

## DF : Decrease of Fidelity

a contemplative device

Many religions have so called “contemplative devices” – the objects of cogitation that offer a unique perspective into nature of reality.

Trinity as a contemplative device was developed in Christianity in late 4<sup>th</sup> century A.D. and widely practice thereafter.

Buddhist had many contemplative devices, one of which is “Nirvana is the sound of the clap one hand makes”

Through attempting to understand these paradoxical statements one could get raptured into transcendence and gain a deeper understanding of reality.

Modern scientists, have similar devices. Consider particle/wave dualism in description of the nature of light or the notion that one atom can everything about the universe.

There is such contemplative device is statistical modeling – degrees of freedom.

**Degrees of Freedom**  
=

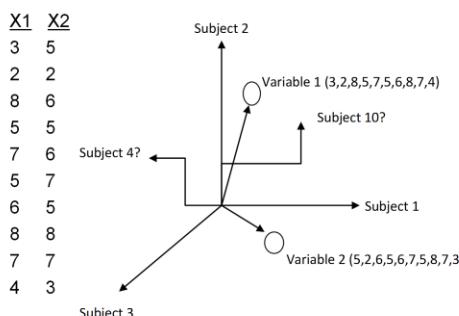
**Decrease of Fidelity**

Explaining modeling through defining DF

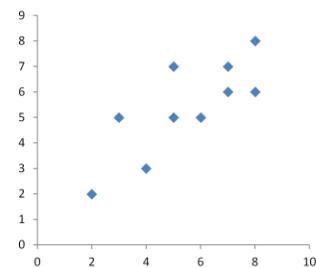
Teaching Students How to Spend Statistical Money to Purchase Effective Behavioral Models:  
Degrees-of-Freedom and Goodness-of-Fit as the Foundation of Psychological Science  
previously...

Joseph Lee Rodgers  
Lunch Bunch, Nov 17, 2011

Fisher Space



Variable Space



Joe spoke of representing data in variable-space and so called, Fisher-space which underlie the statistical thinking of R.A.Fisher

Variable space is the space we are most familiar with. This space places each variable in a separate dimension. This space is useful in describing how subjects differ in respect to variables.

Fisher space, on the other hand, places each subject in its own dimension. This space is useful in describing how variables differ in respect to subjects.

Thus, a variable in a sample is a  $n$ -th dimensional projection into reality observed by the sample of size  $n$ .

Why is this more convenient? It's easier to compare 2 things than 10 ( $N$ ) things.  
Reality is multidimensional, or consists of many perspectives.

However, the multiple dimensionality of fisher space is awkward and inconvenient, and that's why we build models – to simplify descriptions of reality, to make them usable by our minds.

Degrees of freedom were conceptualized as reduction in dimensionality of Fisher space.

Today, I would like to show that Fisher Space is not difficult to conceptualize and offers a (hopefully) easier understanding of the modeling process. But even if this

way of conceputalizing modeling is not “easier” on the face value, thinking about the same thing from different perspectives deepens our understanding of it.

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## Degrees of Freedom

- 1) piece of information.
- 2) a piece of statistical money,  
which can be spent or saved
- 3) The number of df's spent  
estimating a model is (one)  
measure of complexity.

Joe also mentioned other “ways of thinking” about degrees of freedom.

We find it useful to conceptualize the trade-off between fit and degrees of freedom in the allegory of a “purchase”

We say that : We “buy” the accuracy of the model “with” degrees of freedom. We “spend” degrees of freedom.

But I would like you to think about degrees of freedom as what we are after, what we are trying to buy, something that we want to have more of, the “object of desire”. And we pay for this “object” by incurring error. The more the error, the more dearly we pay.

This presentation is the attempt to contribute this line of thinking and to respond to and elaborate on Joe Rodgers’s talk.

# Understanding DF

- Do we ask the right question?

**What is  
“Degrees of Freedom”?**      vs.      **Why is it called  
“Degrees of Freedom”?**

Irresistible Assumptions:

- Terms must be self-explanatory
- The name should suggest the “best” way of understanding the concept
- If the term “Degrees of Freedom” were misleading the scientific community would have surely correct it

In stead of obsessing about the question “what IS” is, we better focus on “what does it do”?

# Understanding DF

What is  
“Degrees of Freedom”?

vs.

Why is it called  
“Degrees of Freedom”?

$Df = N - par$

# independent points



dimensionality

Not important

Does the words of the term determine how we think about it and what we expect it would describe?

Let's not try to explain the name, let's look at what it describes, and perhaps we could find a better name for the concept that guided students towards quicker and more solid understanding of this concept.

None of the answers have necessarily anything to do with “degrees” or “freedom”

# Understanding DF

*Perspective:* DF talks about some completed process.

- Therefore, meaningless outside the context of this process.

Consider:

Study	DF	N	Par
A	40	42	2
B	40	50	10
C	40	80	40

I want to look at DF as evaluation of certain change, a certain transformation

The meaning of DF is relative to the change it describes.

In study A we took 42 pieces of information and changed it into 2.  $DF=40$  tells us something about this change.

Study A – Regression with two parameters

Study B – Two by five ANOVA

Study C – Multilevel dyad intercept only model.

## The Process behind DF

- What change are we talking about?
  - What process/event does DF evaluate?
- Simplification of Reality
- Specifically:  
Simplification of the ***description*** of Reality

Don't ask "What is DF?" but ask "What HAPPENED that DF describe?"

What do we mean by "We gained a degree of freedom"? Isn't it the difference between something and something else? In that case we are talking about the same units that those "something" and "something else" were measuring.

"This simplification decreases the fidelity of our description by 14 degrees"

## Model = simplifies description of reality

- What model **IS**
  - Complexity (#pars, # statements)
  - Form ( $\hat{Y} = \beta_0 + \beta_1 X_1$ )
- What model **DOES** to description
  - Decreases **fidelity**
  - Decreases **accuracy**

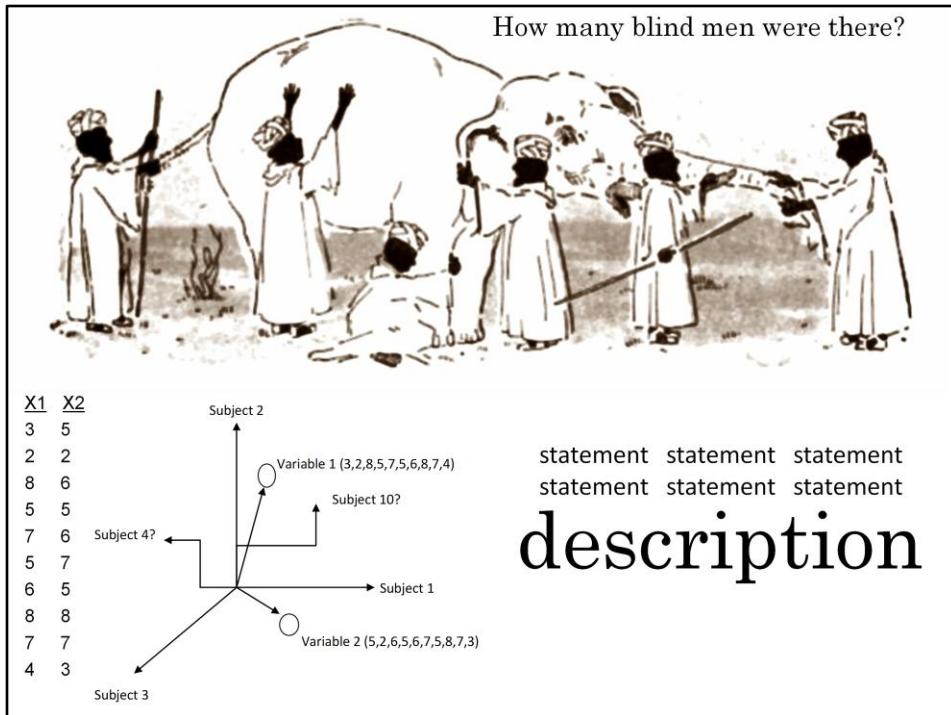
Model takes a simplified copy of the description.

Complexity – number of components, parameters, that model uses in producing the simplified description.

Form – how these components interact to provide a guess about the value of the criterion. Equation.

Models simplify descriptions. That's what they do. To simplify is to decrease the number of statements in the description. To reduce dimensionality of the variable projection into  $n^{\text{th}}$  dimensional Fisher-Space.

What model does to description = is the properties of the description of reality as given by the model.



To ask "What the dimensionality of variable projection into fisher-space?" is to ask "How many blind men were there?" That is, to ask from how many independent points of view the reality has been observed.

It's useful to think in Fisher-Space because one variable represents one description of reality.

Blind men tell us what they observe about reality. Blind men don't talk among each other— their observations are independent, therefore statements exist in their own dimensions.

The report of each is a statement about reality ( a piece of info)

Six blind men give us a 6-dimensional description of reality in respect to the variable (Elephant)

we call a collection of statements about reality a DESCRIPTION of reality. A set of reports from independent observes of reality we call a DESCRIPTION of reality

Description consists of individual statements (to yourself: data points).

Many statements will describe the reality with HI FIDELITY, the fewer the statements the lower the fidelity.

We define COMPLEXITY of description to be the number of statements that comprise it.

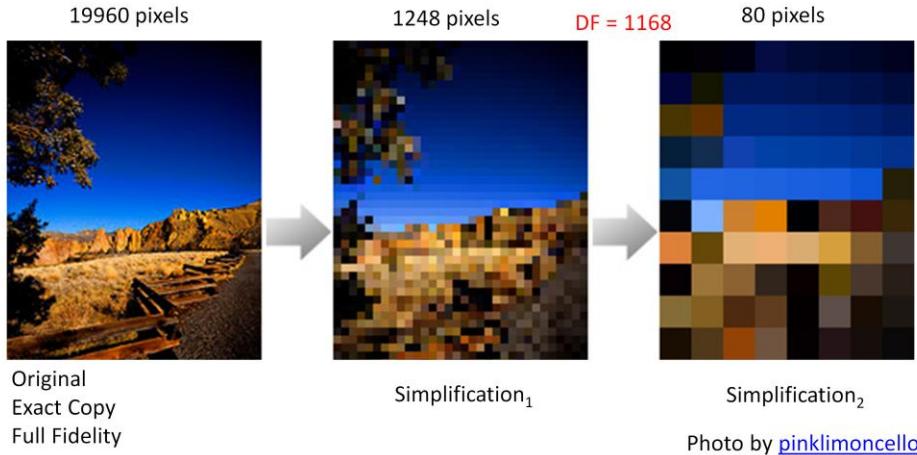
The less complex description are easiest to grasp and decide. KISS principle.

When we simplify a description of reality to suit our needs we reduce the complexity of the description, using fewer statements to describe it.

# Decrease of Fidelity

Change in the number of independent statements

Change in the number of pixels



(Image) Degree of Pixilation

how fine of a resolution do you want to have in our description

It might not make sense to make the picture of reality more fuzzy and ambiguous,  
But in determining whether reality is generally blue or generally brown, this  
simplification could be quite useful.

Especially when he-res images of reality is inconvenient or not available to us

We want our model to have the highest fidelity and the lowest complexity.

When we loose it, we call it Fidelity.

When we talk about how fully we describe reality we'll call the number of statement – Fidelity.

DF=14, the model made 14 less statements than the Full Fidelity description of reality.

DF = how many fewer individuals statements about reality our simplification has.

The change in the number of statements to describe the same reality we'll call a fidelity change, decrease of fidelity.

# Fidelity vs. Accuracy

## Description

Bob is 175 cm tall

Mark is 184 cm tall

Joe is 181 cm tall

DF = 0

SS = 0

Model = exact copy

Bob is 175 cm tall

Mark is 184 cm tall

Joe is 181 cm tall

## Model

No decrease in fidelity, but decrease in accuracy

# Fidelity vs. Accuracy

## Description

Bob is 175 cm tall

Mark is 184 cm tall

Joe is 181 cm tall

$$DF = 0$$

$$SS = 5^2 + 4^2 + 1^2 = 42$$

Bob is 179 cm tall

Mark is 189 cm tall

Joe is 182 cm tall

## Model

No decrease in fidelity, but decrease in accuracy

# Fidelity vs. Accuracy

## Description

Bob is 175 cm tall

Mark is 184 cm tall

Joe is 181 cm tall

Guys are 180 cm tall

$$DF = 2$$

$$SS = 5^2 + 4^2 + 1^2 = 42$$

## Model

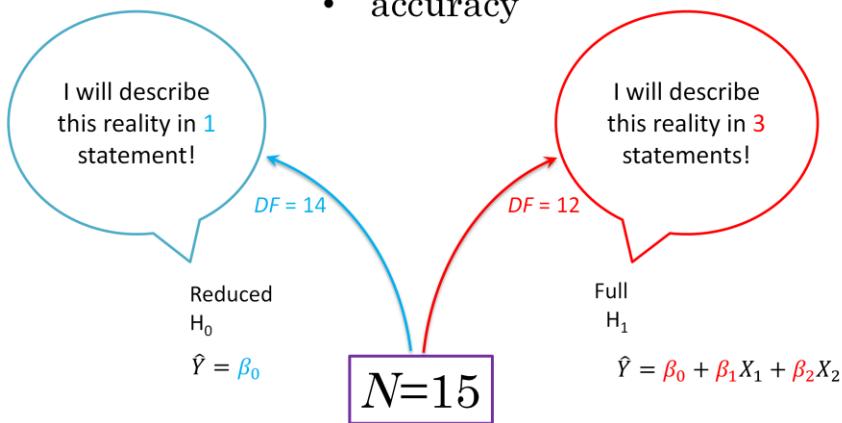
Decrease in fidelity and decrease in accuracy

Description gave you reality in 3 dimensions, but you collapsed it into a single dimension in your model.

So model is how many dimensions you allow in your simplification, and how you direct the projections from higher dimensions into them.

# Model Comparison

- fidelity
- accuracy



So when evaluating rival models we have to understand what they DO to the description of the same reality, in terms of FIDELITY and ACCURACY.

To compare models is to see how fidelity and accuracy relate to each other in each simplification. By nesting models we arrive at rigorous comparison.

The more the model simplifies, the greater the decrease of fidelity (in degrees).

You can think of DF as the amount of work that the model performs, the more, the more work the model does, the better the model is. It simplifies THIS MUCH.

Accuracy, on the other hand could be measured a sum of squared discrepancies between corresponding coordinates.

Source	df	SSQ	MS	F	p
G	2	122.1667	61.0833	29.3200	0.0001
Error	9	18.7500	2.0833		
Total	11	140.9167			

Traditional way to keep track of changes in fidelity and accuracy

Table 1  
*Results From an Age (2) Preposition Type (5)*  
*Mixed-Model Analysis of Variance on Hypothetical Ipsative Scores*

Source	df	SS	MS	F
Between subjects	19	0	—	—
Age	1	0	0	0
Residual	18	0	0	0
Within subjects	80	5.11	—	—
Preposition type	4	4.02	1.010	66.93*
Age Preposition Type	4	0.01	0.002	0.14
Residual	72	1.08	0.015	—
Total	99	5.11	—	—

\* Significant at  $p < .001$ .

## Rodger's Slide

My comment: This supposed to be an “accounting of modeling” easier, this is a tool to keep track of simplifications, to compare them.

The goal of this talk : to present a new way of thinking about degrees of freedom (visual accounting)

This is an actuarial tool created to keep track of the Fisher Space as it undergoes reduction of complexity under the instruction of the model we test.

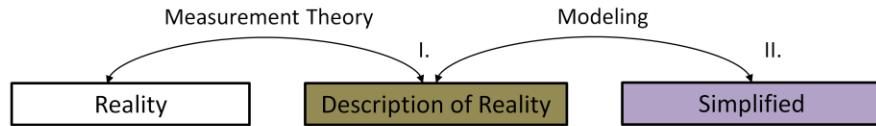
## Today

- I. New way to think about DF as Decrease of Fidelity
- II. Model comparison with new vocabulary
- III. Visual accounting system for model comparison

So in some way, this is not theoretical innovation, but merely a managerial project.

# The Process Behind DF

*simplifying the description of reality*



DF tells us something about what happened when we took a description of reality and CHANGED it somehow to better suit our needs ( clarity, particular question, etc).

So first, scientist must DESCRIBE reality in some way, and then make this description meaningful. It's a two-stage process.

Description of Reality

Simplified

So first, scientist must DESCRIBE reality in some way, and then make this description meaningful. It's a two-stage process.

## Description of Reality

### Triathlon Study

Preparing for Triathlon - What shoes to buy?

Three type of shoes

Which type is easier to put on?

Easier = Faster in seconds

Let's ask some people to put on shoes and describe the reality

Nike	Reebok	Adidas
12	10	8
9	8	7
8	5	5
7	4	3
4	3	2

#### **Describing the reality we witnessed:**

Methods +

Subject #1 tied Nike shoe in 8 seconds

Subject #2 tied Nike shoe in 4 seconds

Subject #3 tied Nike shoe A in 7 seconds

...

The (even more detailed) description in the Methods section references to the reality that was common for all the observations, so no need to repeat for all. Although, those would be the most accurate depiction of reality.

**Verbal**

Subject #1 tied Nike shoe in 8 seconds  
Subject #2 tied Nike shoe in 12 seconds  
Subject #3 tied Nike shoe in 4 seconds  
Subject #4 tied Nike shoe in 9 seconds  
Subject #5 tied Nike shoe in 7 seconds  
Subject #1 tied Reebok shoe in 3 seconds  
Subject #2 tied Reebok shoe in 8 seconds  
Subject #3 tied Reebok shoe in 10 seconds  
Subject #4 tied Reebok shoe in 4 seconds  
Subject #5 tied Reebok shoe in 5 seconds  
Subject #1 tied Adidas shoe in 8 seconds  
Subject #2 tied Adidas shoe in 7 seconds  
Subject #3 tied Adidas shoe in 3 seconds  
Subject #4 tied Adidas shoe in 2 seconds  
Subject #5 tied Adidas shoe in 5 seconds

This is the most precise description of reality possible.

Each line represents an individual statement about reality.

There are 15 statements about reality that we made in this description.

Let see if we can present the same 15 statements in a way that eases our perception

Subject #1 tied Nike shoe in 8 seconds  
Subject #2 tied Nike shoe in 12 seconds  
Subject #3 tied Nike shoe in 4 seconds  
Subject #4 tied Nike shoe in 9 seconds  
Subject #5 tied Nike shoe in 7 seconds  
Subject #1 tied Reebok shoe in 3 seconds  
Subject #2 tied Reebok shoe in 8 seconds  
Subject #3 tied Reebok shoe in 10 seconds  
Subject #4 tied Reebok shoe in 4 seconds  
Subject #5 tied Reebok shoe in 5 seconds  
Subject #1 tied Adidas shoe in 8 seconds  
Subject #2 tied Adidas shoe in 7 seconds  
Subject #3 tied Adidas shoe in 3 seconds  
Subject #4 tied Adidas shoe in 2 seconds  
Subject #5 tied Adidas shoe in 5 seconds

We are not changing the informational content, we merely changing the ease of perception. But the statements carry the same information.

Description of Reality

Hypothetical Study

1	Nike	8
2	Nike	12
3	Nike	4
4	Nike	9
5	Nike	7
1	Reebok	3
2	Reebok	8
3	Reebok	10
4	Reebok	4
5	Reebok	5
1	Adidas	8
2	Adidas	7
3	Adidas	3
4	Adidas	2
5	Adidas	5

Transitional Slide

Description of Reality

Hypothetical Study



1



2



3



4



5

Nike	8
Nike	12
Nike	4
Nike	9
Nike	7
Reebok	3
Reebok	8
Reebok	10
Reebok	4
Reebok	5
Adidas	8
Adidas	7
Adidas	3
Adidas	2
Adidas	5

Transitional Slide

Description of Reality

Hypothetical Study



Nike



Reebok



Adidas



1



2



3



4



5

8

12

4

9

7

3

8

10

4

5

8

7

3

2

5

Transitional Slide

Description of Reality

Hypothetical Study



Nike



Reebok



Adidas



1

12



2

9

10

8



3

8

5

5



4

7

4

3



5

4

3

2

## Description of Reality

### Verbal

Subject #1 tied Nike shoe in 8 seconds  
Subject #2 tied Nike shoe in 4 seconds  
Subject #3 tied Nike shoe A in 7 seconds  
Subject #4 tied Nike shoe A in 5 seconds  
Subject #5 tied Nike shoe in 6 seconds  
Subject #1 tied Reebok shoe in 8 seconds  
Subject #2 tied Reebok shoe in 4 seconds  
Subject #3 tied Reebok shoe A in 7 seconds  
Subject #4 tied Reebok shoe A in 5 seconds  
Subject #5 tied Reebok shoe in 6 seconds  
Subject #1 tied Adidas shoe in 8 seconds  
Subject #2 tied Adidas shoe in 4 seconds  
Subject #3 tied Adidas shoe A in 7 seconds  
Subject #4 tied Adidas shoe A in 5 seconds  
Subject #5 tied Adidas shoe in 6 seconds

### Numeric

Sub	N	R	A
1	12	10	8
2	9	8	7
3	8	5	5
4	7	4	3
5	4	3	2

Notice that we still depict the same 15 units of information.

We preserved only the relevant information, encoding info that pertains to ALL our subjects into representational conventions and rules.

## Description of Reality

### Verbal

Subject #1 tied Nike shoe in 8 seconds  
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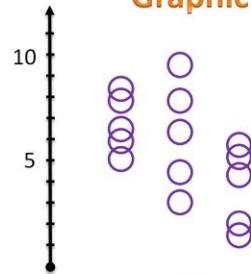


### Numeric

Sub	N	R	A
1	12	10	8
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### Graphic

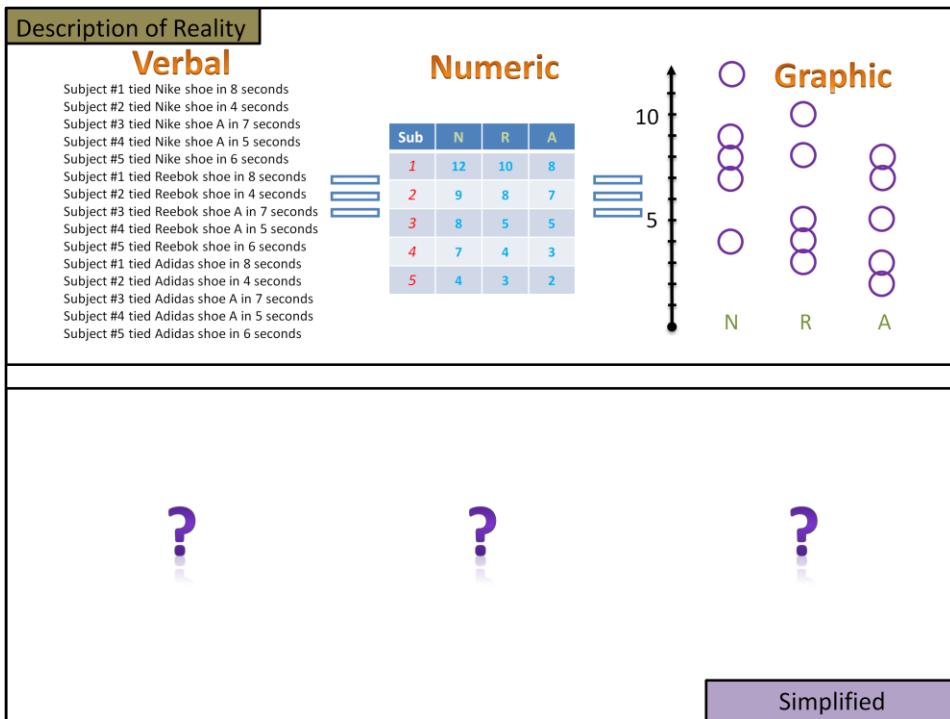


The amount of information in all three expressions is IDENTICAL.  
We used the same number of statements to describe the same reality

If we adopt yet other representational conventions we can depict the same 15 pieces of information graphically.

We will measure information in STATEMENTS about reality.  
The more statements you have about reality the more information you possess.

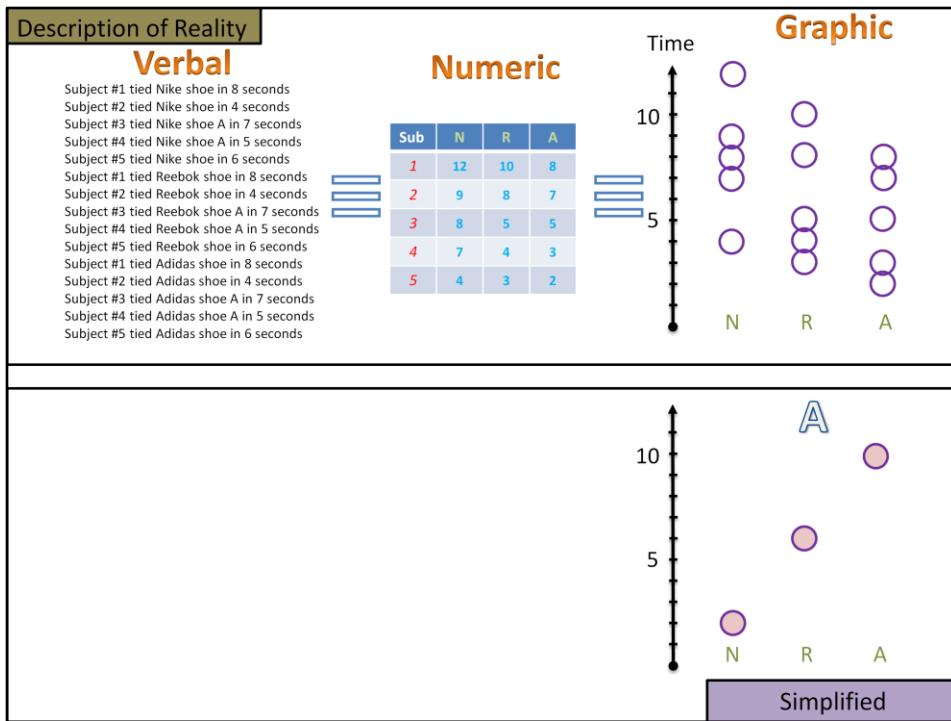
Each of these expressions have different values according to the different purposes.  
But notice that we apply more and more creativity from left to right.  
Practicing Christians should understand this concept better than others ☺



These three sets of 15 statements about witnessed reality are equivalent.  
 It implies that having access to one of them, we are capable of recreating the other two provided we know the rules and conventions of transformations (notation)

To do any sort of reasoning with these statements to see whether they affirm or disproves our hypotheses, we want to keep the number of statements of reasonable size.

To “describe” means to relay the essence. There can’t be objective description. Moreover, when we make a copy of a description, we suit the accuracy of this copy (fidelity) to our research needs. By simplifying, copying the original of the reality we are capturing the essence as we see it and leaving out the cluttering details that rival for our attention.



(NO VERTICAL CORRESPONDENCE)

If we can find a simpler expression of reality in one(Graphic, for example) , we KNOW that there exist representations the other two ( Verbal, Numeric)  
Let's find a simpler way to represent the same information graphically

Add (C)

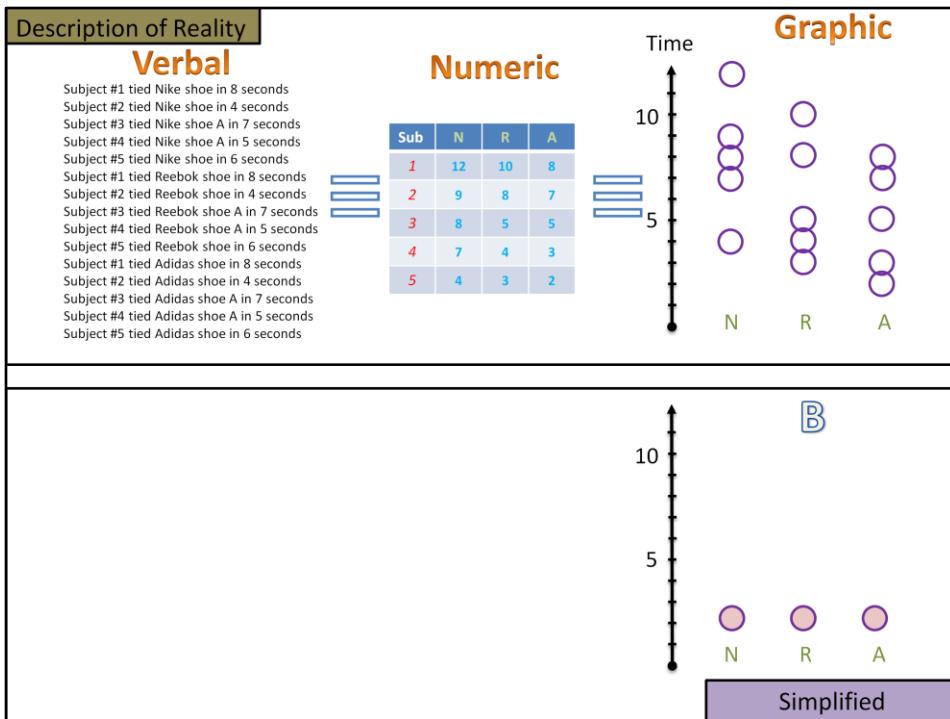
Fair question to ask, does it even talk about the same reality?

(C) is much better representation than (A) or (B). This proves that some representations would be bad one, and some would be good.

This implies that we can measure and compare any two of them using an objective criteria, and based on that criteria choose in favor of one of them, as the more

But even the “best” of possible representation might not cut it. We can compute the “best” but whether it’s an accurate representation of reality is a tougher choice.

Perhaps, I could make you say “Yeah, but....” now it becomes the question of how big of a “but” it is. The claim with the smallest “but” wins.



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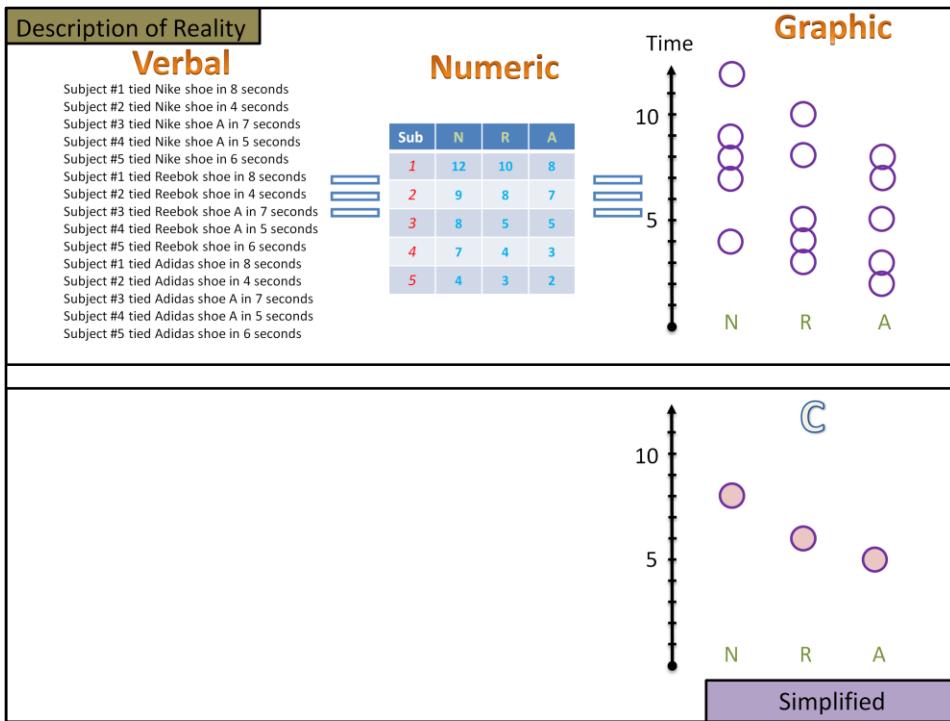
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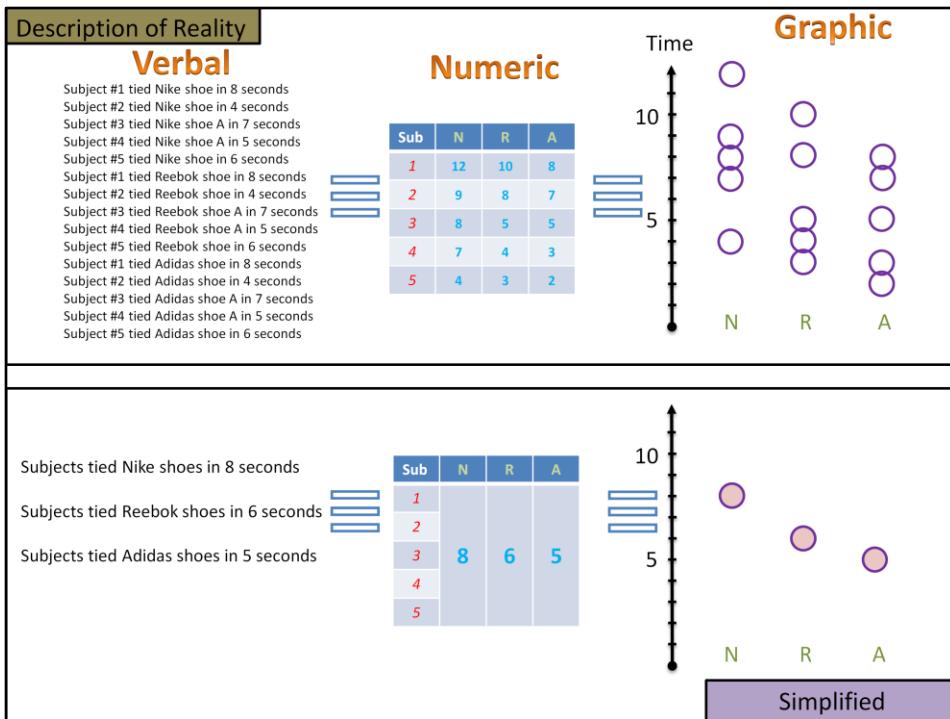
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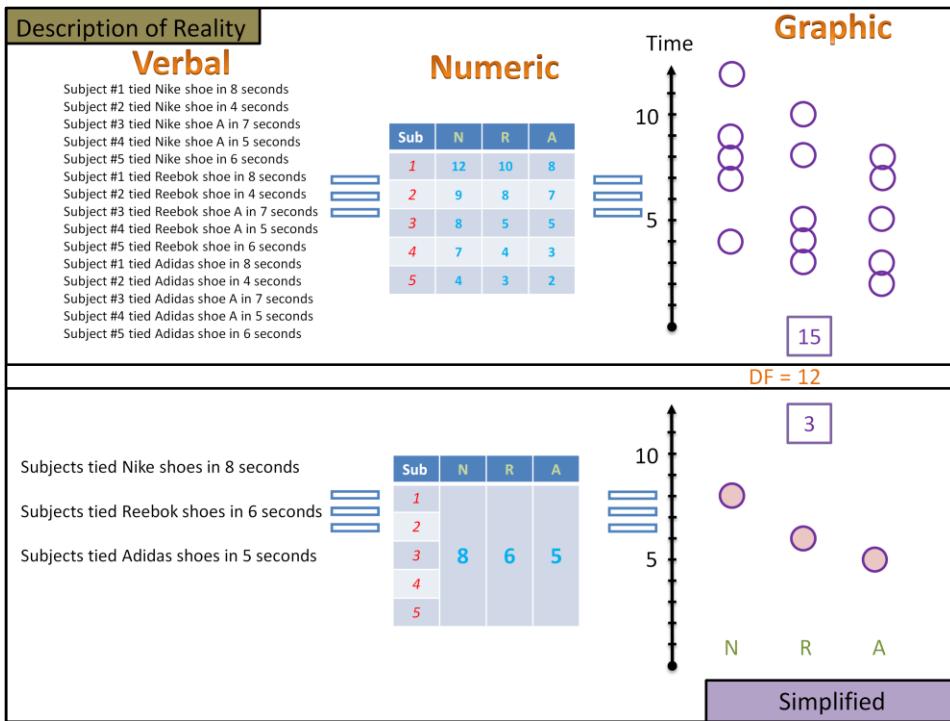
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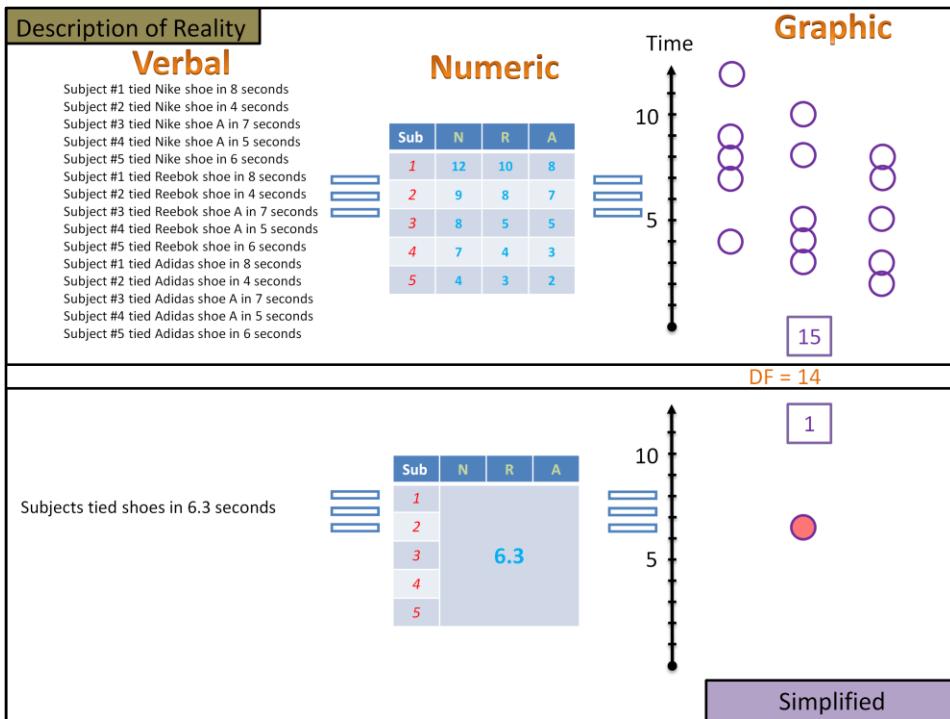
We showed that graphic simplification is a reasonable simplification that does justice to the reality, albeit not without a certain distortion of it.

The equivalence of Graphic Simplified (S) and Numeric (S) has also been demonstrated. We simply applied the representational convention from the top part.

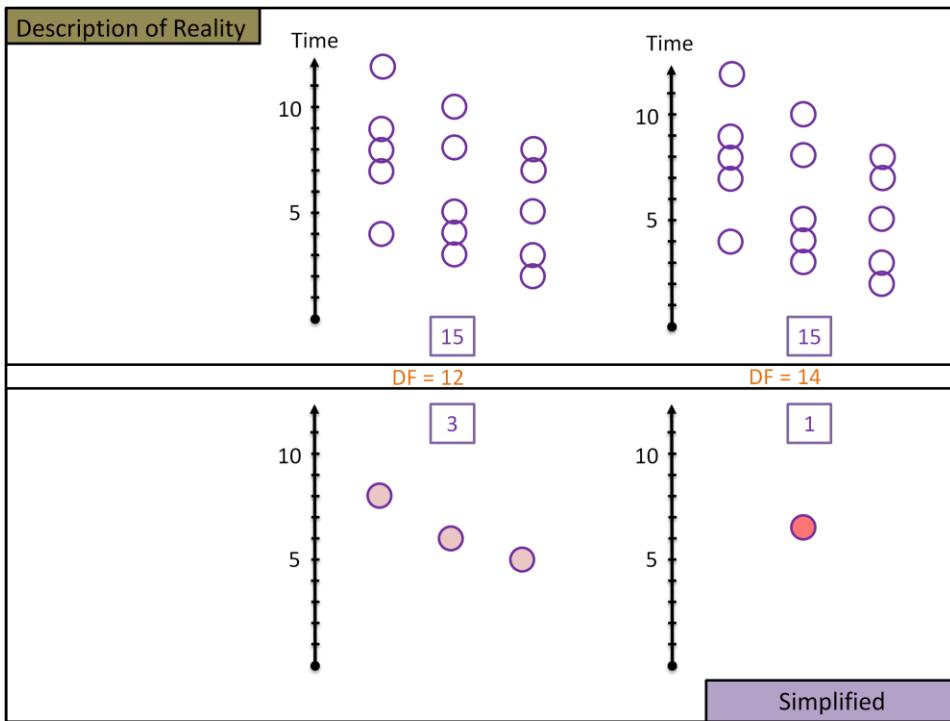
Hence if 1) Graphic (S) is a simpler ( and reasonable) version of Graphic and 2) Numeric(S) is equivalent to Graphic (S) it logically follows that 3) Numeric(S) is a reasonable simplification of Unsimplicated Numeric depiction of reality.



In this particular case...

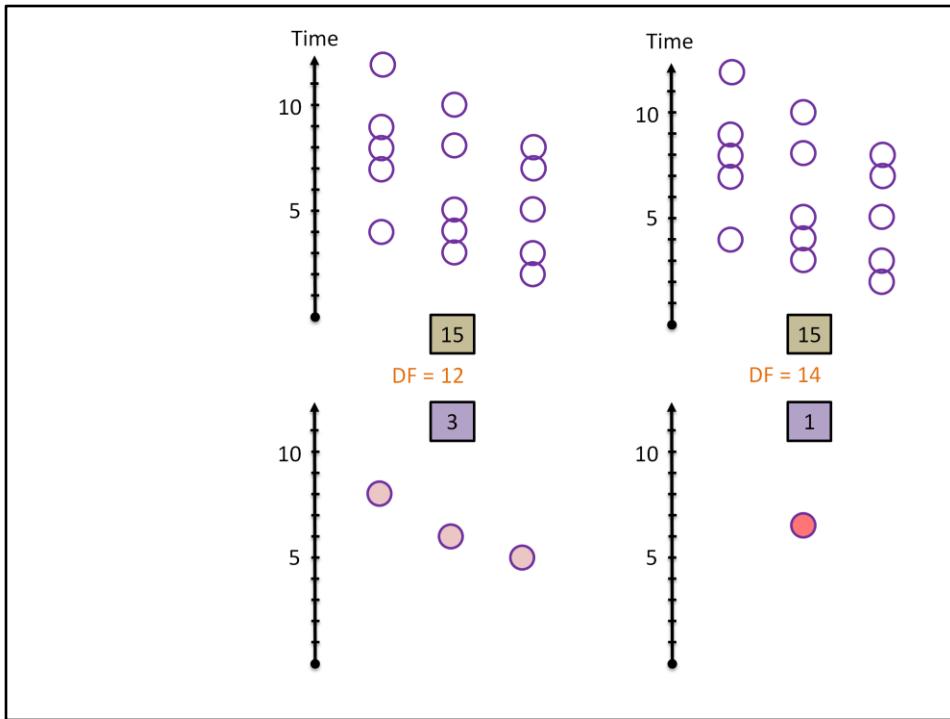


Back and forth slides: First



Comparing Two simplification in terms of fidelity. Next we need to see what inaccuracy was associated with each of these simplifications. But first,

15-3 simplification also can be achieve by formulating a multiple regression with two predictors.



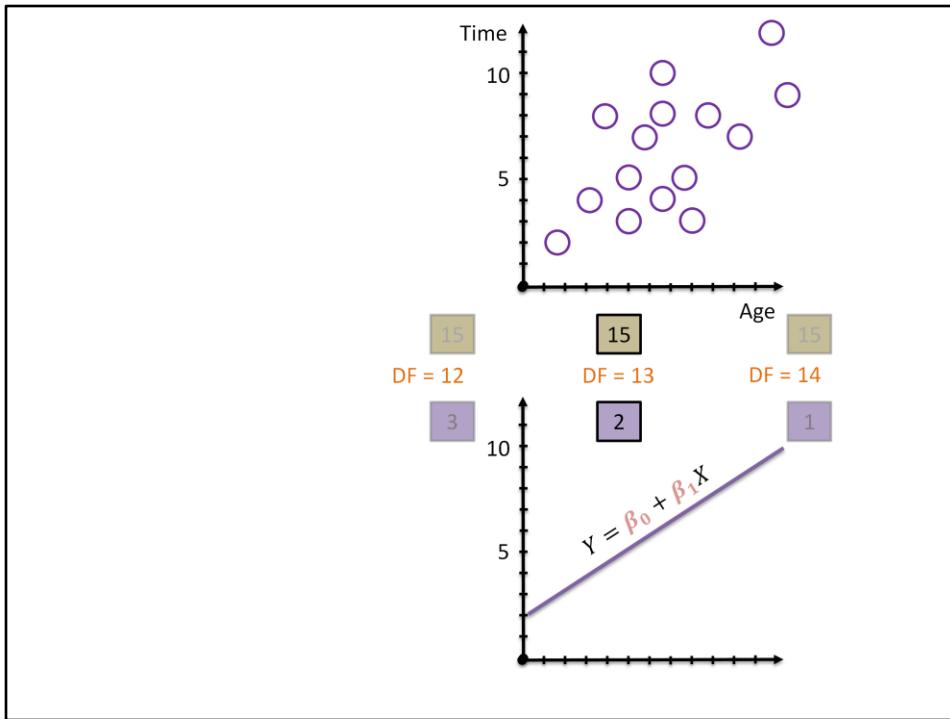
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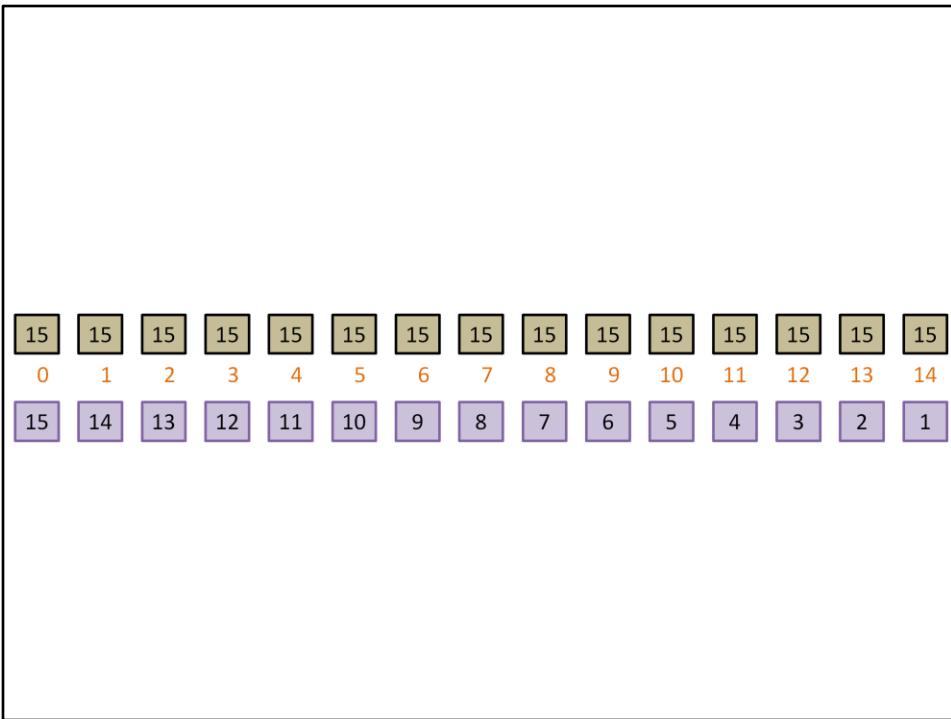
We can imagine a different kind of simplification, and on a different variable (property). For instance their age might influence how fast they tie the shoe.

Fidelity – the number of independent statements that describe reality. Notice, I say nothing of accuracy. Accuracy is a different property of a simplification. Comparing change of rate



We can think of a way to simplify the reality with respect to VARIABLE Experience:  $y = a + bx$

A model that represented 15 statements about reality as 2 statements, reduced the fidelity of the statement by 13 degrees, and accumulated SSE inaccuracy.

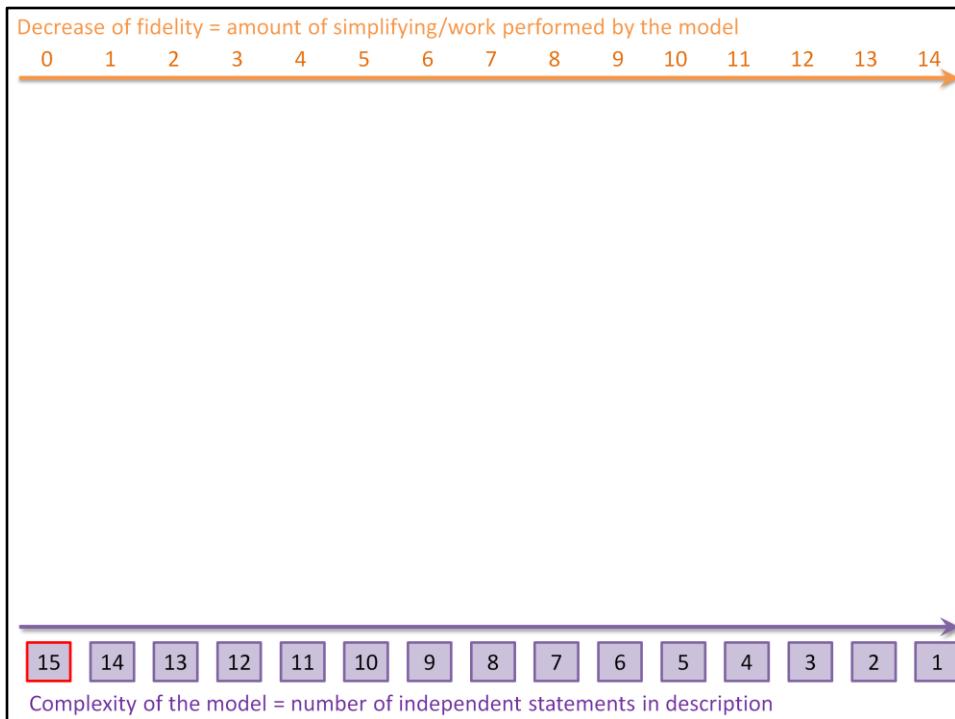


We can imagine a continuum of such simplifications

It can be a nested continuum. Or it could be composed of non-unique simplification with the same loss of fidelity.

For every simplification, we make a copy of reality. We choose what features are of interest and simplify in respect to them. In the process of we lose fidelity of that description. The notion of degrees of (lost) fidelity, quantifies the process of simplification.

There are total of 15 degrees of fidelity in this case. One degree is one observation of reality. The more observations we have, the “truer” the picture of reality, the greater the fidelity of our description of it. Every degree, every data element is the evidence that supports our claim is true ( has fidelity), the greater the fidelity, the more substantiated and “true” the claim is.



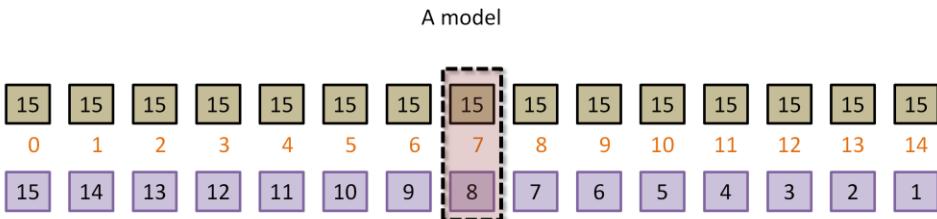
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It can be a nested continuum. Or it could be composed of non-unique simplification with the same loss of fidelity.

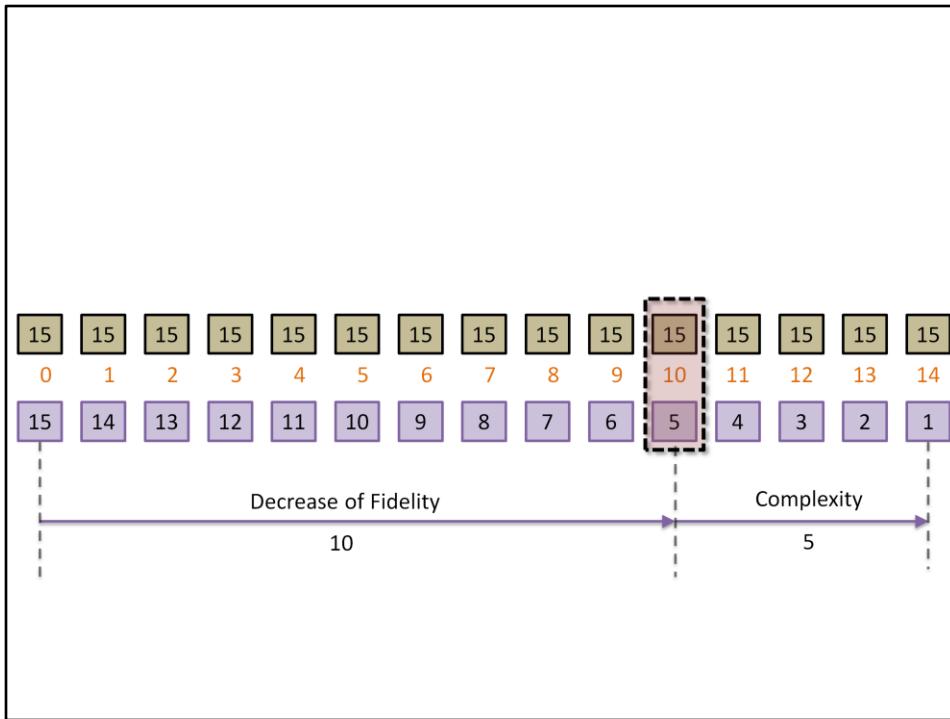
For every simplification, we make a copy of reality. We choose what features are of interest and simplify in respect to them. In the process of we lose fidelity of that description. The notion of degrees of (lost) fidelity, quantifies the process of simplification.

There are total of 15 degrees of fidelity in this case. One degree is one observation of reality. The more observations we have, the “truer” the picture of reality, the greater the fidelity of our description of it. Every degree, every data element is the evidence that supports our claim is true ( has fidelity), the greater the fidelity, the more substantiated and “true” the claim is.

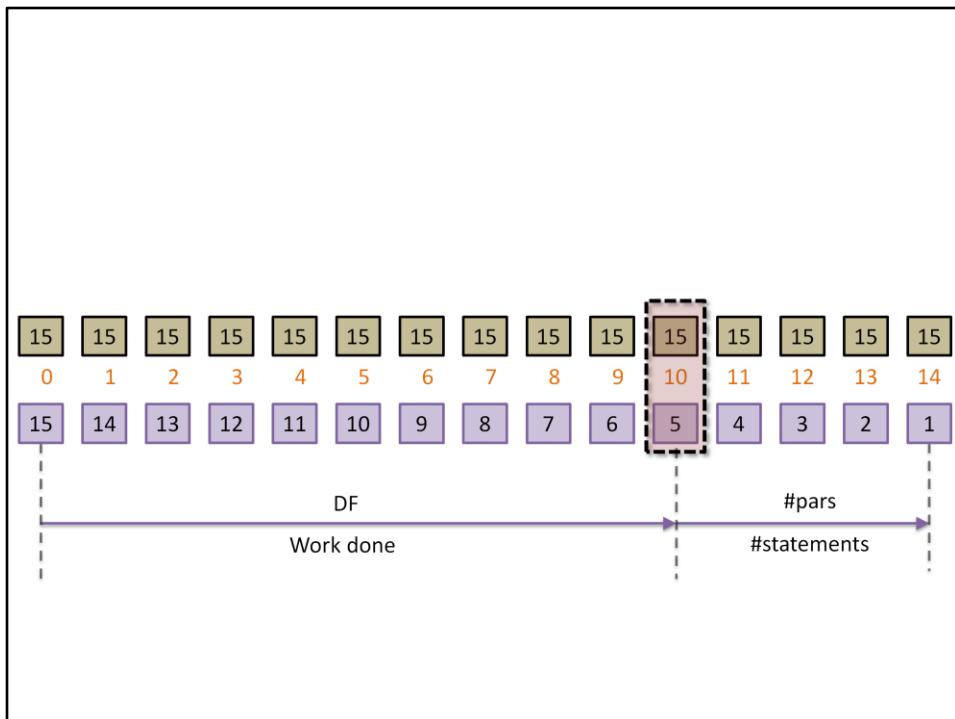
Decrease of fidelity = amount of simplifying/work performed by the model

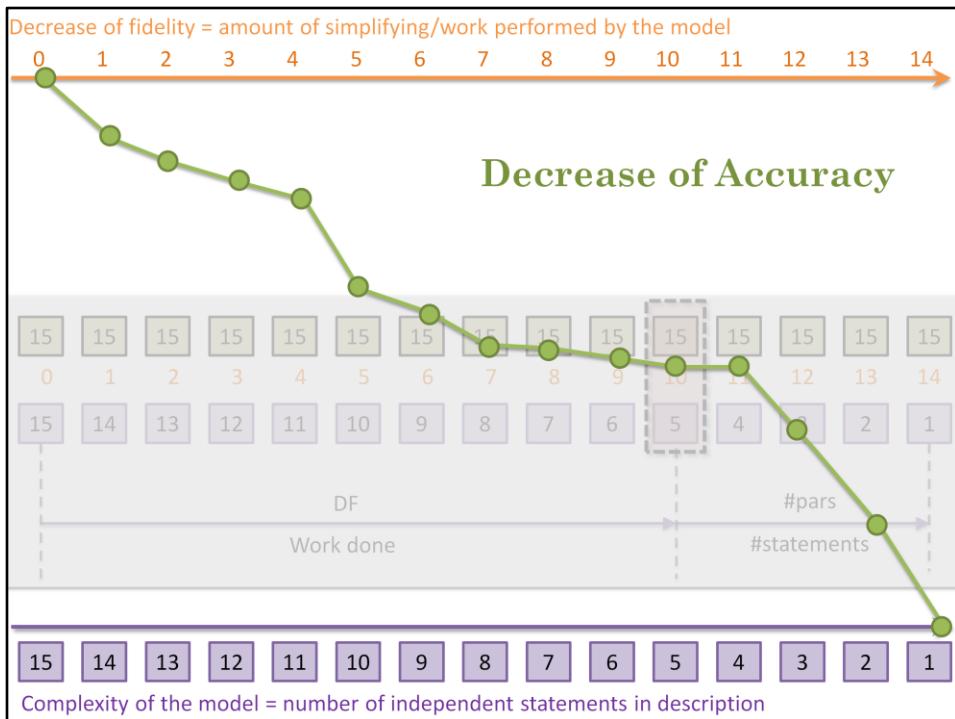


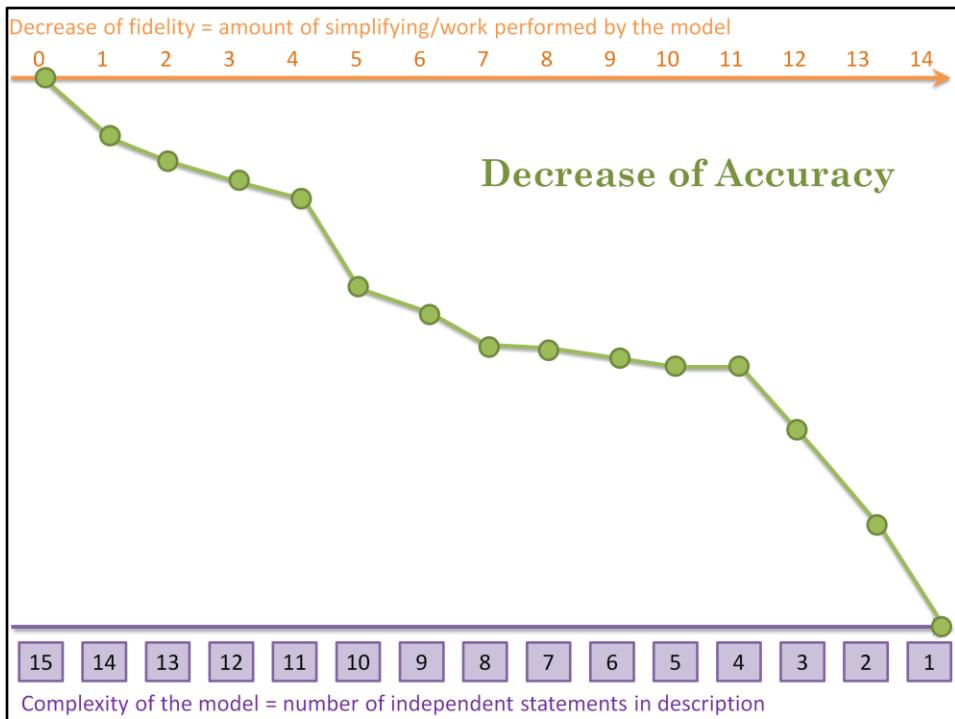
Complexity of the model = number of independent statements in description



We measure parsimony in DF, so it is inaccurate to equate parsimony to model complexity, they are inverse notions.





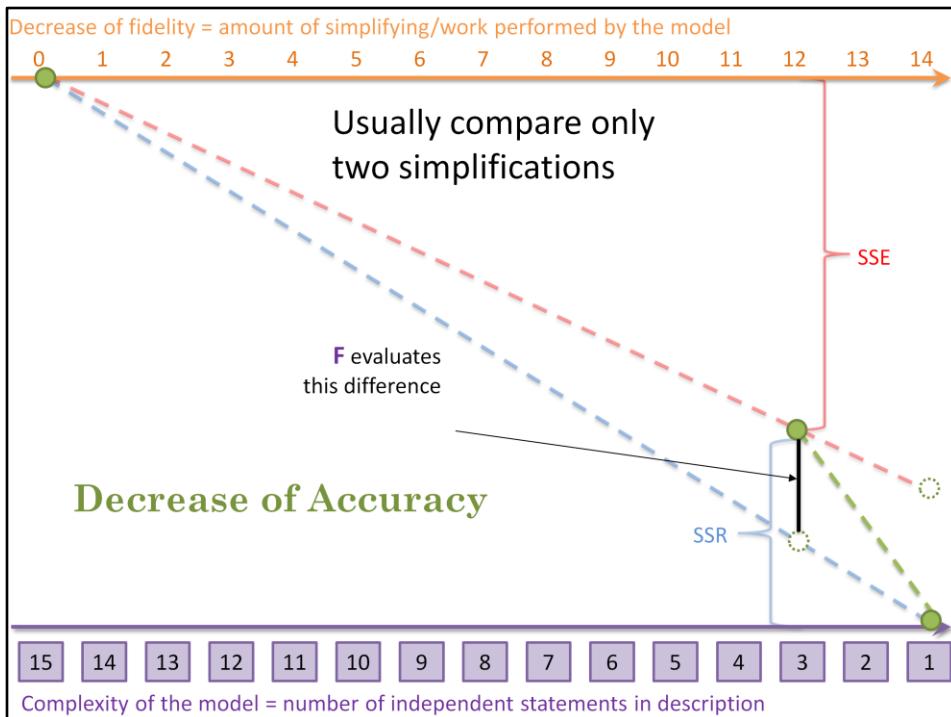


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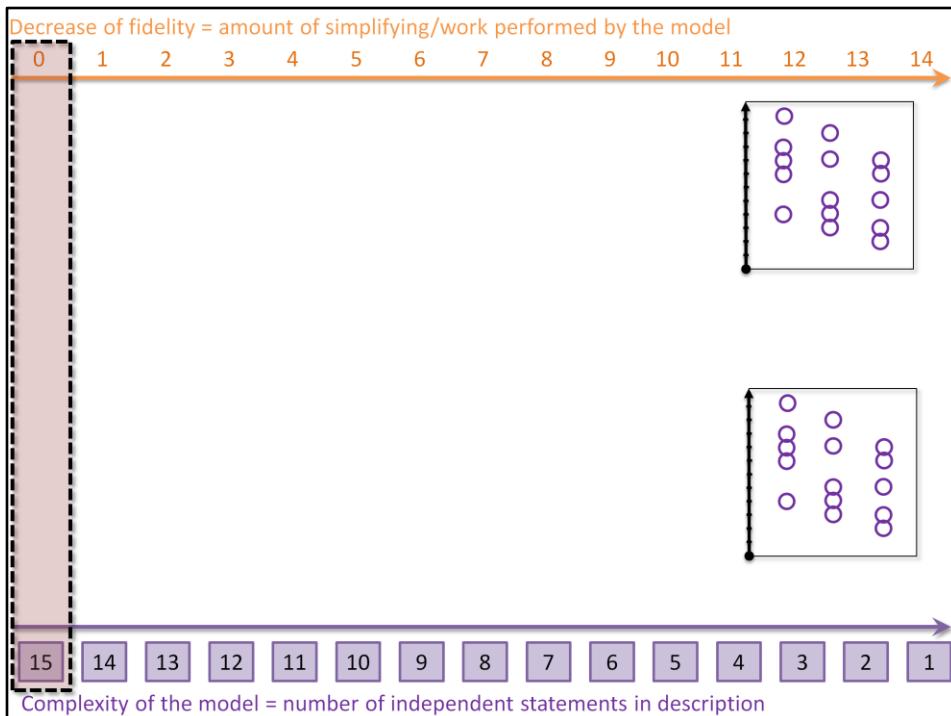
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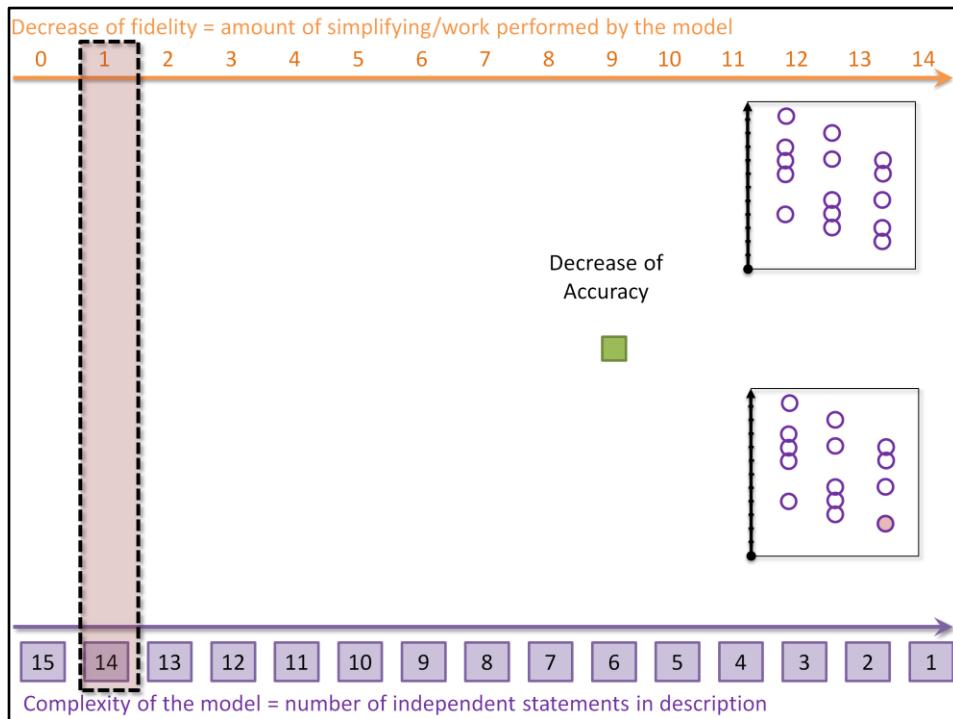
The null model (blue) assumes the constant rate of change for previous simplifications, calculates the average.

For every simplification, we make a copy of reality. We choose what features are of interest and simplify in respect to them. In the process of we lose fidelity of that description. The notion of degrees of (lost) fidelity, quantifies the process of simplification.

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When we make an exact copy of the reality our misrepresentation is NULL.



If we decide that these two scores are sufficiently similar and could be represented by a single value for our convenience, we are decreasing the fidelity of our description of reality by one degree of fidelity.

When fidelity is decreased the inaccuracy of representation increases. Less fidelity = more error.

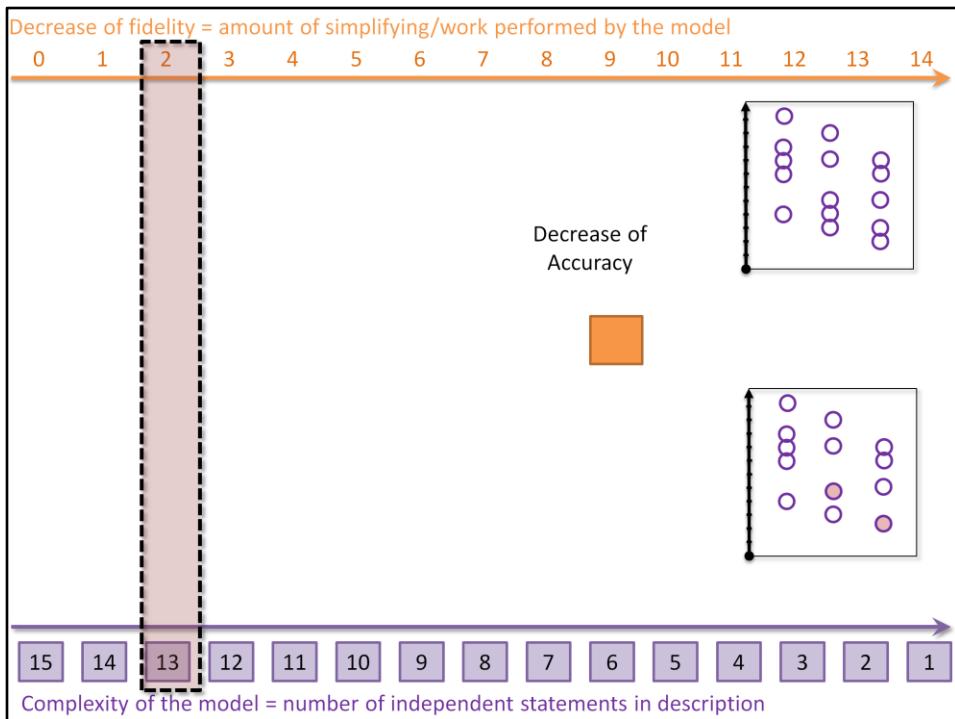
We measure the loss in accuracy by the sum of squared differences between observed and simplified values, literally measuring the simplification.

The fidelity was decreased by 1 (degree), while Accuracy was decreased by (Area of the Square).

The cost of giving up 1 degree of fidelity is (Area of the Square)

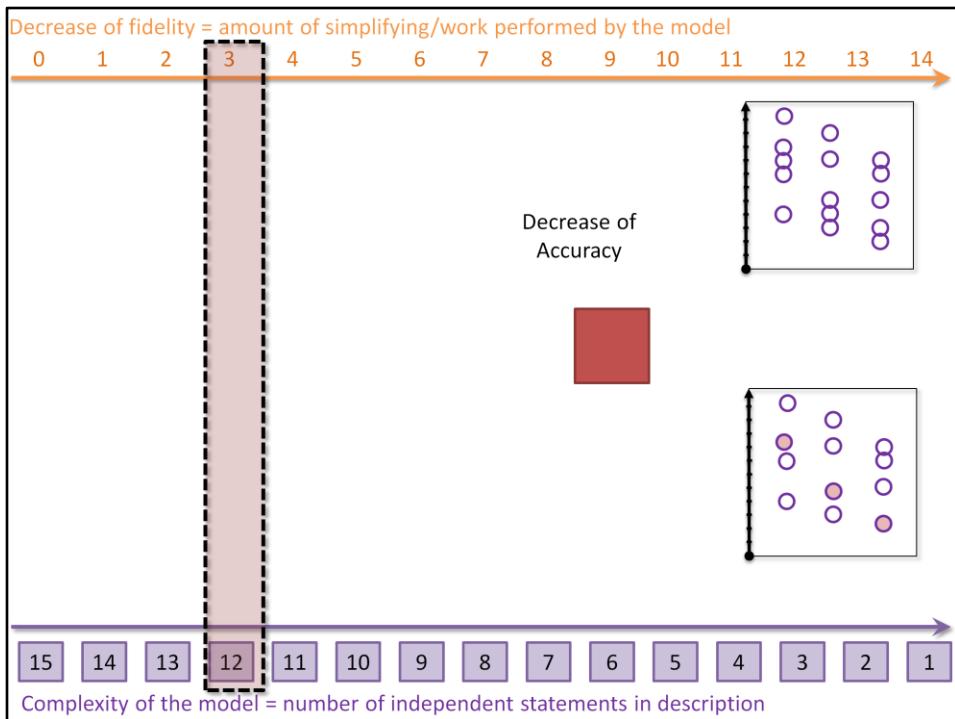
It doesn't seem to make much sense to simplify this reality like that, 15 to 14. However some simplification might be more useful than others. What kind of questions a particular simplification can answer is the domain of research design, which relies on quantitative evaluation of simplifications (hypotheses, models) to answer questions

What simplification possible next?



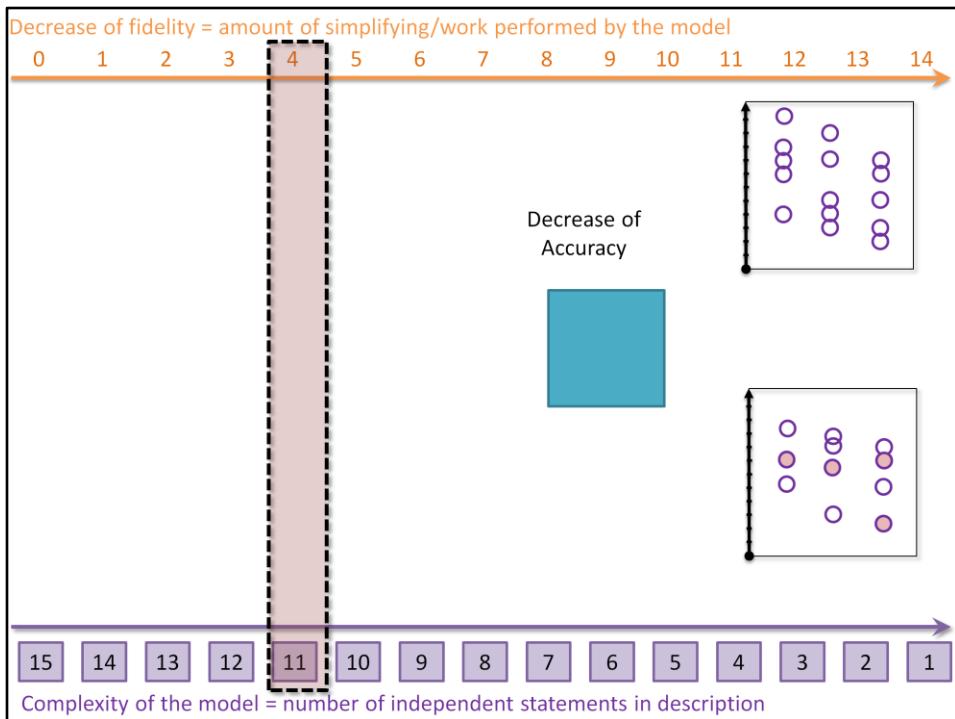
While, not every simplification would be sensible, we must recognize the existence of this simplification continuum.

Any simplification on this continuum would be possible, but not necessarily sensible. But we know that if we make it more simple the “misfit”, error, inaccuracy would increase as well.



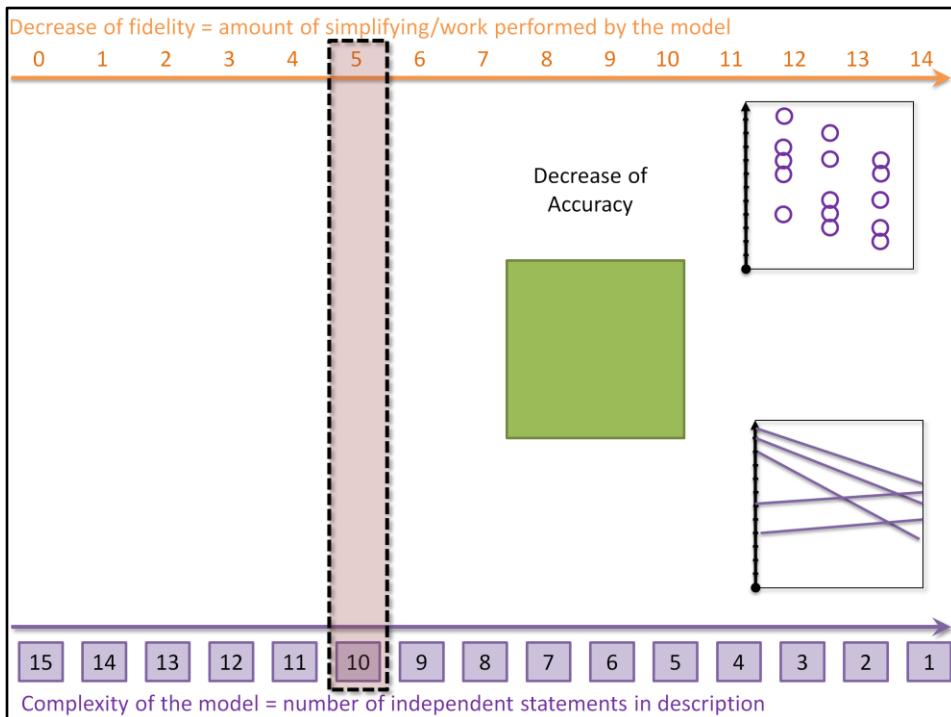
Fidelity was decreased by 3 (degrees), while the Inaccuracy increased by the Area of Square.

The cost of reducing the complexity of description by 3 degrees of fidelity is (Area of Square). Each degree cost MSR.



While, not every simplification would be sensible, we must recognize the existence of this simplification continuum.

Any simplification on this continuum would be possible, but not necessarily sensible. But we know that if we make it more simple the “misfit”, error, inaccuracy would increase as well.



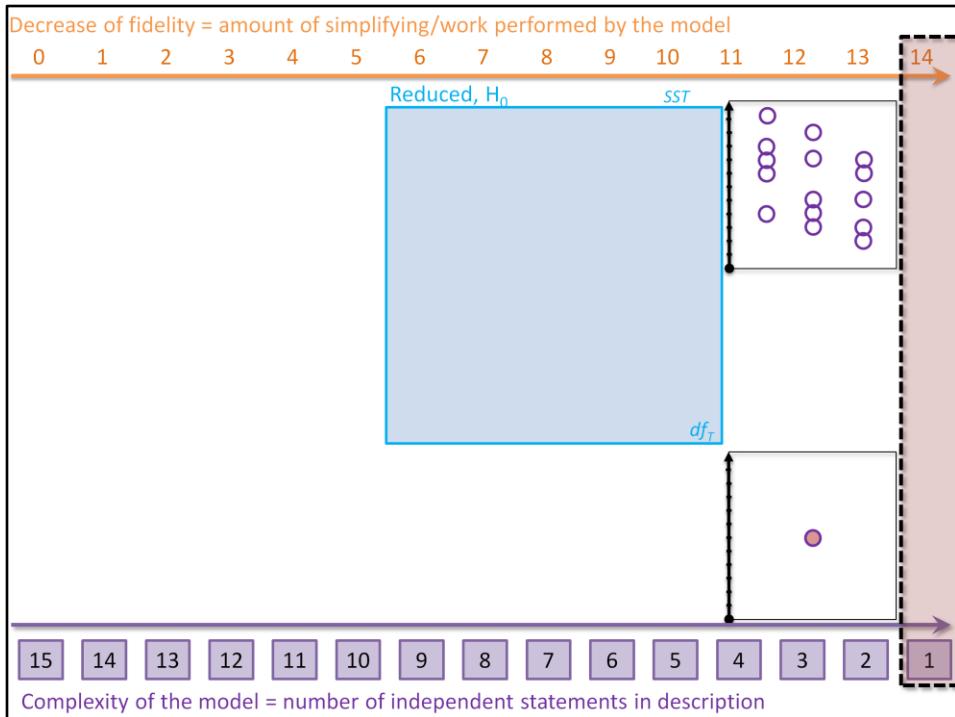
Here, one particular simplification would make sense.

Randomized Block Design

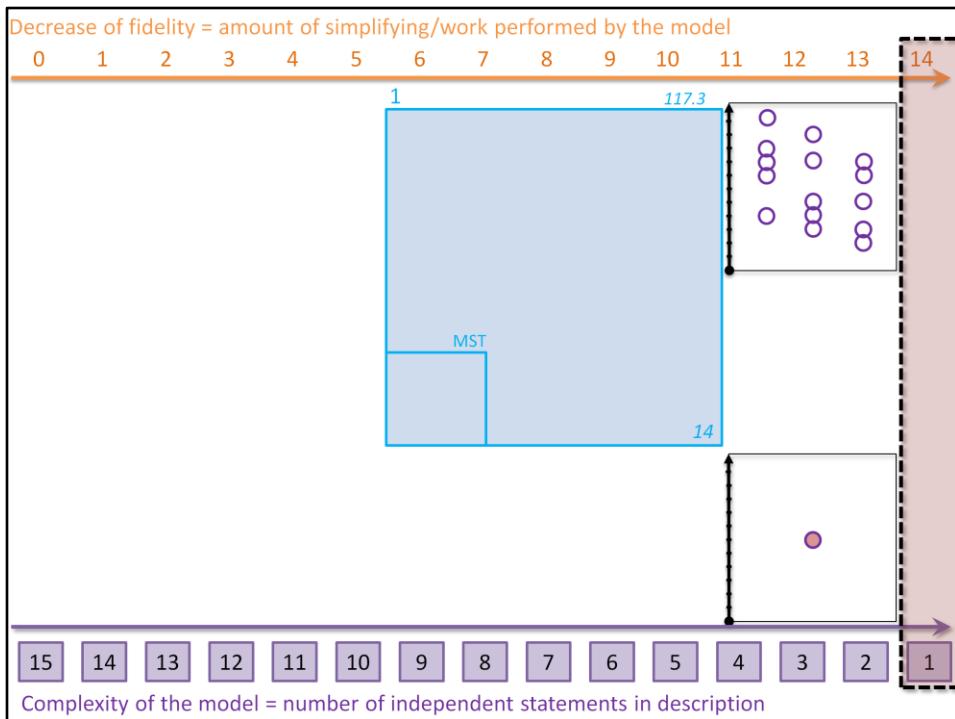
Or

Multilevel Regression

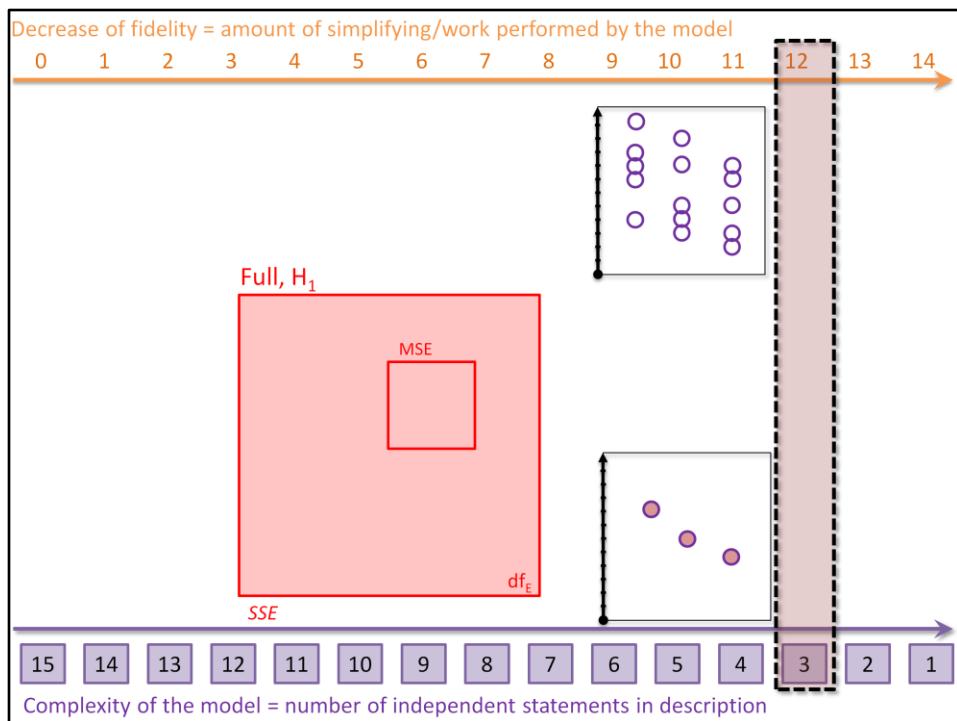
Out of five degrees (units) of fidelity that we lost in this simplification, which one contributed the most to the loss of accuracy? Theoretically, we can find out. But the loss of accuracy varies too, so we use an average, average squared error incurred by a particular simplification.

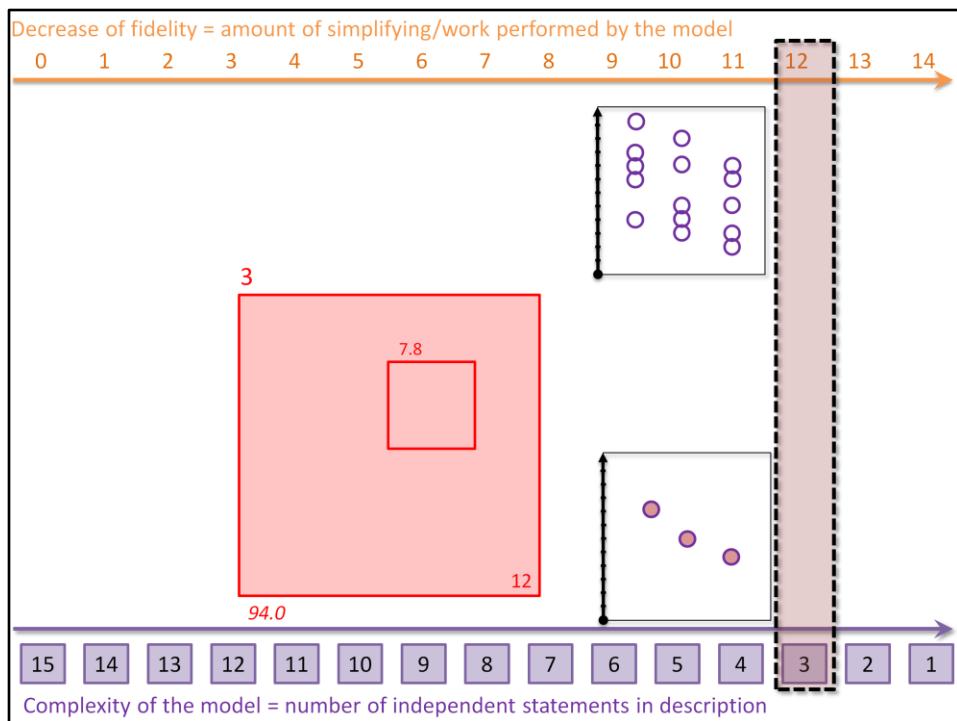


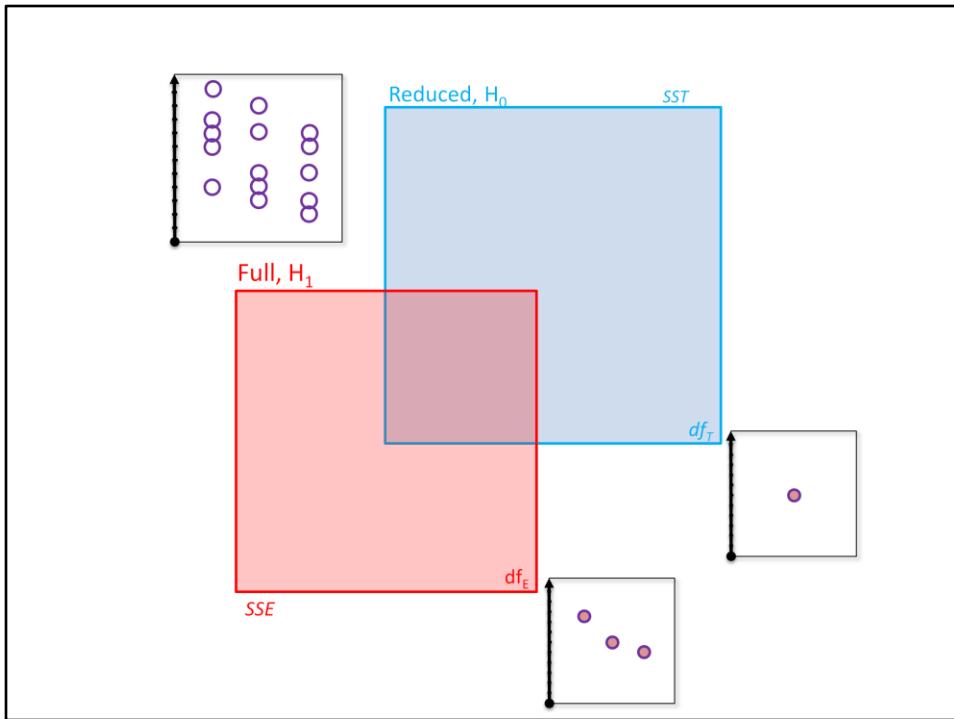
We took description with fidelity of 15, expressed it as 1 , incurred SST inaccuracy, and lost MST accuracy on average for 14 degrees of fidelity lost in simplification.



We took description with fidelity of 15, expressed it as 1 , incurred 117.3inaccuracy, and lost 8.3 accuracy on average for 14 degrees of fidelity lost in simplification.



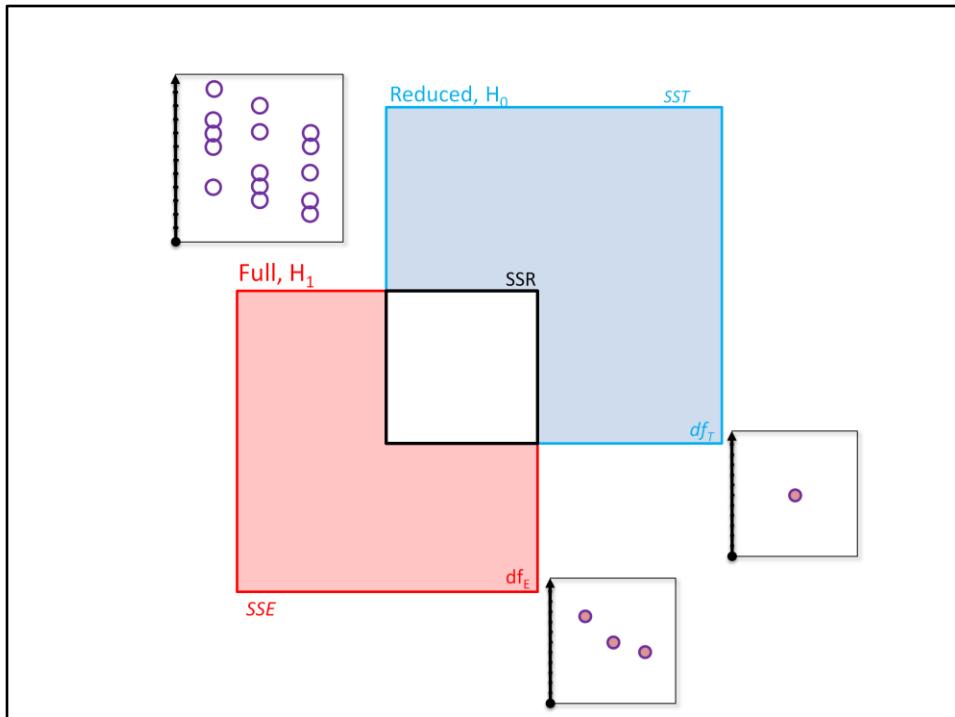




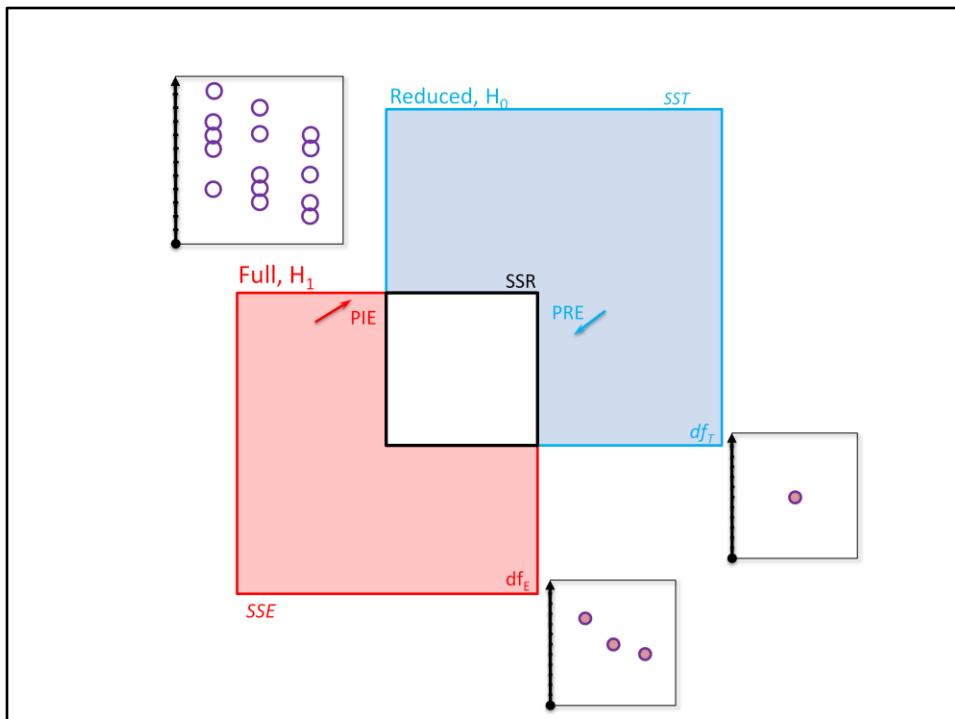
Any hypothesis testing or model evaluation would involve comparing the values that describe this change in accuracy and fidelity.

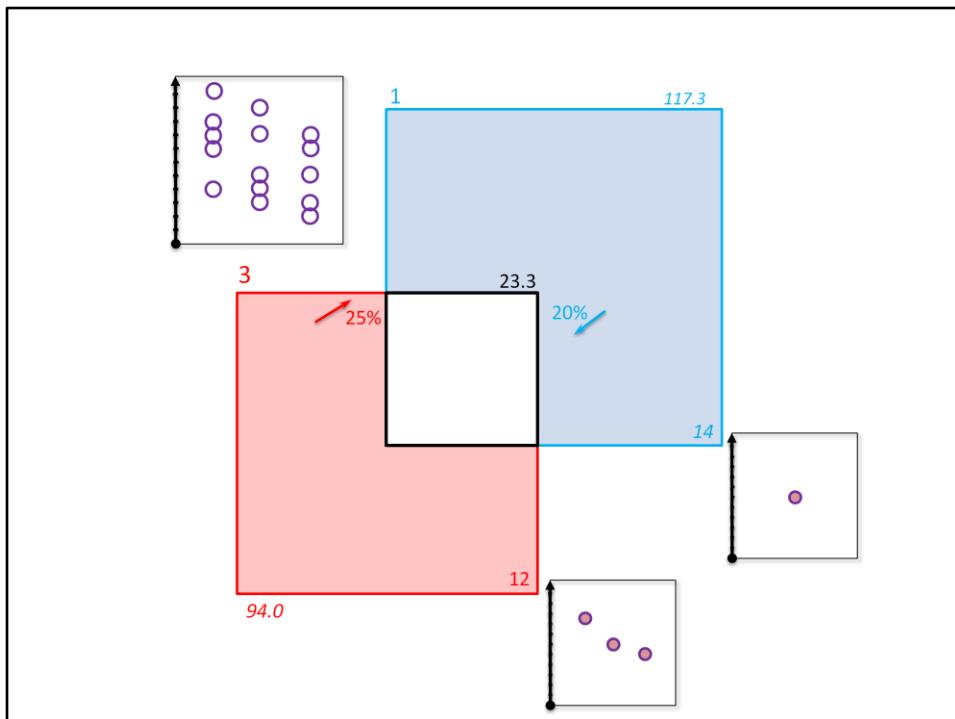
In other words, our analysis of data comes down to analyzing and making sense from the difference of these two inaccuracies.

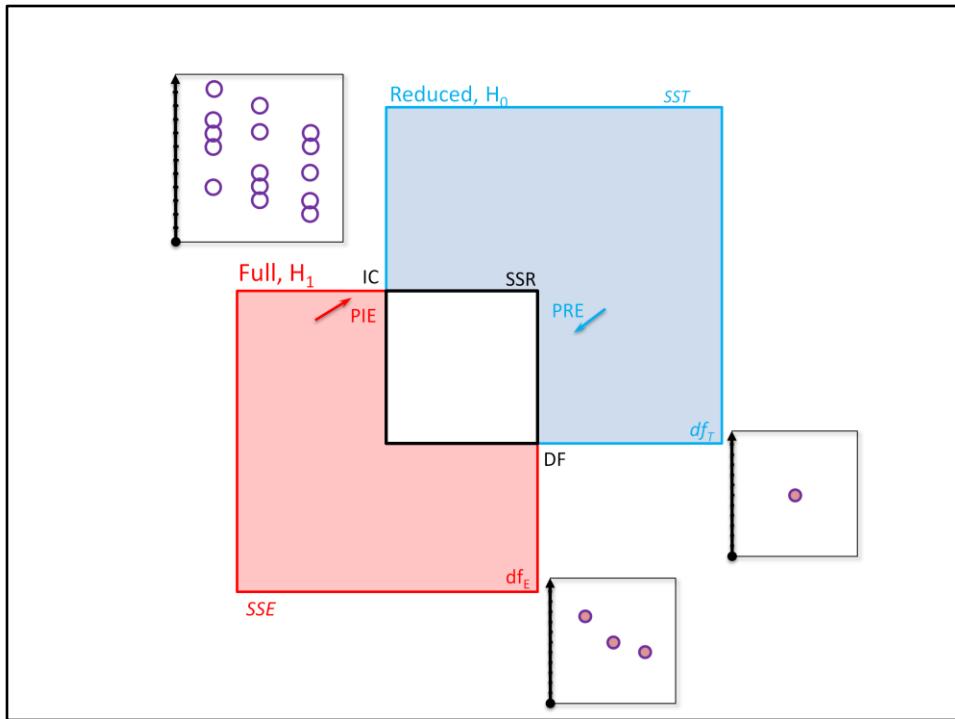
Let's see how we can do that, and how degrees of fidelity help us make sense of it.



In attempt to make sense which simplification is better (accurate, simple) we compare change in inaccuracy to the change in fidelity.  
We are determining the rate of change.





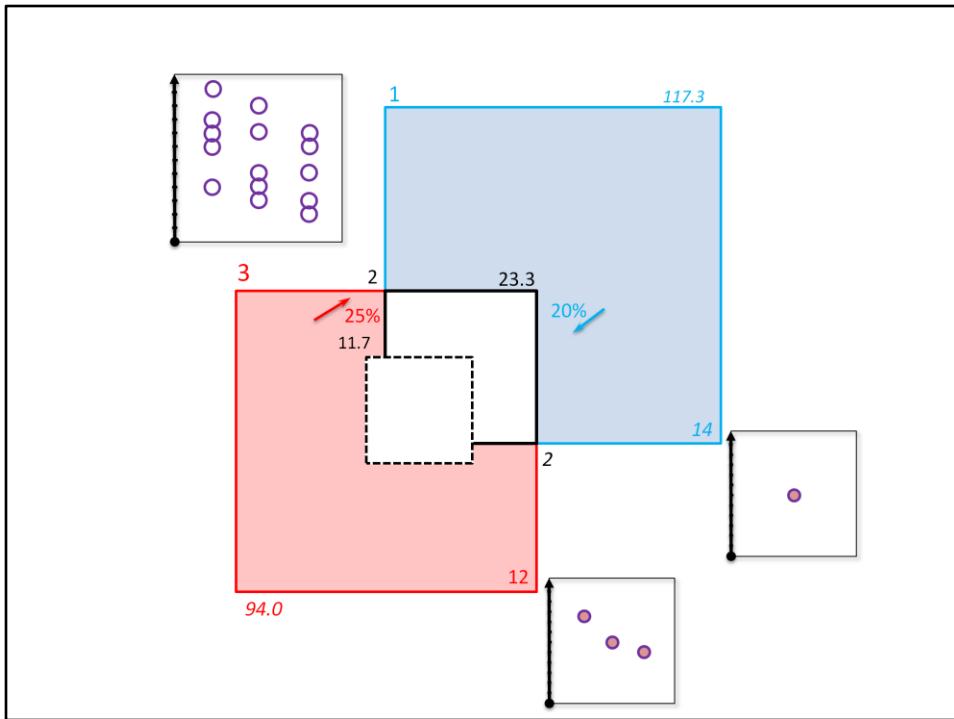


IC – increase in complexity

DF – decrease in fidelity

Some of the info pieces would be more important than others in understanding this aspect of reality ( variable). Model development is a systematic way to remove non-crucial elements from the description of reality

$DF_T$  shows how many degrees of fidelity was lost during simplification,  $SST$  shows how much it cost to simplify like this, measured in the units of accuracy (OLS)



We can think of MSR in two ways:

The cost of having a simpler model is MSR for each degree of simplicity. That's the currency, not DF. Fidelity is what you buy. DF = Degrees of (Lost) Fidelity, count how much simplification was bought ( 14 degrees)

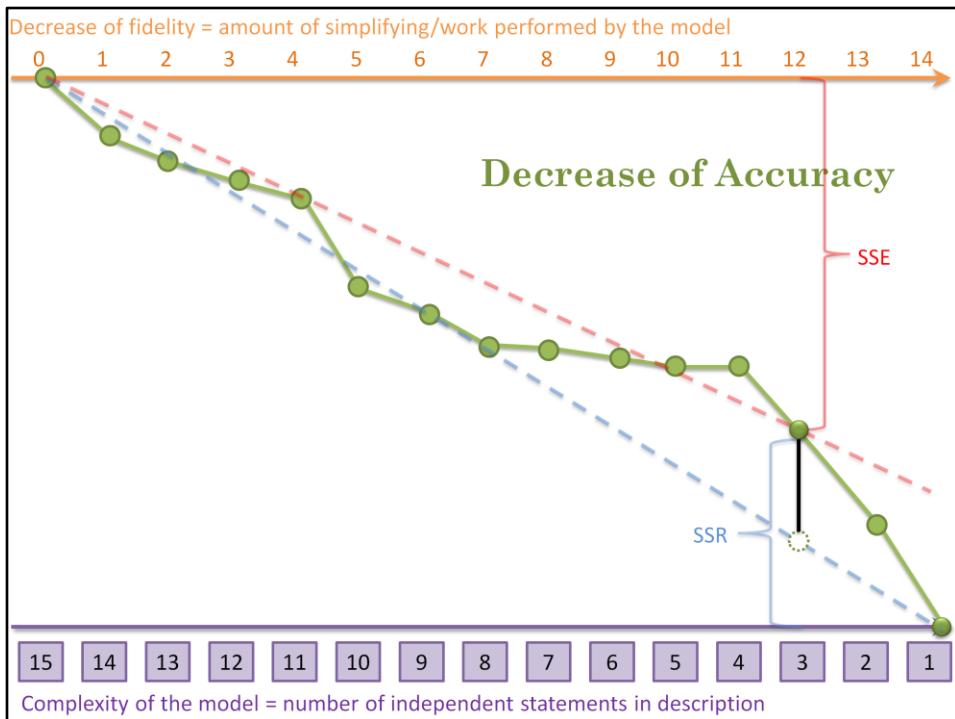
Loss of fidelity is the price for accuracy.

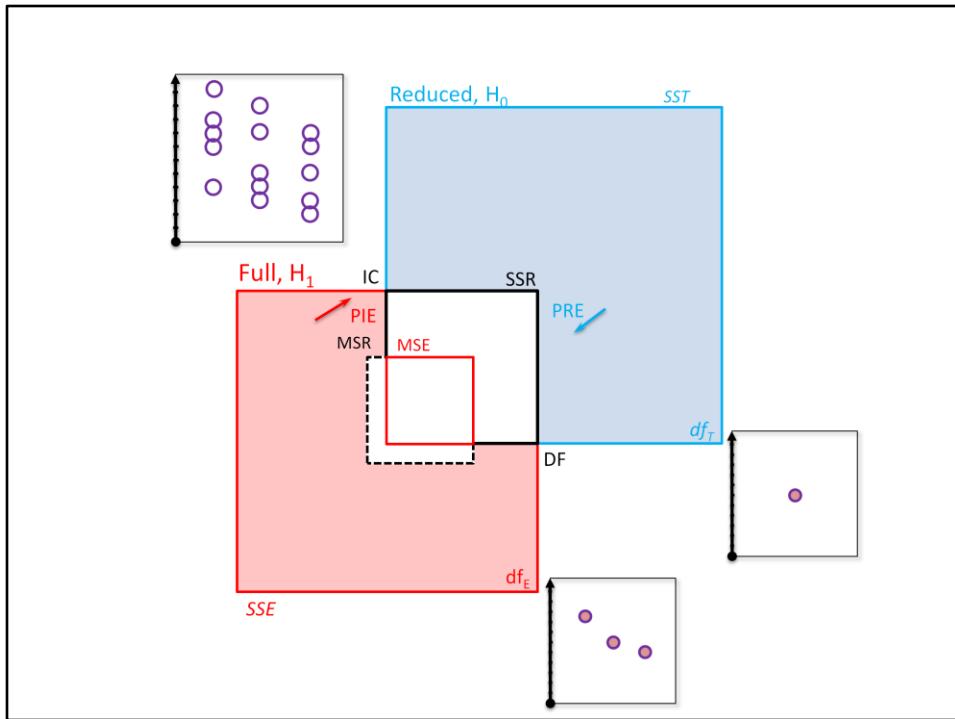
You can increase accuracy without increasing fidelity (better estimation, for example).

You can decrease fidelity without (significantly) decreasing accuracy.

Fidelity was decreased by 14 degrees for this simplification of (description of) reality.

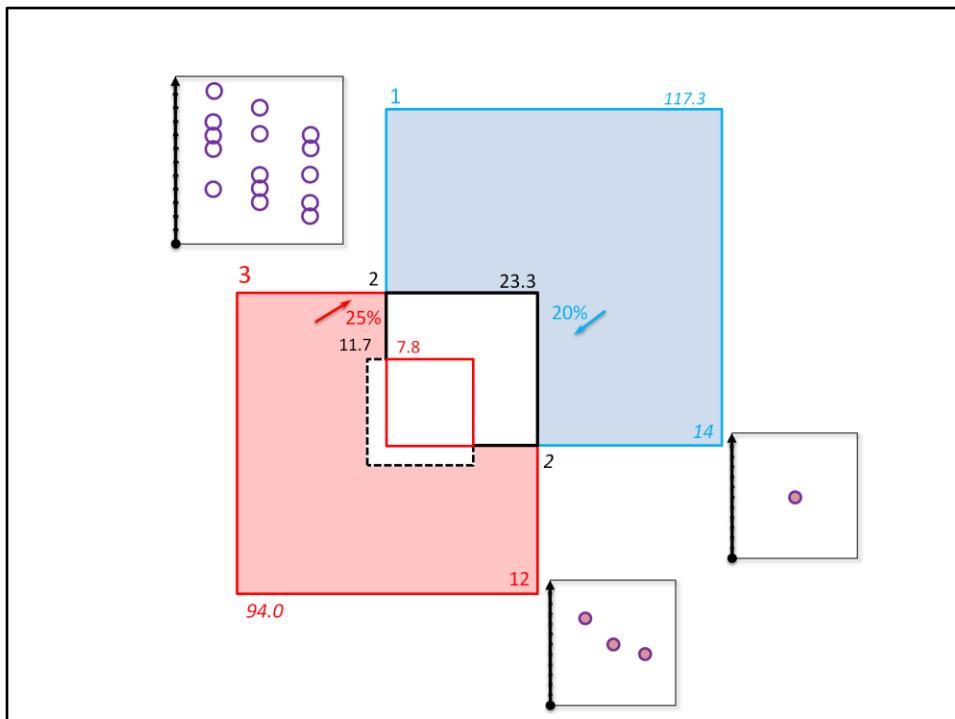
It cost the total of 117.3 units of accuracy

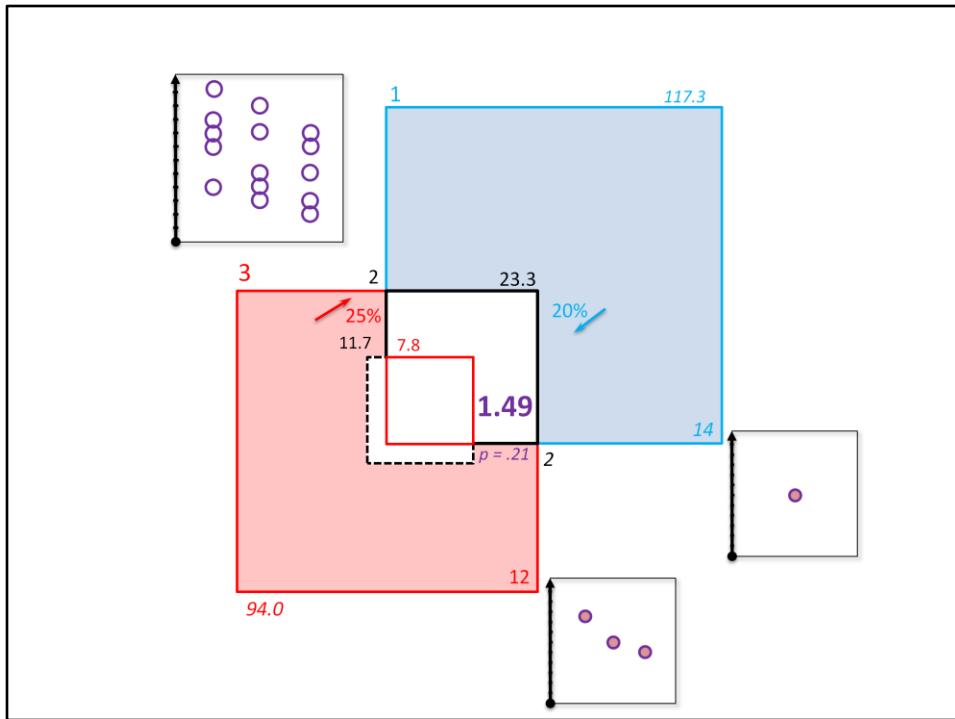




IC – increase in complexity

DF – decrease in fidelity



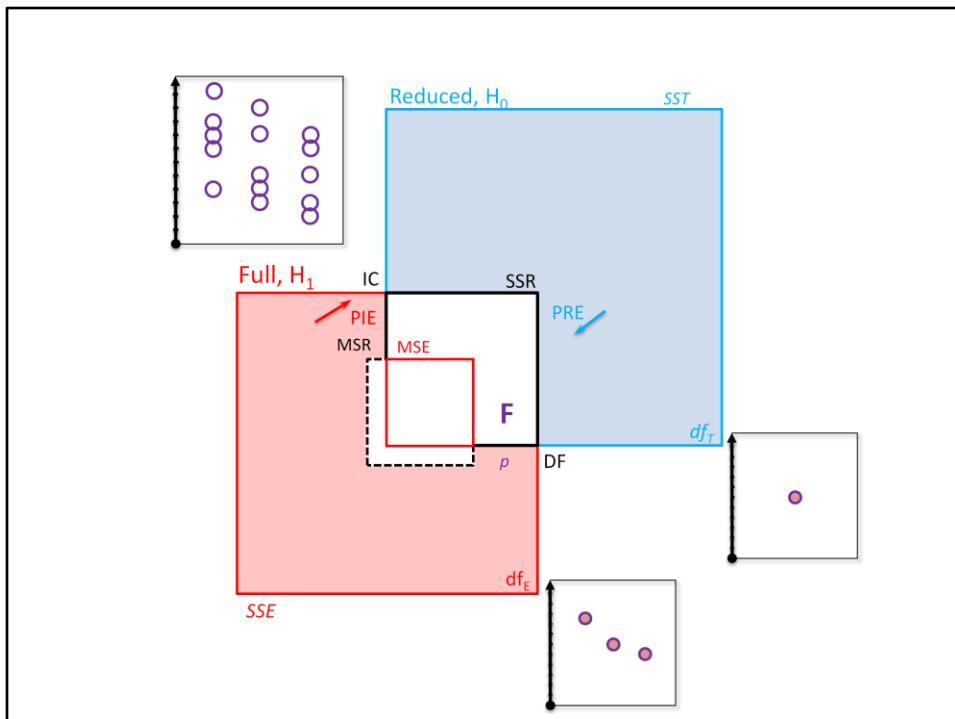


What's a better bargain?

Decrease the fidelity/complexity of the description by 14 and pay 117.3

Or

Decrease the fidelity/complexity of the description by 12 and pay 94.6? What model offers a better rate per degree of simplification?



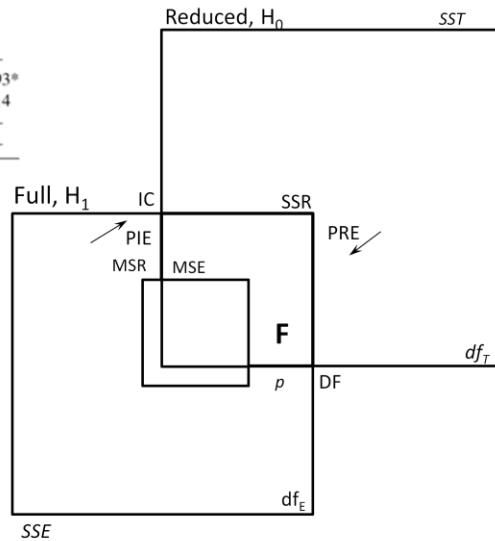
IC – increase in complexity

DF – decrease in fidelity

Table 1  
*Results From an Age (2) Preposition Type (5)  
 Mixed-Model Analysis of Variance on Hypothetical  
 Ipsiative Scores*

Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Between subjects	19	0	—	—
Age	1	0	0	0
Residual	18	0	0	0
Within subjects	80	5.11	—	—
Preposition type	4	4.02	1.010	66.93*
Age Preposition Type	4	0.01	0.002	0.14
Residual	72	1.08	0.015	—
Total	99	5.11	—	—

\* Significant at  $p < .001$ .



IC – increase in complexity

DF – decrease in fidelity

## Summary

- DF = Decrease of Fidelity
- Vocabulary of Fidelity and Accuracy
- Visual accounting system

# Conclusions

- If DF=Decrease of Fidelity (in degrees)
  - Invokes the analogy
  - Need to explain the analogy
  - Explaining = model comparison
- Model properties
  - Accuracy a.k.a. Fit
  - *Complexity, Fidelity*
  - **Decrease of Fidelity** a.k.a. Parsimony

By simply explaining what we mean by “decrease of fidelity”

The higher DF the more parsimonious the model, the more “work” of simplification it completed, the better it is.