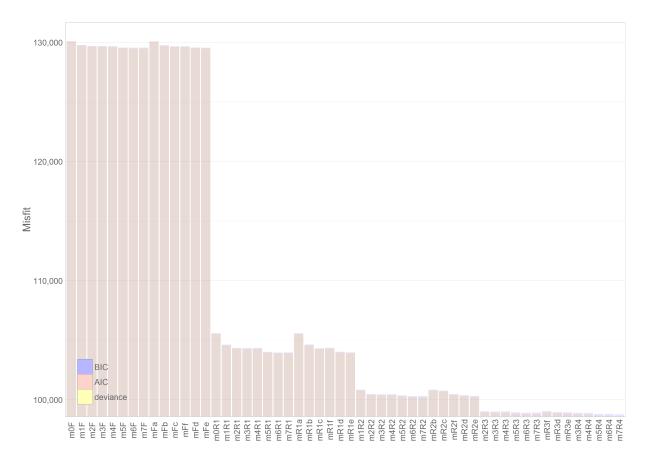
Custom Fit

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1 All models



There are total of 54 distinct models, which can be organized into 4 groups:

- models with fixed effects only
- models with 1 random term
- models with 2 random terms
- models with 3 random temrs

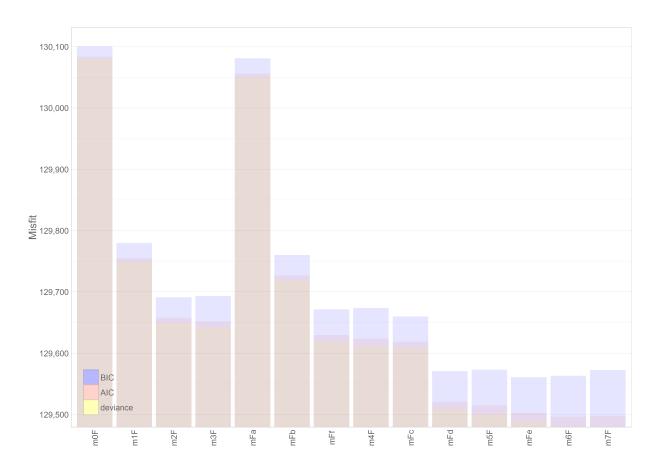
The following layout helps understand how each of the models was constructed. The columns count the number of terms on the first level, the rows shows what predictors are added to the second level. Replace the stars in the name of the model with **F** for models with only the fixed effects and **R1**, **R2**, **R3**, and **R4** for models with the corresponding number of random terms in the second level.

	$oldsymbol{eta}_{0i}$	$eta_{0_i} + eta_{1_i} timec$,	$\beta_{0i} + \beta_{1i}timec_i + \beta_{2i}timec_i^2$	$\beta_{0i} + \beta_{1i}timec_{i} + \beta_{2i}timec_{i}^{2} + \beta_{3i}timec_{i}^{3}$
	m0*	m1*	m2*	m3*
$\gamma_{01}cohort_{i}$	m*a	m*b	m*f	m4*
$\gamma_{01} cohort_i$ $\gamma_{11} cohort_i$		m*c	m*d	m5*
$\gamma_{01}cohort_i$ $\gamma_{11}cohort_i$ $\gamma_{21}cohort_i$			m*e	m6*
$\gamma_{01}cohort_i$ $\gamma_{11}cohort_i$ $\gamma_{21}cohort_i$ $\gamma_{31}cohort_i$				m7*

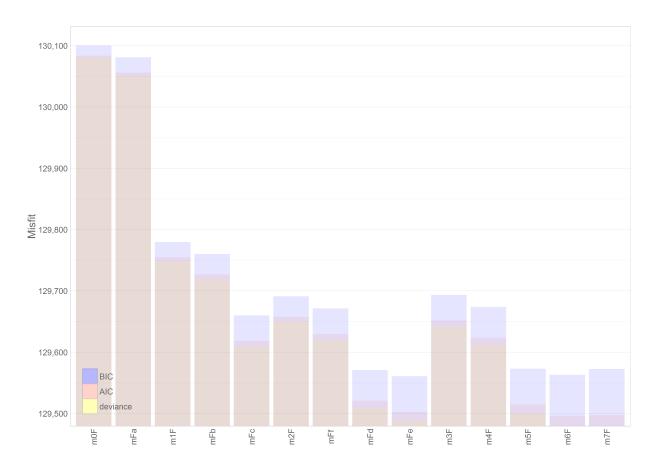
2 Models w/ fixed only

Model Sequence Map	F	0 random terms	
mOF $y_{\varepsilon} = \beta_{w} + \varepsilon_{\varepsilon}$ $\beta_{w} = \gamma_{w}$	m1F $y_i = \beta_{il} + \beta_i timec + \varepsilon_{il}$ $\beta_{il} = \gamma_{il}$ $\beta_{il} = \gamma_{il}$	m2F $y_s = \beta_w + \beta_s timec_s + \beta_s timec_s^2 + \epsilon_w$ $\beta_w = \gamma_{ss}$ $\beta_s = \gamma_{ss}$ $\beta_w = \gamma_{ss}$	m3F $y_t = \beta_{tt} + \beta_t timec_t + \beta_t timec_t^2 + \beta_t timec_t^2 + \varepsilon_t$ $\beta_{tt} = \gamma_{tt}$ $\beta_t = \gamma_{tt}$ $\beta_{tt} = \gamma_{tt}$ $\beta_{tt} = \gamma_{tt}$
mFa $y_{e} = \beta_{m} + \sigma_{e}$ $\beta_{e} = \gamma_{\infty} + \gamma_{s}, cohort,$	mFb $y_{i} = \beta_{ii} + \beta_{i}timec_{i} + \varepsilon_{ii}$ $\beta_{ii} = \gamma_{ii} + \gamma_{i}, cohort_{i}$ $\beta_{ii} = \gamma_{ii}$	$\begin{aligned} \mathbf{mFf} \\ \mathbf{y}_i &= \beta_0 + \beta_0 timec \ , + \beta_0 timec^2, + \varepsilon_0 \\ \beta_0 &= \gamma_0 + \gamma_0 tohor \ t_i \\ \beta_i &= \gamma_0 \\ \beta_0 &= \gamma_0 \end{aligned}$	m4F $\mathbf{y}_t = \beta_{0t} + \beta_t timec_t + \beta_t timec_t^2 + \beta_t timec_t^2 + \mathbf{c}_t$ $\beta_{0t} = y_{0t} + y_{0t} cohort$, $\beta_{1t} = y_{10}$ $\beta_{2t} = y_{2t}$ $\beta_{2t} = y_{2t}$
	MFC $\mathbf{y}_{s} = \beta_{st} + \beta_{s}timec_{s} + \varepsilon_{st}$ $\beta_{tt} = \gamma_{tt} + \gamma_{t}, cohort,$ $\beta_{tt} = \gamma_{tt} + \gamma_{t}, cohort,$	$\begin{aligned} \mathbf{mFd} \\ \mathbf{y}_{\epsilon} &= \beta_{\alpha} + \beta_{\alpha} timsec_{+} + \beta_{\alpha} timsec_{+} + \boldsymbol{\varepsilon}_{\alpha} \\ \boldsymbol{\beta}_{\alpha} &= \boldsymbol{\gamma}_{\alpha} + \boldsymbol{\gamma}_{\alpha} cohort, \\ \boldsymbol{\beta}_{\beta} &= \boldsymbol{\gamma}_{12} + \boldsymbol{\gamma}_{11} cohort, \\ \boldsymbol{\beta}_{22} &= \boldsymbol{\gamma}_{23} \end{aligned}$	m5F $y_{i} = \beta_{ii} + \beta_{i}timec_{i} + \beta_{ii}timec_{i}^{2} + \beta_{i}timec_{i}^{2} + \delta_{i}$ $\beta_{ii} = \gamma_{ii} + \gamma_{ij} cohort_{i}$ $\beta_{ij} = \gamma_{ii} + \gamma_{ij} cohort_{i}$ $\beta_{ij} = \gamma_{ii}$ $\beta_{ij} = \gamma_{ij}$
		mFe $y_{s} = \beta_{u} + \beta_{s}timsec_{s} + \beta_{s}timsec_{s}^{2} + \epsilon_{u}$ $\beta_{u} = y_{u} + y_{u}, cohort,$ $\beta_{u} = y_{u} + y_{u}, cohort,$ $\beta_{u} = y_{u} + y_{z}, cohort,$	m6F $y_{i} = \beta_{ii} + \beta_{i}timec_{i} + \beta_{ii}timec_{i}^{2} + \beta_{i}timec_{i}^{2} + \delta_{i}timec_{i}^{2} + \delta_{i}tim$
			m7F $y_{\ell} = \beta_{n\ell} + \beta_{n\ell}timec_{\ell} + \beta_{n\ell}timec_{\ell}^{2} + \beta_{n\ell}timec_{\ell}^{2} + \delta_{n\ell}timec_{\ell}^{2} + \delta_$

2.1 F by row



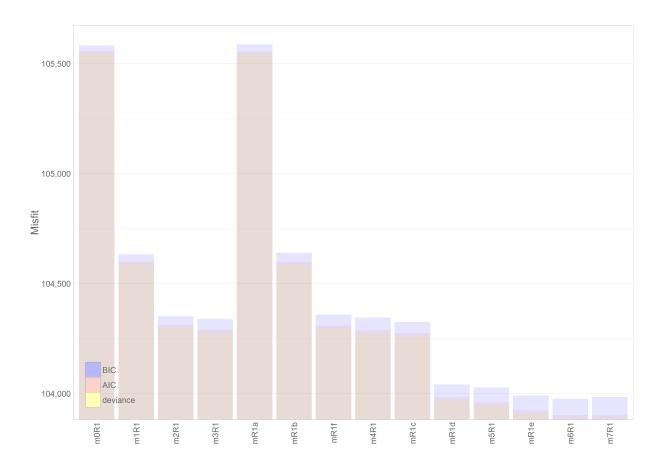
2.2 F by column



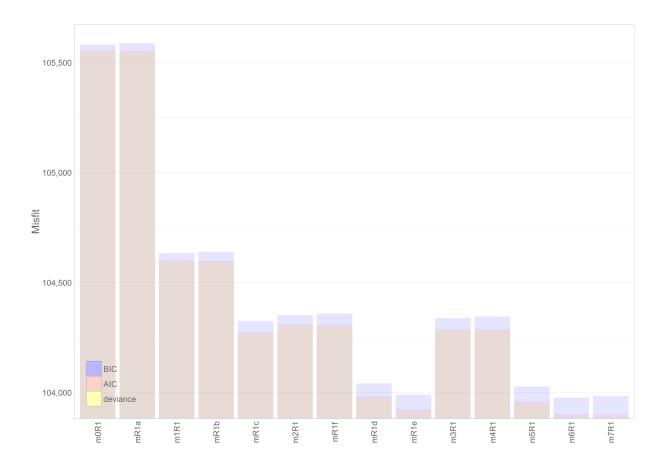
3 Models w/ 1 random

Model Sequence Map R1 1 random term m0R1 m1R1 m2R1 m3R1 $\begin{aligned} &\text{IIIZNI} \\ & \mathbf{y}_{e} = \beta_{o} + \beta_{e} timsc^{2}_{i} + \beta_{e} timsc^{2}_{i} + \boldsymbol{\varepsilon}_{e} \\ & \beta_{o} = \gamma_{o} + u_{o}, \\ & \beta_{i} = \gamma_{i}, \\ & \beta_{i} = \gamma_{i}, \end{aligned}$ $\begin{aligned} & \mathcal{Y}_{s} = \mathcal{G}_{s} + \mathcal{G}_{s}timec^{2}_{s} + \mathcal{G}_{s}timec^{2}_{s} + \mathcal{G}_{s}timec^{2}_{s} + \mathcal{E}_{\varepsilon} \\ & \mathcal{G}_{sc} = \mathcal{Y}_{sc} + \mathbf{u}_{sc} \\ & \mathcal{G}_{sc} = \mathcal{Y}_{sc} \end{aligned}$ $\begin{aligned} \mathbf{y}_{e} &= \beta_{ee} + \beta_{e}timsc + \mathbf{\varepsilon}_{e} \\ \beta_{ee} &= \gamma_{ee} + \mathbf{u}_{ee} \\ \beta_{ef} &= \gamma_{10} \end{aligned}$ $y_t = \beta_{0t} + \varepsilon_{nt}$ $\beta_{0t} = \gamma_{\infty} + u_{0t}$ m4R1 mR1a mR1b mR1f $\begin{aligned} & \mathbf{M4KL} \\ & \mathbf{y}_{c} = \beta_{u} + \beta_{c}timec^{2} + \beta_{c}timec^{2} + \beta_{c}timec^{2} + \mathcal{E}_{c} \\ & \beta_{w} = \gamma_{w} + \gamma_{e}techert_{c} + u_{w} \\ & \beta_{u} = \gamma_{c} \\ & \beta_{u} = \gamma_{c} \\ & \beta_{c} = \gamma_{c} \end{aligned}$ $\begin{aligned} \boldsymbol{y}_{t} &= \boldsymbol{\beta}_{ct} + \boldsymbol{\beta}_{t} timec_{-t} + \boldsymbol{\beta}_{t} timec^{2}_{-t} + \boldsymbol{\xi}_{d} \\ \boldsymbol{\beta}_{ct} &= \boldsymbol{\gamma}_{co} + \boldsymbol{\gamma}_{c,t} cohort_{t} + \boldsymbol{u}_{cc} \\ \boldsymbol{\beta}_{tt} &= \boldsymbol{\gamma}_{to} \end{aligned}$ $y_{ii} = \beta_{0i} + \varepsilon_{ii}$ $y_{\epsilon} = \beta_{o} + \beta_{c}timsc_{\epsilon} + \varepsilon_{\epsilon}$ $\beta_{oi} = \gamma_{oo} + \gamma_{oi} cohort_i + u_{oi}$ $\beta_{tr} = \gamma_{to} + \gamma_{tr} cohort_t + u_{tr}$ $\beta_{ii} = \gamma_{i0}$ $\beta_{2} = \gamma_{20}$ mR1c mR1d m5R1 $\begin{aligned} & \textbf{TISK1} \\ & \mathbf{y}_{\epsilon} = \beta_{\alpha} + \beta_{\alpha} timec^{2}_{\epsilon} + \beta_{\epsilon} timec^{2}_{\epsilon} + \beta_{\epsilon} timec^{2}_{\epsilon} + \mathbf{\varepsilon}_{\epsilon} \\ & \beta_{\alpha} = \gamma_{\alpha} + \gamma_{\alpha} \cdot cohort, + u_{\alpha} \\ & \beta_{\alpha} = \gamma_{\alpha} + \gamma_{\alpha} \cdot cohort, \\ & \beta_{\alpha} = \gamma_{\alpha} + \gamma_{\alpha} \cdot cohort, \\ & \beta_{\alpha} = \gamma_{\alpha} - \gamma_{\alpha} \cdot cohort, \end{aligned}$ $\begin{aligned} & \mathbf{y}_e = \beta_{id} + \beta_i timec_i + \beta_i timec_i + \boldsymbol{\varepsilon}_{e} \\ & \beta_{id} = \gamma_{io} + \gamma_{ii} cohort_i + \boldsymbol{u}_{oi} \\ & \beta_{ii} = \gamma_{io} + \gamma_{ii} cohort_i \\ & \beta_{ii} = \gamma_{io} + \gamma_{ii} cohort_i \end{aligned}$ $\begin{aligned} y_t &= \beta_{ct} + \beta_{tt} timsc_t + \varepsilon_t \\ \beta_{ct} &= \gamma_{co} + \gamma_{ct} cohort_t + u_{ct} \\ \beta_{tt} &= \gamma_{1c} + \gamma_{tt} cohort_t \end{aligned}$ mR1e $\begin{aligned} & \mathbf{y}_{e} = \beta_{w} + \beta_{e} timse \cdot + \beta_{e} timse^{2}, + \mathbf{S}_{e} \\ & \beta_{w} = \gamma_{w} + \gamma_{o} techert, + \mathbf{u}_{w} \\ & \beta_{t} = \gamma_{to} + \gamma_{t} techert, \\ & \beta_{t} = \gamma_{to} + \gamma_{t} techert, \end{aligned}$ $\begin{aligned} \mathbf{y}_{s} &= \beta_{s} + \beta_{s} timse^{s} + \beta_{s} timse^{s} + \beta_{s} timse^{s} + \varepsilon_{s} \\ \beta_{sc} &= \mathbf{y}_{cs} + \mathbf{y}_{c} \cdot cohort_{i} + \mathbf{u}_{sc} \\ \beta_{i} &= \gamma_{is} + \gamma_{i} \cdot cohort_{i} \\ \beta_{i} &= \gamma_{is} + \gamma_{i} \cdot cohort_{i} \\ \beta_{sc} &= \gamma_{sc} + \mathbf{y}_{sc} \cdot cohort_{i} \end{aligned}$ m7R1 $\begin{aligned} & \mathbf{y}_{t} = \boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{t} timse \cdot _{t} + \boldsymbol{\beta}_{u} timse^{2} \cdot_{t} + \boldsymbol{\beta}_{t} timse^{2} \cdot_{t} + \boldsymbol{\varepsilon}_{t} \\ & \boldsymbol{\beta}_{0t} = \boldsymbol{\gamma}_{0t} + \boldsymbol{\gamma}_{0t} cohort, + \mathbf{u}_{0t} \\ & \boldsymbol{\beta}_{tt} = \boldsymbol{\gamma}_{t0} + \boldsymbol{\gamma}_{t}, cohort, \end{aligned}$ $\beta_{2i} = \gamma_{10} + \gamma_{11} cohort,$ $\beta_{2i} = \gamma_{10} + \gamma_{21} cohort,$

3.1 R1 by row



3.2 R1 by column



4 Models w/ 2 random

Model Sequence Map

R2

2 random terms

m1R2

 $y_a = \beta_a + \beta_a timec + \varepsilon_a$ $\beta_{ii} = \gamma_{i0} + u_{ii}$ $\beta_{ii} = \gamma_{i0} + u_{ii}$ $\beta_{ii} = \gamma_{i0} + u_{ii}$

mR2b

 $y_e = \beta_{oi} + \beta_{ii}timsc_i + \varepsilon_e$ $\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + \mathbf{u}_{0i}$ $\beta_{0i} = \gamma_{10} + \mathbf{u}_{1i}$

mR2c

 $\begin{aligned} y_t &= \beta_{ct} + \beta_{tt} timec_t + \varepsilon_t \\ \beta_{ct} &= \gamma_{co} + \gamma_{ct} cohort_t + u_{ct} \\ \beta_{tt} &= \gamma_{10} + \gamma_{1t} cohort_t + u_{tt} \end{aligned}$

m2R2

 $\begin{aligned} & \mathbf{III} \mathbf{L} \mathbf{L} \\ & \mathbf{y}_{e} = \beta_{u} + \beta_{u} timsc^{2}_{v} + \beta_{u} timsc^{2}_{v} + \mathbf{\varepsilon}_{u} \\ & \beta_{u} = \mathbf{y}_{uu} + \mathbf{u}_{u} \\ & \beta_{u} = \mathbf{y}_{u} + \mathbf{u}_{u} \\ & \beta_{u} = \mathbf{y}_{u} \end{aligned}$

mR2f

 $\begin{aligned} \boldsymbol{y}_{t} &= \boldsymbol{\beta}_{ct} + \boldsymbol{\beta}_{t} timec_{-t} + \boldsymbol{\beta}_{t} timec^{2}_{-t} + \boldsymbol{\varepsilon}_{ct} \\ \boldsymbol{\beta}_{ct} &= \boldsymbol{\gamma}_{co} + \boldsymbol{\gamma}_{c_{1}} cohort_{t} + \boldsymbol{u}_{cc} \\ \boldsymbol{\beta}_{tt} &= \boldsymbol{\gamma}_{to} + \boldsymbol{u}_{tt} \end{aligned}$ $\beta_{2i} = \gamma_{20}$

mR2d

 $\begin{aligned} & \mathbf{y}_t = \boldsymbol{\beta}_{tt} + \boldsymbol{\beta}_t timec_t + \boldsymbol{\beta}_t timec_t^2 + \boldsymbol{\varepsilon}_{et} \\ & \boldsymbol{\beta}_{tt} = \boldsymbol{\gamma}_{to} + \boldsymbol{\gamma}_{tt} cohort_t + \boldsymbol{u}_{tt} \\ & \boldsymbol{\beta}_{tt} = \boldsymbol{\gamma}_{to} + \boldsymbol{\gamma}_{tt} cohort_t + \boldsymbol{u}_{tt} \\ & \boldsymbol{\beta}_{tt} = \boldsymbol{\gamma}_{to} + \boldsymbol{\gamma}_{tt} cohort_t + \boldsymbol{u}_{tt} \end{aligned}$

mR2e

 $\begin{aligned} & \mathbf{y}_{e} = \beta_{w} + \beta_{e} timse \cdot + \beta_{e} timse^{2}, + \mathbf{S}_{e} \\ & \beta_{w} = \gamma_{w} + \gamma_{o} techert, + \mathbf{u}_{w} \\ & \beta_{tt} = \gamma_{to} + \gamma_{tt} cohort, + \mathbf{u}_{tt} \\ & \beta_{tt} = \gamma_{to} + \gamma_{tt} cohort, \end{aligned}$

m3R2

 $\begin{aligned} \mathbf{y}_{e} &= \beta_{el} + \beta_{e}timsc_{-i} + \beta_{u}timsc_{-i}^{2} + \beta_{e}timsc_{-i}^{2} + \mathcal{E}_{e} \\ \beta_{el} &= \gamma_{eo} + \mathbf{u}_{el} \\ \beta_{el} &= \gamma_{lo} + \mathbf{u}_{el} \end{aligned}$ $\beta_{2i} = \gamma_{20}$ $\beta_{2i} = \gamma_{20}$ m4R2

 $\begin{aligned} & \mathbf{1114NZ} \\ & \mathbf{y}_t = \mathbf{\beta}_0 + \mathbf{\beta}_t timec^*_t + \mathbf{\beta}_t timec^*_t + \mathbf{\beta}_t timec^*_t + \mathbf{\xi}_t \\ & \mathbf{\beta}_{tt} = \mathbf{y}_{tt} + \mathbf{y}_{tt} cohort_t + \mathbf{u}_{tt} \\ & \mathbf{\beta}_t = \mathbf{y}_{tt} + \mathbf{u}_{tt} \\ & \mathbf{\beta}_t = \mathbf{y}_{tt} = \mathbf{y}_{tt} \end{aligned}$

m5R2 $\begin{aligned} \mathbf{y}_{t} &= \beta_{0} + \beta_{t}timec^{2} + \beta_{2}timec^{2} + \beta_{1}timec^{2} + \varepsilon_{t} \\ \beta_{0} &= \mathbf{y}_{0} + \mathbf{y}_{t}cohort_{t} + \mathbf{u}_{0} \\ \beta_{1} &= \mathbf{y}_{1} + \mathbf{y}_{t}cohort_{t} + \mathbf{u}_{t} \\ \beta_{p} &= \mathbf{y}_{10} + \mathbf{y}_{1}cohort_{t} + \mathbf{u}_{t} \\ \beta_{p} &= \mathbf{y}_{10} \\ \beta_{1} &= \mathbf{y}_{10} \end{aligned}$

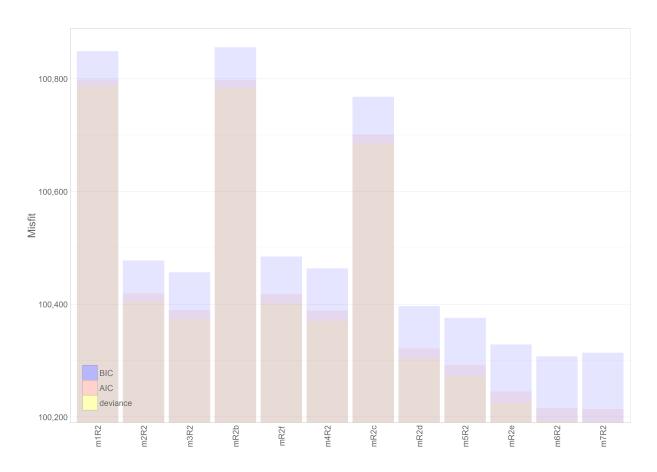
m6R2

 $\begin{aligned} & y_t = \beta_t, timec \ , + \beta_t timec^2 \ , + \beta_t timec^3 \ , + \varepsilon_t \\ & \beta_u = \gamma_u, + \gamma_u cohort_t + u_u \\ & \beta_t = \gamma_u, + \gamma_u cohort_t - u_t \\ & \beta_u = \gamma_u, + \gamma_u cohort_t \\ & \beta_u = \gamma_u, + \gamma_u cohort_t \end{aligned}$

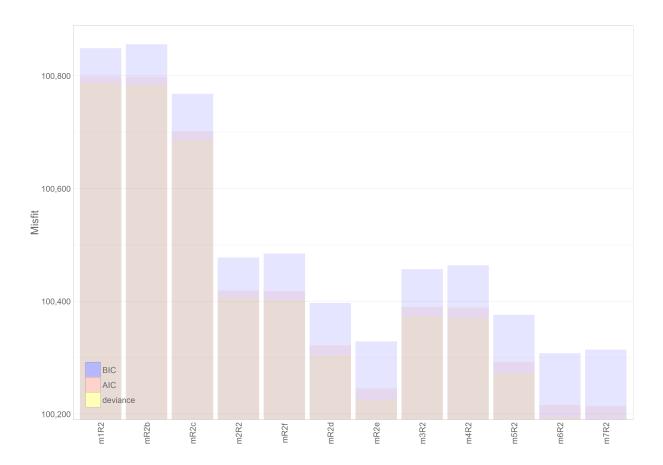
m7R2

 $\begin{aligned} & \mathbf{y}_{t} = \boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{t} timse \cdot _{t} + \boldsymbol{\beta}_{u} timse^{2} \cdot_{t} + \boldsymbol{\beta}_{t} timse^{2} \cdot_{t} + \boldsymbol{\varepsilon}_{t} \\ & \boldsymbol{\beta}_{0t} = \boldsymbol{\gamma}_{0t} + \boldsymbol{\gamma}_{0t} cohort, + \mathbf{u}_{0t} \\ & \boldsymbol{\beta}_{tt} = \boldsymbol{\gamma}_{t0} + \boldsymbol{\gamma}_{t}, cohort, + \mathbf{u}_{tt} \end{aligned}$ $\beta_{2i} = \gamma_{10} + \gamma_{11} cohort,$ $\beta_{2i} = \gamma_{10} + \gamma_{21} cohort,$

4.1 R2 by row



4.2 R2 by column

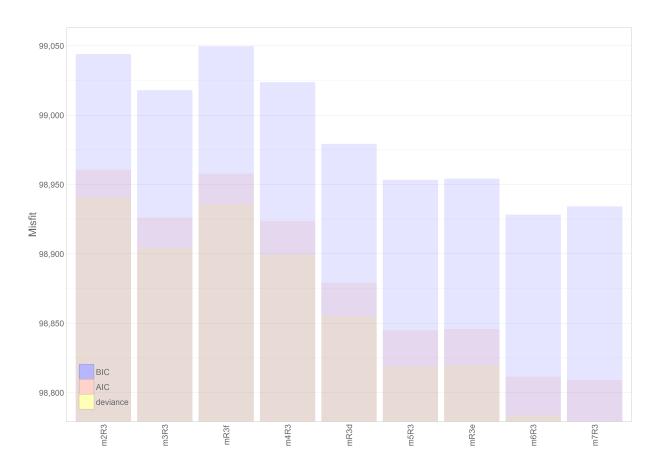


5 Models w/ 3 random

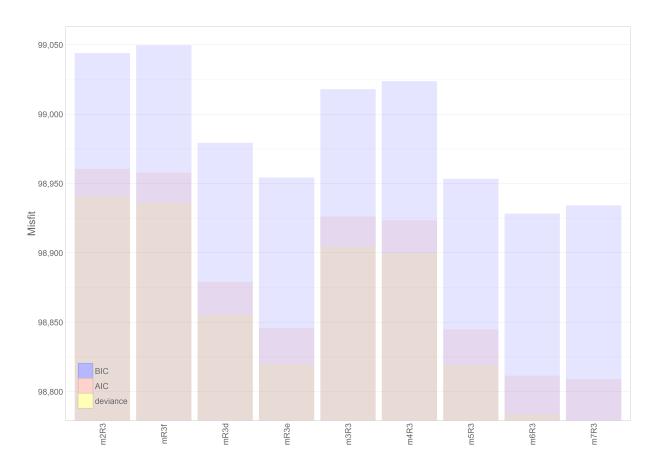
Model Sequence Map R3 3 random terms

```
m2R3
                                                                                                                                                                                                                                                                                                                                                                                                                                     m3R3
\begin{aligned} & \text{III}(\mathbf{A}) \\ & \mathbf{y}_{e} = \beta_{ee} + \beta_{e} \text{times} c_{e} + \beta_{e} \text{times} c_{e}^{2} + \mathbf{\varepsilon}_{e} \\ & \beta_{ee} = \gamma_{ee} + u_{ee} \\ & \beta_{e} = \gamma_{ee} + u_{ee} \\ & \beta_{e} = \gamma_{ee} + u_{ee} \end{aligned}
                                                                                                                                                                                                                                                                                                                                                                                                                                          \begin{aligned} & \mathcal{Y}_{u} = \mathcal{B}_{u} + \mathcal{B}_{u}timec^{2}_{,} + \mathcal{B}_{u}timec^{2}_{,} + \mathcal{B}_{s}_{t}timec^{2}_{,} + \mathcal{E}_{\varepsilon} \\ & \mathcal{B}_{u} = \mathcal{Y}_{us} + \mathbf{u}_{u}, \\ & \mathcal{B}_{z} = \mathcal{Y}_{zs} + \mathbf{u}_{u}, \\ & \mathcal{B}_{z} = \mathcal{Y}_{zs} + \mathbf{u}_{u}, \\ & \mathcal{B}_{z} = \mathcal{Y}_{zs}, \end{aligned}
                                                                                                                                                                                                                                                                                                                                                                                                                                     m4R3
       mR3f
                                                                                                                                                                                                                                                                                                                                                                                                                                          \begin{aligned} & \mathbf{MAKS} \\ & \mathbf{y}_{\epsilon} = \beta_{\alpha} + \beta_{\alpha} timec^{*}_{\epsilon} + \beta_{\alpha} timec^{*}_{\epsilon} + \beta_{\beta} timec^{*}_{\epsilon} + \mathcal{E}_{\epsilon} \\ & \beta_{\alpha} = \gamma_{\alpha} + \gamma_{\alpha} tochort_{\epsilon} + u_{\alpha} \\ & \beta_{\alpha} = \gamma_{\alpha} + \gamma_{\alpha} t_{\alpha} \\ & \beta_{\alpha} = \gamma_{2} + u_{\alpha} \\ & \beta_{\alpha} = \gamma_{2} + u_{\alpha} \end{aligned}
       \begin{aligned} & \mathbf{y}_{e} = \beta_{ec} + \beta_{e}timsc_{e} + \beta_{e}timsc_{e}^{2} + \varepsilon_{e} \\ & \beta_{el} = \gamma_{eo} + \gamma_{e,l}cohort_{l} + \mathbf{u}_{el} \\ & \beta_{el} = \gamma_{l,e} + \mathbf{u}_{el} \end{aligned}
       \beta_{2i} = \gamma_{20} + u_{2i}
       mR3d
                                                                                                                                                                                                                                                                                                                                                                                                                                        m5R3
                                                                                                                                                                                                                                                                                                                                                                                                                                          \begin{aligned} & \text{MSR3} \\ & \mathbf{y}_{\epsilon} = \beta_{u} + \beta_{u} timee^{\epsilon}_{\epsilon} + \beta_{v} timee^{\epsilon}_{\epsilon} + \beta_{v} timee^{\epsilon}_{\epsilon} + \epsilon_{e} \\ & \beta_{u} = \gamma_{u} + \gamma_{v} cohort, + \mathbf{u}_{u} \\ & \beta_{u} = \gamma_{v} + \gamma_{v} cohort, + \mathbf{u}_{u} \\ & \beta_{u} = \gamma_{v} + \gamma_{v} cohort, + \mathbf{u}_{u} \\ & \beta_{u} = \gamma_{v} + \mathbf{u}_{u} \end{aligned}
    \begin{aligned} & \mathbf{y}_t = \beta_{tt} + \beta_t timec_t + \beta_t timec_t^2 + \boldsymbol{\varepsilon}_{et} \\ & \beta_{tt} = \gamma_{t0} + \gamma_{t1} cohort_t + \boldsymbol{u}_{tt} \\ & \beta_{tt} = \gamma_{t0} + \gamma_{t1} cohort_t + \boldsymbol{u}_{tt} \\ & \beta_{tt} = \gamma_{t0} + \boldsymbol{u}_{tt} \end{aligned}
       mR3e
                                                                                                                                                                                                                                                                                                                                                                                                                                        m6R3
       \begin{aligned} & y_e = \beta_w + \beta_e timsec^2_{\ r} + \beta_e timsec^2_{\ r} + \delta_w \\ & \beta_w = \gamma_w + \gamma_{01} cohort_i + \mathbf{u}_w \\ & \beta_{11} = \gamma_{10} + \gamma_{11} cohort_i + \mathbf{u}_{11} \\ & \beta_{12} = \gamma_{20} + \gamma_{21} cohort_i + \mathbf{u}_{21} \end{aligned}
                                                                                                                                                                                                                                                                                                                                                                                                                                     \begin{aligned} & \mathcal{Y}_{s} = \mathcal{B}_{s} + \mathcal{B}_{s} timec^{2} + \mathcal{B}_{s} timec^{2} + \mathcal{B}_{s} timec^{2} + \mathcal{E}_{e} \\ & \mathcal{B}_{sc} = \mathcal{Y}_{sc} + \mathcal{Y}_{sc} cohort_{s} + u_{sc} \\ & \mathcal{B}_{sc} = \mathcal{Y}_{sc} + \mathcal{Y}_{sc} cohort_{s} + u_{sc} \\ & \mathcal{B}_{sc} = \mathcal{Y}_{sc} + \mathcal{Y}_{sc} cohort_{s} + u_{sc} \\ & \mathcal{B}_{sc} = \mathcal{Y}_{sc} + \mathcal{Y}_{sc} cohort_{s} + u_{sc} \end{aligned}
                                                                                                                                                                                                                                                                                                                                                                                                                                          m7R3
                                                                                                                                                                                                                                                                                                                                                                                                                                          \begin{aligned} & \Pi \Pi \Pi \\ & y_t = \beta_0 + \beta_t timec_t + \beta_t timec_t^2 + \beta_t timec_t^2 + \varepsilon_t \\ & \beta_0 = y_{00} + y_{01} cohort_t + u_{01} \\ & \beta_1 = y_{11} + y_{11} cohort_t + u_{11} \\ & \beta_2 = y_{12} + y_{12} cohort_t + u_{12}, \\ & \beta_4 = y_{20} + y_{31} cohort_t \end{aligned}
```

5.1 R3 by row



5.2 R3 by column



6 Models w/ 4 random

Model Sequence Map

R4

4 random terms

```
m3R4

y_i = \beta_0 + \beta_i timee_i + \beta_0 timee_i^2 + \beta_i timee_i^3 + \varepsilon_i
\beta_i = \gamma_i + \gamma_i

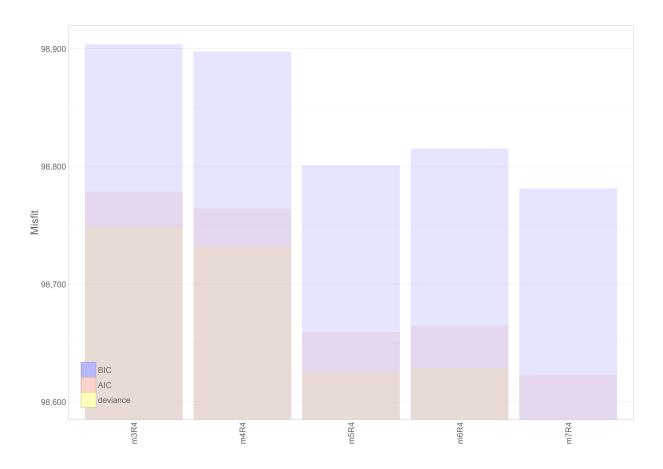
m4R4

y_i = \beta_0 + \beta_i timee_i + \beta_0 timee_i^3 + \beta_i timee_i^3 + \varepsilon_i
\beta_i = \gamma_i + \gamma_i

m5R4

y_i = \beta_0 + \beta_i timee_i + \beta_0 timee_i^3 + \beta_i timee_i^3 + \varepsilon_i
\beta_i = \gamma_i + \gamma_i
\beta_i = \gamma_i + \gamma_
```

6.1 R4 by row



7 General form

$$\begin{aligned} & \mathbf{y}_{l} = \mathbf{\Lambda} \mathbf{\eta}_{l} + \boldsymbol{\epsilon}_{l} & \mathbf{y}_{i} = \mathbf{\Lambda} \boldsymbol{\mu}_{\eta} + \mathbf{\Lambda} \boldsymbol{\Gamma} \mathbf{w}_{i} + \mathbf{\Lambda} \boldsymbol{\zeta}_{i} + \boldsymbol{\epsilon}_{i} & \text{Bollen & Curran (2006)} \\ & \mathbf{y}_{l} = \begin{bmatrix} \mathbf{y}_{l} \\ \mathbf{y}_{l} \\ \vdots \\ \mathbf{y}_{x} \end{bmatrix} \boldsymbol{\eta}_{l} = \begin{bmatrix} \mathbf{y}_{l} \\ \boldsymbol{\beta}_{l} \\ \vdots \\ \boldsymbol{\beta}_{p_{l}} \end{bmatrix} \boldsymbol{\mu}_{l} = \begin{bmatrix} \mathbf{y}_{cl} \\ \boldsymbol{\gamma}_{cl} \\ \boldsymbol{\gamma}_{l} \\ \vdots \\ \boldsymbol{\mu}_{p_{l}} \end{bmatrix} \boldsymbol{\gamma}_{l} = \begin{bmatrix} \mathbf{y}_{cl} \\ \boldsymbol{\gamma}_{cl} \\ \boldsymbol{\gamma}_{l} \\ \vdots \\ \boldsymbol{\gamma}_{p_{l}} \end{bmatrix} \boldsymbol{\gamma}_{l} = \begin{bmatrix} \mathbf{y}_{cl} \\ \boldsymbol{\gamma}_{cl} \\ \boldsymbol{\gamma}_{p_{l}} \\ \vdots \\ \boldsymbol{\gamma}_{p_{l}} \end{bmatrix} \boldsymbol{\gamma}_{l} = \begin{bmatrix} \mathbf{y}_{cl} \\ \boldsymbol{\gamma}_{cl} \\ \boldsymbol{\gamma}_{p_{l}} \end{bmatrix} \boldsymbol{\gamma}_{l} = \begin{bmatrix} \mathbf{y}_{cl} \\ \boldsymbol{\gamma}_{cl} \\ \boldsymbol{\gamma}_{p_{l}} \end{bmatrix} \boldsymbol{\gamma}_{l} = \begin{bmatrix} \mathbf{y}_{cl} \\ \boldsymbol{\gamma}_{cl} \\ \boldsymbol{\gamma}_{p_{l}} \\ \boldsymbol{\gamma}_{p$$

- \mathbf{y}_i A vector of responses of individual i for times T
- Λ Matrix of weights for P functions of time
- $\mathbf{\eta}_i$ Vector of person-specific weights for P time effects
- μ_n Vector of fixed effect estimates (mean/intercept)
- Γ Matrix of fixed effect estimates for ${f w}$, with K predictors
- \mathbf{w}_i Time invariant random predictors of $\mathbf{\eta}_i$
- ζ_i Random effect estimates
- ε_i Residual
- T Total number of time points in the data
- P Total number of time effects estimated in addition to mean/intercept
- K Total number of T predictors \mathbf{w}_i on time effects $\mathbf{\eta}_i$