principal component system

$$\mathbf{x} = \mathbf{B}'\mathbf{y}$$
$$\mathbf{y} = \mathbf{F}\mathbf{x} + \mathbf{e}$$

patterns eigens patterns

$$\mathbf{VDV}'$$

$$\mathbf{R}$$

correlation matrix

$$\mathbf{U}^2 + \mathbf{\Lambda}\mathbf{\Psi}\mathbf{\Lambda}' + \mathbf{E}$$

unique patterns corr patterns error

$$\mathbf{y} = \mathbf{F}\mathbf{x} + \mathbf{e}$$
$$\mathbf{x} = \mathbf{B}'\mathbf{y} + \boldsymbol{\delta}$$

common factor system

## Complexity of Factor Patterns

$\Lambda_1$

| $F_1$ | $F_2$ | $F_3$ |
|---|---|---|
| 0.00 | 0.50 | 0.90 |
| 0.30 | 0.00 | 0.74 |
| 0.00 | 0.25 | 0.79 |
| 0.96 | 0.00 | 0.00 |
| 0.77 | 0.00 | 0.12 |
| 0.73 | 0.00 | 0.00 |
| 0.00 | 0.55 | 0.00 |
| 0.00 | 0.72 | 0.06 |
| 0.00 | 0.88 | 0.00 |

$\Lambda_2$

| $F_1$ | $F_2$ | $F_3$ |
|---|---|---|
| 0.00 | 0.00 | 0.90 |
| 0.00 | 0.00 | 0.74 |
| 0.00 | 0.00 | 0.79 |
| 0.96 | 0.00 | 0.00 |
| 0.77 | 0.00 | 0.00 |
| 0.73 | 0.00 | 0.00 |
| 0.00 | 0.55 | 0.00 |
| 0.00 | 0.72 | 0.00 |
| 0.00 | 0.88 | 0.00 |

$\Lambda_3$

| $F_1$ | $F_2$ | $F_3$ |
|---|---|---|
| 0.00 | 0.00 | 1.00 |
| 0.00 | 0.00 | 1.00 |
| 0.00 | 0.00 | 1.00 |
| 1.00 | 0.00 | 0.00 |
| 1.00 | 0.00 | 0.00 |
| 1.00 | 0.00 | 0.00 |
| 0.00 | 1.00 | 0.00 |
| 0.00 | 1.00 | 0.00 |
| 0.00 | 1.00 | 0.00 |

## Crawford-Ferguson Complexity Function

$$f(\mathbf{L}) = (1-\kappa)\sum_{i=1}^{m} c(\mathbf{s}_{i.}) + \kappa \sum_{j=1}^{p} c(\mathbf{s}_{.j})$$

| | $F_1$ | $F_2$ | $F_3$ | |
|---|---|---|---|---|
| $c(\mathbf{s}_{i.})$ | 0.00 | 0.02 | 0.90 | Word Meaning |
| $c(\mathbf{s}_{i.})$ | 0.17 | 0.01 | 0.74 | Sentence Completion |
| $c(\mathbf{s}_{i.})$ | 0.05 | 0.12 | 0.79 | Odd Words |
| $c(\mathbf{s}_{i.})$ | 0.96 | 0.00 | 0.01 | Mixed Arithmetic |
| $c(\mathbf{s}_{i.})$ | 0.77 | 0.10 | 0.08 | Remainders |
| $c(\mathbf{s}_{i.})$ | 0.73 | 0.10 | 0.17 | Missing Numbers |
| $c(\mathbf{s}_{i.})$ | 0.16 | 0.55 | -0.04 | Gloves |
| $c(\mathbf{s}_{i.})$ | 0.04 | 0.72 | 0.05 | Boots |
| $c(\mathbf{s}_{i.})$ | -0.03 | 0.88 | 0.02 | Hatchets |
| | $c(\mathbf{s}_{.j})$ | $c(\mathbf{s}_{.j})$ | $c(\mathbf{s}_{.j})$ | |

$\Lambda$

| | | |
|---|---|---|
| 0.00 | 0.02 | 0.90 |
| 0.17 | 0.01 | 0.74 |
| 0.05 | 0.12 | 0.79 |
| 0.96 | 0.00 | 0.01 |
| 0.77 | 0.10 | 0.08 |
| 0.73 | 0.10 | 0.17 |
| 0.16 | 0.55 | -0.04 |
| 0.04 | 0.72 | 0.05 |
| -0.03 | 0.88 | 0.02 |

$$\mathbf{R}$$

$$\mathbf{F} \quad \mathbf{F}'$$

$$\mathbf{V}\mathbf{D}^{\frac{1}{2}}\mathbf{D}^{\frac{1}{2}}\mathbf{V}'$$

$$\mathbf{V} \quad \mathbf{D} \quad \mathbf{V}'$$

$$\begin{bmatrix} \mathbf{v}_i & \cdots & \mathbf{v}_r \\ \vdots & | & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} c_i & & \\ & \ddots & \\ & & c_r \end{bmatrix} \begin{bmatrix} \mathbf{v}_i & \cdots & \cdots \\ \vdots & — & \cdots \\ \mathbf{v}_r & \cdots & \cdots \end{bmatrix}$$

$$\underset{N\times N}{\mathbf{R}}\,\mathbf{v}_i = c_i\mathbf{v}_i$$

$$\mathbf{x} = \mathbf{B}'\mathbf{y}$$

$$\mathbf{y} = \mathbf{F}\mathbf{x} + \mathbf{e}$$

$$\mathbf{V}\mathbf{D}\mathbf{V}'$$

$$\mathbf{R}$$

$$\mathbf{U}^2 + \mathbf{\Lambda}\mathbf{\Psi}\mathbf{\Lambda}' + \mathbf{E}$$

$$\mathbf{y} = \mathbf{F}\mathbf{x} + \mathbf{e}$$

$$\mathbf{x} = \mathbf{B}'\mathbf{y} + \boldsymbol{\delta}$$

$r$ = rank of $\mathbf{R}$

$N$ – number of variables/columns in $\mathbf{R}$

$r$ – # positive/nonzero eigenvalues of N × N matrix $\mathbf{R}$