Model sequence report

Contents

1 About

The origin of the data for the model

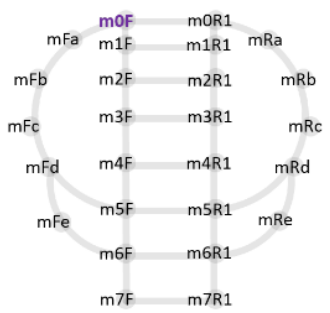
```
numID<- 9022 # highest id value (max = 9022)
### Define the data that will populate the model
ds<- dsL %>%  # chose conditions to apply in creating dataset for modeling
  dplyr::filter(id < numID) %.% # 1:9022
  dplyr::filter(year %in% c(2000:2011)) %.% # 1997:2011
  dplyr::filter(sample %in% c(1)) %.% # 0-Oversample; 1-Cross-Sectional
  dplyr::filter(race %in% c(4)) %.% # 1-Black; 2-Hispanis; 3-Mixed; 4-White
  dplyr::filter(byear %in% c(1980:1984)) %.% # birth year 1980:1984
  dplyr::filter(ave(!is.na(attend), id, FUN = all)) %.% # only complete trajectories
  dplyr::mutate( # compute new variables
    age= year-byear, # definition of age to be used in the model
    timec=year-2000, # metric of time is rounds of NSLY97 in years, centered at 2000
    timec2= timec^2,
    timec3= timec^3,
    #         timec= age-16, # metric of time is bilogical age in years, centered at 16
    #         timec2= timec^2,
    #         timec3= timec^3,#
    cohort=byear-1980) %.% # age difference, years younger (unit - 1 cohort away)
  dplyr::select( # assemble the dataset for modeling
    id, sample, race, byear,cohort, # Time Invariant variables
    year,
    age, timec,timec2,timec3, attend)  # Time Variant variables
```

2 Model specification

General case of LCM and model sequence are defined

```
############################
```

2.1 m0F – Fix Coef



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Tt} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^n \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \vdots \\ \varepsilon_{Ti} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

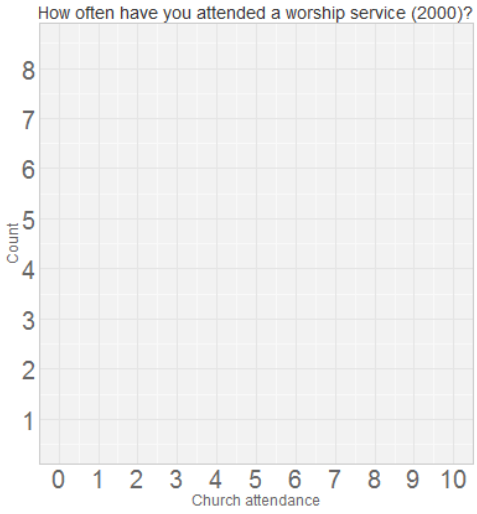
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

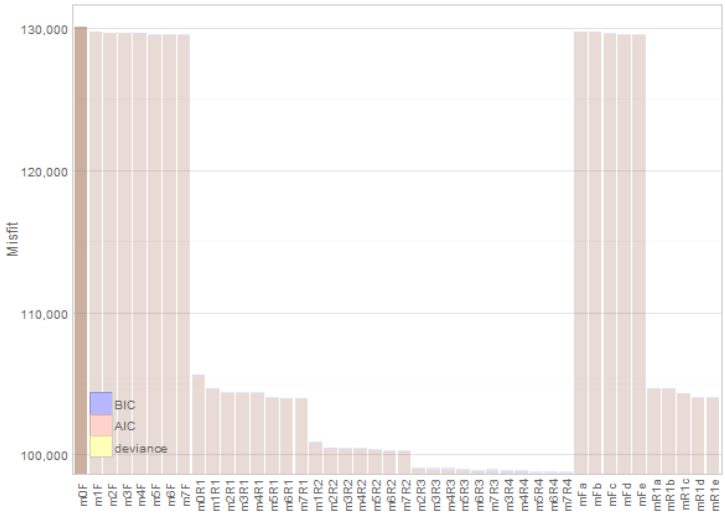
Snijders & Bosker (2011)

$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

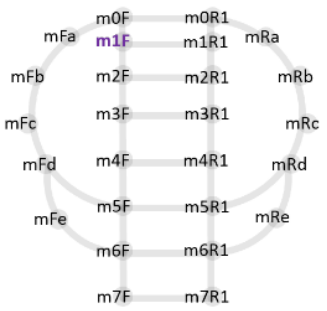
$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.80	0.01	251.55		0.00				1.96
timec					0.00				1.96
timec2									
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





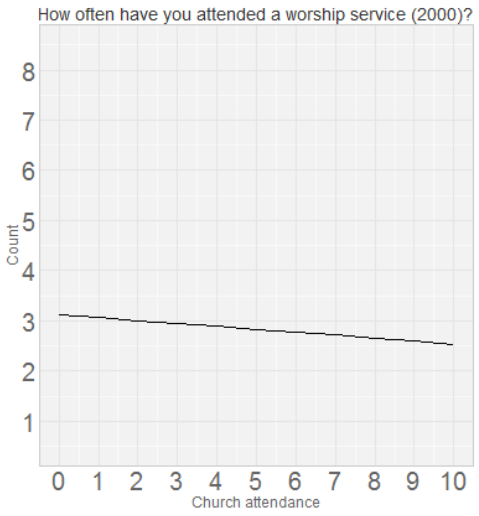
2.2 m1F

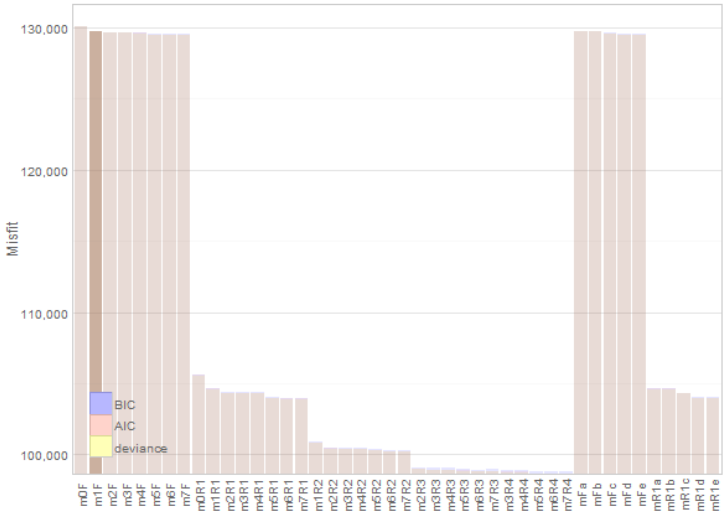


Bollen & Curran (2006)

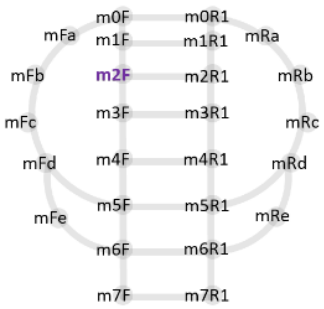
Snijders & Bosker (2011)

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.12	0.02	149.89		0.00				1.95
timec	-0.06	0.00	-18.24		0.00				1.95
timec2									
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.3 m2F



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

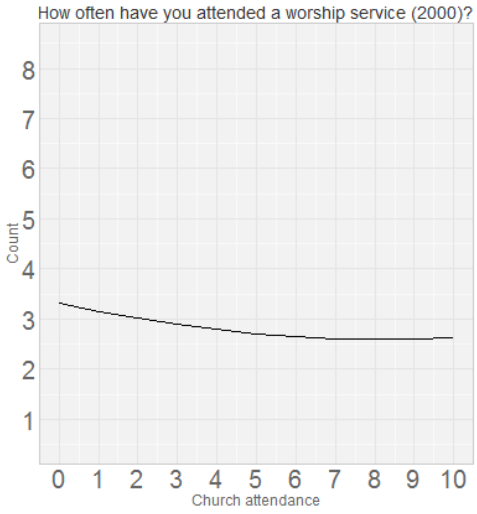
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

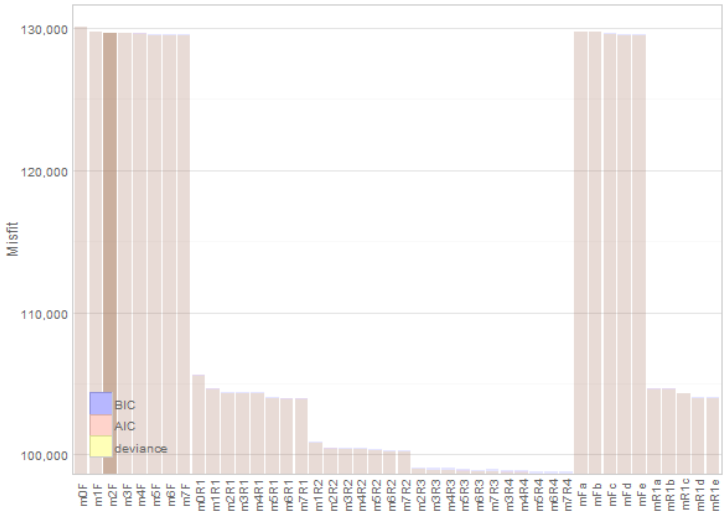
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

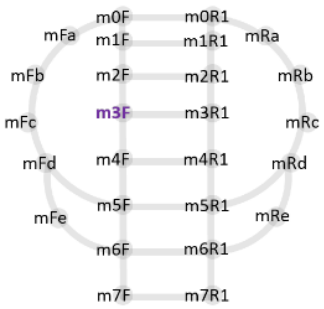
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.32	0.03	117.01		0.00				1.95
timec	-0.17	0.01	-14.49		0.00				1.95
timec2	0.01	0.00	9.96		0.00				1.95
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





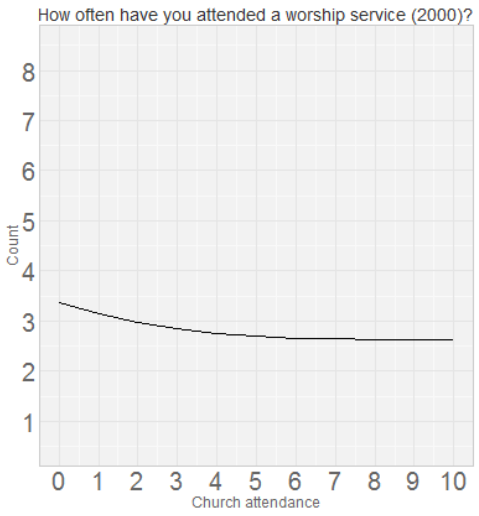
2.4 m3F

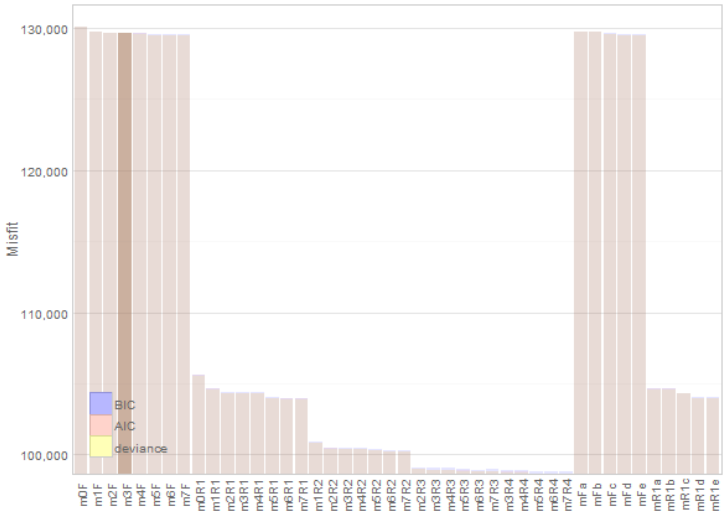


Bollen & Curran (2006)

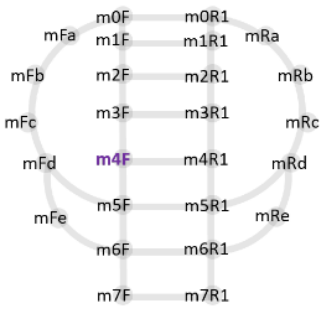
Snijders & Bosker (2011)

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.37	0.03	100.87		0.00				1.95
timec	-0.24	0.03	-8.87		0.00				1.95
timec2	0.03	0.01	4.55		0.00				1.95
timec3	-0.00	0.00	-2.84		0.00				1.95
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.5 m4F



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^K \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

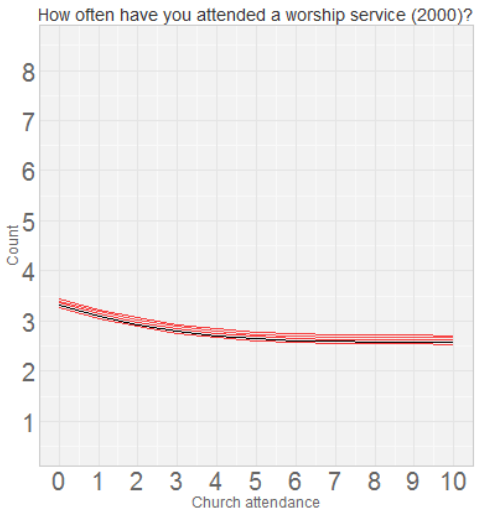
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

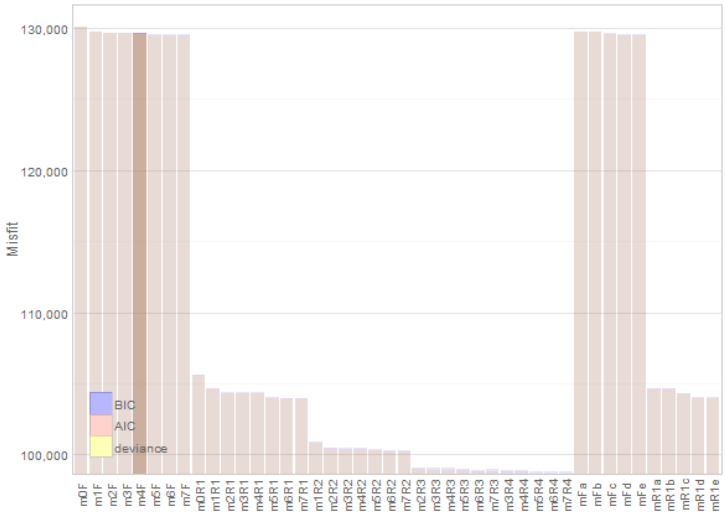
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

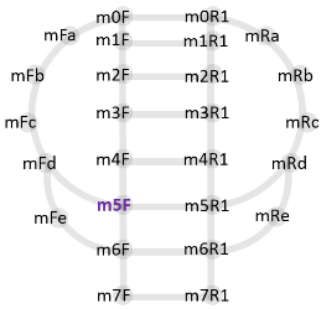
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.27	0.04	87.84		0.00				1.95
timec	−0.24	0.03	−8.87		0.00				1.95
timec2	0.03	0.01	4.55		0.00				1.95
timec3	−0.00	0.00	−2.84		0.00				1.95
cohort	0.04	0.01	5.48		0.00				1.95
timec:cohort									
timec2:cohort									
timec3:cohort									





2.6 m5F

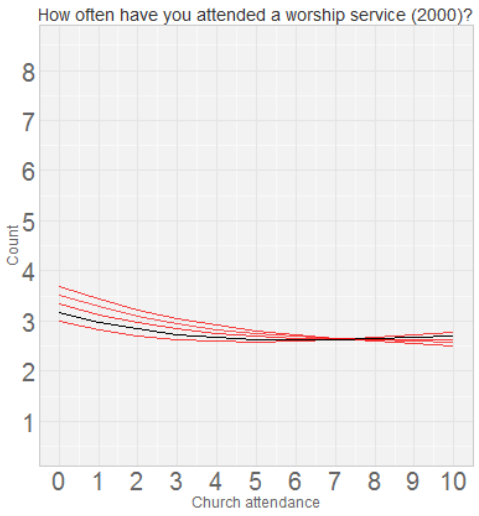


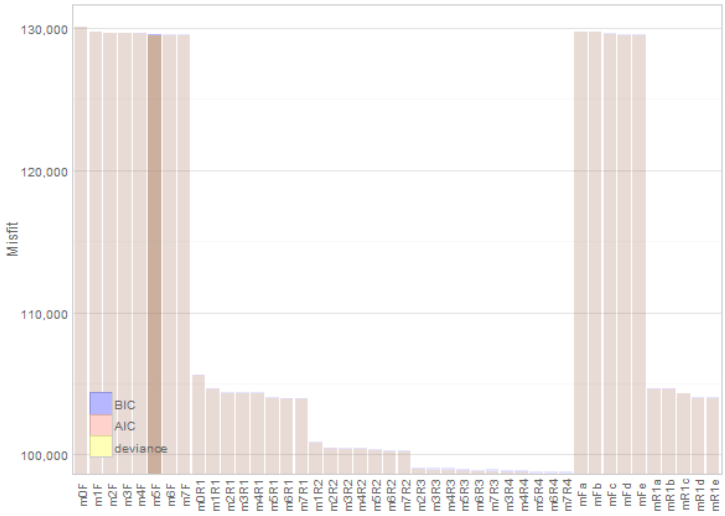
Bollen & Curran (2006)

Snijders & Bosker (2011)

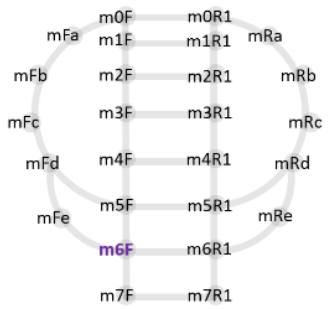
	Estimate	Std.Error	t.value
(Intercept)	3.00	0.05	65.59
timec	-0.19	0.03	-6.93
timec2	0.03	0.01	4.56
timec3	-0.00	0.00	-2.85
cohort	0.18	0.01	11.84
timec:cohort	-0.02	0.00	-10.54
timec2:cohort			
timec3:cohort			

sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
	0.00				1.94
	0.00				1.94
	0.00				1.94
	0.00				1.94
	0.00				1.94
	0.00				1.94





2.7 m6F



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Tt} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^n \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \vdots \\ \varepsilon_{Ti} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

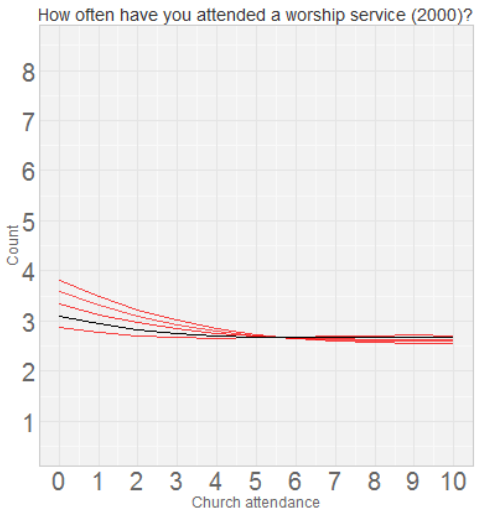
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

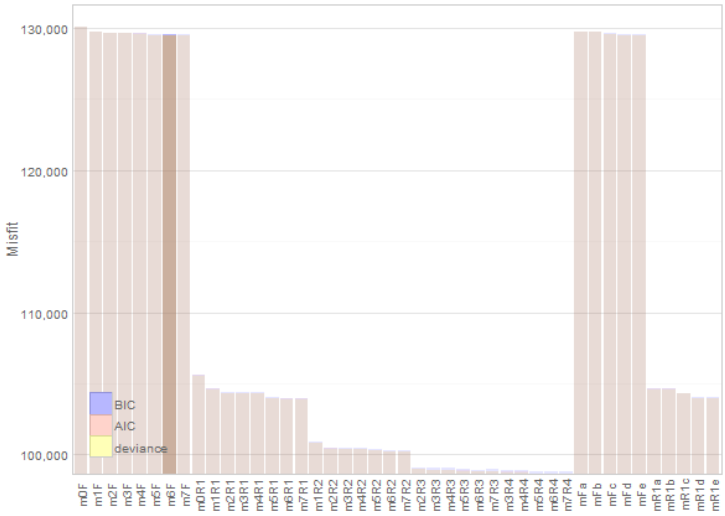
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], \left[\sigma^2\right]\right)$$

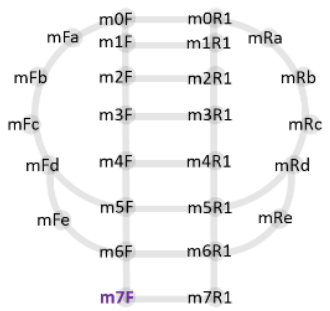
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.86	0.05	53.03		0.00				1.94
timec	-0.11	0.03	-3.50		0.00				1.94
timec2	0.02	0.01	3.25		0.00				1.94
timec3	-0.00	0.00	-2.85		0.00				1.94
cohort	0.24	0.02	11.76		0.00				1.94
timec:cohort	-0.06	0.01	-7.17		0.00				1.94
timec2:cohort	0.00	0.00	4.51		0.00				1.94
timec3:cohort									





2.8 m7F



$$\mathbf{y}_i = \boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$
$$\boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$
$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^K \end{bmatrix} \begin{bmatrix} \eta_{0i} \\ \eta_{1i} \\ \vdots \\ \eta_{Pi} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$\begin{bmatrix} \eta_{0i} \\ \eta_{1i} \\ \vdots \\ \eta_{Pi} \end{bmatrix} = \begin{bmatrix} \mu_{\eta_0} \\ \mu_{\eta_1} \\ \vdots \\ \mu_{\eta_P} \end{bmatrix} + \begin{bmatrix} \gamma_{00} & \gamma_{01} & \cdots & \gamma_{0K} \\ \gamma_{10} & \gamma_{11} & \cdots & \gamma_{1K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{P0} & \gamma_{P1} & \cdots & \gamma_{PK} \end{bmatrix} \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} + \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \\ \vdots \\ \zeta_{Pi} \end{bmatrix}, \quad \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \\ \vdots \\ \zeta_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right)$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

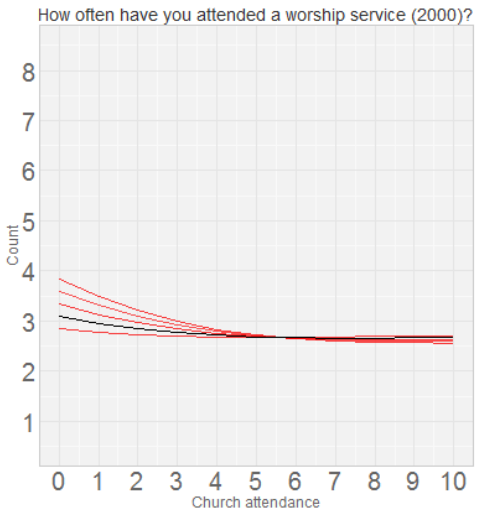
Snijders & Bosker (2011)

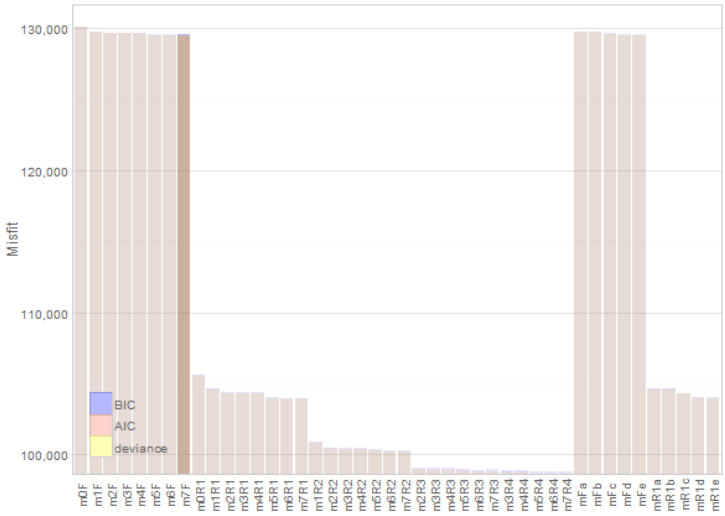
$$\varepsilon_{ti} \sim N\left(\left[0\right], \left[\sigma^2\right]\right)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$
$$\vdots \quad \vdots \quad \vdots$$
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

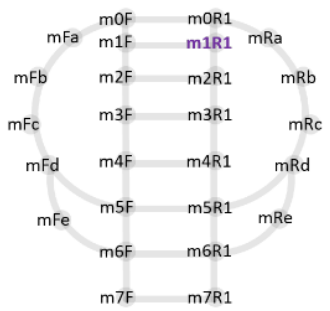
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.84	0.06	47.18		0.00				1.94
timec	-0.08	0.05	-1.57		0.00				1.94
timec2	0.01	0.01	1.04		0.00				1.94
timec3	-0.00	0.00	-0.75		0.00				1.94
cohort	0.25	0.02	10.51		0.00				1.94
timec:cohort	-0.08	0.02	-4.02		0.00				1.94
timec2:cohort	0.01	0.00	1.77		0.00				1.94
timec3:cohort	-0.00	0.00	-1.00		0.00				1.94





2.9 m1R1 – 1 Ran Coef



Bollen & Curran (2006)

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

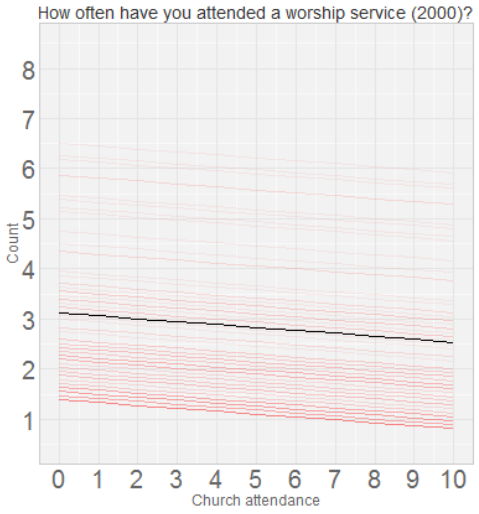
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

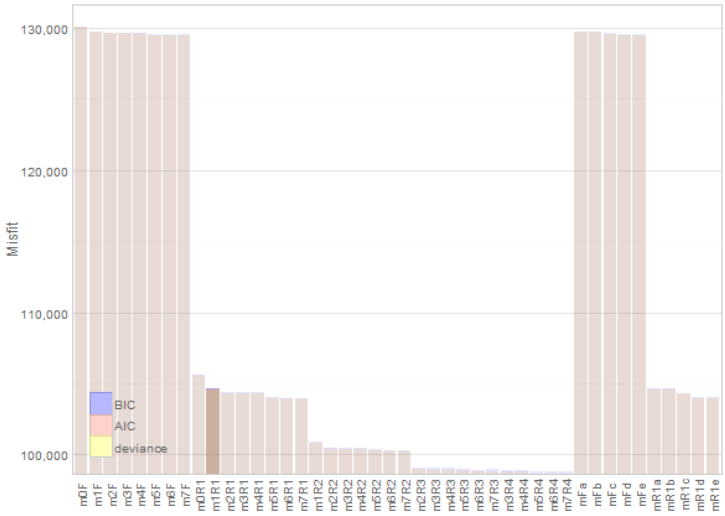
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

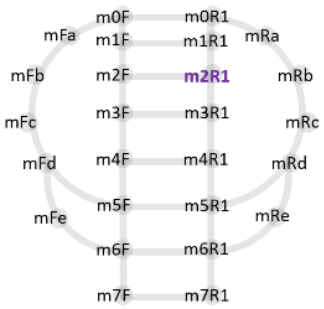
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.12	0.03	93.40	1.58	2.51				1.14
timec	-0.06	0.00	-31.22						1.14
timec2									
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.10 m2R1



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta PK} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

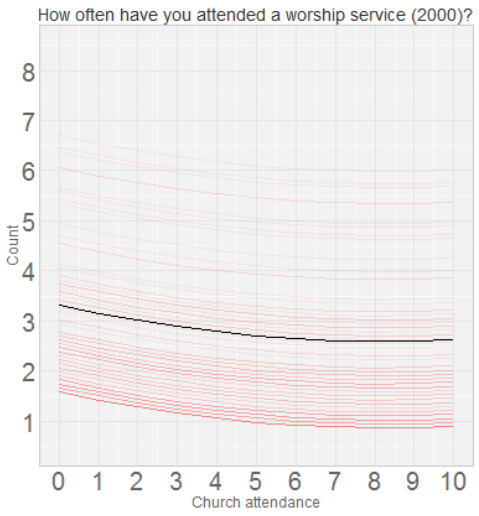
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

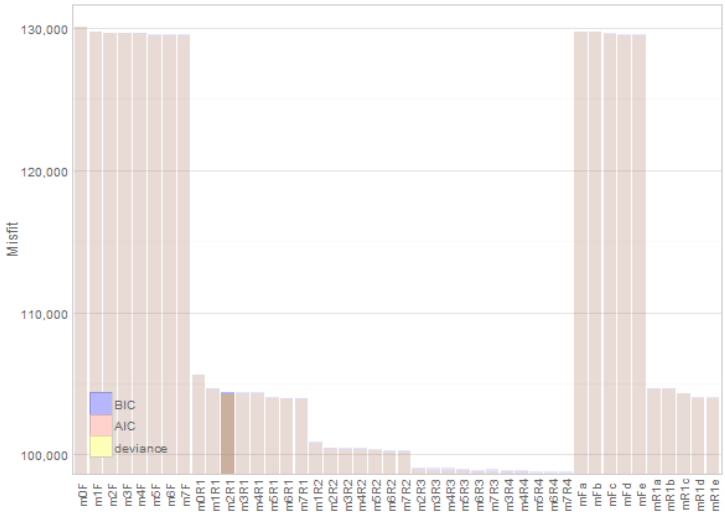
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

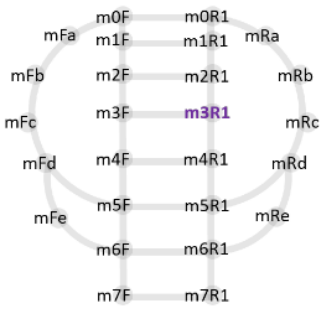
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.32	0.04	94.05	1.59	2.51				1.13
timec	−0.17	0.01	−24.88						1.13
timec2	0.01	0.00	17.11						1.13
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.11 m3R1



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta PK} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

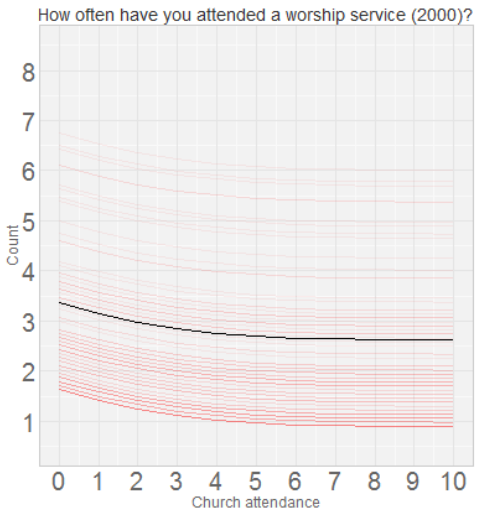
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

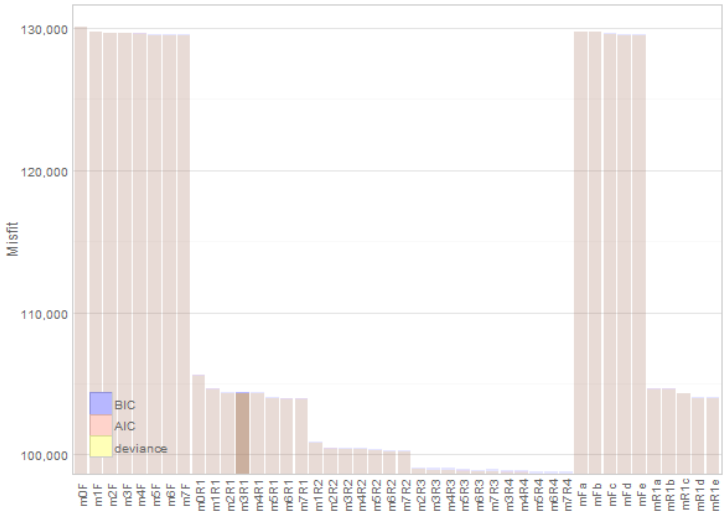
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

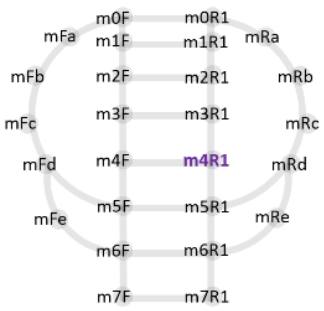
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.37	0.04	91.67	1.59	2.51				1.13
timec	-0.24	0.02	-15.24						1.13
timec2	0.03	0.00	7.82						1.13
timec3	-0.00	0.00	-4.88						1.13
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.12 m4R1



$$\mathbf{y}_i = \boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i \qquad \mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

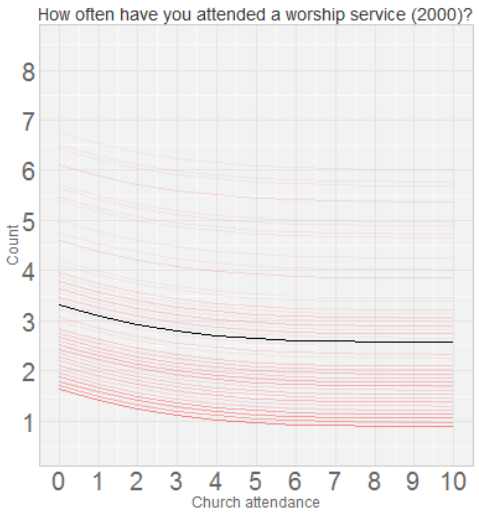
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

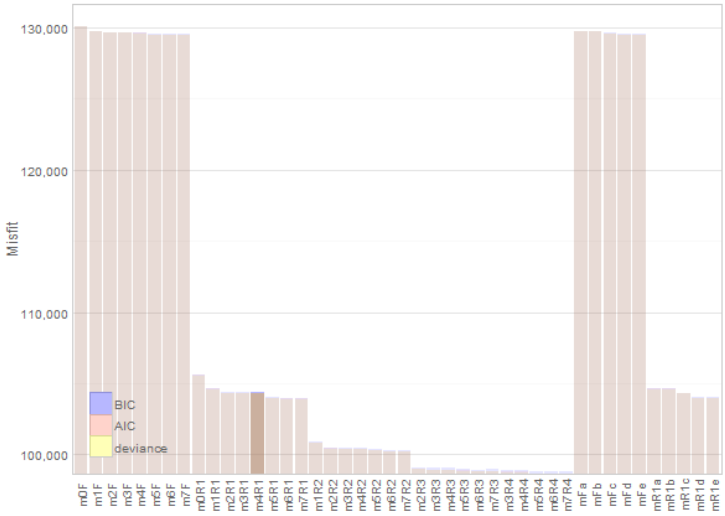
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

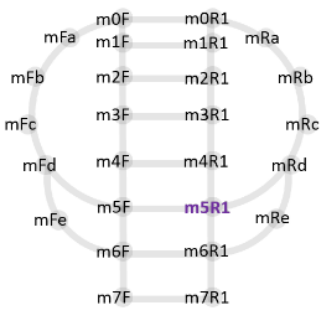
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.27	0.06	54.28	1.58	2.51				1.13
timec	-0.24	0.02	-15.24						1.13
timec2	0.03	0.00	7.82						1.13
timec3	-0.00	0.00	-4.88						1.13
cohort	0.04	0.02	1.90						1.13
timec:cohort									
timec2:cohort									
timec3:cohort									





2.13 m5R1



Bollen & Curran (2006)

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

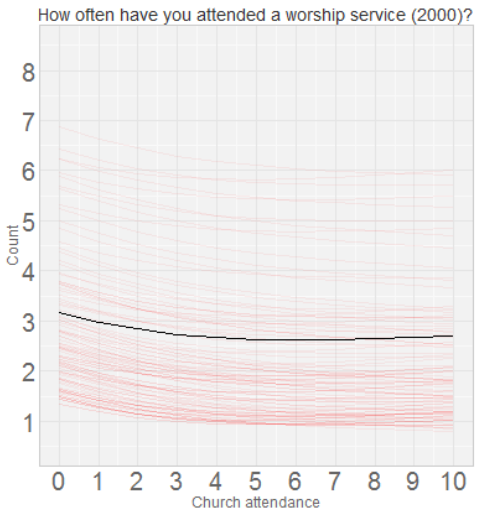
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

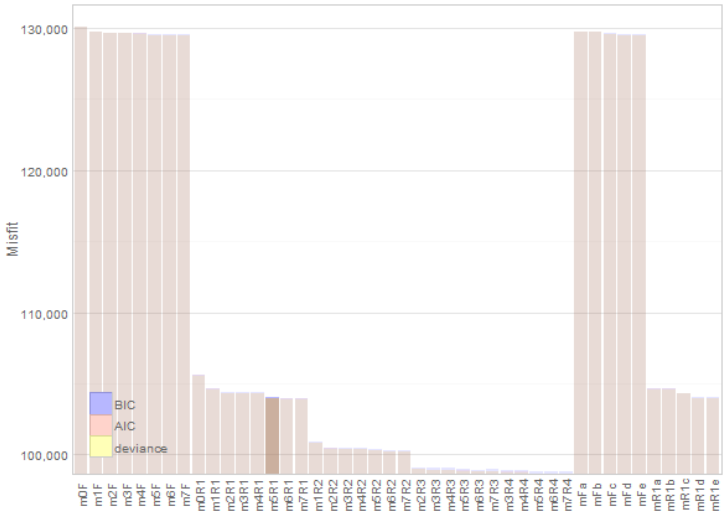
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

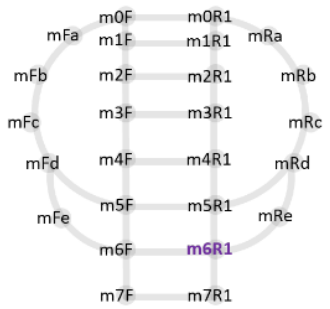
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.00	0.06	48.14	1.58	2.51				1.13
timec	−0.19	0.02	−11.96						1.13
timec2	0.03	0.00	7.86						1.13
timec3	−0.00	0.00	−4.91						1.13
cohort	0.18	0.02	7.36						1.13
timec:cohort	−0.02	0.00	−18.17						1.13
timec2:cohort									
timec3:cohort									





2.14 m6R1



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

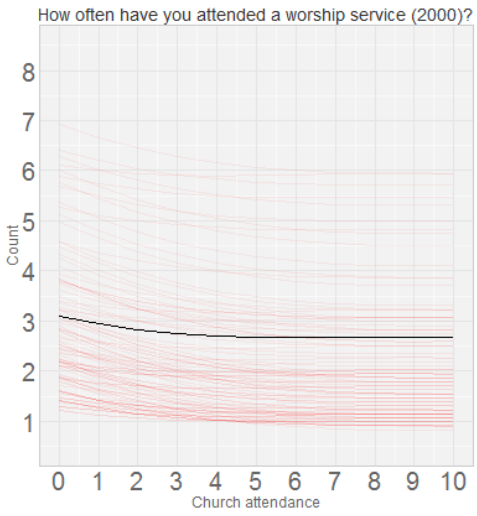
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

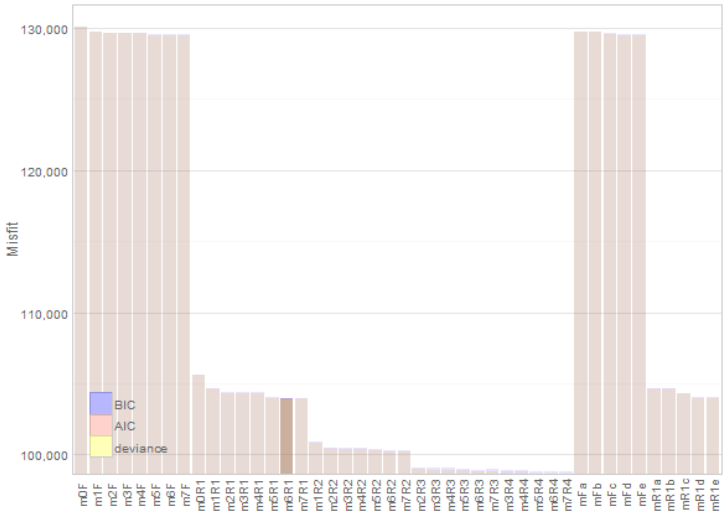
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

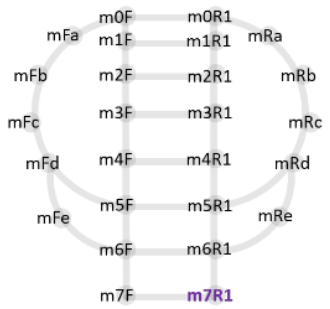
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.86	0.06	44.47	1.58	2.51				1.13
timec	−0.11	0.02	−6.05						1.13
timec2	0.02	0.00	5.62						1.13
timec3	−0.00	0.00	−4.92						1.13
cohort	0.24	0.03	9.45						1.13
timec:cohort	−0.06	0.00	−12.37						1.13
timec2:cohort	0.00	0.00	7.79						1.13
timec3:cohort									





2.15 m7R1



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1 \beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1 \beta P} & \cdots & \psi_{\beta P \beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

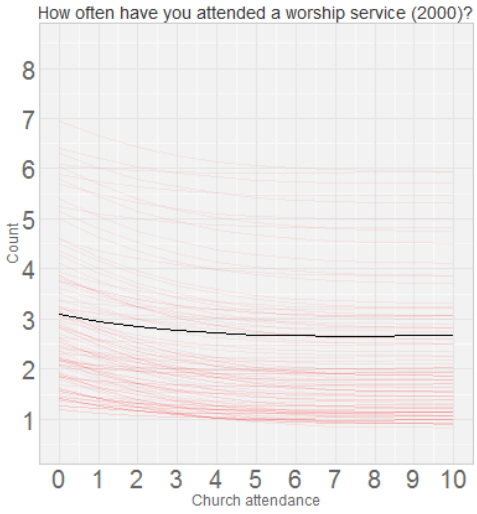
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

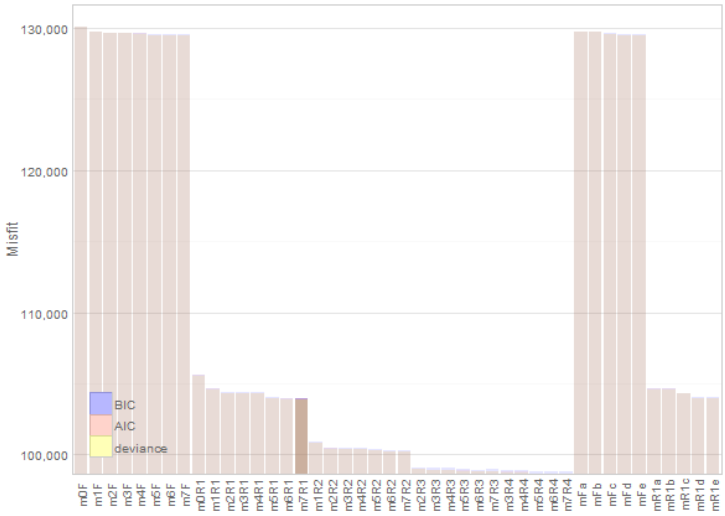
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

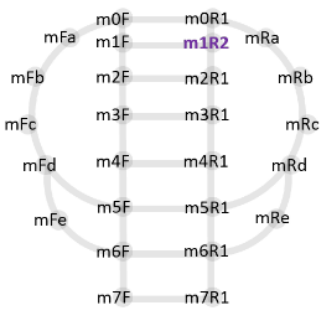
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.84	0.07	42.87	1.58	2.51				1.13
timec	-0.08	0.03	-2.71						1.13
timec2	0.01	0.01	1.80						1.13
timec3	-0.00	0.00	-1.29						1.13
cohort	0.25	0.03	9.55						1.13
timec:cohort	-0.08	0.01	-6.94						1.13
timec2:cohort	0.01	0.00	3.06						1.13
timec3:cohort	-0.00	0.00	-1.72						1.13





2.16 m1R2 – 2 Ran Coef



Bollen & Curran (2006)

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

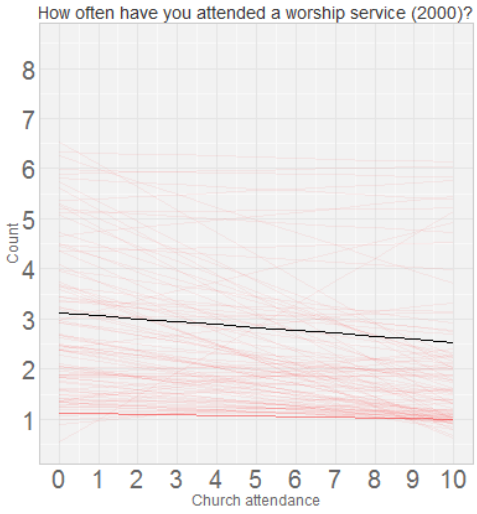
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

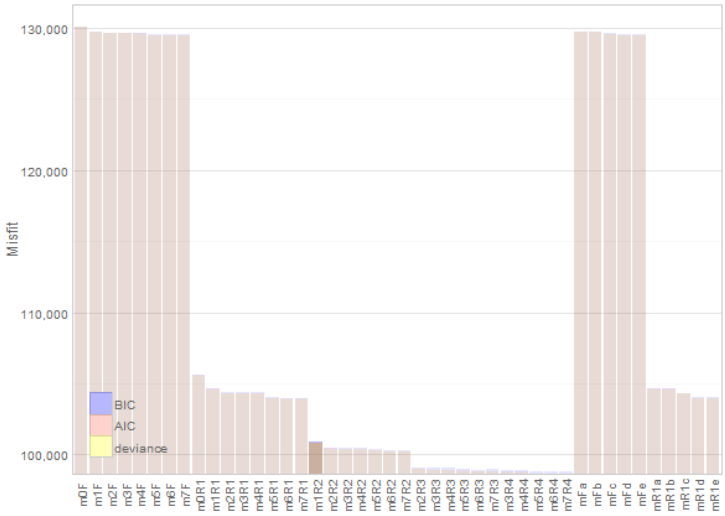
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

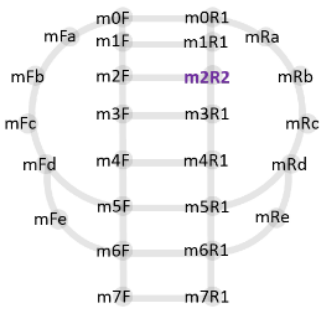
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.12	0.04	83.91	1.82	3.29	-0.13			1.00
timec	-0.06	0.00	-17.06	0.15	-0.13	0.02			1.00
timec2									
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.17 m2R2



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

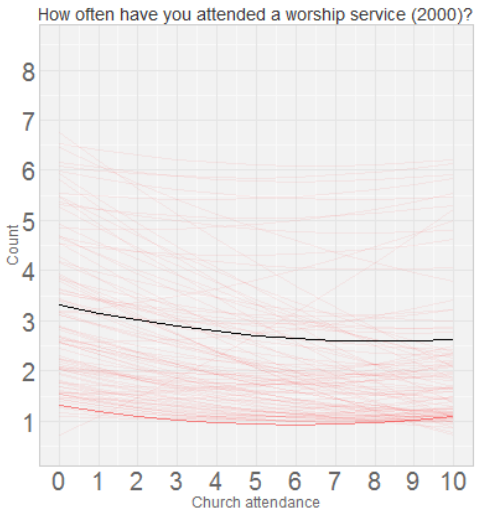
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

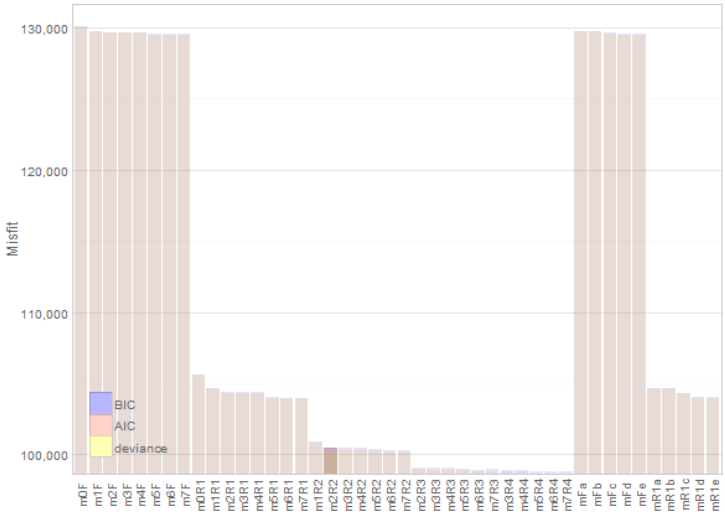
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N([0],[\sigma^2])$$

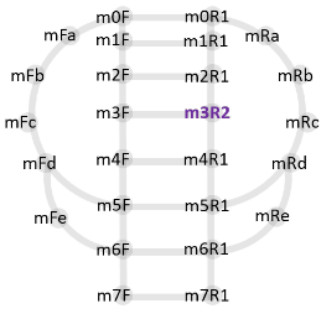
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.32	0.04	86.13	1.82	3.30	-0.13			0.99
timec	-0.17	0.01	-25.54	0.15	-0.13	0.02			0.99
timec2	0.01	0.00	19.61						0.99
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.18 m3R2



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i1} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

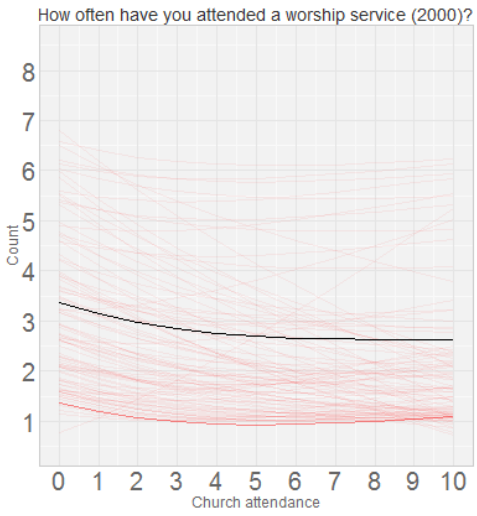
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

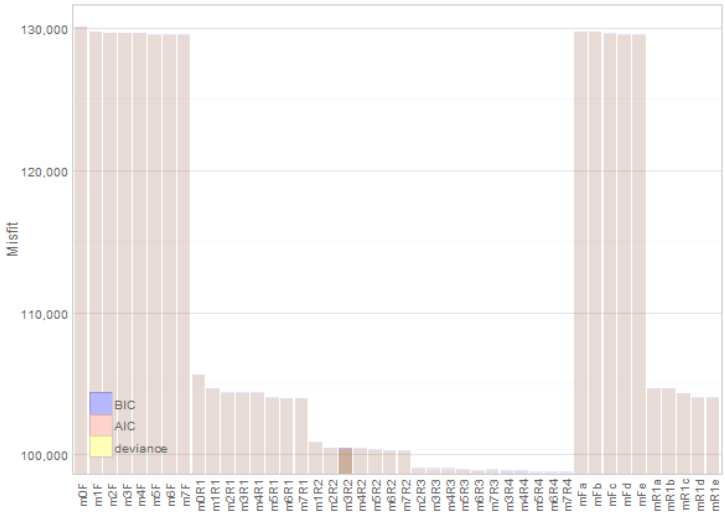
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

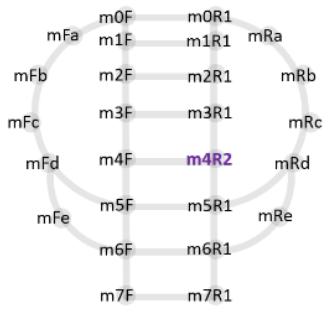
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.37	0.04	85.17	1.82	3.30	-0.13			0.99
timec	-0.24	0.01	-17.07	0.15	-0.13	0.02			0.99
timec2	0.03	0.00	8.96						0.99
timec3	-0.00	0.00	-5.60						0.99
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.19 m4R2



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Tt} \end{bmatrix} \quad \boldsymbol{\eta}_t = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_t = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_t = \begin{bmatrix} \zeta_{\alpha} \\ \zeta_{\beta 1} \\ \vdots \\ \zeta_{\beta P} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Tt} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

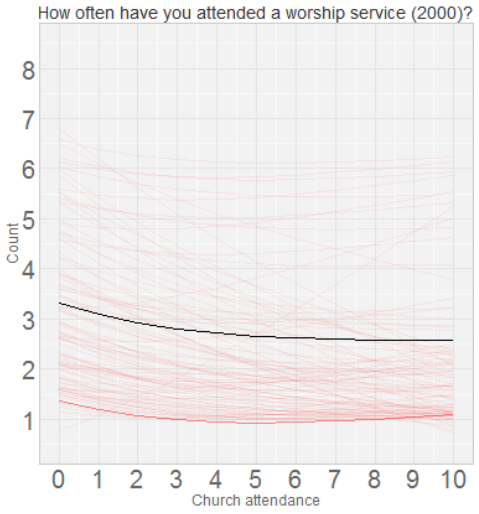
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

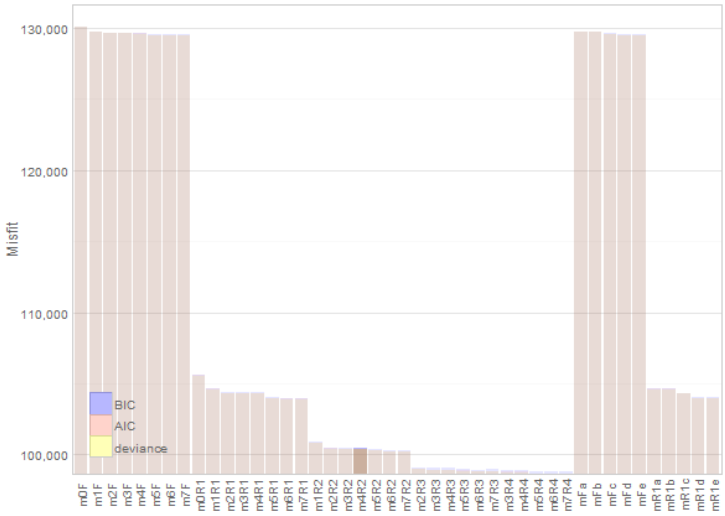
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

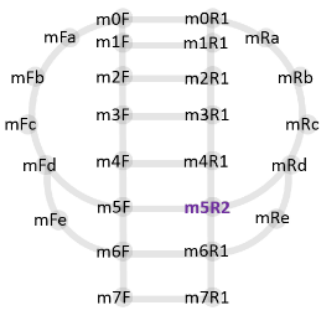
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.28	0.06	52.87	1.81	3.27	-0.13			0.99
timec	-0.24	0.01	-17.07	0.15	-0.13	0.02			0.99
timec2	0.03	0.00	8.96						0.99
timec3	-0.00	0.00	-5.60						0.99
cohort	0.04	0.02	1.83						0.99
timec:cohort									
timec2:cohort									
timec3:cohort									





2.20 m5R2



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Tt} \end{bmatrix} \quad \boldsymbol{\eta}_t = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_t = \begin{bmatrix} w_{1t} \\ w_{2t} \\ \vdots \\ w_{Kt} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_t = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Tt} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

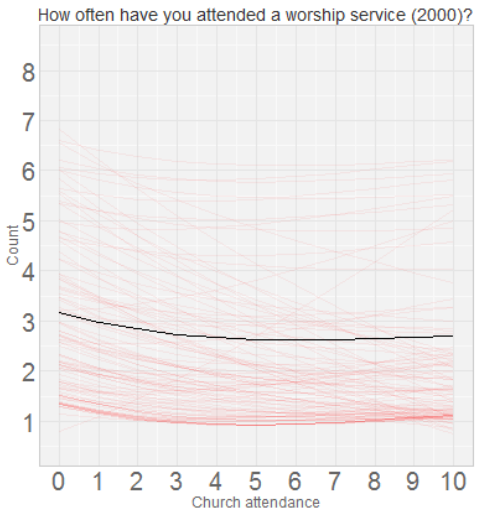
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

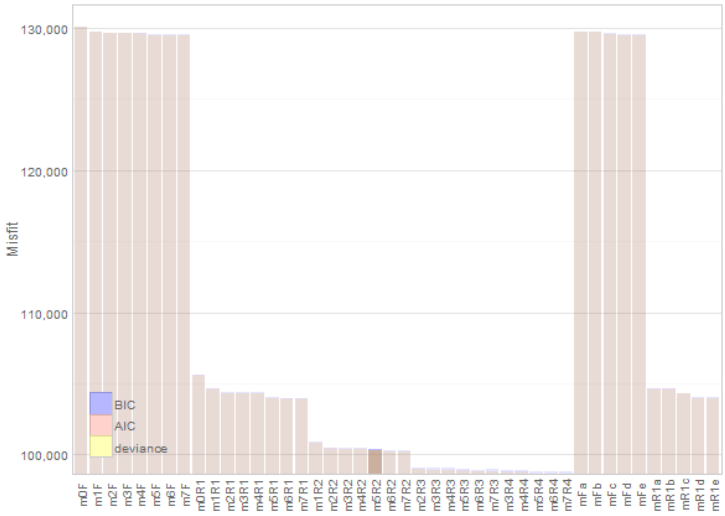
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

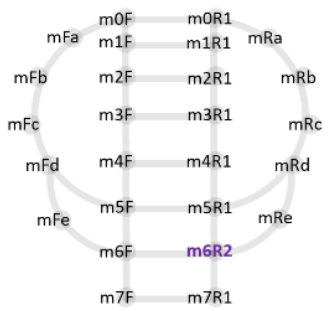
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.00	0.07	44.02	1.80	3.24	-0.13			0.99
timec	-0.19	0.02	-12.75	0.15	-0.13	0.02			0.99
timec2	0.03	0.00	8.96						0.99
timec3	-0.00	0.00	-5.60						0.99
cohort	0.18	0.03	6.66						0.99
timec:cohort	-0.02	0.00	-10.01						0.99
timec2:cohort									
timec3:cohort									





2.21 m6R2



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P0} & \gamma_{\beta P1} & \cdots & \gamma_{\beta PK} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

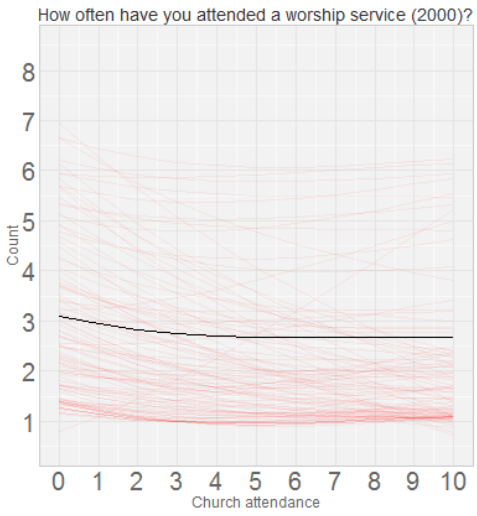
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

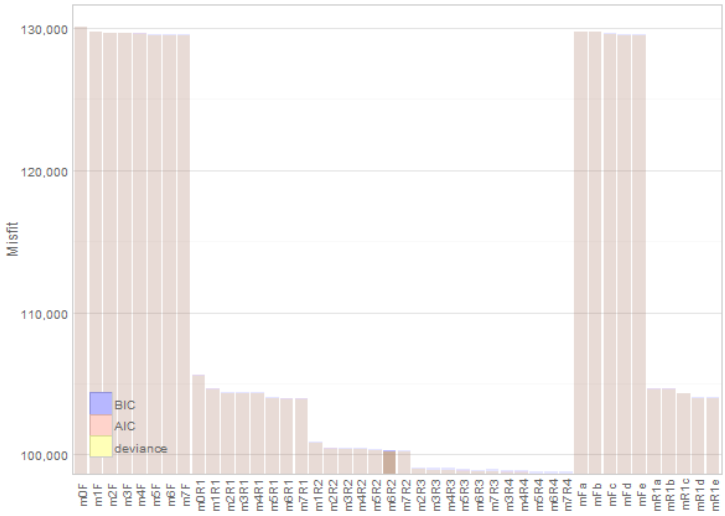
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

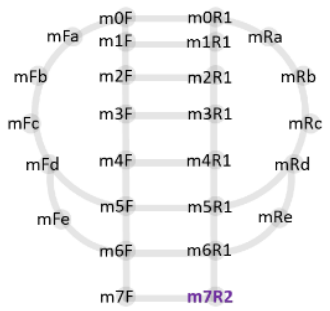
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.86	0.07	41.16	1.80	3.24	-0.13			0.99
timec	-0.11	0.02	-6.56	0.15	-0.13	0.02			0.99
timec2	0.02	0.00	6.40						0.99
timec3	-0.00	0.00	-5.61						0.99
cohort	0.24	0.03	8.70						0.99
timec:cohort	-0.06	0.00	-12.68						0.99
timec2:cohort	0.00	0.00	8.88						0.99
timec3:cohort									





2.22 m7R2



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

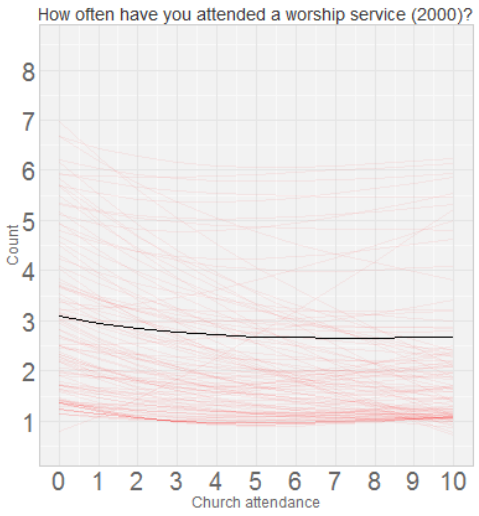
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

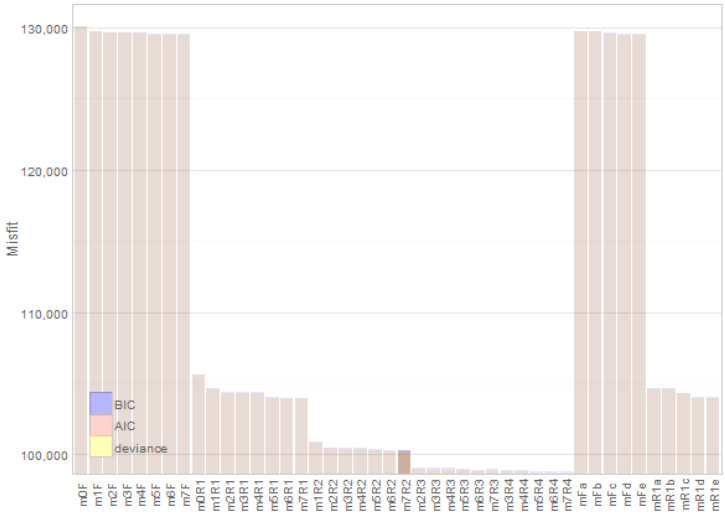
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N([0],[\sigma^2])$$

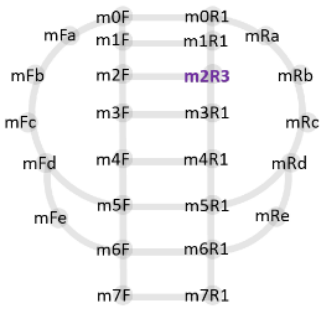
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.84	0.07	40.04	1.80	3.24	-0.13			0.99
timec	-0.08	0.03	-3.02	0.15	-0.13	0.02			0.99
timec2	0.01	0.01	2.05						0.99
timec3	-0.00	0.00	-1.47						0.99
cohort	0.25	0.03	8.92						0.99
timec:cohort	-0.08	0.01	-7.74						0.99
timec2:cohort	0.01	0.00	3.49						0.99
timec3:cohort	-0.00	0.00	-1.96						0.99





2.23 m2R3 – 3 Ran Coef



Bollen & Curran (2006)

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

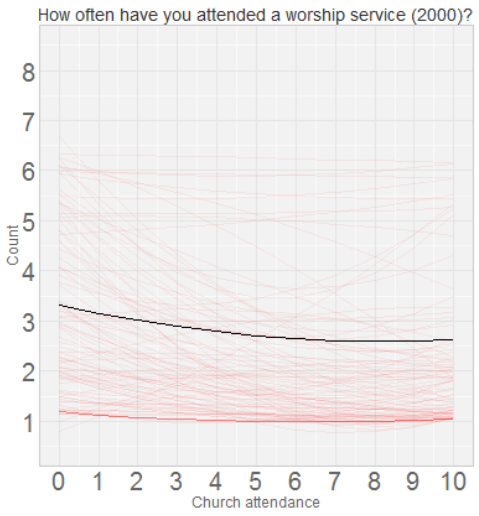
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

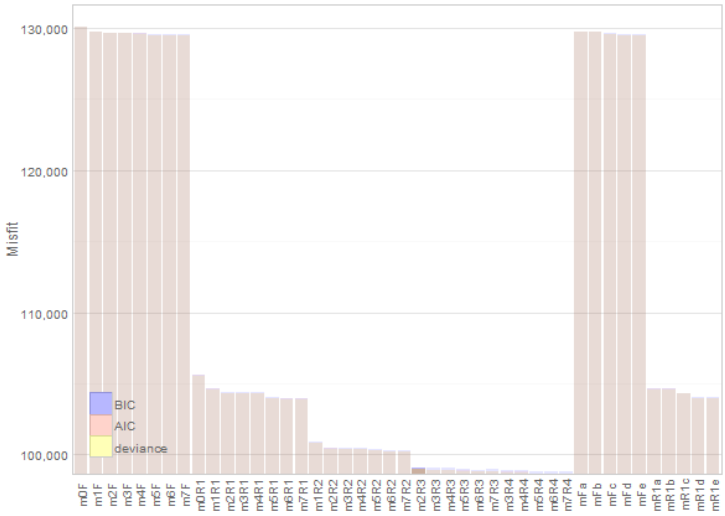
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

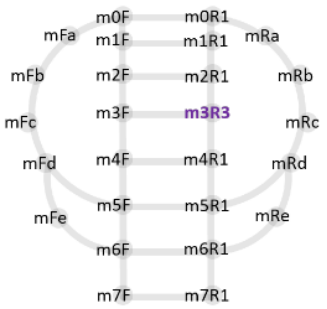
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.32	0.04	82.10	1.94	3.76	-0.39	0.02		0.92
timec	-0.17	0.01	-17.36	0.42	-0.39	0.18	-0.01		0.92
timec2	0.01	0.00	13.12	0.03	0.02	-0.01	0.00		0.92
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.24 m3R3



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} + \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$\begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} + \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 1 0} & \gamma_{\beta 1 1} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} + \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix}$$

$$\begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right)$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

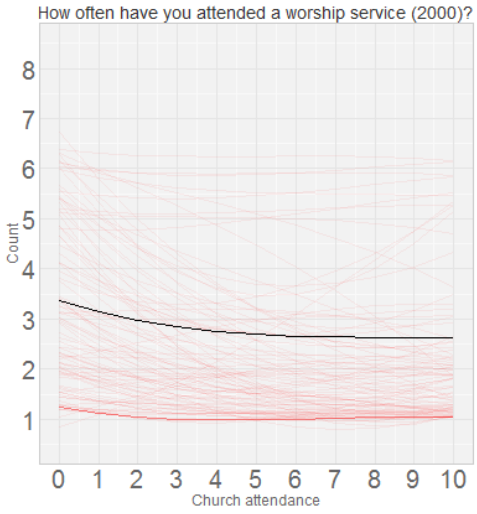
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

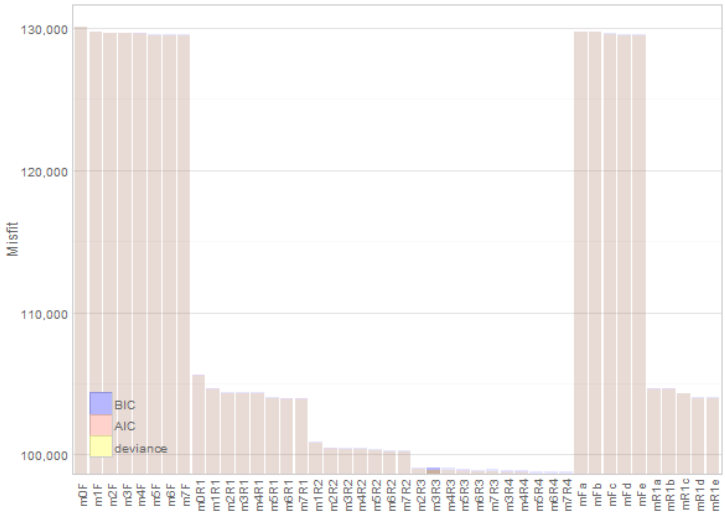
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

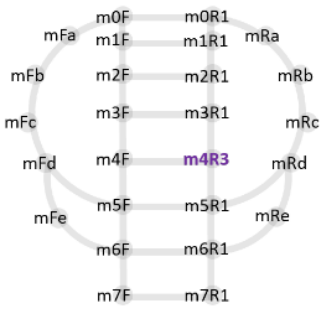
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.37	0.04	81.64	1.94	3.76	-0.39	0.02		0.92
timec	-0.24	0.02	-15.87	0.42	-0.39	0.18	-0.01		0.92
timec2	0.03	0.00	9.42	0.03	0.02	-0.01	0.00		0.92
timec3	-0.00	0.00	-6.03						0.92
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.25 m4R3



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta PK} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

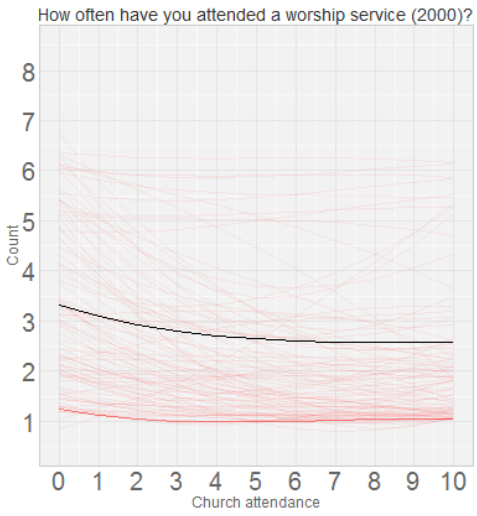
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

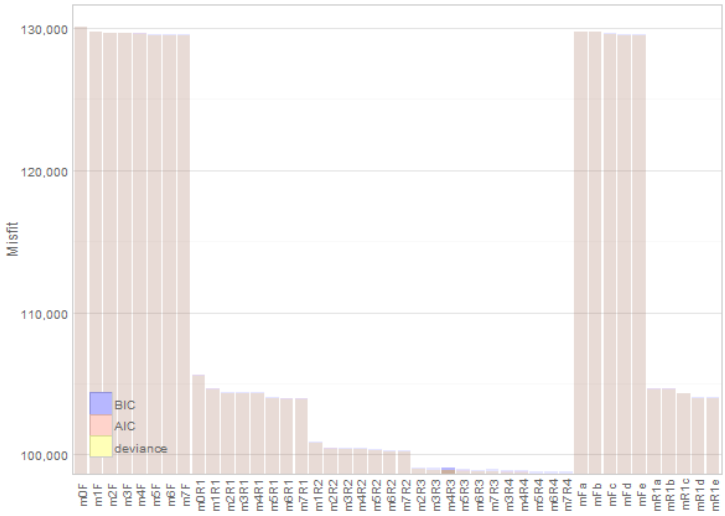
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

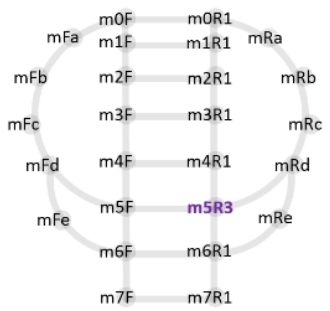
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.26	0.06	51.82	1.93	3.72	-0.39	0.02		0.92
timec	-0.24	0.02	-15.87	0.42	-0.39	0.18	-0.01		0.92
timec2	0.03	0.00	9.42	0.03	0.02	-0.01	0.00		0.92
timec3	-0.00	0.00	-6.03						0.92
cohort	0.05	0.02	2.21						0.92
timec:cohort									
timec2:cohort									
timec3:cohort									





2.26 m5R3

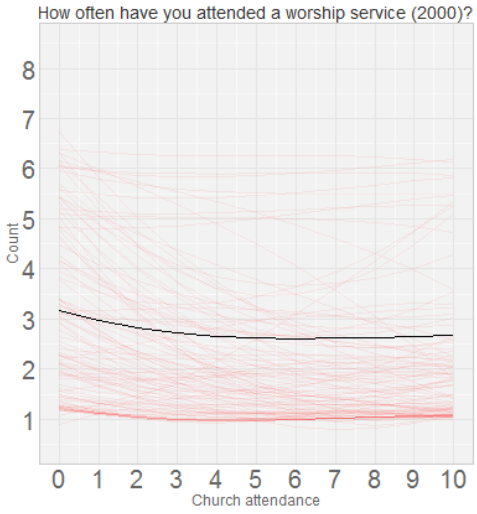


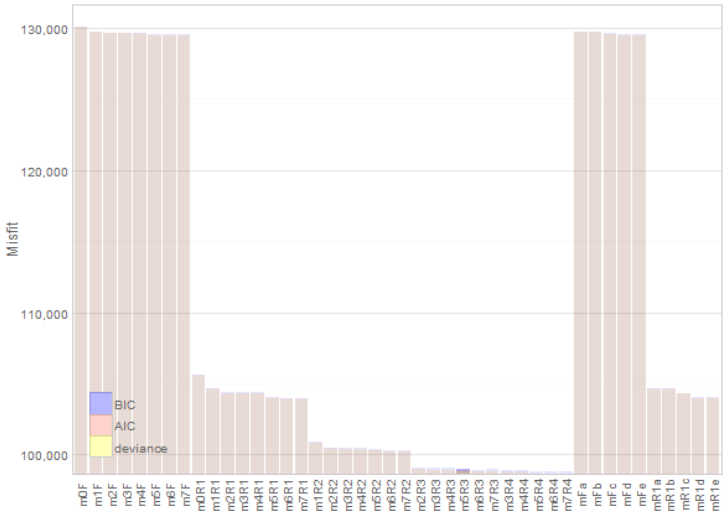
Bollen & Curran (2006)

Snijders & Bosker (2011)

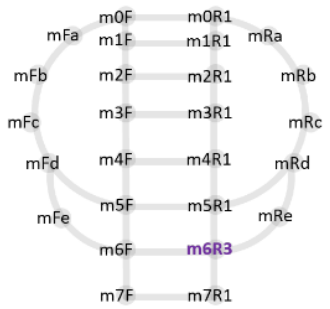
	Estimate	Std.Error	t.value
(Intercept)	3.00	0.07	43.53
timec	-0.20	0.02	-12.31
timec2	0.03	0.00	9.42
timec3	-0.00	0.00	-6.03
cohort	0.17	0.03	6.58
timec:cohort	-0.02	0.00	-9.12
timec2:cohort			
timec3:cohort			

sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
1.91	3.66	-0.37	0.02		0.92
0.41	-0.37	0.17	-0.01		0.92
0.03	0.02	-0.01	0.00		0.92
					0.92
					0.92
					0.92





2.27 m6R3



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

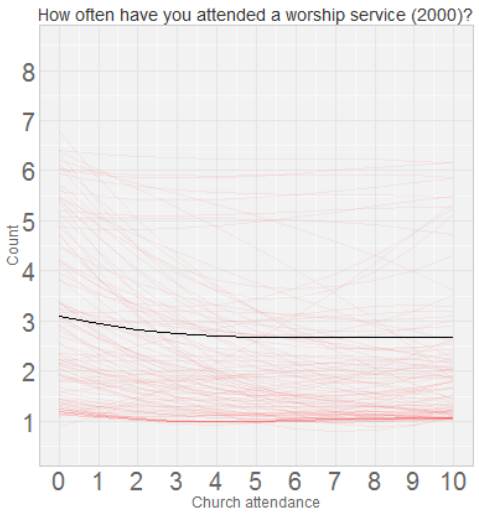
Snijders & Bosker (2011)

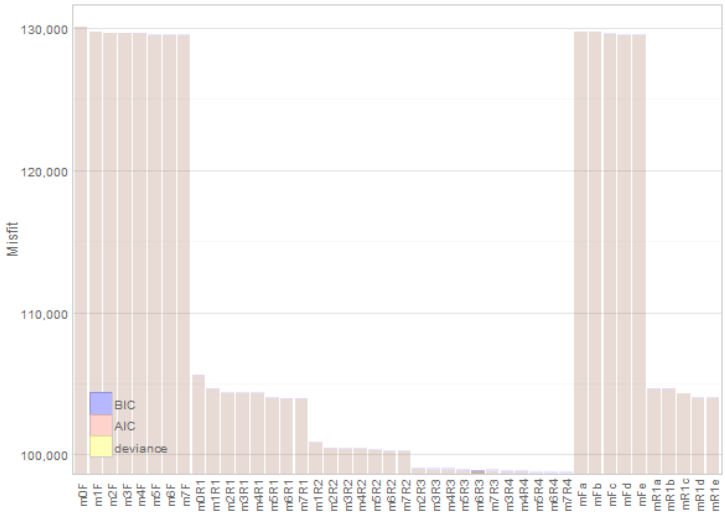
$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

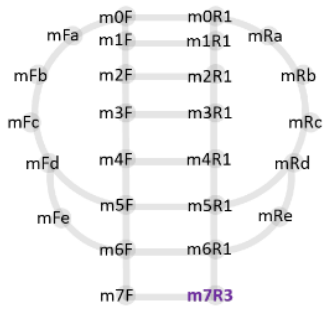
	Estimate	Std.Error	t.value
(Intercept)	2.86	0.07	39.51
timec	-0.11	0.02	-5.40
timec2	0.02	0.00	6.43
timec3	-0.00	0.00	-6.03
cohort	0.24	0.03	8.34
timec:cohort	-0.06	0.01	-8.69
timec2:cohort	0.00	0.00	5.97
timec3:cohort			

sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
1.91	3.65	-0.36	0.02		0.92
0.41	-0.36	0.17	-0.01		0.92
0.03	0.02	-0.01	0.00		0.92
					0.92
					0.92
					0.92
					0.92





2.28 m7R3



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$
$$\boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$
$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{Pt} + \varepsilon_{ti}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$
$$\vdots \quad \vdots \quad \vdots$$
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

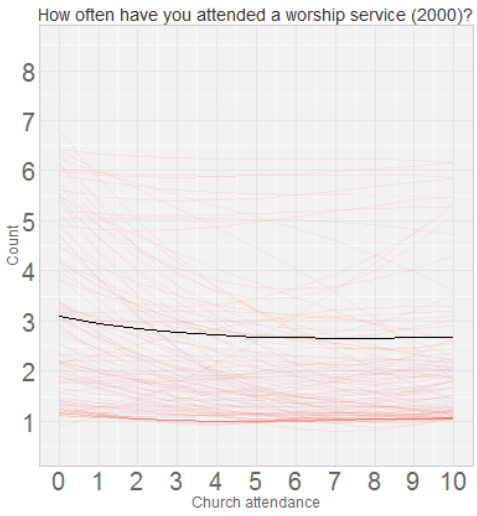
Snijders & Bosker (2011)

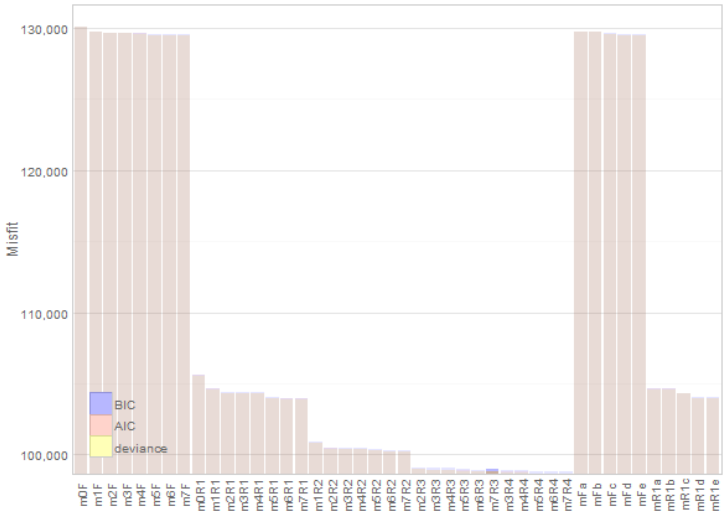
$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

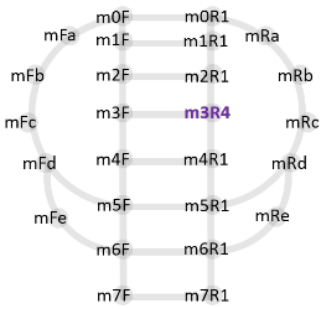
	Estimate	Std.Error	t.value
(Intercept)	2.84	0.07	38.57
timec	-0.08	0.03	-2.82
timec2	0.01	0.01	2.16
timec3	-0.00	0.00	-1.58
cohort	0.25	0.03	8.60
timec:cohort	-0.08	0.01	-7.22
timec2:cohort	0.01	0.00	3.67
timec3:cohort	-0.00	0.00	-2.11

sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
1.91	3.65	-0.36	0.02		0.92
0.41	-0.36	0.17	-0.01		0.92
0.03	0.02	-0.01	0.00		0.92
					0.92
					0.92
					0.92
					0.92
					0.92





2.29 m3R4 – 4 Ran Coef



Bollen & Curran (2006)

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

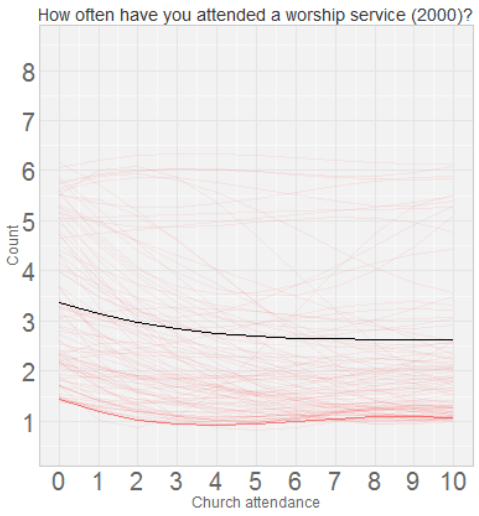
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

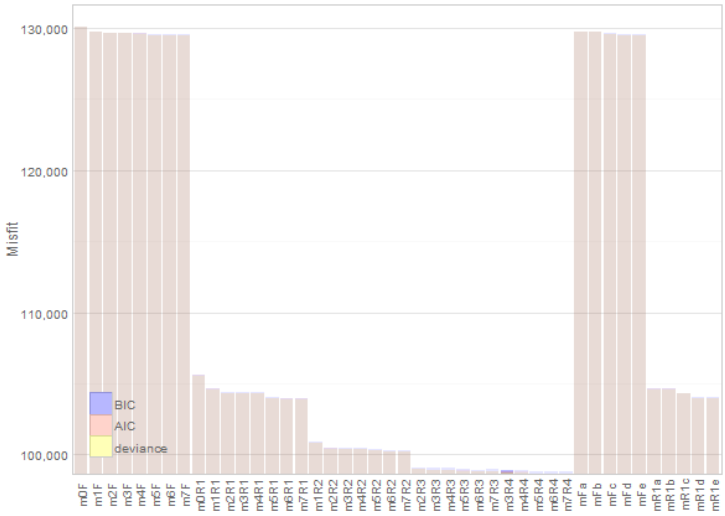
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

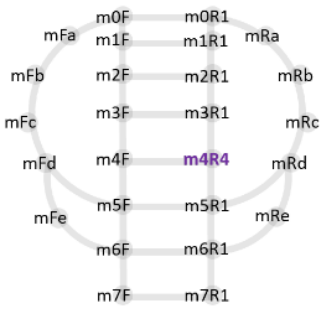
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.37	0.03	99.80	1.53	2.33	0.00	-0.04	0.00	0.90
timec	-0.24	0.02	-13.10	0.69	0.00	0.48	-0.09	0.00	0.90
timec2	0.03	0.00	6.96	0.14	-0.04	-0.09	0.02	-0.00	0.90
timec3	-0.00	0.00	-4.48	0.01	0.00	0.00	-0.00	0.00	0.90
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.30 m4R4



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Tt} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & \psi_{\alpha\beta 1} & \cdots & \psi_{\alpha\beta P} \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & \cdots & \psi_{\beta 1\beta P} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \vdots \\ \varepsilon_{Ti} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

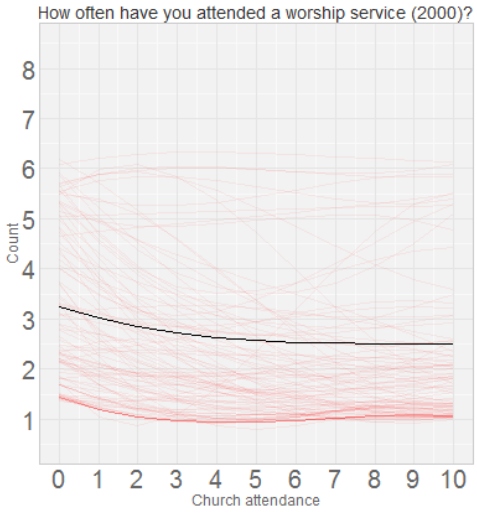
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

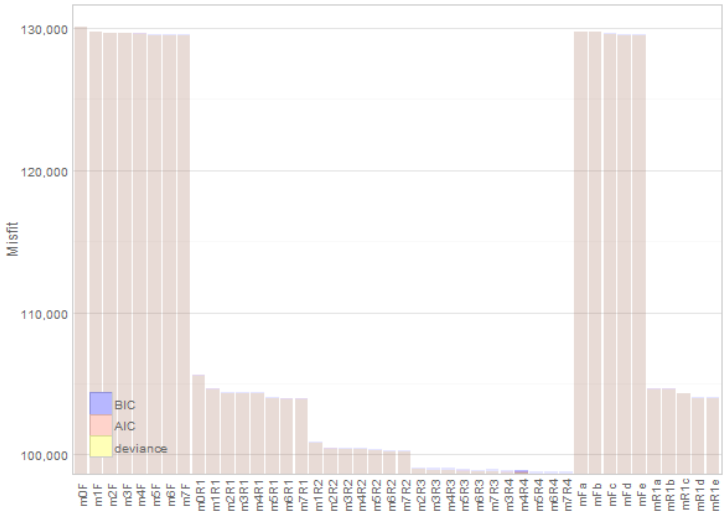
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

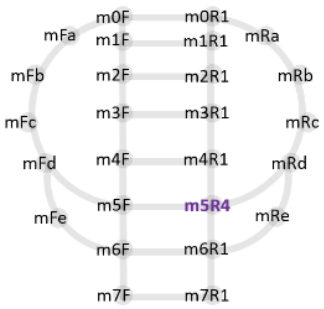
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.13	0.05	57.83	1.49	2.21	-0.06	-0.02	0.00	0.90
timec	-0.24	0.02	-12.81	0.72	-0.06	0.52	-0.10	0.00	0.90
timec2	0.03	0.00	6.88	0.14	-0.02	-0.10	0.02	-0.00	0.90
timec3	-0.00	0.00	-4.46	0.01	0.00	0.00	-0.00	0.00	0.90
cohort	0.11	0.02	5.39						0.90
timec:cohort									
timec2:cohort									
timec3:cohort									





2.31 m5R4



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Tt} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \vdots \\ \varepsilon_{Ti} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

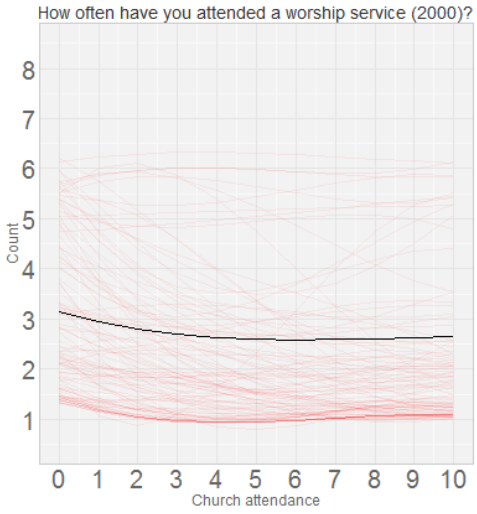
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

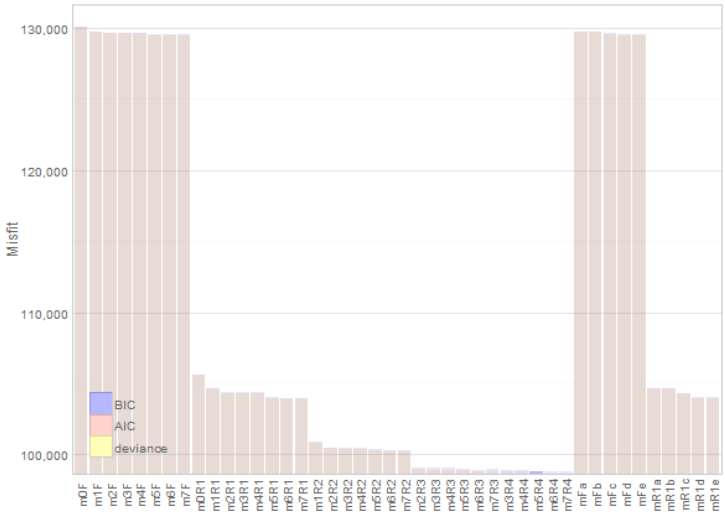
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

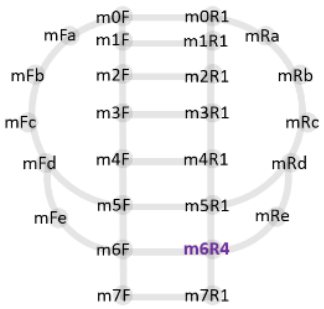
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.93	0.06	50.56	1.49	2.21	-0.09	-0.01	0.00	0.90
timec	-0.19	0.02	-9.81	0.74	-0.09	0.55	-0.10	0.01	0.90
timec2	0.03	0.00	6.78	0.15	-0.01	-0.10	0.02	-0.00	0.90
timec3	-0.00	0.00	-4.40	0.01	0.00	0.01	-0.00	0.00	0.90
cohort	0.21	0.02	9.25						0.90
timec:cohort	-0.02	0.00	-10.34						0.90
timec2:cohort									
timec3:cohort									





2.32 m6R4



Bollen & Curran (2006)

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

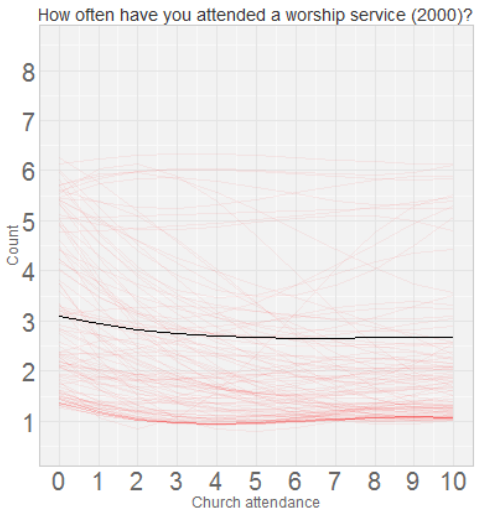
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

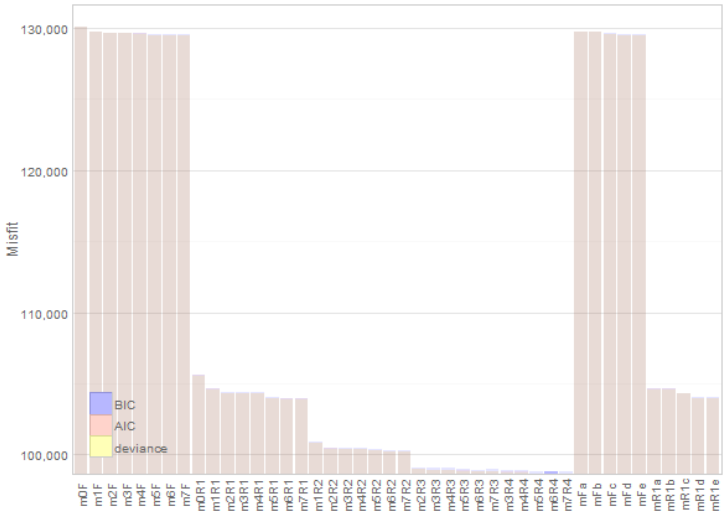
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

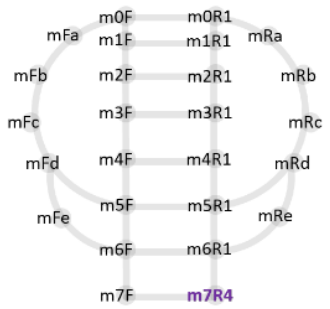
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.85	0.06	47.93	1.48	2.20	-0.08	-0.01	0.00	0.90
timec	-0.12	0.02	-4.83	0.77	-0.08	0.59	-0.11	0.01	0.90
timec2	0.02	0.00	4.78	0.15	-0.01	-0.11	0.02	-0.00	0.90
timec3	-0.00	0.00	-4.30	0.01	0.00	0.01	-0.00	0.00	0.90
cohort	0.25	0.02	10.51						0.90
timec:cohort	-0.06	0.01	-8.97						0.90
timec2:cohort	0.00	0.00	5.86						0.90
timec3:cohort									





2.33 m7R4



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta PK} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

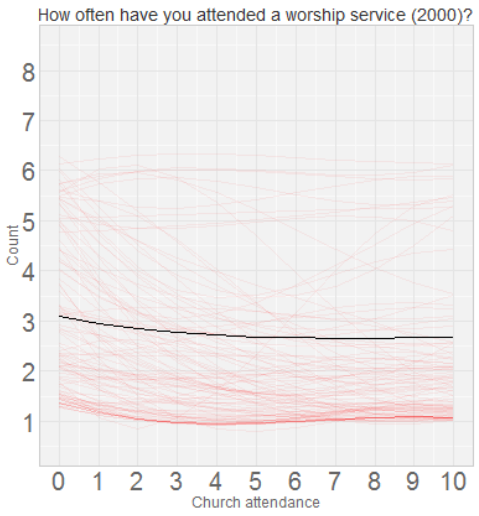
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

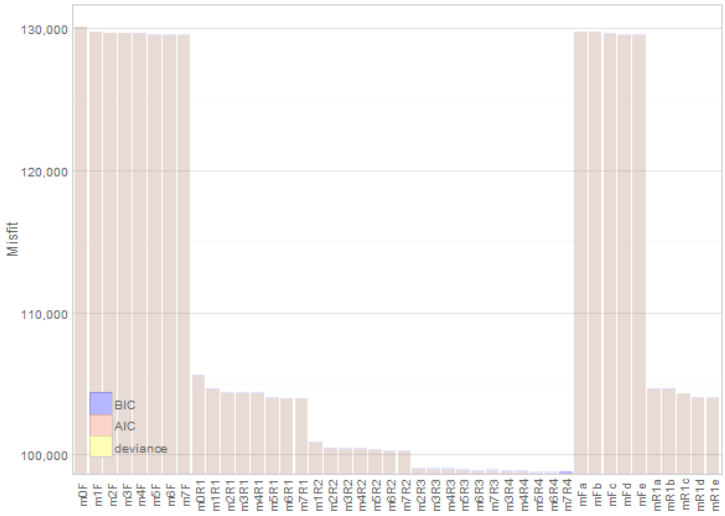
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

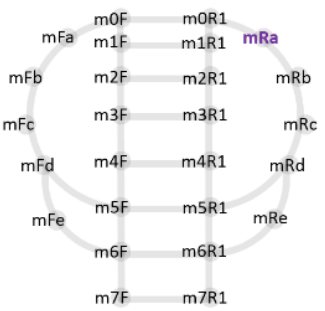
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.84	0.06	47.19	1.50	2.25	-0.07	-0.02	0.00	0.90
timec	-0.08	0.03	-2.22	0.74	-0.07	0.55	-0.10	0.01	0.90
timec2	0.01	0.01	1.54	0.15	-0.02	-0.10	0.02	-0.00	0.90
timec3	-0.00	0.00	-1.14	0.01	0.00	0.01	-0.00	0.00	0.90
cohort	0.25	0.02	10.51						0.90
timec:cohort	-0.08	0.01	-5.69						0.90
timec2:cohort	0.01	0.00	2.62						0.90
timec3:cohort	-0.00	0.00	-1.53						0.90





2.34 mR1a – Experiment R1



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{i\alpha} \\ \zeta_{i\beta 1} \\ \vdots \\ \zeta_{i\beta P} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti} \qquad \varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

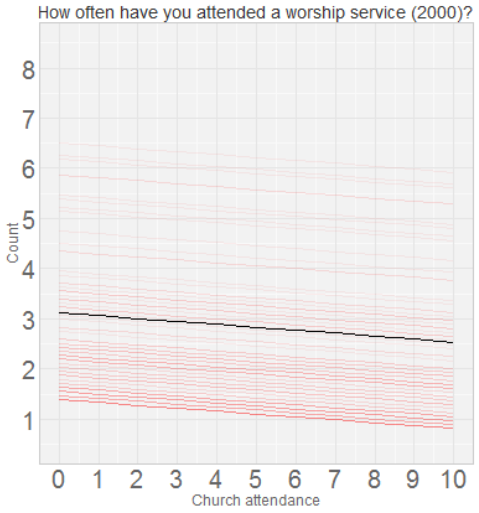
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

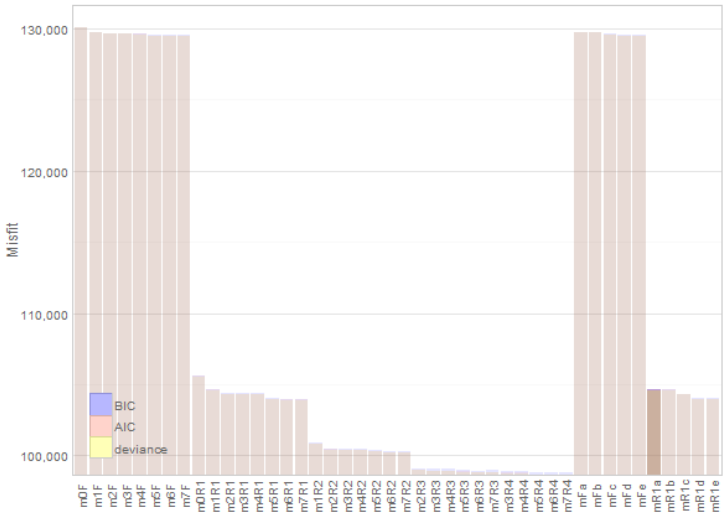
$$\vdots \quad \vdots \quad \vdots$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

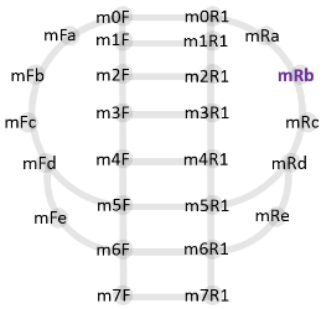
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.12	0.03	93.40	1.58	2.51				1.14
timec	-0.06	0.00	-31.22						1.14
timec2									
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.35 mR1b



$$\mathbf{y}_i = \boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

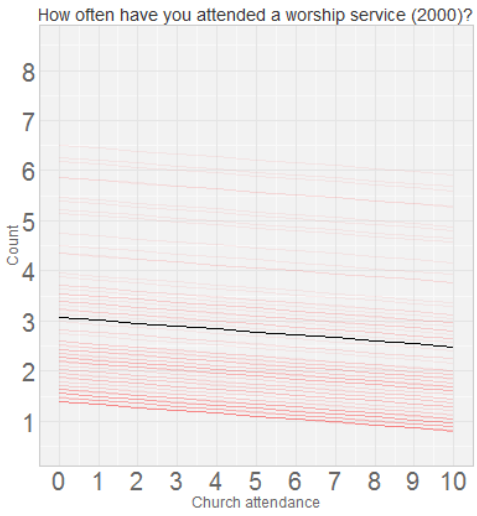
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

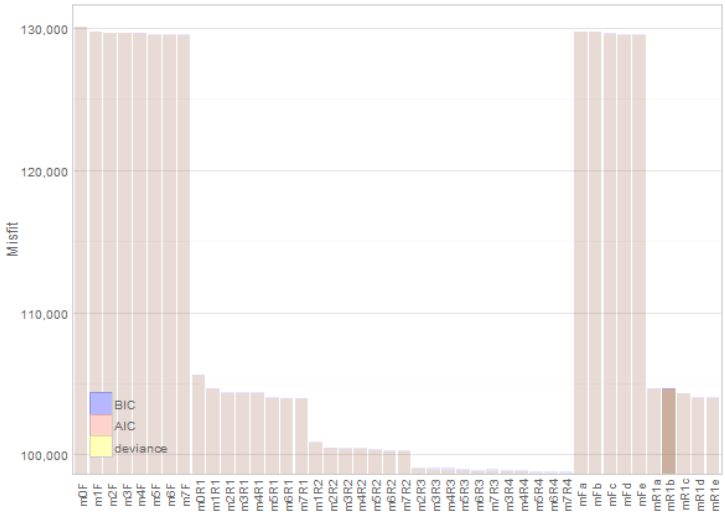
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

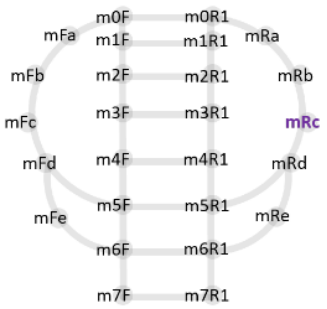
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.03	0.06	51.94	1.58	2.51				1.14
timec	−0.06	0.00	−31.22						1.14
timec2									
timec3									
cohort	0.04	0.02	1.90						1.14
timec:cohort									
timec2:cohort									
timec3:cohort									





2.36 mR1c



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^q \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i1} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

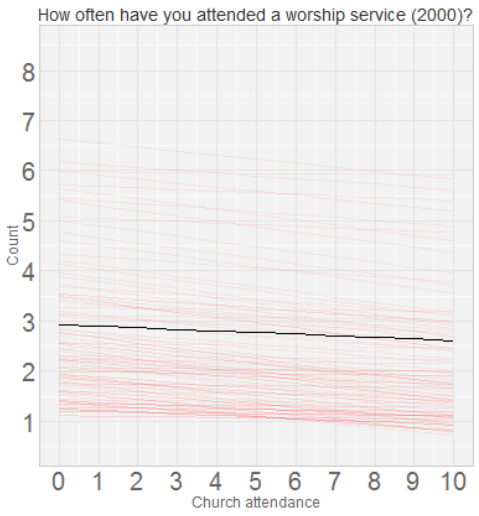
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

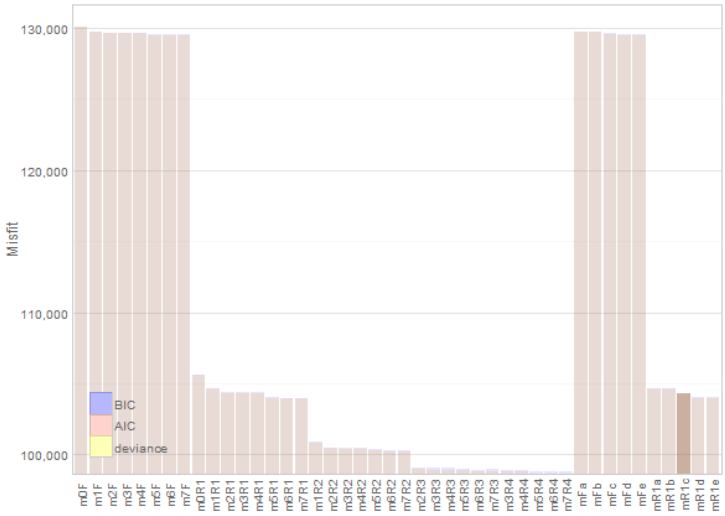
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

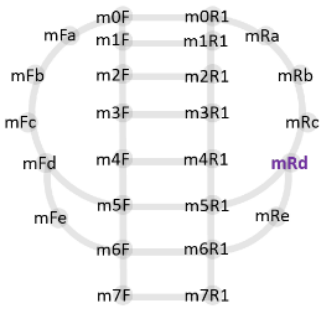
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.75	0.06	45.60	1.58	2.51				1.13
timec	-0.01	0.00	-2.32						1.13
timec2									
timec3									
cohort	0.18	0.02	7.36						1.13
timec:cohort	-0.02	0.00	-18.07						1.13
timec2:cohort									
timec3:cohort									





2.37 mR1d



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i1} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

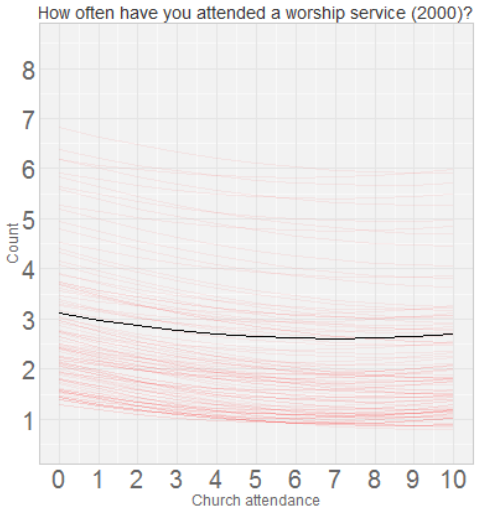
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

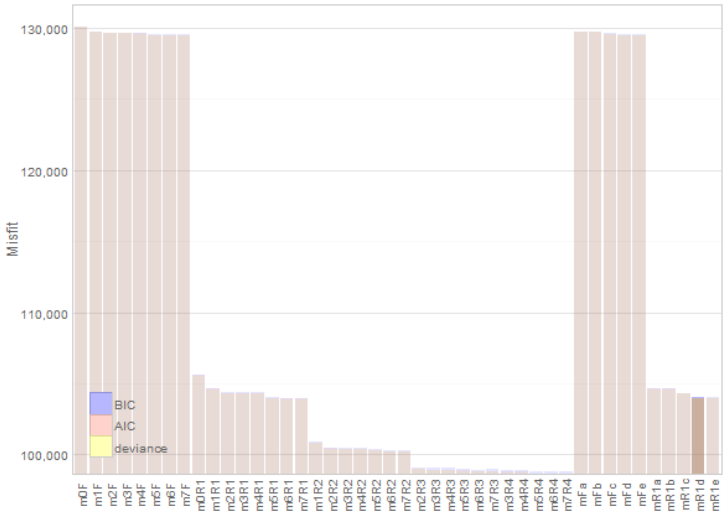
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

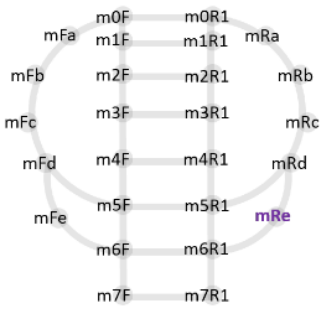
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.94	0.06	47.98	1.58	2.51				1.13
timec	-0.12	0.01	-16.42						1.13
timec2	0.01	0.00	17.21						1.13
timec3									
cohort	0.18	0.02	7.36						1.13
timec:cohort	-0.02	0.00	-18.17						1.13
timec2:cohort									
timec3:cohort									





2.38 mR1e



$$\mathbf{y}_i = \boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} = \begin{bmatrix} \eta_{0i} \\ \eta_{1i} \\ \vdots \\ \eta_{Pi} \end{bmatrix} = \begin{bmatrix} \mu_{\eta_0} \\ \mu_{\eta_1} \\ \vdots \\ \mu_{\eta_P} \end{bmatrix} \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{01} & \gamma_{02} & \cdots & \gamma_{0K} \\ \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{P1} & \gamma_{P2} & \cdots & \gamma_{PK} \end{bmatrix} \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{0i} \\ \zeta_{\beta 1} \\ \vdots \\ \zeta_{\beta P} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

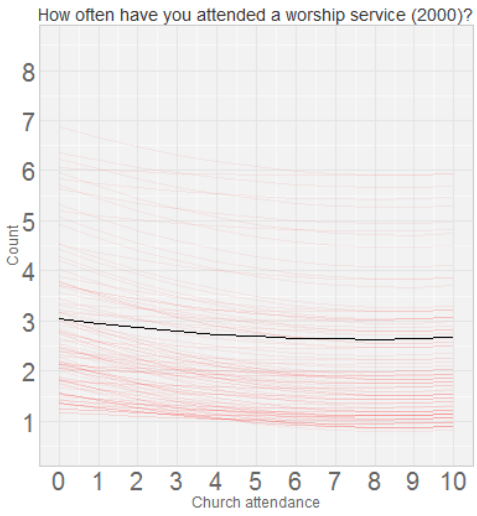
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

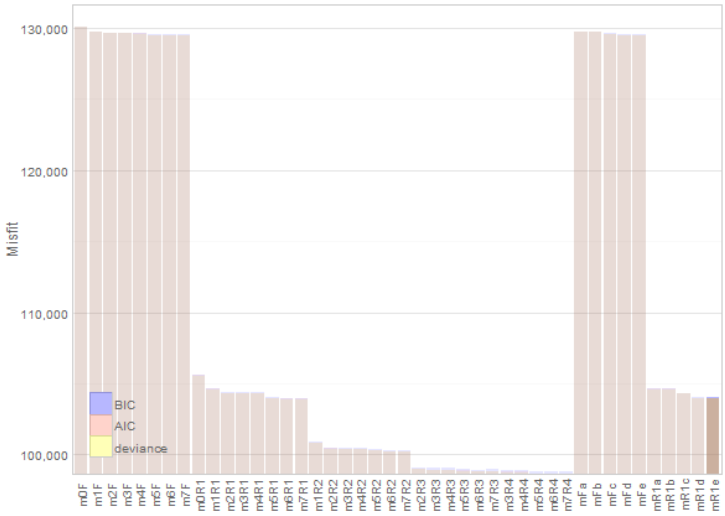
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

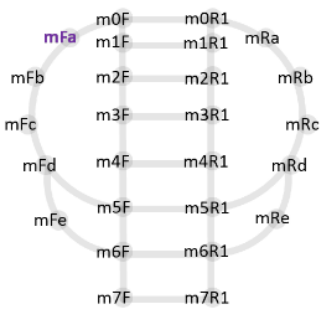
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.81	0.06	44.25	1.58	2.51				1.13
timec	-0.04	0.01	-3.56						1.13
timec2	0.00	0.00	3.05						1.13
timec3									
cohort	0.24	0.03	9.44						1.13
timec:cohort	-0.06	0.00	-12.37						1.13
timec2:cohort	0.00	0.00	7.78						1.13
timec3:cohort									





2.39 mFa – Experiment F



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 11} & \gamma_{\beta 12} & \cdots & \gamma_{\beta 1K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P1} & \gamma_{\beta P2} & \cdots & \gamma_{\beta PK} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^P \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_\alpha \\ \zeta_{\beta 1} \\ \vdots \\ \zeta_{\beta P} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \Psi_{\alpha\alpha} & & & \\ \Psi_{\alpha\beta 1} & \Psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \Psi_{\alpha\beta P} & \Psi_{\beta 1\beta P} & \cdots & \Psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

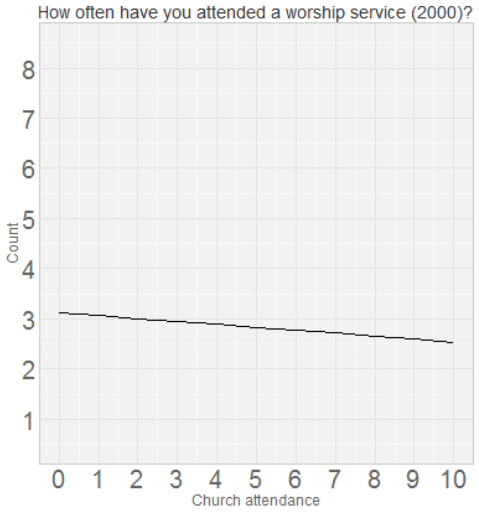
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

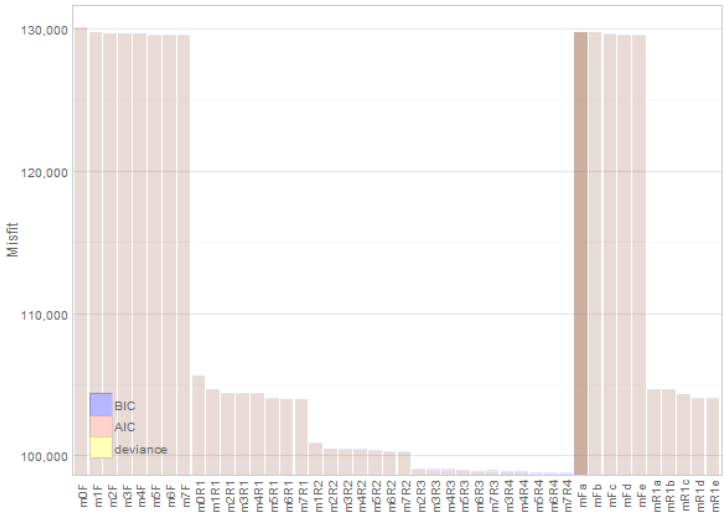
$$\vdots \quad \vdots \quad \vdots$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

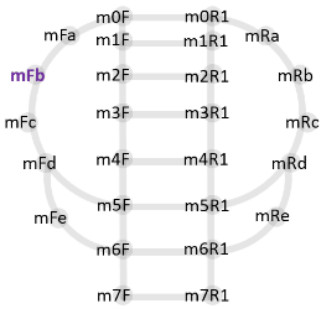
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.12	0.02	149.89		0.00				1.95
timec	-0.06	0.00	-18.24		0.00				1.95
timec2									
timec3									
cohort									
timec:cohort									
timec2:cohort									
timec3:cohort									





2.40 mFb



$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\eta}_t = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\zeta}_t$$

$$\mathbf{y}_t = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_t + \boldsymbol{\Lambda}\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t$$

Bollen & Curran (2006)

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Tt} \end{bmatrix} \quad \boldsymbol{\eta}_t = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_t = \begin{bmatrix} w_{1t} \\ w_{2t} \\ \vdots \\ w_{Kt} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^n \end{bmatrix} \quad \boldsymbol{\zeta}_t = \begin{bmatrix} \zeta_{\alpha} \\ \zeta_{\beta 1} \\ \vdots \\ \zeta_{\beta P} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Tt} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

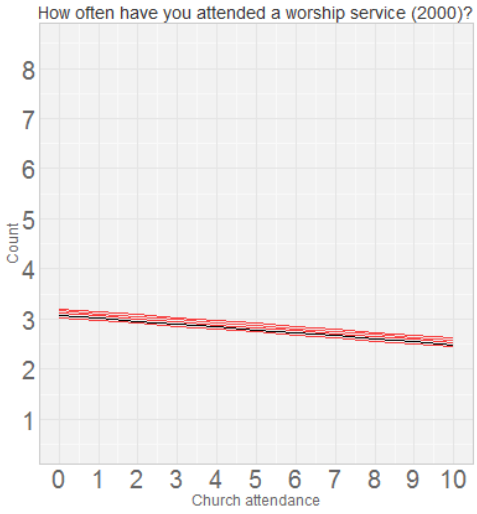
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

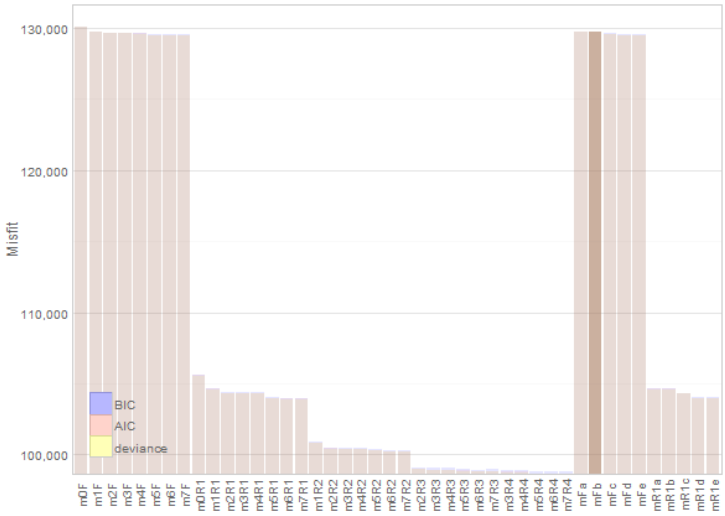
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

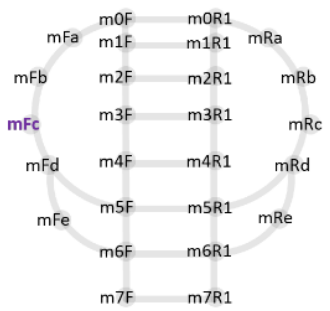
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	3.03	0.03	113.65		0.00				1.95
timec	−0.06	0.00	−18.25		0.00				1.95
timec2									
timec3									
cohort	0.04	0.01	5.47		0.00				1.95
timec:cohort									
timec2:cohort									
timec3:cohort									





2.41 mFc



$$\mathbf{y}_i = \boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$
$$\boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$
$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^K \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

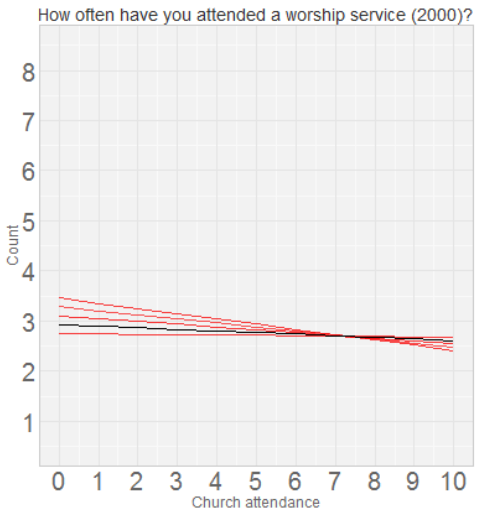
$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$
$$\vdots \quad \vdots \quad \vdots$$
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

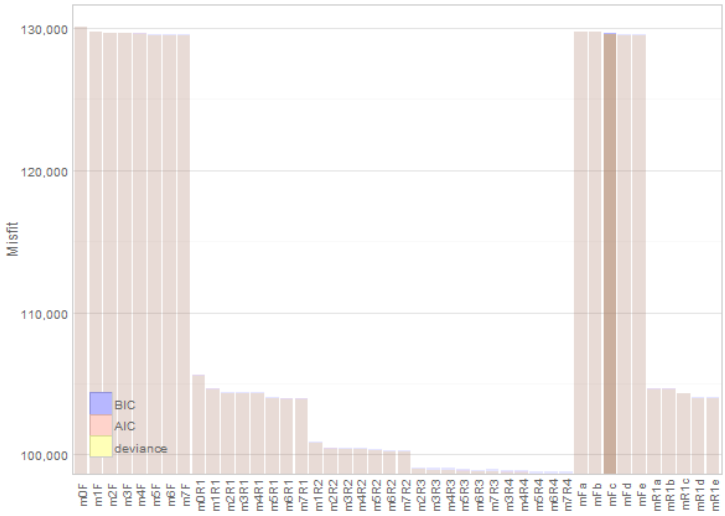
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

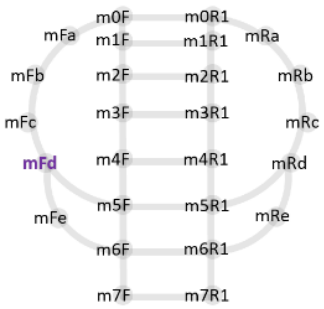
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.75	0.04	73.25		0.00				1.95
timec	-0.01	0.01	-1.35		0.00				1.95
timec2									
timec3									
cohort	0.18	0.01	11.82		0.00				1.95
timec:cohort	-0.02	0.00	-10.52		0.00				1.95
timec2:cohort									
timec3:cohort									





2.42 mFd



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \qquad \boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 0} & \gamma_{\alpha 1} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 10} & \gamma_{\beta 11} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 0} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^n \end{bmatrix} \quad \boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_{\alpha i} \\ \zeta_{\beta 1 i} \\ \vdots \\ \zeta_{\beta P i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1\beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1\beta P} & \cdots & \psi_{\beta P\beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{Pt} + \varepsilon_{ti}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\vdots \quad \vdots \quad \vdots$$

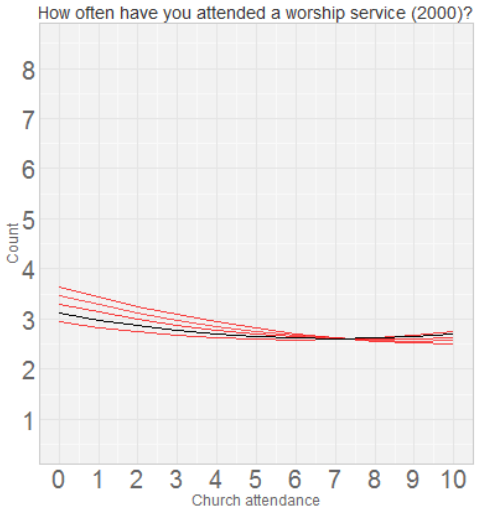
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

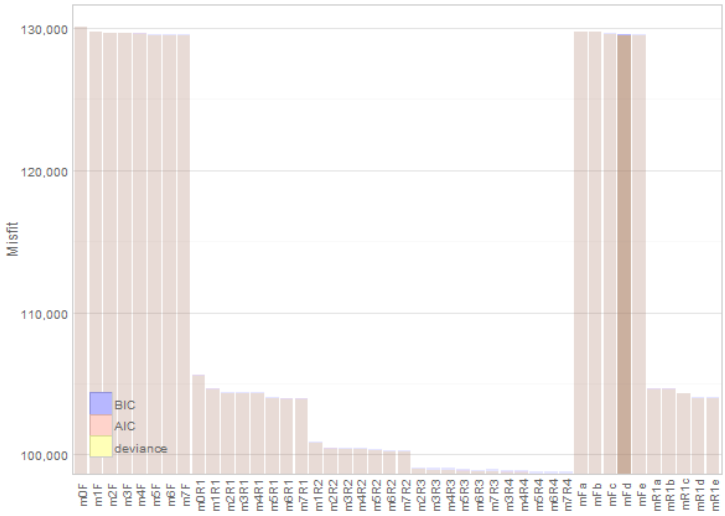
Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

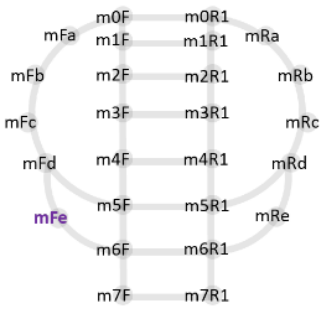
$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.94	0.04	69.87		0.00				1.95
timec	-0.12	0.01	-9.53		0.00				1.95
timec2	0.01	0.00	9.98		0.00				1.95
timec3									
cohort	0.18	0.01	11.84		0.00				1.95
timec:cohort	-0.02	0.00	-10.54		0.00				1.95
timec2:cohort									
timec3:cohort									





2.43 mFe



Bollen & Curran (2006)

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Snijders & Bosker (2011)

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{pi} time_{pt} + \varepsilon_{ti}$$

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$

$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

	Estimate	Std.Error	t.value	sdRE	intVarRE	timecVarRE	timec2VarRE	timec3VarRE	sigma
(Intercept)	2.81	0.05	55.10		0.00				1.94
timec	-0.04	0.02	-2.06		0.00				1.94
timec2	0.00	0.00	1.77		0.00				1.94
timec3									
cohort	0.24	0.02	11.76		0.00				1.94
timec:cohort	-0.06	0.01	-7.17		0.00				1.94
timec2:cohort	0.00	0.00	4.51		0.00				1.94
timec3:cohort