

$$\begin{array}{lll} \mathbf{y}_{ii} &= \alpha_{i} + \beta_{ii} \mathrm{age}_{t} + \delta_{2} \, \mathrm{TVC}_{ii} + \mathcal{E}_{ii} & \mathrm{MODEL:} \\ \alpha_{i} &= \mu_{\alpha} + \gamma_{01} \mathrm{TIC}_{i} + \mathcal{C}_{ai} & ! \, \mathrm{Grows BY} \\ \beta_{ii} &= \mu_{\beta_{i}} + \gamma_{11} \mathrm{TIC}_{i} + \mathcal{C}_{\beta_{ii}} & \mathrm{Int} & \mathrm{by y8@1 \ y9@1} \\ \mathbf{y}_{i} &= \Lambda \mathbf{\eta}_{i} + \Delta \mathbf{z}_{ai} + \mathbf{\epsilon}_{i} & \mathrm{Int \ by y8@0 \ y9@1} \\ \mathbf{y}_{10} &= \mathbf{\eta}_{i} + \mathbf{\Gamma} \mathbf{w}_{i} + \mathbf{v}_{i} \\ \mathbf{y}_{i0} &= \begin{pmatrix} \delta_{i8} \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \delta_{i8} \\ \delta_{i9} \\ \delta_{i10} \\ \delta_{i11} \end{pmatrix} + \begin{pmatrix} \delta_{i8} \\ \delta_{i9} \\ \delta_{i0} \\ \delta_{i11} \end{pmatrix} \begin{pmatrix} \mathcal{E}_{i8} \\ \delta_{i9} \\ \delta_{i10} \\ \delta_{i11} \end{pmatrix} \begin{pmatrix} \mathcal{E}_{i8} \\ \mathcal{E}_{i0} \\ \mathcal{E}_{i10} \\ \mathcal{E}_{i10} \end{pmatrix} + \begin{pmatrix} \gamma_{01} \\ \gamma$$

$$\begin{array}{lll} \mathbf{y}_{ii} &= \alpha_{i} + \beta_{ii} \mathrm{age}_{t} + \delta_{2i} \mathrm{TVC}_{ii} + \mathcal{E}_{ii} & \mathrm{MODEL:} \\ \alpha_{i} &= \mu_{\alpha} + \gamma_{01} \mathrm{TIC}_{i} + \mathcal{C}_{ai} & ! \mathrm{Grows BY} \\ \beta_{ii} &= \mu_{\beta_{i}} + \gamma_{11} \mathrm{TIC}_{i} + \mathcal{C}_{\beta_{ii}} & \mathrm{Int} & \mathrm{by y8@1 y9@1} \\ \mathbf{y}_{i} &= \Lambda \mathbf{\eta}_{i} + \Delta \mathbf{z}_{ai} + \mathbf{\epsilon}_{i} & \mathrm{Int} & \mathrm{by y8@0 y9@1} \\ \mathbf{y}_{i0} &= \mathbf{\eta}_{i} + \mathbf{\Gamma} \mathbf{w}_{i} + \mathbf{v}_{i} \\ \mathbf{y}_{i0} &= \begin{pmatrix} \mathbf{j}_{i} \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \delta_{i} \\ \delta_{i} \\ \delta_{i} \\ \delta_{i} \\ \delta_{i} \end{pmatrix} + \begin{pmatrix} \delta_{i} \\ \delta$$

$$\begin{array}{lll} \mathbf{y}_{ii} &= \alpha_{i} + \beta_{ii} \operatorname{age}_{i} + \mathcal{E}_{ii} \\ \alpha_{i} &= \mu_{\alpha} + \gamma_{01} \operatorname{TIC}_{i} + \zeta_{\alpha i} \\ \beta_{ii} &= \mu_{\beta_{i}} + \gamma_{11} \operatorname{TIC}_{i} + \zeta_{\beta ii} \\ \mathbf{y}_{i} &= \Lambda \mathbf{\eta}_{i} + \mathbf{\epsilon}_{i} \\ \mathbf{y}_{ij} &= \Lambda \mathbf{\eta}_{i} + \mathbf{\epsilon}_{i} \\ \mathbf{y}_{i0} &= \begin{pmatrix} I & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \mathcal{E}_{ii} \\ \mathcal{E}_{i0} \\ \mathcal{E}_{i11} \end{pmatrix} + \begin{pmatrix} \mathcal{E}_{ii} \\ \mathcal{E}_{i0} \\ \mathcal{E}_{i11} \end{pmatrix} & \text{Int by y8@1 y9@1 } \\ y10@1 y11@1; \\ \text{linear by y8@0 y9@1 } \\ y10@2 y11@3; \\ \text{linear on TIC;} \\ \text{linear on TIC;} \\ \text{Make residual variances} \neq \\ & \text{Add TIC of slope} \\ & \text{Add TIC of intercept} \\ & \text{Add TIC of intercept} \\ & \text{Make residual variances} \neq \\ & \text{Add TIC of intercept} \\ & \text{Make residual variances} \neq \\ & \text{Add TIC of intercept} \\ & \text{Make residual variances} \neq \\ & \text{Add TIC of intercept} \\ & \text{Make residual variances} \neq \\ & \text{Have intercept & slope covary} \\ & & \text{Engles} \\ & \text{Find } \\ & \mathcal{E}_{\beta_{i}} \\ & \sim N \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \mathcal{W}_{\alpha\alpha} \\ \mathcal{W}_{\beta_{i}} \\ \mathcal{W}_{\beta_{i}} \\ & \mathcal{W}$$

```
y_{it} = \alpha_i + \beta_{1i} age_t + \varepsilon_{it}
                                                                                                       MODEL:
                                                                                                                                                                                   Add unique TVC for each time
                                                                                                       ! Grows BY
\alpha_i = \mu_{\alpha} + \zeta_{\alpha i}
                                                                                                                       by y8@1 y9@1
\beta_{1i} = \mu_{\beta 1} + \zeta_{\beta 1i}
                                                                                                       Int
                                                                                                                                                                                   Add same TVC for all times
                                                                                                                           y10@1 y11@1;
 \mathbf{y}_i = \mathbf{\Lambda} \mathbf{\eta}_i + \mathbf{\varepsilon}_i
                                                                                                       linear by y8@0 y9@1
 \mathbf{\eta}_i = \mathbf{\mu}_{\eta} + \mathbf{\zeta}_i
                                                                                                                                                                                   Add TIC of slope
                                                                                                                           y10@2 y11@3;
         \begin{pmatrix} y_{i8} \\ y_{i9} \\ y_{i10} \\ y_{i10} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i8} \\ \varepsilon_{i9} \\ \varepsilon_{i10} \\ \varepsilon \end{pmatrix} 
                                                                                                       ! Regressed ON
                                                                                                                                                                                   Add TIC of intercept
                                                                                                                                                                                   Make residual variances ≠
           \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} = \begin{pmatrix} \mu_{\alpha} \\ \mu_{\beta 1} \end{pmatrix} + \begin{pmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1i} \end{pmatrix} 
                                                                                                                                                                                   Have intercept & slope covary
        \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \end{bmatrix} \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \psi_{\alpha \alpha} \\ \psi_{\alpha \beta 1} & \psi_{\beta 1 \beta 1} \end{pmatrix}
                                                                                                                                                                                   Make slope random
                                                                                                       ! Correlated WITH
                                                                                                      int with linear;
                                                                                                      ! Intercepts
                                                                                                                                                                                   Make intercept random
                                                                                                      [y8-y11@0];
                                                                                                       [int];
                                                                                                                                                                                   Add fixed slope
                                                                                                       [linear];
\mathbf{y}_{i} = \mathbf{\Lambda} \mathbf{\mu}_{\eta} + \mathbf{\Lambda} \mathbf{\zeta}_{i} + \mathbf{\varepsilon}_{i}
                                                                                                       ! Residual Variances
                                                                                                                                                                                   Add fixed intercept
y_{it} = \mu_{\alpha} + \mu_{\beta 1} age_t + \varepsilon_{it}
                                                                                                       y8-y11;
                                                                                                       int;
                                                                                                                                                                                   Null
                        +\zeta_{\beta 1i}age<sub>t</sub>
                                                                                                       linear;
```

```
y_{it} = \alpha_i + \beta_{1i} age_t + \varepsilon_{it}
                                                                                                                      MODEL:
                                                                                                                                                                                                             Add unique TVC for each time
                                                                                                                      ! Grows BY
 \alpha_i = \mu_{\alpha} + \zeta_{\alpha i}
                                                                                                                                        by y8@1 y9@1
\beta_{1i} = \mu_{\beta 1} + \zeta_{\beta 1i}
                                                                                                                      Int
                                                                                                                                                                                                             Add same TVC for all times
                                                                                                                                             y10@1 y11@1;
 \mathbf{y}_i = \mathbf{\Lambda} \mathbf{\eta}_i + \mathbf{\varepsilon}_i
                                                                                                                      linear by y8@0 y9@1

\eta_i = \mu_{\eta} + \zeta_i

                                                                                                                                                                                                             Add TIC of slope
                                                                                                                                             y10@2 y11@3;
          \begin{pmatrix} y_{i8} \\ y_{i9} \\ y_{i10} \\ y_{i1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i8} \\ \varepsilon_{i9} \\ \varepsilon_{i10} \\ \varepsilon_{i1} \end{pmatrix}
                                                                                                                      ! Regressed ON
                                                                                                                                                                                                             Add TIC of intercept
                                                                                                                                                                                                              Make residual variances ≠
            \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} = \begin{pmatrix} \mu_{\alpha} \\ \mu_{\beta 1} \end{pmatrix} + \begin{pmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1i} \end{pmatrix} 
                                                                                                                                                                                                              Have intercept & slope covary
          \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \end{bmatrix} \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \psi_{\alpha \alpha} \\ \psi_{\alpha \beta 1} \end{pmatrix} \psi_{\beta 1 \beta 1}
                                                                                                                                                                                                             Make slope random
                                                                                                                      ! Correlated WITH
                                                                                                                     int with linear;
          \begin{bmatrix} \boldsymbol{\varepsilon}_{i8} \\ \boldsymbol{\varepsilon}_{i9} \\ \boldsymbol{\varepsilon}_{i10} \\ \boldsymbol{\varepsilon}_{i11} \end{bmatrix} \sim N \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \boldsymbol{\theta}_{\varepsilon} \\ \boldsymbol{\theta}_{\varepsilon} \\ \boldsymbol{\theta}_{\varepsilon} \\ \boldsymbol{\theta}_{\varepsilon} \end{pmatrix}
                                                                                                                   ! Intercepts
                                                                                                                                                                                                             Make intercept random
                                                                                                                    [y8-y11@0];
                                                                                                                      [int];
                                                                                                                                                                                                             Add fixed slope
                                                                                                                      [linear];
 \mathbf{y}_{i} = \mathbf{\Lambda} \mathbf{\mu}_{\eta} + \mathbf{\Lambda} \mathbf{\zeta}_{i} + \mathbf{\varepsilon}_{i}
                                                                                                                      ! Residual Variances
                                                                                                                                                                                                             Add fixed intercept
y_{it} = \mu_{\alpha} + \mu_{\beta 1} age_t + \varepsilon_{it}
                                                                                                                      y8-y11(epsilon);
                                                                                                                      int;
                                                                                                                                                                                                             Null
                            +\zeta_{\beta li}age<sub>t</sub>
                                                                                                                      linear;
```

```
y_{it} = \alpha_i + \beta_{1i} age_t + \varepsilon_{it}
                                                                                                                      MODEL:
                                                                                                                                                                                                             Add unique TVC for each time
                                                                                                                      ! Grows BY
\alpha_i = \mu_{\alpha} + \zeta_{ai}
                                                                                                                                        by y8@1 y9@1
\beta_{1i} = \mu_{\beta 1} + \zeta_{\beta 1i}
                                                                                                                      Int
                                                                                                                                                                                                             Add same TVC for all times
                                                                                                                                            y10@1 y11@1;
 \mathbf{y}_i = \mathbf{\Lambda} \mathbf{\eta}_i + \mathbf{\varepsilon}_i
                                                                                                                      linear by y8@0 y9@1
 \mathbf{\eta}_i = \mathbf{\mu}_{\eta} + \mathbf{\zeta}_i
                                                                                                                                                                                                             Add TIC of slope
                                                                                                                                            y10@2 y11@3;
          \begin{pmatrix} y_{i8} \\ y_{i9} \\ y_{i10} \\ y_{i1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i8} \\ \varepsilon_{i9} \\ \varepsilon_{i10} \\ \varepsilon_{i1} \end{pmatrix}
                                                                                                                      ! Regressed ON
                                                                                                                                                                                                             Add TIC of intercept
                                                                                                                                                                                                             Make residual variances ≠
            \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} = \begin{pmatrix} \mu_{\alpha} \\ \mu_{\beta 1} \end{pmatrix} + \begin{pmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1i} \end{pmatrix} 
                                                                                                                                                                                                             Have intercept & slope covary
          \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \end{bmatrix} \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \psi_{\alpha \alpha} \\ \psi_{\beta 1 \beta 1} \end{pmatrix}
                                                                                                                                                                                                             Make slope random
                                                                                                                      ! Correlated WITH
                                                                                                                     int with linear@0;
          \begin{bmatrix} \boldsymbol{\varepsilon}_{i8} \\ \boldsymbol{\varepsilon}_{i9} \\ \boldsymbol{\varepsilon}_{i10} \\ \boldsymbol{\varepsilon}_{i11} \end{bmatrix} \sim N \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \boldsymbol{\theta}_{\varepsilon} \\ \boldsymbol{\theta}_{\varepsilon} \\ \boldsymbol{\theta}_{\varepsilon} \\ \boldsymbol{\theta}_{\varepsilon} \end{pmatrix}
                                                                                                                     ! Intercepts
                                                                                                                                                                                                             Make intercept random
                                                                                                                    [y8-y11@0];
                                                                                                                      [int];
                                                                                                                                                                                                             Add fixed slope
                                                                                                                      [linear];
 \mathbf{y}_{i} = \mathbf{\Lambda} \mathbf{\mu}_{\eta} + \mathbf{\Lambda} \mathbf{\zeta}_{i} + \mathbf{\varepsilon}_{i}
                                                                                                                      ! Residual Variances
                                                                                                                                                                                                             Add fixed intercept
y_{it} = \mu_{\alpha} + \mu_{\beta 1} age_t + \varepsilon_{it}
                                                                                                                      y8-y11(epsilon);
                                                                                                                      int;
                                                                                                                                                                                                             Null
                           +\zeta_{\beta 1i}age<sub>t</sub>
                                                                                                                      linear;
```

```
y_{it} = \alpha_i + \beta_{1i} age_t + \varepsilon_{it}
                                                                                                        MODEL:
                                                                                                                                                                                     Add unique TVC for each time
                                                                                                        ! Grows BY
\alpha_i = \mu_{\alpha} + \zeta_{ai}
                                                                                                                        by y8@1 y9@1
\beta_{1i} = \mu_{\beta 1} +
                                                                                                        Int
                                                                                                                                                                                     Add same TVC for all times
                                                                                                                            y10@1 y11@1;
 \mathbf{y}_i = \mathbf{\Lambda} \mathbf{\eta}_i + \mathbf{\varepsilon}_i
                                                                                                        linear by y8@0 y9@1

\eta_i = \mu_{\eta} + \zeta_i

                                                                                                                                                                                     Add TIC of slope
                                                                                                                            y10@2 y11@3;
         \begin{pmatrix} y_{i8} \\ y_{i9} \\ y_{i10} \\ y_{ord} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i8} \\ \varepsilon_{i9} \\ \varepsilon_{i10} \\ \varepsilon_{ord} \end{pmatrix}
                                                                                                        ! Regressed ON
                                                                                                                                                                                     Add TIC of intercept
                                                                                                                                                                                      Make residual variances ≠
          \begin{pmatrix} \alpha_i \\ \beta_{1i} \end{pmatrix} = \begin{pmatrix} \mu_{\alpha} \\ \mu_{\beta 1} \end{pmatrix} + \begin{pmatrix} \zeta_{\alpha i} \\ \end{pmatrix}
                                                                                                                                                                                      Have intercept & slope covary
         \left| \begin{array}{c} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \end{array} \right| \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \left[ \begin{array}{c} \psi_{\alpha \alpha} \\ \end{array} \right] 
                                                                                                                                                                                     Make slope random
                                                                                                        ! Correlated WITH
       \begin{bmatrix} \varepsilon_{i8} \\ \varepsilon_{i9} \\ \varepsilon_{i10} \end{bmatrix} \sim N \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \theta_{\varepsilon} \qquad \theta_{\varepsilon} \qquad \theta_{\varepsilon} 
                                                                                                        int with linear@0;
                                                                                                      ! Intercepts
                                                                                                                                                                                      Make intercept random
                                                                                                      [y8-y11@0];
                                                                                                        [int];
                                                                                                                                                                                     Add fixed slope
                                                                                                        [linear];
\mathbf{y}_{i} = \mathbf{\Lambda} \mathbf{\mu}_{\eta} + \mathbf{\Lambda} \mathbf{\zeta}_{i} + \mathbf{\varepsilon}_{i}
                                                                                                        ! Residual Variances
                                                                                                                                                                                     Add fixed intercept
y_{it} = \mu_{\alpha} + \mu_{\beta 1} age_t + \varepsilon_{it}
                                                                                                        y8-y11(epsilon);
                                                                                                        int;
                                                                                                                                                                                     Null
                                                                                                        linear@0;
```

```
y_{it} = \alpha_i + \beta_{1i} age_t + \varepsilon_{it}
                                                                                         MODEL:
                                                                                                                                                            Add unique TVC for each time
                                                                                         ! Grows BY
\alpha_i = \mu_{\alpha}
\beta_{1i} = \mu_{\beta 1}
                                                                                                       by y8@1 y9@1
                                                                                         Int
                                                                                                                                                            Add same TVC for all times
                                                                                                           y10@1 y11@1;
 \mathbf{y}_i = \mathbf{\Lambda} \mathbf{\eta}_i + \mathbf{\varepsilon}_i
                                                                                         linear by y8@0 y9@1
 \eta_i = \mu_{\eta}
                                                                                                                                                            Add TIC of slope
                                                                                                           y10@2 y11@3;
        \begin{pmatrix} \mathbf{y}_{i8} \\ \mathbf{y}_{i9} \\ \mathbf{y}_{i10} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_{i} \\ \boldsymbol{\beta}_{1i} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_{i8} \\ \boldsymbol{\varepsilon}_{i9} \\ \boldsymbol{\varepsilon}_{i10} \\ \boldsymbol{\varepsilon}_{i10} \end{pmatrix}
                                                                                         ! Regressed ON
                                                                                                                                                            Add TIC of intercept
                                                                                                                                                            Make residual variances ≠
                                                                                                                                                            Have intercept & slope covary
         \left| \begin{array}{c} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \end{array} \right| \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, 
                                                                                                                                                            Make slope random
                                                                                         ! Correlated WITH
                                                                                         int with linear@0;
                                                                                       ! Intercepts
                                                                                                                                                            Make intercept random
                                                                                       [y8-y11@0];
                                                                                         [int];
                                                                                                                                                           Add fixed slope
                                                                                         [linear];
\mathbf{y}_i = \mathbf{\Lambda} \mathbf{\mu}_{\eta} + \mathbf{\varepsilon}_i
                                                                                         ! Residual Variances
                                                                                                                                                           Add fixed intercept
y_{it} = \mu_{\alpha} + \mu_{\beta 1} age_t + \varepsilon_{it}
                                                                                         y8-y11(epsilon);
                                                                                         int@0;
                                                                                                                                                            Null
                                                                                         linear@0;
```



