14.02 – Fall 2018
Recitation 3

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Part 1: Dealing with dynamic systems
(With and without phase diagrams)
Application to Problem set 3.2

Dynamic Systems

- Up to now, we have studied *steady states*, i.e. systems' **resting points**;
- E.g. in IS Income Y is a function of G. We change G to G', and find corresponding Y', no study of *how* we get there *over time*;
- In dynamic systems, we are interested in the *transition dynamics* of variables;
- Express variables at **time** $t+1$ as a function of the same variable at t and other *forcing terms*;
- E.g. in problem set 3:

$$u_{t+1} = (1 - f - s)u_t + s$$

- In future classes, economic growth $Y_{t+1} = f(Y_t)$

What we are interested in

1. Does the system **converge** somewhere? I.e. Does it have a *steady state* and what is it?
2. How do variables evolve, starting from some arbitrary point? *Here is where the dynamics is!*
3. What conditions are required for convergence to steady state?

The Steady State

- Given some stability conditions (i.e. variables do not explode), dynamic systems have a **steady state**;
- A steady state is a *resting point*, the main variables of interest *do not move any longer*;
- Equivalently, the system reaches a *stationary equilibrium*;
- Example in the Pset: the steady state unemployment rate defined as

$$u^* \text{ s.t. } u_{t+1} = u_t = u^*$$

- How to find it? Impose the condition above!

$$u^* = (1 - f - s)u^* + s \;\Rightarrow\; u^* = \frac{s}{f + s}$$

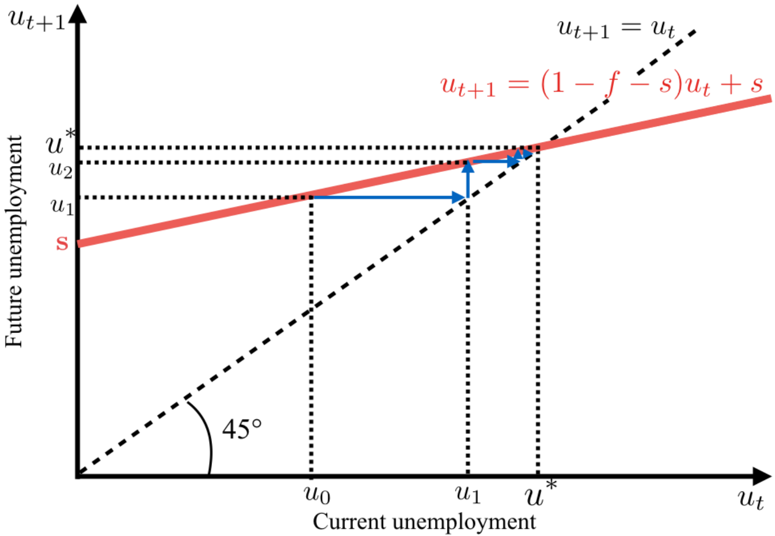
Phase Diagrams: Seeing the Dynamics

- For now just an extra, will use more at the end of the class;
- Tool to visualize the dynamics, i.e. *how variables converge to steady states*;
  - Plot variable at $t+1$ as a function of variable at time t (e.g. $u_{t+1} = f(u_t)$) together with 45-degree line;
  - **Intersection** of the lines gives the *steady state*.

Starting from arbitrary value at time t, represent the dynamics with arrows:

1. Use the schedule $u_{t+1} = f(u_t)$ to read future value of variable given the present;
2. Then we move one period into the future... u_{t+1} is now u_t! *Graphically: use the 45-degree line to read the future value of the variable on the horizontal axis*
3. Go back to (1) and repeat until tired.

Unemployment example: case $0 < (1 - f - s) < 1$



The graph in a table

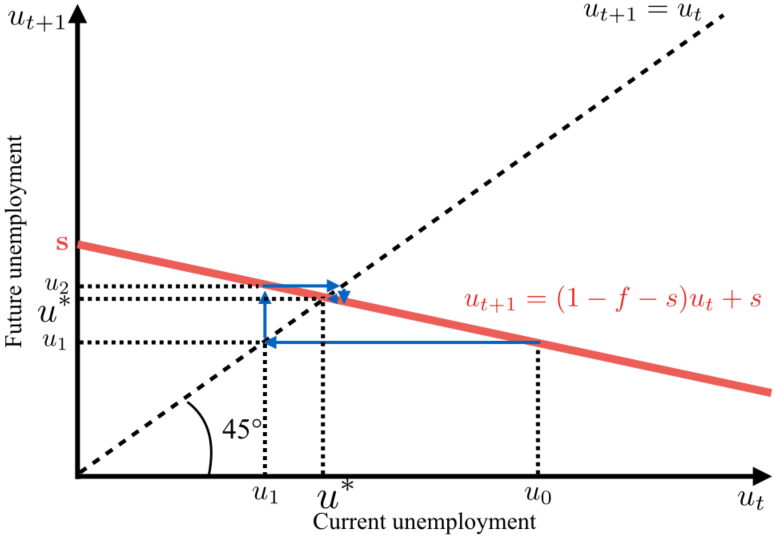
Some numbers consistent with the graph:

- Assume $f = .3$, $s = .6$;
- Dynamics: $u_t = .1 \times u_{t-1} + .6$;
- Steady State: $u^* = u_t = u_{t+1} = 2/3 \approx .6667$;

Suppose we start at $u_0 = .3$

t	u_t	u_{t+1}
0	.30	.63
1	.63	.663
2	.663	.6663
3	.6663	.66663
	...	

Unemployment example: case $-1 < (1 - f - s) < 0$



What if $|1 - f - s| \geq 1$?

- The system has **no steady state**... The dynamics is *explosive* or *periodic* (oscillation between a set of values);
- Remember analogous condition $0 < 1 - c_1 < 1$ for the equilibrium on goods market;
- Only economically reasonable scenario: $1 - f - s = -1$ given by $f = s = 1$. Start form $u_0 = 0$:

$$u_1 = -1 * u_0 + 1 = 1$$

- $u_2 = 0$, $u_3 = 1, u_4 = 0$... no convergence!

No worries if all this is unclear right now... you will see it again when we cover growth, but useful to have it in the back of your mind...

$n_t + u_t = 1$, $0 < f, s < 1 \;\Rightarrow |1 - f - s| < 1$, job finding and separation rates.

1. Given
$$u_{t+1} = (1 - f)u_t + s \times n_t$$
write an expression for n_{t+1} as a function of u_t, n_t, s and f;
2. Write u_{t+1} as a function of u_t, f, s;
3. Compute the “stationary” value of unemployment rate u^*;
4. Compute $u_{t+1} - u^*$ as function of $u_t - u^*$. Explain why u_t converges to u^*. In what sense is u^* an equilibrium value ?
5. Assume that in Portugal, $s^{Port} = 0.005$ and $f^{Port} = 0.045$ and that in the United States, $s^{US} = 0.05$ and $f^{US} = 0.45$. Compare u^* in both countries.
Is u sufficient to compare workers' conditions across countries ?

Chapter 7 Review
The Labor Market
(And application to 3.3.6)

Labor Market Equilibrium

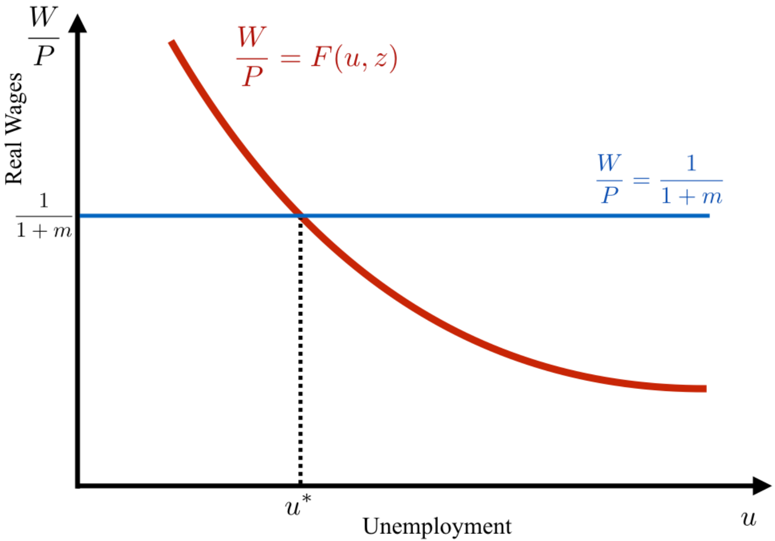
- The objects of interest are:
  - Equilibrium **real wage** $\frac{W}{P}$;
  - Equilibrium **unemployment** u.
- Use a *qualitative* graph: real wage on vertical axis, u on horizontal;
- Key Equations:

$$\frac{W}{P} = \frac{1}{1+m} \tag{PS}$$

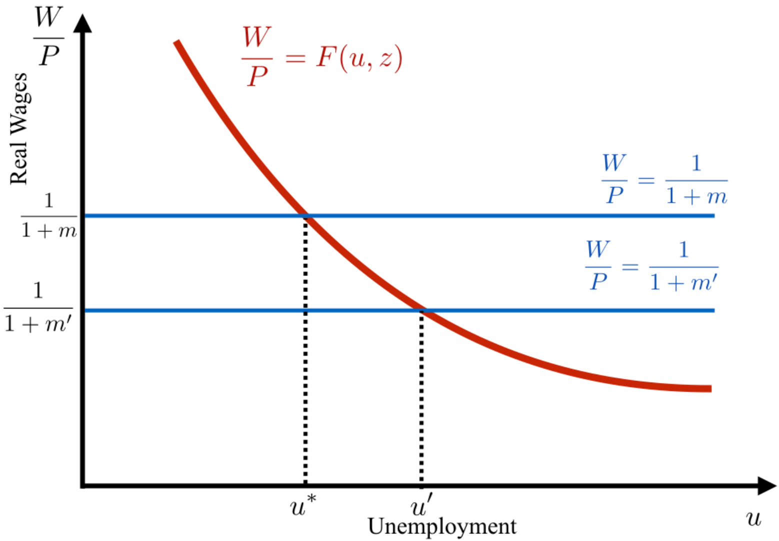
$$\frac{W}{P} = F(\underset{-}{u}, \underset{+}{z}) \tag{WS}$$

- (PS) Comes from *price setting* by firms $P = W(1+m)$;
- (WS) from *wage setting* by workers $W = P\ F(u, z)$, with F capturing factors affecting *bargaining power*.

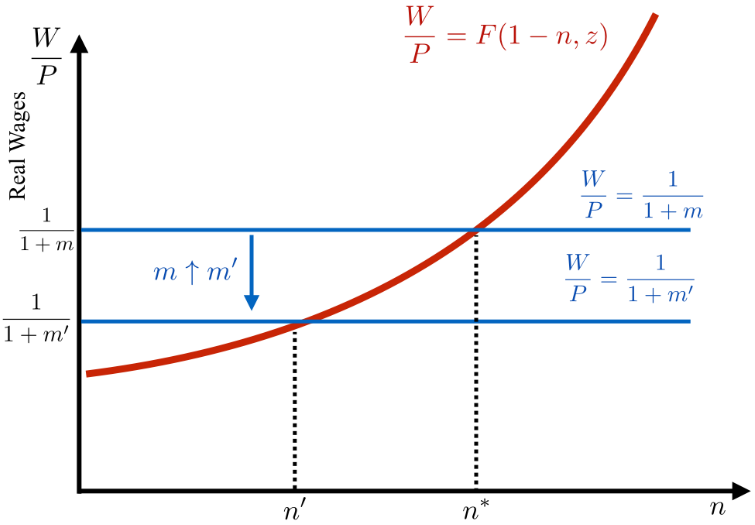
Equilibrium Graphically



Comparative Statics - Increase in Markup m



Comparative Statics for Employment n - Increase in Markup m



Comparative Statics for Employment n - Increase in unemployment benefits z