

Dipole DM: Model Definitions

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1 Dipole Dark Matter Model (DIPoleDM_FULL_UFO)

The Dipole DM model extends the SM by adding three Dirac Fermions (χ_2 , χ_1 and χ_0) and a real scalar (ϕ), all singlets under the SM gauge group. The Lagrangian of the model is given by:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\phi + \mathcal{L}_\chi + \mathcal{L}_{\text{dipole}} + \mathcal{L}_{H\chi}, \quad (1.1)$$

where \mathcal{L}_{SM} represents the SM Lagrangian and

$$\mathcal{L}_\phi = (\partial^\mu \phi)^2 - \mu_2^2 |\phi|^2 - \lambda_2 \phi^4 - \lambda_3 \phi^2 |H|^2, \quad (1.2)$$

$$\mathcal{L}_\chi = i\bar{\chi}_i \not{\partial} \chi_i - \tilde{M}_{ij} \bar{\chi}_i \chi_j - (y_\chi)_{ij} \bar{\chi}_i \chi_j \phi, \quad (1.3)$$

$$\mathcal{L}_{\text{dipole}} = \frac{(C_{\gamma\chi\chi})_{ij}}{\Lambda} \bar{\chi}_i \sigma^{\mu\nu} \chi_j F_{\mu\nu}, \quad (1.4)$$

$$\mathcal{L}_{H\chi} = \frac{(C_{H\chi\chi})_{ij}}{\Lambda} \bar{\chi}_i \chi_j |H|^2. \quad (1.5)$$

In the equations above, $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ and H represents the Higgs doublet.

The model Lagrangian given in Eq. (1.1) contains the scalar potential:

$$V_{H,\phi} = \mu_1^2 |H|^2 + \lambda_1 |H|^4 + \mu_2^2 \phi^2 + \lambda_2 \phi^4 + \lambda_3 \phi^2 |H|^2. \quad (1.6)$$

Assuming that both H and ϕ develop vevs, $\langle \phi \rangle = v_D/\sqrt{2}$ and $\langle H \rangle = v/\sqrt{2}$, we obtain the mass eigenstates h and S :

$$\begin{aligned} h &= (\sqrt{2}H^0 - v) \cos \alpha - (\sqrt{2}\phi - v_D) \sin \alpha, \\ S &= (\sqrt{2}\phi - v_D) \cos \alpha + (\sqrt{2}H^0 - v) \sin \alpha. \end{aligned} \quad (1.7)$$

Where the mixing angle (α) is given by

$$\tan(2\alpha) \equiv \frac{\lambda_3 v v_D}{\lambda_1 v^2 - \lambda_2 v_D^2}. \quad (1.8)$$

and with masses:

$$m_{S,h}^2 = \lambda_1 v^2 + \lambda_2 v_D^2 \mp (\lambda_1 v^2 - \lambda_2 v_D^2) \sqrt{1 + \tan^2(2\alpha)}, \quad (1.9)$$

Figure 1. Production cross-section for the dark scalar S as a function of its mass.

Using the above equations, we can express the quartic couplings (λ_i) and the mass parameters (μ_i) in terms of the physical masses, mixing angle and vevs:

$$\lambda_1 = \frac{1}{2v^2} (\cos^2 \alpha m_h^2 + m_S^2 \sin^2 \alpha), \quad (1.10)$$

$$\lambda_2 = \frac{1}{2v_D^2} (\cos^2 \alpha m_S^2 + m_h^2 \sin^2 \alpha), \quad (1.11)$$

$$\lambda_3 = \frac{1}{v_D v} (m_S^2 - m_h^2) \sin \alpha \cos \alpha, \quad (1.12)$$

$$\mu_1^2 = - \left(\lambda_1 v^2 + \lambda_3 \frac{v_D^2}{2} \right), \quad (1.13)$$

$$\mu_2^2 = - \left(\lambda_2 v_D^2 + \lambda_3 \frac{v^2}{2} \right) \quad (1.14)$$

Therefore, the parameters of the scalar potential ($\mu_{1,2}$ and $\lambda_{1,2,3}$) can be replaced by m_h, m_S, v, v_D and $\sin \alpha$.

Finally, the dark fermion mass matrix \tilde{M}_{ij} is defined in the mass eigenstate basis:

$$\tilde{M}_{ij} = M_i \delta_{ij} - \frac{v_D}{\sqrt{2}} (y_\chi)_{ij} - \frac{v^2}{2\Lambda} (C_{H\chi\chi})_{ij}$$

where M_i are the physical masses.

In addition to the Lagrangian in Eq. (1.1), the effective $h - G - G$ and $S - G - G$ couplings induced by a top quark loop were also included as effective operators:

$$\mathcal{L}_{GG\phi} = \frac{g_s^2}{48\pi^2 v} \cos \alpha F(m_h^2/m_t^2) G^{\mu\nu} G_{\mu\nu} h + \frac{g_s^2}{48\pi^2 v} \sin \alpha F(m_S^2/m_t^2) G^{\mu\nu} G_{\mu\nu} S$$

where $F(x)$ is the effective loop function.

Model Parameters

The model parameters are given in Table 1 as well as their naming convention in the UFO model and their default values. The $C_{\gamma\chi\chi}$ and $C_{H\chi\chi}$ matrices are assumed to be real and symmetric.

Feynman rules

The relevant Feynman rules for the BSM particles are given in Table 2 below. Here, f represents any of the SM fermions and q any SM quark.

The production of S at the LHC mainly occurs due to gluon-fusion and is proportional to $\sin \alpha$. In Fig. 1 we show the $\sigma(pp \rightarrow S)$ as a function of m_S for $\sin \alpha = 0.2$ and a center of mass energy of 13.6 TeV.

Table 1. Model parameters, their respective names in the UFO model and default values.

Parameter	UFO name	Default Value
Λ	LambdaUV	5 TeV
$(C_{\gamma\chi\chi})_{22}$	Caxx2	0
$(C_{\gamma\chi\chi})_{11}$	Caxx1	0
$(C_{\gamma\chi\chi})_{00}$	Caxx0	0
$(C_{\gamma\chi\chi})_{10}$	Caxx10	0.1
$(C_{\gamma\chi\chi})_{20}$	Caxx20	0.1
$(C_{\gamma\chi\chi})_{21}$	Caxx21	0.1
$(C_{H\chi\chi})_{22}$	Chxx2	0
$(C_{H\chi\chi})_{11}$	Chxx1	0
$(C_{H\chi\chi})_{00}$	Chxx0	0
$(C_{H\chi\chi})_{10}$	Chxx10	0.1
$(C_{H\chi\chi})_{20}$	Chxx20	0.1
$(C_{H\chi\chi})_{21}$	Chxx21	0.1
$(y_\chi)_{22}$	ychi2	1.0
$(y_\chi)_{11}$	ychi1	1.0
$(y_\chi)_{00}$	ychi0	0
$(y_\chi)_{10}$	ychi10	0
$(y_\chi)_{20}$	ychi20	0
$(y_\chi)_{21}$	ychi21	0
$\sin \alpha$	sina	0.2
v_D	vevD	1 TeV
m_S	MSd	1.5 TeV
M_0	M0	425 GeV
M_1	M1	500 GeV
M_2	M2	510 GeV

Interaction	Vertex term
$\chi_i \chi_j h$	$\frac{i}{\sqrt{2}}(y_\chi)_{ij} \sin \alpha - i \frac{v}{\Lambda}(C_{H\chi\chi})_{ij} \cos \alpha$
$\chi_i \chi_j A^\mu$	$-i \frac{1}{\Lambda}(C_{\gamma\chi\chi})_{ij}(\gamma^\mu \not{p} - \not{p} \gamma^\mu)$
$S \chi_i \chi_j$	$-\frac{i}{\sqrt{2}}(y_\chi)_{ij} \cos \alpha - i \frac{v}{\Lambda}(C_{H\chi\chi})_{ij} \sin \alpha$
$S h h$	$-i \frac{m_S^2}{2v} \left(1 + 2 \frac{m_h^2}{m_S^2}\right) \left(\cos \alpha + 2 \frac{v}{v_D} \sin \alpha\right) \sin(2\alpha)$
$S f \bar{f}$	$-i \frac{m_f}{v} \sin \alpha$
$S W_\mu^- W_\nu^+$	$2i g^{\mu\nu} \frac{m_W^2}{v} \sin \alpha$
$S Z_\mu Z_\nu$	$2i g^{\mu\nu} \frac{m_Z^2}{v} \sin \alpha$
$S G_\mu G_\nu$	$i \frac{g_s^2}{12\pi^2 v} \sin \alpha F(m_S^2/m_t^2)(p_1^\mu p_2^\nu - g^{\mu\nu} p_1 \cdot p_2)$

Table 2. Feynman rules for the relevant interactions in the full model.

1.1 Minimal Couplings to Higgs (DIPoleDM2_MINIMALH_UFO)

The minimal scenario assumes that χ_0 only couples through the H effective operator and the diagonal entries are zero:

$$(C_{\gamma\chi\chi})_{ij} = 0 \quad (1.15)$$

$$(C_{H\chi\chi})_{ij} = 0, \text{ if } i = j \quad (1.16)$$

$$(y_\chi)_{ij} = 0, \text{ if } i = 0 \text{ or } j = 0 \quad (1.17)$$

With the above assumptions the Lagrangian simplifies to:

$$\mathcal{L}_\chi = i\bar{\chi}_i \not{\partial} \chi_i - \tilde{M}_{ij} \bar{\chi}_i \chi_j - (y_\chi)_{11} \bar{\chi}_1 \chi_1 \phi - (y_\chi)_{22} \bar{\chi}_2 \chi_2 \phi - ((y_\chi)_{21} \bar{\chi}_2 \chi_1 \phi + h.c.) \quad (1.18)$$

$$+ \frac{(C_{H\chi\chi})_{01}}{\Lambda} (\bar{\chi}_0 \chi_1 |H|^2 + h.c.) \quad (1.19)$$

resulting in the vertices given in Table 3.

Interaction	Vertex term (Minimal H)
$h \chi_1 \chi_1$	$\frac{i}{\sqrt{2}} (y_\chi)_{11} \sin \alpha$
$S \chi_1 \chi_1$	$-\frac{i}{\sqrt{2}} (y_\chi)_{11} \cos \alpha$
$\chi_1 \chi_0 h$	$-i \frac{v}{\Lambda} (C_{H\chi\chi})_{01} \cos \alpha$
$S \chi_1 \chi_0$	$-i \frac{v}{\Lambda} (C_{H\chi\chi})_{01} \sin \alpha$
$S G_\mu G_\nu$	$i \frac{g_s^2}{12\pi^2 v} \sin \alpha F(m_S^2/m_t^2) (p_1^\mu p_2^\nu - g^{\mu\nu} p_1 \cdot p_2)$

Table 3. Feynman rules for the relevant interactions in the **minimal** model with couplings to Higgs. The couplings between S and two SM particles are the same as in Table 2 and have been omitted.

In this scenario the heavy fermion can only decay through the effective operator, hence:

$$\Gamma(\chi_1 \rightarrow h\chi_0) = \cos^2 \alpha \frac{C_{H\chi\chi}^2}{16\pi} \frac{v^2}{\Lambda^2} M_1 \left[\left(1 + \frac{M_0}{M_1} \right)^2 - \frac{m_h^2}{M_1^2} \right] \sqrt{\lambda \left(1, \frac{M_0^2}{M_1^2}, \frac{m_h^2}{M_1^2} \right)} \quad (1.20)$$

$$\Gamma(\chi_1 \rightarrow S\chi_0) = \sin^2 \alpha \frac{C_{H\chi\chi}^2}{16\pi} \frac{v^2}{\Lambda^2} M_1 \left[\left(1 + \frac{M_0}{M_1} \right)^2 - \frac{m_S^2}{M_1^2} \right] \sqrt{\lambda \left(1, \frac{M_0^2}{M_1^2}, \frac{m_S^2}{M_1^2} \right)}, \quad (1.21)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$.

Note that the decay to $S + \chi_0$ is suppressed by $\sin \alpha$ and will be subdominant as long as $m_h \lesssim m_S$. Furthermore, if $\Delta m = m_{\chi_1} - m_{\chi_0} < m_h$ the decay is kinematically suppressed and takes place through an off-shell Higgs (h^*). In Fig. 2 we show the branching ratios as a function of Δm for $m_{\chi_1} = 500$ GeV. For the extremely compressed scenario ($\Delta m < 2$ GeV) only the decay to gluons ($h^* \rightarrow gg$) is kinematically allowed, resulting in a 100% BR in this channel. Once decays to τs are allowed, they become the dominant channel. For $\Delta m \gtrsim 10$ GeV the decays $h^* \rightarrow b\bar{b}$ are open and rapidly become the main decay channel up to $\Delta m > m_h$, where the decay is 100% to an on-shell Higgs. Finally, in Fig. 3 we show the decay length as a function of Δm , where we see that it can take a wide range of

Figure 2. Branching ratios for χ_1 decays in the minimal H scenario as a function of $\Delta m = m_{\chi_1} - m_{\chi_0}$. Only channels with a branching ratio larger than 1% are shown.

Figure 3. Proper decay length for χ_1 in the minimal H scenario as a function of $\Delta m = m_{\chi_1} - m_{\chi_0}$.

values depending on the mass compression. In particular, once the $b\bar{b}$ channel is open the decay length falls below 10 m. It is important to notice, however, that the lifetime scales as $\tau \propto (\Lambda/C_{H\chi\chi})^2$ and will be further suppressed for larger values of $\Lambda/C_{H\chi\chi}$.

1.2 Minimal Couplings to Photon (DIPOLEDM_MINIMALA_UFO)

The minimal scenario assumes that χ_0 only couples through the $F^{\mu\nu}$ effective operator and the diagonal entries are zero:

$$(C_{\gamma\chi\chi})_{ij} = 0, \text{ if } i = j \quad (1.22)$$

$$(C_{H\chi\chi})_{ij} = 0 \quad (1.23)$$

$$(y_\chi)_{ij} = 0, \text{ if } i = 0 \text{ or } j = 0 \quad (1.24)$$

With the above assumptions the Lagrangian simplifies to:

$$\mathcal{L}_\chi = i\bar{\chi}_i \not{\partial} \chi_i - \tilde{M}_{ij} \bar{\chi}_i \chi_j - (y_\chi)_{11} \bar{\chi}_1 \chi_1 \phi + \frac{(C_{\gamma\chi\chi})_{01}}{\Lambda} (\bar{\chi}_0 \sigma^{\mu\nu} \chi_1 + h.c.) F_{\mu\nu} \quad (1.25)$$

resulting in the vertices given in Table 4.

Interaction	Vertex term (Minimal A)
$h \chi_1 \chi_1$	$\frac{i}{\sqrt{2}} (y_\chi)_{11} \sin \alpha$
$S \chi_1 \chi_1$	$-\frac{i}{\sqrt{2}} (y_\chi)_{11} \cos \alpha$
$\chi_1 \chi_0 A^\mu$	$-i \frac{1}{\Lambda} (C_{\gamma\chi\chi})_{01} (\gamma^\mu \not{p} - \not{p} \gamma^\mu)$
$S G_\mu G_\nu$	$i \frac{g_s^2}{12\pi^2 v} \sin \alpha F(m_S^2/m_t^2) (p_1^\mu p_2^\nu - g^{\mu\nu} p_1 \cdot p_2)$

Table 4. Feynman rules for the relevant interactions in the **minimal** model with couplings to photons. The couplings between S and two SM particles are the same as in Table 2 and have been omitted.

In this scenario the heavy fermion can only decay through the effective operator, hence:

$$\Gamma(\chi_1 \rightarrow \gamma \chi_0) = \frac{C_{\gamma\chi\chi}^2}{2\pi} \frac{M_1^2}{\Lambda^2} M_1 \left(1 - \frac{M_0^2}{M_1^2}\right)^3 \quad (1.26)$$

Since this is the only allowed channel it has always 100% branching ratio and the lifetime for the compressed scenario is given by:

$$c\tau = 1.55 \times 10^{-6} \text{ m} \left(\frac{0.1}{C_{\gamma\chi\chi}}\right)^2 \left(\frac{\Lambda}{10 \text{ TeV}}\right)^2 \left(\frac{1 \text{ GeV}}{\Delta m}\right)^3, \quad (1.27)$$

where we have assumed $M_0 \lesssim M_1$, so $\Delta m^2 = M_1^2 - M_0^2 \simeq 2M_1 \Delta m$. In Fig. 4 we show the decay length as a function of Δm . As we can see, even for $\Lambda = 100 \text{ TeV}$ the decay is prompt as long as $\Delta m > 1 \text{ GeV}$.

Figure 4. Proper decay length for χ_1 in the minimal photon scenario as a function of $\Delta m = m_{\chi_1} - m_{\chi_0}$.