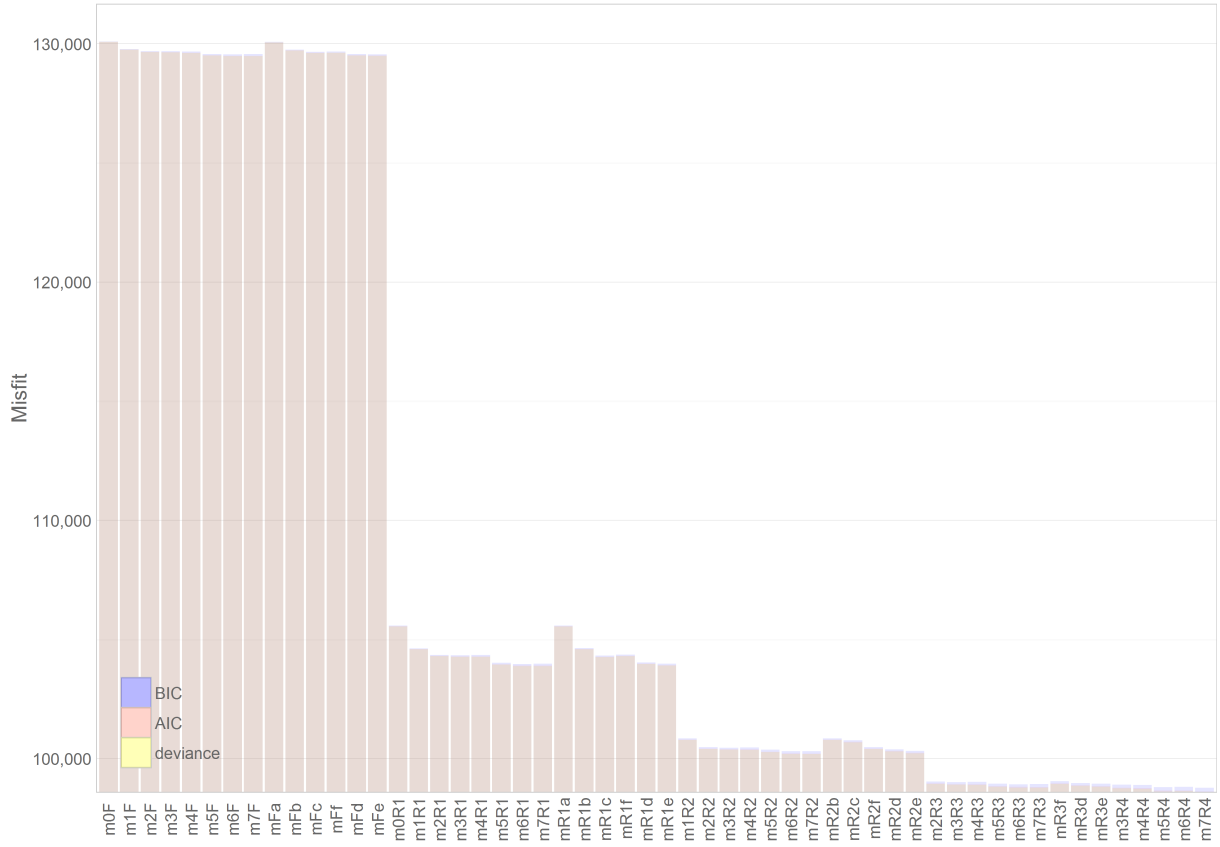
Custom Fit

Contents

1 All models



There are total of 54 distinct models, which can be organized into 4 groups:

- models with fixed effects only
- models with 1 random term
- models with 2 random terms
- models with 3 random temrs

The following layout helps understand how each of the models was constructed. The columns count the number of terms on the first level, the rows shows what predictors are added to the second level. Replace the stars in the name of the model with **F** for models with only the fixed effects and **R1**, **R2**, **R3**, and **R4** for models with the corresponding number of random terms in the second level.

	β_{0i}	$\beta_{0i} + \beta_{1i} timec_{ti}$	$\beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti}$	$\beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \beta_{3i} timec^3_{ti}$
	m0*	m1*	m2*	m3*
$\gamma_{01} cohort_i$	m*a	m*b	m*f	m4*
$\gamma_{01} cohort_i$ $\gamma_{11} cohort_i$		m*c	m*d	m5*
$\gamma_{01} cohort_i$ $\gamma_{11} cohort_i$ $\gamma_{21} cohort_i$			m*e	m6*
$\gamma_{01} cohort_i$ $\gamma_{11} cohort_i$ $\gamma_{21} cohort_i$ $\gamma_{31} cohort_i$				m7*

2 Models w/ fixed only

Model Sequence Map F

0 random terms

m0F

$y_{ti} = \beta_{0i} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00}$

m1F

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00}$
$\beta_{1i} = \gamma_{10}$

mFa

$y_{ti} = \beta_{0i} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$

mFb

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10}$

mFc

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$

m2F

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00}$
$\beta_{1i} = \gamma_{10}$
$\beta_{2i} = \gamma_{20}$

mFf

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10}$
$\beta_{2i} = \gamma_{20}$

mFd

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$
$\beta_{2i} = \gamma_{20}$

mFe

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$
$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$

m3F

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \beta_{3i} timec^3_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00}$
$\beta_{1i} = \gamma_{10}$
$\beta_{2i} = \gamma_{20}$
$\beta_{3i} = \gamma_{30}$

m4F

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \beta_{3i} timec^3_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10}$
$\beta_{2i} = \gamma_{20}$
$\beta_{3i} = \gamma_{30}$

m5F

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \beta_{3i} timec^3_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$
$\beta_{2i} = \gamma_{20}$
$\beta_{3i} = \gamma_{30}$

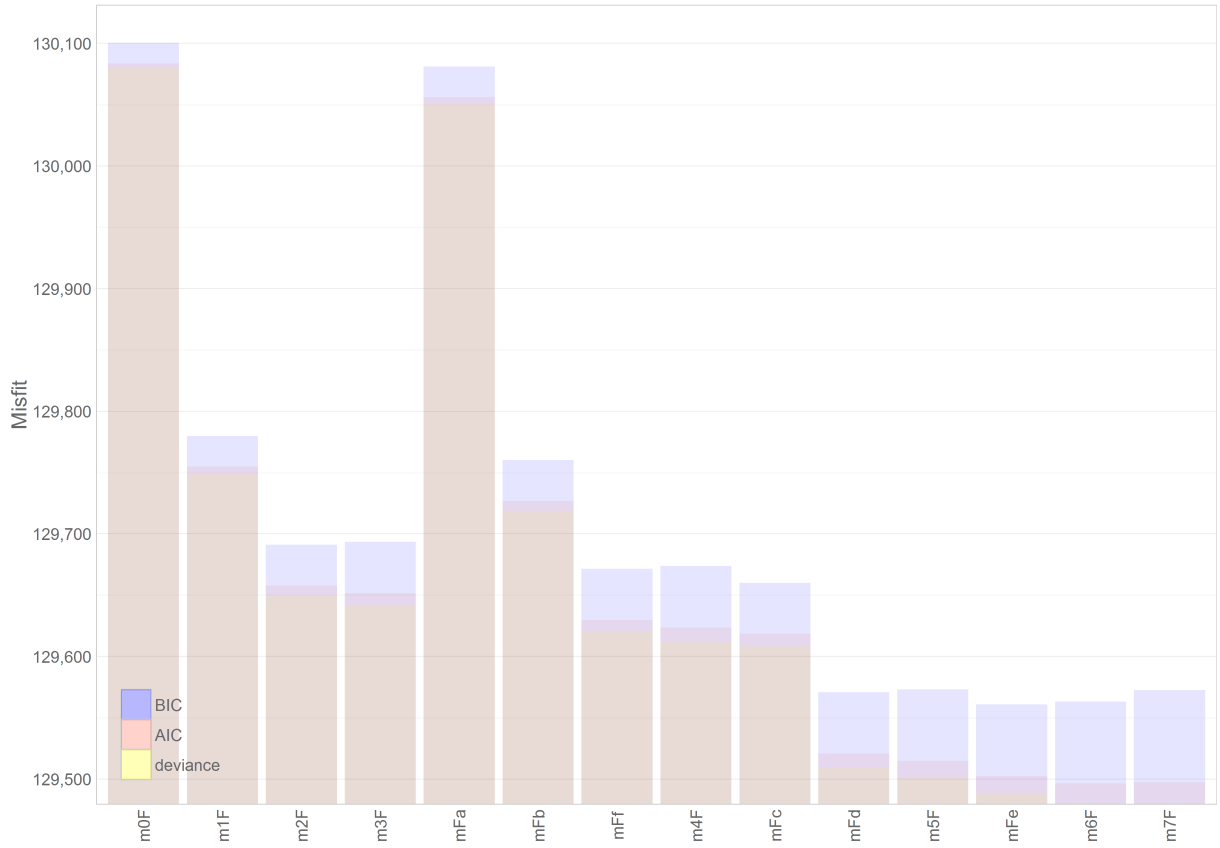
m6F

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \beta_{3i} timec^3_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$
$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$
$\beta_{3i} = \gamma_{30}$

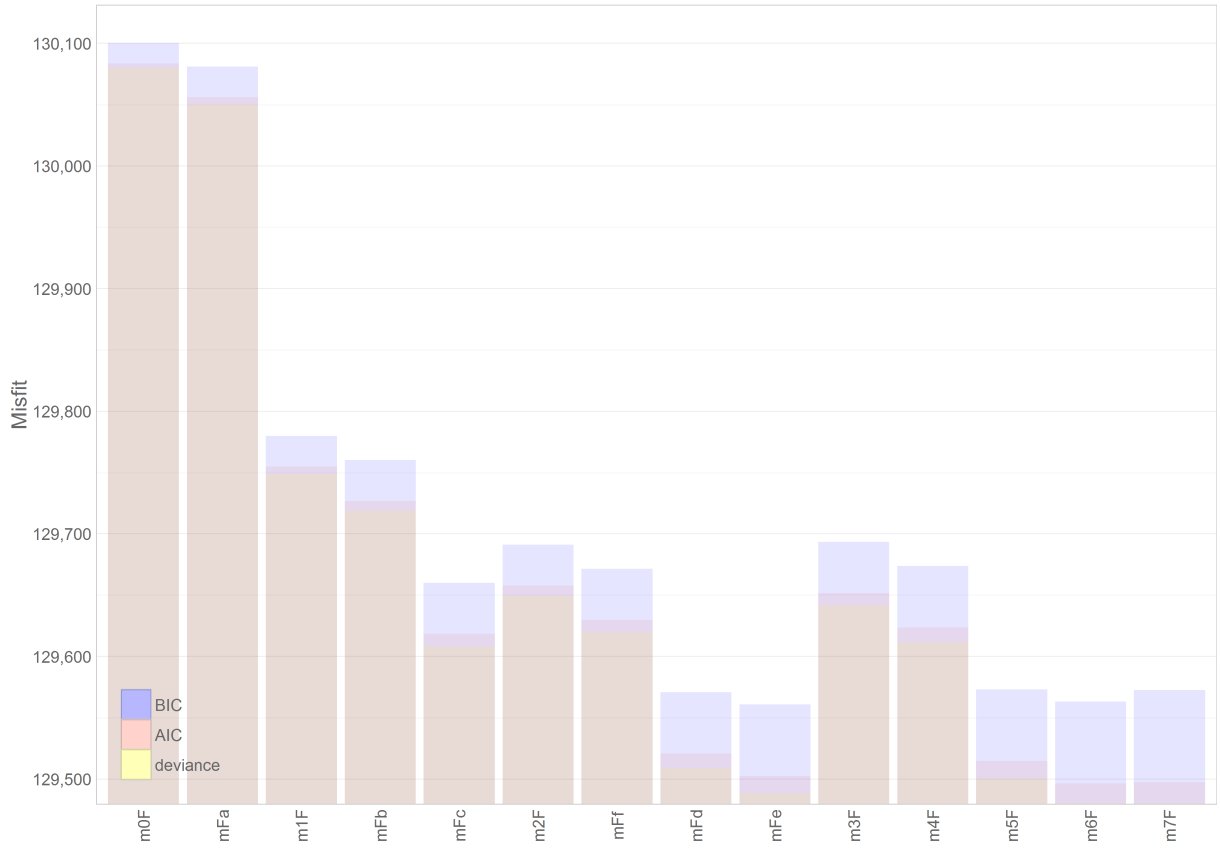
m7F

$y_{ti} = \beta_{0i} + \beta_{1i} timec_{ti} + \beta_{2i} timec^2_{ti} + \beta_{3i} timec^3_{ti} + \varepsilon_{ti}$
$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i$
$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$
$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$
$\beta_{3i} = \gamma_{30} + \gamma_{31} cohort_i$

2.1 F by row


130,100
130,000
129,900
129,800
129,700
129,600
129,500
Misfit
BIC
AIC
deviance
m0F
m1F
m2F
m3F
mFa
mFb
mFf
m4F
mFc
mFd
m5F
mFe
m6F
m7F

2.2 F by column



3 Models w/ 1 random

Model Sequence Map R1

m0R1

$$y_{it} = \beta_{0i} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$

m1R1

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10}$$

mR1a

$$y_{it} = \beta_{0i} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$

mR1b

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10}$$

mR1c

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$$

1 random term

m2R1

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10}$$
$$\beta_{2i} = \gamma_{20}$$

mR1f

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10}$$
$$\beta_{2i} = \gamma_{20}$$

mR1d

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$$
$$\beta_{2i} = \gamma_{20}$$

mR1e

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$$

m3R1

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \beta_{3i} timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10}$$
$$\beta_{2i} = \gamma_{20}$$
$$\beta_{3i} = \gamma_{30}$$

m4R1

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \beta_{3i} timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10}$$
$$\beta_{2i} = \gamma_{20}$$
$$\beta_{3i} = \gamma_{30}$$

m5R1

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \beta_{3i} timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$$
$$\beta_{2i} = \gamma_{20}$$
$$\beta_{3i} = \gamma_{30}$$

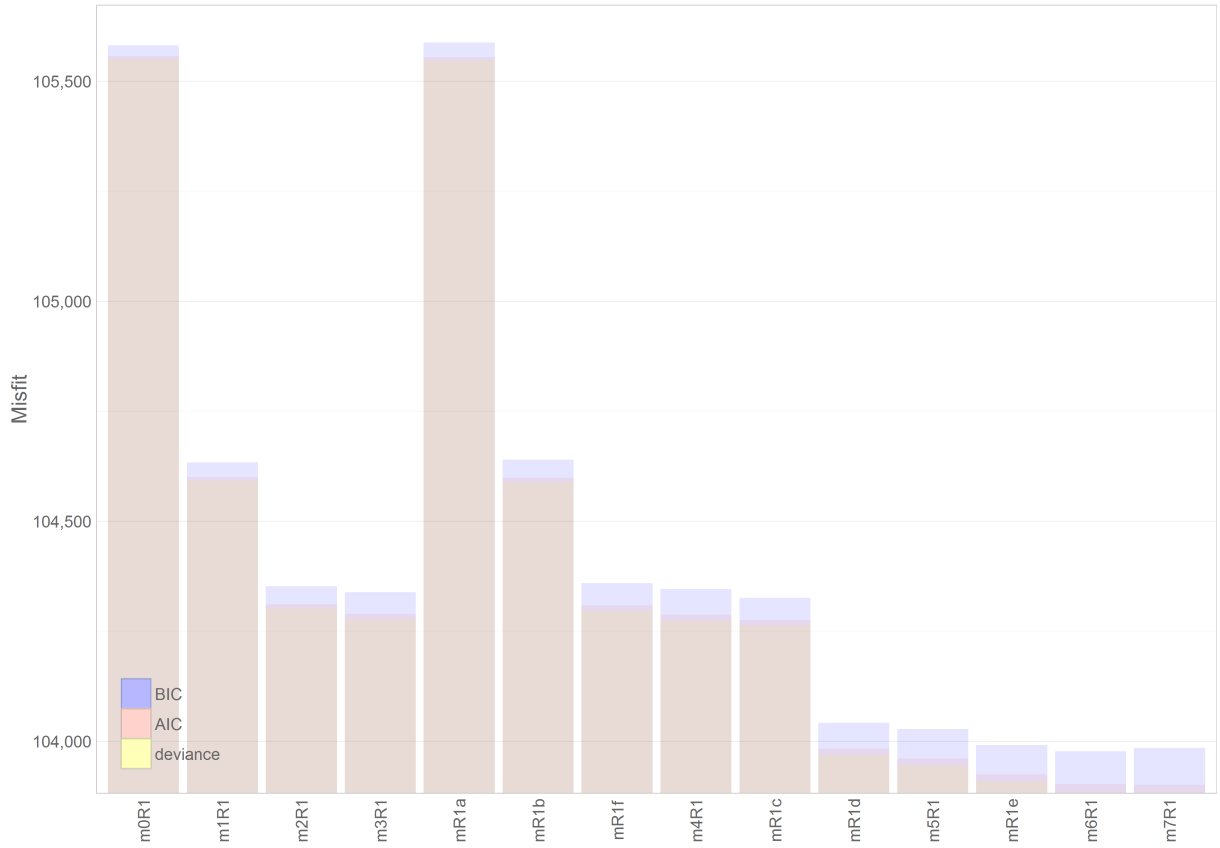
m6R1

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \beta_{3i} timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$$
$$\beta_{3i} = \gamma_{30}$$

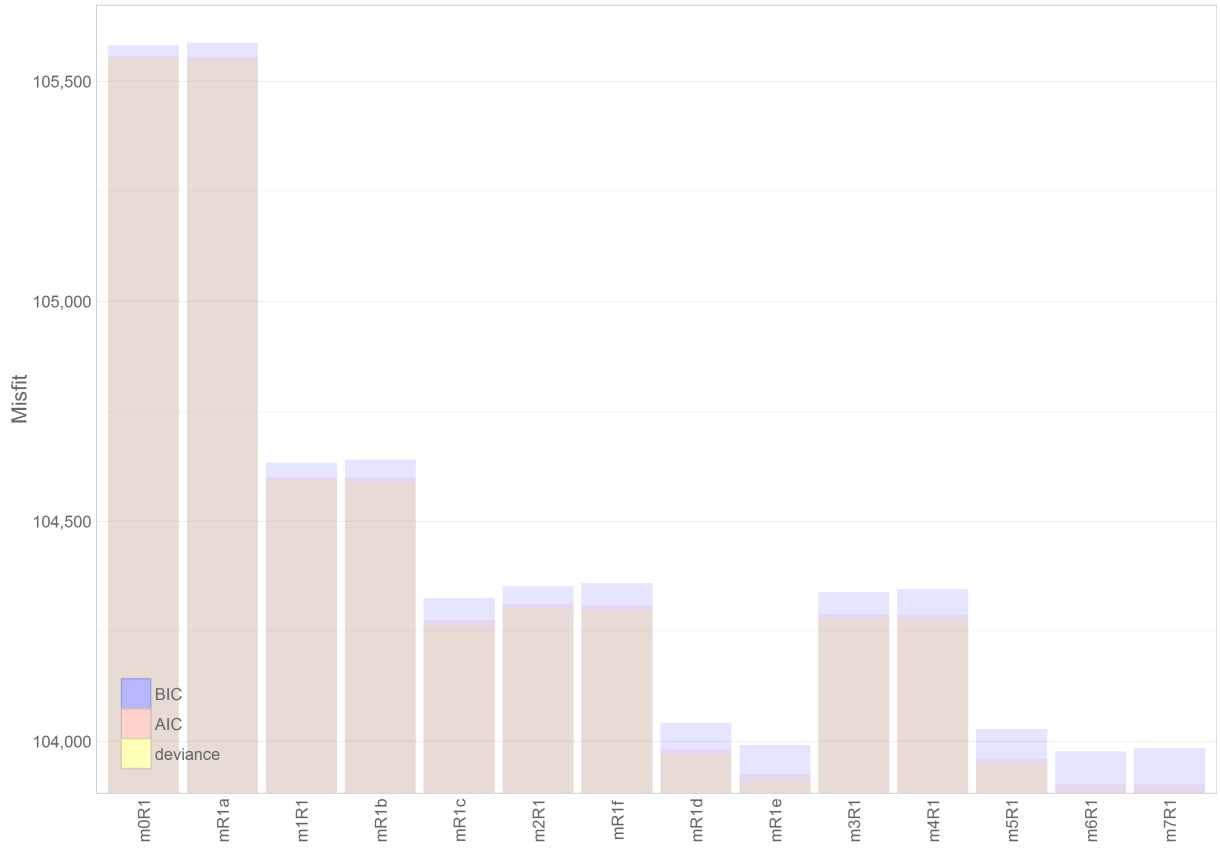
m7R1

$$y_{it} = \beta_{0i} + \beta_{1i} timec_{it} + \beta_{2i} timec^2_{it} + \beta_{3i} timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$$
$$\beta_{3i} = \gamma_{30} + \gamma_{31} cohort_i$$

3.1 R1 by row


105,500
105,000
104,500
104,000
Misfit
BIC
AIC
deviance
m0R1
m1R1
m2R1
m3R1
mR1a
mR1b
mR1f
m4R1
mR1c
mR1d
m5R1
mR1e
m6R1
m7R1

3.2 R1 by column



4 Models w/ 2 random

Model Sequence Map R2

2 random terms

m1R2

$$y_{it} = \beta_{0i} + \beta_{1i} timec_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$

mR2b

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$

mR2c

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$

m2R2

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \beta_2 timec^2_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20}$$

mR2f

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \beta_2 timec^2_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20}$$

mR2d

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \beta_2 timec^2_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20}$$

mR2e

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \beta_2 timec^2_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$$

m3R2

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \beta_{2i} timec^2_t + \beta_3 timec^3_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20}$$
$$\beta_{3i} = \gamma_{30}$$

m4R2

$$y_{it} = \beta_{0i} + \beta_{1i} timec_t + \beta_{2i} timec^2_t + \beta_3 timec^3_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20}$$
$$\beta_{3i} = \gamma_{30}$$

m5R2

$$y_{it} = \beta_{0i} + \beta_{1i} timec_t + \beta_{2i} timec^2_t + \beta_3 timec^3_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20}$$
$$\beta_{3i} = \gamma_{30}$$

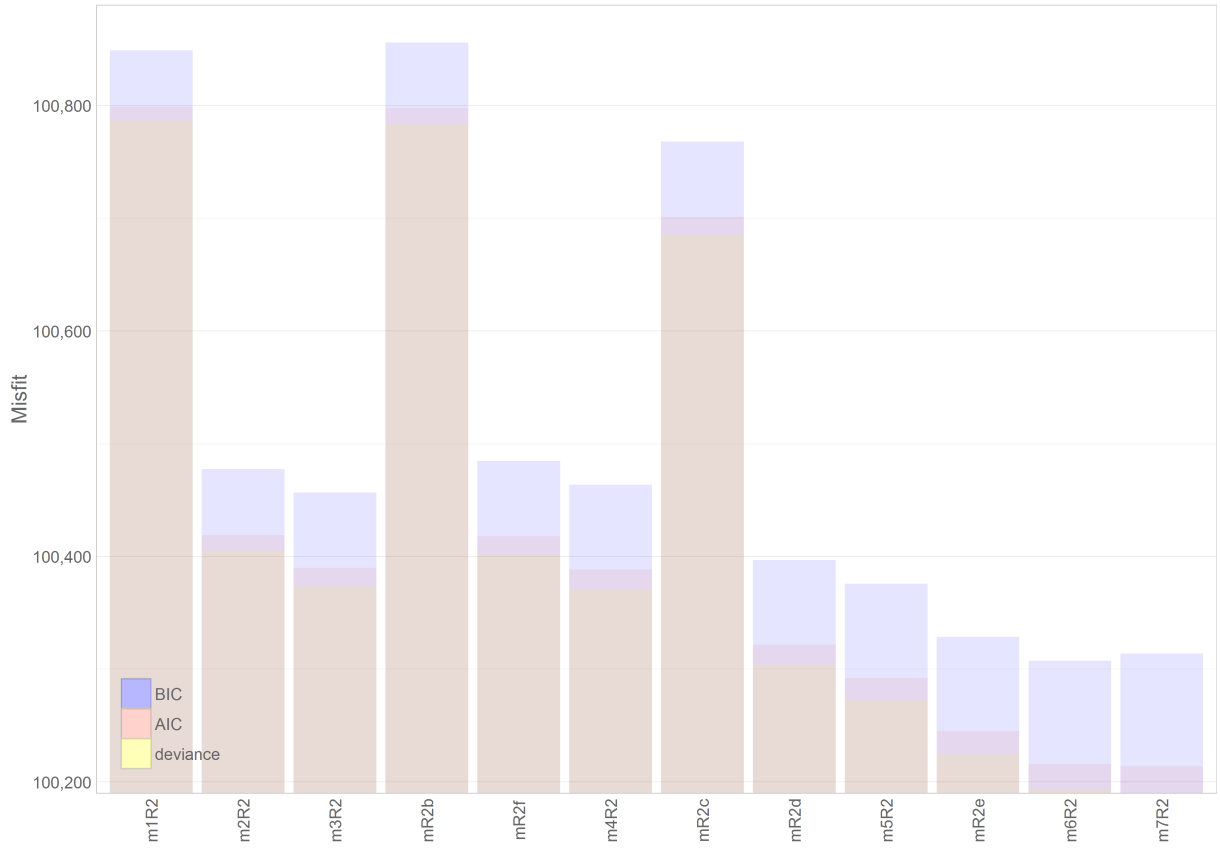
m6R2

$$y_{it} = \beta_{0i} + \beta_1 timec_t + \beta_{2i} timec^2_t + \beta_3 timec^3_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$$
$$\beta_{3i} = \gamma_{30}$$

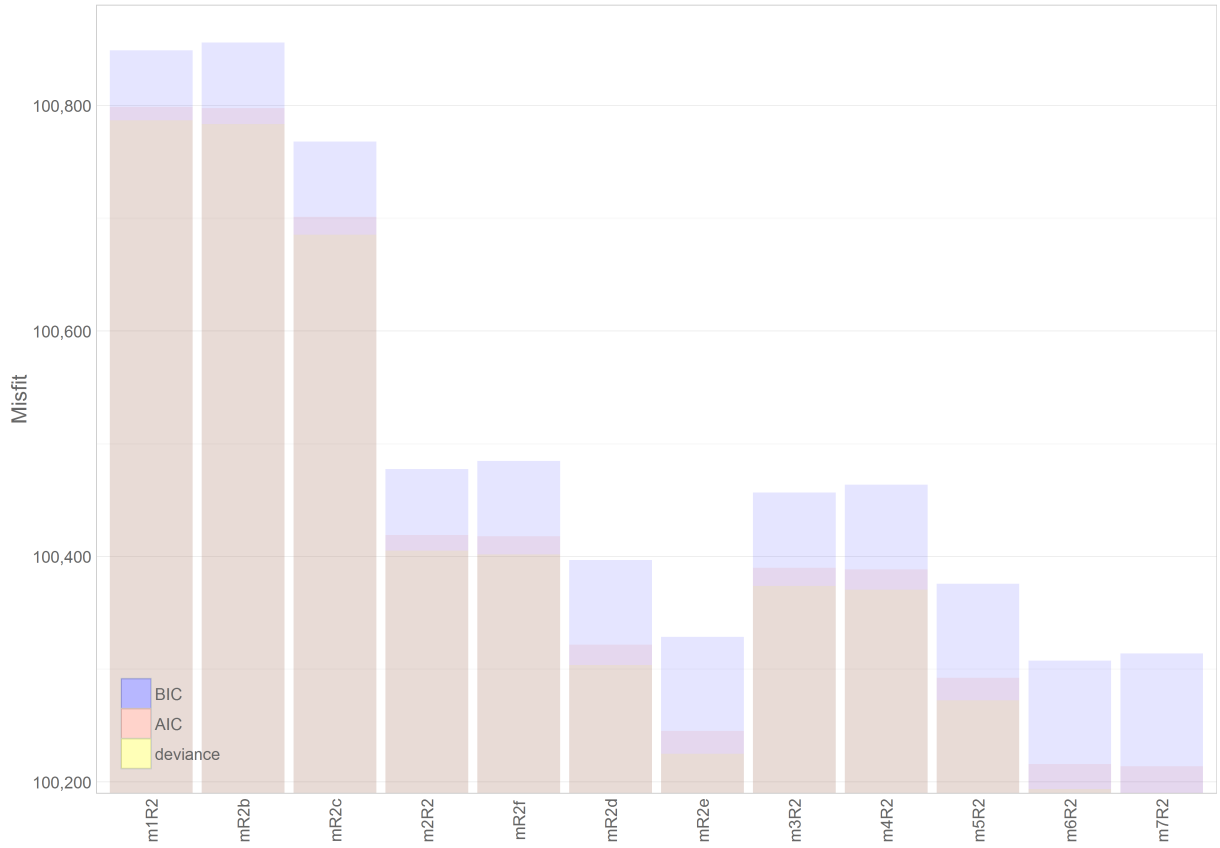
m7R2

$$y_{it} = \beta_{0i} + \beta_{1i} timec_t + \beta_{2i} timec^2_t + \beta_3 timec^3_t + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i$$
$$\beta_{3i} = \gamma_{30} + \gamma_{31} cohort_i$$

4.1 R2 by row


Misfit
100,800
100,600
100,400
100,200
BIC
AIC
deviance
m1R2
m2R2
m3R2
mR2b
mR2f
m4R2
mR2c
mR2d
m5R2
mR2e
m6R2
m7R2

4.2 R2 by column



5 Models w/ 3 random

Model Sequence Map R3

3 random terms

m2R3

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_2 timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$

mR3f

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_2 timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$

mR3d

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_2 timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$

mR3e

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_2 timec^2_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i + u_{2i}$$

m3R3

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_{2i} timec^2_{it} + \beta_3 timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$
$$\beta_{3i} = \gamma_{30}$$

m4R3

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_{2i} timec^2_{it} + \beta_3 timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$
$$\beta_{3i} = \gamma_{30}$$

m5R3

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_{2i} timec^2_{it} + \beta_3 timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$
$$\beta_{3i} = \gamma_{30}$$

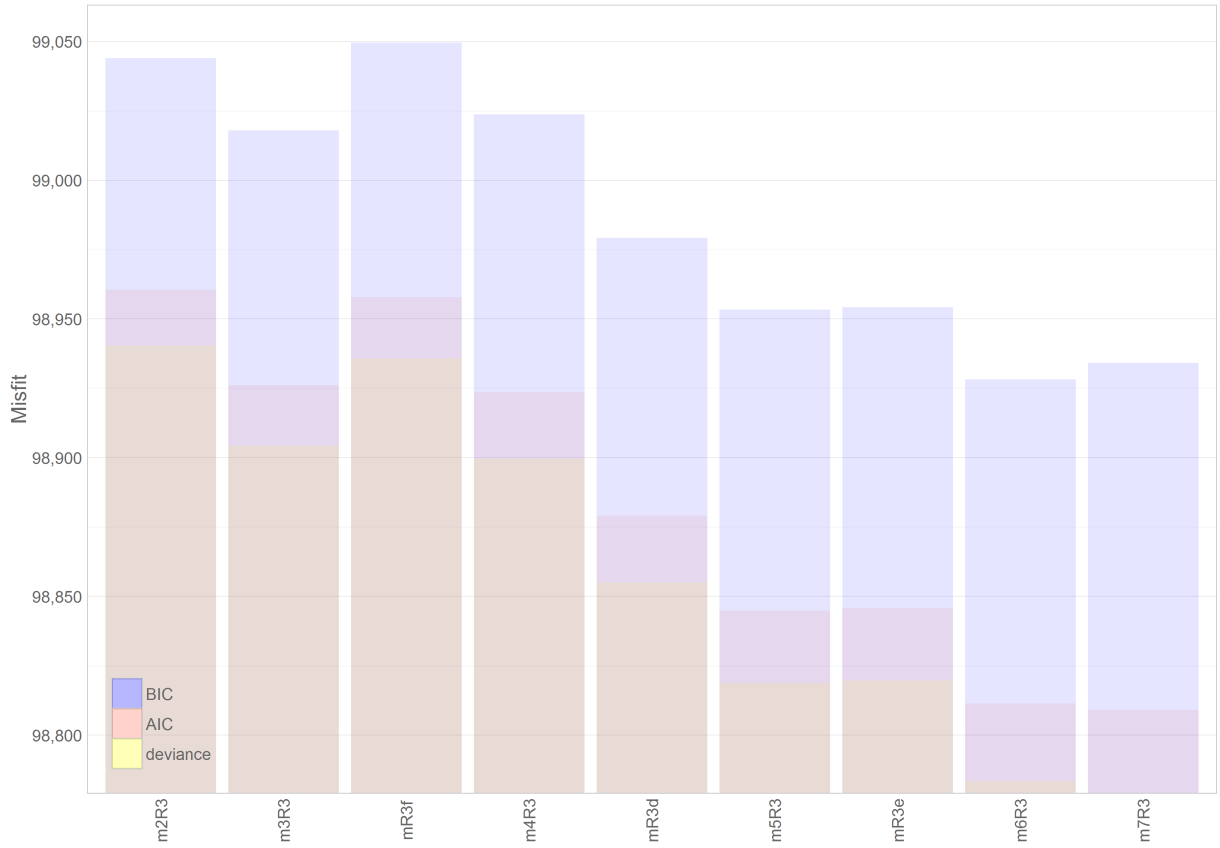
m6R3

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_{2i} timec^2_{it} + \beta_3 timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i + u_{2i}$$
$$\beta_{3i} = \gamma_{30}$$

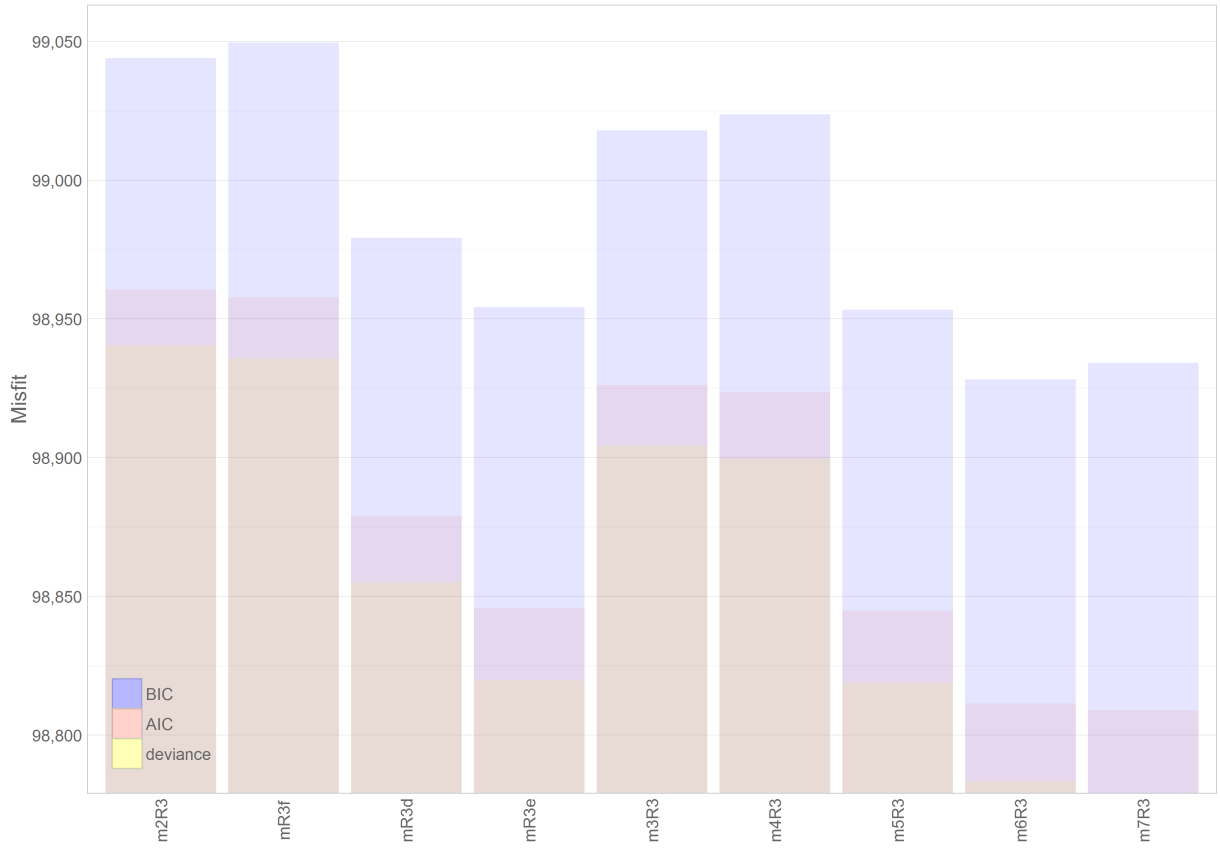
m7R3

$$y_{it} = \beta_{0i} + \beta_1 timec_{it} + \beta_{2i} timec^2_{it} + \beta_3 timec^3_{it} + \varepsilon_{it}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01} cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21} cohort_i + u_{2i}$$
$$\beta_{3i} = \gamma_{30} + \gamma_{31} cohort_i$$

5.1 R3 by row


99,050
99,000
98,950
98,900
98,850
98,800
Misfit
BIC
AIC
deviance
m2R3
m3R3
mR3f
m4R3
mR3d
m5R3
mR3e
m6R3
m7R3

5.2 R3 by column



6 Models w/ 4 random

Model Sequence Map R4 4 random terms

m3R4

$$y_{ij} = \beta_{0i} + \beta_{1i}timsc_{ij} + \beta_{2i}timsc^2_{ij} + \beta_{3i}timsc^3_{ij} + \varepsilon_{ij}$$
$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$
$$\beta_{3i} = \gamma_{30} + u_{3i}$$

m4R4

$$y_{ij} = \beta_{0i} + \beta_{1i}timsc_{ij} + \beta_{2i}timsc^2_{ij} + \beta_{3i}timsc^3_{ij} + \varepsilon_{ij}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01}cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$
$$\beta_{3i} = \gamma_{30} + u_{3i}$$

m5R4

$$y_{ij} = \beta_{0i} + \beta_{1i}timsc_{ij} + \beta_{2i}timsc^2_{ij} + \beta_{3i}timsc^3_{ij} + \varepsilon_{ij}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01}cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$
$$\beta_{3i} = \gamma_{30} + u_{3i}$$

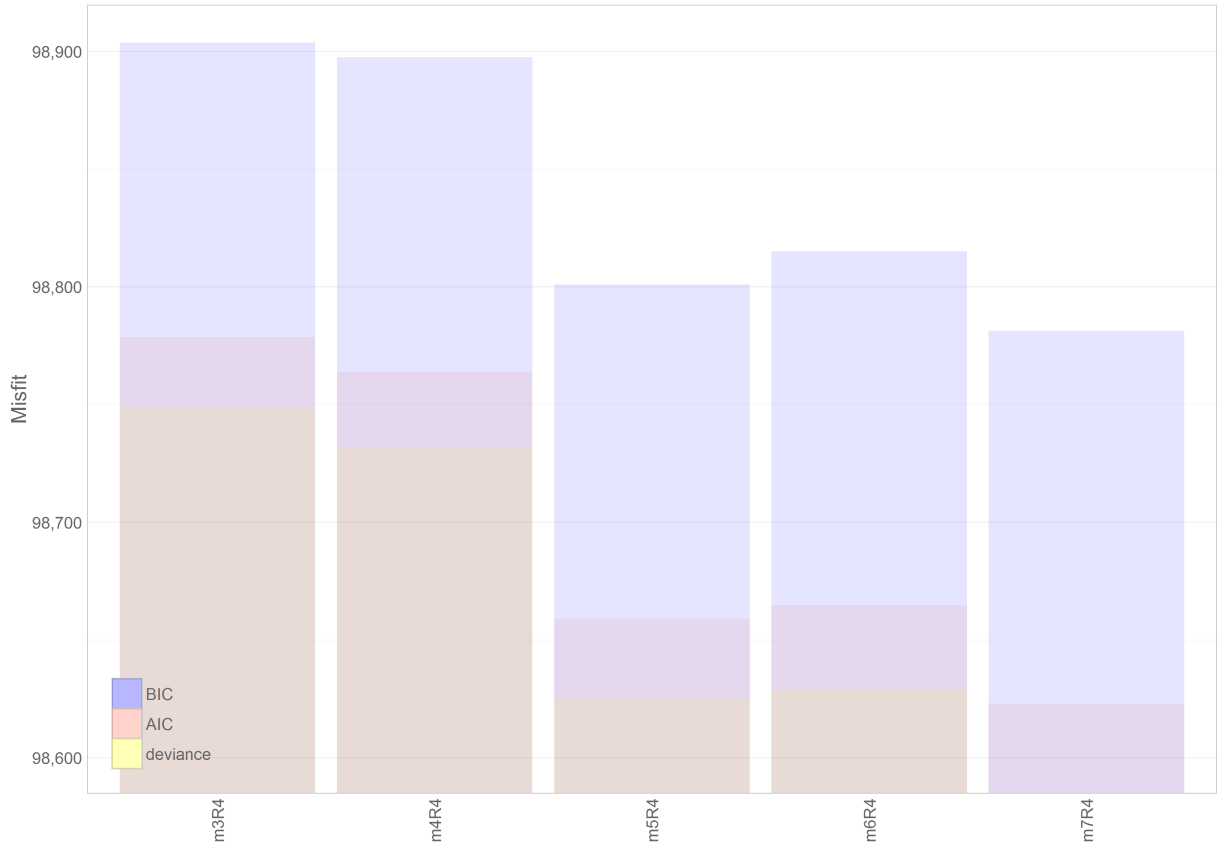
m6R4

$$y_{ij} = \beta_{0i} + \beta_{1i}timsc_{ij} + \beta_{2i}timsc^2_{ij} + \beta_{3i}timsc^3_{ij} + \varepsilon_{ij}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01}cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21}cohort_i + u_{2i}$$
$$\beta_{3i} = \gamma_{30} + u_{3i}$$

m7R4

$$y_{ij} = \beta_{0i} + \beta_{1i}timsc_{ij} + \beta_{2i}timsc^2_{ij} + \beta_{3i}timsc^3_{ij} + \varepsilon_{ij}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01}cohort_i + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}cohort_i + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21}cohort_i + u_{2i}$$
$$\beta_{3i} = \gamma_{30} + \gamma_{31}cohort_i + u_{3i}$$

6.1 R4 by row


98,900
98,800
98,700
98,600
Misfit
BIC
AIC
deviance
m3R4
m4R4
m5R4
m6R4
m7R4

7 General form

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$
$$\boldsymbol{\eta}_i = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\zeta}_i$$

$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{w}_i + \boldsymbol{\Lambda}\boldsymbol{\zeta}_i + \boldsymbol{\varepsilon}_i$$

Bollen & Curran (2006)

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \boldsymbol{\eta}_i = \begin{bmatrix} \alpha_i \\ \beta_{1i} \\ \vdots \\ \beta_{Pi} \end{bmatrix} \quad \boldsymbol{\mu}_\eta = \begin{bmatrix} \mu_\alpha \\ \mu_{\beta 1} \\ \vdots \\ \mu_{\beta P} \end{bmatrix} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{\alpha 1} & \gamma_{\alpha 2} & \cdots & \gamma_{\alpha K} \\ \gamma_{\beta 1 1} & \gamma_{\beta 1 2} & \cdots & \gamma_{\beta 1 K} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\beta P 1} & \gamma_{\beta P 1} & \cdots & \gamma_{\beta P K} \end{bmatrix} \quad \mathbf{w}_i = \begin{bmatrix} w_{1i} \\ w_{2i} \\ \vdots \\ w_{Ki} \end{bmatrix} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (T-1)^1 & \cdots & (T-1)^n \end{bmatrix}$$

$$\boldsymbol{\zeta}_i = \begin{bmatrix} \zeta_\alpha \\ \zeta_{\beta 1} \\ \vdots \\ \zeta_{\beta P} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{\alpha\alpha} & & & \\ \psi_{\alpha\beta 1} & \psi_{\beta 1 \beta 1} & & \\ \vdots & \vdots & \ddots & \\ \psi_{\alpha\beta P} & \psi_{\beta 1 \beta P} & \cdots & \psi_{\beta P \beta P} \end{bmatrix} \right) \quad \boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i1} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{1t} + \beta_{2i} time_{2t} \ldots + \beta_{Pi} time_{Pt} + \varepsilon_{ti}$$

Snijders & Bosker (2011)

$$\varepsilon_{ti} \sim N\left([0], [\sigma^2]\right)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{01} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11} w_{1i} + \gamma_{12} w_{2i} + \ldots + \gamma_{0K} w_{Ki} + u_{1i}$$
$$\vdots \quad \vdots \quad \vdots$$
$$\beta_{Pi} = \gamma_{P0} + \gamma_{P1} w_{1i} + \gamma_{P2} w_{2i} + \ldots + \gamma_{PK} w_{Ki} + u_{Ki}$$

$$\begin{bmatrix} u_{0i} \\ u_{1i} \\ \vdots \\ u_{Pi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & & & \\ \tau_{10} & \tau_{11} & & \\ \vdots & \vdots & \ddots & \\ \tau_{P0} & \tau_{P1} & \cdots & \tau_{PK} \end{bmatrix} \right)$$

$\mathbf{y}_i$ - A vector of responses of individual *i* for times T

$\boldsymbol{\Lambda}$ - Matrix of weights for P functions of time

$\boldsymbol{\eta}_i$ - Vector of person-specific weights for P time effects

$\boldsymbol{\mu}_\eta$ - Vector of fixed effect estimates (mean/intercept)

$\boldsymbol{\Gamma}$ - Matrix of fixed effect estimates for $\mathbf{w}_i$ with K predictors

$\mathbf{w}_i$ - Time invariant random predictors of $\boldsymbol{\eta}_i$

$\boldsymbol{\zeta}_i$ - Random effect estimates

$\boldsymbol{\varepsilon}_i$ - Residual

T - Total number of time points in the data

P - Total number of time effects estimated in addition to mean/intercept

K - Total number of T predictors $\mathbf{w}_i$ on time effects $\boldsymbol{\eta}_i$