : \langle d, o\rangle \sim \langle d, o'\rangle \Longrightarrow o \approx o'$

In words: for all dimensions d and any two outcomes o, o', if o is equally good along d as o' is, then o and o' are equally good, all dimensions considered. **DPE** captures the plausible thought that, provided outcomes are equivalent across all relevant dimensions of goodness, then the outcomes are overall equivalently good. (Note the analogous thought behind the principle in the context of social choice: provided two outcomes make all individuals equally well off, the outcomes are equivalent overall.)

The second constraint is Dimensioned Strict Pareto (**DSP**):

Dimensioned Strict Pareto.
$\forall o, o' \in \mathcal{O} : (\forall d \in D : \langle d, o\rangle \succeq \langle d, o'\rangle) \& (\exists d' \in D : \langle d', o\rangle \succ \langle d', o'\rangle) \Longrightarrow o > o'$

In words: for any two outcomes o, o', if o is at least as good along d as o' is for all d, and there is a dimension d' such that o is better along d' than o' is, then o is better than o', all dimensions considered. **DSP** captures the equally plausible thought that, provided one outcome is better than another along one dimension and equal (or possibly better) on others, then it is better overall. (Note the analogous thought behind the principle in the context of social choice: provided one outcome is better than another for one individual and equal to (or possibly better than) it for other individuals, then the former is better overall.)

To introduce the third constraint, we need some definitions related to permutations of dimensions. A permutation π is a bijective map from $\mathcal{D}$ onto itself; let $Sym(\mathcal{D})$ be the set of all permutations of $\mathcal{D}$. For a sequence of dimensions $(d_1, \ldots, d_n)$ containing all $d \in \mathcal{D}$,

o determines a sequence $(o(d_1), \ldots, o(d_n))$, as above. Where π is a permutation of $\mathcal{D}$, let $\pi(o)(\cdot)$ be a function such that $\pi(o)(d) = o(\pi(d))$. The function $\pi(o)(\cdot)$, like $o(\cdot)$, determines a sequence $(\pi(o)(d_1), \ldots, \pi(o)(d_n))$, which we call simply $\pi(o)$. Permutation Invariance (**PI**) then is:

Permutation Invariance.
$\forall \pi \in Sym(\mathcal{D}) : o \geqslant o' \implies \pi(o) \geqslant \pi(o')$

In words: for all ways of permuting the domain of dimensions, if o is as good as o', all dimensions considered, then $\pi(o)$ is as good as $\pi(o')$, all dimensions considered. Another way of rendering **PI** is by saying that orderings are preserved under permutations of dimensions, or that the order of dimensions in a sequence does not matter for the goodness ordering over outcomes. **PI** captures the thought that the goodness ordering over outcomes does not depend on the identity of dimensions themselves, thus capturing a constraint of *impartiality* (more on this in Section 3.3). Although **PI** is less intuitive than **DPE** and **DSP**, it is still a plausible constraint on the goodness ordering.

For bookkeeping purposes, we also formalise completeness for overall goodness orderings and infinitary multidimensionalism **(IM)**.

$\geq$-Completeness	**Infinitary Multidimensionalism**
$\forall o, o' \in \mathcal{O} : o \geq o' \ \vee \ o' \geq o$	$\lvert\mathcal{D}\rvert = \lvert\mathbb{N}\rvert$

Two outcomes o and o' witness the falsity of $\geq$-Completeness if neither outcome is at least as good as the other (i.e., if neither outcome is better than or equivalent to another).

A result first suggested by Askell (2018) and streamlined by Goodman and Lederman (MS) in an analogous setting is that:[11]

Fact 1. Provided $(\mathcal{D} \times \mathcal{O}, \geq)$ is sufficiently rich,[12] **IM**, **DPE**, **DSP**, **PI**, and **$\geq$-Completeness** are inconsistent.

Proof. Let $A, A^-, B \subseteq \mathcal{D}$ be infinite and s.t. $A \cap B = \emptyset$ and $A^- \subset A$. Let $o, o^-, u \in \mathcal{O}$ be as follows:

$$o(d) = \begin{cases} 1 & \text{if } d \in A \\ 0 & \text{if } d \notin A \end{cases} \qquad o^-(d) = \begin{cases} 1 & \text{if } d \in A^- \\ 0 & \text{if } d \notin A^- \end{cases} \qquad u(d) = \begin{cases} 1 & \text{if } d \in B \\ 0 & \text{if } d \notin B \end{cases}$$

By **DSP**, $o > o^-$.

[11] See Goodman (2025) for a related result.

[12] That $(\mathcal{D} \times \mathcal{O}, \geq)$ is sufficiently rich means that there are outcomes o, o^-, and u that compare such that they induce the distributions $o(\cdot)$, $o^-(\cdot)$, and $u(\cdot)$. We discuss this provision in Section 3.1.

Suppose $o > u$. By the Axiom of Choice (AC), there is a permutation π on $\mathcal{D}$ s.t. $im(\pi|_A) = B$ and $im(\pi|_B) = A$. By **PI**, $\pi(o) > \pi(u)$. Note that $\forall d \in \mathcal{D} : \langle d, \pi(o)\rangle \sim \langle d, u\rangle$.[13] For analogous reasons, $\forall d \in \mathcal{D} : \langle d, \pi(u)\rangle \sim \langle d, o\rangle$. By **DPE**, $\pi(o) \approx u$ and $\pi(u) \approx o$. But then $u \approx \pi(o) > \pi(u) \approx o$, and so $u > o$. Contradiction; so $o \not> u$. Parallel reasoning establishes that $u \not> o$. By $\geq$-Completeness, it holds that $o \approx u$.

Suppose, now, that $u > o^-$. By AC, there is a permutation π' on $\mathcal{D}$ s.t. $im(\pi'|_{A^-}) = B$ and $im(\pi'|_B) = A^-$. By **PI**, $\pi'(u) > \pi'(o^-)$. By **DPE** (as above), $\pi'(o^-) \approx u$ and $\pi'(u) \approx o^-$. But then $\pi'(o^-) \approx u > o^- \approx \pi'(u)$, and so $\pi'(o^-) > \pi'(u)$. Contradiction; so $u \not> o^-$. Parallel reasoning establishes that $o^- \not> u$. By $\geq$-Completeness, it holds that $o^- \approx u$.

But then it holds that $o \approx u \approx o^-$ and so that $o \approx o^-$. Contradiction.

An immediate corollary of **Fact 1** is:

Corollary 1. **IM**, **DPE**, **DSP**, and **PI** entail that $\geq$ is incomplete.

Considering the motivation for our framework and the relevant principles given above, **Corollary 1** may be interpreted as stating that, if there are infinitely many ways in which an outcome can be good, then the overall goodness ordering on outcomes is incomplete, and so even against the background of relatively weak assumptions about Pareto-tradeoffs and permutations. Provided one accepts the requisite assumptions, then, there appears to be no guarantee that a non-primitive goodness ordering over outcomes (i.e., one determined on the basis of multidimensional considerations) is complete. The next section considers the requisite assumptions in further detail.

3

The result of Section 2.2 indicates that one cannot accept Infinitary Multidimensionalism, Dimensioned Pareto Equivalence, Dimensioned Strict Pareto, and Permutation Invariance without renouncing $\geq$-Completeness. This may be taken to suggest that overall goodness orderings are incomplete, or that at least some outcomes are incomparable. While some have found this conclusion intuitive independently of this result (see the references in Section 1), recent arguments suggest that, necessarily, preferential orderings are complete

[13] Consider the case $d \in B$; then $\rho(\langle d, u\rangle) = 1$ by the definition of $u(d)$; we also have that $\rho(\langle d, \pi(o)\rangle) = 1$, since $\pi(d) \in A$ and $\pi(o)(d) = o(\pi(d)) = 1$ for any $d \in B$; but then $\langle d, u\rangle \sim \langle d, \pi(o)\rangle$ for any $d \in B$. Consider the case $d \notin B$; then $\rho(\langle d, u\rangle) = 0$; we also have that $\rho(\langle d, \pi(o)\rangle) = 0$ since $\pi(d) \notin A$ and $\pi(o)(d) = o(\pi(d)) = 0$ for any $d \notin B$; but then $\langle d, u\rangle \sim \langle d, \pi(o)\rangle$ for any $d \notin B$.

(Tarsney et al., in press),[14] or that, necessarily, all outcomes are comparable (Dorr et al., 2023). The result may then also be taken to imply that one of the premises is false. We should thus further scrutinise the implicit assumptions of our framework (Sec. 3.1), the Pareto principles **DPE** and **DSP** (Sec. 3.2), the permutation-theoretic constraint **PI** (Sec. 3.3), and Infinitary Multdimensionalism (Sec. 3.4). Anticipating our discussion, we consider the premises independently defensible, and thus defend the conclusion of **Corollary 1**.

3.1 Background Assumptions

Before discussing the premises, we consider some implicit assumptions of our framework.

First, we assumed inter-dimensional comparability, taking at face value comparisons of outcomes with respect to how good they are along different dimensions.[15] At worst, it may be objected that such comparisons are nonsensical, akin to how comparisons of one's speed and one's height are nonsensical. Even if we admit that they make sense, moreover, such comparisons may appear unavailable across all dimensions, e.g. when comparing the knowledge afforded by an academic career and the social contribution afforded by a medical career. Such worries threaten to undermine our framework, and with it the hypothesis that 'good' is multidimensional in the first place, and should be addressed before continuing.

Two things can be said in response. Note, first, that overall comparisons taking multiple dimensions into account (just like overall comparisons taking multiple individuals into account) plausibly require assuming inter-dimensional (interpersonal) comparability.[16] Indeed, the absence of inter-dimensional comparability would significantly undermine the informativity of our framework, in the sense of leaving many overall comparisons unsettled. Of course, constraints like **DPE** and **DSP** do not appeal to inter-dimensional comparisons and are compatible with inter-dimensional non-comparability. However, **DPE** and **DSP** are also weak constraints that are bound to leave a lot of overall comparisons unsettled. Provided we need to aggregate over relevant dimensions in some principled way, appealing only to goodness-related differences between outcomes, assuming inter-dimensional comparability is necessary. It is also worth noting, moreover, that inter-dimensional *non-*

[14] An adjacent but different argument for completeness emerges from the discussions in Lederman (in press, MS).

[15] We thus assume the falsity of any view on which dimensions of goodness are in principle incomparable. See Walzer (1983) for one prominent defence of this view.

[16] The assumption regarding inter*personal* comparability has been widely discussed in the literature on social choice, with the considered view being that at least some interpersonal comparisons are required to avoid trivial social welfare functionals (see Baccelli, 2023, for further references and discussion).

comparability plausibly leads to failures of $\geq$-Completeness, and so that relaxing this assumption does not avoid the conclusion that some outcomes are incomparable.

Furthermore, conceptually speaking, it should be stressed that we are *not* assuming that different dimensions are directly comparable. Rather, our framework requires that different outcomes are comparable *in terms of goodness* along different dimensions. For instance, we need never directly compare the knowledge of an academic and the social contribution of a doctor, but rather the goodness of an academic career with respect to the knowledge it affords and the goodness of a medical career with respect to the social contribution it affords. Insofar as there is a genuine difference between such comparisons, it is possible to reject the former as nonsensical and accept the latter as a necessary idealising assumption in formal theorising.

Second, we assumed that the primitive ordering over dimension-outcome pairs is reflexive, transitive, and complete. Reflexivity and transitivity are standard constraints on weak orderings and plausibly are ineliminable components of the meaning of the 'at least as good as' and similar relations; as such, we accept them without further argument. Completeness, however, is much more controversial and may be rejected. Nevertheless, the plausibility of completeness for $\geq$ is *not* at issue in our result. Indeed, assuming completeness for $\geq$ is dialectically appropriate, since **Corollary 1** suggests that $\geq$-Completeness fails *even when* the primitive ordering is complete. More loosely put, even if all fine-grained comparison between dimension-outcome pairs are settled, the overall ordering over outcomes may still fail to settle how some outcomes compare. In a word, completeness for $\geq$ is a concession to friends of $\geq$-Completeness, and thus dialectically appropriate.

Finally, in our statement of **Fact 1** we assumed that the ordered set $(\mathcal{D} \times \mathcal{O}, \geq)$ is sufficiently rich. For the purposes of the result, sufficient richness amounts to there being three outcomes o, o^-, and u such that the relation $\geq$ over dimension-outcome pairs induces the corresponding distributions $o(\cdot)$, $o^-(\cdot)$, and $u(\cdot)$. Although this is best seen as a formal characteristic of the proof, it might still be found objectionable. One might object that, since there are not infinitely many dimensions of goodness, there cannot be outcomes such as those required by our stipulation of sufficient richness. While this objection possibly has bite, it supposes the falsity of **IM**, the discussion of which we defer to Section 3.4. Supposing one accepts **IM**, on the contrary, there seems little reason to object to the stipulation of sufficient richness. In fact, provided the existence of some dimension is independent from that of others, a plausible assumption of 'plenitude', to the effect that any space of outcomes is instantiated, implies the existence of outcomes $o(\cdot)$, $o^-(\cdot)$, and $u(\cdot)$ – which is just what was supposed.[17]

[17] We outline a more intuitive rendering of such a space of outcomes by way of outlining our preferred account of incomparability in Section 4.2.

3.2 *Dimensioned Pareto Principles*

We now move on to the explicit premises. This sub-section considers the Pareto principles **DPE** and **DSP**.

As noted previously, both **DPE** and **DSP** are plausible principles about how the overall goodness ordering supervenes on fine-grained dimension-outcome comparisons. In a multidimensional setting, two outcomes being equally good along all dimensions seems to be just the kind of consideration that shows that the outcomes are equally good. Further, an outcome being better than another on at least one dimension, with all else being equal, likewise seems to be just the kind of consideration that shows the former outcome is better than the latter. But this is just what **DPE** and **DSP** state. In light of their plausibility, we could go further and argue that **DPE** and **DSP** are non-negotiable constraints on goodness orderings that depend on dimensional considerations.

It should also be noted, as suggested above, that **DPE** and **DSP** are weak constraints: (i) they do not by themselves require the strong framework we are working with and (ii) they under-specify many comparisons. As for (i), **DPE** and **DSP** do not require inter-dimensional comparisons: given any comparison between outcomes o, o' determined by either of these principles, the same comparison holds in a model with only *intra*-dimensionally defined $\geq$. As for (ii), **DPE** and **DSP** stay silent on any comparison where o is better than o' along some d and the opposite holds for some other d' (regardless on how o and o' compare along other dimensions). As such, they stay neutral on *all* trade-offs between dimensions. In our setting, this is a point in their favour since even relatively weak principles suffice to undermine $\geq$-Completeness.

Both **DPE** and **DSP** have been met with objections, however.

One kind of objection maintains that **DSP** makes false predictions, e.g. in cases where improvements lead to 'imbalance' between dimensions. In a recent paper, Hedden and Muñoz (2024) make headway on multidimensionalism about goodness and defend **DSP** ('Strong Pareto', in their terminology) from this and similar counterexamples. In the interest of economy, we do not rehearse their arguments here and simply accept their strategy for explaining away the counterexamples. Independently of this strategy, however, we should note that renouncing **DSP** on account of such counterexamples appears ill-advised in light of **DSP**'s simplicity and plausibility.

Another kind of objection, transposed to our setting from the literature on social choice, maintains that (i) **DPE** makes intuitively false predictions in infinitary cases (see e.g. Askell, 2018; Goodman & Lederman, MS) and that (ii) **DSP** is inconsistent, in infinitary contexts, with a permutation-theoretic principle similar to (but importantly different from) **PI** (Nebel, 2025, Sec. 3.1.). These issues have garnered significant attention in the literature on social choice and doing justice to them would take us far afield. Before developing our favoured responses, we anticipate by saying that, in both cases, giving up the relevant Pareto principle appears to forfeit simplicity and strength in the interest of conserving either shaky pre-theoretic intuitions or independently questionable constraints. While abductive considerations are seldom clear-cut, we take the following two issues to be comparatively straightforward.

(i) Consider the following distributions of goodness corresponding to outcomes o and o' (supposing the sequences continue in the obvious way):

$$\begin{array}{cccccccccccc} & d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 & d_9 & \dots \\ o & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & \dots \\ o' & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & \dots \end{array}$$

Pre-theoretically, it appears that o' is a better than o overall since for each dimension along which o is better than o' (e.g. d_3), there are two dimensions along which o' is better than o (e.g. d_1 and d_2). However, since there exists a permutation mapping the dimensions to which $o(\cdot)$ assigns 1 to those to which $o'(\cdot)$ assigns 1 and vice versa, **DPE** and **PI** jointly entail that o and o' are equally good, all dimensions considered.[18] Insofar as the pre-theoretic judgment that o' is better than o is to be trusted, then, this could count as a strike against **DPE** (at least if **PI** is accepted, the discussion of which we defer to Section 3.3).

Supposing the same 'pattern' is instantiated in a case with only *finitely* many dimensions of goodness, what the relevant pre-theoretic intuition tracks is the fact that there are twice as many (or simply more) dimensions along which o' is better than those at which o is better. However, although the intuition is reliable in finitary cases, it founders in infinitary ones: that the relevant permutation exists shows that the cardinality judgment in question is mistaken. More intuitively put, that the sequence determined by o' may be obtained from a permutation of the sequence determined by o plausibly indicates that, in the infinitary case, the difference between o and o' is due to how the dimensions are arranged, rather than a difference in the number of dimensions 'supporting' each outcome. Thus, regardless of its reliability in finitary cases, the intuition should be resisted in infinitary cases. Arguably, then, the verdict of **DPE** in this case holds up under scrutiny.[19]

[18] Here's why. Let $O = \{d \in \mathcal{D} \mid o(d) = 1\}$ and $O' = \{d \in \mathcal{D} \mid o'(d) = 1\}$. Suppose $o > o'$. By AC, there is a permutation $\pi(\cdot)$ s.t. $im(\pi\mid_O) = O'$ and $im(\pi\mid_{O'}) = O$. By **PI**, $\pi(o) > \pi(o')$. Note that $\forall d \in \mathcal{D} : \langle d, \pi(o)\rangle \sim \langle d, o'\rangle$ and that $\forall d \in \mathcal{D} : \langle d, \pi(o')\rangle \sim \langle d, o\rangle$. By **DPE**, $\pi(o) \approx o'$ and $\pi(o') \approx o$. But then $o' \approx \pi(o) > \pi(o') \approx o$. Contradiction; so $o \not> o'$. Parallel reasoning establishes that $o' \not> o$. By $\geq$-Completeness, $o \approx o'$.

[19] Discussing the same example in the setting of social choice, Goodman and Lederman (MS) similarly conclude that '[o]nce it is stipulated that the case involves the same people at the same wellbeing levels, our intuitions clearly side with Pareto Indifference, and against the density-driven judgment' (p. 10).

(ii) To introduce the second issue, we need another constraint about permutations. Call the following principle, intuitively stipulating that an outcome and its permutation (more precisely, and outcome's distribution and its permutation) are equally good, Permutation Equivalence (**PE**):

Permutation Equivalence. $\forall \pi \in Sym(\mathcal{D}), \forall o \in \mathcal{O} : o \approx \pi(o)$.

Consider, now, the following two distributions of goodness corresponding to outcomes o and o', with $A, B \subseteq \mathcal{D}$ disjoint, infinite sets and $d \in \mathcal{D} \setminus (A \cup B)$:

	A	B	d
o	1	0	0
o'	1	0	1

By **DSP**, we get that $o' > o$. However, there exists a permutation π such that $im(\pi|_{A\cup\{d\}}) = A$, and so it holds by **DPE** that $o \approx \pi(o')$. Since it holds, by **PE**, that $\pi(o') \approx o'$, it follows that $o \approx o'$. But this contradicts the verdict of **DSP** that $o' > o$. Insofar as **PE** is a plausible constraint on the overall goodness ordering, then, this could count as a strike against **DSP**.

There are strong indications, however, that **PE** is not a plausible permutation-related constraint on goodness orderings (Askell, 2018; Nebel, 2025; Goodman & Lederman, MS). While similar to **PI**, **PE** is a stronger principle maintaining that any permutation of any distribution is equally good as the original distribution. Given that permutations preserve only information about the number of dimensions at each level of goodness in some distribution, **PE** amounts to the claim that any two outcomes with an equivalent number of dimensions at each level of goodness are equally good. As such, **PE** implies that information not captured by this 'metric' (e.g. the number of dimensions by which an outcome 'improves' upon another) is irrelevant to axiological comparisons (*cf.* Muñoz (MSb) for this point in an analogous setting). However, precisely such information is behind the relevant judgment that o' is better than o. In other words, the fact that d is at a higher 'level of goodness' in o' compared to o (i.e., that o' 'improves' upon o along d) is a plausible reason for thinking o' is better than o. Provided this kind of information matters for axiological comparisons, **PE** should be rejected.

In both cases, then, our Dimensioned Pareto principles conflict either with shaky intuitions (i.e., the intuition, in (i), that o' is better than o) or with independently questionable principles (i.e., **PE** in (ii)). Arguably, the Dimensioned Pareto principles hold up under such scrutiny. Given their simplicity and general plausibility, then, we conclude that **DPE** and **DSP** should not be given up in the interest of avoiding the conclusion of **Corollary 1**.

3.3 *Permutation Invariance*

This section considers the premise **PI**.

Recall that **PI** may be glossed as saying that goodness orderings are preserved under permutations of dimensions, or that shifting the labels of dimensions in a sequence does not matter for the goodness ordering. Alternatively, **PI** may be glossed as saying that the overall goodness ordering depends entirely on the qualitative aspects of fine-grained comparisons between dimension-outcome pairs: provided two fine-grained orderings differ *only* in the labels they assign to the dimensions, they agree on how any two outcomes compare overall.

To see how **PI** works, consider the following example (we consider a finite case for illustration). Let o be an artist's career and o' a corporate career; moreover, let the relevant dimensions of goodness be a := authenticity, b := belonging, and c := consistency. Suppose that the artist's career is better for the agent along a and b, the corporate career better along c, and also that the artist's career is better along a than each career in terms of any other dimension. (We stay neutral, as above, on whether these comparisons are 'subjective' or 'objective'.) The following distributions are then induced over o and o':

	a	b	c
o	2	1	0
o'	1	0	1

Suppose o is better than o' overall for the agent. Consider two further careers, o^* and o'^*, such that o^* is better than o'^* in terms of a and c, o'^* is better than o^* in terms of b, and o^* is better in terms of c than both careers combined. Note that o^* and o'^* just are o and o' under a permutation of dimensions $\pi : a \mapsto b, b \mapsto c, c \mapsto a$. Qualitatively speaking (i.e., not taking into account which dimension is which), it may be said that o just is o^*, that o' just is o'^*, and that o^* and o'^* compare in the same way as o and o' do. We may tabulate o^*, o'^* ($\pi(o)$ and $\pi(o')$) as follows:

	a	b	c
o^* $[\pi(o)]$	1	0	2
o'^* $[\pi(o')]$	0	1	1

PI implies, in this case, that o^* is better than o'^* overall. This may be motivated by appealing to the qualitative identities of o and o^*, o' and o'^*, and the fact that o compares to o' in the same way that o^* compares to o'^*. More precisely, there is i) one dimension along which o (o^*) is better by one welfare level, ii) one dimension along which o (o^*) is worse by one welfare level, and iii) one dimension along which o (o^*) attains a higher welfare level than the other dimensioned outcomes, and also is better by one welfare level. Provided these considerations support the claim that o is better than o', they also support the claim that o^* is better than o'^*. But this is just what **PI** predicts.

Is **PI** a plausible constraint on goodness orderings in our multidimensional setting? To address this, note that a principle like **PI** amounts to a constraint of *impartiality* in the setting of social choice (May, 1952; A. Sen, 1984, Ch. 5*). For an overall goodness ordering to be impartial between individuals just is for it to be insensitive to the identities of individuals, and thus to who attains which level of welfare in which outcome. We may say that the impartiality of a social welfare functional amounts to treating all individuals equally (i.e., not privileging any individual's welfare over those of others). Pursuing the analogy, **PI** amounts to a constraint of *dimensioned* impartiality, to the effect that all dimensions are considered on a par (i.e., that the goodness along some dimension is not privileged over that along another). With this in mind, we may consider the plausibility of **PI** by considering whether impartiality about dimensions of goodness is required.

While the requirement of impartiality for individuals appears uncontroversial (perhaps on grounds of fairness, or similar considerations), the corresponding requirement for dimensions appears less so. Unlike with individuals, there appears to be no immediate reason against treating dimensions partially, in the sense of according some more importance than others, or even holding to a strict order of importance between 'competing' dimensions. Indeed, it seems perfectly permissible for an individual's goodness ordering to be determined via a hierarchy of dimensions, each dimension taking precedence over the one following it when it comes to comparing outcomes overall. Such 'lexicographic' rules are obviously conflict with **PI**, and hence one of them is to be rejected. To illustrate on our example, it seems perfectly permissible that the goodness ordering for some individual privileges belonging over other dimensions of goodness. (As before, we may still stay neutral on whether what is better for the individual is subjective or objective, since the judgment of permissibility persists on both renderings.) This is easily seen to conflict with **PI**. If b is privileged, o is better than o', since o is better along b than o' is, and o'^* is better than o^* since o'^* is better along b than o^* is. But this conflicts with the verdict of **PI** that o^* is better than o'^*. Thus, provided the overall goodness ordering is determined via lexicographic rules (such as the rule that b is privileged), **PI** is false.

The permissibility of dimensioned partiality in a multidimensionalist setting is open to doubt, however.[20] First, partiality makes some strange predictions against the backdrop of **IM**. Suppose that $\mathcal{D}$ is infinite and that g is privileged over any other $d \in \mathcal{D}$. Consider the

[20]We here focus on the multidimensionality of goodness, but our arguments plausibly generalise.

following distributions:

$$\begin{array}{cccccccccc} & g & d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 & \dots \\ o & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ o' & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & \dots \end{array}$$

Insofar as g is privileged, o is predicted to be better than o', and so despite o' being better along d_i than o (for any i) and better along d_i than o is along g (for any i). But this is implausible: if o' is better than o in infinitely many ways except one, surely o' is better than o overall, no matter what dimension is privileged for assessing overall goodness. Note that this generalises for any lexicographic order and distributions with n dimensions tied and a tie-breaking dimension d_{n+1}:

$$\begin{array}{ccccccccccccc} & \dots & d_{n-1} & d_n & d_{n+1} & d_{n+2} & d_{n+3} & d_{n+4} & d_{n+5} & d_{n+6} & d_{n+7} & \dots \\ o & \dots & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ o' & \dots & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & \dots \end{array}$$

In any such case, o is predicted to be better than o', and yet the prediction is implausible for reasons similar to those above. Insofar as these intuitive judgments have weight, this casts some doubt on the permissibility of partiality.

While compelling, these intuitions may well be distrusted (especially in light of our caution about appealing to intuitions in Section 3.2). Nevertheless, more principled considerations against partiality are also available in our setting. In most ordinary circumstances, partiality conflicts with a natural motivation for (infinitary) multidimensionalism about goodness. Recall that multidimensionalism about goodness holds that how good an outcome is depends on how good it is along different dimensions of goodness. The infinitary variant of multidimensionalism adds to this that there are infinitely many *relevant* dimensions of goodness. There are at least two sources of conflict between infinitary multidimensionalism and lexicographic rankings of dimensions of goodness. Here is a sketch:

1) The position of some dimension d_i in the lexicographic ranking is proportional to the extent to which it factors into the overall goodness ordering. The higher d_i ranks in the ordering, the more overall comparisons between pairs of outcomes it decides; the lower it ranks, the less so. In a lexicographic setting, the set of comparisons decided by a dimension just is its relevance for the overall goodness ordering. We may assume that, in most ordinary contexts, some finite set of dimensions suffices to determine the overall goodness ordering, with the other dimensions being idle. It is plausible, then, that those idle dimensions are *practically irrelevant* in ordinary contexts. But this conflicts with there being infinitely many relevant ways for outcomes to be good.

2) Say that one dimension-outcome ordering is more fine-grained than another if it induces more levels of goodness than the other.[21] The fineness-of-grain of a dimension-outcome ordering is proportional to the extent to which the high-ranking dimensions factor into the overall ordering. The more fine-grained the base ordering is, the more comparisons between pairs of outcomes are determined by the high-ranking dimensions; the less fine-grained, the less so. Plausibly, as the base ordering gets finer, the set of relevant dimensions (in the above sense) gets lower and tends to a single dimension. Provided the dimension-outcome ordering is fine-grained enough, then, this conflicts not only with infinitary multidimensionalism, but with multidimensionalism in general.

Arguably, then, ranking dimensions lexicographically undermines the motivation for (infinitary) multidimensionalism. In a word: provided some dimensions are privileged, most dimensions are irrelevant in ordinary contexts. Conversely, provided (infinitely) many dimensions are relevant, lexicographic rankings of dimensions are impermissible.

We submit that these are strong considerations against the permissibility of partiality about dimensions, and thus in favour of impartiality and, with it, **PI**. Although the truth of **PI** is still an open question in a multidimensional setting (see Muñoz, MSa), we provisionally accept **PI** and conclude that it should not be given up in the interest of avoiding the conclusion of **Corollary 1**.

3.4 Infinitely Many Dimensions of Goodness

The previous two sections defended some explicit premises of the argument against $\geq$-Completeness. We saw that **DSP**, **DPE**, and **PI** are plausible constraints on the overall goodness ordering, and that the independent reasons for rejecting them are defeasible. Assuming these constraints are true, we get the preliminary result that failures of completeness follow from infinitary multidimensionalism. More formally put, if **IM** is true, $\geq$-Completeness fails (i.e., there are at least two incomparable outcomes). Although this result is interesting in itself, the plausibility of **IM** should also be further considered.

IM may easily be seen as the weakest link in the argument for incompleteness based on **Corollary 1**. In fact, a sustained defence of the existence of infinitely many dimensions of goodness has, to our knowledge, not yet been given, and multidimensionalism about goodness in general is (as noted in fn. 6) a marginal position in the axiological literature. Nevertheless, the question of how many dimensions of goodness there are is still an open question (see Muñoz, MSa). Provided one is committed to $\geq$-Completeness, our defence of

[21]Formally: $\geq$ is more fine-grained than $\geq'$ if $|im(\rho^{\geq}(\cdot))| > |im(\rho^{\geq'}(\cdot))|$.

DSP, **DPE**, and **PI** could also be taken as an argument against **IM**. Moreover, if one finds **IM** independently implausible, this argument may further reinforce one's commitment to the existence of only finitely many ways in which an outcome can be good. Regardless of how compelling one finds this line of argument, strategies for defending **IM** should still be considered.

To gauge the strength of considerations against **IM**, consider a *prima facie* plausible argument appealing to the practice of practical reasoning. We assume, as throughout this article, that practical reasoning appeals to goodness orderings, which in turn depend on dimensions of goodness and comparisons between dimension-outcome pairs. The argument is as follows:

P1. If dimensions of goodness just are those considerations that factor in practical reasoning, then there are only finitely many dimensions of goodness.

P2. Dimensions of goodness just are those considerations that factor in practical reasoning.

C. There are only finitely many dimensions of goodness. (from **P1**, **P2**)

While initially compelling, it is difficult to jointly justify the premises on any one rendering of what dimensions of goodness are. In fact, the argument fails to go through both on a realist and an anti-realist approach to the ontological question. For clarity, we take realism about dimensions of goodness to be the view that dimensions of goodness exist independently of practical reasoners, and anti-realism to be the negation of realism. While this is only a rough statement of the relevant positions, it should suffice for our purposes.

First, suppose anti-realism is true, so that which dimensions of goodness exist depends on which considerations factor in practical reasoning. **P2** is then true by hypothesis. **P1** is false on an anti-realist approach, however. Judging by our current evidence, the universe may be infinite and may contain infinitely many individuals (see Bostrom, 2011; Askell, 2018, Ch. 1). Supposing these individuals are like us in relevant ways, they engage in practical reasoning in ways similar to us. But then, since infinitely many individuals engage in practical reasoning (of a kind relevantly similar to ours), it is not plausible that only finitely many dimensions of goodness are ever employed in practical reasoning. On the contrary, supposing the circumstances of at least some of these individuals are importantly different from ours (as is plausible, in an infinite universe), it is natural to assume that infinitely many dimensions of goodness are employed in practical reasoning. Thus, if anti-realism is true, **P1** is false and the argument unsound.

Second, suppose realism is true, so that which dimensions of goodness exist is independent of practical reasoning. **P2** is then arguably false. On a realist-friendly construal of **P2**, whatever factors in practical reasoning contingently includes all dimensions of goodness there are. In other words, the fact that our practical reasoning appeals to all the independently existing dimensions of goodness is modally fragile.[22] So construed, however, **P2** is arguably false. For it is much more plausible to assume that our practical reasoning does not in fact exhaust the axiological space, and that there exist ways of being good that are yet to factor in our practical reasoning. (Even if true, moreover, **P2** is epistemically opaque, for there is no reliable way for us to come to know it.) Thus, if realism is true, the antecedent is false and the argument unsound.

On both a realist and an anti-realist construal of dimensions of goodness, then, the argument against infinitary multidimensionalism fails to go through. Insofar as no other construal of dimensions of goodness makes both **P1** and **P2** true, then, we may consider the argument unsuccessful.

Our responses to the argument, moreover, serve to motivate the possibility of **IM**. In effect, it emerges that the possibility of **IM** is motivated both on a realist and an anti-realist construal of dimensions of goodness. As these two positions are plausibly exhaustive, this suggests that **IM** is possible *simpliciter*. On the one hand, if anti-realism is true, an infinite universe and infinitely many individuals entail the existence of infinitely many dimensions of goodness; since it is possible that the universe is infinite, it is possible that infinitely many dimensions of goodness exist; hence, **IM** is possibly true. Alternatively, if realism is true, it is implausible to assume that the axiological space is exhausted by the finitely many dimensions of goodness employed in practical reasoning; in fact, it is more natural to assume the contrary, or that the axiological space contains infinitely many dimensions of goodness; hence, **IM** is possibly true. Regardless of the construal, then, **IM** is arguably possible.

Although the *possibility* of **IM** suffices to establish that $\geq$-Completeness *possibly* fails, we might want a more categorical verdict on the central premise of our argument. Independent considerations can indeed motivate such a verdict. Recall that in Section 2.1 we stayed neutral on the ontological aspect of dimensions of goodness, or the question of what dimensions of goodness *are*. Consider the following two ways of approaching the ontological question.

- On a coarse-grained rendering, each dimension is a separate *kind* of goodness an outcome can have, where each kind encompasses more specific features of the outcome relevant for this kind. For instance, utility may plausibly be seen as a kind of good-

[22]We here disregard appeals to intuitive capacities that would allow us to identify all dimensions of goodness and thus make the (hypothetical) truth of **P2** modally robust.

ness, or a broad way in which an outcome can be good, with each outcome's utility encompassing the utilities of the outcome's 'parts'.

- On a fine-grained rendering, each dimension is a specific *feature* of a given outcome or a specific source of its goodness. (Note that such features plausibly 'instantiate' the kinds of goodness mentioned above.) For instance, a given outcome may be considered with respect to the number of apples and oranges it affords, with these features instantiating possibly different kinds of goodness.

While these renderings are different, they are not incompatible. Indeed, both renderings are routinely used in practical reasoning, in the sense that outcomes are sometimes compared with respect to what kinds of value they afford and sometimes with respect to their features. This is unsurprising given that the fine-grained construal refines the coarse-grained construal, in the sense that the former includes *more* information than the latter. Depending on the salient purpose, then, one may adopt either the less informative coarse-grained rendering or the more informative fine-grained rendering.

Going even further, the coarse-grained rendering arguably supervenes on the fine-grained rendering. More intuitively, the kinds of value an outcome has depend on its features, so that specifying the outcome's features suffices to specify its kinds of value. For instance, suppose that a given outcome's relevant features include the number of apples and oranges it affords, with a corresponding utility value for each feature, and that other features of the outcome are neutral on the dimension of utility. Specifying the utility the apple- and orange-features then suffices to specify how the outcome fares along utility (i.e., one of its kinds of goodness).

Insofar as they entirely determine an outcome's kinds of value, the outcome's features are arguably metaphysically fundamental. Moreover, insofar as they are more informative than the outcome's kinds of value, the outcome's features are arguably epistemically fundamental.[23] The metaphysical and epistemic fundamentality of features with respect to kinds of value suggests that, at least insofar as we seek to address the question of cardinality at the fundamental level, we should seek to address the question of how many evaluatively relevant features an outcome has (as opposed to how many kinds of value it has).

IM is an overwhelmingly plausible answer to this question.[24] Recall that an outcome, on our approach, represents a complete specification of the evaluatively relevant ways some world is. Since an outcome specifies all there is to specify about some world, and each

[23]This is not to say that kinds of value are unimportant; as noted above, they might have heuristic or computational significance in light of their comparative simplicity.

[24]This is not to say that **IM** is excluded on the coarse-grained rendering. Moreover, the argument for the possibility of **IM** given previously is neutral between these renderings, and thus holds regardless of whether we construe dimensions of goodness as kinds of value, features of outcomes, or yet in some third way.

world is ordinarily supposed to consist in infinitely many features, there are infinitely many dimensions of goodness *qua* evaluatively relevant features of some given outcome. Thus, provided the fine-grained rendering is fundamental, **IM** is a plausible hypothesis.

Let us take stock of our conclusions on the score of **IM**. We noted, first, that **IM** is *prima facie* implausible, and motivated a tentative argument against it. We suggested, however, that the argument fails to go through on both the objective and the subjective construal of dimensions of goodness, and that this counts against it. We argued, furthermore, that both construals in fact motivate at least the possibility, if not the truth, of infinitary multidimensionalism. Finally, we fleshed out a coarse- and fine-grained rendering of dimensions of goodness, argued that the latter is metaphysically and epistemically fundamental, and argued that the fine-grained rendering entails **IM**. We conclude from this that **IM** is at least possibly true and likely true *simpliciter*.

Where does this leave us with respect to the overarching argument for incompleteness? The initial implausibility of **IM** might have suggested that **IM** should be rejected in favour of preserving ≥-Completeness. The foregoing considerations show, however, that **IM** holds up under argumentative scrutiny and is independently defensible from at least two directions. This not only undermines the argument against **IM** from ≥-Completeness, but also leaves friends of ≥-Completeness who seek to reject **IM** with the burden of resisting the arguments presented above. More positively, given our defence of **IM** (and our previous defence of **DPE**, **DSP**, and **PI**), we are led to conclude that ≥-Completeness fails. Moreover, even if **IM** is only admitted as a possibility, we are still led to conclude that ≥-Completeness *possibly* fails. In sum: ≥-Completeness (possibly) fails, *pace* Dorr et al. (2023), Tarsney et al. (in press).

4

Section 3 defended Dimensioned Pareto Equivalence, Dimensioned Strict Pareto, Permutation Invariance, and (the possibility of) Infinitary Multidimensionalism. We thus conclude that ≥-Completeness (possibly) fails, or that there (possibly) exist pairs of incomparable outcomes. This section (i) outlines how our argument differs from a prominent argument to the same conclusion and (ii) proposes an explanation of incomparability between outcomes that fares better than those previously proposed in the literature.

4.1 *The Small Improvement Argument*

To see how our argument differs from the prominent Small Improvement Argument (SIA) against completeness (first proposed by Chang (1997)),[25] we sketch it briefly. (We here employ an example by Hedden (2024) and stay neutral on the underlying framework in which the relevant claims may be formalised.) Suppose you find yourself in a burning house and you can save either your wedding album or a first edition copy of Descartes' *Meditations*. Saving the album is (possibly) not better than saving the Descartes, and that saving the Descartes is (possibly) not better than saving the album. Note that, if two outcomes are equally good, improving one of them makes the improved outcome better than the other. Saving the album and saving the Descartes are not equally good since, (it is possible that) saving the album with a $10 bill inside it is not better than saving the Descartes. But then (it is possible that) the two outcomes are incomparable: neither outcome is better than the other, and they are not equally good.

We should note, first, that our argument for (possible) failures of completeness is different in form from the SIA to the same conclusion. Our argument, for its part, derives failures of completeness from infinitary multidimensionalism about goodness and independently plausible principles. As such, it does not and need not appeal to intuitions about comparisons between outcomes and the putative intuitive plausibility of the existence of incomparable outcomes. The SIA, on the contrary, hinges on such intuitions: we consider, first, two outcomes o and o' and are supposed to judge that (possibly) neither is better than the other; we consider, second, a small improvement to one of the outcomes (e.g. o^+) and are supposed to judge, of o^+ and o', that (possibly) neither is better than the other. The arguments are thus importantly different.

Second, and relatedly, some immediate objections to the SIA are non-starters against our argument. Here's one such objection. While implausible as a description of the preferences of actual agents, completeness might be seen as a simple and strong modelling assumption. Indeed, orthodox decision theory assumes that an agent's preferences are complete. To accept the intuition that some outcomes are incomparable and modify orthodox decision theory to accommodate it is a case of *overfitting* the decision-theoretic model to some intuition-based data (cf. Williamson, 2024). Particularly, instead of adopting an ideologically simpler theory (e.g. by taking all putatively incomparable outcomes as equally good), one introduces a new primitive relation (namely, incomparability) to accommodate the relevant intuitions.[26] Insofar as overfitting is objectionable in such cases, this counts against the SIA.

[25] Variants of this argument have been explored by De Sousa (1974), Hare (2010), *inter alia*.

[26] *Cf.* Chang (2017), who introduces the notion of *parity* as a primitive to denote a relation between two outcomes such that neither is at least as good as the other.

While we consider this a compelling consideration against incomparability in cases such as that above, and the SIA particularly, the overfitting objection is not effective against our argument. As our argument does not appeal to intuitions, it specifically avoids the charge of overfitting the model to the data. Moreover, the argument shows that incompleteness is a non-trivial consequence of a package of seemingly independent principles and a position on the cardinality of dimensions of goodness. As such, the argument creates a problem for those who are sympathetic to **IM** and the principles **DPE**, **DSP**, and **PI**. For friends of $\geq$-Completeness, then, dealing with our argument is not simply a matter of 'equating' putatively incomparable outcomes, but also of rejecting one of the principles $\geq$-Completeness is inconsistent with and justifying one's choice accordingly.

Before continuing, we should note that our argument significantly alters the dialectic in the current literature. As noted previously, recent arguments have defended $\geq$-Completeness by appealing to (a) the intuitive validity of natural language arguments involving comparatives, along with the implausibility of some consequences of incomparability (Dorr et al., 2023), and (b) the inconsistency between incomparability and independently plausible principles about choice under uncertainty (Tarsney et al., in press; Lederman, MS). In the absence of independent arguments for incompleteness, such theoretical considerations may be thought to undermine its pre-theoretical appeal. By showing that incompleteness is a consequence of seemingly independent commitments about the nature of goodness, our argument bolsters the case for incompleteness and opens new choice-points for friends of $\geq$-Completeness.

4.2 *Explaining Incomparability*

Our argument is not only different from the most prominent argument for incompleteness, but also allows a novel and natural explanation of incomparability.

To appreciate this, we survey some explanations of incompleteness proposed in the literature. (We here rely on the discussion in Chang (2017).) (i) One way to explain incomparability is via *ignorance*: some two outcomes are incomparable because the agent is ignorant of all evaluative facts pertaining to the outcomes. (ii) Another way is via *incommensurability*: some two outcomes are incomparable because some dimensions of goodness (e.g., human life) have a special status that makes them incomparable with other dimensions (e.g. pleasure). (iii) Yet another way, similar to the previous, is via more basic *incomparability*: some two outcomes are incomparable because the goodness of outcomes cannot, as a matter of principle, be compared along different dimensions (see Section 3.1 for further discussion).

Regardless of their other faults, our argument suggests that these explanations are not even extensionally correct. Note that we assumed that the agent has access to all the relevant evaluative facts, or that all the dimensions of goodness are accessible to the agent; furthermore, we assumed that outcomes are intra- *and* inter-dimensionally comparable, so that there is neither incommensurability nor incomparability in the fine-grained ordering over dimension-outcome pairs.[27] We saw, however, that incomparability arises even in such settings. In other words, regardless of how hospitable the background is to $\geq$-Completeness, the principle still fails to hold. But this shows that incomparability is *consistent* with knowledge of all evaluative facts and complete fine-grained comparability of outcomes along all relevant dimensions, which conflicts with the explanations above.

Our setting provides a natural explanation of incomparability without appealing to ignorance, incommensurability, or basic incomparability. On our account, incomparability is attributable to a particular way two outcomes may compare when infinitely many dimensions are relevant for their comparison. More specifically, in a multidimensionalist setting, there might be two outcomes that entirely differ as to their infinitely many 'upsides' (i.e., dimensions along which they are better than the other). But this plausibly suggests that the two are incomparable: if the upsides of two outcomes are entirely different, and there are infinitely many such upsides, there appears to be no principled basis for comparison.

Arguably, this is confluent with our intuitions about incomparability. Cases of incomparability seem to be cases in which the competing considerations (in the sense of the outcomes' upsides) entirely fail to settle the comparison in favour of either outcome. Otherwise put, cases of incomparability are those in which the underlying dimensional comparisons offer competing but entirely distinct considerations in favour of either outcome. In such cases, neither outcome has a 'decisive lead' over the other (thus ruling out betterness) and the dimensions speaking in their favour fail to 'even out' (thus ruling out equal goodness).

Although this explanation is sketchy, the proof of **Fact 1** illustrates the sort of case we have in mind. As such, the proof is not only a proof of inconsistency but also serves a constructive example of incomparability. Recall, first, that since o is better than o^- along at least one dimension (and equal or better along others), o is better than o^-. (Our defence of **DSP** in Section 3.2 allows us to admit this verdict as true.) Recall, second, that u is equated in goodness with o, since neither can be better than the other on permutation-theoretic grounds (and similarly for u and o^-). But this is a patently inconsistent trio, and one is pressured to give up either of these equivalences. Although giving up either equivalence suffices for incomparability, parity of reasoning suggests both that u is incomparable with o *and* that u is incomparable with o^-. This is unsurprising, on our explanation: the considerations in favour of o – i.e., the dimensions in A – and those in favour of u – i.e., the dimensions in B –

[27] While both assumptions are implausible, recall that they are arguably dialectically appropriate (see Section 2.1).

are completely different (because disjoint), and so do not allow any principled comparison between o and u. (The case is similar for o^- and u, *mutatis mutandis*.)

It is worth noting that, while our explanation predicts pervasive failures of completeness, such failures are neatly circumscribed to a certain type of comparative situation. Since there may, in principle, be infinitely many cases in which infinitely many distinct dimensions favour either of two outcomes, the extent of incomparability is in principle unbounded. However, for all that we have said, this is the only type of case in which incomparable outcomes may be found. Specifically, infinitary multidimensionalism fails to predict incomparability whenever (i) there are only finitely many dimensions in favour of either outcome or (ii) there is any overlap in dimensions favouring some two outcomes. While unbounded, then, incomparability is safely circumscribed to a special case.

Before concluding, we should note that our explanation vindicates the core intuition behind the SIA: namely, the notion that a small improvement to one outcome in an incomparable pair should not tip the scales in favour of the improved outcome. Our setting thus vindicates a central motivation for incompleteness without appealing to it directly. Note, first, that o may be seen as a small improvement to o^-, in the sense of being better than o^- along all dimensions in the set $A \setminus A^-$ and not worse along any dimension in A^-.[28] (For familiar reasons, we easily get the result that such an improvement is better than what it improves upon.) By the reasoning above, we get that both o^- and o are incomparable with u. On the hypothesis that o^- is incomparable with u, then, we get the result that its improvement o is likewise incomparable with u.[29]

This concludes the comparison of our argument with the prominent Small Improvement Argument for incompleteness and our explanation of incomparability. On the former score, we submit that our argument has a decisive edge over the SIA due to its non-reliance on intuitions, and that it contributes to the dialectic in the current literature by creating new choice-points for friends of $\geq$-Completeness. On the latter, we submit that our explanation has more purchase than the ones offered previously, that it is independently plausible, and that it (indirectly) vindicates the core intuitions behind SIA.

5

The view that some outcomes cannot be compared with respect to how good they are enjoys pre-theoretic appeal and has found broad support. In recent years, however, independent

[28] To be sure, the relevant improvement may also be 'large', since $A \setminus A^-$ can contain arbitrarily (finitely) many dimensions.

[29] Our explanation also vindicates the related prediction that incomparability is intransitive: while o^- is incomparable with u and u is incomparable with o, o^- need not be incomparable with o.

arguments against incompleteness have emerged, appealing to considerations as different as the logic of comparatives and choice under uncertainty. Such theoretical arguments shift the balance of reasons in favour of completeness, arguably undermining intuitive support for the alternative. In the absence of further motivation, it is reasonable to conclude that personal goodness orderings are (necessarily) complete, or that all outcomes are (necessarily) comparable.

This article proposed a new argument for incompleteness appealing to the multidimensionality of 'goodness'. Drawing on a result from an analogous setting, we outlined the result that an infinitary domain of dimensions of goodness, paired with two weak Pareto principles and a principle about permutations, is inconsistent with completeness for the overall goodness ordering. More intuitively, the argument is that the infinitary richness of the axiological space excludes settling how any two outcomes compare, all things considered. By justifying the background principles and motivating (the possibility of) infinitary multidimensionalism, we argued for (the possibility of) incompleteness.

It is useful to summarise how this article contributes to different strands of the literature on goodness and value. First: in discussing the argument for incompleteness, we began addressing some open questions on the multidimensionality of goodness (see Muñoz, MSa, for an overview of some open questions). For one, we touched on the ontological question of *what* dimensions of goodness are (Section 3.4); we specified that they can be construed both as kinds of value and more specific features of outcomes, and argued that features of outcomes are fundamental. For another, we discussed the question of *how many* dimensions of goodness there are (Section 3.4); we dispelled the initial implausibility of an infinitary axiological space and offered independent considerations in favour thinking there (possibly) are infinitely many dimensions. For yet another, we discussed the plausibility of (im)partiality for dimensions (Section 3.3); although it appears permissible to favour some dimensions over others, we argued that partiality conflicts with multidimensionalism in ordinary circumstances and suggested that it be rejected.

Second: our argument shows that a commitment to completeness equally commits one to non-trivial claims about the nature of the axiological space and as such opens new choice-points for friends of completeness. Defenders of incompleteness must reject at least one of **IM**, **DSP**, **DPE**, and **PI** and justify this choice against our defence of these principles in Section 3.

Finally, our argument also improves on the existing arguments for incompleteness and offers deeper insight into the phenomenon of incompleteness. As for the former, our argument importantly differs from the Small Improvement Argument in that it motivates incompleteness by appeal to independently plausible principles, rather than by appeal to intuitions (Section 4.1). As for the latter, the background axiological setting may also be used to motivate an explanation of incompleteness, on which incompleteness arises whenever there are two disjoint sets of considerations in favour of either outcome (Section 4.2).

While reflection on the multidimensionality of goodness is still in its early stages, we hope to have shown that it is a fertile avenue of research with significant consequences for axiological theorising. In particular, we conclude that the question about goodness comparisons discussed in this article is far from being settled in favour of completeness, and that incompleteness is still in the running.

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