Single model

Contents


```
modelNumber<- "m10"
numID<- 10000

modnum<-cat(modelNumber)

m10

dsL<-readRDS("./Data/Derived/dsL.rds")
ds<- dsL %>% dplyr::filter(id %in% c(1:numID),year %in% c(2000:2011)) %>%
  dplyr::mutate(timec=year-2000, timec2= timec^2, timec3= timec^3,
                agec= round( (agemon/12),0)-16) %>%
  dplyr::select(id,year,attend, timec,timec2, timec3, agec)
head(ds, 20)
```

```
   id year attend timec timec2 timec3 agec
1   1 2000      1     0      0      0    3
2   1 2001      6     1      1      1    4
3   1 2002      2     2      4      8    5
4   1 2003      1     3      9     27    6
5   1 2004      1     4     16     64    7
6   1 2005      1     5     25    125    8
7   1 2006      1     6     36    216    9
8   1 2007      1     7     49    343   10
9   1 2008      1     8     64    512   11
10  1 2009      1     9     81    729   12
11  1 2010      1    10    100   1000   13
12  1 2011      1    11    121   1331   14
13  2 2000      2     0      0      0    2
14  2 2001      2     1      1      1    3
15  2 2002      1     2      4      8    4
16  2 2003      1     3      9     27    5
```

```
17  2 2004      2     4     16     64    6
18  2 2005      2     5     25    125    8
19  2 2006     NA     6     36    216   NA
20  2 2007     NA     7     49    343   NA
```

```
modnum <-lmer (attend ~
                1  + agec + timec + timec2 + timec3
              + agec:timec +agec:timec2 + agec:timec3
              + (1 + timec + timec2 + timec3 | id),
              data = ds, REML=0)
```

```
Warning: convergence code 1 from bobyqa: bobyqa -- maximum number of function evaluations exceeded
Warning: Model failed to converge with max|grad| = 48731.7 (tol = 0.002)
Warning: the condition has length > 1 and only the first element will be used
Warning: Model is nearly unidentifiable: very large eigenvalue
 - Rescale variables?;Model is nearly unidentifiable: large eigenvalue ratio
 - Rescale variables?
```

```
model<- modnum
```

```
summary(model)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: attend ~ 1 + agec + timec + timec2 + timec3 + agec:timec + agec:timec2 +
    agec:timec3 + (1 + timec + timec2 + timec3 | id)
   Data: ds

     AIC      BIC   logLik deviance df.resid
  315388   315567  -157675   315350    89654

Scaled residuals:
   Min     1Q Median     3Q    Max
-4.607 -0.448 -0.073  0.391  5.134

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept) 2.723830 1.6504
          timec       0.934653 0.9668    0.00
          timec2      0.043489 0.2085   -0.13 -0.96
          timec3      0.000139 0.0118    0.17  0.91 -0.99
 Residual             1.132133 1.0640
Number of obs: 89673, groups: id, 8761

Fixed effects:
             Estimate Std. Error t value
(Intercept)  3.81e+00   3.80e-02   100.5
agec        -1.79e-01   1.26e-02   -14.2
timec       -2.17e-01   2.57e-02    -8.5
timec2      -7.16e-03   6.39e-03    -1.1
timec3       1.06e-03   4.72e-04     2.3
agec:timec   6.24e-02   4.80e-03    13.0
agec:timec2 -5.20e-03   7.33e-04    -7.1
agec:timec3  1.38e-04   3.89e-05     3.6
```

```
Correlation of Fixed Effects:
            (Intr) agec   timec  timec2 timec3 agc:tm agc:t2
agec        -0.830
timec        0.041 -0.238
timec2      -0.091  0.247 -0.729
timec3       0.117 -0.243  0.511 -0.934
agec:timec   0.459 -0.486 -0.507  0.013  0.190
agec:timec2 -0.286  0.232  0.723 -0.452  0.236 -0.857
agec:timec3  0.155 -0.058 -0.724  0.734 -0.619  0.566 -0.891
```

0.1 Model formula

```
model@call
```

```
lmer(formula = attend ~ 1 + agec + timec + timec2 + timec3 +
    agec:timec + agec:timec2 + agec:timec3 + (1 + timec + timec2 +
    timec3 | id), data = ds, REML = 0)
```

0.2 Fit and Information indices

```
# get indicies
mInfo<-summary(model)$AICtab
mInfo["N"]<- model@devcomp$dims["N"] # number of datapoints, verify
mInfo["p"]<- model@devcomp$dims["p"] # number of estimated parameters, verify
mInfo["ids"]<- (summary(model))$ngrps # number of units on level-2, here: individuals
mInfo
```

```
     AIC      BIC   logLik deviance df.resid        N        p      ids
  315388   315567  -157675   315350    89654    89673        8     8761
```

0.3 Random Effects (RE)

0.3.1 Matrix of RE

```
# extract RE covariance matrix
mREcov<-     data.frame(     summary(model)$varcor$id   ) # covariance matrix of RE
mREcor<-  data.frame(attr(summary(model)$varcor$id,"correlation")) # corrleation matrix of RE
mRE<-   data.frame(sd= (attr(summary(model)$varcor$id,"stddev")))
mRE$var<- mRE$sd^2
mRE<-mRE[c("var","sd")]
mRE
```

```
                 var     sd
(Intercept) 2.7238296 1.6504
timec       0.9346528 0.9668
timec2      0.0434892 0.2085
timec3      0.0001393 0.0118
```

```
mREcov
```

```
            X.Intercept.      timec    timec2     timec3
(Intercept)    2.7238296  0.0002616 -0.045266  0.0033807
timec          0.0002616  0.9346528 -0.192771  0.0103415
timec2        -0.0452662 -0.1927714  0.043489 -0.0024324
timec3         0.0033807  0.0103415 -0.002432  0.0001393
```

0.3.2 extracting RE for each individual

```
RE<- lme4:::ranef.merMod(model)$id
head(RE,6)
```

```
  (Intercept)   timec   timec2     timec3
1     -0.4933 -0.1637 -0.01643  0.0020082
2     -1.2389 -0.2469  0.07768 -0.0048398
3     -0.4170 -0.8988  0.26248 -0.0159721
4     -1.0949  0.1718 -0.04857  0.0039100
5     -0.1242 -0.5760  0.11263 -0.0057958
6     -0.3819  0.8317 -0.04011 -0.0001786
```

```
# however
cor(RE)  # not the same as mRE, find out why
```

```
            (Intercept)     timec  timec2  timec3
(Intercept)    1.000000  0.001894 -0.1651  0.2214
timec          0.001894  1.000000 -0.9288  0.8575
timec2        -0.165126 -0.928763  1.0000 -0.9849
timec3         0.221435  0.857521 -0.9849  1.0000
```

```
var(RE)  # not the same as mRE, find out why
```

```
            (Intercept)     timec     timec2     timec3
(Intercept)    2.212613  0.001709 -0.0311576  2.331e-03
timec          0.001709  0.368047 -0.0714745  3.681e-03
timec2        -0.031158 -0.071475  0.0160912 -8.841e-04
timec3         0.002331  0.003681 -0.0008841  5.008e-05
```

0.4 Fixed Effects (FE)

0.4.1 estimate of the FE

```
# similar ways to extract FE estimates, #3 is the fullest
FE<- summary(model)$coefficients
FE
```

```
              Estimate Std. Error t value
(Intercept)  3.8143107  3.795e-02 100.500
agec        -0.1788465  1.260e-02 -14.192
timec       -0.2168952  2.566e-02  -8.453
timec2      -0.0071627  6.385e-03  -1.122
timec3       0.0010643  4.725e-04   2.253
agec:timec   0.0623689  4.799e-03  12.997
agec:timec2 -0.0052026  7.335e-04  -7.093
agec:timec3  0.0001384  3.892e-05   3.557
```

0.4.2 Matrix of FE

```
mFE<- (summary(model)$vcov@factors$correlation) # notice that this is object of
mFE
```

```
8 x 8 Matrix of class "corMatrix"
            (Intercept)    agec    timec   timec2  timec3 agec:timec agec:timec2 agec:timec3
(Intercept)     1.00000 -0.8300  0.04065 -0.09081  0.1173    0.45897     -0.2859      0.1554
agec           -0.83002  1.0000 -0.23760  0.24722 -0.2434   -0.48574      0.2319     -0.0583
timec           0.04065 -0.2376  1.00000 -0.72854  0.5114   -0.50654      0.7229     -0.7243
timec2         -0.09081  0.2472 -0.72854  1.00000 -0.9340    0.01336     -0.4519      0.7341
timec3          0.11730 -0.2434  0.51140 -0.93403  1.0000    0.18993      0.2360     -0.6188
agec:timec      0.45897 -0.4857 -0.50654  0.01336  0.1899    1.00000     -0.8574      0.5656
agec:timec2    -0.28586  0.2319  0.72292 -0.45195  0.2360   -0.85740      1.0000     -0.8907
agec:timec3     0.15537 -0.0583 -0.72434  0.73407 -0.6188    0.56562     -0.8907      1.0000
```

0.5 Prediction and Residuals

```
dsp<- data.frame(getME(model,"X")) # no Y, only predictors (with interaction terms)
dsp$id<-getME(model,"flist")$id # first level grouping factor, individual
dsp$y<-getME(model,"y") # response vector
dsp$yHat<- predict(model) # predicted values
dsp$resid<- lme4:::residuals.merMod(model)
identical (  dsp$y-dsp$yHat, dsp$resid)
```

```
[1] TRUE
```

```
head(dsp,13)
```

```
   X.Intercept. agec timec timec2 timec3 agec.timec agec.timec2 agec.timec3 id y   yHat   resid
1             1    3     0      0      0          0           0           0  1 1 2.7845 -1.78447
2             1    4     1      1      1          4           4           4  1 6 2.4338  3.56624
3             1    5     2      4      8         10          20          40  1 2 2.1210 -0.12104
4             1    6     3      9     27         18          54         162  1 1 1.8410 -0.84100
5             1    7     4     16     64         28         112         448  1 1 1.5917 -0.59168
6             1    8     5     25    125         40         200        1000  1 1 1.3744 -0.37440
7             1    9     6     36    216         54         324        1944  1 1 1.1938 -0.19382
8             1   10     7     49    343         70         490        3430  1 1 1.0579 -0.05794
9             1   11     8     64    512         88         704        5632  1 1 0.9781  0.02194
10            1   12     9     81    729        108         972        8748  1 1 0.9688  0.03118
11            1   13    10    100   1000        130        1300       13000  1 1 1.0482 -0.04816
12            1   14    11    121   1331        154        1694       18634  1 1 1.2374 -0.23736
13            1    2     0      0      0          0           0           0  2 2 2.2177 -0.21770
```

Getting the standard error of residuals

```
sigma<-sigma(model) # std.error of residuals <- this methods is preferred
# however
SDR<-sd(dsp$resid) # not the same as sigma(model) = find out why
identical (sigma, SDR) # WHY?
```

```
[1] FALSE
```

```
# however, compare
sigma
```

```
[1] 1.064
```

```
SDR
```

```
[1] 0.9375
```

```
sqrt(sigma/SDR)
```

```
[1] 1.065
```

0.6 Conditional values

Predictions form fixed effects only, no individual variability is used in calculation

```
FE <- fixef(model)
# use fixed effects estimates to find conditional predictions
dsp$yPar<-(
  (FE["(Intercept)"])          +(FE["agec"]*dsp$agec)
  +(FE["timec"]*dsp$timec)     +(FE["agec:timec"]*dsp$agec*dsp$timec)
  +(FE["timec2"]*dsp$timec2)   +(FE["agec:timec2"]*dsp$agec*dsp$timec2)
  +(FE["timec3"]*dsp$timec3)   +(FE["agec:timec3"]*dsp$agec*dsp$timec3)
  )
```

0.7 List of availible elements

```
summary(model)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: attend ~ 1 + agec + timec + timec2 + timec3 + agec:timec + agec:timec2 +
    agec:timec3 + (1 + timec + timec2 + timec3 | id)
   Data: ds

     AIC      BIC   logLik deviance df.resid
  315388   315567  -157675   315350    89654

Scaled residuals:
   Min     1Q Median     3Q    Max
-4.607 -0.448 -0.073  0.391  5.134

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept) 2.723830 1.6504
          timec       0.934653 0.9668    0.00
          timec2      0.043489 0.2085   -0.13 -0.96
          timec3      0.000139 0.0118    0.17  0.91 -0.99
 Residual             1.132133 1.0640
Number of obs: 89673, groups: id, 8761
```

```
Fixed effects:
             Estimate Std. Error t value
(Intercept)  3.81e+00   3.80e-02   100.5
agec        -1.79e-01   1.26e-02   -14.2
timec       -2.17e-01   2.57e-02    -8.5
timec2      -7.16e-03   6.39e-03    -1.1
timec3       1.06e-03   4.72e-04     2.3
agec:timec   6.24e-02   4.80e-03    13.0
agec:timec2 -5.20e-03   7.33e-04    -7.1
agec:timec3  1.38e-04   3.89e-05     3.6

Correlation of Fixed Effects:
            (Intr) agec   timec  timec2 timec3 agc:tm agc:t2
agec        -0.830
timec        0.041 -0.238
timec2      -0.091  0.247 -0.729
timec3       0.117 -0.243  0.511 -0.934
agec:timec   0.459 -0.486 -0.507  0.013  0.190
agec:timec2 -0.286  0.232  0.723 -0.452  0.236 -0.857
agec:timec3  0.155 -0.058 -0.724  0.734 -0.619  0.566 -0.891

mInfo # model information indices

     AIC      BIC   logLik deviance df.resid        N        p      ids
  315388   315567  -157675   315350    89654    89673        8     8761

mRE  # variances and standard deviations of random effects

                 var     sd
(Intercept) 2.7238296 1.6504
timec       0.9346528 0.9668
timec2      0.0434892 0.2085
timec3      0.0001393 0.0118

mREcov # covariance matrix of Random Effects

            X.Intercept.      timec    timec2     timec3
(Intercept)    2.7238296  0.0002616 -0.045266  0.0033807
timec          0.0002616  0.9346528 -0.192771  0.0103415
timec2        -0.0452662 -0.1927714  0.043489 -0.0024324
timec3         0.0033807  0.0103415 -0.002432  0.0001393

mREcor # correlation  matrix of Random Effects

            X.Intercept.     timec  timec2  timec3
(Intercept)     1.000000  0.000164 -0.1315  0.1736
timec           0.000164  1.000000 -0.9562  0.9065
timec2         -0.131520 -0.956151  1.0000 -0.9884
timec3          0.173582  0.906471 -0.9884  1.0000

FE # estimates of Fixed Effects, SE, t-value
```

```
(Intercept)       agec      timec     timec2     timec3  agec:timec agec:timec2 agec:timec3
 3.8143107 -0.1788465 -0.2168952 -0.0071627  0.0010643  0.0623689 -0.0052026  0.0001384
```

```
mFE # matrix of correlations among Fixed Effects
```

```
8 x 8 Matrix of class "corMatrix"
           (Intercept)    agec    timec   timec2  timec3 agec:timec agec:timec2 agec:timec3
(Intercept)     1.00000 -0.8300  0.04065 -0.09081  0.1173    0.45897     -0.2859      0.1554
agec           -0.83002  1.0000 -0.23760  0.24722 -0.2434   -0.48574      0.2319     -0.0583
timec           0.04065 -0.2376  1.00000 -0.72854  0.5114   -0.50654      0.7229     -0.7243
timec2         -0.09081  0.2472 -0.72854  1.00000 -0.9340    0.01336     -0.4519      0.7341
timec3          0.11730 -0.2434  0.51140 -0.93403  1.0000    0.18993      0.2360     -0.6188
agec:timec      0.45897 -0.4857 -0.50654  0.01336  0.1899    1.00000     -0.8574      0.5656
agec:timec2    -0.28586  0.2319  0.72292 -0.45195  0.2360   -0.85740      1.0000     -0.8907
agec:timec3     0.15537 -0.0583 -0.72434  0.73407 -0.6188    0.56562     -0.8907      1.0000
```

```
sigma # standard deviation of residual
```

```
[1] 1.064
```

```
head(dsp,13) # input + output + residual + conditional
```

```
   X.Intercept. agec timec timec2 timec3 agec.timec agec.timec2 agec.timec3 id y   yHat   resid yPar
1             1    3     0      0      0         0          0          0  1 1 2.7845 -1.78447 3.278
2             1    4     1      1      1         4          4          4  1 6 2.4338  3.56624 3.105
3             1    5     2      4      8        10         20         40  1 2 2.1210 -0.12104 2.991
4             1    6     3      9     27        18         54        162  1 1 1.8410 -0.84100 2.919
5             1    7     4     16     64        28        112        448  1 1 1.5917 -0.59168 2.874
6             1    8     5     25    125        40        200       1000  1 1 1.3744 -0.37440 2.846
7             1    9     6     36    216        54        324       1944  1 1 1.1938 -0.19382 2.827
8             1   10     7     49    343        70        490       3430  1 1 1.0579 -0.05794 2.813
9             1   11     8     64    512        88        704       5632  1 1 0.9781  0.02194 2.804
10            1   12     9     81    729       108        972       8748  1 1 0.9688  0.03118 2.802
11            1   13    10    100   1000       130       1300      13000  1 1 1.0482 -0.04816 2.813
12            1   14    11    121   1331       154       1694      18634  1 1 1.2374 -0.23736 2.846
13            1    2     0      0      0         0          0          0  2 2 2.2177 -0.21770 3.457
```

```
modelSave<-cat(paste0(modelNumber,"S"))
```

```
m10S
```

```
modS<- list(mInfo,mRE,mREcov,mREcor,FE,mFE,sigma,dsp) # to save in an external file
str(modS)
```

```
List of 8
 $ : Named num [1:8] 315388 315567 -157675 315350 89654 ...
  ..- attr(*, "names")= chr [1:8] "AIC" "BIC" "logLik" "deviance" ...
 $ :'data.frame':   4 obs. of  2 variables:
  ..$ var: num [1:4] 2.72383 0.934653 0.043489 0.000139
  ..$ sd : num [1:4] 1.6504 0.9668 0.2085 0.0118
 $ :'data.frame':   4 obs. of  4 variables:
```

```
 ..$ X.Intercept.: num [1:4] 2.72383 0.000262 -0.045266 0.003381
 ..$ timec       : num [1:4] 0.000262 0.934653 -0.192771 0.010342
 ..$ timec2      : num [1:4] -0.04527 -0.19277 0.04349 -0.00243
 ..$ timec3      : num [1:4] 0.003381 0.010342 -0.002432 0.000139
$ :'data.frame':   4 obs. of  4 variables:
 ..$ X.Intercept.: num [1:4] 1 0.000164 -0.13152 0.173582
 ..$ timec       : num [1:4] 0.000164 1 -0.956151 0.906471
 ..$ timec2      : num [1:4] -0.132 -0.956 1 -0.988
 ..$ timec3      : num [1:4] 0.174 0.906 -0.988 1
$ : Named num [1:8] 3.81431 -0.17885 -0.2169 -0.00716 0.00106 ...
 ..- attr(*, "names")= chr [1:8] "(Intercept)" "agec" "timec" "timec2" ...
$ :Formal class 'corMatrix' [package "Matrix"] with 6 slots
 .. ..@ sd      : num [1:8] 0.037953 0.012602 0.025658 0.006385 0.000472 ...
 .. ..@ x       : num [1:64] 1 -0.83 0.0407 -0.0908 0.1173 ...
 .. ..@ Dim     : int [1:2] 8 8
 .. ..@ Dimnames:List of 2
 .. .. ..$ : chr [1:8] "(Intercept)" "agec" "timec" "timec2" ...
 .. .. ..$ : chr [1:8] "(Intercept)" "agec" "timec" "timec2" ...
 .. ..@ uplo    : chr "U"
 .. ..@ factors :List of 1
 .. .. ..$ Cholesky:Formal class 'Cholesky' [package "Matrix"] with 5 slots
 .. .. .. .. ..@ x       : num [1:64] 1 0 0 0 0 ...
 .. .. .. .. ..@ Dim     : int [1:2] 8 8
 .. .. .. .. ..@ Dimnames:List of 2
 .. .. .. .. .. ..$ : NULL
 .. .. .. .. .. ..$ : NULL
 .. .. .. .. ..@ uplo    : chr "U"
 .. .. .. .. ..@ diag    : chr "N"
$ : num 1.06
$ :'data.frame':   89673 obs. of  13 variables:
 ..$ X.Intercept.: num [1:89673] 1 1 1 1 1 1 1 1 1 1 ...
 ..$ agec        : num [1:89673] 3 4 5 6 7 8 9 10 11 12 ...
 ..$ timec       : num [1:89673] 0 1 2 3 4 5 6 7 8 9 ...
 ..$ timec2      : num [1:89673] 0 1 4 9 16 25 36 49 64 81 ...
 ..$ timec3      : num [1:89673] 0 1 8 27 64 125 216 343 512 729 ...
 ..$ agec.timec  : num [1:89673] 0 4 10 18 28 40 54 70 88 108 ...
 ..$ agec.timec2 : num [1:89673] 0 4 20 54 112 200 324 490 704 972 ...
 ..$ agec.timec3 : num [1:89673] 0 4 40 162 448 ...
 ..$ id          : Factor w/ 8761 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 ..$ y           : num [1:89673] 1 6 2 1 1 1 1 1 1 1 ...
 ..$ yHat        : num [1:89673] 2.78 2.43 2.12 1.84 1.59 ...
 ..$ resid       : num [1:89673] -1.784 3.566 -0.121 -0.841 -0.592 ...
 ..$ yPar        : num [1:89673] 3.28 3.11 2.99 2.92 2.87 ...
```