Single model

Contents


```
dsL<-readRDS("./Data/Derived/dsL.rds")
ds<- dsL %>% dplyr::filter(id %in% c(1:200),year %in% c(2000:2011)) %>%
  dplyr::mutate(timec=year-2000, timec2= timec^2, timec3= timec^3,
                agec= round( (agemon/12),0)-16) %>%
  dplyr::select(id,year,attend, timec,timec2, timec3, agec)
head(ds, 20)
```

```
   id year attend timec timec2 timec3 agec
1   1 2000      1     0      0      0    3
2   1 2001      6     1      1      1    4
3   1 2002      2     2      4      8    5
4   1 2003      1     3      9     27    6
5   1 2004      1     4     16     64    7
6   1 2005      1     5     25    125    8
7   1 2006      1     6     36    216    9
8   1 2007      1     7     49    343   10
9   1 2008      1     8     64    512   11
10  1 2009      1     9     81    729   12
11  1 2010      1    10    100   1000   13
12  1 2011      1    11    121   1331   14
13  2 2000      2     0      0      0    2
14  2 2001      2     1      1      1    3
15  2 2002      1     2      4      8    4
16  2 2003      1     3      9     27    5
17  2 2004      2     4     16     64    6
18  2 2005      2     5     25    125    8
19  2 2006     NA     6     36    216   NA
20  2 2007     NA     7     49    343   NA
```

```
modelNumber<- "m10"
modnum<-cat(modelNumber)
```

```
m10

modnum <-lmer (attend ~
                1  + agec + timec + timec2 + timec3
              + agec:timec +agec:timec2 + agec:timec3
              + (1 + timec + timec2 + timec3 | id),
              data = ds, REML=0)

Warning: convergence code 1 from bobyqa: bobyqa -- maximum number of function evaluations exceeded
Warning: Model failed to converge with max|grad| = 680.408 (tol = 0.002)
Warning: the condition has length > 1 and only the first element will be used
Warning: Model is nearly unidentifiable: very large eigenvalue
 - Rescale variables?;Model is nearly unidentifiable: large eigenvalue ratio
 - Rescale variables?

model<- modnum

summary(model)

Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: attend ~ 1 + agec + timec + timec2 + timec3 + agec:timec + agec:timec2 +
    agec:timec3 + (1 + timec + timec2 + timec3 | id)
   Data: ds

     AIC      BIC   logLik deviance df.resid
    6490     6595    -3226     6452     1854

Scaled residuals:
   Min     1Q Median     3Q    Max
-3.559 -0.442 -0.097  0.375  4.992

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept) 1.53e+00 1.23574
          timec       6.74e-01 0.82117  -0.12
          timec2      2.77e-02 0.16635   0.00 -0.95
          timec3      8.21e-05 0.00906   0.03  0.90 -0.98
 Residual             1.14e+00 1.06840
Number of obs: 1873, groups: id, 191

Fixed effects:
             Estimate Std. Error t value
(Intercept)  3.37e+00   2.22e-01   15.16
agec        -1.33e-01   7.49e-02   -1.78
timec       -1.32e-01   1.64e-01   -0.80
timec2      -1.95e-02   4.20e-02   -0.46
timec3       7.12e-04   3.21e-03    0.22
agec:timec   3.83e-02   3.26e-02    1.17
agec:timec2 -1.41e-03   5.00e-03   -0.28
agec:timec3  2.02e-05   2.64e-04    0.08

Correlation of Fixed Effects:
            (Intr) agec   timec  timec2 timec3 agc:tm agc:t2
```

```
agec        -0.843
timec       -0.085 -0.147
timec2      -0.008  0.198 -0.698
timec3       0.038 -0.186  0.455 -0.927
agec:timec   0.490 -0.504 -0.558  0.015  0.199
agec:timec2 -0.338  0.275  0.756 -0.449  0.215 -0.865
agec:timec3  0.223 -0.121 -0.739  0.745 -0.617  0.573 -0.886
```

0.1 Model formula

```
model@call
```

```
lmer(formula = attend ~ 1 + agec + timec + timec2 + timec3 +
    agec:timec + agec:timec2 + agec:timec3 + (1 + timec + timec2 +
    timec3 | id), data = ds, REML = 0)
```

0.2 Fit and Information indices

```
# get indicies
mInfo<-summary(model)$AICtab
mInfo["N"]<- model@devcomp$dims["N"] # number of datapoints, verify
mInfo["p"]<- model@devcomp$dims["p"] # number of estimated parameters, verify
mInfo["ids"]<- (summary(model))$ngrps # number of units on level-2, here: individuals
mInfo
```

```
     AIC      BIC   logLik deviance df.resid        N        p      ids
    6490     6595    -3226     6452     1854     1873        8      191
```

0.3 Random Effects (RE)

0.3.1 Matrix of RE

```
# extract RE covariance matrix
mREcov<-     data.frame(     summary(model)$varcor$id   ) # covariance matrix of RE
mREcor<-  data.frame(attr(summary(model)$varcor$id,"correlation")) # corrleation matrix of RE
mRE<-   data.frame(sd= (attr(summary(model)$varcor$id,"stddev")))
mRE$var<- mRE$sd^2
mRE<-mRE[c("var","sd")]
mRE
```

```
                 var       sd
(Intercept) 1.527e+00 1.235744
timec       6.743e-01 0.821170
timec2      2.767e-02 0.166350
timec3      8.205e-05 0.009058
```

```
mREcov
```

```
            X.Intercept.     timec     timec2     timec3
(Intercept)    1.5270624 -0.119554  0.0002804  2.908e-04
timec         -0.1195542  0.674320 -0.1302896  6.658e-03
timec2         0.0002804 -0.130290  0.0276723 -1.483e-03
timec3         0.0002908  0.006658 -0.0014833  8.205e-05
```

0.3.2 extracting RE for each individual

```
RE<- lme4:::ranef.merMod(model)$id
head(RE,6)
```

```
  (Intercept)   timec    timec2    timec3
1     -0.1998 -0.1948 -0.005072  0.001247
2     -0.8681 -0.2208  0.064432 -0.003808
3     -0.1842 -0.6816  0.197827 -0.011676
4     -0.7941  0.1096 -0.031264  0.002802
5      0.1088 -0.4967  0.096427 -0.004834
6     -0.1551  0.9127 -0.060165  0.001026
```

```
# however
cor(RE)  # not the same as mRE, find out why
```

```
            (Intercept)   timec  timec2  timec3
(Intercept)      1.0000  0.0319 -0.1996  0.2342
timec            0.0319  1.0000 -0.9257  0.8486
timec2          -0.1996 -0.9257  1.0000 -0.9817
timec3           0.2342  0.8486 -0.9817  1.0000
```

```
var(RE)  # not the same as mRE, find out why
```

```
            (Intercept)     timec     timec2     timec3
(Intercept)    1.487037  0.019434 -0.0233827  0.0014309
timec          0.019434  0.249575 -0.0444308  0.0021240
timec2        -0.023383 -0.044431  0.0092298 -0.0004726
timec3         0.001431  0.002124 -0.0004726  0.0000251
```

0.4 Fixed Effects (FE)

0.4.1 estimate of the FE

```
# similar ways to extract FE estimates, #3 is the fullest
FE<- summary(model)$coefficients
FE
```

```
              Estimate Std. Error  t value
(Intercept)  3.365e+00  0.2220560 15.15503
agec        -1.332e-01  0.0748883 -1.77801
timec       -1.320e-01  0.1641525 -0.80443
timec2      -1.951e-02  0.0419778 -0.46471
timec3       7.124e-04  0.0032064  0.22218
agec:timec   3.830e-02  0.0326349  1.17370
agec:timec2 -1.409e-03  0.0050003 -0.28168
agec:timec3  2.016e-05  0.0002644  0.07626
```

0.4.2 Matrix of FE

```
mFE<- (summary(model)$vcov@factors$correlation) # notice that this is object of
mFE
```

```
8 x 8 Matrix of class "corMatrix"
            (Intercept)    agec   timec    timec2   timec3 agec:timec agec:timec2 agec:timec3
(Intercept)    1.000000 -0.8431 -0.0847 -0.007885  0.03823    0.49037     -0.3380      0.2233
agec          -0.843145  1.0000 -0.1466  0.198315 -0.18560   -0.50436      0.2747     -0.1210
timec         -0.084700 -0.1466  1.0000 -0.698075  0.45470   -0.55822      0.7563     -0.7388
timec2        -0.007885  0.1983 -0.6981  1.000000 -0.92735    0.01546     -0.4489      0.7448
timec3         0.038229 -0.1856  0.4547 -0.927346  1.00000    0.19911      0.2154     -0.6169
agec:timec     0.490370 -0.5044 -0.5582  0.015461  0.19911    1.00000     -0.8654      0.5728
agec:timec2   -0.337972  0.2747  0.7563 -0.448850  0.21543   -0.86543      1.0000     -0.8860
agec:timec3    0.223305 -0.1210 -0.7388  0.744776 -0.61695    0.57280     -0.8860      1.0000
```

0.5 Prediction and Residuals

```
dsp<- data.frame(getME(model,"X")) # no Y, only predictors (with interaction terms)
dsp$id<-getME(model,"flist")$id # first level grouping factor, individual
dsp$y<-getME(model,"y") # response vector
dsp$yHat<- predict(model) # predicted values
dsp$resid<- lme4:::residuals.merMod(model)
identical (  dsp$y-dsp$yHat, dsp$resid)
```

```
[1] TRUE
```

```
head(dsp,13)
```

```
   X.Intercept. agec timec timec2 timec3 agec.timec agec.timec2 agec.timec3 id y  yHat    resid
1             1    3     0      0      0          0           0           0  1 1 2.766 -1.766010
2             1    4     1      1      1          4           4           4  1 6 2.431  3.568956
3             1    5     2      4      8         10          20          40  1 2 2.119 -0.119025
4             1    6     3      9     27         18          54         162  1 1 1.834 -0.834349
5             1    7     4     16     64         28         112         448  1 1 1.582 -0.581892
6             1    8     5     25    125         40         200        1000  1 1 1.367 -0.367016
7             1    9     6     36    216         54         324        1944  1 1 1.196 -0.195566
8             1   10     7     49    343         70         490        3430  1 1 1.074 -0.073873
9             1   11     8     64    512         88         704        5632  1 1 1.009 -0.008749
10            1   12     9     81    729        108         972        8748  1 1 1.007 -0.007492
11            1   13    10    100   1000        130        1300       13000  1 1 1.078 -0.077881
12            1   14    11    121   1331        154        1694       18634  1 1 1.228 -0.228184
13            1    2     0      0      0          0           0           0  2 2 2.231 -0.230870
```

Getting the standard error of residuals

```
sigma<-sigma(model) # std.error of residuals <- this methods is preferred
# however
SDR<-sd(dsp$resid) # not the same as sigma(model) = find out why
identical (sigma, SDR) # WHY?
```

```
[1] FALSE
```

```
# however, compare
sigma
```

```
[1] 1.068
```

```
SDR
```

```
[1] 0.9335
```

```
sqrt(sigma/SDR)
```

```
[1] 1.07
```

0.6 Conditional values

Predictions form fixed effects only, no individual variability is used in calculation

```
FE <- fixef(model)
# use fixed effects estimates to find conditional predictions
dsp$yPar<-(
  (FE["(Intercept)"])          +(FE["agec"]*dsp$agec)
  +(FE["timec"]*dsp$timec)     +(FE["agec:timec"]*dsp$agec*dsp$timec)
  +(FE["timec2"]*dsp$timec2)   +(FE["agec:timec2"]*dsp$agec*dsp$timec2)
  +(FE["timec3"]*dsp$timec3)   +(FE["agec:timec3"]*dsp$agec*dsp$timec3)
  )
```

0.7 List of availible elements

```
summary(model)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: attend ~ 1 + agec + timec + timec2 + timec3 + agec:timec + agec:timec2 +
    agec:timec3 + (1 + timec + timec2 + timec3 | id)
   Data: ds

     AIC      BIC   logLik deviance df.resid
    6490     6595    -3226     6452     1854

Scaled residuals:
   Min     1Q Median     3Q    Max
-3.559 -0.442 -0.097  0.375  4.992

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept) 1.53e+00 1.23574
          timec       6.74e-01 0.82117  -0.12
          timec2      2.77e-02 0.16635   0.00 -0.95
          timec3      8.21e-05 0.00906   0.03  0.90 -0.98
 Residual             1.14e+00 1.06840
Number of obs: 1873, groups: id, 191
```

```
Fixed effects:
             Estimate Std. Error t value
(Intercept)  3.37e+00   2.22e-01   15.16
agec        -1.33e-01   7.49e-02   -1.78
timec       -1.32e-01   1.64e-01   -0.80
timec2      -1.95e-02   4.20e-02   -0.46
timec3       7.12e-04   3.21e-03    0.22
agec:timec   3.83e-02   3.26e-02    1.17
agec:timec2 -1.41e-03   5.00e-03   -0.28
agec:timec3  2.02e-05   2.64e-04    0.08

Correlation of Fixed Effects:
            (Intr) agec   timec  timec2 timec3 agc:tm agc:t2
agec        -0.843
timec       -0.085 -0.147
timec2      -0.008  0.198 -0.698
timec3       0.038 -0.186  0.455 -0.927
agec:timec   0.490 -0.504 -0.558  0.015  0.199
agec:timec2 -0.338  0.275  0.756 -0.449  0.215 -0.865
agec:timec3  0.223 -0.121 -0.739  0.745 -0.617  0.573 -0.886

mInfo # model information indices

     AIC      BIC   logLik deviance df.resid        N        p      ids
    6490     6595    -3226     6452     1854     1873        8      191

mRE  # variances and standard deviations of random effects

                 var       sd
(Intercept) 1.527e+00 1.235744
timec       6.743e-01 0.821170
timec2      2.767e-02 0.166350
timec3      8.205e-05 0.009058

mREcov # covariance matrix of Random Effects

            X.Intercept.     timec     timec2     timec3
(Intercept)    1.5270624 -0.119554  0.0002804  2.908e-04
timec         -0.1195542  0.674320 -0.1302896  6.658e-03
timec2         0.0002804 -0.130290  0.0276723 -1.483e-03
timec3         0.0002908  0.006658 -0.0014833  8.205e-05

mREcor # correlation  matrix of Random Effects

            X.Intercept.   timec    timec2   timec3
(Intercept)     1.000000 -0.1178  0.001364  0.02598
timec          -0.117816  1.0000 -0.953793  0.89509
timec2          0.001364 -0.9538  1.000000 -0.98438
timec3          0.025983  0.8951 -0.984383  1.00000

FE # estimates of Fixed Effects, SE, t-value
```

```
(Intercept)       agec      timec     timec2     timec3  agec:timec agec:timec2 agec:timec3
 3.365e+00 -1.332e-01 -1.320e-01 -1.951e-02  7.124e-04  3.830e-02 -1.409e-03  2.016e-05
```

```
mFE # matrix of correlations among Fixed Effects
```

```
8 x 8 Matrix of class "corMatrix"
           (Intercept)    agec   timec    timec2   timec3 agec:timec agec:timec2 agec:timec3
(Intercept)    1.000000 -0.8431 -0.0847 -0.007885  0.03823    0.49037     -0.3380      0.2233
agec          -0.843145  1.0000 -0.1466  0.198315 -0.18560   -0.50436      0.2747     -0.1210
timec         -0.084700 -0.1466  1.0000 -0.698075  0.45470   -0.55822      0.7563     -0.7388
timec2        -0.007885  0.1983 -0.6981  1.000000 -0.92735    0.01546     -0.4489      0.7448
timec3         0.038229 -0.1856  0.4547 -0.927346  1.00000    0.19911      0.2154     -0.6169
agec:timec     0.490370 -0.5044 -0.5582  0.015461  0.19911    1.00000     -0.8654      0.5728
agec:timec2   -0.337972  0.2747  0.7563 -0.448850  0.21543   -0.86543      1.0000     -0.8860
agec:timec3    0.223305 -0.1210 -0.7388  0.744776 -0.61695    0.57280     -0.8860      1.0000
```

```
sigma # standard deviation of residual
```

```
[1] 1.068
```

```
head(dsp,13) # input + output + residual + conditional
```

```
   X.Intercept. agec timec timec2 timec3 agec.timec agec.timec2 agec.timec3 id y  yHat    resid yPar
1             1    3     0      0      0         0          0          0  1 1 2.766 -1.766010 2.966
2             1    4     1      1      1         4          4          4  1 6 2.431  3.568956 2.829
3             1    5     2      4      8        10         20         40  1 2 2.119 -0.119025 2.719
4             1    6     3      9     27        18         54        162  1 1 1.834 -0.834349 2.631
5             1    7     4     16     64        28        112        448  1 1 1.582 -0.581892 2.562
6             1    8     5     25    125        40        200       1000  1 1 1.367 -0.367016 2.512
7             1    9     6     36    216        54        324       1944  1 1 1.196 -0.195566 2.477
8             1   10     7     49    343        70        490       3430  1 1 1.074 -0.073873 2.458
9             1   11     8     64    512        88        704       5632  1 1 1.009 -0.008749 2.453
10            1   12     9     81    729       108        972       8748  1 1 1.007 -0.007492 2.462
11            1   13    10    100   1000       130       1300      13000  1 1 1.078 -0.077881 2.486
12            1   14    11    121   1331       154       1694      18634  1 1 1.228 -0.228184 2.525
13            1    2     0      0      0         0          0          0  2 2 2.231 -0.230870 3.099
```

```
modelSave<-cat(paste0(modelNumber,"S"))
```

```
m10S
```

```
modS<- list(mInfo,mRE,mREcov,mREcor,FE,mFE,sigma,dsp) # model save
str(modS)
```

```
List of 8
 $ : Named num [1:8] 6490 6595 -3226 6452 1854 ...
  ..- attr(*, "names")= chr [1:8] "AIC" "BIC" "logLik" "deviance" ...
 $ :'data.frame':   4 obs. of  2 variables:
  ..$ var: num [1:4] 1.53 6.74e-01 2.77e-02 8.21e-05
  ..$ sd : num [1:4] 1.23574 0.82117 0.16635 0.00906
 $ :'data.frame':   4 obs. of  4 variables:
```

```
 ..$ X.Intercept.: num [1:4] 1.527062 -0.119554 0.00028 0.000291
 ..$ timec       : num [1:4] -0.11955 0.67432 -0.13029 0.00666
 ..$ timec2      : num [1:4] 0.00028 -0.13029 0.02767 -0.00148
 ..$ timec3      : num [1:4] 2.91e-04 6.66e-03 -1.48e-03 8.21e-05
$ :'data.frame':   4 obs. of  4 variables:
 ..$ X.Intercept.: num [1:4] 1 -0.11782 0.00136 0.02598
 ..$ timec       : num [1:4] -0.118 1 -0.954 0.895
 ..$ timec2      : num [1:4] 0.00136 -0.95379 1 -0.98438
 ..$ timec3      : num [1:4] 0.026 0.895 -0.984 1
$ : Named num [1:8] 3.365265 -0.133152 -0.13205 -0.019508 0.000712 ...
 ..- attr(*, "names")= chr [1:8] "(Intercept)" "agec" "timec" "timec2" ...
$ :Formal class 'corMatrix' [package "Matrix"] with 6 slots
 .. ..@ sd      : num [1:8] 0.22206 0.07489 0.16415 0.04198 0.00321 ...
 .. ..@ x       : num [1:64] 1 -0.84315 -0.0847 -0.00788 0.03823 ...
 .. ..@ Dim     : int [1:2] 8 8
 .. ..@ Dimnames:List of 2
 .. .. ..$ : chr [1:8] "(Intercept)" "agec" "timec" "timec2" ...
 .. .. ..$ : chr [1:8] "(Intercept)" "agec" "timec" "timec2" ...
 .. ..@ uplo    : chr "U"
 .. ..@ factors :List of 1
 .. .. ..$ Cholesky:Formal class 'Cholesky' [package "Matrix"] with 5 slots
 .. .. .. .. ..@ x       : num [1:64] 1 0 0 0 0 ...
 .. .. .. .. ..@ Dim     : int [1:2] 8 8
 .. .. .. .. ..@ Dimnames:List of 2
 .. .. .. .. .. ..$ : NULL
 .. .. .. .. .. ..$ : NULL
 .. .. .. .. ..@ uplo    : chr "U"
 .. .. .. .. ..@ diag    : chr "N"
$ : num 1.07
$ :'data.frame':   1873 obs. of  13 variables:
 ..$ X.Intercept.: num [1:1873] 1 1 1 1 1 1 1 1 1 1 ...
 ..$ agec        : num [1:1873] 3 4 5 6 7 8 9 10 11 12 ...
 ..$ timec       : num [1:1873] 0 1 2 3 4 5 6 7 8 9 ...
 ..$ timec2      : num [1:1873] 0 1 4 9 16 25 36 49 64 81 ...
 ..$ timec3      : num [1:1873] 0 1 8 27 64 125 216 343 512 729 ...
 ..$ agec.timec  : num [1:1873] 0 4 10 18 28 40 54 70 88 108 ...
 ..$ agec.timec2 : num [1:1873] 0 4 20 54 112 200 324 490 704 972 ...
 ..$ agec.timec3 : num [1:1873] 0 4 40 162 448 ...
 ..$ id          : Factor w/ 191 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 ..$ y           : num [1:1873] 1 6 2 1 1 1 1 1 1 1 ...
 ..$ yHat        : num [1:1873] 2.77 2.43 2.12 1.83 1.58 ...
 ..$ resid       : num [1:1873] -1.766 3.569 -0.119 -0.834 -0.582 ...
 ..$ yPar        : num [1:1873] 2.97 2.83 2.72 2.63 2.56 ...
```