

(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

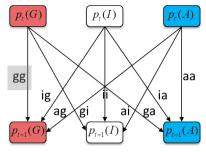
$$p_{t+1}(G) = p_{t}(G) - k_{gi}p_{t}(G) - k_{ga}p_{t}(G) + k_{io}p_{t}(I) + k_{ao}p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - k_{ag} p_t(A) - k_{ai} p_t(A)$$

$$+ k_{ga} p_t(G) + k_{ia} p_t(I)$$

(b) Those who come



(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$

$$aa = (p_{\bullet}(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga = k_{\alpha a} p_{t}(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_{t}(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

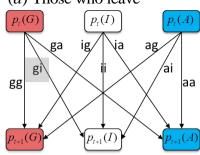
ig

ii

ia

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

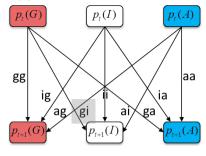
$$p_{t+1}(G) = p_{t}(G) - k_{gi}p_{t}(G) - k_{ga}p_{t}(G) + k_{io}p_{t}(I) + k_{ao}p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - k_{ag} p_t(A) - k_{ai} p_t(A)$$

$$+ \, k_{ga} \, p_t(G) + k_{ia} \, p_t(I)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$

$$aa = (p_t(A) - \overline{ag - ai})$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga = k_{ga} p_t(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_{t}(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

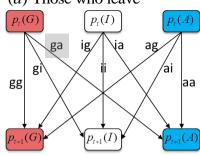
ig

ii

ia

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year $p_{t+1}(G) = gg + ig + ag$

$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

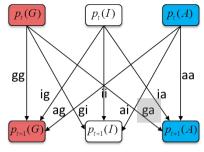
(g) Counting with Diffusion

$$p_{t+1}(G) = p_{t}(G) - k_{gi}p_{t}(G) - k_{ga}p_{t}(G) + k_{ig}p_{t}(I) + k_{ag}p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_{t}(A) - k_{ag} p_{t}(A) - k_{ai} p_{t}(A) + k_{oa} p_{t}(G) + k_{ia} p_{t}(I)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_r(G) - gi - ga)$

$$aa = (p_t(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga = k_{\sigma a} p_{t}(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_{t}(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

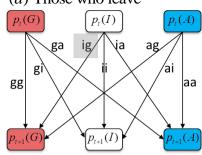
ig

ii

ia

ag

ai



(e) Counting transitions

 $p_{i+1}(G) = p_i(G) - gi - ga$

$$+ig+ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year $p_{t+1}(G) = gg + ig + ag$

$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

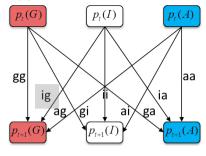
$$p_{t+1}(G) = p_{t}(G) - k_{gi}p_{t}(G) - k_{ga}p_{t}(G) + k_{go}p_{t}(I) + k_{go}p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{_{t+1}}(A) = p_{_t}(A) - k_{_{ag}} p_{_t}(A) - k_{_{ai}} p_{_t}(A)$$

$$+ \, k_{ga} \, p_t(G) + k_{ia} \, p_t(I)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_i(G) - gi - ga)$

$$aa = (p_{t}(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga = k_{oa} p_{t}(G)$$

$$ig = k_{io} p_{t}(I)$$

$$ia = k_{ia} p_{t}(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_t(A)$$

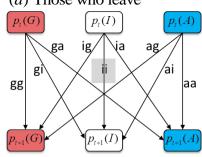
 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg gi ga

ig ii ia

ag ai aa



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

 $p_{t+1}(I) = ii + gi + ai$
 $p_{t+1}(A) = aa + ga + ia$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

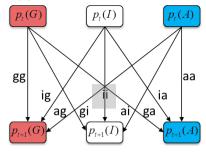
$$p_{t+1}(G) = p_{t}(G) - k_{gi}p_{t}(G) - k_{ga}p_{t}(G) + k_{io}p_{t}(I) + k_{ao}p_{t}(A)$$

$$p_{_{t+1}}(I) = 1 - p_{_{t+1}}(G) - p_{_{t+1}}(A)$$

$$p_{t+1}(A) = p_t(A) - k_{ag} p_t(A) - k_{ai} p_t(A)$$

$$+ \, k_{ga} \, p_{\scriptscriptstyle t}(G) + k_{ia} \, p_{\scriptscriptstyle t}(I)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_i(G) - gi - ga)$

$$aa = (p_t(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$
$$-aa - ig - ag - ai$$

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga = k_{ga} p_{t}(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_{t}(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

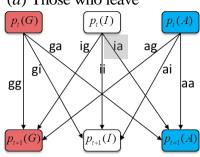




ia

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

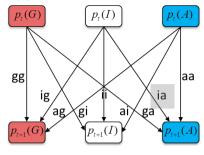
$$p_{t+1}(G) = p_{t}(G) - k_{gi} p_{t}(G) - k_{ga} p_{t}(G) + k_{go} p_{t}(I) + k_{go} p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - k_{ag} p_t(A) - k_{ai} p_t(A)$$

$$+ \, k_{ga} \, p_t(G) + k_{ia} \, p_t(I)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$

$$ii = 1 - gg - gi - ga - ia$$

-aa-ig-ag-ai

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga = k_{ga} p_t(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_{t}(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

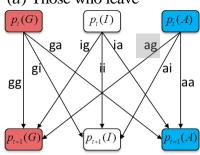
ig

ii

ia

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga$$
$$+ ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-\overline{p_{t+1}(A)}$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

$$p_{t+1}(G) = p_{t}(G) - k_{gi} p_{t}(G) - k_{ga} p_{t}(G)$$

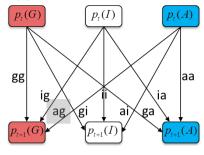
$$+ k_{ig} p_{t}(I) + k_{ag} p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_{t}(A) - k_{ag} p_{t}(A) - k_{ai} p_{t}(A)$$

$$+ \, k_{ga} \, p_t(G) + k_{ia} \, p_t(I)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia-aa - ig - ag - ai

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$
$$ga = k_{ga} p_t(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_t(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_{\scriptscriptstyle t}(A)$$

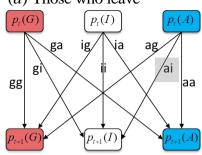
 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg gi ga

ig ii ia

ag ai aa



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

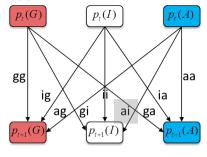
$$p_{t+1}(G) = p_{t}(G) - k_{gi}p_{t}(G) - k_{ga}p_{t}(G) + k_{io}p_{t}(I) + k_{ao}p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - k_{as} p_t(A) - k_{ai} p_t(A)$$

$$+k_{ga}p_{t}(G)+k_{ia}p_{t}(I)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia

-aa-ig-ag-ai

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga=k_{_{ga}}p_{_t}(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_t(I)$$

$$ag = k_{ag} p_{t}(A)$$

$$ai = k_{ai} p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

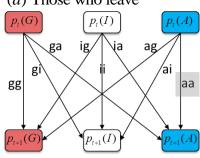
gg gi

g

i

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Diffusion

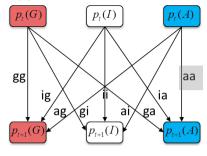
$$p_{t+1}(G) = p_{t}(G) - k_{gi}p_{t}(G) - k_{ga}p_{t}(G) + k_{go}p_{t}(I) + k_{go}p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{_{t+1}}(A) = p_{_t}(A) - k_{_{ag}} p_{_t}(A) - k_{_{ai}} p_{_t}(A)$$

$$+ \, k_{ga} \, p_t(G) + k_{ia} \, p_t(I)$$

(b) Those who come



(f) Redundancy/Constraints

$$gg = (p_t(G) - gi - ga)$$

$$aa = (p_t(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Diffusion predicts transitions:

$$gi = k_{gi} p_t(G)$$

$$ga = k_{\sigma a} p_{t}(G)$$

$$ig = k_{ig} p_t(I)$$

$$ia = k_{ia} p_{t}(I)$$

$$ag = k_{ag} p_t(A)$$

$$ai = k_{ai} p_t(A)$$

 $p_t(G)$ = proportion of *Goers*_t (at time t) $p_t(A)$ – proportion of *Absentees*_t (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

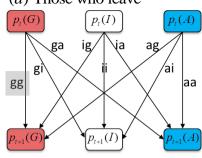
ig

ii

ic

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

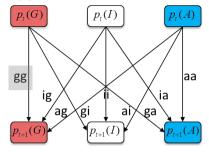
Contagion

Resilience

(g) Counting with Contagion

$$\begin{aligned} p_{t+1}(G) &= p_t(G) - T_{gi} p_t(G) p_t(I) - T_{ga} p_t(G) p_t(A) \\ &+ T_{ig} p_t(I) p_t(G) + T_{ag} p_t(A) p_t(G) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_t(A) - T_{ag} p_t(A) p_t(G) - T_{ai} p_t(A) p_t(I) \\ &+ T_{ga} p_t(G) p_t(A) + T_{ia} p_t(I) p_t(A) \end{aligned}$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$

$$aa = (p_t(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

-aa-ig-ag-ai

(h) Contagion predicts transitions:

$$gi = T_{gi} p_t(G) p_t(I)$$

$$ga = T_{ga} p_t(G) p_t(A)$$

$$ig = T_{ig} p_t(I) p_t(G)$$

$$ia = T_{ia} p_{t}(I) p_{t}(A)$$

$$ag = T_{ag} p_t(A) p_t(G)$$

$$ai = T_{ai} p_t(A) p_t(I)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

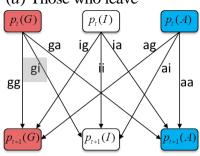
ig

ii

ia

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year $p_{t+1}(G) = gg + ig + ag$

$$p_{t+1}(I) = ii + gi + ai$$

 $p_{t+1}(A) = aa + ga + ia$

Diffusion

Contagion

Resilience

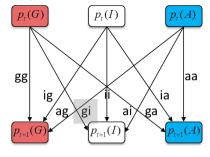
(g) Counting with Contagion

$$\begin{aligned} p_{t+1}(G) &= p_{t}(G) - T_{gi}p_{t}(G)p_{t}(I) - T_{ga}p_{t}(G)p_{t}(A) \\ &+ T_{ig}p_{t}(I)p_{t}(G) + T_{ag}p_{t}(A)p_{t}(G) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \end{aligned}$$

$$p_{t+1}(A) = p_t(A) - T_{ag} p_t(A) p_t(G) - T_{ai} p_t(A) p_t(I)$$

$$+ T_{ga} p_t(G) p_t(A) + T_{ia} p_t(I) p_t(A)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$

$$aa = (p_{\star}(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Contagion predicts transitions:

$$gi = T_{gi} p_t(G) p_t(I)$$

$$ga = T_{ga} p_t(G) p_t(A)$$

$$ig = T_{ig} p_t(I) p_t(G)$$

$$ia = T_{ia} p_t(I) p_t(A)$$

$$ag = T_{ag} p_t(A) p_t(G)$$

$$ai = T_{ai} p_t(A) p_t(I)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

ig

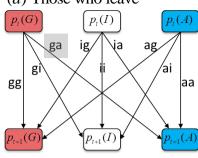
ii

ia

ag

ai

ac



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga$$
$$+ ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

$$+ ga + ia$$

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year $p_{t+1}(G) = gg + ig + ag$

$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

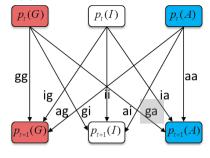
Resilience

(g) Counting with Contagion

$$p_{t+1}(G) = p_{t}(G) - T_{gi}p_{t}(G)p_{t}(I) - T_{ga}p_{t}(G)p_{t}(A) + T_{ig}p_{t}(I)p_{t}(G) + T_{ag}p_{t}(A)p_{t}(G)$$

$$\begin{aligned} p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_t(A) - T_{ag} p_t(A) p_t(G) - T_{ai} p_t(A) p_t(I) \\ &+ T_{oa} p_t(G) p_t(A) + T_{ia} p_t(I) p_t(A) \end{aligned}$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$

$$aa = (p_t(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Contagion predicts transitions:

$$gi = T_{gi} p_t(G) p_t(I)$$

$$ga = T_{\sigma a} p_{t}(G) p_{t}(A)$$

$$ig = T_{ig} p_t(I) p_t(G)$$

$$ia = T_{ia}p_t(I)p_t(A)$$

$$ag = T_{ag} p_t(A) p_t(G)$$

$$ai = T_{ai} p_t(A) p_t(I)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

ig

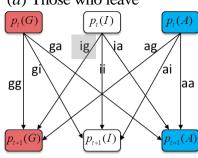
ii

ic

ag

ai

ac



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A)=p_{t}(A)-ag-ai$$

$$+ga+ia$$

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

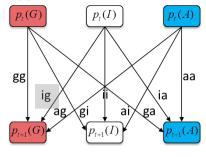
Resilience

(g) Counting with Contagion

$$p_{t+1}(G) = p_{t}(G) - T_{gi}p_{t}(G)p_{t}(I) - T_{ga}p_{t}(G)p_{t}(A)$$
$$+ T_{iv}p_{t}(I)p_{t}(G) + T_{av}p_{t}(A)p_{t}(G)$$

$$\begin{aligned} p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_t(A) - T_{ag} p_t(A) p_t(G) - T_{ai} p_t(A) p_t(I) \\ &+ T_{oa} p_t(G) p_t(A) + T_{ia} p_t(I) p_t(A) \end{aligned}$$

(b) Those who come

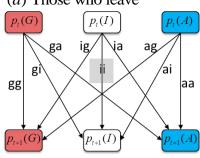


(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia-aa - ig - ag - ai

(h) Contagion predicts transitions: $gi = T_{gi} p_t(G) p_t(I)$ $ga = T_{ga} p_t(G) p_t(A)$ $ig = T_{ig} p_t(I) p_t(G)$ $ia = T_{ia} p_t(I) p_t(A)$ $ag = T_{ag} p_t(A) p_t(G)$ $ai = T_{ai} p_t(A) p_t(I)$ $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg gi go ig ii ia ag ai ac



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year $p_{+1}(G) = gg + ig + ag$

$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Contagion

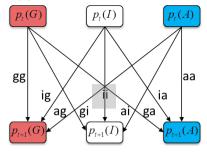
$$p_{t+1}(G) = p_{t}(G) - T_{gi}p_{t}(G)p_{t}(I) - T_{ga}p_{t}(G)p_{t}(A)$$
$$+ T_{io}p_{t}(I)p_{t}(G) + T_{ao}p_{t}(A)p_{t}(G)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - T_{ag} p_t(A) p_t(G) - T_{ai} p_t(A) p_t(I)$$

$$+ T_{ga} p_t(G) p_t(A) + T_{ia} p_t(I) p_t(A)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_r(G) - gi - ga)$

$$aa = (p_{\iota}(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Contagion predicts transitions:

$$gi = T_{gi} p_t(G) p_t(I)$$

$$ga = T_{\sigma a} p_{t}(G) p_{t}(A)$$

$$ig = T_{ig} p_t(I) p_t(G)$$

$$ia = T_{ia} p_{t}(I) p_{t}(A)$$

$$ag = T_{ag} p_t(A) p_t(G)$$

$$ai = T_{ai} p_t(A) p_t(I)$$

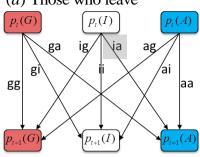
 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg gi

ig ii ic

ag ai ad



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

$$+ ga + ia$$

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

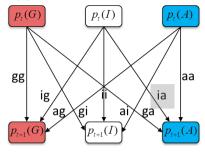
Resilience

(g) Counting with Contagion

$$\begin{split} p_{t+1}(G) &= p_{t}(G) - T_{gi} p_{t}(G) p_{t}(I) - T_{ga} p_{t}(G) p_{t}(A) \\ &+ T_{ig} p_{t}(I) p_{t}(G) + T_{ag} p_{t}(A) p_{t}(G) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_{t}(A) - T_{ae} p_{t}(A) p_{t}(G) - T_{ai} p_{t}(A) p_{t}(I) \end{split}$$

 $+T_{oa}p_{t}(G)p_{t}(A)+T_{ia}p_{t}(I)p_{t}(A)$

(b) Those who come

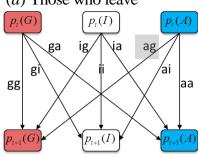


(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia-aa - ig - ag - ai

(h) Contagion predicts transitions: $gi = T_{gi} p_t(G) p_t(I)$ $ga = T_{ga} p_t(G) p_t(A)$ $ig = T_{ig} p_t(I) p_t(G)$ $ia = T_{ia} p_t(I) p_t(A)$ $ag = T_{ag} p_t(A) p_t(G)$ $ai = T_{ai} p_t(A) p_t(I)$ $p_{t}(G)$ = proportion of *Goers*_t (at time t) $p_{t}(A)$ – proportion of *Absentees*_t (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg gi ga
ig ii ia
ag ai aa



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-\overline{p_{t+1}(A)}$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

$$+ ga + ia$$

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

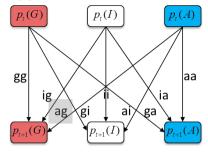
(g) Counting with Contagion

$$p_{t+1}(G) = p_{t}(G) - T_{gi}p_{t}(G)p_{t}(I) - T_{ga}p_{t}(G)p_{t}(A)$$
$$+ T_{io}p_{t}(I)p_{t}(G) + T_{ao}p_{t}(A)p_{t}(G)$$

$$\begin{aligned} p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_{t}(A) - T_{ao} p_{t}(A) p_{t}(G) - T_{ai} p_{t}(A) p_{t}(I) \end{aligned}$$

$$+T_{ga}p_{t}(G)p_{t}(A)+T_{ia}p_{t}(I)p_{t}(A)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_{\bullet}(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia

-aa-ig-ag-ai

(h) Contagion predicts transitions:

$$gi = T_{gi} p_t(G) p_t(I)$$

$$ga = T \quad p(G) p(A)$$

$$ga = T_{ga} p_t(G) p_t(A)$$

$$ig = T_{ig} p_t(I) p_t(G)$$

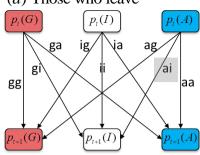
$$ia = T_{ia} p_t(I) p_t(A)$$

$$ag = T_{ag} p_t(A) p_t(G)$$

$$ai = T_{ai} p_t(A) p_t(I)$$

 $p_{\cdot}(G)$ = proportion of *Goers*, (at time t) $p_{\bullet}(A)$ – proportion of *Absentees*, (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$, who stay $Goers_{t+1}$ gi – Goers, who become Irregulars... ga - Goers, who become Absentees... ig - Irregulars, who become Goers, $ii-Irregulars_{t+1}$ who stay $Irregulars_{t+1}$ $ia-Irregulars_t$ who become Absentees_{t+1} $ag - Absentees_t$ who become Goers_{t+1} ai - Absentees, who become Irregulars, $aa - Absentees_t$, who stay $Absentees_{t+1}$



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Contagion

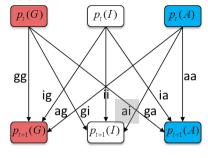
$$p_{t+1}(G) = p_{t}(G) - T_{gi}p_{t}(G)p_{t}(I) - T_{ga}p_{t}(G)p_{t}(A)$$
$$+ T_{io}p_{t}(I)p_{t}(G) + T_{ao}p_{t}(A)p_{t}(G)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - T_{ae} p_t(A) p_t(G) - T_{ai} p_t(A) p_t(I)$$

$$+ T_{ga} p_t(G) p_t(A) + T_{ia} p_t(I) p_t(A)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$

$$aa = (p_t(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Contagion predicts transitions:

$$gi = T_{gi} p_t(G) p_t(I)$$

$$ga = T_{\sigma a} p_{t}(G) p_{t}(A)$$

$$ig = T_{ig} p_t(I) p_t(G)$$

$$ia = T_{ia} p_{t}(I) p_{t}(A)$$

$$ag = T_{ag} p_t(A) p_t(G)$$

$$ai = T_{ai} p_t(A) p_t(I)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

gc

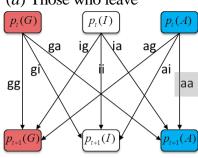
ig

ii

ic

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

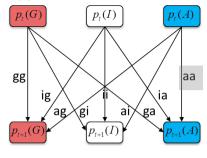
Resilience

(g) Counting with Contagion

$$\begin{split} p_{t+1}(G) &= p_t(G) - T_{gi} p_t(G) p_t(I) - T_{ga} p_t(G) p_t(A) \\ &+ T_{ig} p_t(I) p_t(G) + T_{ag} p_t(A) p_t(G) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_t(A) - T_{ag} p_t(A) p_t(G) - T_{ai} p_t(A) p_t(I) \end{split}$$

$$+T_{ga}p_{t}(G)p_{t}(A)+T_{ia}p_{t}(I)p_{t}(A)$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia

-aa-ig-ag-ai

(h) Contagion predicts transitions: $gi = T_{gi} p_i(G) p_i(I)$

$$ga = T_{ga} p_t(G) p_t(A)$$

$$ig = T_{ig} p_t(I) p_t(G)$$
$$ia = T_{ig} p_t(I) p_t(A)$$

$$ag = T_{ag} p_t(A) p_t(G)$$

 $ai = T_{ai} p_t(A) p_t(I)$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

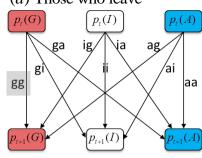
gg gi

g = i

1

ag

ai



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

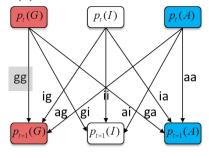
Contagion

Resilience

(g) Counting with Resilience

$$\begin{aligned} p_{t+1}(G) &= p_t(G) - K_{gi} p_t(G) p_t(G) - K_{ga} p_t(G) p_t(G) \\ &+ K_{ig} p_t(I) p_t(I) + K_{ag} p_t(A) p_t(A) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_t(A) - K_{ag} p_t(A) p_t(A) - K_{ai} p_t(A) p_t(A) \\ &+ K_{aa} p_t(G) p_t(G) + K_{ia} p_t(I) p_t(I) \end{aligned}$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia -aa - ig - ag - ai

(h) Resilience predicts transitions: $gi = K_{gi} p_i(G) p_i(G)$

$$ga = K_{ga} p_t(G) p_t(G)$$

$$ig = K_{ig} p_t(I) p_t(I)$$

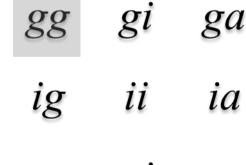
$$ia = K_{ia} p_t(I) p_t(I)$$

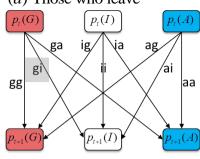
$$ag = K_{ag} p_{t}(A) p_{t}(A)$$

$$ai = K_{ai} p_t(A) p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$





(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(g) Counting with Resilience

$$p_{t+1}(G) = p_{t}(G) - K_{gi}p_{t}(G)p_{t}(G) - K_{ga}p_{t}(G)p_{t}(G)$$

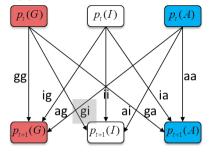
$$+ K_{ig}p_{t}(I)p_{t}(I) + K_{ag}p_{t}(A)p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_{t}(A) - K_{ag}p_{t}(A)p_{t}(A) - K_{ai}p_{t}(A)p_{t}(A)$$

 $+K_{oa}p_{t}(G)p_{t}(G)+K_{ia}p_{t}(I)p_{t}(I)$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$

$$aa = (p_{\iota}(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

-aa-ig-ag-ai

(h) Resilience predicts transitions:

$$gi = K_{gi} p_t(G) p_t(G)$$
$$ga = K_{ga} p_t(G) p_t(G)$$

$$ig = K_{ig} p_t(I) p_t(I)$$

$$ia = K_{ia} p_t(I) p_t(I)$$

$$ag = K_{ag} p_{t}(A) p_{t}(A)$$

$$ai = K_{ai} p_t(A) p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

ig

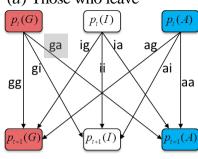
ii

ia

ag

ai

ac



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga$$
$$+ ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$
$$+ ga + ia$$

(g) Counting with Resilience

$$p_{t+1}(G) = p_{t}(G) - K_{gi} p_{t}(G) p_{t}(G) - K_{ga} p_{t}(G) p_{t}(G)$$

$$+ K_{ig} p_{t}(I) p_{t}(I) + K_{ag} p_{t}(A) p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_{t}(A) - K_{ag} p_{t}(A) p_{t}(A) - K_{ai} p_{t}(A) p_{t}(A)$$

 $+K_{\sigma a}p_{t}(G)p_{t}(G)+K_{ia}p_{t}(I)p_{t}(I)$

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

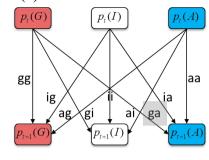
(d) Next year $p_{+1}(G) = gg + ig + ag$ $p_{t+1}(I) = ii + gi + ai$ $p_{t+1}(A) = aa + ga + ia$

Diffusion

Contagion

Resilience

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_{\iota}(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia-aa-ig-ag-ai

(h) Resilience predicts transitions:

$$gi = K_{gi} p_t(G) p_t(G)$$

$$ga = K_{ga} p_t(G) p_t(G)$$

$$ig = K_{ig} p_t(I) p_t(I)$$

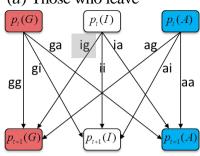
$$ia = K_{ia} p_t(I) p_t(I)$$

$$ag = K_{ao} p_t(A) p_t(A)$$

 $ai = K_{ai} p_t(A) p_t(A)$

 $p_{\cdot}(G)$ = proportion of *Goers*, (at time t) $p_{\bullet}(A)$ – proportion of *Absentees*, (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$, who stay $Goers_{t+1}$ gi – Goers, who become Irregulars... ga-Goers, who become Absentees, ig - Irregulars, who become Goers, $ii-Irregulars_{t+1}$ who stay $Irregulars_{t+1}$ $ia-Irregulars_t$ who become Absentees_{t+1} $ag - Absentees_t$ who become Goers_{t+1} ai - Absentees, who become Irregulars, $aa - Absentees_t$, who stay $Absentees_{t+1}$



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga$$
$$+ ig + ag$$
$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

$$+ ga + ia$$

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

 $p_{t+1}(I) = ii + gi + ai$
 $p_{t+1}(A) = aa + ga + ia$

Diffusion

Contagion

Resilience

(g) Counting with Resilience

$$p_{t+1}(G) = p_{t}(G) - K_{gi} p_{t}(G) p_{t}(G) - K_{ga} p_{t}(G) p_{t}(G)$$

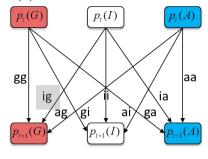
$$+ K_{ig} p_{t}(I) p_{t}(I) + K_{ag} p_{t}(A) p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_{t}(A) - K_{ag} p_{t}(A) p_{t}(A) - K_{ai} p_{t}(A) p_{t}(A)$$

 $+K_{oa}p_{t}(G)p_{t}(G)+K_{ia}p_{t}(I)p_{t}(I)$

(b) Those who come

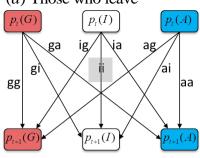


(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia-aa - ig - ag - ai

(h) Resilience predicts transitions: $gi = K_{gi}p_t(G)p_t(G)$ $ga = K_{ga}p_t(G)p_t(G)$ $ig = K_{ig}p_t(I)p_t(I)$ $ia = K_{ia}p_t(I)p_t(I)$ $ag = K_{ag}p_t(A)p_t(A)$ $ai = K_{oi}p_t(A)p_t(A)$ $p_t(G)$ = proportion of *Goers*_t (at time t) $p_t(A)$ – proportion of *Absentees*_t (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

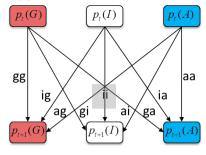
Resilience

(g) Counting with Resilience

$$\begin{aligned} p_{t+1}(G) &= p_{t}(G) - K_{gi} p_{t}(G) p_{t}(G) - K_{ga} p_{t}(G) p_{t}(G) \\ &+ K_{ig} p_{t}(I) p_{t}(I) + K_{ag} p_{t}(A) p_{t}(A) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_{t}(A) - K_{ag} p_{t}(A) p_{t}(A) - K_{ai} p_{t}(A) p_{t}(A) \end{aligned}$$

 $+K_{oa}p_{t}(G)p_{t}(G)+K_{ia}p_{t}(I)p_{t}(I)$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$

$$aa = (p_{\iota}(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$
$$-aa - ig - ag - ai$$

(h) Resilience predicts transitions:

$$gi = K_{gi} p_t(G) p_t(G)$$
$$ga = K_{ga} p_t(G) p_t(G)$$

$$ga = \mathbf{K}_{ga} p_t(G) p_t(G)$$

$$ig = K_{ig} p_t(I) p_t(I)$$

$$ia = K_{ia} p_t(I) p_t(I)$$

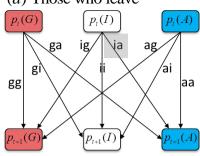
$$ag = K_{ag} p_t(A) p_t(A)$$

$$ai = K_{ai} p_t(A) p_t(A)$$

 $p_{\cdot}(G)$ = proportion of *Goers*, (at time t) $p_{\bullet}(A)$ – proportion of *Absentees*, (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$, who stay $Goers_{t+1}$ gi – Goers, who become Irregulars... ga - Goers, who become Absentees... ig - Irregulars, who become Goers, *ii* – *Irregulars*, who stay *Irregulars*, $ia-Irregulars_t$ who become Absentees_{t+1} $ag - Absentees_t$ who become Goers_{t+1} ai - Absentees, who become Irregulars, $aa - Absentees_t$, who stay $Absentees_{t+1}$

> iilg



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

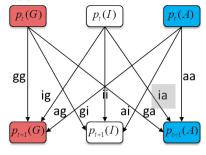
Contagion

Resilience

(g) Counting with Resilience

$$\begin{aligned} p_{t+1}(G) &= p_{t}(G) - K_{gi} p_{t}(G) p_{t}(G) - K_{ga} p_{t}(G) p_{t}(G) \\ &+ K_{ig} p_{t}(I) p_{t}(I) + K_{ag} p_{t}(A) p_{t}(A) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_{t}(A) - K_{ag} p_{t}(A) p_{t}(A) - K_{ai} p_{t}(A) p_{t}(A) \\ &+ K_{ag} p_{t}(G) p_{t}(G) + K_{ig} p_{t}(I) p_{t}(I) \end{aligned}$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_{\iota}(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia

-aa-ig-ag-ai

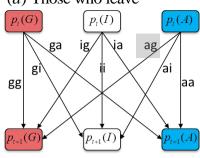
(h) Resilience predicts transitions: $gi = K_{gi} p_t(G) p_t(G)$ $ga = K_{\sigma a} p_{\tau}(G) p_{\tau}(G)$ $ig = K_{ig} p_t(I) p_t(I)$ $ia = K_{ia} p_{t}(I) p_{t}(I)$ $ag = K_{a\sigma} p_t(A) p_t(A)$

 $ai = K_{ai} p_{t}(A) p_{t}(A)$

 $p_{\cdot}(G)$ = proportion of *Goers*, (at time t) $p_{\bullet}(A)$ – proportion of *Absentees*, (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$, who stay $Goers_{t+1}$ gi – Goers, who become Irregulars... ga - Goers, who become Absentees... ig - Irregulars, who become Goers... $ii-Irregulars_{t+1}$ who stay $Irregulars_{t+1}$ ia – Irregulars, who become Absentees, $ag - Absentees_t$ who become Goers_{t+1} ai - Absentees, who become Irregulars, $aa - Absentees_t$, who stay $Absentees_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-\overline{p_{t+1}(A)}$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ga+ia

(g) Counting with Resilience

$$p_{t+1}(G) = p_{t}(G) - K_{gi}p_{t}(G)p_{t}(G) - K_{ga}p_{t}(G)p_{t}(G)$$

$$+ K_{ig}p_{t}(I)p_{t}(I) + K_{ag}p_{t}(A)p_{t}(A)$$

$$p_{t+1}(I) = 1 - p_{t+1}(G) - p_{t+1}(A)$$

$$p_{t+1}(A) = p_{t}(A) - K_{ag}p_{t}(A)p_{t}(A) - K_{ai}p_{t}(A)p_{t}(A)$$

 $+K_{oa}p_{t}(G)p_{t}(G)+K_{ia}p_{t}(I)p_{t}(I)$

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

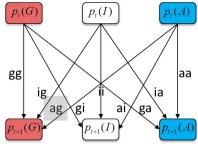
$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

Resilience

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia-aa-ig-ag-ai

(h) Resilience predicts transitions:

$$gi = K_{gi} p_t(G) p_t(G)$$

$$ga = K_{ga} p_t(G) p_t(G)$$

$$ig = K_{ig} p_t(I) p_t(I)$$

$$ia = K_{ia} p_t(I) p_t(I)$$

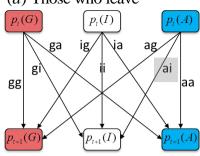
$$ag = K_{ag} p_t(A) p_t(A)$$

 $ai = K_{ai} p_{t}(A) p_{t}(A)$

 $p_{\cdot}(G)$ = proportion of *Goers*, (at time t) $p_{\bullet}(A)$ – proportion of *Absentees*, (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$, who stay $Goers_{t+1}$ gi – Goers, who become Irregulars... ga - Goers, who become Absentees... ig - Irregulars, who become Goers, $ii-Irregulars_{t+1}$ who stay $Irregulars_{t+1}$ $ia-Irregulars_t$ who become Absentees_{t+1} $ag - Absentees_t$ who become Goers_{t+1} ai - Absentees, who become Irregulars, $aa - Absentees_t$, who stay $Absentees_{t+1}$

gg	gi	ga
ig	ii	ia
ag	ai	aa



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga$$
$$+ ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year

$$p_{t+1}(G) = gg + ig + ag$$

$$p_{t+1}(I) = ii + gi + ai$$

$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

Contagion

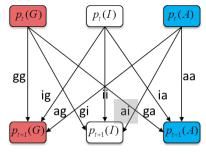
Resilience

(g) Counting with Resilience

$$\begin{split} p_{t+1}(G) &= p_{t}(G) - K_{gi}p_{t}(G)p_{t}(G) - K_{ga}p_{t}(G)p_{t}(G) \\ &+ K_{ig}p_{t}(I)p_{t}(I) + K_{ag}p_{t}(A)p_{t}(A) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_{t}(A) - K_{ag}p_{t}(A)p_{t}(A) - K_{ai}p_{t}(A)p_{t}(A) \end{split}$$

 $+K_{oa}p_{t}(G)p_{t}(G)+K_{ia}p_{t}(I)p_{t}(I)$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_t(G) - gi - ga)$ $aa = (p_t(A) - ag - ai)$ ii = 1 - gg - gi - ga - ia

-aa-ig-ag-ai

(h) Resilience predicts transitions:

$$gi = K_{gi} p_t(G) p_t(G)$$

$$ga = K_{ga} p_t(G) p_t(G)$$

$$ig = K_{ig} p_t(I) p_t(I)$$

$$ia = K_{ia} p_t(I) p_t(I)$$

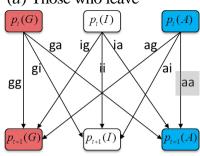
$$ag = K_{ag} p_t(A) p_t(A)$$

 $ai = K_{ai} p_t(A) p_t(A)$

 $p_{t}(G)$ = proportion of *Goers*_t (at time t) $p_{t}(A)$ – proportion of *Absentees*_t (at time t) $p_{t+1}(I)$ = proportion of *Irregulars*_{t+1}

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

ig ii ia
ag ai ac



(e) Counting transitions

$$p_{t+1}(G) = p_t(G) - gi - ga + ig + ag$$

$$p_{t+1}(I)=1-p_{t+1}(G)-p_{t+1}(A)$$

$$p_{t+1}(A) = p_t(A) - ag - ai$$

+ ga + ia

(c) This year

$$p_{t}(G) = gg + gi + ga$$

$$p_{t}(I) = ig + ii + ia$$

$$p_{t}(A) = ag + ai + aa$$

(d) Next year $p_{+1}(G) = gg + ig + ag$

$$p_{t+1}(I) = ii + gi + ai$$
$$p_{t+1}(A) = aa + ga + ia$$

Diffusion

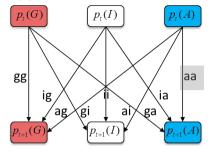
Contagion

Resilience

(g) Counting with Resilience

$$\begin{aligned} p_{t+1}(G) &= p_t(G) - K_{gi} p_t(G) p_t(G) - K_{ga} p_t(G) p_t(G) \\ &+ K_{ig} p_t(I) p_t(I) + K_{ag} p_t(A) p_t(A) \\ p_{t+1}(I) &= 1 - p_{t+1}(G) - p_{t+1}(A) \\ p_{t+1}(A) &= p_t(A) - K_{ag} p_t(A) p_t(A) - K_{ai} p_t(A) p_t(A) \\ &+ K_{ag} p_t(G) p_t(G) + K_{ig} p_t(I) p_t(I) \end{aligned}$$

(b) Those who come



(f) Redundancy/Constraints $gg = (p_r(G) - gi - ga)$

$$aa = (p_t(A) - ag - ai)$$

$$ii = 1 - gg - gi - ga - ia$$

$$-aa-ig-ag-ai$$

(h) Resilience predicts transitions:

$$gi = K_{gi} p_t(G) p_t(G)$$

$$ga = K_{\sigma a} p_{t}(G) p_{t}(G)$$

$$ig = K_{ig} p_t(I) p_t(I)$$

$$ia = K_{ia} p_{t}(I) p_{t}(I)$$

$$ag = K_{ao} p_t(A) p_t(A)$$

$$ai = K_{ai} p_t(A) p_t(A)$$

 $p_t(G)$ = proportion of $Goers_t$ (at time t) $p_t(A)$ – proportion of $Absentees_t$ (at time t) $p_{t+1}(I)$ = proportion of $Irregulars_{t+1}$

 $gg - Goers_t$ who stay $Goers_{t+1}$ $gi - Goers_t$ who become $Irregulars_{t+1}$ $ga - Goers_t$ who become $Absentees_{t+1}$ $ig - Irregulars_t$ who become $Goers_{t+1}$ $ii - Irregulars_t$ who stay $Irregulars_{t+1}$ $ia - Irregulars_t$ who become $Absentees_{t+1}$ $ag - Absentees_t$ who become $Goers_{t+1}$ $ai - Absentees_t$ who become $Irregulars_{t+1}$ $aa - Absentees_t$ who stay $Absentees_{t+1}$

gg

gi

ga

ig

ii

ia

ag

ai