

Two Singlet scalar Dark Matter
Lagrangian, Rotations and Interactions for eigenstates 'GaugeES'
including Renormalization Group Equations

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1 Fields

1.1 Gauge Fields

Name	$SU(N)$	Coupling	Name
B	$U(1)$	g_1	hypercharge
W	$SU(2)$	g_2	left
g	$SU(3)$	g_3	color

1.2 Matter Superfields

Name	Spin	Generations	$(U(1) \otimes SU(2) \otimes SU(3))$
H	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
S1	0	1	$(0, \mathbf{1}, \mathbf{1})$
S2	0	1	$(0, \mathbf{1}, \mathbf{1})$
q	$\frac{1}{2}$	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
l	$\frac{1}{2}$	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
d	$\frac{1}{2}$	3	$(\frac{1}{3}, \mathbf{1}, \mathbf{\bar{3}})$
u	$\frac{1}{2}$	3	$(-\frac{2}{3}, \mathbf{1}, \mathbf{\bar{3}})$
e	$\frac{1}{2}$	3	$(1, \mathbf{1}, \mathbf{1})$

2 Lagrangian

2.1 Input Lagrangian for Eigenstates GaugeES

$$\begin{aligned}
L = & -\frac{1}{2}m_1^2 S_1^2 - \lambda_{14} S_1^4 - \lambda_{31} S_1^3 S_2 - \frac{1}{2}m_2^2 S_2^2 - \lambda_{22} S_1^2 S_2^2 - \lambda_{13} S_1 S_2^3 - \lambda_{24} S_2^4 \\
& - \mu^2 |H^0|^2 - \lambda_{H1} S_1^2 |H^0|^2 - \lambda_{H12} S_1 S_2 |H^0|^2 - \lambda_{H2} S_2^2 |H^0|^2 - \mu^2 |H^+|^2 \\
& - \lambda_{H1} S_1^2 |H^+|^2 - \lambda_{H12} S_1 S_2 |H^+|^2 - \lambda_{H2} S_2^2 |H^+|^2 - \frac{1}{2}\lambda |H^0|^4 - \frac{1}{2}\lambda |H^+|^4 - H^+ \lambda |H^0|^2 H^- \\
& - H^0 d_{L,k\gamma}^* Y_{d,jk}^* \delta_{\beta\gamma} d_{R,j\beta} - H^+ u_{L,k\gamma}^* Y_{d,jk}^* \delta_{\beta\gamma} d_{R,j\beta} - H^0 e_{L,k}^* Y_{e,jk}^* e_{R,j} \\
& - H^+ \nu_{L,k}^* Y_{e,jk}^* e_{R,j} - H^- d_{L,k\gamma}^* Y_{u,jk}^* \delta_{\beta\gamma} u_{R,j\beta} + H^{0,*} u_{L,k\gamma}^* Y_{u,jk}^* \delta_{\beta\gamma} u_{R,j\beta} \\
& - H^{0,*} d_{R,j\beta}^* \delta_{\beta\gamma} d_{L,k\gamma} Y_{d,jk} - H^- d_{R,j\beta}^* \delta_{\beta\gamma} u_{L,k\gamma} Y_{d,jk} - H^{0,*} e_{R,j}^* e_{L,k} Y_{e,jk} \\
& - H^- e_{R,j}^* \nu_{L,k} Y_{e,jk} - H^+ u_{R,j\beta}^* \delta_{\beta\gamma} d_{L,k\gamma} Y_{u,jk} + H^0 u_{R,j\beta}^* \delta_{\beta\gamma} u_{L,k\gamma} Y_{u,jk}
\end{aligned} \tag{1}$$

2.2 Gauge fixing terms

2.2.1 Gauge fixing terms for eigenstates 'GaugeES'

$$L_{GF} = -\frac{1}{2}|\partial_\mu B|^2\xi_B^{-1} - \frac{1}{2}|\partial_\mu g|^2\xi_g^{-1} - \frac{1}{2}|\partial_\mu W|^2\xi_W^{-1} \quad (2)$$

2.2.2 Gauge fixing terms for eigenstates 'EWSB'

$$\begin{aligned} L_{GF} = & -\frac{1}{2}|\partial_\mu g|^2\xi_g^{-1} - \frac{1}{2}|\partial_\mu \gamma|^2\xi_\gamma^{-1} - \left| -\frac{i}{2}g_2 H^+ v \xi_{W^+} + \partial_\mu W^+ \right|^2 \xi_{W^+}^{-1} \\ & - \frac{1}{2} \left| -\frac{1}{2}A^0 v \xi_Z \left(g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) + \partial_\mu Z \right|^2 \xi_Z^{-1} \end{aligned} \quad (3)$$

2.3 Fields integrated out

None

3 Renormalization Group Equations

3.1 Anomalous Dimensions for fermions

$$\gamma_d^{(1)} = \frac{1}{15} \left(20g_3^2 + g_1^2 \right) \mathbf{1Xi} \delta_{1,\text{col}2} + Y_d Y_d^\dagger \quad (4)$$

$$\begin{aligned} \gamma_d^{(2)} = & \frac{1}{120} \left(-2\delta_{1,\text{col}2} \left(15 \left(Y_d Y_d^\dagger Y_d Y_d^\dagger + Y_d Y_u^\dagger Y_u Y_d^\dagger \right) + \mathbf{1} \left(16g_1^2 g_3^2 - 20g_3^4 \left(24\text{Xi} + 3\text{Xi}^2 + 43 \right) + 25g_1^4 \right) \right) \right. \\ & \left. + Y_d Y_d^\dagger \left(133g_1^2 - 180\text{Tr} \left(Y_e Y_e^\dagger \right) - 540\text{Tr} \left(Y_d Y_d^\dagger \right) - 640g_3^2 + 765g_2^2 \right) \right) \end{aligned} \quad (5)$$

$$\gamma_e^{(1)} = \frac{3}{5} g_1^2 \mathbf{1Xi} + Y_e Y_e^\dagger \quad (6)$$

$$\gamma_e^{(2)} = \frac{1}{200} \left(-15Y_e Y_e^\dagger \left(11g_1^2 + 20\text{Tr} \left(Y_e Y_e^\dagger \right) + 60\text{Tr} \left(Y_d Y_d^\dagger \right) - 85g_2^2 \right) - 50Y_e Y_e^\dagger Y_e Y_e^\dagger - 846g_1^4 \mathbf{1} \right) \quad (7)$$

$$\gamma_u^{(1)} = \frac{4}{15} \left(5g_3^2 + g_1^2 \right) \mathbf{1Xi} \delta_{1,\text{col}2} + Y_u Y_u^\dagger \quad (8)$$

$$\begin{aligned} \gamma_u^{(2)} = & \frac{1}{600} \left(-2\delta_{1,\text{col}2} \left(41 \left(131g_1^4 - 25g_3^4 \left(24\text{Xi} + 3\text{Xi}^2 + 43 \right) + 80g_1^2 g_3^2 \right) + 75 \left(Y_u Y_d^\dagger Y_d Y_u^\dagger + Y_u Y_u^\dagger Y_u Y_u^\dagger \right) \right) \right. \\ & \left. + 5Y_u Y_u^\dagger \left(-180\text{Tr} \left(Y_e Y_e^\dagger \right) + 49g_1^2 - 540\text{Tr} \left(Y_d Y_d^\dagger \right) - 640g_3^2 + 765g_2^2 \right) \right) \end{aligned} \quad (9)$$

$$\gamma_l^{(1)} = \frac{1}{2} Y_e^T Y_e^* + \frac{3}{20} \left(5g_2^2 + g_1^2 \right) \mathbf{1Xi} \delta_{1,\text{lef}2} \quad (10)$$

$$\begin{aligned} \gamma_l^{(2)} = & \frac{1}{160} \left(-6 \left(11g_1^2 + 35g_2^2 \right) Y_e^T Y_e^* - 40Y_e^T Y_e^* Y_e^T Y_e^* \right. \\ & \left. - 3\delta_{1,\text{lef}2} \left(-8Y_e^T Y_e^* \left(-15\text{Tr} \left(Y_d Y_d^\dagger \right) + 45g_2^2 - 5\text{Tr} \left(Y_e Y_e^\dagger \right) + 9g_1^2 \right) + \mathbf{1} \left(18g_1^2 g_2^2 + 51g_1^4 - 5g_2^4 \left(32\text{Xi} + 4\text{Xi}^2 + 41 \right) \right) \right) \right) \end{aligned} \quad (11)$$

$$\gamma_q^{(1)} = \frac{1}{60} \delta_{\text{lef}1,\text{lef}2} \left(30 \left(\left(\epsilon^{1\text{lef}1} \epsilon^{1\text{lef}2} + \epsilon^{2\text{lef}1} \epsilon^{2\text{lef}2} \right) Y_u^T Y_u^* + Y_d^T Y_d^* \right) + \left(45g_2^2 + 80g_3^2 + g_1^2 \right) \mathbf{1Xi} \delta_{1,\text{col}2} \right) \quad (12)$$

$$\begin{aligned}
\gamma_q^{(2)} = & -\frac{1}{2400}\delta_{\text{lef1,lef2}}\left(\delta_{1,\text{col2}}\left(\mathbf{1}\left(10g_1^2\left(16g_3^2+9g_2^2\right)+247g_1^4-25\left(-288g_2^2g_3^2+32g_3^4\left(24\text{Xi}+3\text{Xi}^2+43\right)+9g_2^4\left(32\text{Xi}+4\text{Xi}^2+41\right)\right)\right)\right.\right. \\
& +600\left(\left(\epsilon^{1\text{lef1}}\epsilon^{1\text{lef2}}+\epsilon^{2\text{lef1}}\epsilon^{2\text{lef2}}\right)Y_u^TY_u^*Y_u^TY_u^*+Y_d^TY_d^*Y_d^TY_d^*\right) \\
& -10\left(Y_u^TY_u^*\left(\epsilon^{1\text{lef1}}\left(162\left(5g_2^2+g_1^2\right)\epsilon^{2\text{lef2}}+\epsilon^{1\text{lef2}}\left(139g_1^2-180\text{Tr}\left(Y_eY_e^\dagger\right)+495g_2^2-540\text{Tr}\left(Y_dY_d^\dagger\right)-640g_3^2\right)\right)\right.\right. \\
& +\epsilon^{2\text{lef1}}\left(162\left(5g_2^2+g_1^2\right)\epsilon^{1\text{lef2}}+\epsilon^{2\text{lef2}}\left(139g_1^2-180\text{Tr}\left(Y_eY_e^\dagger\right)+495g_2^2-540\text{Tr}\left(Y_dY_d^\dagger\right)-640g_3^2\right)\right) \\
& \left.\left.+Y_d^TY_d^*\left(-1080\text{Tr}\left(Y_dY_d^\dagger\right)+2925g_2^2-360\text{Tr}\left(Y_eY_e^\dagger\right)+637g_1^2-640g_3^2\right)\right)\right) \quad (13)
\end{aligned}$$

3.2 Anomalous Dimensions for real components of scalars

$$\gamma_{\text{IH}}^{(1)} = -3\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) + 3\text{Tr}\left(Y_dY_d^\dagger\right) + \frac{3}{20}\left(5g_2^2+g_1^2\right)\left(-3+\text{Xi}\right) + \text{Tr}\left(Y_eY_e^\dagger\right) \quad (14)$$

$$\begin{aligned}
\gamma_{\text{IH}}^{(2)} = & \frac{1293}{800}g_1^4 + \frac{27}{80}g_1^2g_2^2 - \frac{271}{32}g_2^4 + \frac{3}{2}\lambda^2 + \lambda_{H1}^2 + \frac{1}{2}\lambda_{H12}^2 + \lambda_{H2}^2 + 3g_2^4\text{Xi} + \frac{3}{8}g_2^4\text{Xi}^2 \\
& + \frac{5}{8}\left(32g_3^2+9g_2^2+g_1^2\right)\text{Tr}\left(Y_dY_d^\dagger\right) + \frac{15}{8}\left(g_1^2+g_2^2\right)\text{Tr}\left(Y_eY_e^\dagger\right) - \frac{17}{8}g_1^2\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - \frac{45}{8}g_2^2\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) - 20g_3^2\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) + \frac{3}{4}\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) \\
& - \frac{3}{4}\epsilon^{2\text{lef2}}\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) - \frac{9}{4}\text{Tr}\left(Y_eY_e^\dagger Y_eY_e^\dagger\right) + \frac{27}{4}\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger Y_uY_u^\dagger\right) \quad (15)
\end{aligned}$$

$$\gamma_{\text{rH}}^{(1)} = -3\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) + 3\text{Tr}\left(Y_dY_d^\dagger\right) + \frac{3}{20}\left(5g_2^2+g_1^2\right)\left(-3+\text{Xi}\right) + \text{Tr}\left(Y_eY_e^\dagger\right) \quad (16)$$

$$\begin{aligned}
\gamma_{\text{rH}}^{(2)} = & \frac{1293}{800}g_1^4 + \frac{27}{80}g_1^2g_2^2 - \frac{271}{32}g_2^4 + \frac{3}{2}\lambda^2 + \lambda_{H1}^2 + \frac{1}{2}\lambda_{H12}^2 + \lambda_{H2}^2 + 3g_2^4\text{Xi} + \frac{3}{8}g_2^4\text{Xi}^2 \\
& + \frac{5}{8}\left(32g_3^2+9g_2^2+g_1^2\right)\text{Tr}\left(Y_dY_d^\dagger\right) + \frac{15}{8}\left(g_1^2+g_2^2\right)\text{Tr}\left(Y_eY_e^\dagger\right) - \frac{17}{8}g_1^2\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - \frac{45}{8}g_2^2\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) - 20g_3^2\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) + \frac{3}{4}\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) \\
& - \frac{3}{4}\epsilon^{2\text{lef2}}\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) - \frac{9}{4}\text{Tr}\left(Y_eY_e^\dagger Y_eY_e^\dagger\right) + \frac{27}{4}\epsilon^{2\text{lef2}}\text{Tr}\left(Y_uY_u^\dagger Y_uY_u^\dagger\right) \quad (17)
\end{aligned}$$

$$\gamma_{\text{rS1}}^{(1)} = 0 \quad (18)$$

$$\gamma_{\text{rS1}}^{(2)} = 3\lambda_{13}^2 + 48\lambda_{14}^2 + 4\lambda_{22}^2 + 4\lambda_{H1}^2 + 9\lambda_{31}^2 + \lambda_{H12}^2 \quad (19)$$

$$\gamma_{\text{rS2}}^{(1)} = 0 \quad (20)$$

$$\gamma_{\text{rS2}}^{(2)} = 3\lambda_{31}^2 + 48\lambda_{24}^2 + 4\lambda_{22}^2 + 4\lambda_{H2}^2 + 9\lambda_{13}^2 + \lambda_{H12}^2 \quad (21)$$

3.3 Gauge Couplings

$$\beta_{g_1}^{(1)} = \frac{41}{10}g_1^3 \quad (22)$$

$$\beta_{g_1}^{(2)} = \frac{1}{50}g_1^3\left(135g_2^2+199g_1^2-25\text{Tr}\left(Y_dY_d^\dagger\right)+440g_3^2-75\text{Tr}\left(Y_eY_e^\dagger\right)-85\text{Tr}\left(Y_uY_u^\dagger\right)\right) \quad (23)$$

$$\beta_{g_2}^{(1)} = -\frac{19}{6}g_2^3 \quad (24)$$

$$\beta_{g_2}^{(2)} = \frac{1}{30}g_2^3 \left(-15\text{Tr}(Y_e Y_e^\dagger) + 175g_2^2 + 27g_1^2 + 360g_3^2 - 45\text{Tr}(Y_d Y_d^\dagger) - 45\text{Tr}(Y_u Y_u^\dagger) \right) \quad (25)$$

$$\beta_{g_3}^{(1)} = -7g_3^3 \quad (26)$$

$$\beta_{g_3}^{(2)} = -\frac{1}{10}g_3^3 \left(-11g_1^2 + 20\text{Tr}(Y_d Y_d^\dagger) + 20\text{Tr}(Y_u Y_u^\dagger) + 260g_3^2 - 45g_2^2 \right) \quad (27)$$

3.4 Quartic scalar couplings

$$\beta_{\lambda_{14}}^{(1)} = 2\lambda_{22}^2 + 2\lambda_{H1}^2 + 72\lambda_{14}^2 + 9\lambda_{31}^2 \quad (28)$$

$$\begin{aligned} \beta_{\lambda_{14}}^{(2)} = & +12\lambda_{13}^2\lambda_{14} - 3264\lambda_{14}^3 - 36\lambda_{13}^2\lambda_{22} - 80\lambda_{14}\lambda_{22}^2 - 32\lambda_{22}^3 - 66\lambda_{13}\lambda_{22}\lambda_{31} + 12\lambda_{13}\lambda_{24}\lambda_{31} - 816\lambda_{14}\lambda_{31}^2 \\ & - 174\lambda_{22}\lambda_{31}^2 + \frac{12}{5}g_1^2\lambda_{H1}^2 + 12g_2^2\lambda_{H1}^2 - 80\lambda_{14}\lambda_{H1}^2 - 16\lambda_{H1}^3 - 22\lambda_{31}\lambda_{H1}\lambda_{H12} + 4\lambda_{14}\lambda_{H12}^2 - 4\lambda_{22}\lambda_{H12}^2 \\ & - 4\lambda_{H1}\lambda_{H12}^2 + 2\lambda_{31}\lambda_{H12}\lambda_{H2} - 12\lambda_{H1}^2\text{Tr}(Y_d Y_d^\dagger) - 4\lambda_{H1}^2\text{Tr}(Y_e Y_e^\dagger) - 12\lambda_{H1}^2\text{Tr}(Y_u Y_u^\dagger) \end{aligned} \quad (29)$$

$$\beta_{\lambda_{31}}^{(1)} = 4 \left(18\lambda_{14}\lambda_{31} + 3\lambda_{13}\lambda_{22} + 6\lambda_{22}\lambda_{31} + \lambda_{H1}\lambda_{H12} \right) \quad (30)$$

$$\begin{aligned} \beta_{\lambda_{31}}^{(2)} = & -\frac{2}{5} \left(270\lambda_{13}^3 + 660\lambda_{13}\lambda_{14}\lambda_{22} + 690\lambda_{13}\lambda_{22}^2 - 120\lambda_{13}\lambda_{14}\lambda_{24} + 660\lambda_{13}\lambda_{22}\lambda_{24} + 225\lambda_{13}^2\lambda_{31} + 8160\lambda_{14}^2\lambda_{31} + 3480\lambda_{14}\lambda_{22}\lambda_{31} \right. \\ & + 890\lambda_{22}^2\lambda_{31} + 720\lambda_{22}\lambda_{24}\lambda_{31} - 120\lambda_{24}^2\lambda_{31} + 810\lambda_{13}\lambda_{31}^2 + 735\lambda_{31}^3 + 90\lambda_{31}\lambda_{H1}^2 - 12g_1^2\lambda_{H1}\lambda_{H12} - 60g_2^2\lambda_{H1}\lambda_{H12} \\ & + 220\lambda_{14}\lambda_{H1}\lambda_{H12} + 70\lambda_{22}\lambda_{H1}\lambda_{H12} + 80\lambda_{H1}^2\lambda_{H12} + 30\lambda_{13}\lambda_{H12}^2 + 20\lambda_{31}\lambda_{H12}^2 + 10\lambda_{H12}^3 + 120\lambda_{31}\lambda_{H1}\lambda_{H2} - 20\lambda_{14}\lambda_{H12}\lambda_{H2} \\ & + 30\lambda_{22}\lambda_{H12}\lambda_{H2} + 40\lambda_{H1}\lambda_{H12}\lambda_{H2} - 10\lambda_{31}\lambda_{H2}^2 + 60\lambda_{H1}\lambda_{H12}\text{Tr}(Y_d Y_d^\dagger) + 20\lambda_{H1}\lambda_{H12}\text{Tr}(Y_e Y_e^\dagger) \\ & \left. + 60\lambda_{H1}\lambda_{H12}\text{Tr}(Y_u Y_u^\dagger) \right) \end{aligned} \quad (31)$$

$$\beta_{\lambda_{22}}^{(1)} = 2 \left(12\lambda_{14}\lambda_{22} + 12\lambda_{22}\lambda_{24} + 2\lambda_{H1}\lambda_{H2} + 8\lambda_{22}^2 + 9\lambda_{13}^2 + 9\lambda_{13}\lambda_{31} + 9\lambda_{31}^2 + \lambda_{H12}^2 \right) \quad (32)$$

$$\begin{aligned} \beta_{\lambda_{22}}^{(2)} = & -\frac{2}{5} \left(540\lambda_{13}^2\lambda_{14} + 1335\lambda_{13}^2\lambda_{22} + 1200\lambda_{14}^2\lambda_{22} + 1440\lambda_{14}\lambda_{22}^2 + 360\lambda_{22}^3 + 2610\lambda_{13}^2\lambda_{24} + 1440\lambda_{22}^2\lambda_{24} + 1200\lambda_{22}\lambda_{24}^2 \right. \\ & + 990\lambda_{13}\lambda_{14}\lambda_{31} + 2070\lambda_{13}\lambda_{22}\lambda_{31} + 990\lambda_{13}\lambda_{24}\lambda_{31} + 2610\lambda_{14}\lambda_{31}^2 + 1335\lambda_{22}\lambda_{31}^2 + 540\lambda_{24}\lambda_{31}^2 + 20\lambda_{22}\lambda_{H1}^2 \\ & + 45\lambda_{13}\lambda_{H1}\lambda_{H12} + 105\lambda_{31}\lambda_{H1}\lambda_{H12} - 6g_1^2\lambda_{H12}^2 - 30g_2^2\lambda_{H12}^2 + 60\lambda_{14}\lambda_{H12}^2 + 30\lambda_{22}\lambda_{H12}^2 + 60\lambda_{24}\lambda_{H12}^2 + 50\lambda_{H1}\lambda_{H12}^2 \\ & - 12g_1^2\lambda_{H1}\lambda_{H2} - 60g_2^2\lambda_{H1}\lambda_{H2} + 160\lambda_{22}\lambda_{H1}\lambda_{H2} + 40\lambda_{H1}^2\lambda_{H2} + 105\lambda_{13}\lambda_{H12}\lambda_{H2} + 45\lambda_{31}\lambda_{H12}\lambda_{H2} + 50\lambda_{H12}^2\lambda_{H2} \\ & + 20\lambda_{22}\lambda_{H2}^2 + 40\lambda_{H1}\lambda_{H2}^2 + 30 \left(2\lambda_{H1}\lambda_{H2} + \lambda_{H12}^2 \right) \text{Tr}(Y_d Y_d^\dagger) + 10 \left(2\lambda_{H1}\lambda_{H2} + \lambda_{H12}^2 \right) \text{Tr}(Y_e Y_e^\dagger) + 30\lambda_{H12}^2 \text{Tr}(Y_u Y_u^\dagger) \\ & \left. + 60\lambda_{H1}\lambda_{H2} \text{Tr}(Y_u Y_u^\dagger) \right) \end{aligned} \quad (33)$$

$$\begin{aligned} \beta_{\lambda_{H1}}^{(1)} = & -\frac{9}{10}g_1^2\lambda_{H1} - \frac{9}{2}g_2^2\lambda_{H1} + 24\lambda_{14}\lambda_{H1} + 6\lambda\lambda_{H1} + 8\lambda_{H1}^2 + 6\lambda_{31}\lambda_{H12} + 2\lambda_{H12}^2 + 4\lambda_{22}\lambda_{H2} + 6\lambda_{H1}\text{Tr}(Y_d Y_d^\dagger) \\ & + 2\lambda_{H1}\text{Tr}(Y_e Y_e^\dagger) + 6\lambda_{H1}\text{Tr}(Y_u Y_u^\dagger) \end{aligned} \quad (34)$$

$$\beta_{\lambda_{H1}}^{(2)} = +\frac{1671}{400}g_1^4\lambda_{H1} + \frac{9}{8}g_1^2g_2^2\lambda_{H1} - \frac{145}{16}g_2^4\lambda_{H1} + 6\lambda_{13}^2\lambda_{H1} - 480\lambda_{14}^2\lambda_{H1} - 8\lambda_{22}^2\lambda_{H1} - 54\lambda_{31}^2\lambda_{H1} + \frac{36}{5}g_1^2\lambda\lambda_{H1}$$

$$\begin{aligned}
& + 36g_2^2\lambda\lambda_{H1} - 15\lambda^2\lambda_{H1} + \frac{6}{5}g_1^2\lambda_{H1}^2 + 6g_2^2\lambda_{H1}^2 - 288\lambda_{14}\lambda_{H1}^2 - 72\lambda\lambda_{H1}^2 - 42\lambda_{H1}^3 - 18\lambda_{13}\lambda_{22}\lambda_{H12} \\
& + 12\lambda_{13}\lambda_{24}\lambda_{H12} - 132\lambda_{14}\lambda_{31}\lambda_{H12} - 42\lambda_{22}\lambda_{31}\lambda_{H12} - 96\lambda_{31}\lambda_{H1}\lambda_{H12} + \frac{3}{10}g_1^2\lambda_{H12}^2 + \frac{3}{2}g_2^2\lambda_{H12}^2 - 24\lambda_{14}\lambda_{H12}^2 \\
& - 20\lambda_{22}\lambda_{H12}^2 - 18\lambda\lambda_{H12}^2 - 21\lambda_{H1}\lambda_{H12}^2 - 36\lambda_{13}^2\lambda_{H2} - 32\lambda_{22}^2\lambda_{H2} - 36\lambda_{31}^2\lambda_{H2} - 32\lambda_{22}\lambda_{H1}\lambda_{H2} - 24\lambda_{13}\lambda_{H12}\lambda_{H2} \\
& - 24\lambda_{31}\lambda_{H12}\lambda_{H2} - 10\lambda_{H12}^2\lambda_{H2} - 16\lambda_{22}\lambda_{H2}^2 - 2\lambda_{H1}\lambda_{H2}^2 \\
& + \left(-24\lambda_{H1}^2 - 36\lambda\lambda_{H1} + 40g_3^2\lambda_{H1} - 6\lambda_{H12}^2 + \frac{45}{4}g_2^2\lambda_{H1} + \frac{5}{4}g_1^2\lambda_{H1}\right)\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + \frac{1}{4}\left(15g_1^2\lambda_{H1} + 15g_2^2\lambda_{H1} - 8\left(4\lambda_{H1}^2 + 6\lambda\lambda_{H1} + \lambda_{H12}^2\right)\right)\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{17}{4}g_1^2\lambda_{H1}\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& + \frac{45}{4}g_2^2\lambda_{H1}\text{Tr}\left(Y_u Y_u^\dagger\right) + 40g_3^2\lambda_{H1}\text{Tr}\left(Y_u Y_u^\dagger\right) - 36\lambda\lambda_{H1}\text{Tr}\left(Y_u Y_u^\dagger\right) - 24\lambda_{H1}^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& - 6\lambda_{H12}^2\text{Tr}\left(Y_u Y_u^\dagger\right) - \frac{27}{2}\lambda_{H1}\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 21\lambda_{H1}\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{9}{2}\lambda_{H1}\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) \\
& - \frac{27}{2}\lambda_{H1}\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)
\end{aligned} \tag{35}$$

$$\beta_{\lambda_{13}}^{(1)} = 4\left(3\lambda_{22}\lambda_{31} + 6\lambda_{13}\left(3\lambda_{24} + \lambda_{22}\right) + \lambda_{H12}\lambda_{H2}\right) \tag{36}$$

$$\begin{aligned}
\beta_{\lambda_{13}}^{(2)} = & -\frac{2}{5}\left(735\lambda_{13}^3 - 120\lambda_{13}\lambda_{14}^2 + 720\lambda_{13}\lambda_{14}\lambda_{22} + 890\lambda_{13}\lambda_{22}^2 + 3480\lambda_{13}\lambda_{22}\lambda_{24} + 8160\lambda_{13}\lambda_{24}^2 + 810\lambda_{13}^2\lambda_{31} + 660\lambda_{14}\lambda_{22}\lambda_{31} \right. \\
& + 690\lambda_{22}^2\lambda_{31} - 120\lambda_{14}\lambda_{24}\lambda_{31} + 660\lambda_{22}\lambda_{24}\lambda_{31} + 225\lambda_{13}\lambda_{31}^2 + 270\lambda_{31}^3 - 10\lambda_{13}\lambda_{H1}^2 + 30\lambda_{22}\lambda_{H1}\lambda_{H12} - 20\lambda_{24}\lambda_{H1}\lambda_{H12} \\
& + 20\lambda_{13}\lambda_{H12}^2 + 30\lambda_{31}\lambda_{H12}^2 + 10\lambda_{H12}^3 + 120\lambda_{13}\lambda_{H1}\lambda_{H2} - 12g_1^2\lambda_{H12}\lambda_{H2} - 60g_2^2\lambda_{H12}\lambda_{H2} + 70\lambda_{22}\lambda_{H12}\lambda_{H2} + 220\lambda_{24}\lambda_{H12}\lambda_{H2} \\
& \left. + 40\lambda_{H1}\lambda_{H12}\lambda_{H2} + 90\lambda_{13}\lambda_{H2}^2 + 80\lambda_{H12}\lambda_{H2}^2 + 60\lambda_{H12}\lambda_{H2}\text{Tr}\left(Y_d Y_d^\dagger\right) + 20\lambda_{H12}\lambda_{H2}\text{Tr}\left(Y_e Y_e^\dagger\right) + 60\lambda_{H12}\lambda_{H2}\text{Tr}\left(Y_u Y_u^\dagger\right)\right)
\end{aligned} \tag{37}$$

$$\begin{aligned}
\beta_{\lambda_{H12}}^{(1)} = & +12\lambda_{31}\lambda_{H1} - \frac{9}{10}g_1^2\lambda_{H12} - \frac{9}{2}g_2^2\lambda_{H12} + 8\lambda_{22}\lambda_{H12} + 6\lambda\lambda_{H12} + 8\lambda_{H1}\lambda_{H12} + 12\lambda_{13}\lambda_{H2} + 8\lambda_{H12}\lambda_{H2} + 6\lambda_{H12}\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + 2\lambda_{H12}\text{Tr}\left(Y_e Y_e^\dagger\right) + 6\lambda_{H12}\text{Tr}\left(Y_u Y_u^\dagger\right)
\end{aligned} \tag{38}$$

$$\begin{aligned}
\beta_{\lambda_{H12}}^{(2)} = & -36\lambda_{13}\lambda_{22}\lambda_{H1} + 24\lambda_{13}\lambda_{24}\lambda_{H1} - 264\lambda_{14}\lambda_{31}\lambda_{H1} - 84\lambda_{22}\lambda_{31}\lambda_{H1} - 96\lambda_{31}\lambda_{H1}^2 + \frac{1671}{400}g_1^4\lambda_{H12} + \frac{9}{8}g_1^2g_2^2\lambda_{H12} \\
& - \frac{145}{16}g_2^4\lambda_{H12} - 24\lambda_{13}^2\lambda_{H12} + 48\lambda_{14}^2\lambda_{H12} - 96\lambda_{14}\lambda_{22}\lambda_{H12} - 24\lambda_{22}^2\lambda_{H12} - 96\lambda_{22}\lambda_{24}\lambda_{H12} + 48\lambda_{24}^2\lambda_{H12} - 72\lambda_{13}\lambda_{31}\lambda_{H12} \\
& - 24\lambda_{31}^2\lambda_{H12} + \frac{36}{5}g_1^2\lambda\lambda_{H12} + 36g_2^2\lambda\lambda_{H12} - 15\lambda^2\lambda_{H12} + \frac{6}{5}g_1^2\lambda_{H1}\lambda_{H12} + 6g_2^2\lambda_{H1}\lambda_{H12} - 96\lambda_{14}\lambda_{H1}\lambda_{H12} \\
& - 80\lambda_{22}\lambda_{H1}\lambda_{H12} - 72\lambda\lambda_{H1}\lambda_{H12} - 42\lambda_{H1}^2\lambda_{H12} - 36\lambda_{13}\lambda_{H12}^2 - 36\lambda_{31}\lambda_{H12}^2 - 11\lambda_{H12}^3 - 84\lambda_{13}\lambda_{22}\lambda_{H2} - 264\lambda_{13}\lambda_{24}\lambda_{H2} \\
& + 24\lambda_{14}\lambda_{31}\lambda_{H2} - 36\lambda_{22}\lambda_{31}\lambda_{H2} - 48\lambda_{13}\lambda_{H1}\lambda_{H2} - 48\lambda_{31}\lambda_{H1}\lambda_{H2} + \frac{6}{5}g_1^2\lambda_{H12}\lambda_{H2} + 6g_2^2\lambda_{H12}\lambda_{H2} - 80\lambda_{22}\lambda_{H12}\lambda_{H2} \\
& - 96\lambda_{24}\lambda_{H12}\lambda_{H2} - 72\lambda\lambda_{H12}\lambda_{H2} - 40\lambda_{H1}\lambda_{H12}\lambda_{H2} - 96\lambda_{13}\lambda_{H2}^2 - 42\lambda_{H12}\lambda_{H2}^2 \\
& + \frac{1}{4}\lambda_{H12}\left(16\left(10g_3^2 - 6\left(\lambda_{H1} + \lambda_{H2}\right) - 9\lambda\right) + 45g_2^2 + 5g_1^2\right)\text{Tr}\left(Y_d Y_d^\dagger\right) \\
& + \frac{1}{4}\lambda_{H12}\left(15g_1^2 + 15g_2^2 - 32\left(\lambda_{H1} + \lambda_{H2}\right) - 48\lambda\right)\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{17}{4}g_1^2\lambda_{H12}\text{Tr}\left(Y_u Y_u^\dagger\right) + \frac{45}{4}g_2^2\lambda_{H12}\text{Tr}\left(Y_u Y_u^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& + 40g_3^2\lambda_{H12}\text{Tr}\left(Y_uY_u^\dagger\right) - 36\lambda\lambda_{H12}\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_{H1}\lambda_{H12}\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_{H12}\lambda_{H2}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - \frac{27}{2}\lambda_{H12}\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) - 21\lambda_{H12}\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) - \frac{9}{2}\lambda_{H12}\text{Tr}\left(Y_eY_e^\dagger Y_eY_e^\dagger\right) - \frac{27}{2}\lambda_{H12}\text{Tr}\left(Y_uY_u^\dagger Y_uY_u^\dagger\right)
\end{aligned} \tag{39}$$

$$\beta_{\lambda_{24}}^{(1)} = 2\left(36\lambda_{24}^2 + \lambda_{22}^2 + \lambda_{H2}^2\right) + 9\lambda_{13}^2 \tag{40}$$

$$\begin{aligned}
\beta_{\lambda_{24}}^{(2)} = & -174\lambda_{13}^2\lambda_{22} - 32\lambda_{22}^3 - 816\lambda_{13}^2\lambda_{24} - 80\lambda_{22}^2\lambda_{24} - 3264\lambda_{24}^3 + 12\lambda_{13}\lambda_{14}\lambda_{31} - 66\lambda_{13}\lambda_{22}\lambda_{31} - 36\lambda_{22}\lambda_{31}^2 \\
& + 12\lambda_{24}\lambda_{31}^2 + 2\lambda_{13}\lambda_{H1}\lambda_{H12} - 4\lambda_{22}\lambda_{H12}^2 + 4\lambda_{24}\lambda_{H12}^2 - 22\lambda_{13}\lambda_{H12}\lambda_{H2} - 4\lambda_{H12}^2\lambda_{H2} + \frac{12}{5}g_1^2\lambda_{H2}^2 + 12g_2^2\lambda_{H2}^2 \\
& - 80\lambda_{24}\lambda_{H2}^2 - 16\lambda_{H2}^3 - 12\lambda_{H2}^2\text{Tr}\left(Y_dY_d^\dagger\right) - 4\lambda_{H2}^2\text{Tr}\left(Y_eY_e^\dagger\right) - 12\lambda_{H2}^2\text{Tr}\left(Y_uY_u^\dagger\right)
\end{aligned} \tag{41}$$

$$\begin{aligned}
\beta_{\lambda_{H2}}^{(1)} = & + 4\lambda_{22}\lambda_{H1} + 6\lambda_{13}\lambda_{H12} + 2\lambda_{H12}^2 - \frac{9}{10}g_1^2\lambda_{H2} - \frac{9}{2}g_2^2\lambda_{H2} + 24\lambda_{24}\lambda_{H2} + 6\lambda\lambda_{H2} + 8\lambda_{H2}^2 + 6\lambda_{H2}\text{Tr}\left(Y_dY_d^\dagger\right) \\
& + 2\lambda_{H2}\text{Tr}\left(Y_eY_e^\dagger\right) + 6\lambda_{H2}\text{Tr}\left(Y_uY_u^\dagger\right)
\end{aligned} \tag{42}$$

$$\begin{aligned}
\beta_{\lambda_{H2}}^{(2)} = & -36\lambda_{13}^2\lambda_{H1} - 32\lambda_{22}^2\lambda_{H1} - 36\lambda_{31}^2\lambda_{H1} - 16\lambda_{22}\lambda_{H1}^2 - 42\lambda_{13}\lambda_{22}\lambda_{H12} - 132\lambda_{13}\lambda_{24}\lambda_{H12} + 12\lambda_{14}\lambda_{31}\lambda_{H12} - 18\lambda_{22}\lambda_{31}\lambda_{H12} \\
& - 24\lambda_{13}\lambda_{H1}\lambda_{H12} - 24\lambda_{31}\lambda_{H1}\lambda_{H12} + \frac{3}{10}g_1^2\lambda_{H12}^2 + \frac{3}{2}g_2^2\lambda_{H12}^2 - 20\lambda_{22}\lambda_{H12}^2 - 24\lambda_{24}\lambda_{H12}^2 - 18\lambda\lambda_{H12}^2 - 10\lambda_{H1}\lambda_{H12}^2 \\
& + \frac{1671}{400}g_1^4\lambda_{H2} + \frac{9}{8}g_1^2g_2^2\lambda_{H2} - \frac{145}{16}g_2^4\lambda_{H2} - 54\lambda_{13}^2\lambda_{H2} - 8\lambda_{22}^2\lambda_{H2} - 480\lambda_{24}^2\lambda_{H2} + 6\lambda_{31}^2\lambda_{H2} + \frac{36}{5}g_1^2\lambda\lambda_{H2} \\
& + 36g_2^2\lambda\lambda_{H2} - 15\lambda^2\lambda_{H2} - 32\lambda_{22}\lambda_{H1}\lambda_{H2} - 2\lambda_{H1}^2\lambda_{H2} - 96\lambda_{13}\lambda_{H12}\lambda_{H2} - 21\lambda_{H12}^2\lambda_{H2} + \frac{6}{5}g_1^2\lambda_{H2}^2 + 6g_2^2\lambda_{H2}^2 \\
& - 288\lambda_{24}\lambda_{H2}^2 - 72\lambda\lambda_{H2}^2 - 42\lambda_{H2}^3 + \left(-6\lambda_{H12}^2 + \frac{1}{4}\left(-144\lambda + 160g_3^2 + 45g_2^2 + 5g_1^2 - 96\lambda_{H2}\right)\lambda_{H2}\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - \frac{1}{4}\left(8\lambda_{H12}^2 + \lambda_{H2}\left(-15g_1^2 - 15g_2^2 + 32\lambda_{H2} + 48\lambda\right)\right)\text{Tr}\left(Y_eY_e^\dagger\right) - 6\lambda_{H12}^2\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{17}{4}g_1^2\lambda_{H2}\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{45}{4}g_2^2\lambda_{H2}\text{Tr}\left(Y_uY_u^\dagger\right) + 40g_3^2\lambda_{H2}\text{Tr}\left(Y_uY_u^\dagger\right) - 36\lambda\lambda_{H2}\text{Tr}\left(Y_uY_u^\dagger\right) - 24\lambda_{H2}^2\text{Tr}\left(Y_uY_u^\dagger\right) \\
& - \frac{27}{2}\lambda_{H2}\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) - 21\lambda_{H2}\text{Tr}\left(Y_dY_u^\dagger Y_uY_d^\dagger\right) - \frac{9}{2}\lambda_{H2}\text{Tr}\left(Y_eY_e^\dagger Y_eY_e^\dagger\right) - \frac{27}{2}\lambda_{H2}\text{Tr}\left(Y_uY_u^\dagger Y_uY_u^\dagger\right)
\end{aligned} \tag{43}$$

$$\begin{aligned}
\beta_{\lambda}^{(1)} = & + \frac{27}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{9}{4}g_2^4 - \frac{9}{5}g_1^2\lambda - 9g_2^2\lambda + 12\lambda^2 + 4\lambda_{H1}^2 + 2\lambda_{H12}^2 + 4\lambda_{H2}^2 + 12\lambda\text{Tr}\left(Y_dY_d^\dagger\right) \\
& + 4\lambda\text{Tr}\left(Y_eY_e^\dagger\right) + 12\lambda\text{Tr}\left(Y_uY_u^\dagger\right) - 12\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) - 4\text{Tr}\left(Y_eY_e^\dagger Y_eY_e^\dagger\right) - 12\text{Tr}\left(Y_uY_u^\dagger Y_uY_u^\dagger\right)
\end{aligned} \tag{44}$$

$$\begin{aligned}
\beta_{\lambda}^{(2)} = & -\frac{3411}{1000}g_1^6 - \frac{1677}{200}g_1^4g_2^2 - \frac{289}{40}g_1^2g_2^4 + \frac{305}{8}g_2^6 + \frac{1887}{200}g_1^4\lambda + \frac{117}{20}g_1^2g_2^2\lambda - \frac{73}{8}g_2^4\lambda + \frac{54}{5}g_1^2\lambda^2 + 54g_2^2\lambda^2 \\
& - 78\lambda^3 - 20\lambda\lambda_{H1}^2 - 32\lambda_{H1}^3 - 10\lambda\lambda_{H12}^2 - 24\lambda_{H1}\lambda_{H12}^2 - 24\lambda_{H12}^2\lambda_{H2} - 20\lambda\lambda_{H2}^2 - 32\lambda_{H2}^3 \\
& + \frac{1}{10}\left(225g_2^2\lambda - 45g_2^4 + 80\left(10g_3^2 - 9\lambda\right)\lambda + 9g_1^4 + g_1^2\left(25\lambda + 54g_2^2\right)\right)\text{Tr}\left(Y_dY_d^\dagger\right) \\
& - \frac{3}{10}\left(15g_1^4 + 5\left(16\lambda^2 - 5g_2^2\lambda + g_2^4\right) - g_1^2\left(22g_2^2 + 25\lambda\right)\right)\text{Tr}\left(Y_eY_e^\dagger\right) - \frac{171}{50}g_1^4\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + \frac{63}{5}g_1^2g_2^2\text{Tr}\left(Y_uY_u^\dagger\right) - \frac{9}{2}g_2^4\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{17}{2}g_1^2\lambda\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{45}{2}g_2^2\lambda\text{Tr}\left(Y_uY_u^\dagger\right) \\
& + 80g_3^2\lambda\text{Tr}\left(Y_uY_u^\dagger\right) - 72\lambda^2\text{Tr}\left(Y_uY_u^\dagger\right) + \frac{8}{5}g_1^2\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right) - 64g_3^2\text{Tr}\left(Y_dY_d^\dagger Y_dY_d^\dagger\right)
\end{aligned}$$

$$\begin{aligned}
& -3\lambda\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 42\lambda\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{24}{5}g_1^2\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - \lambda\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) \\
& - \frac{16}{5}g_1^2\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) - 64g_3^2\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) - 3\lambda\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + 60\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& - 24\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u Y_d^\dagger\right) + 12\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d Y_d^\dagger\right) - 12\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u Y_d^\dagger\right) \\
& + 20\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger\right) + 60\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger\right)
\end{aligned} \tag{45}$$

3.5 Yukawa Couplings

$$\begin{aligned}
\beta_{Y_u}^{(1)} = & -\frac{3}{2}\left(-Y_u Y_u^\dagger Y_u + Y_u Y_d^\dagger Y_d\right) \\
& + Y_u\left(3\text{Tr}\left(Y_d Y_d^\dagger\right) + 3\text{Tr}\left(Y_u Y_u^\dagger\right) - 8g_3^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(Y_e Y_e^\dagger\right)\right)
\end{aligned} \tag{46}$$

$$\begin{aligned}
\beta_{Y_u}^{(2)} = & +\frac{1}{80}\left(20\left(11Y_u Y_d^\dagger Y_d Y_d^\dagger Y_d - 4Y_u Y_u^\dagger Y_u Y_d^\dagger Y_d + 6Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u - Y_u Y_d^\dagger Y_d Y_u^\dagger Y_u\right)\right. \\
& + Y_u Y_u^\dagger Y_u\left(1280g_3^2 - 180\text{Tr}\left(Y_e Y_e^\dagger\right) + 223g_1^2 - 480\lambda - 540\text{Tr}\left(Y_d Y_d^\dagger\right) - 540\text{Tr}\left(Y_u Y_u^\dagger\right) + 675g_2^2\right) \\
& + Y_u Y_d^\dagger Y_d\left(100\text{Tr}\left(Y_e Y_e^\dagger\right) - 1280g_3^2 + 300\text{Tr}\left(Y_d Y_d^\dagger\right) + 300\text{Tr}\left(Y_u Y_u^\dagger\right) - 43g_1^2 + 45g_2^2\right) \\
& + Y_u\left(\frac{1187}{600}g_1^4 - \frac{9}{20}g_1^2 g_2^2 - \frac{23}{4}g_2^4 + \frac{19}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + \frac{3}{2}\lambda^2 + \lambda_{H1}^2 + \frac{1}{2}\lambda_{H12}^2 + \lambda_{H2}^2\right. \\
& + \frac{5}{8}\left(32g_3^2 + 9g_2^2 + g_1^2\right)\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{15}{8}\left(g_1^2 + g_2^2\right)\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{17}{8}g_1^2\text{Tr}\left(Y_u Y_u^\dagger\right) + \frac{45}{8}g_2^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& \left. + 20g_3^2\text{Tr}\left(Y_u Y_u^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) + \frac{3}{2}\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{9}{4}\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)\right)
\end{aligned} \tag{47}$$

$$\begin{aligned}
\beta_{Y_d}^{(1)} = & \frac{1}{4}\left(6\left(-Y_d Y_u^\dagger Y_u + Y_d Y_d^\dagger Y_d\right)\right. \\
& \left.- Y_d\left(-12\text{Tr}\left(Y_d Y_d^\dagger\right) - 12\text{Tr}\left(Y_u Y_u^\dagger\right) + 32g_3^2 - 4\text{Tr}\left(Y_e Y_e^\dagger\right) + 9g_2^2 + g_1^2\right)\right)
\end{aligned} \tag{48}$$

$$\begin{aligned}
\beta_{Y_d}^{(2)} = & +\frac{1}{80}\left(20\left(11Y_d Y_u^\dagger Y_u Y_u^\dagger Y_u - 4Y_d Y_d^\dagger Y_d Y_u^\dagger Y_u + 6Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d - Y_d Y_u^\dagger Y_u Y_d^\dagger Y_d\right)\right. \\
& + Y_d Y_d^\dagger Y_d\left(1280g_3^2 - 180\text{Tr}\left(Y_e Y_e^\dagger\right) + 187g_1^2 - 480\lambda - 540\text{Tr}\left(Y_d Y_d^\dagger\right) - 540\text{Tr}\left(Y_u Y_u^\dagger\right) + 675g_2^2\right) \\
& + Y_d Y_u^\dagger Y_u\left(100\text{Tr}\left(Y_e Y_e^\dagger\right) - 1280g_3^2 + 300\text{Tr}\left(Y_d Y_d^\dagger\right) + 300\text{Tr}\left(Y_u Y_u^\dagger\right) + 45g_2^2 - 79g_1^2\right) \\
& + Y_d\left(-\frac{127}{600}g_1^4 - \frac{27}{20}g_1^2 g_2^2 - \frac{23}{4}g_2^4 + \frac{31}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4 + \frac{3}{2}\lambda^2 + \lambda_{H1}^2 + \frac{1}{2}\lambda_{H12}^2 + \lambda_{H2}^2\right. \\
& + \frac{5}{8}\left(32g_3^2 + 9g_2^2 + g_1^2\right)\text{Tr}\left(Y_d Y_d^\dagger\right) + \frac{15}{8}\left(g_1^2 + g_2^2\right)\text{Tr}\left(Y_e Y_e^\dagger\right) + \frac{17}{8}g_1^2\text{Tr}\left(Y_u Y_u^\dagger\right) + \frac{45}{8}g_2^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& \left. + 20g_3^2\text{Tr}\left(Y_u Y_u^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) + \frac{3}{2}\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - \frac{9}{4}\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right) - \frac{27}{4}\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)\right)
\end{aligned} \tag{49}$$

$$\beta_{Y_e}^{(1)} = \frac{3}{2}Y_e Y_e^\dagger Y_e + Y_e\left(3\text{Tr}\left(Y_d Y_d^\dagger\right) + 3\text{Tr}\left(Y_u Y_u^\dagger\right) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(Y_e Y_e^\dagger\right)\right) \tag{50}$$

$$\beta_{Y_e}^{(2)} = +\frac{3}{80}\left(40Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e\right)$$

$$\begin{aligned}
& + Y_e Y_e^\dagger Y_e \left(129g_1^2 - 160\lambda - 180\text{Tr}(Y_d Y_d^\dagger) - 180\text{Tr}(Y_u Y_u^\dagger) + 225g_2^2 - 60\text{Tr}(Y_e Y_e^\dagger) \right) \\
& + \frac{1}{200} Y_e \left(1371g_1^4 + 270g_1^2 g_2^2 - 1150g_2^4 + 300\lambda^2 + 200\lambda_{H1}^2 + 100\lambda_{H12}^2 + 200\lambda_{H2}^2 + 125(32g_3^2 + 9g_2^2 + g_1^2) \text{Tr}(Y_d Y_d^\dagger) \right. \\
& + 375(g_1^2 + g_2^2) \text{Tr}(Y_e Y_e^\dagger) + 425g_1^2 \text{Tr}(Y_u Y_u^\dagger) + 1125g_2^2 \text{Tr}(Y_u Y_u^\dagger) + 4000g_3^2 \text{Tr}(Y_u Y_u^\dagger) \\
& \left. - 1350\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) + 300\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - 450\text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - 1350\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) \right) \quad (51)
\end{aligned}$$

3.6 Scalar Mass Terms

$$\beta_{m_1^2}^{(1)} = 4(2\lambda_{H1}\mu^2 + 6\lambda_{14}m_1^2 + \lambda_{22}m_2^2) \quad (52)$$

$$\begin{aligned}
\beta_{m_1^2}^{(2)} = & +6\lambda_{13}^2 m_1^2 - 480\lambda_{14}^2 m_1^2 - 8\lambda_{22}^2 m_1^2 - 54\lambda_{31}^2 m_1^2 - 8\lambda_{H1}^2 m_1^2 + 2\lambda_{H12}^2 m_1^2 - 36\lambda_{13}^2 m_2^2 - 32\lambda_{22}^2 m_2^2 \\
& - 36\lambda_{31}^2 m_2^2 - 4\lambda_{H12}^2 m_2^2 + \frac{48}{5}g_1^2 \lambda_{H1}\mu^2 + 48g_2^2 \lambda_{H1}\mu^2 - 32\lambda_{H1}^2 \mu^2 - 8\lambda_{H12}^2 \mu^2 - 48\lambda_{H1}\mu^2 \text{Tr}(Y_d Y_d^\dagger) \\
& - 16\lambda_{H1}\mu^2 \text{Tr}(Y_e Y_e^\dagger) - 48\lambda_{H1}\mu^2 \text{Tr}(Y_u Y_u^\dagger) \quad (53)
\end{aligned}$$

$$\beta_{m_2^2}^{(1)} = 4(2\lambda_{H2}\mu^2 + 6\lambda_{24}m_2^2 + \lambda_{22}m_1^2) \quad (54)$$

$$\begin{aligned}
\beta_{m_2^2}^{(2)} = & -36\lambda_{13}^2 m_1^2 - 32\lambda_{22}^2 m_1^2 - 36\lambda_{31}^2 m_1^2 - 4\lambda_{H12}^2 m_1^2 - 54\lambda_{13}^2 m_2^2 - 8\lambda_{22}^2 m_2^2 - 480\lambda_{24}^2 m_2^2 + 6\lambda_{31}^2 m_2^2 \\
& + 2\lambda_{H12}^2 m_2^2 - 8\lambda_{H2}^2 m_2^2 - 8\lambda_{H12}^2 \mu^2 + \frac{48}{5}g_1^2 \lambda_{H2}\mu^2 + 48g_2^2 \lambda_{H2}\mu^2 - 32\lambda_{H2}^2 \mu^2 - 48\lambda_{H2}\mu^2 \text{Tr}(Y_d Y_d^\dagger) \\
& - 16\lambda_{H2}\mu^2 \text{Tr}(Y_e Y_e^\dagger) - 48\lambda_{H2}\mu^2 \text{Tr}(Y_u Y_u^\dagger) \quad (55)
\end{aligned}$$

$$\beta_{\mu^2}^{(1)} = 2\lambda_{H1}m_1^2 + 2\lambda_{H2}m_2^2 + 2\mu^2 \text{Tr}(Y_e Y_e^\dagger) + 6\lambda\mu^2 + 6\mu^2 \text{Tr}(Y_d Y_d^\dagger) + 6\mu^2 \text{Tr}(Y_u Y_u^\dagger) - \frac{9}{10}g_1^2 \mu^2 - \frac{9}{2}g_2^2 \mu^2 \quad (56)$$

$$\begin{aligned}
\beta_{\mu^2}^{(2)} = & -8\lambda_{H1}^2 m_1^2 - 2\lambda_{H12}^2 m_1^2 - 2\lambda_{H12}^2 m_2^2 - 8\lambda_{H2}^2 m_2^2 + \frac{1671}{400}g_1^4 \mu^2 + \frac{9}{8}g_1^2 g_2^2 \mu^2 - \frac{145}{16}g_2^4 \mu^2 + \frac{36}{5}g_1^2 \lambda \mu^2 \\
& + 36g_2^2 \lambda \mu^2 - 15\lambda^2 \mu^2 - 2\lambda_{H1}^2 \mu^2 - \lambda_{H12}^2 \mu^2 - 2\lambda_{H2}^2 \mu^2 \\
& + \frac{1}{4}(-144\lambda + 160g_3^2 + 45g_2^2 + 5g_1^2)\mu^2 \text{Tr}(Y_d Y_d^\dagger) + \frac{3}{4}(-16\lambda + 5g_1^2 + 5g_2^2)\mu^2 \text{Tr}(Y_e Y_e^\dagger) \\
& + \frac{17}{4}g_1^2 \mu^2 \text{Tr}(Y_u Y_u^\dagger) + \frac{45}{4}g_2^2 \mu^2 \text{Tr}(Y_u Y_u^\dagger) + 40g_3^2 \mu^2 \text{Tr}(Y_u Y_u^\dagger) - 36\lambda \mu^2 \text{Tr}(Y_u Y_u^\dagger) \\
& - \frac{27}{2}\mu^2 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 21\mu^2 \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) - \frac{9}{2}\mu^2 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - \frac{27}{2}\mu^2 \text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) \quad (57)
\end{aligned}$$

3.7 Vacuum expectation values

$$\beta_v^{(1)} = \frac{1}{20}v(15g_2^2 \text{Xi} - 20\text{Tr}(Y_e Y_e^\dagger) + 3g_1^2 \text{Xi} + 45g_2^2 - 60\text{Tr}(Y_d Y_d^\dagger) - 60\text{Tr}(Y_u Y_u^\dagger) + 9g_1^2) \quad (58)$$

$$\begin{aligned}
\beta_v^{(2)} = & \frac{1}{800}v(-1293g_1^4 - 270g_1^2 g_2^2 + 6775g_2^4 - 1200\lambda^2 - 800\lambda_{H1}^2 - 400\lambda_{H12}^2 - 800\lambda_{H2}^2 + 36g_1^4 \text{Xi} + 360g_1^2 g_2^2 \text{Xi} \\
& + 2700g_2^4 \text{Xi} + 36g_1^4 \text{Xi}^2 + 360g_1^2 g_2^2 \text{Xi}^2 - 20(45g_2^2(4\text{Xi} + 5) + 800g_3^2 + g_1^2(36\text{Xi} + 25))\text{Tr}(Y_d Y_d^\dagger)
\end{aligned}$$

$$\begin{aligned}
& -60\left(5g_2^2(4X_i+5)+g_1^2(4X_i+25)\right)\text{Tr}\left(Y_e Y_e^\dagger\right)-1700g_1^2\text{Tr}\left(Y_u Y_u^\dagger\right)-4500g_2^2\text{Tr}\left(Y_u Y_u^\dagger\right) \\
& -16000g_3^2\text{Tr}\left(Y_u Y_u^\dagger\right)-720g_1^2X_i\text{Tr}\left(Y_u Y_u^\dagger\right)-3600g_2^2X_i\text{Tr}\left(Y_u Y_u^\dagger\right)+5400\text{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) \\
& -1200\text{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right)+1800\text{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right)+5400\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)
\end{aligned} \tag{59}$$

4 Field Rotations

4.1 Rotations in gauge sector for eigenstates 'EWSB'

$$\begin{pmatrix} B_\rho \\ W_{3\rho} \end{pmatrix} = Z^{\gamma Z} \begin{pmatrix} \gamma_\rho \\ Z_\rho \end{pmatrix} \tag{60}$$

$$\begin{pmatrix} W_{1\rho} \\ W_{2\rho} \end{pmatrix} = Z^W \begin{pmatrix} W_\rho^+ \\ W_\rho^+ \end{pmatrix} \tag{61}$$

$$\tag{62}$$

The mixing matrices are parametrized by

$$Z^{\gamma Z} = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix} \tag{63}$$

$$Z^W = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ i\frac{1}{\sqrt{2}} & -i\frac{1}{\sqrt{2}} \end{pmatrix} \tag{64}$$

$$\tag{65}$$

4.2 Rotations in Mass sector for eigenstates 'EWSB'

4.2.1 Mass Matrices for Scalars

- No Scalar Mixings

4.2.2 Mass Matrices for Fermions

- **Mass matrix for Down-Quarks**, Basis: $(d_{L,\alpha_1}), (d_{R,\beta_1}^*)$

$$m_d = \left(\frac{1}{\sqrt{2}} v \delta_{\alpha_1 \beta_1} Y_d^T \right) \tag{66}$$

This matrix is diagonalized by U_L^d and U_R^d

$$U_L^{d,*} m_d U_R^{d,\dagger} = m_d^{dia} \tag{67}$$

with

$$d_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{d,*} D_{L,j\alpha} \quad (68)$$

$$d_{R,i\alpha} = \sum_{t_2} U_{R,ij}^d D_{R,j\alpha}^* \quad (69)$$

- **Mass matrix for Up-Quarks**, Basis: $(u_{L,\alpha_1}), (u_{R,\beta_1}^*)$

$$m_u = \left(-\frac{1}{\sqrt{2}} v \delta_{\alpha_1 \beta_1} Y_u^T \right) \quad (70)$$

This matrix is diagonalized by U_L^u and U_R^u

$$U_L^{u,*} m_u U_R^{u,\dagger} = m_u^{dia} \quad (71)$$

with

$$u_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{u,*} U_{L,j\alpha} \quad (72)$$

$$u_{R,i\alpha} = \sum_{t_2} U_{R,ij}^u U_{R,j\alpha}^* \quad (73)$$

- **Mass matrix for Leptons**, Basis: $(e_L), (e_R^*)$

$$m_e = \left(\frac{1}{\sqrt{2}} v Y_e^T \right) \quad (74)$$

This matrix is diagonalized by U_L^e and U_R^e

$$U_L^{e,*} m_e U_R^{e,\dagger} = m_e^{dia} \quad (75)$$

with

$$e_{L,i} = \sum_{t_2} U_{L,ji}^{e,*} E_{L,j} \quad (76)$$

$$e_{R,i} = \sum_{t_2} U_{R,ij}^e E_{R,j}^* \quad (77)$$

5 Vacuum Expectation Values

$$H^0 = \frac{1}{\sqrt{2}} h + \frac{1}{\sqrt{2}} v + i \frac{1}{\sqrt{2}} A^0 \quad (78)$$

6 Tadpole Equations

$$\frac{\partial V}{\partial h} = \frac{1}{2} \lambda v^3 + \mu^2 v \quad (79)$$

7 Particle content for eigenstates 'GaugeES'

Name	Type	complex/real	Generations	Indices
H^0	Scalar	complex	1	
H^+	Scalar	complex	1	
S_1	Scalar	real	1	
S_2	Scalar	real	1	
B	Vector	real	1	lorentz, 4
W	Vector	real	3	generation, 3, lorentz, 4
g	Vector	real	1	color, 8, lorentz, 4
η^B	Ghost	real	1	
η^W	Ghost	real	3	generation, 3
η^G	Ghost	real	1	color, 8

8 Interactions for eigenstates 'GaugeES'

8.1 Three Scalar-Interaction

8.2 Two Scalar-One Vector Boson-Interaction

$$-\frac{i}{2}g_1\left(-p_\mu^{H^{0,*}}+p_\mu^{H^0}\right) \quad (80)$$

$$-\frac{i}{2}g_2\sigma_{2,2}^k\left(-p_\mu^{H^{0,*}}+p_\mu^{H^0}\right) \tag{81}$$

$$-\frac{i}{2}g_2\sigma_{1,2}^k\left(-p_\mu^{H^-}+p_\mu^{H^0}\right) \tag{82}$$

$$-\frac{i}{2}g_1\left(-p_\mu^{H^-}+p_\mu^{H^+}\right) \tag{83}$$

$$-\frac{i}{2}g_2\sigma_{2,1}^k\left(-p_\mu^{H^{0,*}}+p_\mu^{H^+}\right) \quad (84)$$

$$-\frac{i}{2}g_2\sigma_{1,1}^k\left(-p_\mu^{H^-}+p_\mu^{H^+}\right) \quad (85)$$

- 8.3 One Scalar-Two Vector Boson-Interaction**
- 8.4 Two Fermion-One Vector Boson-Interaction**
- 8.5 Two Fermion-One Scalar Boson-Interaction**
- 8.6 Three Vector Boson-Interaction**

$$g_3 f_{\alpha,\beta,\gamma} \left(g_{\rho\mu} \left(-p_\sigma^{g\gamma\mu} + p_\sigma^{g\alpha\rho} \right) + g_{\rho\sigma} \left(-p_\mu^{g\alpha\rho} + p_\mu^{g\beta\sigma} \right) + g_{\sigma\mu} \left(-p_\rho^{g\beta\sigma} + p_\rho^{g\gamma\mu} \right) \right) \quad (86)$$

$$g_2 \epsilon_{i,j,k} \left(g_{\rho\mu} \left(-p_\sigma^{W_{k\mu}} + p_\sigma^{W_{i\rho}} \right) + g_{\rho\sigma} \left(-p_\mu^{W_{i\rho}} + p_\mu^{W_{j\sigma}} \right) + g_{\sigma\mu} \left(-p_\rho^{W_{j\sigma}} + p_\rho^{W_{k\mu}} \right) \right) \quad (87)$$

8.7 Four Scalar-Interaction

$$-2i\lambda \quad (88)$$

$$-i\lambda \quad (89)$$

$$-2i\lambda_{H1} \tag{90}$$

$$-i\lambda_{H12} \tag{91}$$

$$-2i\lambda_{H2} \tag{92}$$

$$-2i\lambda \tag{93}$$

$$-2i\lambda_{H1} \tag{94}$$

$$-i\lambda_{H12} \tag{95}$$

$$-2i\lambda_{H2} \tag{96}$$

$$-24i\lambda_{14} \tag{97}$$

$$-6i\lambda_{31} \tag{98}$$

$$-4i\lambda_{22} \tag{99}$$

$$-6i\lambda_{13} \tag{100}$$

$$-24i\lambda_{24} \tag{101}$$

8.8 Two Scalar-Two Vector Boson-Interaction

$$\frac{i}{2}g_1^2(g_{\mu\nu}) \tag{102}$$

$$\frac{i}{2}g_1g_2\sigma_{2,2}^l(g_{\mu\nu}) \tag{103}$$

$$\frac{i}{2}g_1g_2\sigma_{1,2}^l(g_{\mu\nu}) \tag{104}$$

$$\left(\frac{i}{2}g_2^2\sigma_{2,2}^k\sigma_{2,2}^l + \frac{i}{4}g_2^2\sigma_{1,2}^k\sigma_{2,1}^l + \frac{i}{4}g_2^2\sigma_{2,1}^k\sigma_{1,2}^l\right)(g_{\mu\nu}) \quad (105)$$

$$\left(\frac{i}{4}g_2^2\sigma_{1,1}^k\sigma_{1,2}^l + \frac{i}{4}g_2^2\sigma_{1,2}^k\sigma_{1,1}^l + \frac{i}{4}g_2^2\sigma_{1,2}^k\sigma_{2,2}^l + \frac{i}{4}g_2^2\sigma_{2,2}^k\sigma_{1,2}^l\right)(g_{\mu\nu}) \quad (106)$$

$$\frac{i}{2}g_1^2(g_{\mu\nu}) \quad (107)$$

$$\frac{i}{2}g_1g_2\sigma_{2,1}^l\left(g_{\mu\nu}\right) \tag{108}$$

$$\frac{i}{2}g_1g_2\sigma_{1,1}^l\left(g_{\mu\nu}\right) \tag{109}$$

$$\left(\frac{i}{4}g_2^2\sigma_{1,1}^k\sigma_{2,1}^l+\frac{i}{4}g_2^2\sigma_{2,1}^k\sigma_{1,1}^l+\frac{i}{4}g_2^2\sigma_{2,1}^k\sigma_{2,2}^l+\frac{i}{4}g_2^2\sigma_{2,2}^k\sigma_{2,1}^l\right)\left(g_{\mu\nu}\right) \tag{110}$$

$$\left(\frac{i}{2}g_2^2\sigma_{1,1}^k\sigma_{1,1}^l + \frac{i}{4}g_2^2\sigma_{1,2}^k\sigma_{2,1}^l + \frac{i}{4}g_2^2\sigma_{2,1}^k\sigma_{1,2}^l\right)(g_{\mu\nu}) \quad (111)$$

8.9 Four Vector Boson-Interaction

$$ig_3^2\left(-\sum_{a=1}^8 f_{\alpha,\delta,a}f_{\beta,\gamma,a} - \sum_{a=1}^8 f_{\alpha,\gamma,a}f_{\beta,\delta,a}\right)(g_{\rho\sigma}g_{\mu\nu}) \quad (112)$$

$$+ ig_3^2\left(-\sum_{a=1}^8 f_{\alpha,\beta,a}f_{\gamma,\delta,a} + \sum_{a=1}^8 f_{\alpha,\delta,a}f_{\beta,\gamma,a}\right)(g_{\rho\mu}g_{\sigma\nu}) \quad (113)$$

$$+ ig_3^2\left(\sum_{a=1}^8 f_{\alpha,\gamma,a}f_{\beta,\delta,a} + \sum_{a=1}^8 f_{\alpha,\beta,a}f_{\gamma,\delta,a}\right)(g_{\rho\nu}g_{\sigma\mu}) \quad (114)$$

$$ig_2^2 \left(-\epsilon_{1,i,k}\epsilon_{1,j,l} - \epsilon_{1,i,l}\epsilon_{1,j,k} - \epsilon_{2,i,k}\epsilon_{2,j,l} - \epsilon_{2,i,l}\epsilon_{2,j,k} - \epsilon_{3,i,k}\epsilon_{3,j,l} - \epsilon_{3,i,l}\epsilon_{3,j,k} \right) (g_{\rho\sigma}g_{\mu\nu}) \quad (115)$$

$$+ ig_2^2 \left(-\epsilon_{1,i,j}\epsilon_{1,k,l} + \epsilon_{1,i,l}\epsilon_{1,j,k} - \epsilon_{2,i,j}\epsilon_{2,k,l} + \epsilon_{2,i,l}\epsilon_{2,j,k} - \epsilon_{3,i,j}\epsilon_{3,k,l} + \epsilon_{3,i,l}\epsilon_{3,j,k} \right) (g_{\rho\mu}g_{\sigma\nu}) \quad (116)$$

$$+ ig_2^2 \left(\epsilon_{1,i,j}\epsilon_{1,k,l} + \epsilon_{1,i,k}\epsilon_{1,j,l} + \epsilon_{2,i,j}\epsilon_{2,k,l} + \epsilon_{2,i,k}\epsilon_{2,j,l} + \epsilon_{3,i,j}\epsilon_{3,k,l} + \epsilon_{3,i,k}\epsilon_{3,j,l} \right) (g_{\rho\nu}g_{\sigma\mu}) \quad (117)$$

8.10 Two Ghosts-One Vector Boson-Interaction

$$g_3 f_{\alpha,\beta,\gamma} \left(p_\mu^{\eta_\beta^G} \right) \quad (118)$$

$$g_2 \epsilon_{i,j,k} \left(p_\mu^{\eta_j^W} \right) \quad (119)$$

8.11 Two Ghosts-One Scalar-Interaction

9 Clebsch-Gordan Coefficients