

# Dipole DM: Model Definitions

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## Contents

### 1 Dipole Dark Matter Model

The Dipole DM model extends the SM by adding two Dirac Fermions ( $\chi_1$  and  $\chi_0$ ) and a real scalar ( $\phi$ ), all singlets under the SM gauge group. The Lagrangian of the model is given by:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\phi + \mathcal{L}_\chi + \mathcal{L}_{\text{dipole}} + \mathcal{L}_{H\chi}, \quad (1.1)$$

where  $\mathcal{L}_{\text{SM}}$  represents the SM Lagrangian and

$$\mathcal{L}_\phi = (\partial^\mu \phi)^2 - \mu_2^2 |\phi|^2 - \lambda_2 \phi^4 - \lambda_3 \phi^2 |H|^2, \quad (1.2)$$

$$\mathcal{L}_\chi = i \bar{\chi}_i \not{\partial} \chi_i - \tilde{M}_{ij} \bar{\chi}_i \chi_j - (y_\chi)_{ij} \bar{\chi}_i \chi_j \phi, \quad (1.3)$$

$$\mathcal{L}_{\text{dipole}} = \frac{(C_{\gamma\chi\chi})_{ij}}{\Lambda} \bar{\chi}_i \sigma^{\mu\nu} \chi_j F_{\mu\nu}, \quad (1.4)$$

$$\mathcal{L}_{H\chi} = \frac{(C_{H\chi\chi})_{ij}}{\Lambda} \bar{\chi}_i \chi_j |H|^2. \quad (1.5)$$

In the equations above,  $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$  and  $H$  represents the Higgs doublet.

The model Lagrangian given in Eq. (1.1) contains the scalar potential:

$$V_{H,\phi} = \mu_1^2 |H|^2 + \lambda_1 |H|^4 + \mu_2^2 |\phi|^2 + \lambda_2 |\phi|^4 + \lambda_3 |\phi|^2 |H|^2. \quad (1.6)$$

Assuming that both  $H$  and  $\phi$  develop vevs,  $\langle \phi \rangle = v_D/\sqrt{2}$  and  $\langle H \rangle = v/\sqrt{2}$ , we obtain the mass eigenstates  $h$  and  $S$ :

$$\begin{aligned} h &= (\sqrt{2}H^0 - v) \cos \alpha - (\sqrt{2}\phi - v_D) \sin \alpha, \\ S &= (\sqrt{2}\phi - v_D) \cos \alpha + (\sqrt{2}H^0 - v) \sin \alpha. \end{aligned} \quad (1.7)$$

Where the mixing angle ( $\alpha$ ) is given by

$$\tan(2\alpha) \equiv \frac{\lambda_3 v v_D}{\lambda_1 v^2 - \lambda_2 v_D^2}. \quad (1.8)$$

and with masses:

$$m_{S,h}^2 = \lambda_1 v^2 + \lambda_2 v_D^2 \mp (\lambda_1 v^2 - \lambda_2 v_D^2) \sqrt{1 + \tan^2(2\alpha)}, \quad (1.9)$$

Using the above equations, we can express the quartic couplings ( $\lambda_i$ ) and the mass parameters ( $\mu_i$ ) in terms of the physical masses, mixing angle and vevs:

$$\lambda_1 = \frac{1}{2v^2} (\cos^2 \alpha m_h^2 + m_S^2 \sin^2 \alpha), \quad (1.10)$$

$$\lambda_2 = \frac{1}{2v_D^2} (\cos^2 \alpha m_S^2 + m_h^2 \sin^2 \alpha), \quad (1.11)$$

$$\lambda_3 = \frac{1}{v_D v} (m_S^2 - m_h^2) \sin \alpha \cos \alpha, \quad (1.12)$$

$$\mu_1^2 = - \left( \lambda_1 v^2 + \lambda_3 \frac{v_D^2}{2} \right), \quad (1.13)$$

$$\mu_2^2 = - \left( \lambda_2 v_D^2 + \lambda_3 \frac{v^2}{2} \right) \quad (1.14)$$

Therefore, the parameters of the scalar potential ( $\mu_{1,2}$  and  $\lambda_{1,2,3}$ ) can be replaced by  $m_h, m_S, v, v_D$  and  $\sin \alpha$ .

Finally, the dark fermion mass matrix  $\tilde{M}_{ij}$  is defined in the mass eigenstate basis:

$$\tilde{M}_{ij} = M_i \delta_{ij} - \frac{v_D}{\sqrt{2}} (y_\chi)_{ij} - \frac{v^2}{2\Lambda} (C_{H\chi\chi})_{ij}$$

where  $M_i$  are the physical masses.

In addition to the Lagrangian in Eq. (1.1), the effective  $h - G - G$  and  $S - G - G$  couplings induced by a top quark loop were also included as effective operators:

$$\mathcal{L}_{GG\phi} = -\frac{1}{4} \cos \alpha F(m_h^2/m_t^2) G^{\mu\nu} G_{\mu\nu} h - \frac{1}{4} \sin \alpha F(m_S^2/m_t^2) G^{\mu\nu} G_{\mu\nu} S$$

where  $F(x)$  is the effective loop function.

## Model Parameters

The model parameters are given in Table Table 1 as well as their naming convention in the UFO model and their default values. The  $C_{\gamma\chi\chi}$  and  $C_{H\chi\chi}$  matrices are assumed to be real and symmetric.

## Feynman rules

The relevant Feynman rules for the BSM particles are given in Table 2 below. Here,  $f$  represents any of the SM fermions and  $q$  any SM quark.

## 2 Minimal Dipole Model

The minimal scenario assumes that  $\chi_0$  only couples through the effective operators and these have zero entries in the diagonal:

$$(C_{\gamma\chi\chi})_{ij} = (C_{H\chi\chi})_{ij} = 0, \text{ if } i = j \quad (2.1)$$

$$(y_\chi)_{ij} = 0, \text{ if } i = 0 \text{ or } j = 0 \quad (2.2)$$

**Table 1.** Model parameters, their respective names in the UFO model and default values.

Parameter	UFO name	Default Value
$\Lambda$	LambdaUV	5 TeV
$(C_{\gamma\chi\chi})_{11}$	Caxx1	0
$(C_{\gamma\chi\chi})_{00}$	Caxx0	0
$(C_{\gamma\chi\chi})_{10}$	Caxx10	0.1
$(C_{H\chi\chi})_{11}$	Chxx1	0
$(C_{H\chi\chi})_{00}$	Chxx0	0
$(C_{H\chi\chi})_{10}$	Chxx10	0.1
$(y_\chi)_{11}$	ychi1	1.0
$(y_\chi)_{00}$	ychi0	0
$(y_\chi)_{10}$	ychi10	0
$\sin \alpha$	sina	0.2
$v_D$	vevD	1 TeV

**Table 2.** Feynman rules for the relevant interactions.

Interaction	Vertex term
$\chi_i \chi_j h$	$\frac{i}{\sqrt{2}}(y_\chi)_{ij} \sin \alpha - i \frac{v}{\Lambda}(C_{H\chi\chi})_{ij} \cos \alpha$
$\chi_i \chi_j A^\mu$	$-i \frac{1}{\Lambda}(C_{\gamma\chi\chi})_{ij}(\gamma^\mu \not{p} - \not{p} \gamma^\mu)$
$S \chi_i \chi_j$	$-\frac{i}{\sqrt{2}}(y_\chi)_{ij} \cos \alpha - i \frac{v}{\Lambda}(C_{H\chi\chi})_{ij} \sin \alpha$
$S h h$	$-i \frac{m_S^2}{2v} \left(1 + 2 \frac{m_h^2}{m_S^2}\right) \left(\cos \alpha + 2 \frac{v}{v_D} \sin \alpha\right) \sin(2\alpha)$
$S f \bar{f}$	$-i \frac{m_f}{v} \sin \alpha$
$S W_\mu^- W_\nu^+$	$2i g^{\mu\nu} \frac{m_W^2}{v} \sin \alpha$
$S Z_\mu Z_\nu$	$2i g^{\mu\nu} \frac{m_Z^2}{v} \sin \alpha$

With the above assumptions the Lagrangian simplifies to:

$$\mathcal{L}_\chi = \bar{\chi}_i \left( i \not{\partial} - \tilde{M}_i \right) \chi_i - (y_\chi)_{11} \bar{\chi}_1 \chi_1 \phi, \quad (2.3)$$

$$\mathcal{L}_{\text{dipole}} = \frac{(C_{\gamma\chi\chi})_{01}}{\Lambda} \bar{\chi}_0 \sigma^{\mu\nu} \chi_1 F_{\mu\nu} + h.c., \quad (2.4)$$

$$\mathcal{L}_{H\chi} = \frac{(C_{H\chi\chi})_{01}}{\Lambda} \bar{\chi}_0 \chi_1 |H|^2 + h.c.. \quad (2.5)$$

resulting in the following vertices:

$$\chi_1 \chi_1 h : \frac{i}{\sqrt{2}}(y_\chi)_{11} \sin \alpha, \quad S \chi_1 \chi_1 : -\frac{i}{\sqrt{2}}(y_\chi)_{11} \cos \alpha \quad (2.6)$$

$$\chi_1 \chi_0 h : -i \frac{v}{\Lambda}(C_{H\chi\chi})_{01} \cos \alpha, \quad S \chi_1 \chi_0 : -i \frac{v}{\Lambda}(C_{H\chi\chi})_{01} \sin \alpha \quad (2.7)$$

$$\chi_1 \chi_0 A^\mu : -i \frac{1}{\Lambda}(C_{\gamma\chi\chi})_{01}(\gamma^\mu \not{p} - \not{p} \gamma^\mu) \quad (2.8)$$