Dipole DM: Model Definitions

Andre Lessa,[a] **Jose Zurita,**[b]

[a] *Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, Santo André, 09210-580 SP, Brazil*

E-mail: andre.lessa@ufabc.edu.br

Contents

Contents

1 Dipole Dark Matter Model (DipoleDM_full_UFO)

The Dipole DM model extends the SM by adding three Dirac Fermions (χ_2, χ_1 and χ_0) and a real scalar (ϕ), all singlets under the SM gauge group. The Lagrangian of the model is given by:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_\phi + \mathcal{L}_\chi + \mathcal{L}_{\rm dipole} + \mathcal{L}_{H\chi}, \tag{1.1}$$

where $\mathcal{L}_{\rm SM}$ represents the SM Lagrangian and

$$\mathcal{L}_\phi = (\partial^\mu \phi)^2 - \mu_2^2 |\phi|^2 - \lambda_2 \phi^4 - \lambda_3 \phi^2 |H|^2 \,, \tag{1.2}$$

$$\mathcal{L}_\chi = i\overline{\chi}_i \not{\partial} \chi_i - \tilde{M}_{ij} \overline{\chi}_i \chi_j - (y_\chi)_{ij} \, \overline{\chi}_i \chi_j \phi \,, \tag{1.3}$$

$$\mathcal{L}_{\rm dipole} = \frac{(C_{\gamma\chi\chi})_{ij}}{\Lambda} \overline{\chi}_i \sigma^{\mu\nu} \chi_j F_{\mu\nu} \,, \tag{1.4}$$

$$\mathcal{L}_{H\chi} = \frac{(C_{H\chi\chi})_{ij}}{\Lambda} \overline{\chi}_i \chi_j |H|^2 \,. \tag{1.5}$$

In the equations above, $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$ and H represents the Higgs doublet.

The model Lagrangian given in Eq. (1.1) contains the scalar potential:

$$V_{H,\phi} = \mu_1^2 |H|^2 + \lambda_1 |H|^4 + \mu_2^2 \phi^2 + \lambda_2 \phi^4 + \lambda_3 \phi^2 |H|^2 \,. \tag{1.6}$$

Assuming that both H and ϕ develop vevs, $\langle \phi \rangle = v_D/\sqrt{2}$ and $\langle H \rangle = v/\sqrt{2}$, we obtain the mass eigenstates h and S:

$$\begin{aligned} h &= (\sqrt{2}H^0 - v)\cos\alpha - (\sqrt{2}\phi - v_D)\sin\alpha \,, \\ S &= (\sqrt{2}\phi - v_D)\cos\alpha + (\sqrt{2}H^0 - v)\sin\alpha \,. \end{aligned} \tag{1.7}$$

Where the mixing angle (α) is given by

$$\tan(2\alpha) \equiv \frac{\lambda_3 v v_D}{\lambda_1 v^2 - \lambda_2 v_D^2} \,. \tag{1.8}$$

and with masses:

$$m_{S,h}^2 = \lambda_1 v^2 + \lambda_2 v_D^2 \mp \left(\lambda_1 v^2 - \lambda_2 v_D^2\right) \sqrt{1 + \tan^2(2\alpha)} \,, \tag{1.9}$$

Figure 1. Production cross-section for the dark scalar S as a function of its mass.

Using the above equations, we can express the quartic couplings (λ_i) and the mass parameters (μ_i) in terms of the physical masses, mixing angle and vevs:

$$\lambda_1 = \frac{1}{2v^2}\left(\cos^2\alpha\, m_h^2 + m_S^2\sin^2\alpha\right), \tag{1.10}$$

$$\lambda_2 = \frac{1}{2v_D^2}\left(\cos^2\alpha\, m_S^2 + m_h^2\sin^2\alpha\right), \tag{1.11}$$

$$\lambda_3 = \frac{1}{v_D v}\left(m_S^2 - m_h^2\right)\sin\alpha\cos\alpha\ , \tag{1.12}$$

$$\mu_1^2 = -\left(\lambda_1 v^2 + \lambda_3\frac{v_D^2}{2}\right), \tag{1.13}$$

$$\mu_2^2 = -\left(\lambda_2 v_D^2 + \lambda_3\frac{v^2}{2}\right) \tag{1.14}$$

Therefore, the parameters of the scalar potential ($\mu_{1,2}$ and $\lambda_{1,2,3}$) can be replaced by m_h, m_S, v, v_D and $\sin\alpha$.

Finally, the dark fermion mass matrix $\tilde{M}_{ij}$ is defined in the mass eigenstate basis:

$$\tilde{M}_{ij} = M_i\delta_{ij} - \frac{v_D}{\sqrt{2}}(y_\chi)_{ij} - \frac{v^2}{2\Lambda}(C_{H\chi\chi})_{ij}$$

where M_i are the physical masses.

In addition to the Lagrangian in Eq. (1.1), the effective $h - G - G$ and $S - G - G$ couplings induced by a top quark loop were also included as effective operators:

$$\mathcal{L}_{GG\phi} = \frac{g_s^2}{48\pi^2 v}\cos\alpha F(m_h^2/m_t^2)G^{\mu\nu}G_{\mu\nu}h + \frac{g_s^2}{48\pi^2 v}\sin\alpha F(m_S^2/m_t^2)G^{\mu\nu}G_{\mu\nu}S$$

where $F(x)$ is the effective loop function.

Model Parameters

The model parameters are given in Table 1 as well as their naming convention in the UFO model and their default values. The $C_{\gamma\chi\chi}$ and $C_{H\chi\chi}$ matrices are assumed to be real and symmetric.

Feynman rules

The relevant Feynman rules for the BSM particles are given in Table 2 below. Here, f represents any of the SM fermions and q any SM quark.

The production of S at the LHC mainly occurs due to gluon-fusion and is proportional to $\sin\alpha$. In Fig. 1 we show the $\sigma(pp \to S)$ as a function of m_S for $\sin\alpha = 0.2$ and a center of mass energy of 13.6 TeV.

Table 1. Model parameters, their respective names in the UFO model and default values.

Parameter	UFO name	Default Value
Λ	LambdaUV	5 TeV
$(C_{\gamma\chi\chi})_{22}$	Caxx2	0
$(C_{\gamma\chi\chi})_{11}$	Caxx1	0
$(C_{\gamma\chi\chi})_{00}$	Caxx0	0
$(C_{\gamma\chi\chi})_{10}$	Caxx10	0.1
$(C_{\gamma\chi\chi})_{20}$	Caxx20	0.1
$(C_{\gamma\chi\chi})_{21}$	Caxx21	0.1
$(C_{H\chi\chi})_{22}$	Chxx2	0
$(C_{H\chi\chi})_{11}$	Chxx1	0
$(C_{H\chi\chi})_{00}$	Chxx0	0
$(C_{H\chi\chi})_{10}$	Chxx10	0.1
$(C_{H\chi\chi})_{20}$	Chxx20	0.1
$(C_{H\chi\chi})_{21}$	Chxx21	0.1
$(y_\chi)_{22}$	ychi2	1.0
$(y_\chi)_{11}$	ychi1	1.0
$(y_\chi)_{00}$	ychi0	0
$(y_\chi)_{10}$	ychi10	0
$(y_\chi)_{20}$	ychi20	0
$(y_\chi)_{21}$	ychi21	0
$\sin\alpha$	sina	0.2
v_D	vevD	1 TeV
m_S	MSd	1.5 TeV
M_0	M0	425 GeV
M_1	M1	500 GeV
M_2	M2	510 GeV

Interaction	Vertex term
$\chi_i\,\chi_j\,h$	$\frac{i}{\sqrt{2}}(y_\chi)_{ij}\sin\alpha - i\frac{v}{\Lambda}(C_{H\chi\chi})_{ij}\cos\alpha$
$\chi_i\,\chi_j\,A^\mu$	$-i\frac{1}{\Lambda}(C_{\gamma\chi\chi})_{ij}(\gamma^\mu\not{p} - \not{p}\gamma^\mu)$
$S\,\chi_i\,\chi_j$	$-\frac{i}{\sqrt{2}}(y_\chi)_{ij}\cos\alpha - i\frac{v}{\Lambda}(C_{H\chi\chi})_{ij}\sin\alpha$
$S\,h\,h$	$-i\frac{m_S^2}{2v}\left(1+2\frac{m_h^2}{m_S^2}\right)\left(\cos\alpha+2\frac{v}{v_D}\sin\alpha\right)\sin(2\alpha)$
$S\,f\,\bar{f}$	$-i\frac{m_f}{v}\sin\alpha$
$S\,W_\mu^-\,W_\nu^+$	$2ig^{\mu\nu}\frac{m_W^2}{v}\sin\alpha$
$S\,Z_\mu\,Z_\nu$	$2ig^{\mu\nu}\frac{m_Z^2}{v}\sin\alpha$
$S\,G_\mu\,G_\nu$	$i\frac{g_s^2}{12\pi^2 v}\sin\alpha F(m_S^2/m_t^2)(p_1^\mu p_2^\nu - g^{\mu\nu}p_1\cdot p_2)$

Table 2. Feynman rules for the relevant interactions in the full model.

1.1 Minimal Couplings to Higgs (DipoleDM2_minimalH_UFO)

The minimal scenario assumes that χ_0 only couples through the H effective operator and the diagonal entries are zero:

$$(C_{\gamma\chi\chi})_{ij} = 0 \tag{1.15}$$

$$(C_{H\chi\chi})_{ij} = 0 \text{ , if } i = j \text{ or } i = 2 \text{ or } j = 2 \tag{1.16}$$

$$(y_\chi)_{ij} = 0 \text{ , if } i = j \text{ or } i = 1, j = 0 \tag{1.17}$$

With the above assumptions the Lagrangian simplifies to:

$$\mathcal{L}_\chi = \overline{\chi}_i \left(i\not{\partial} - M_i\right) \chi_i - \left[(y_\chi)_{21}\, \overline{\chi}_2 \chi_1 \phi + (y_\chi)_{20}\, \overline{\chi}_2 \chi_0 \phi + \frac{(C_{H\chi\chi})_{01}}{\Lambda} \overline{\chi}_0 \chi_1 |H|^2 + h.c. \right] \tag{1.18}$$

resulting in the vertices given in Table 3.

Interaction	Vertex term (Minimal H)
$h\,\chi_1\,\chi_1$	$\frac{i}{\sqrt{2}}(y_\chi)_{11}\sin\alpha$
$S\,\chi_1\,\chi_1$	$-\frac{i}{\sqrt{2}}(y_\chi)_{11}\cos\alpha$
$\chi_1\,\chi_0\,h$	$-i\frac{v}{\Lambda}(C_{H\chi\chi})_{01}\cos\alpha$
$S\,\chi_1\,\chi_0$	$-i\frac{v}{\Lambda}(C_{H\chi\chi})_{01}\sin\alpha$
$S\,G_\mu\,G_\nu$	$i\frac{g_s^2}{12\pi^2 v}\sin\alpha F(m_S^2/m_t^2)(p_1^\mu p_2^\nu - g^{\mu\nu} p_1\cdot p_2)$

Table 3. Feynman rules for the relevant interactions in the **minimal** model with couplings to Higgs. The couplings between S and two SM particles are the same as in Table 2 and have been omitted.

In this scenario the heavy fermion can only decay through the effective operator, hence:

$$\Gamma(\chi_1 \to h\chi_0) = \cos^2\alpha \frac{C_{H\chi\chi}^2}{16\pi}\frac{v^2}{\Lambda^2} M_1 \left[\left(1+\frac{M_0}{M_1}\right)^2 - \frac{m_h^2}{M_1^2}\right] \sqrt{\lambda\left(1, \frac{M_0^2}{M_1^2}, \frac{m_h^2}{M_1^2}\right)} \tag{1.19}$$

$$\Gamma(\chi_1 \to S\chi_0) = \sin^2\alpha \frac{C_{H\chi\chi}^2}{16\pi}\frac{v^2}{\Lambda^2} M_1 \left[\left(1+\frac{M_0}{M_1}\right)^2 - \frac{m_S^2}{M_1^2}\right] \sqrt{\lambda\left(1, \frac{M_0^2}{M_1^2}, \frac{m_S^2}{M_1^2}\right)}\,, \tag{1.20}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$.

Note that the decay to $S+\chi_0$ is suppressed by $\sin\alpha$ and will be subdominant as long as $m_h \lesssim m_S$. Furthermore, if $\Delta m = m_{\chi_1} - m_{\chi_0} < m_h$ the decay is kinematically suppressed and takes place through an off-shell Higgs (h^*). In Fig. 2 we show the branching ratios as a function of Δm for $m_{\chi_1} = 500$ GeV. For the extremely compressed scenario ($\Delta m < 2$ GeV) only the decay to gluons ($h^* \to gg$) is kinematically allowed, resulting in a 100% BR in this channel. Once decays to τs are allowed, they become the dominant channel. For $\Delta m \gtrsim 10$ GeV the decays $h^* \to b\bar{b}$ are open and rapidly become the main decay channel up to $\Delta m > m_h$, where the decay is 100% to an on-shell Higgs. Finally, in Fig. 3 we show the decay length as a function of Δm, where we see that it can take a wide range of values depending on the mass compression. In particular, once the $b\bar{b}$ channel is open the decay length falls below 10 m. It is important to notice, however, that the lifetime scales as $\tau \propto (\Lambda/C_{H\chi\chi})^2$ and will be further suppressed for larger values of $\Lambda/C_{H\chi\chi}$.

Figure 2. Branching ratios for χ_1 decays in the minimal H scenario as a function of $\Delta m = m_{\chi_1} - m_{\chi_0}$. Only channels with a branching ratio larger than 1% are shown.

Figure 3. Proper decay length for χ_1 in the minimal H scenario as a function of $\Delta m = m_{\chi_1} - m_{\chi_0}$.

1.2 Minimal Couplings to Photon (DipoleDM_minimalA_UFO)

The minimal scenario assumes that χ_0 only couples through the $F^{\mu\nu}$ effective operator and the diagonal entries are zero:

$$(C_{\gamma\chi\chi})_{ij} = 0 \text{ , if } i = j \tag{1.21}$$

$$(C_{H\chi\chi})_{ij} = 0 \tag{1.22}$$

$$(y_\chi)_{ij} = 0 \text{ , if } i = 0 \text{ or } j = 0 \tag{1.23}$$

With the above assumptions the Lagrangian simplifies to:

$$\mathcal{L}_\chi = i\overline{\chi}_i \not{\partial} \chi_i - \tilde{M}_{ij}\overline{\chi}_i\chi_j - (y_\chi)_{11}\,\overline{\chi}_1\chi_1\phi + \frac{(C_{\gamma\chi\chi})_{01}}{\Lambda}\left(\overline{\chi}_0\sigma^{\mu\nu}\chi_1 + h.c.\right)F_{\mu\nu} \tag{1.24}$$

resulting in the vertices given in Table 4.

Interaction	**Vertex term (Minimal A)**
$h\,\chi_1\,\chi_1$	$\frac{i}{\sqrt{2}}(y_\chi)_{11}\sin\alpha$
$S\,\chi_1\,\chi_1$	$-\frac{i}{\sqrt{2}}(y_\chi)_{11}\cos\alpha$
$\chi_1\,\chi_0\,A^\mu$	$-i\frac{1}{\Lambda}(C_{\gamma\chi\chi})_{01}(\gamma^\mu\not{p} - \not{p}\gamma^\mu)$
$S\,G_\mu\,G_\nu$	$i\frac{g_s^2}{12\pi^2 v}\sin\alpha F(m_S^2/m_t^2)(p_1^\mu p_2^\nu - g^{\mu\nu}p_1\cdot p_2)$

Table 4. Feynman rules for the relevant interactions in the **minimal** model with couplings to photons. The couplings between S and two SM particles are the same as in Table 2 and have been omitted.

In this scenario the heavy fermion can only decay through the effective operator, hence:

$$\Gamma(\chi_1 \to \gamma\chi_0) = \frac{C_{\gamma\chi\chi}^2}{2\pi}\frac{M_1^2}{\Lambda^2}M_1\left(1 - \frac{M_0^2}{M_1^2}\right)^3 \tag{1.25}$$

Since this is the only allowed channel it has always 100% branching ratio and the lifetime for the compressed scenario is given by:

$$c\tau = 1.55\times 10^{-6}\text{ m}\left(\frac{0.1}{C_{\gamma\chi\chi}}\right)^2\left(\frac{\Lambda}{10\text{ TeV}}\right)^2\left(\frac{1\text{ GeV}}{\Delta m}\right)^3, \tag{1.26}$$

where we have assumed $M_0 \lesssim M_1$, so $\Delta m^2 = M_1^2 - M_0^2 \simeq 2M_1\Delta m$. In Fig. 4 we show the decay length as a function of Δm. As we can see, even for $\Lambda = 100$ TeV the decay is prompt as long as $\Delta m > 1$ GeV.

Figure 4. Proper decay length for χ_1 in the minimal photon scenario as a function of $\Delta m = m_{\chi_1} - m_{\chi_0}$.