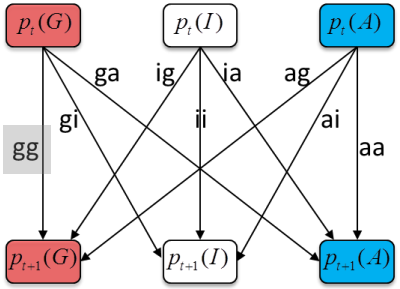
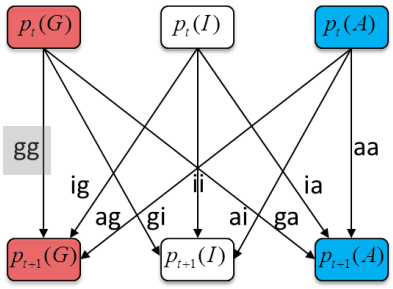
(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

$p_t(G)$ = proportion of $Goers_t$ (at time t)

$p_t(A)$ – proportion of $Absentees_t$ (at time t)

$p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

ga – $Goers_t$ who become $Absentees_{t+1}$

ig – $Irregulars_t$ who become $Goers_{t+1}$

ii – $Irregulars_t$ who stay $Irregulars_{t+1}$

ia – $Irregulars_t$ who become $Absentees_{t+1}$

ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

aa – $Absentees_t$ who stay $Absentees_{t+1}$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

gg *gi* *ga*

ig *ii* *ia*

ag *ai* *aa*

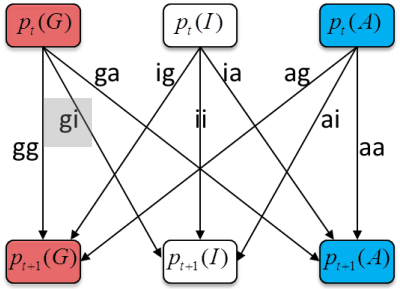
(*g*) Counting with Diffusion

$$p_{t+1}(G) = p_t(G) - k_{gi}\,p_t(G) - k_{ga}\,p_t(G) + k_{ig}\,p_t(I) + k_{ag}\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - k_{ag}\,p_t(A) - k_{ai}\,p_t(A) + k_{ga}\,p_t(G) + k_{ia}\,p_t(I)$$

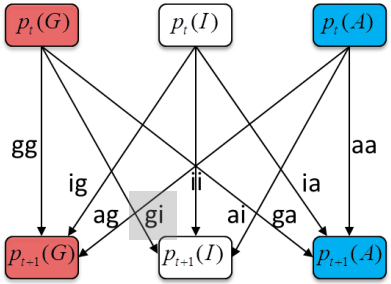
(*h*) Diffusion predicts transitions:

$$gi = k_{gi}\,p_t(G)$$
$$ga = k_{ga}\,p_t(G)$$
$$ig = k_{ig}\,p_t(I)$$
$$ia = k_{ia}\,p_t(I)$$
$$ag = k_{ag}\,p_t(A)$$
$$ai = k_{ai}\,p_t(A)$$

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

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$p_t(G)$ = proportion of $Goers_t$ (at time t)

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gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

ga – $Goers_t$ who become $Absentees_{t+1}$

ig – $Irregulars_t$ who become $Goers_{t+1}$

ii – $Irregulars_t$ who stay $Irregulars_{t+1}$

ia – $Irregulars_t$ who become $Absentees_{t+1}$

ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

aa – $Absentees_t$ who stay $Absentees_{t+1}$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
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Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Diffusion

$$p_{t+1}(G) = p_t(G) - k_{gi}\,p_t(G) - k_{ga}\,p_t(G) + k_{ig}\,p_t(I) + k_{ag}\,p_t(A)$$
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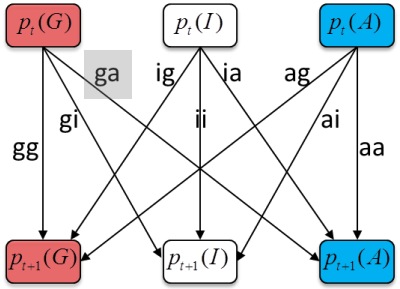
$$ag = k_{ag}\,p_t(A)$$
$$ai = k_{ai}\,p_t(A)$$

gg	*gi*	*ga*
ig	*ii*	*ia*
ag	*ai*	*aa*

(*a*) Those who leave



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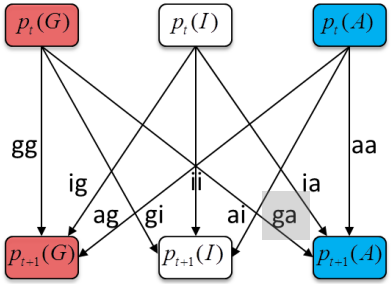

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

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$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

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(*c*) This year

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Diffusion

Contagion

Resilience

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$$gg = (p_t(G) - gi - ga)$$
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$$p_{t+1}(G) = p_t(G) - k_{gi}\,p_t(G) - k_{ga}\,p_t(G) + k_{ig}\,p_t(I) + k_{ag}\,p_t(A)$$
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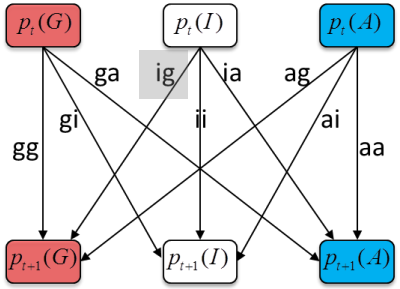
$$ai = k_{ai}\,p_t(A)$$

gg	*gi*	*ga*
ig	*ii*	*ia*
ag	*ai*	*aa*

(*a*) Those who leave



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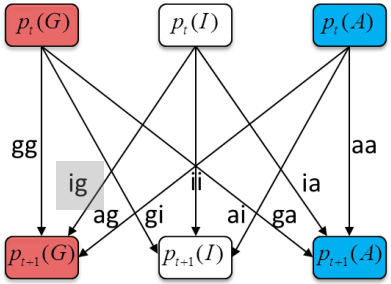

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Diffusion

Contagion

Resilience

(*g*) Counting with Diffusion

$$p_{t+1}(G) = p_t(G) - k_{gi}\,p_t(G) - k_{ga}\,p_t(G) + k_{ig}\,p_t(I) + k_{ag}\,p_t(A)$$
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(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
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$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

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ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

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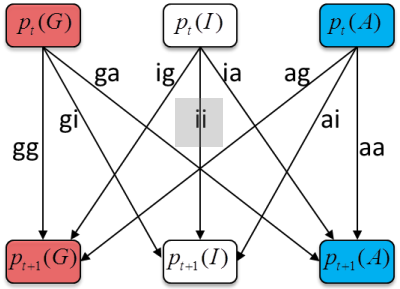
gg	gi	ga
ig	ii	ia
ag	ai	aa

(a) Those who leave



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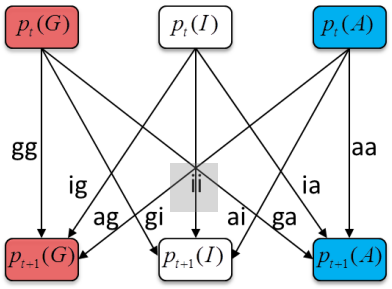

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Diffusion

Contagion

Resilience

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$$gg = (p_t(G) - gi - ga)$$
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(g) Counting with Diffusion

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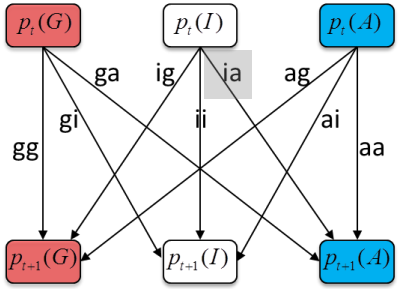
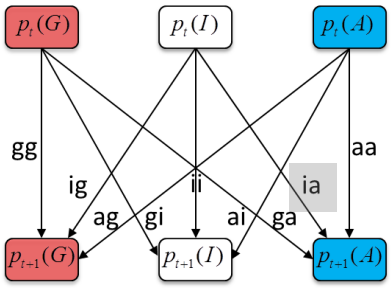
gg	gi	ga
ig	ii	ia
ag	ai	aa

(*a*) Those who leave



(*b*) Those who come



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Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
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gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

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gg	gi	ga
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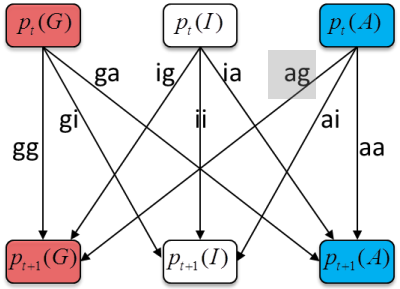

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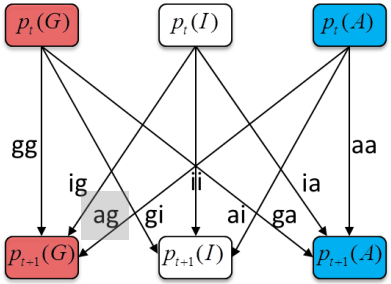

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Diffusion

Contagion

Resilience

(*g*) Counting with Diffusion

$$p_{t+1}(G) = p_t(G) - k_{gi}\,p_t(G) - k_{ga}\,p_t(G) + k_{ig}\,p_t(I) + k_{ag}\,p_t(A)$$
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(*f*) Redundancy/Constraints

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gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

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ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

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gg	gi	ga
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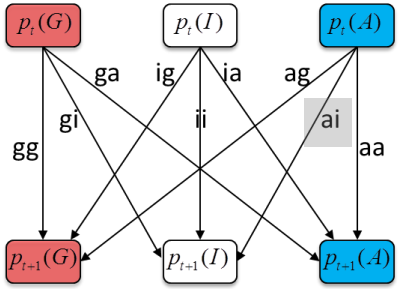
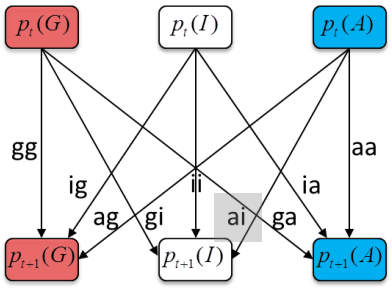


(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*g*) Counting with Diffusion

$$p_{t+1}(G) = p_t(G) - k_{gi}\,p_t(G) - k_{ga}\,p_t(G) + k_{ig}\,p_t(I) + k_{ag}\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - k_{ag}\,p_t(A) - k_{ai}\,p_t(A) + k_{ga}\,p_t(G) + k_{ia}\,p_t(I)$$

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*h*) Diffusion predicts transitions:

$$gi = k_{gi}\,p_t(G)$$
$$ga = k_{ga}\,p_t(G)$$
$$ig = k_{ig}\,p_t(I)$$
$$ia = k_{ia}\,p_t(I)$$
$$ag = k_{ag}\,p_t(A)$$
$$ai = k_{ai}\,p_t(A)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

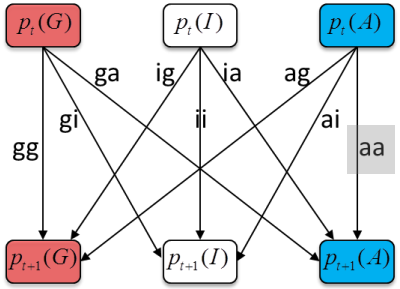
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

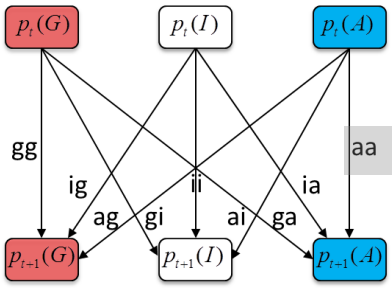
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Diffusion

$$p_{t+1}(G) = p_t(G) - k_{gi}\,p_t(G) - k_{ga}\,p_t(G) + k_{ig}\,p_t(I) + k_{ag}\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - k_{ag}\,p_t(A) - k_{ai}\,p_t(A) + k_{ga}\,p_t(G) + k_{ia}\,p_t(I)$$

(*h*) Diffusion predicts transitions:

$$gi = k_{gi}\,p_t(G)$$
$$ga = k_{ga}\,p_t(G)$$
$$ig = k_{ig}\,p_t(I)$$
$$ia = k_{ia}\,p_t(I)$$
$$ag = k_{ag}\,p_t(A)$$
$$ai = k_{ai}\,p_t(A)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

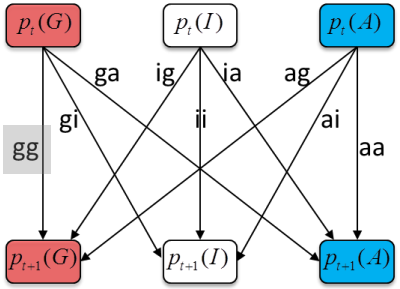
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg gi ga

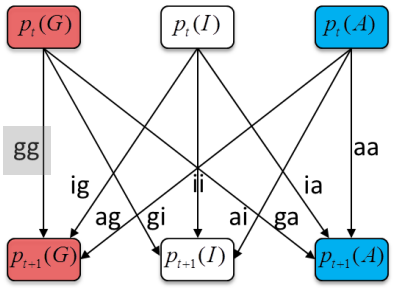
ig ii ia

ag ai aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

$p_t(G)$ = proportion of $Goers_t$ (at time t)

$p_t(A)$ – proportion of $Absentees_t$ (at time t)

$p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

ga – $Goers_t$ who become $Absentees_{t+1}$

ig – $Irregulars_t$ who become $Goers_{t+1}$

ii – $Irregulars_t$ who stay $Irregulars_{t+1}$

ia – $Irregulars_t$ who become $Absentees_{t+1}$

ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

aa – $Absentees_t$ who stay $Absentees_{t+1}$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

gg *gi* *ga*

ig *ii* *ia*

ag *ai* *aa*

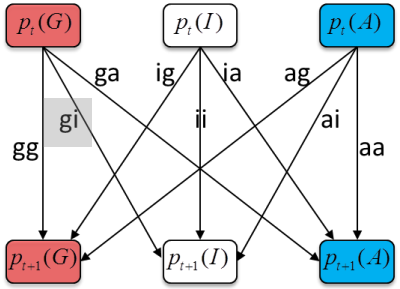
(*g*) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

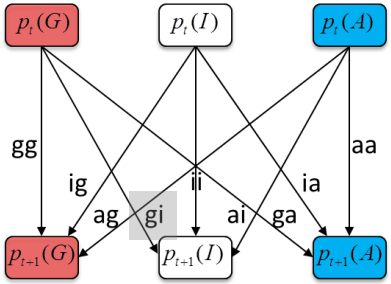
(*h*) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

(a) Those who leave



(b) Those who come



(c) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

(h) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

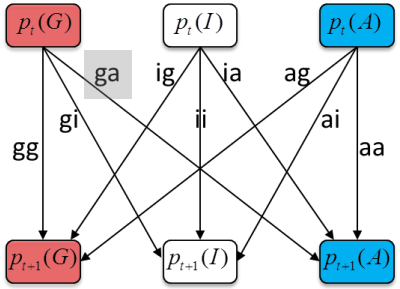
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

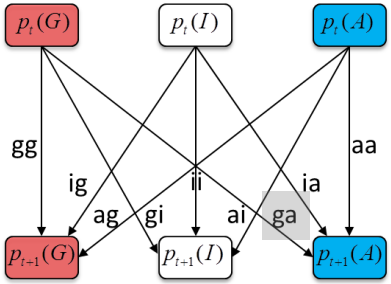
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

(*h*) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

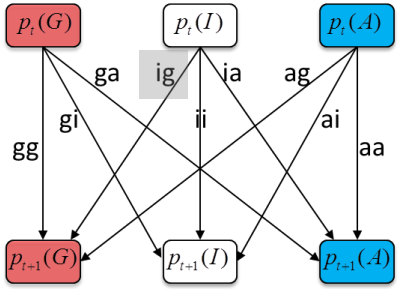
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

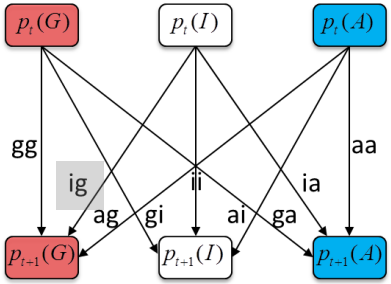
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	*gi*	*ga*
ig	*ii*	*ia*
ag	*ai*	*aa*

(a) Those who leave



(b) Those who come



(c) This year

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(d) Next year

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(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
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Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

(h) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
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$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

$p_t(G)$ = proportion of $Goers_t$ (at time t)

$p_t(A)$ – proportion of $Absentees_t$ (at time t)

$p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

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ia – $Irregulars_t$ who become $Absentees_{t+1}$

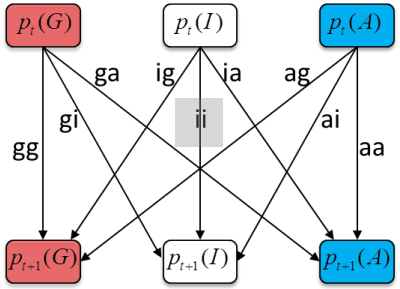
ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

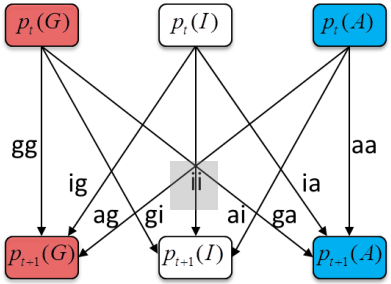
aa – $Absentees_t$ who stay $Absentees_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
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$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
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$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

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ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

gg gi ga

ig ii ia

ag ai aa

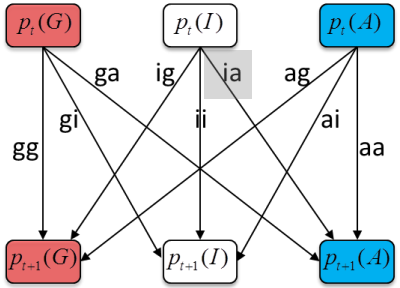
(*g*) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

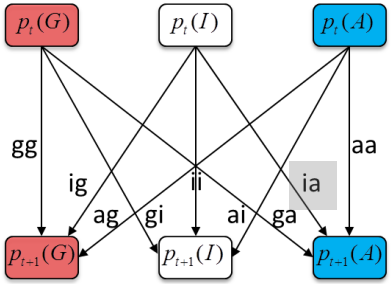
(*h*) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

(a) Those who leave



(b) Those who come



(c) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

(h) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

$p_t(G)$ = proportion of $Goers_t$ (at time t)

$p_t(A)$ – proportion of $Absentees_t$ (at time t)

$p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

ga – $Goers_t$ who become $Absentees_{t+1}$

ig – $Irregulars_t$ who become $Goers_{t+1}$

ii – $Irregulars_t$ who stay $Irregulars_{t+1}$

ia – $Irregulars_t$ who become $Absentees_{t+1}$

ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

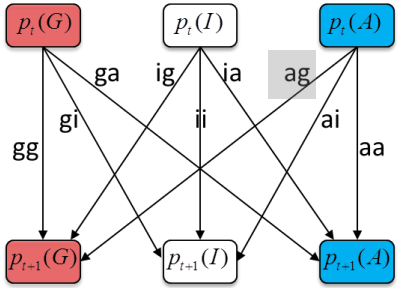
aa – $Absentees_t$ who stay $Absentees_{t+1}$

gg gi ga

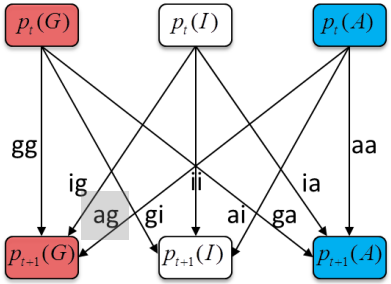
ig ii ia

ag ai aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

(*h*) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

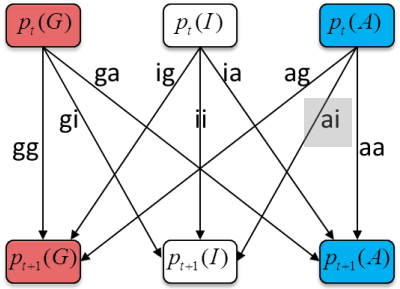
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

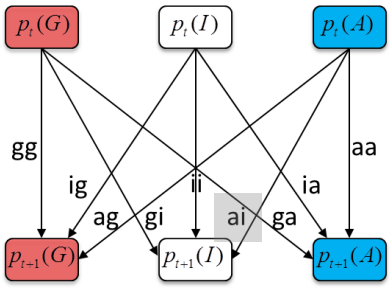
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(a) Those who leave



(b) Those who come



(c) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

(h) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

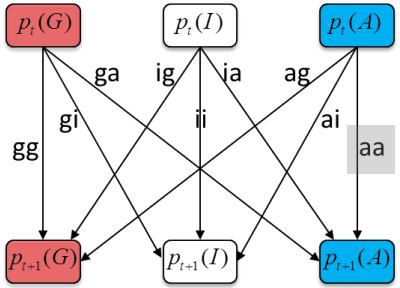
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

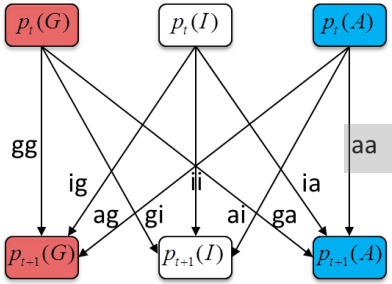
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(a) Those who leave



(b) Those who come



(c) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Contagion

$$p_{t+1}(G) = p_t(G) - T_{gi}\,p_t(G)\,p_t(I) - T_{ga}\,p_t(G)\,p_t(A) + T_{ig}\,p_t(I)\,p_t(G) + T_{ag}\,p_t(A)\,p_t(G)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - T_{ag}\,p_t(A)\,p_t(G) - T_{ai}\,p_t(A)\,p_t(I) + T_{ga}\,p_t(G)\,p_t(A) + T_{ia}\,p_t(I)\,p_t(A)$$

(h) Contagion predicts transitions:

$$gi = T_{gi}\,p_t(G)\,p_t(I)$$
$$ga = T_{ga}\,p_t(G)\,p_t(A)$$
$$ig = T_{ig}\,p_t(I)\,p_t(G)$$
$$ia = T_{ia}\,p_t(I)\,p_t(A)$$
$$ag = T_{ag}\,p_t(A)\,p_t(G)$$
$$ai = T_{ai}\,p_t(A)\,p_t(I)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

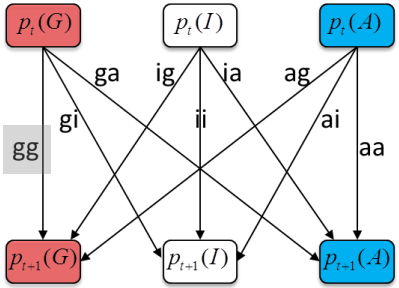
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

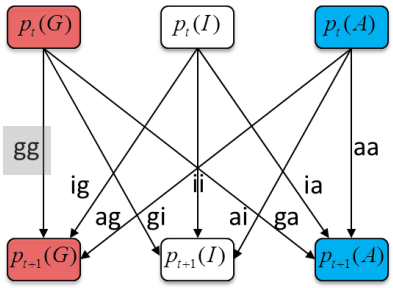
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(a) Those who leave



(b) Those who come



(c) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Resilience

$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

(h) Resilience predicts transitions:

$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

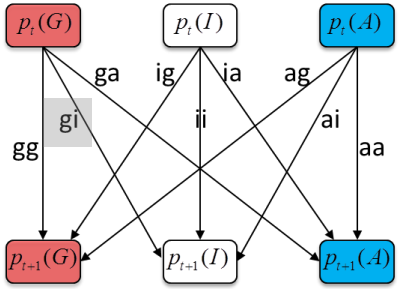
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

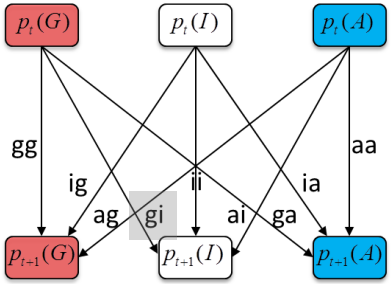
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(a) Those who leave



(b) Those who come



(c) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Resilience

$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

(h) Resilience predicts transitions:

$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

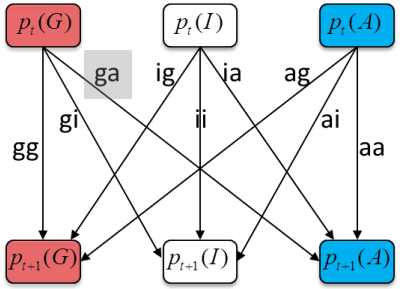
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

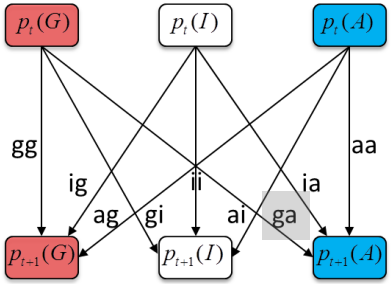
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	*gi*	*ga*
ig	*ii*	*ia*
ag	*ai*	*aa*

(*a*) Those who leave



(*b*) Those who come



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$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Resilience

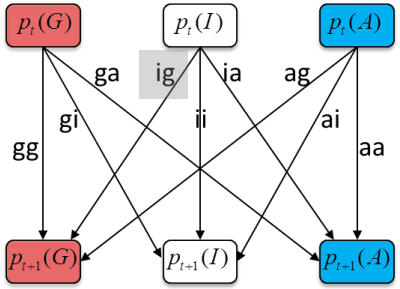
$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

(*h*) Resilience predicts transitions:

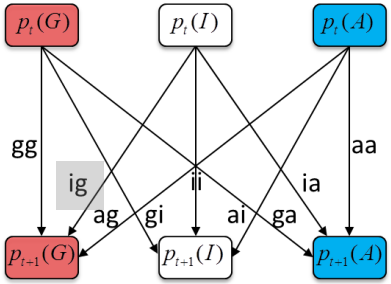
$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(a) Those who leave



(b) Those who come



(c) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(g) Counting with Resilience

$$p_{t+1}(G) = p_t(G) - K_{gi}p_t(G)p_t(G) - K_{ga}p_t(G)p_t(G) + K_{ig}p_t(I)p_t(I) + K_{ag}p_t(A)p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}p_t(A)p_t(A) - K_{ai}p_t(A)p_t(A) + K_{ga}p_t(G)p_t(G) + K_{ia}p_t(I)p_t(I)$$

(h) Resilience predicts transitions:

$$gi = K_{gi}p_t(G)p_t(G)$$
$$ga = K_{ga}p_t(G)p_t(G)$$
$$ig = K_{ig}p_t(I)p_t(I)$$
$$ia = K_{ia}p_t(I)p_t(I)$$
$$ag = K_{ag}p_t(A)p_t(A)$$
$$ai = K_{ai}p_t(A)p_t(A)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

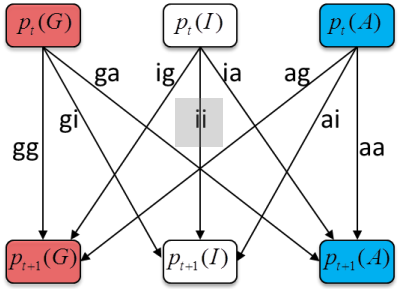
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

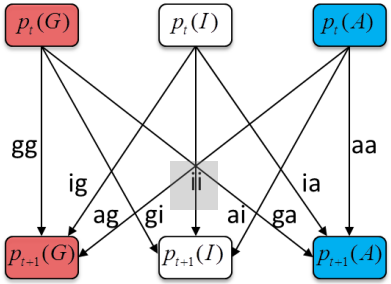
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

$p_t(G)$ = proportion of $Goers_t$ (at time t)

$p_t(A)$ – proportion of $Absentees_t$ (at time t)

$p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

ga – $Goers_t$ who become $Absentees_{t+1}$

ig – $Irregulars_t$ who become $Goers_{t+1}$

ii – $Irregulars_t$ who stay $Irregulars_{t+1}$

ia – $Irregulars_t$ who become $Absentees_{t+1}$

ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

aa – $Absentees_t$ who stay $Absentees_{t+1}$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Resilience

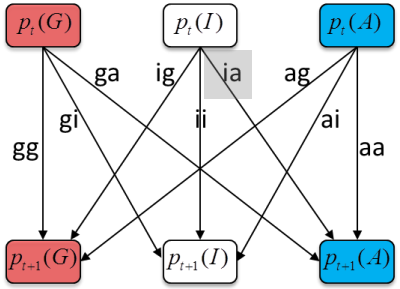
$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

(*h*) Resilience predicts transitions:

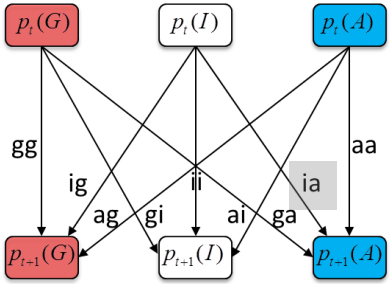
$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Resilience

$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

(*h*) Resilience predicts transitions:

$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

ga – *Goers*$_t$ who become *Absentees*$_{t+1}$

ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

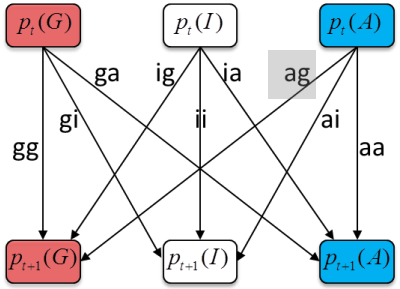
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

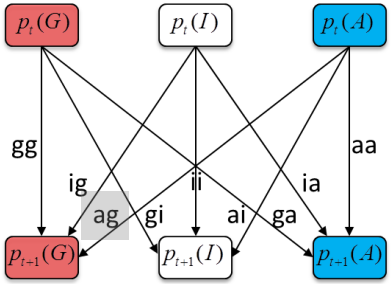
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	*gi*	*ga*
ig	*ii*	*ia*
ag	*ai*	*aa*

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

$$p_t(G) = gg + gi + ga$$
$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

$p_t(G)$ = proportion of $Goers_t$ (at time t)

$p_t(A)$ – proportion of $Absentees_t$ (at time t)

$p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

ga – $Goers_t$ who become $Absentees_{t+1}$

ig – $Irregulars_t$ who become $Goers_{t+1}$

ii – $Irregulars_t$ who stay $Irregulars_{t+1}$

ia – $Irregulars_t$ who become $Absentees_{t+1}$

ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

aa – $Absentees_t$ who stay $Absentees_{t+1}$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

gg *gi* *ga*

ig *ii* *ia*

ag *ai* *aa*

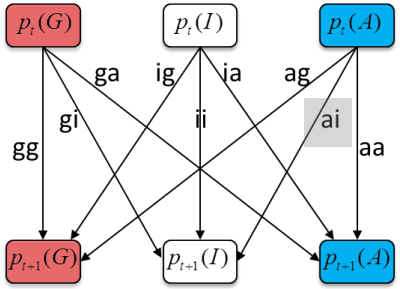
(*g*) Counting with Resilience

$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

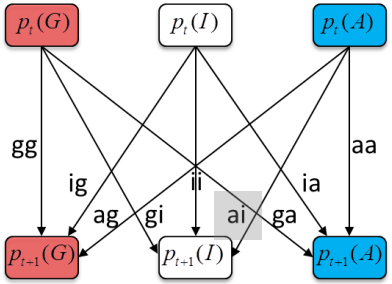
(*h*) Resilience predicts transitions:

$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

(a) Those who leave



(b) Those who come



(c) This year

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$$p_t(I) = ig + ii + ia$$
$$p_t(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$
$$p_{t+1}(I) = ii + gi + ai$$
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(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
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Diffusion

Contagion

Resilience

(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
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(g) Counting with Resilience

$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

(h) Resilience predicts transitions:

$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

$p_t(G)$ = proportion of *Goers*$_t$ (at time t)

$p_t(A)$ – proportion of *Absentees*$_t$ (at time t)

$p_{t+1}(I)$ = proportion of *Irregulars*$_{t+1}$

gg – *Goers*$_t$ who stay *Goers*$_{t+1}$

gi – *Goers*$_t$ who become *Irregulars*$_{t+1}$

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ig – *Irregulars*$_t$ who become *Goers*$_{t+1}$

ii – *Irregulars*$_t$ who stay *Irregulars*$_{t+1}$

ia – *Irregulars*$_t$ who become *Absentees*$_{t+1}$

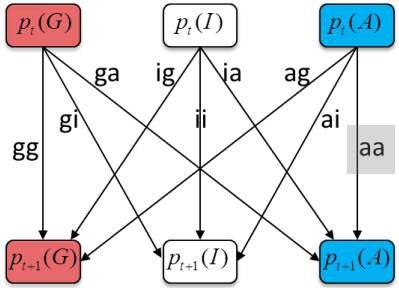
ag – *Absentees*$_t$ who become *Goers*$_{t+1}$

ai – *Absentees*$_t$ who become *Irregulars*$_{t+1}$

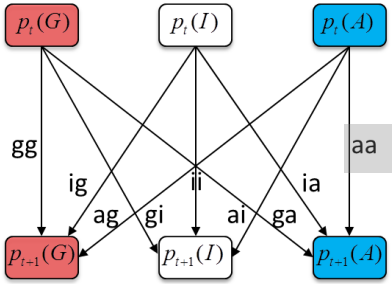
aa – *Absentees*$_t$ who stay *Absentees*$_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa

(*a*) Those who leave



(*b*) Those who come



(*c*) This year

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(*d*) Next year

$$p_{t+1}(G) = gg + ig + ag$$
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$p_t(G)$ = proportion of $Goers_t$ (at time t)

$p_t(A)$ – proportion of $Absentees_t$ (at time t)

$p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

gg – $Goers_t$ who stay $Goers_{t+1}$

gi – $Goers_t$ who become $Irregulars_{t+1}$

ga – $Goers_t$ who become $Absentees_{t+1}$

ig – $Irregulars_t$ who become $Goers_{t+1}$

ii – $Irregulars_t$ who stay $Irregulars_{t+1}$

ia – $Irregulars_t$ who become $Absentees_{t+1}$

ag – $Absentees_t$ who become $Goers_{t+1}$

ai – $Absentees_t$ who become $Irregulars_{t+1}$

aa – $Absentees_t$ who stay $Absentees_{t+1}$

(*e*) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - ag - ai + ga + ia$$

Diffusion

Contagion

Resilience

(*f*) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$
$$aa = (p_t(A) - ag - ai)$$
$$ii = 1 - gg - gi - ga - ia - aa - ig - ag - ai$$

(*g*) Counting with Resilience

$$p_{t+1}(G) = p_t(G) - K_{gi}\,p_t(G)\,p_t(G) - K_{ga}\,p_t(G)\,p_t(G) + K_{ig}\,p_t(I)\,p_t(I) + K_{ag}\,p_t(A)\,p_t(A)$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$
$$p_{t+1}(A) = p_t(A) - K_{ag}\,p_t(A)\,p_t(A) - K_{ai}\,p_t(A)\,p_t(A) + K_{ga}\,p_t(G)\,p_t(G) + K_{ia}\,p_t(I)\,p_t(I)$$

(*h*) Resilience predicts transitions:

$$gi = K_{gi}\,p_t(G)\,p_t(G)$$
$$ga = K_{ga}\,p_t(G)\,p_t(G)$$
$$ig = K_{ig}\,p_t(I)\,p_t(I)$$
$$ia = K_{ia}\,p_t(I)\,p_t(I)$$
$$ag = K_{ag}\,p_t(A)\,p_t(A)$$
$$ai = K_{ai}\,p_t(A)\,p_t(A)$$

gg	*gi*	*ga*
ig	*ii*	*ia*
ag	*ai*	*aa*