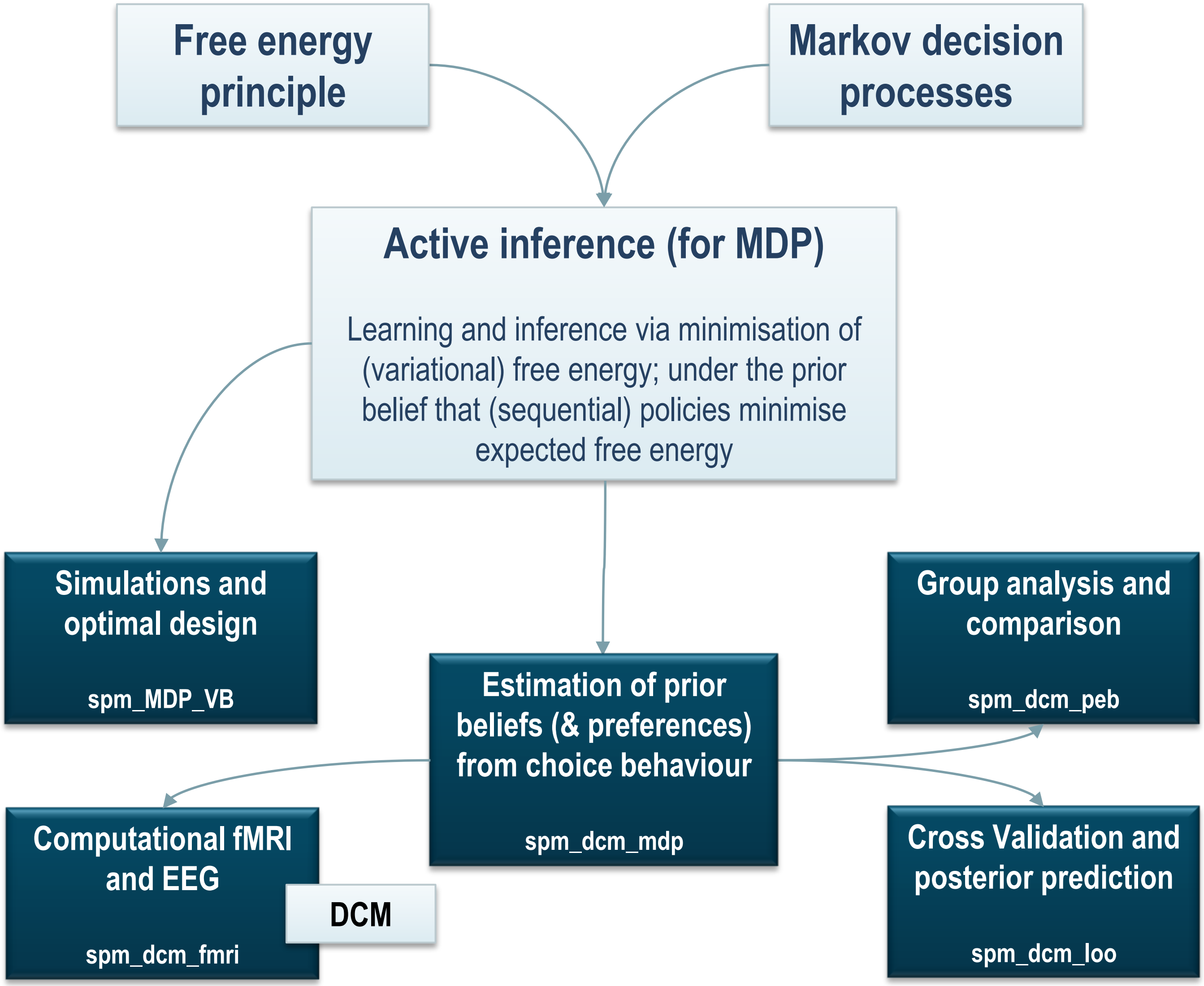
Predictive coding & active inference

Karl Friston, University College London

Abstract: I will talk about a formal treatment of choice behaviour based on the premise that agents minimise the expected free energy of future outcomes. Crucially, the negative free energy or quality of a policy can be decomposed into extrinsic and epistemic (intrinsic) value. Minimising expected free energy is therefore equivalent to maximising extrinsic value or expected utility (defined in terms of prior preferences or goals), while maximising information gain or intrinsic value; i.e., reducing uncertainty about the causes of valuable outcomes. The resulting scheme resolves the exploration-exploitation dilemma: epistemic value is maximised until there is no further information gain, after which exploitation is assured through maximisation of extrinsic value. This is formally consistent with the Infomax principle, generalising formulations of active vision based upon salience (Bayesian surprise) and optimal decisions based on expected utility and risk sensitive (KL) control. Furthermore, as with previous active inference formulations of discrete (Markovian) problems; ad hoc softmax parameters become the expected (Bayes-optimal) precision of beliefs about – or confidence in – policies. We focus on the basic theory – illustrating the minimisation of expected free energy using simulations. A key aspect of this minimisation is the similarity of precision updates and dopaminergic discharges observed in conditioning paradigms.

Key words: active inference · cognitive · dynamics · free energy · epistemic value · self-organization

overview


Free energy principle
Markov decision processes
Active inference (for MDP)
Learning and inference via minimisation of (variational) free energy; under the prior belief that (sequential) policies minimise expected free energy
Simulations and optimal design
spm_MDP_VB
Estimation of prior beliefs (& preferences) from choice behaviour
spm_dcm_mdp
Group analysis and comparison
spm_dcm_peb
Computational fMRI and EEG
spm_dcm_fmri
DCM
Cross Validation and posterior prediction
spm_dcm_loo

Active inference

Action and the path of least resistance
Generative models and active inference
From principles to process theories
Some empirical predictions

Imagine you are an owl – and you are hungry…

Optimal action depends on the state of the world

$$u_t^* = \arg\max V(s_{t+1} \mid u_t)$$

Optimal action depends on *beliefs about* the state of the world

$$u_t^* = \arg\min F(Q(s_{t+1}) \mid u_t)$$

Optimal action depends on beliefs about the state of the world *and subsequent action*

$$\pi^* = \arg\min \sum_{\tau > t} F(Q(s_\tau) \mid \pi)$$

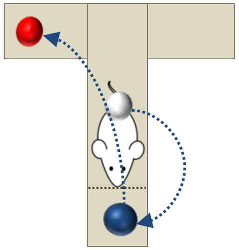
$$u_\tau = \pi(\tau)$$

Bellman's Optimality Principle:

- Optimal control theory
- Bayesian decision theory
- Reinforcement learning
- ....

Hamilton's Principle of least Action:

- The free energy principle
- Active inference
- Active learning
- ....



Prior beliefs about policies

$$\ln P(\pi \mid \gamma) = -\gamma \cdot \widehat{\mathbf{F}} : \widehat{\mathbf{F}} = \sum_{\tau} F(\pi, \tau)$$

Quality of a policy = (negative) expected free energy

$$\begin{aligned}
-F(\pi,\tau) &= E_{Q(o_\tau, s_\tau \mid \pi)}[\ln P(o_\tau, s_\tau \mid \pi)] + H[Q(s_\tau \mid \pi)] \\
&= E_{Q(o_\tau, s_\tau \mid \pi)}[\ln Q(s_\tau \mid o_\tau, \pi) + \ln P(o_\tau \mid m) - \ln Q(s_\tau \mid \pi)] \\
&= \underbrace{E_{Q(o_\tau \mid \pi)}[\ln P(o_\tau \mid m)]}_{\text{Extrinsic value}} + \underbrace{E_{Q(o_\tau \mid \pi)}[D[Q(s_\tau \mid o_\tau, \pi) \,\|\, Q(s_\tau \mid \pi)]]}_{\text{Epistemic value or information gain}}
\end{aligned}$$

Bayesian surprise and Infomax

In the absence of prior beliefs about outcomes:

$$= \underbrace{E_{Q(o_\tau \mid \pi)}[D[Q(s_\tau \mid o_\tau, \pi) \,\|\, Q(s_\tau \mid \pi)]]}_{\text{Bayesian surprise}}$$

$$= \underbrace{D[Q(s_\tau, o_\tau \mid \pi) \,\|\, Q(s_\tau \mid \pi) Q(o_\tau \mid \pi)]}_{\text{Predicted mutual information}}$$

KL or risk-sensitive control

In the absence of ambiguity (known states):

$$\begin{aligned}
&= E_{Q(s_\tau \mid \pi)}[\ln P(s_\tau \mid \pi) - \ln Q(s_\tau \mid \pi)] \\
&= -\underbrace{D[Q(s_\tau \mid \pi) \,\|\, P(s_\tau \mid \pi)]}_{\text{Predicted divergence}}
\end{aligned}$$

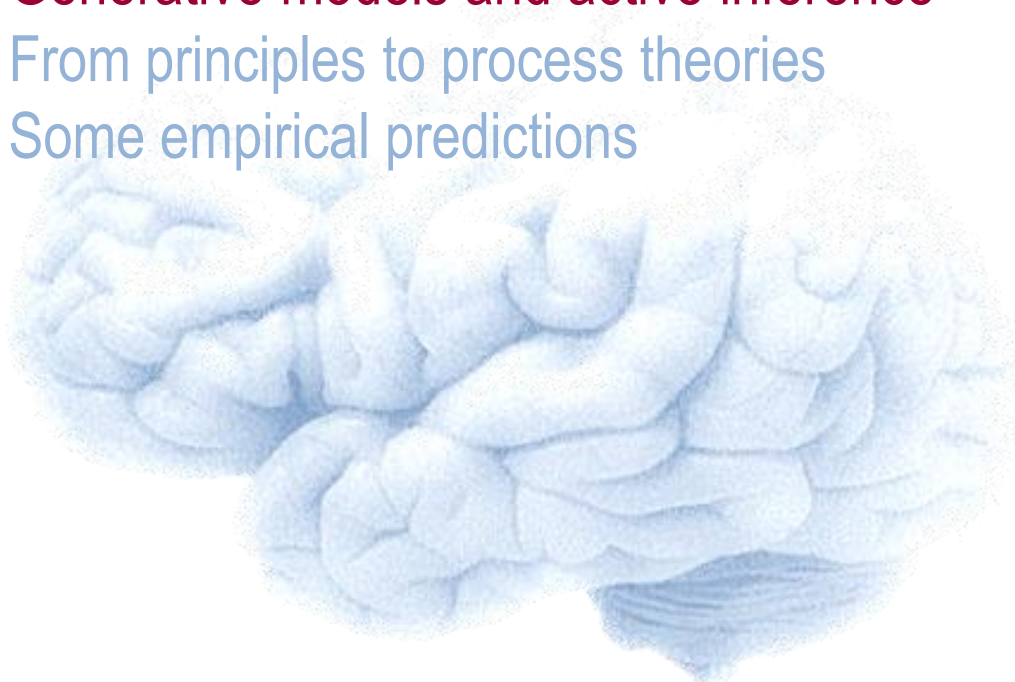
Expected utility theory

In the absence of uncertainty or risk:

$$= \underbrace{E_{Q(o_\tau \mid \pi)}[\ln P(o_\tau \mid m)]}_{\text{Extrinsic value}}$$

Overview

Action and the path of least resistance
Generative models and active inference
From principles to process theories
Some empirical predictions

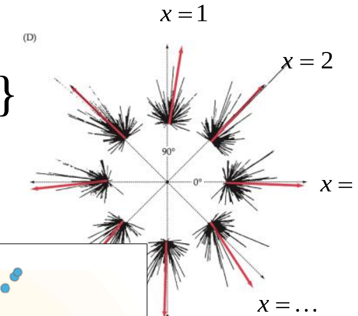


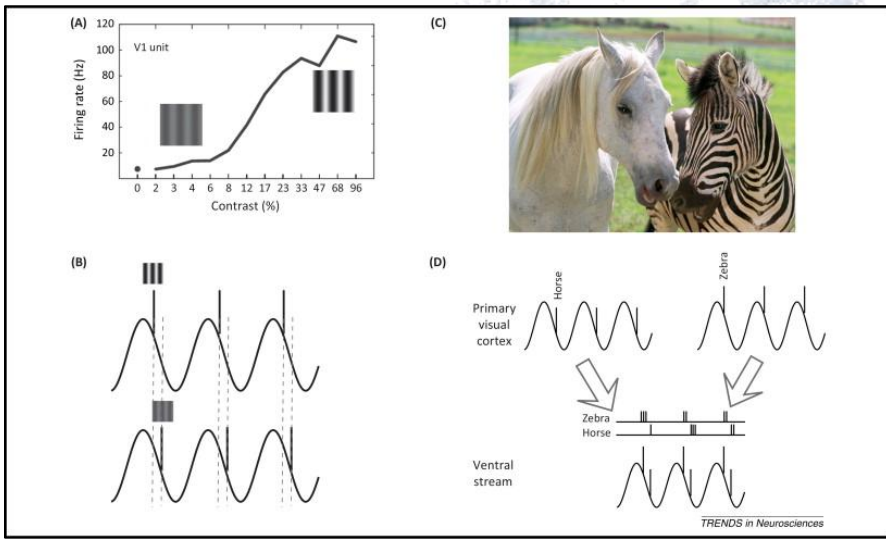
Continuous or discrete state-space models?

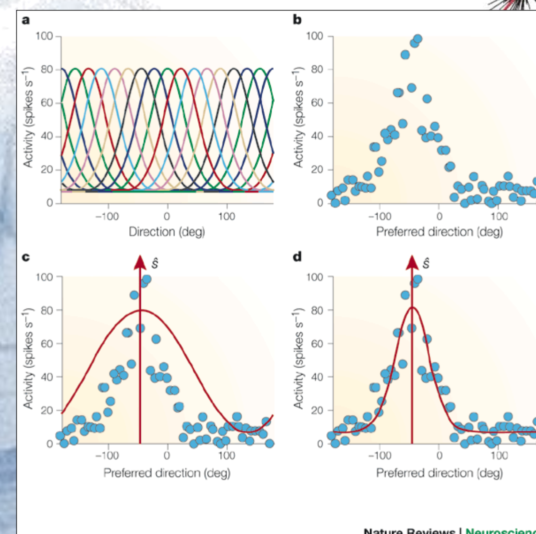
States and their
Sufficient statistics

$$s \in \{1,\ldots,N\}$$

$$\mu \in [0,1]$$





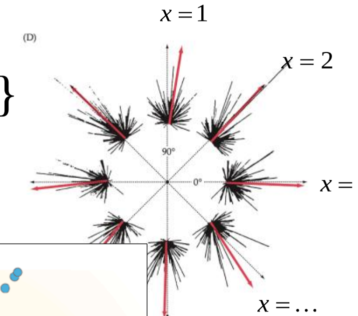


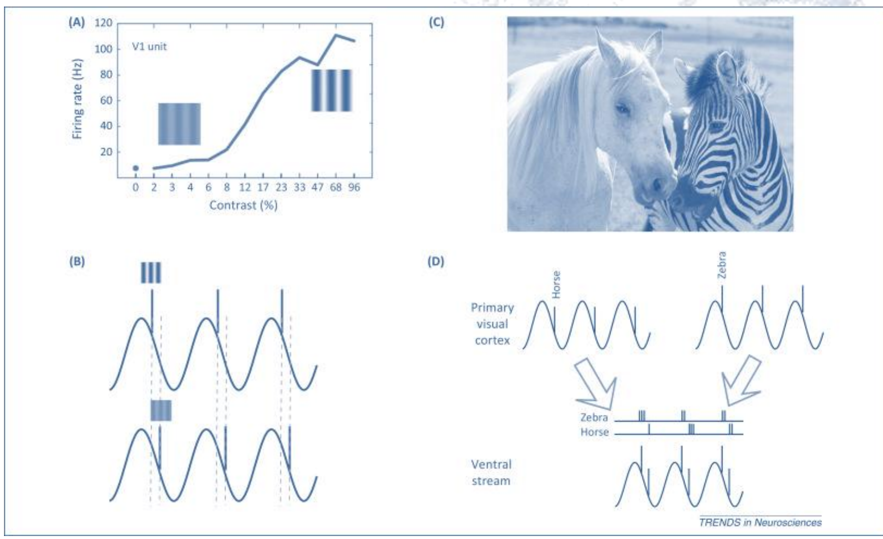
$$\mu^* = \arg\min F(Q(s \mid \mu)) \Rightarrow Q(s \mid \mu) \approx P(s \mid o, m)$$

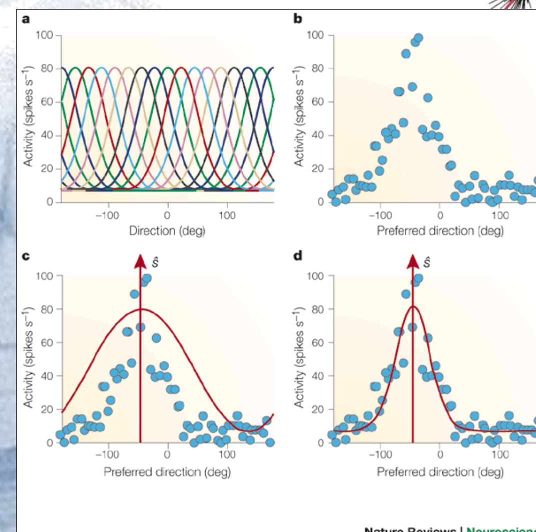
Continuous or discrete state-space models?

$$x \in \{1,\ldots,N\}$$

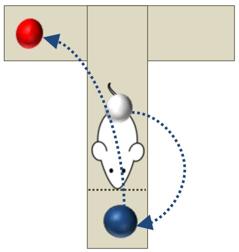
$$\mu \in [0,1]$$







$$\mu^* = \arg\min F(Q(s\,|\,\mu)) \Rightarrow Q(s\,|\,\mu) \approx P(s\,|\,o,m)$$



A (Markovian) generative model

$$P(\tilde{o},\tilde{s},\pi,\gamma) = P(\tilde{o}\,|\,\tilde{s})P(\tilde{s}\,|\,\pi)P(\pi\,|\,\gamma)P(\gamma)$$

$$P(\tilde{o}\,|\,\tilde{s}) = P(o_0\,|\,s_0)P(o_1\,|\,s_1)\ldots P(o_t\,|\,s_t)$$

$$P(o_t\,|\,s_t) = \mathbf{A}$$

Likelihood

$$P(\tilde{s}\,|\,\pi) = P(s_t\,|\,s_{t-1},\pi)\ldots P(s_1\,|\,s_0,\pi)P(s_0)$$

$$P(s_{t+1}\,|\,s_t,\pi>0) = \mathbf{B}(u=\pi(t))$$

$$P(s_{t+1}\,|\,s_t,\pi=0) = \mathbf{C}$$

$$P(s_0) = \mathbf{D}$$

Empirical priors – hidden states

– control states

$$P(\pi\,|\,\gamma) = \sigma(-\gamma\cdot\widehat{\mathbf{F}})$$

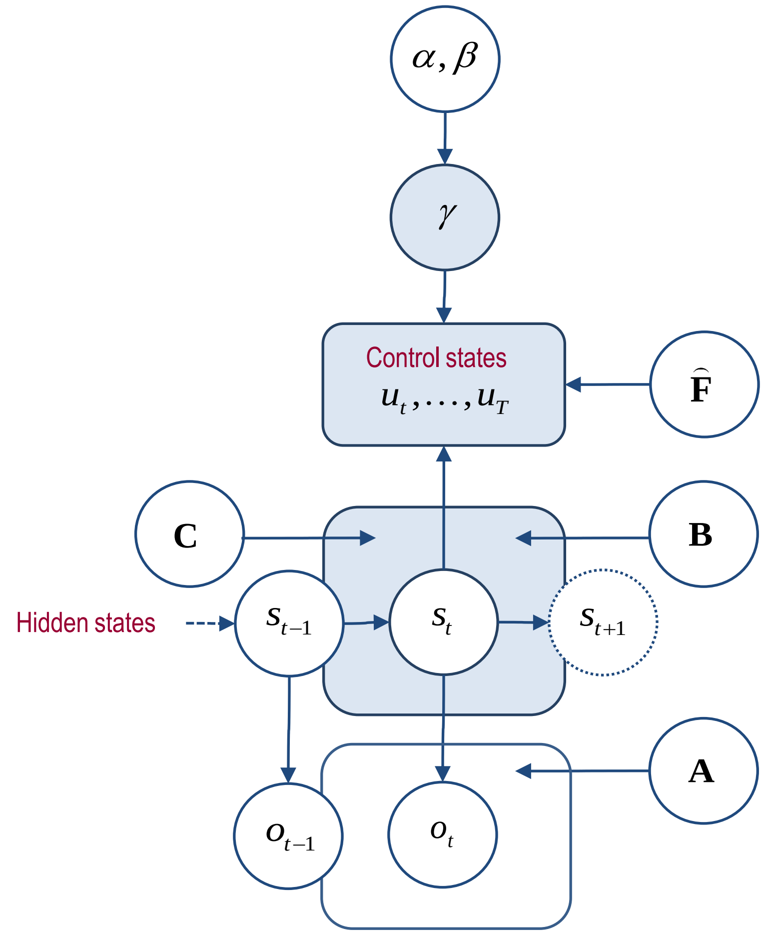
$$\widehat{\mathbf{F}} = \sum\nolimits_{\tau} E_{Q(o_\tau,s_\tau|\pi)}[\ln P(o_\tau,s_\tau\,|\,\pi) - \ln Q(s_\tau\,|\,\pi)]$$

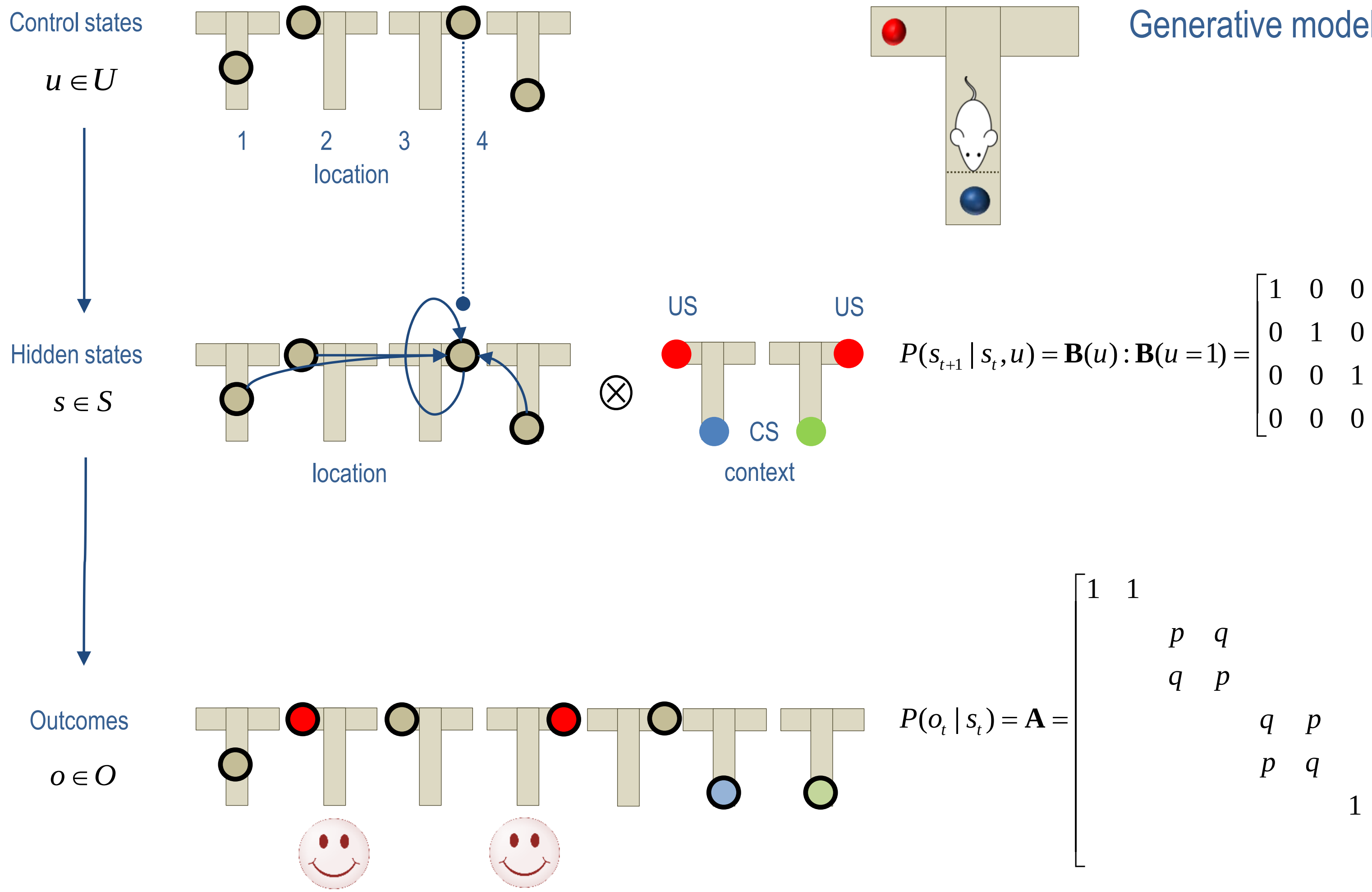
$$P(\mathbf{C}) = Dir(c)$$

$$P(\mathbf{D}) = Dir(d)$$

$$P(\gamma) = \Gamma(\alpha,\beta)$$

Full priors





$$P(s_{t+1} \mid s_t, u) = \mathbf{B}(u) : \mathbf{B}(u=1) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ldots$$

$$P(o_t \mid s_t) = \mathbf{A} = \begin{bmatrix} 1 & 1 & & & & & & \\ & & p & q & & & & \\ & & q & p & & & & \\ & & & & q & p & & \\ & & & & p & q & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{bmatrix} : q = 1 - p$$

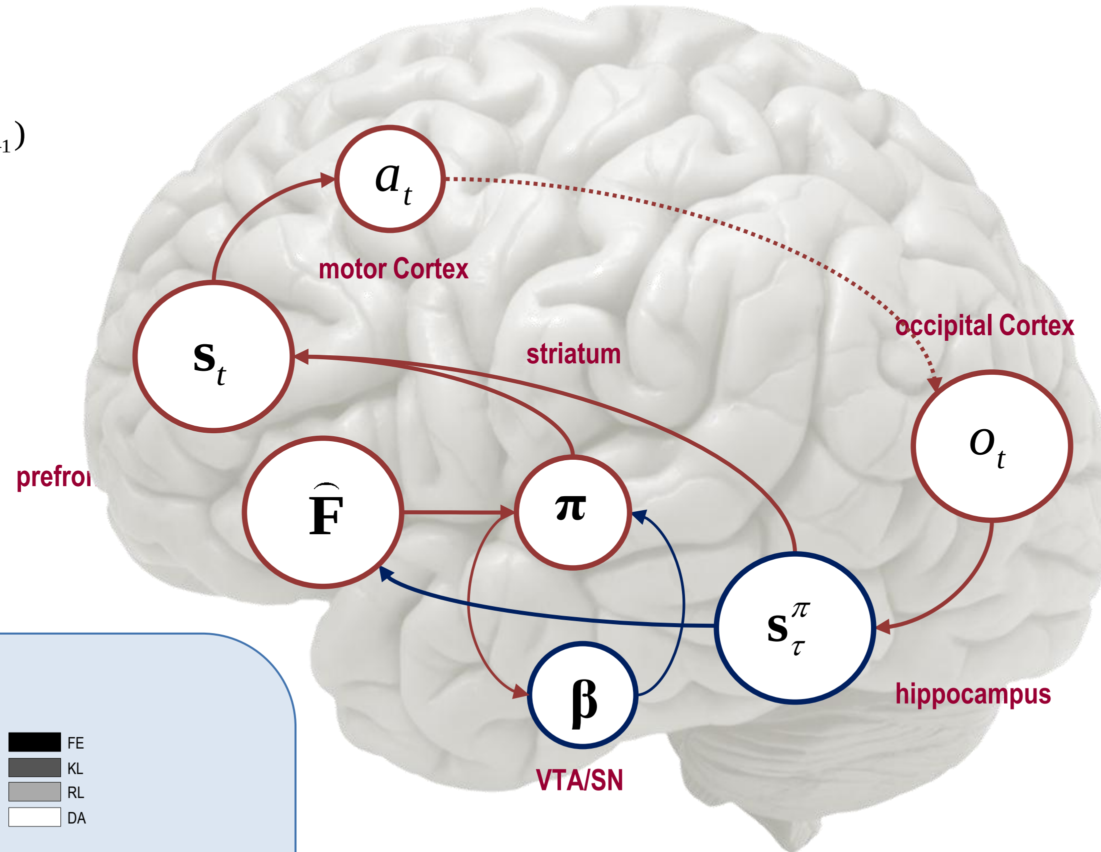
$$\ln P(o_t) = \begin{bmatrix} 0 & 3 & -3 & 3 & -3 & 0 & 0 \end{bmatrix}^T$$

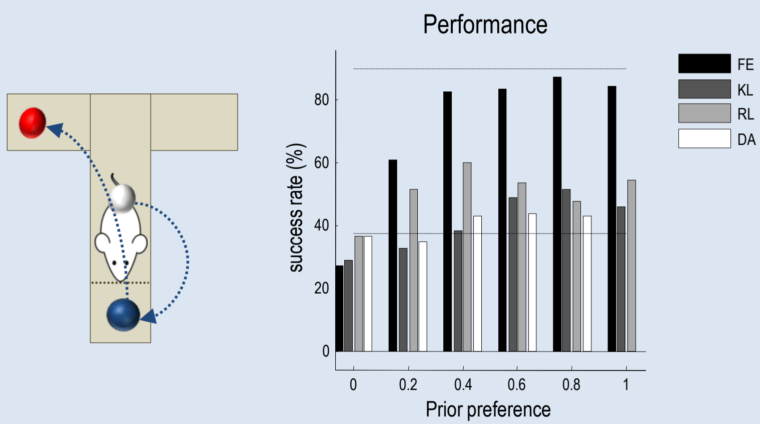
Variational updates

Perception	$\mathbf{s}_t^{\pi} = \sigma(\widehat{\mathbf{A}} \cdot o_t + \widehat{\mathbf{B}}_{t-1}^{\pi} \mathbf{s}_{t-1}^{\pi} + \widehat{\mathbf{B}}_t^{\pi} \cdot \mathbf{s}_{t+1}^{\pi})$
Action selection	$\boldsymbol{\pi} = \sigma(-\mathbf{F} - \boldsymbol{\gamma} \cdot \widehat{\mathbf{F}})$
Incentive salience	$\boldsymbol{\beta} = \beta + (\boldsymbol{\pi} - \boldsymbol{\pi}_0) \cdot \widehat{\mathbf{F}}$

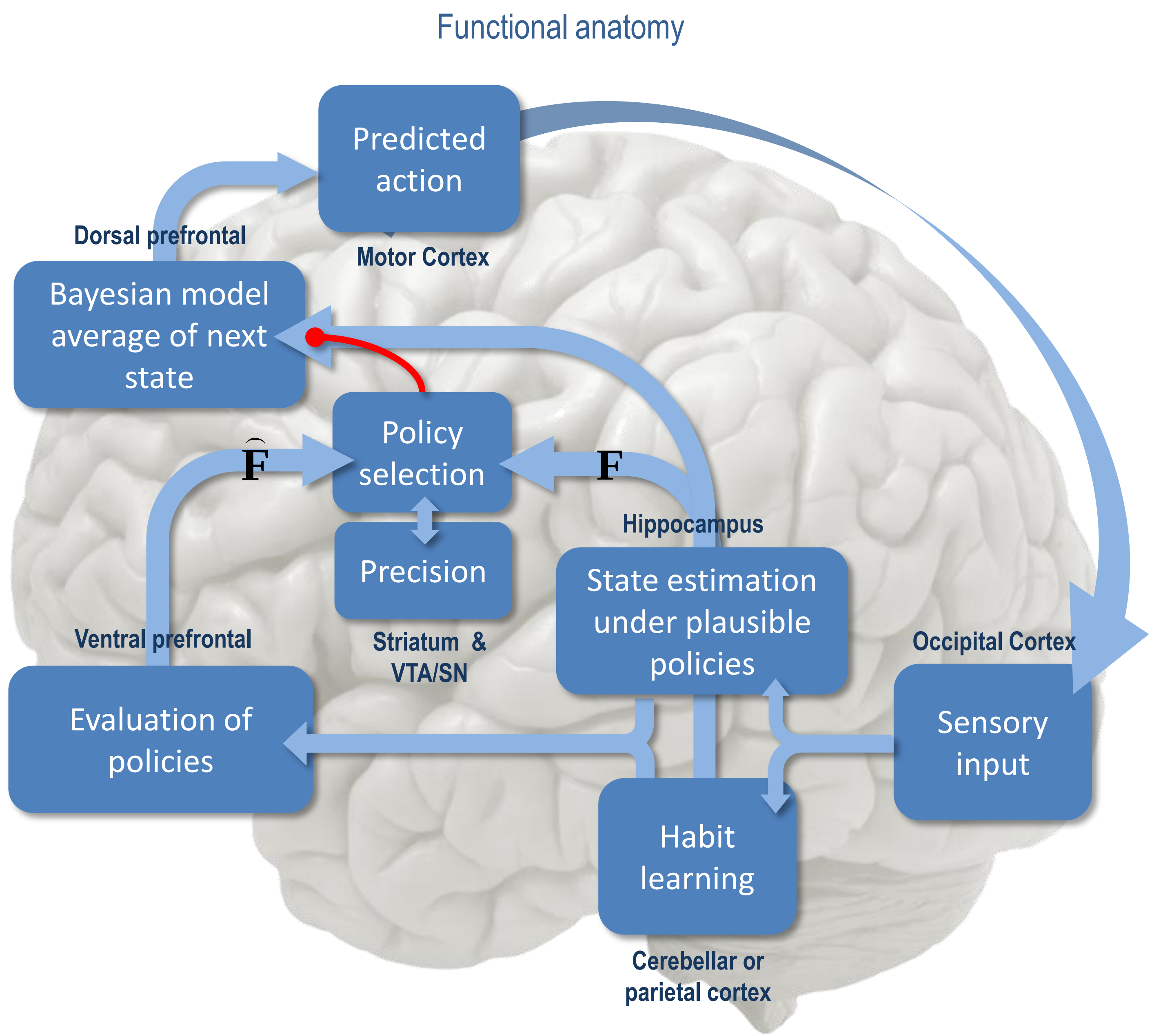
$$\tilde{\mu} = \arg\min F(o_1, \ldots, o_t, \tilde{\mu})$$

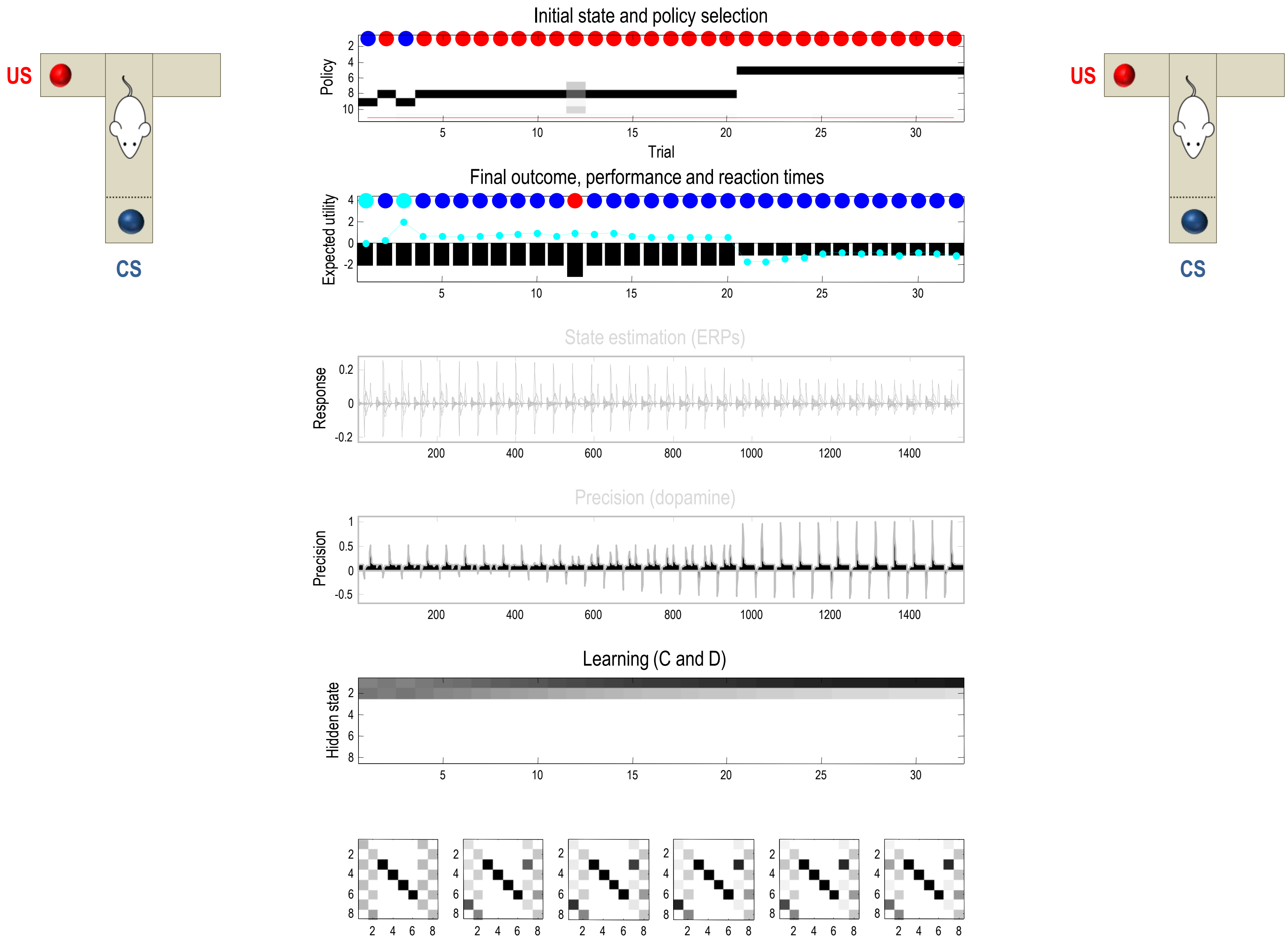
Functional anatomy





Simulated behaviour

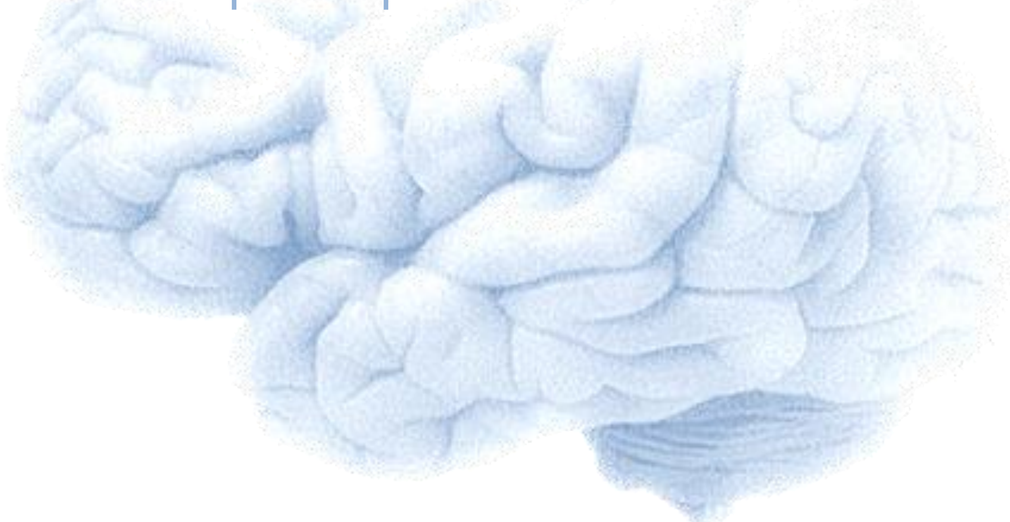

Functional anatomy
Predicted action
Motor Cortex
Dorsal prefrontal
Bayesian model average of next state
Policy selection
$\hat{\mathbf{F}}$
$\mathbf{F}$
Precision
Striatum & VTA/SN
Hippocampus
State estimation under plausible policies
Ventral prefrontal
Evaluation of policies
Occipital Cortex
Sensory input
Habit learning
Cerebellar or parietal cortex



US
CS
Initial state and policy selection
Policy
Trial
Final outcome, performance and reaction times
Expected utility
State estimation (ERPs)
Response
Precision (dopamine)
Precision
Learning (C and D)
Hidden state
US
CS

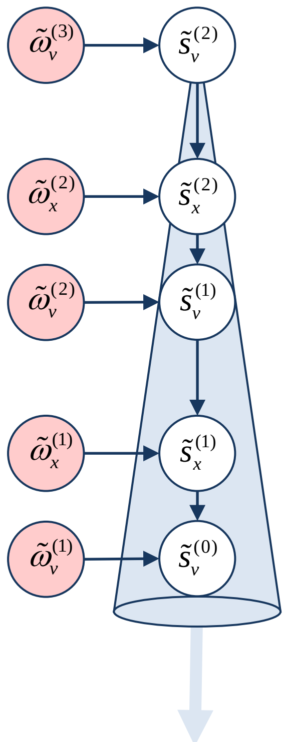
Overview

Action and the path of least resistance
Generative models and active inference
From principles to process theories
Some empirical predictions



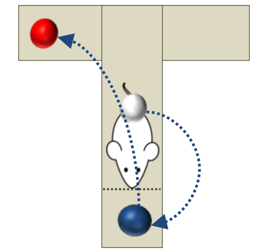
Generative models

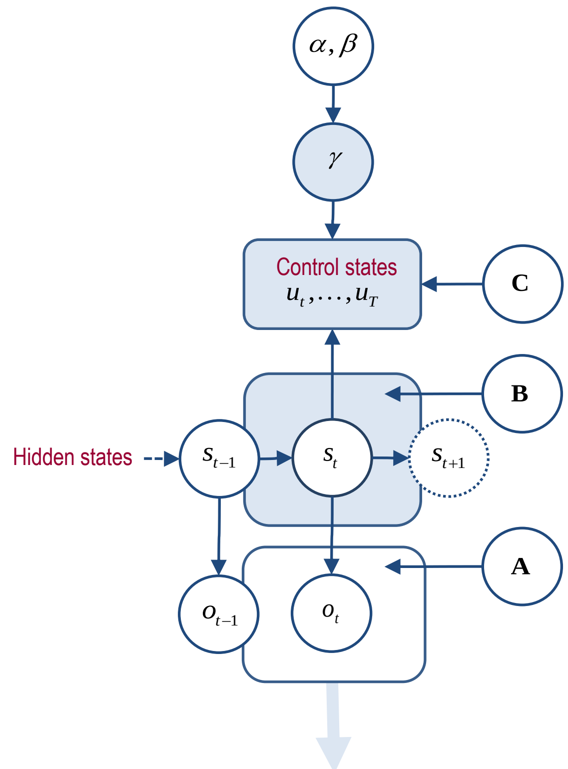
Continuous states



Bayesian filtering
(predictive coding)

Discrete states





Variational Bayes
(belief updating)

$$\dot{\tilde{\mathbf{s}}}^{(i)} = D\tilde{\mathbf{s}}^{(i)} - \nabla\tilde{\varepsilon}^{(i)} \cdot \Pi^{(i)} \cdot \tilde{\varepsilon}^{(i)} \xrightarrow{\dot{\tilde{s}}=0} \mathbf{s}_t^{\pi} = \sigma(\widehat{\mathbf{A}} \cdot o_t + \widehat{\mathbf{B}}_{t-1}^{\pi}\mathbf{s}_{t-1}^{\pi} + \widehat{\mathbf{B}}_t^{\pi} \cdot \mathbf{s}_{t+1}^{\pi})$$

Variational updates

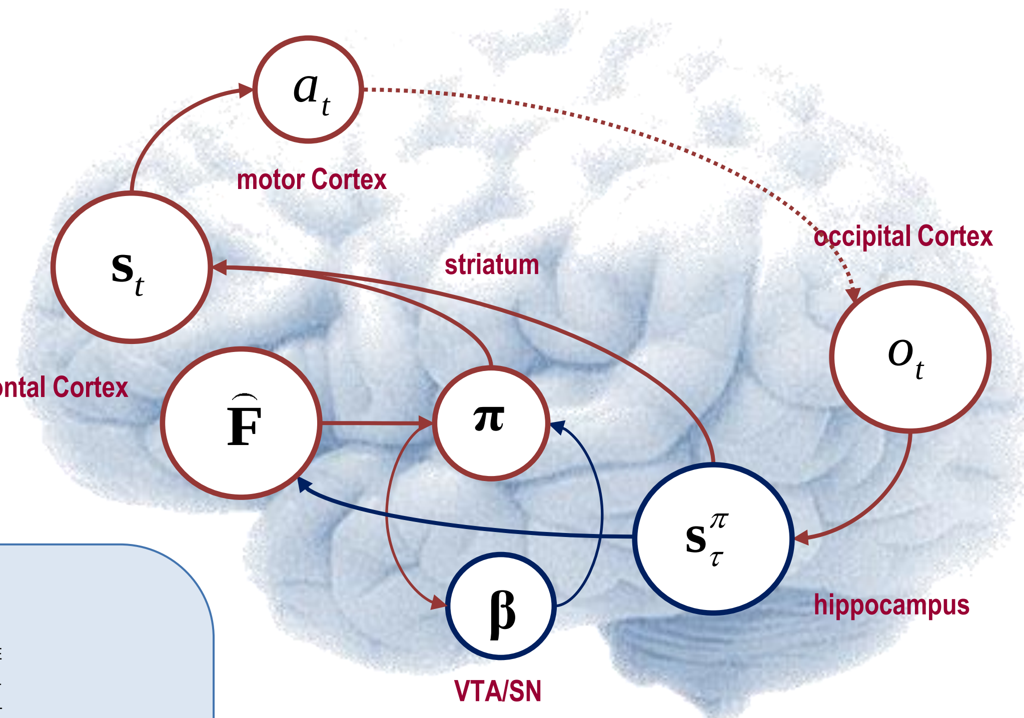
Perception $\mathbf{s}_t^\pi = \sigma(\widehat{\mathbf{A}}\cdot o_t + \widehat{\mathbf{B}}_{t-1}^\pi \mathbf{s}_{t-1}^\pi + \widehat{\mathbf{B}}_t^\pi \cdot \mathbf{s}_{t+1}^\pi)$

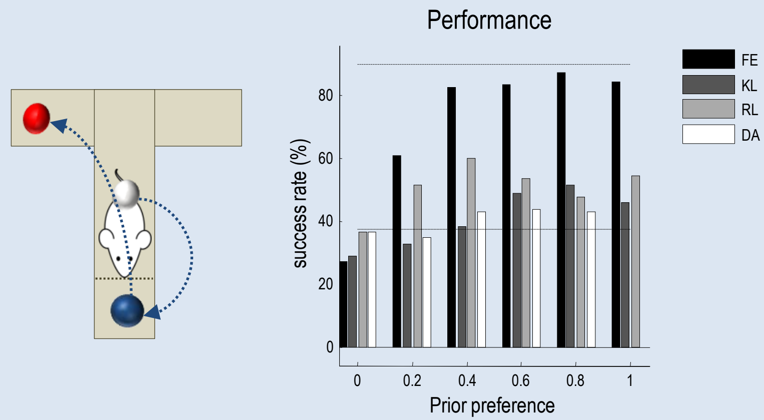
Action selection $\boldsymbol{\pi} = \sigma(-\mathbf{F} - \boldsymbol{\gamma}\cdot\widehat{\mathbf{F}})$

Incentive salience $\boldsymbol{\beta} = \beta + (\boldsymbol{\pi}-\boldsymbol{\pi}_0)\cdot\widehat{\mathbf{F}}$

$$\tilde{\mu} = \arg\min F(o_1,\ldots,o_t,\tilde{\mu})$$

Functional anatomy





Simulated behaviour

Variational updating

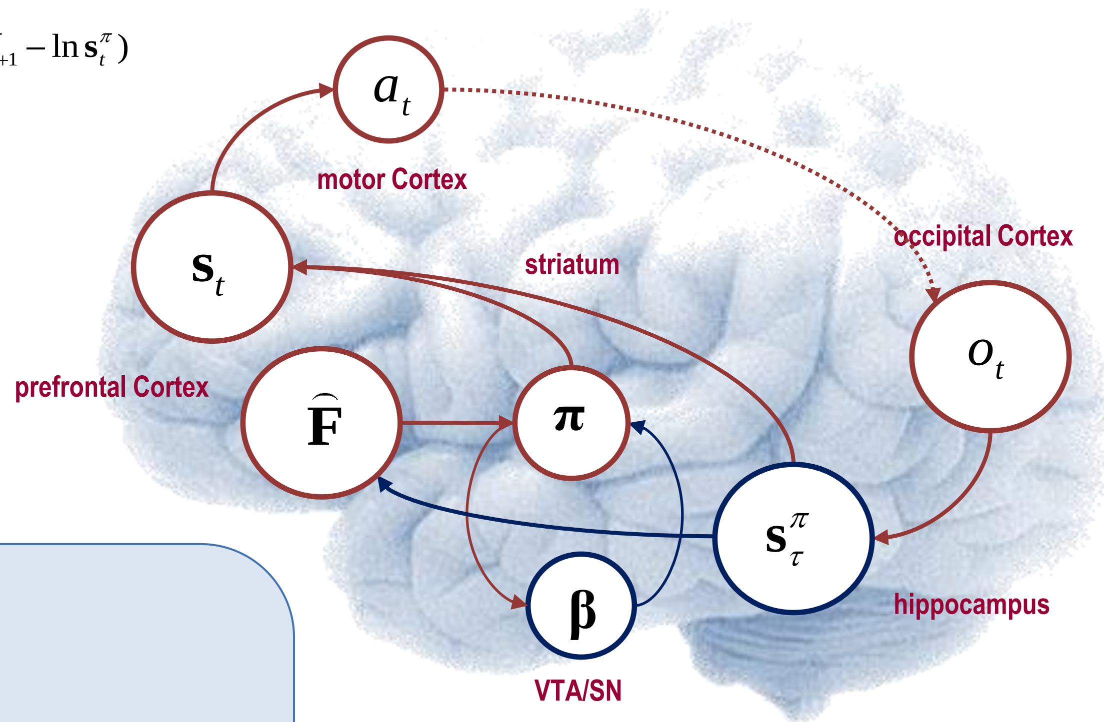
Perception $\dot{\mathbf{s}}_t^{\pi} = \sigma'(\widehat{\mathbf{A}} \cdot o_t + \widehat{\mathbf{B}}_{t-1}^{\pi} \mathbf{s}_{t-1}^{\pi} + \widehat{\mathbf{B}}_t^{\pi} \cdot \mathbf{s}_{t+1}^{\pi} - \ln \mathbf{s}_t^{\pi})$

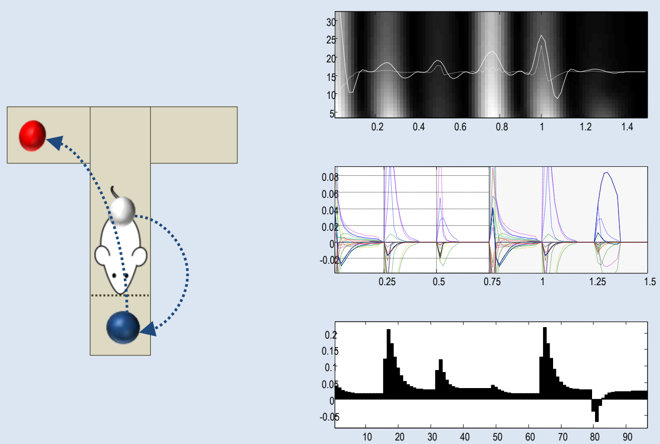
Action selection $\boldsymbol{\pi} = \sigma(-\mathbf{F} - \boldsymbol{\gamma} \cdot \widehat{\mathbf{F}})$

Incentive salience $\dot{\boldsymbol{\beta}} = \beta + (\boldsymbol{\pi} - \boldsymbol{\pi}_0) \cdot \widehat{\mathbf{F}} - \boldsymbol{\beta}$

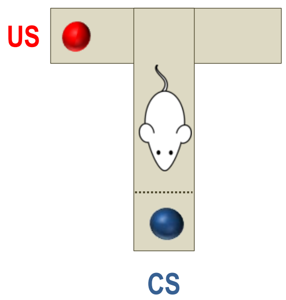
$$\dot{\tilde{\mu}} = -\nabla F(o_1, \ldots, o_t, \tilde{\mu})$$

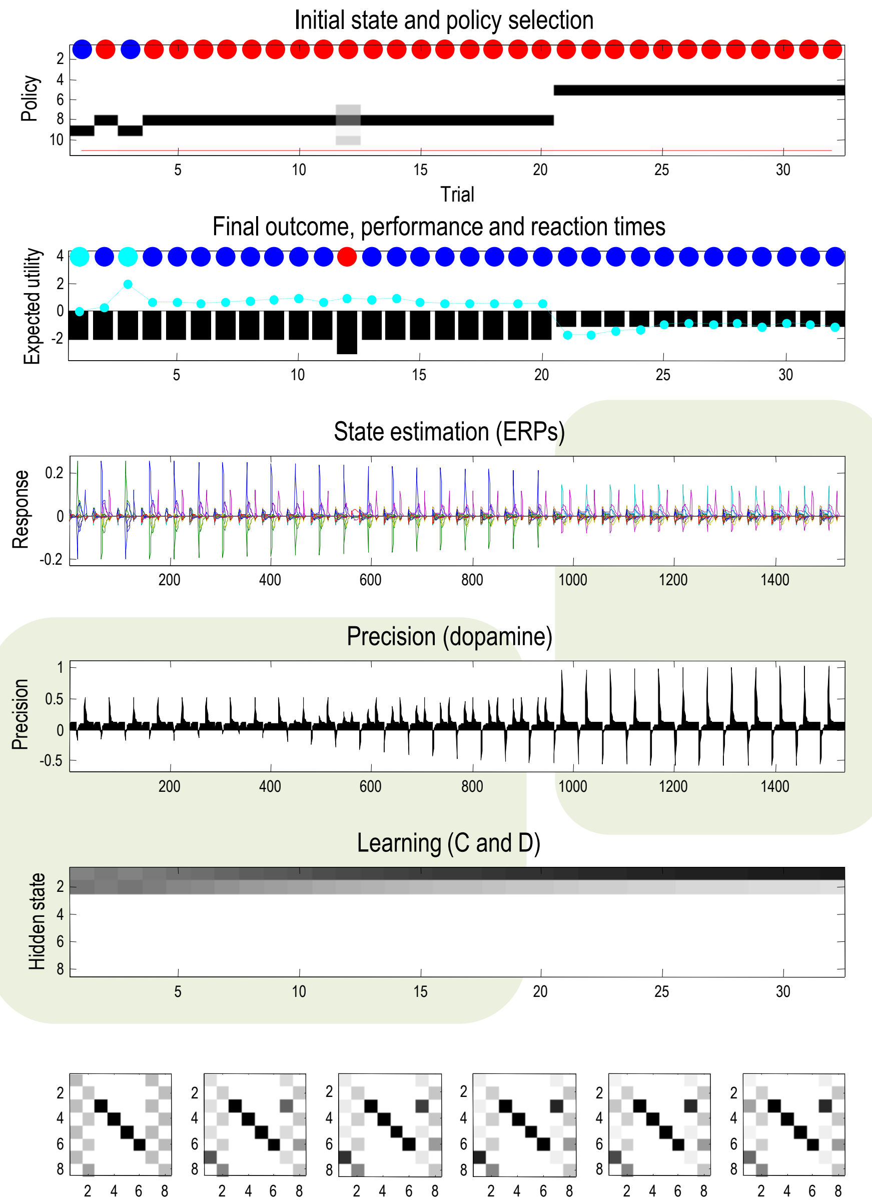
Functional anatomy





Simulated neuronal behaviour





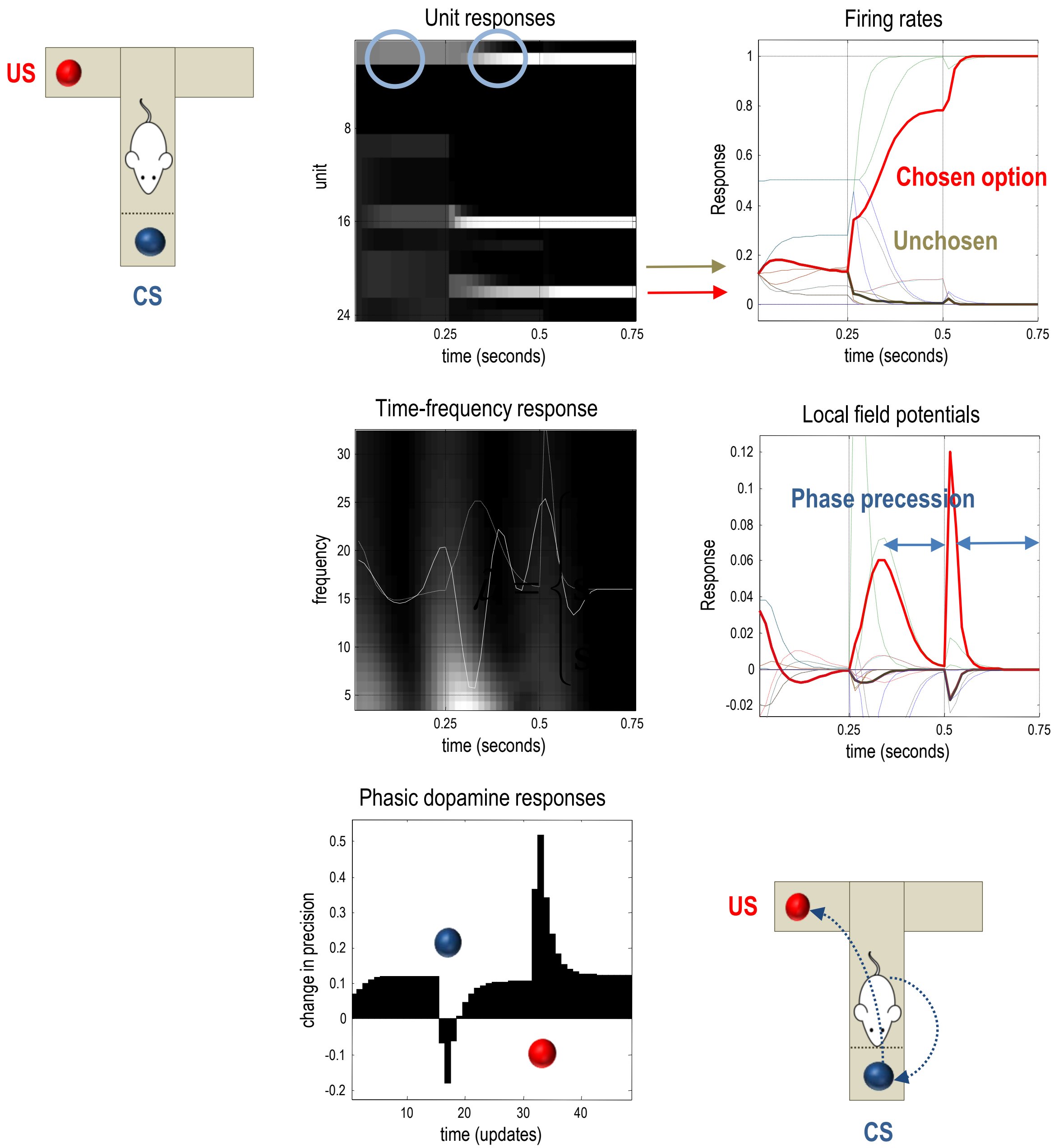
$$\left.\begin{aligned} \mathbf{s}_1^\pi &= \sigma(\hat{\mathbf{A}}\cdot o_1 + \hat{\mathbf{D}} + \hat{\mathbf{B}}_1^\pi \cdot \mathbf{s}_2^\pi) \\ \mathbf{s}_2^\pi &= \sigma(\hat{\mathbf{A}}\cdot o_2 + \hat{\mathbf{B}}_1^\pi \mathbf{s}_1^\pi + \hat{\mathbf{B}}_2^\pi \cdot \mathbf{s}_3^\pi) \\ &\vdots \end{aligned}\right\} \text{Perceptual inference (state estimation)}$$

$$\left.\begin{aligned} \boldsymbol{\pi} &= \sigma(\hat{\mathbf{E}} - \mathbf{F} - \boldsymbol{\gamma}\cdot\hat{\mathbf{F}}) \\ \boldsymbol{\beta} &= \beta + (\boldsymbol{\pi} - \boldsymbol{\pi}_0)\cdot\hat{\mathbf{F}} \\ \boldsymbol{\gamma} &= 1/\boldsymbol{\beta} \end{aligned}\right\} \text{Policy selection}$$

$$\left.\begin{aligned} \hat{\mathbf{C}} &= \psi(\mathbf{c}) - \psi(\mathbf{c}^0) \quad &: \mathbf{c} = c + \textstyle\sum_\tau \mathbf{s}_\tau^0 \otimes \mathbf{s}_{\tau-1}^0 \\ \hat{\mathbf{D}} &= \psi(\mathbf{d}) - \psi(\mathbf{d}^0) \quad &: \mathbf{d} = d + \mathbf{s}_1 \end{aligned}\right\} \text{Learning}$$

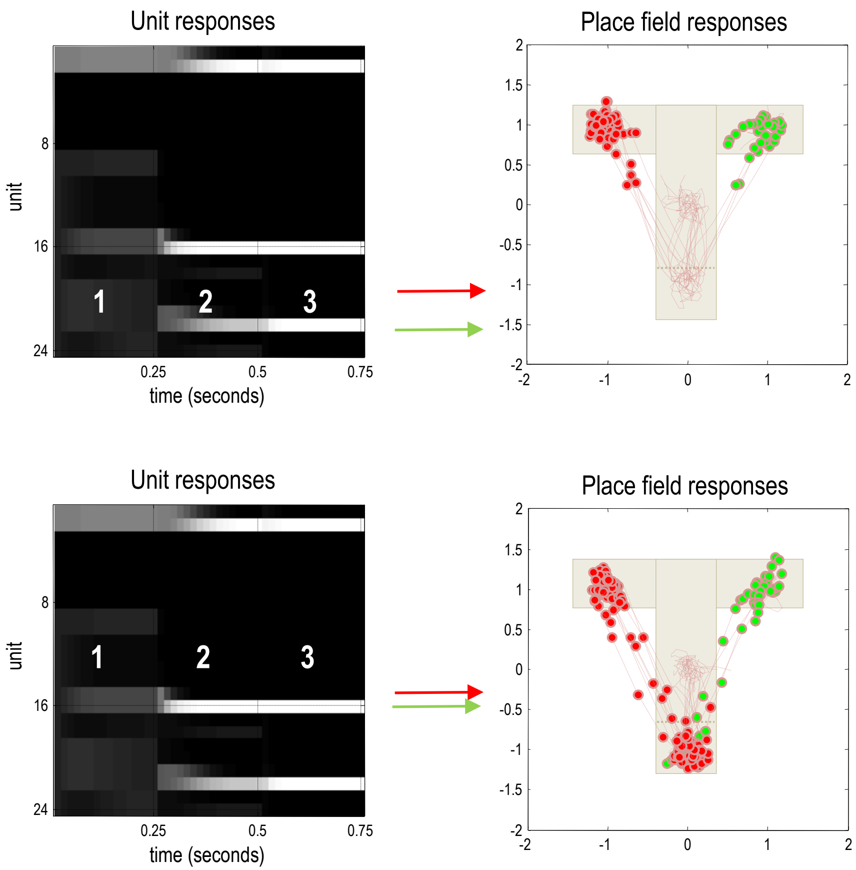
Overview

Action and the path of least resistance
Generative models and active inference
From principles to process theories
Some empirical predictions

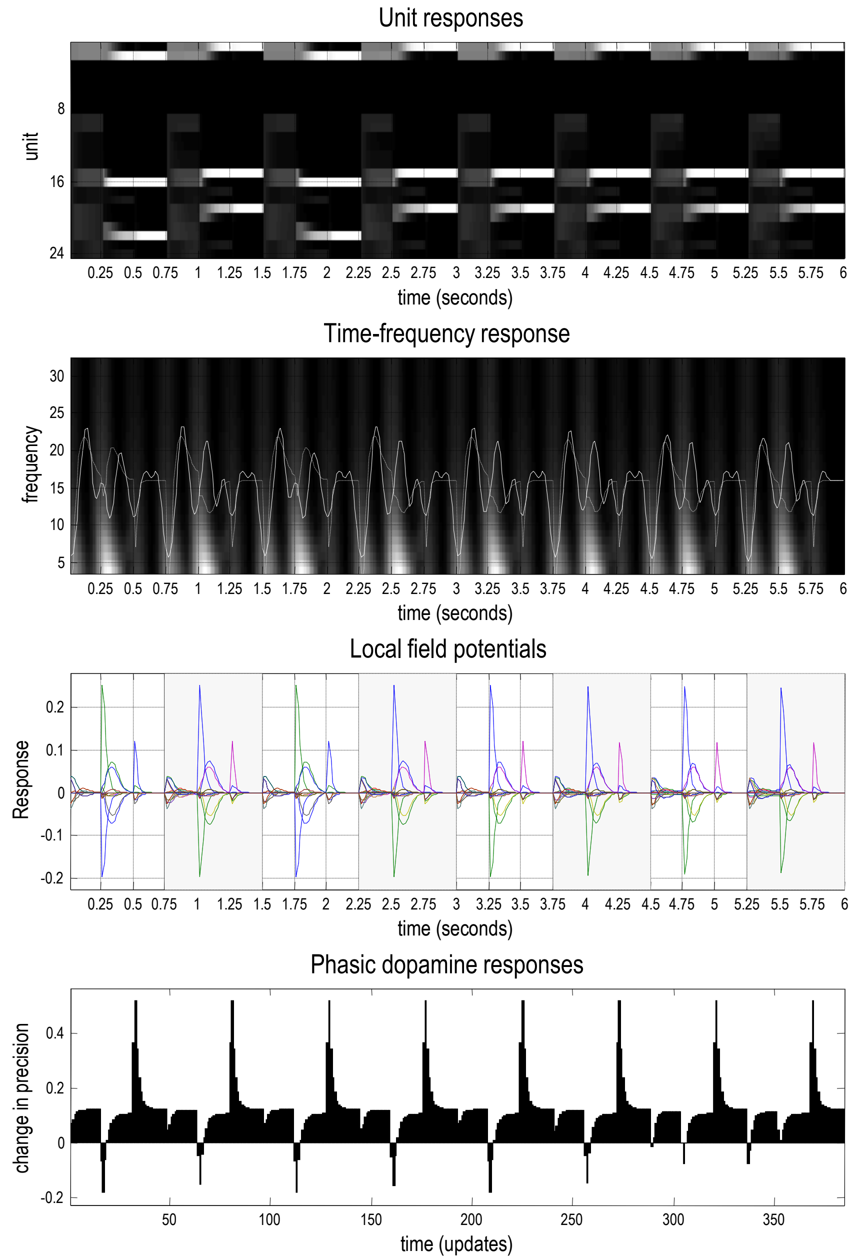


Evidence accumulation
Phase precession
Place cell activity
Cross frequency coupling
Perceptual categorisation
Oddball (MMN) responses
Violation (P300) responses
Dopamine transfer
Reversal learning
Devaluation

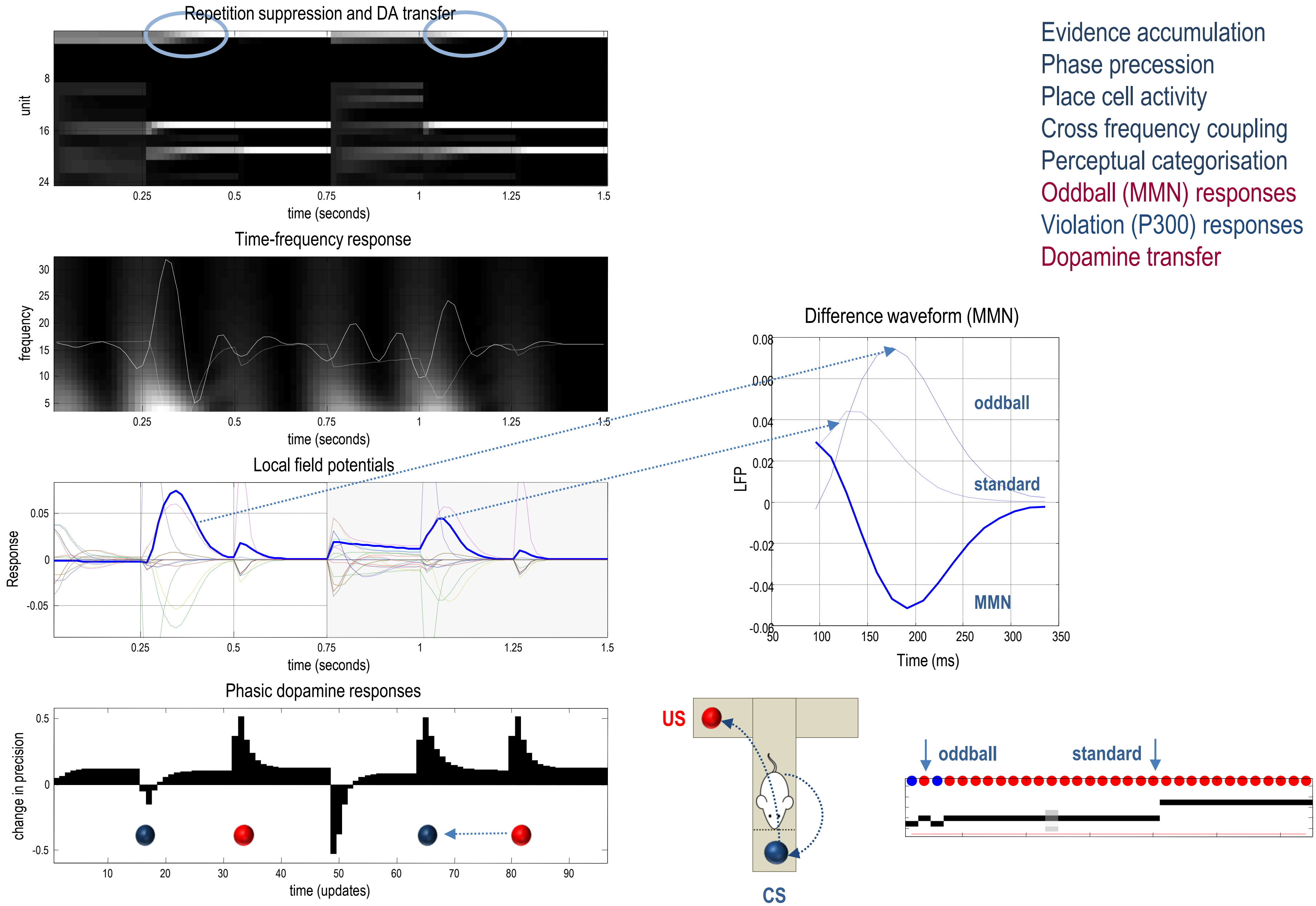
$$\tilde{\mu} = \begin{cases} \mathbf{s}_{t=1} \\ \mathbf{s}_{t=2} \\ \mathbf{s}_{t=3} \end{cases}$$



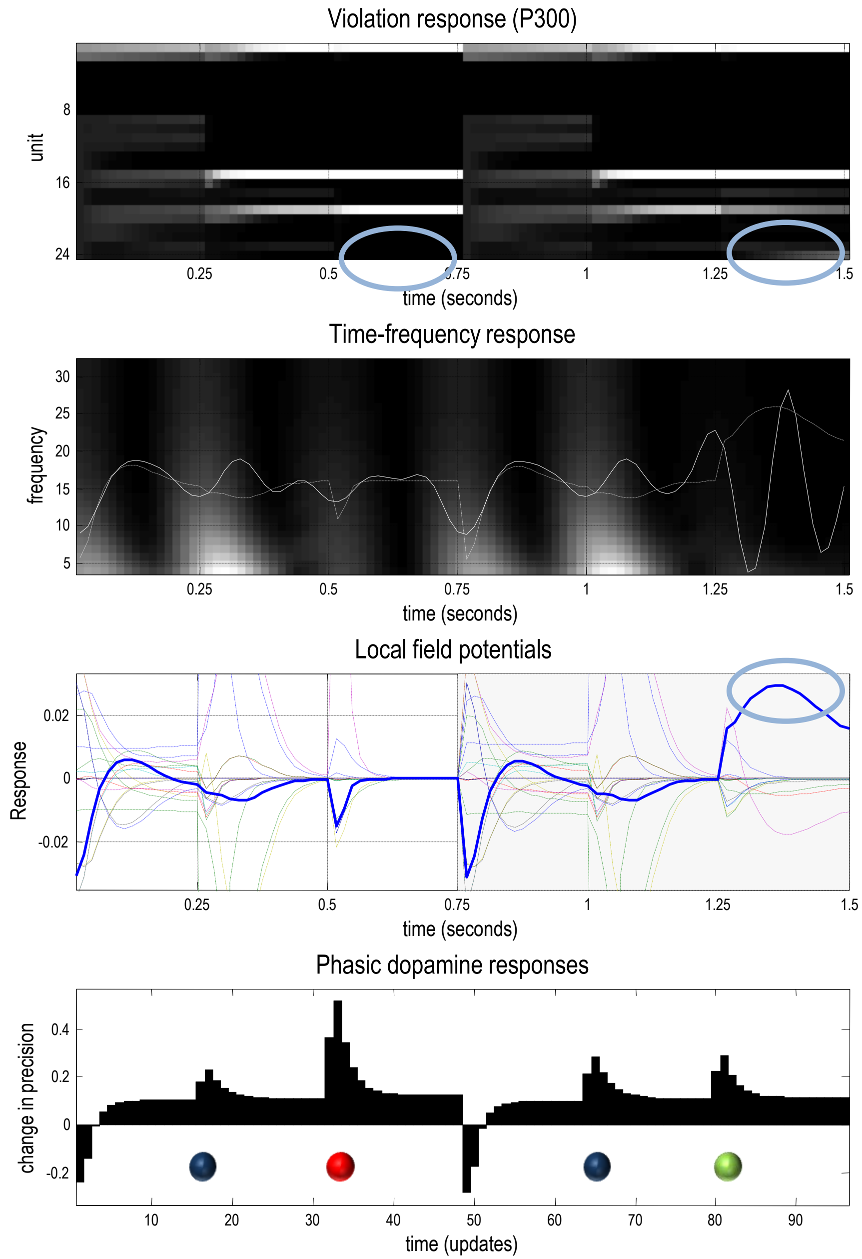
Evidence accumulation
Phase precession
Place cell activity
Cross frequency coupling
Perceptual categorisation
Oddball (MMN) responses
Violation (P300) responses
Dopamine transfer
Reversal learning
Devaluation

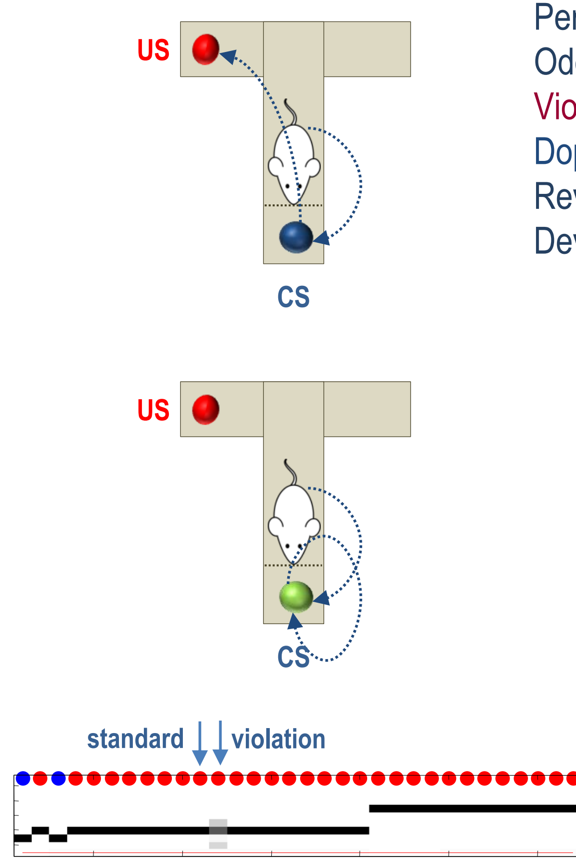


Evidence accumulation
Phase precession
Place cell activity
Cross frequency coupling
Perceptual categorisation
Oddball (MMN) responses
Violation (P300) responses
Dopamine transfer
Reversal learning
Devaluation

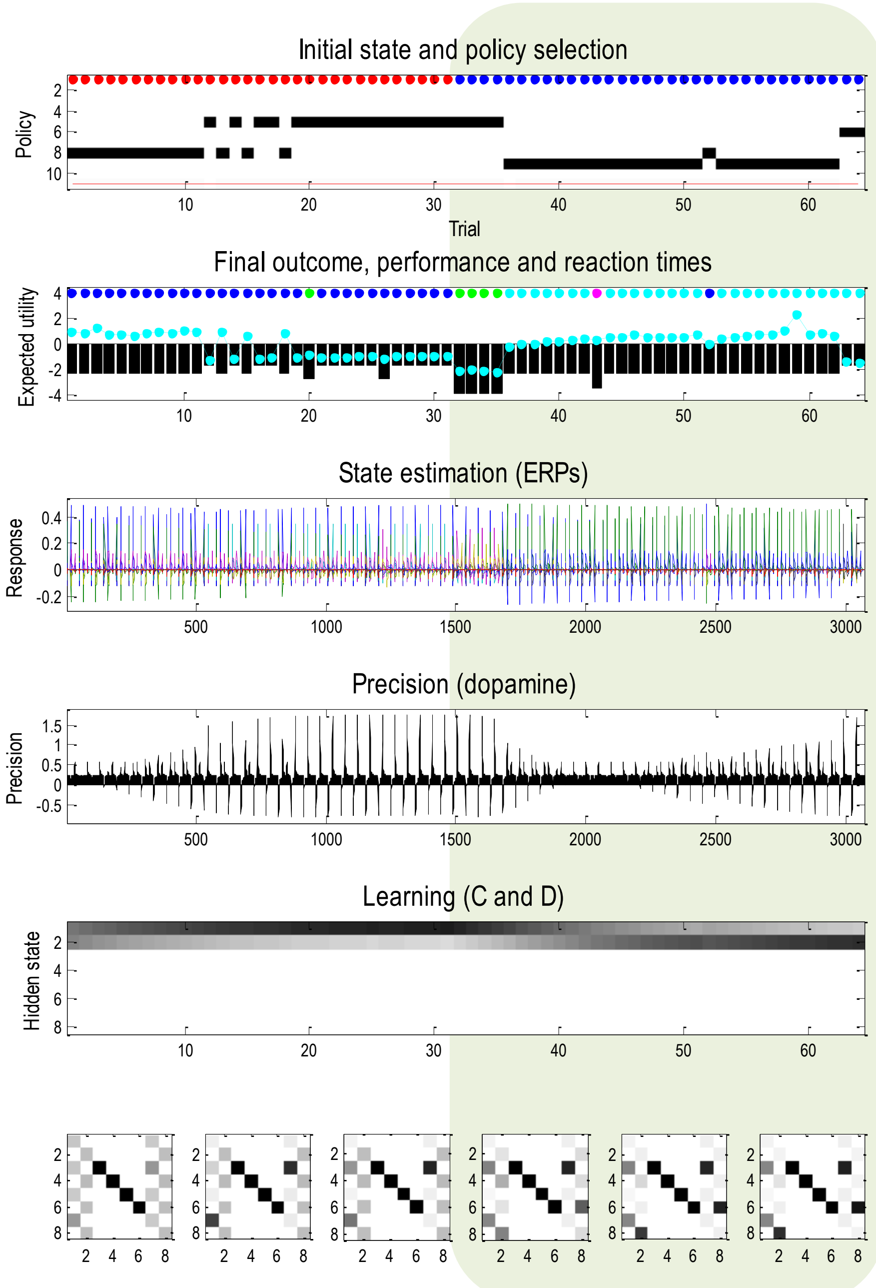


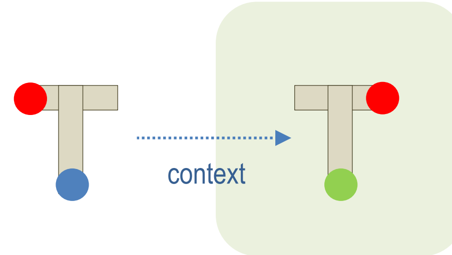
Repetition suppression and DA transfer
unit
time (seconds)
Time-frequency response
frequency
time (seconds)
Local field potentials
Response
time (seconds)
Phasic dopamine responses
change in precision
time (updates)
Evidence accumulation
Phase precession
Place cell activity
Cross frequency coupling
Perceptual categorisation
Oddball (MMN) responses
Violation (P300) responses
Dopamine transfer
Difference waveform (MMN)
LFP
Time (ms)
oddball
standard
MMN
US
CS
oddball
standard



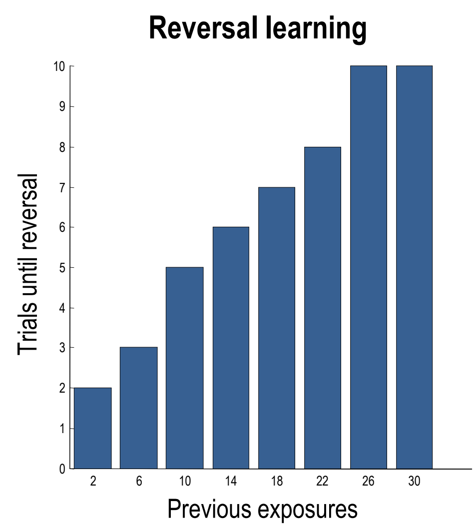


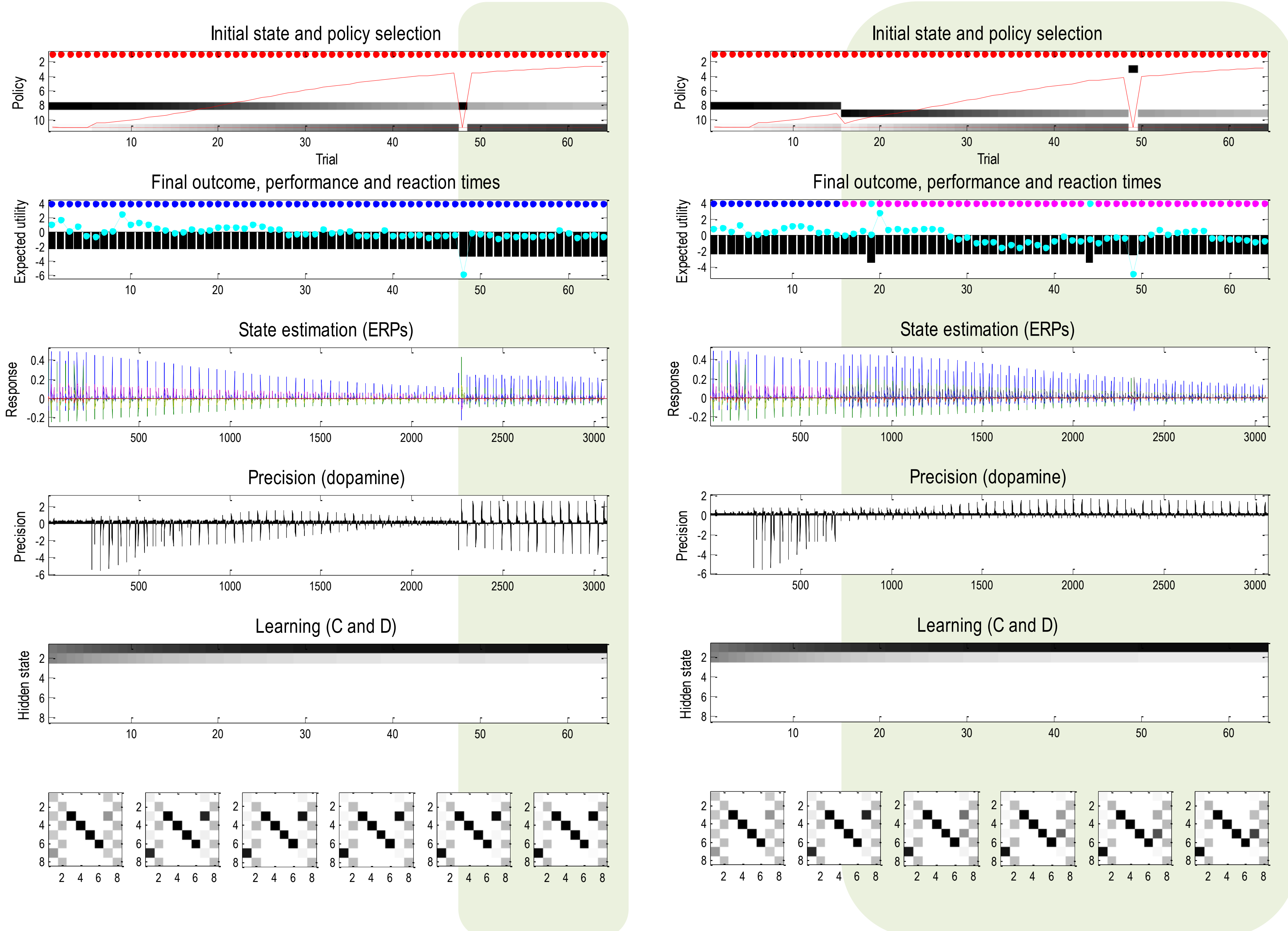
Evidence accumulation
Phase precession
Place cell activity
Cross frequency coupling
Perceptual categorisation
Oddball (MMN) responses
Violation (P300) responses
Dopamine transfer
Reversal learning
Devaluation





Evidence accumulation
Phase precession
Place cell activity
Cross frequency coupling
Perceptual categorisation
Oddball (MMN) responses
Violation (P300) responses
Dopamine transfer
Reversal learning
Devaluation





Initial state and policy selection
Policy
Trial
Final outcome, performance and reaction times
Expected utility
State estimation (ERPs)
Response
Precision (dopamine)
Precision
Learning (C and D)
Hidden state
Initial state and policy selection
Policy
Trial
Final outcome, performance and reaction times
Expected utility
State estimation (ERPs)
Response
Precision (dopamine)
Precision
Learning (C and D)
Hidden state

summary

Free energy principle

Markov decision processes

Active inference (for MDP)

Learning and inference via minimisation of (variational) free energy; under the prior belief that (sequential) policies minimise expected free energy

Explanatory scope

- Evidence accumulation
- Phase precession
- Place cell activity
- Cross frequency coupling
- Perceptual categorisation
- Oddball (MMN) responses
- Violation (P300) responses
- Dopamine transfer
- Reversal learning
- Devaluation

summary

Free energy principle

Markov decision processes

Active inference (for MDP)

Learning and inference via minimisation of (variational) free energy; under the prior belief that (sequential) policies minimise expected free energy

Simulations and optimal design

spm_MDP_VB

Estimation of prior beliefs (& preferences) from choice behaviour

spm_dcm_mdp

Group analysis and comparison

spm_dcm_peb

Computational fMRI and EEG

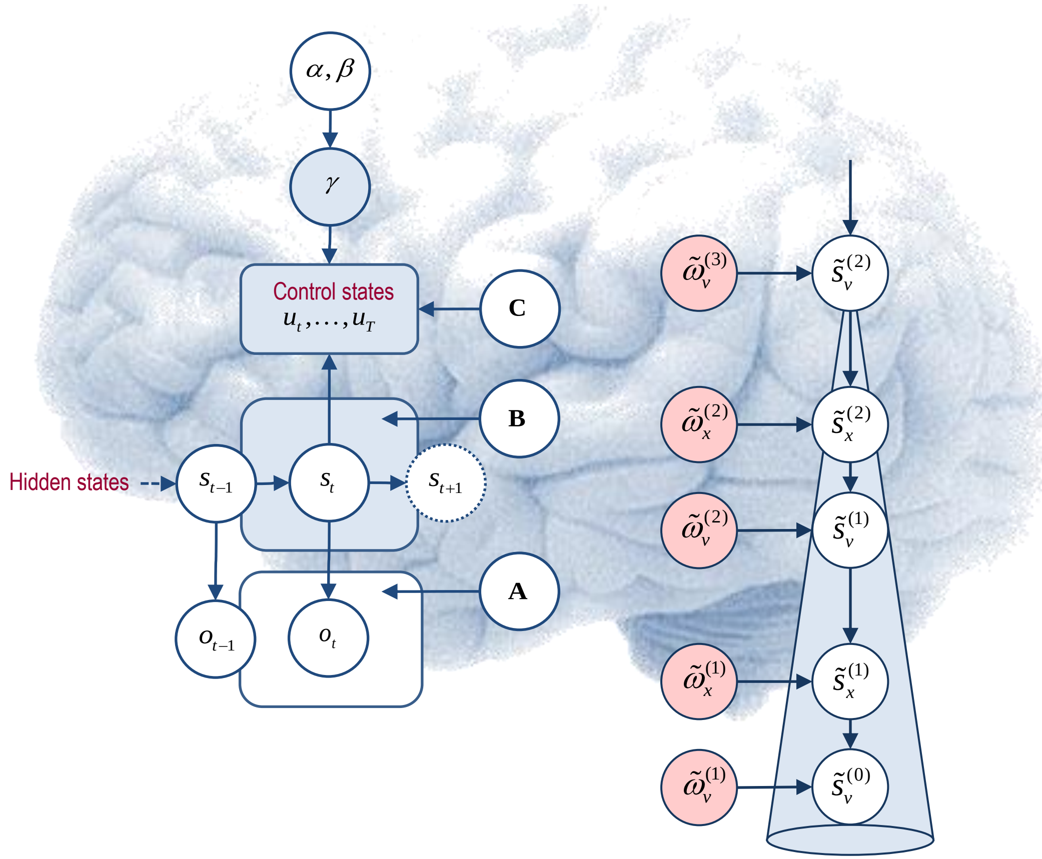
spm_dcm_fmri

DCM

Cross Validation and posterior prediction

spm_dcm_loo

Continuous or discrete state space models or both?



Thank you

And thanks to collaborators:

Rick Adams
Ryszard Auksztulewicz
Andre Bastos
Sven Bestmann
Harriet Brown
Jean Daunizeau
Mark Edwards
Chris Frith
Thomas FitzGerald
Xiaosi Gu
Stefan Kiebel
James Kilner
Christoph Mathys
Jérémie Mattout
Rosalyn Moran
Dimitri Ognibene
Sasha Ondobaka
Will Penny
Giovanni Pezzulo
Lisa Quattrocki Knight
Francesco Rigoli
Klaas Stephan
Philipp Schwartenbeck

And colleagues:

Micah Allen
Felix Blankenburg
Andy Clark
Peter Dayan
Ray Dolan
Allan Hobson
Paul Fletcher
Pascal Fries
Geoffrey Hinton
James Hopkins
Jakob Hohwy
Mateus Joffily
Henry Kennedy
Simon McGregor
Read Montague
Tobias Nolte
Anil Seth
Mark Solms
Paul Verschure

And many others