

# Competing for Inventors: Market Concentration and the Misallocation of Innovative Talent

Andrea Manera

October 5, 2021

# Motivation

- ▶ R&D is key to innovation and growth

Romer (1987,1990), Aghion and Howitt (1992), Grossman and Helpman (1991)

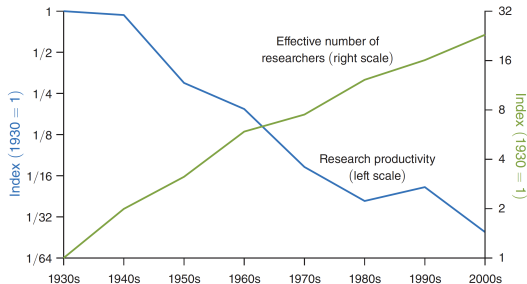
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Source: Bloom et al. (2020)

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- ▶ R&D productivity and growth have fallen  
Bloom et al. (2020), Fernald et al. (2014), Gordon (2016)
- ▶ Concerns over inventors' misallocation  
Acemoglu et al. (2018, 2021)

Tech Giants Are  
Paying Huge Salaries  
for Scarce A.I. Talent

Source: The New York Times (2017)

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## 3. Schumpeterian model with *defensive innovation*:

- ◇ Explain mechanism
- ◇ Discuss policy

# Related Literature

- ▶ *Trends in innovation and R&D productivity*

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- ▶ *Models of Schumpeterian and Defensive innovation*

Aghion and Howitt (1992), Acemoglu and Akcigit (2012), Abrams et al. (2018), Jo (2019)

# Data Construction



# Empirical Analysis Objectives

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  - ◇ Evaluate effects of increased concentration on R&D productivity

# Data Sources

- ▶ USPTO (patent-year) and Goldschlag et al. (2016):
  - ◇ patent citation and disambiguated inventor id's, 1975-present;
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  - ◇ patent classification by NAICS of application (1978-2016)
- ▶ Economic Census and Keil (2017) (5-year-NAICS)
  - ◇ NAICS 4-digit concentration measure: HHI and HHI lower bound
  - ◇ 157 NAICS 4-digit sectors out of 304 business sectors
  - ◇ Most of manufacturing, retail, warehousing, telecommunications, publishing
  - ◇ Output per worker growth



# “Knowledge Markets”

▶ *Ideally:*

◇ set of product markets with same *required knowledge* innovate

# “Knowledge Markets”

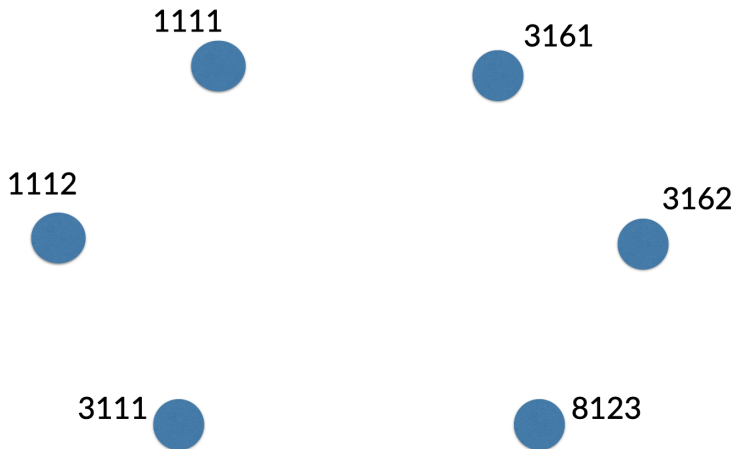
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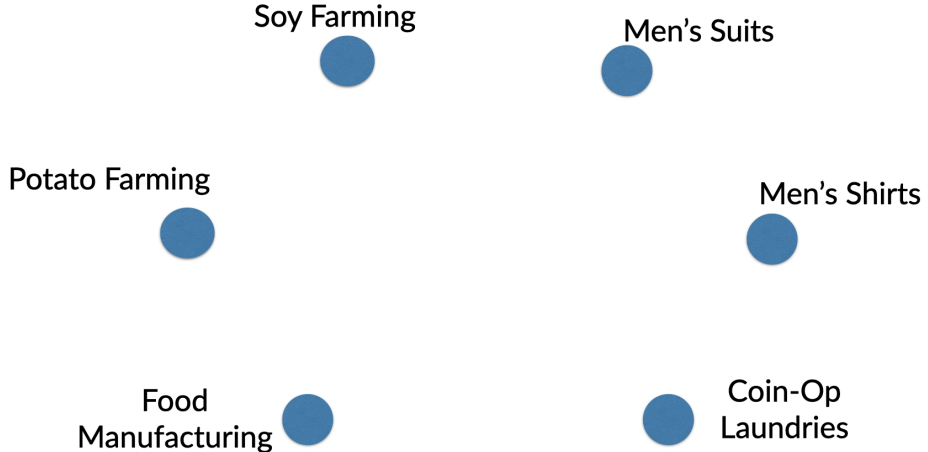
▶ *In practice:*

- ◇ NAICS 4-digit sectors connected by flows of inventors
- ◇ Identify flows from patents with disambiguated inventors
- ◇ Group NAICS that have strongest connections

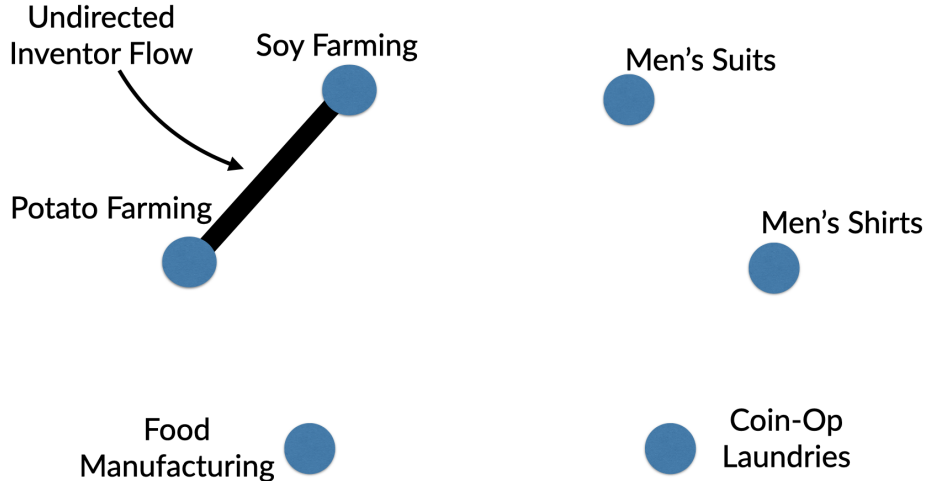
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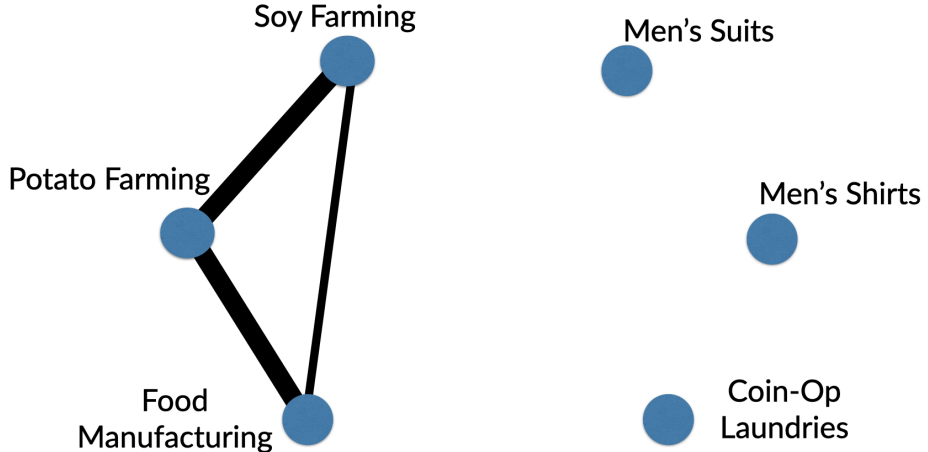
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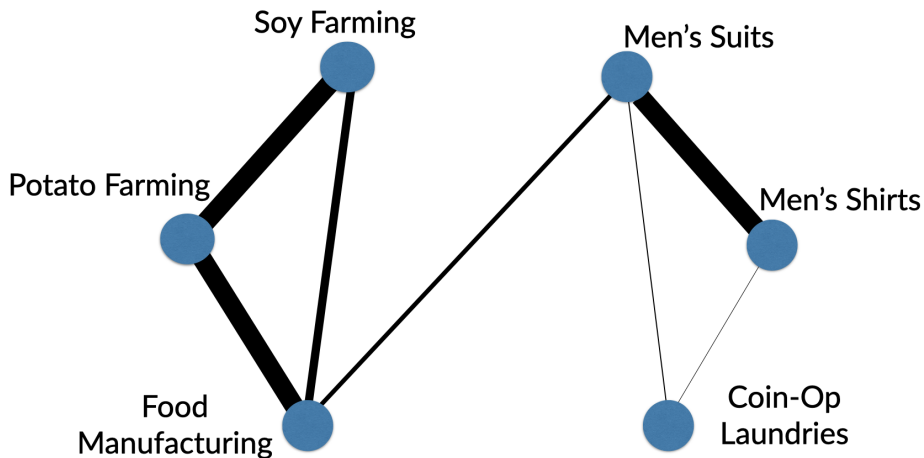
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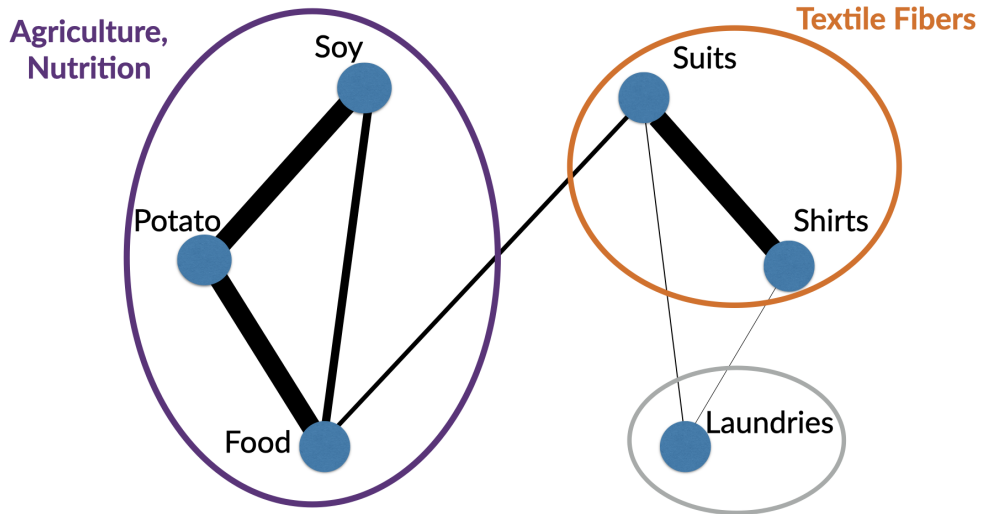
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## Constructing Inventor Flows

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# Weighting Flows: “Effective Inventors”

- ▶ Flows shall adjust for productivity of inventors who move

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## ► *Effective inventor flow from sector 1 to 2:*

$$flow_{1 \rightarrow 2, t} = \sum_i \# \{i' \text{'s transitions } 1 \rightarrow 2 \text{ in } t\} \cdot \alpha_i$$

# Detecting Knowledge Markets: Network Weights

- ▶ Total undirected flows:

$$\text{flow}_{12} = \sum_t (\text{flow}_{1 \rightarrow 2,t} + \text{flow}_{2 \rightarrow 1,t})$$

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- ▶ Modularity: density of links *within* communities versus *between*

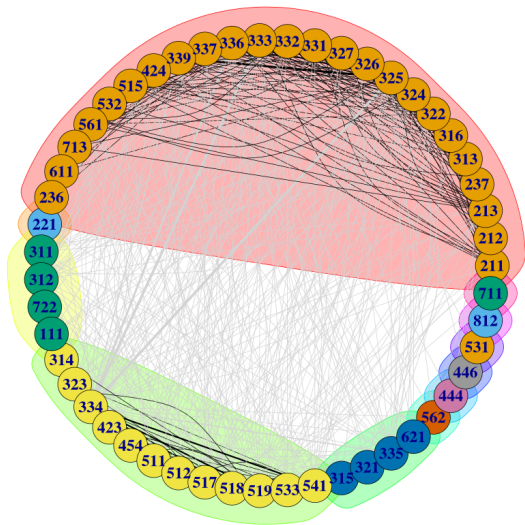
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- ▶ Result: 10 non-singleton sets of NAICS 4-digit that share inventors with non-missing HHI

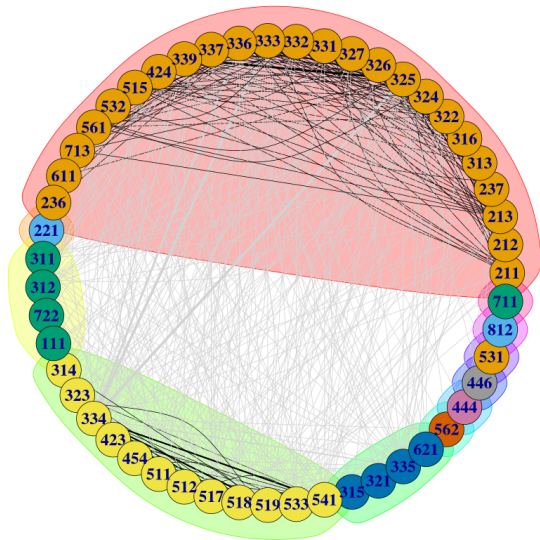
## Visualization at 3-digit NAICS





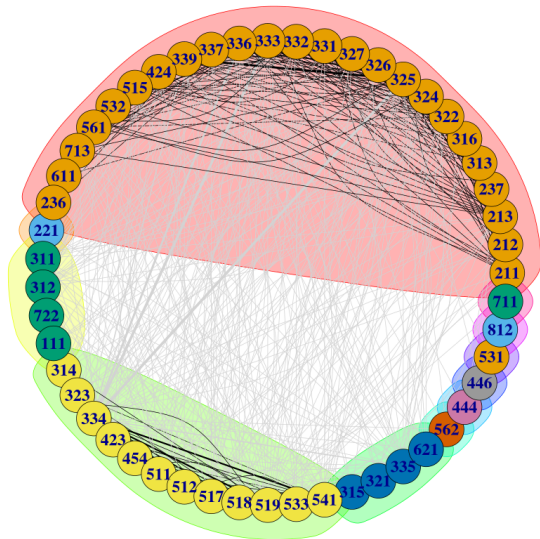
# Features of Flows and Knowledge Markets

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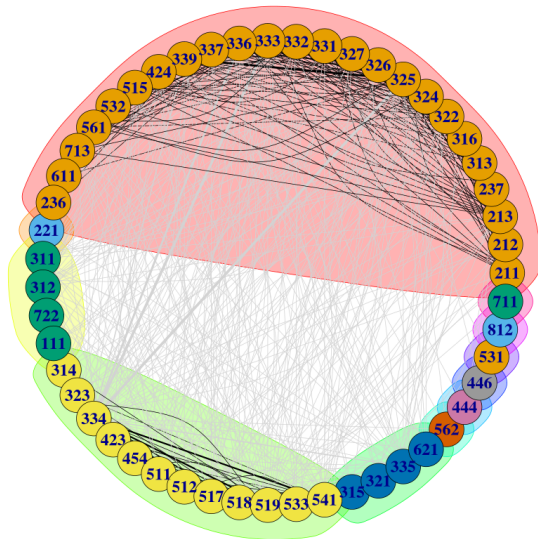
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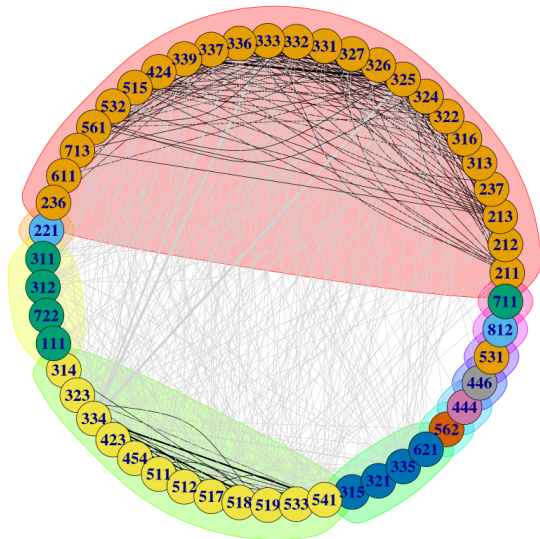
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Computer Products, Telecommunications, Data Processing



# Empirical Analysis

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- ▶ Look within *knowledge* markets, across products:
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- ▶ Look within *product* markets:
  - ◇ Effect of increased concentration on inventors' productivity
  - ◇ Correlation between patent metrics and changes in inventors' share



# Variable Definition

- ▶ Sector  $p$ 's share of effective inventors in knowledge market,  $k$ :

$$\text{Inventor Share}_{p,t} \equiv \frac{\sum_{p(i,t)=p} \alpha_i}{\sum_{k(i,t)=k} \alpha_i},$$

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- ▶ Size of sectors or firms: real sales, real sales per company (Economic Census)

# Specification

- ▶ Long-difference 2012-1997 at NAICS 4-digit sector,  $p$ :

$$\Delta \text{Share}_{p, 2012-1997} = f_k \mathbf{1}\{p \in k\} + \beta \Delta \text{HHI}_{p, 2012-1997} + \gamma \Delta \text{Size}_{p, 2012-1997} + \varepsilon_p,$$

- ▶  $f_k \mathbf{1}\{p \in k\}$ : sector  $p$  belongs to knowledge market  $k$
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- ▶ Weighted by sales, robust standard errors

# Main Specification Results

► No Controls

► Census HHI

► Trim Outliers

► Raw Inventors

	$\Delta$ Inventor Share (pp)	
	(1)	(2)
$\Delta$ <u>HHI</u>	26.093*	22.509*
	(10.696)	(10.848)
$\Delta$ log Sales	0.914**	0.548*
	(0.278)	(0.243)
Knowledge Market FE		✓
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	157	153

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

- $\Rightarrow$  1 s.d. increase in HHI: .033  $\Rightarrow$   $\uparrow$  .858pp inventor share (.55 s.d.)
- Compares to average inventors' share of 1.16pp, median .37pp



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  - ◇ IV analysis using Mercatus regulation data [▶ Details](#)

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- ▶ Significantly decreases
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- ◇ Inventors' productivity (next slide)

- ◇ Patents' forward citations ▶ Table

# Fall in Inventors' Productivity

► Robustness to Outliers

	$\Delta$ Growth/Inventor (pp)	
	(1)	(2)
$\Delta$ HHI	-0.332** (0.113)	-0.292* (0.123)
$\Delta \log$ Sales		-0.052* (0.021)
Knowledge Market FE	✓	✓
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	101	101

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

## Back-of-the-envelope Loss from Misallocation

- ▶ Median change in annual output per worker growth in sample :  $-2.73\text{pp}$
- ▶ Implied loss from misallocation for the median sector with  $\Delta$  Inventor Share  $> 0$ :

$$\Delta g = \frac{\Delta \left( \frac{g}{\text{Inventor}} \right)}{\Delta \text{HHI}} \times \Delta \text{HHI} \times \text{Inventors}$$

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- ▶ Within product markets, increasing concentration:
  - ◇ Lowers inventors' productivity (growth per inventor)
  - ◇ Increases share of inventors at top firms
  - ◇ Lowers forward citations



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  - ◇ Optimal to subsidize entrants' R&D in concentrated sectors
  - ◇ Cost-neutral policy gives up to .28pp higher annual growth (+10%)

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► Concentration v. Markup

Consumption good is C-D of intermediates:

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Bertrand competition, incumbent sets  $p_t(i) = c_t(i)$ , realizes monopoly profits:

$$\Pi_t = \left( \frac{\phi - 1}{\phi} \right) c_t(i) y_t(i) = \left( \frac{\phi - 1}{\phi} \right) Y_t.$$

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Normalized incumbents' profits are constant:

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- ▶  $\alpha_I \frac{x_I^\gamma}{\gamma}$  total incumbents' inventors
- ▶ Fix  $w^{RD} = 1$  for now

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$$\rho v(\omega) = \left( \frac{\phi - 1}{\phi} \right) + \delta (v(\mathbf{1}) - v(\omega)) - x_{e,\omega} (v(\omega))$$

- ▶  $x_{e,1}, x_{e,\omega}$ : total research intensity of entrants
- ▶  $\delta$ : depreciation of patent wall (patent protection)
- ▶ Gain from innovation depends on the difference between  $x_{e,\omega}$  and  $x_{e,1}$

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- ▶ Finite demand IFF:

$$x_{e,\Omega} = \frac{v(1)}{\zeta \Omega}$$

# Growth and Inventors' Productivity

► Equilibrium Definition

Stationary distribution with constant growth: ► Derivation

$$\mu = \left[ \mu_1 \quad \mu_\omega \quad \mu_{e,1} \quad \mu_{e,\omega} \right]$$

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Inventors' productivity (growth per inventor):

$$\frac{g}{L^{RD}} = \eta \frac{x_{e,\omega} \mu_{e,\omega} + x_{e,1} \mu_{e,1} + \lambda x_I \mu_1}{\zeta (\omega x_{e,\omega} \mu_{e,\omega} + x_{e,1} \mu_{e,1}) + \alpha_I \frac{x_I^\gamma}{\gamma} \mu_1}.$$

# One-Sector Equilibrium Proposition

## Proposition

*Consider the above model with a perfectly elastic production and R&D labor supply.*

*Assume  $\delta, \lambda = 0$ , and*

$$\sqrt{\frac{\phi - 1}{\phi}} \left( \frac{\alpha_I - \zeta \omega (\omega - 1)}{\alpha_I \zeta \omega} \right) > \rho.$$

*An increase in the markup factor  $m \equiv \frac{\phi - 1}{\phi}$ , increases incumbents' and entrants' R&D and the incumbents' share of total R&D labor, and decreases inventor productivity,  $g/L^{RD}$ .*



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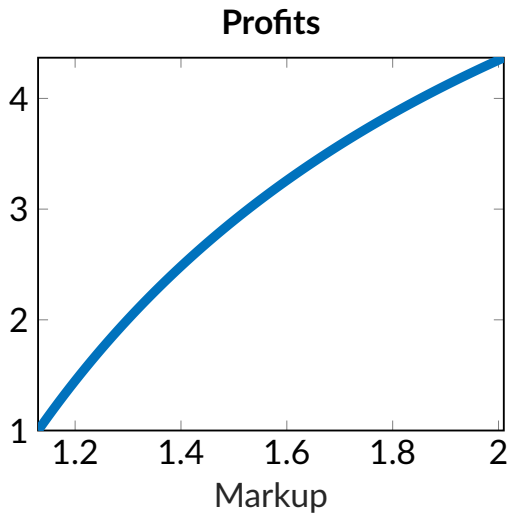
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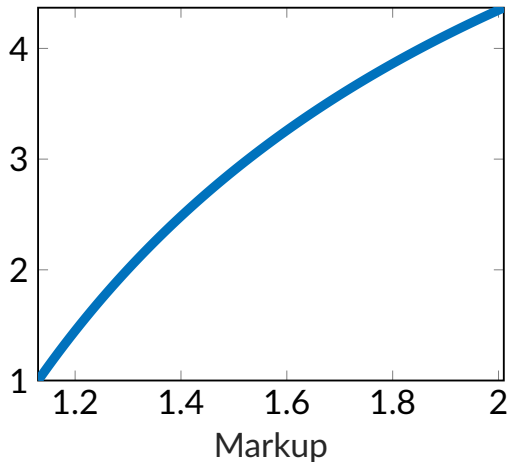
- Sufficient conditions, holds for wider range of parameters in simulations

## Single Sector Comparative Statics: Markup Increase

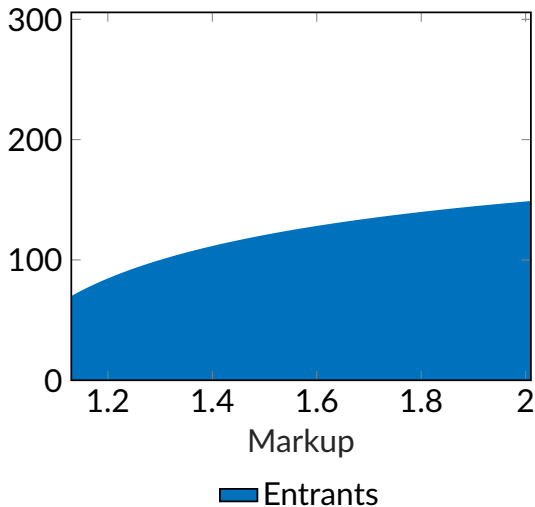


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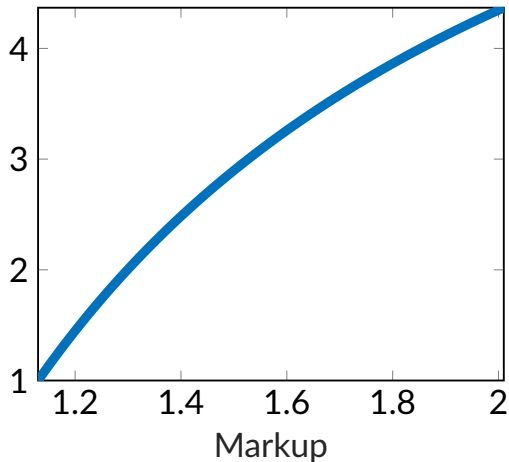


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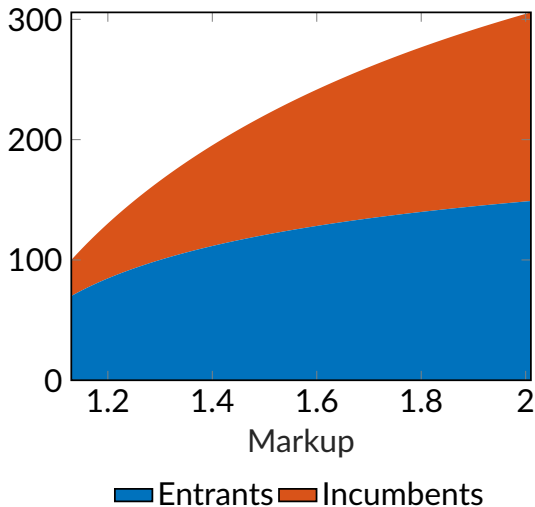


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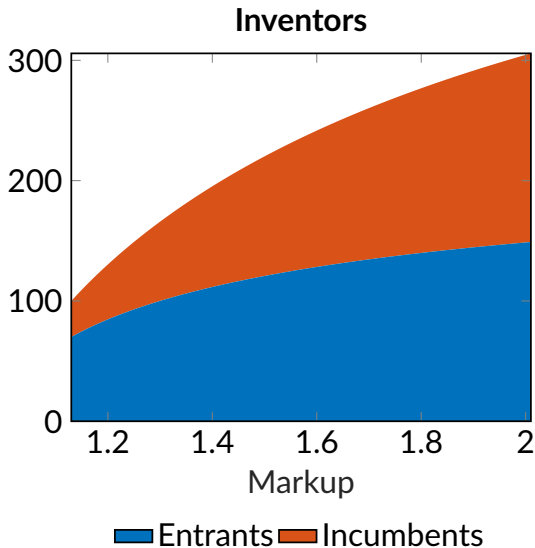
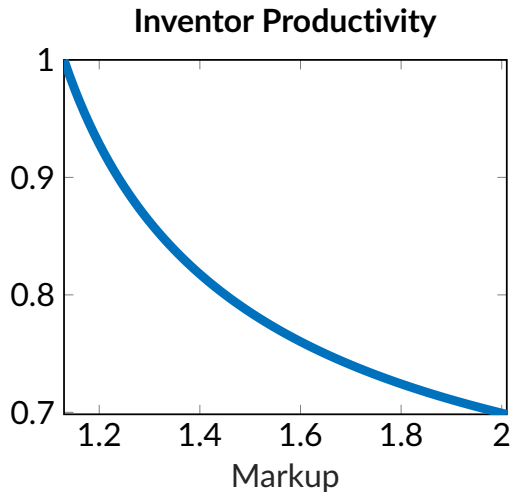
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# Calibration and Policy

# Two-Sectors, Inventor Market Equilibrium

- ▶ Two Cobb-Douglas sectors as above:

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- ▶ Normalize  $L^{\text{RD},s} = 100$ .

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►  $\tau, s$ : corp. tax 23% and R&D subsidy 19% (Akcigit et al., 2019)

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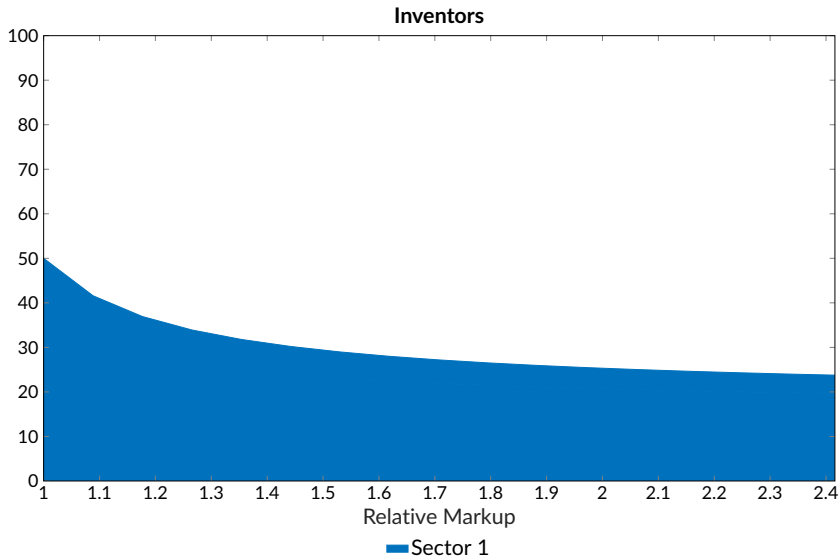
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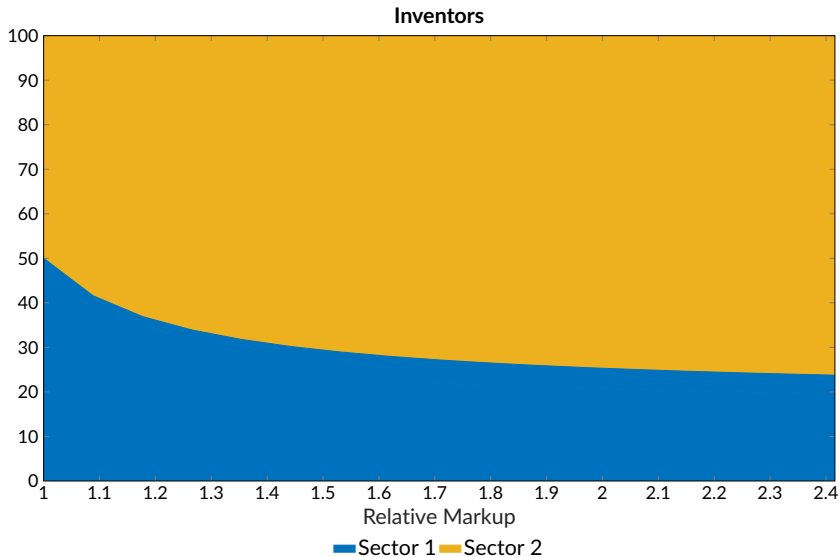
Parameter Name	Symbol	Value	Target
Incumbent Costs	$\alpha_I$	6.67	Top 10% Firms' Inventor Share, 1997: 30.3%
Entrants' Costs	$\zeta$	3.38	Business R&D Share over GDP, 1997: 1.81%
Innovation Step	$\eta$	0.0024	Average Annual Growth, 1997-2012: 2.45%

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# CS: Increase in Sector 2 Markup

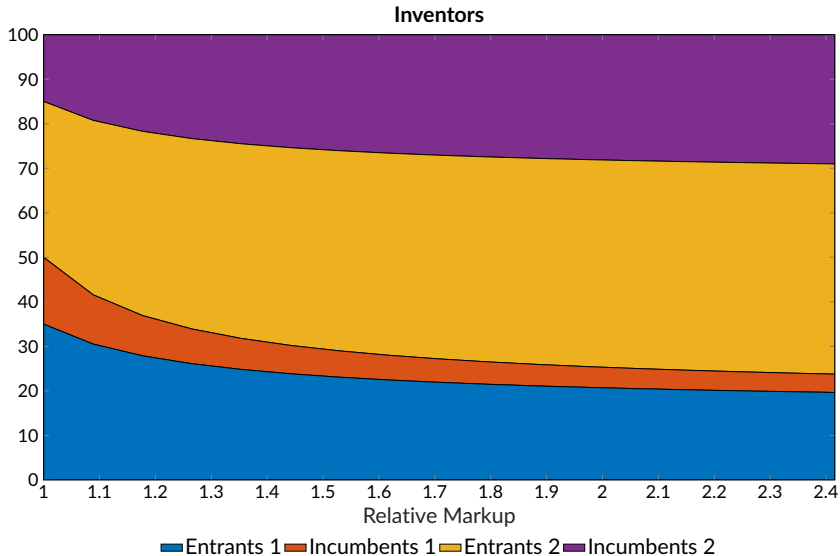


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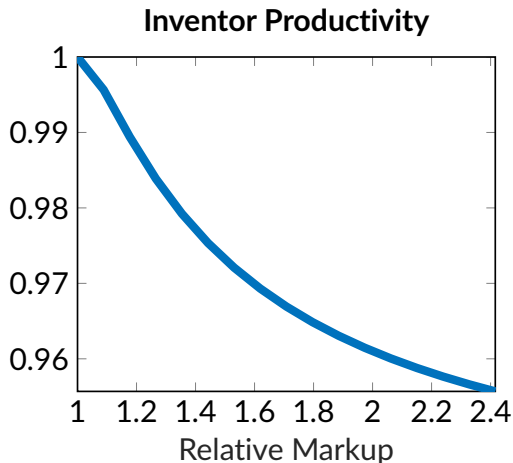
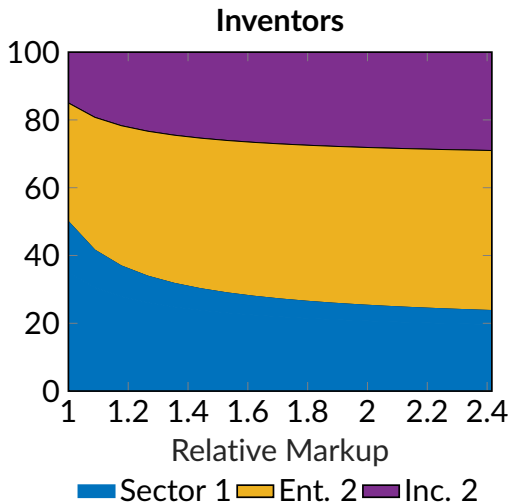




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  - ◇ sector- and position-specific R&D subsidies,  $s_{I,i}, s_{e,i}$

# Growth-Maximizing Policy

- ▶ Planner wishes to maximize constant growth rate
- ▶ Instruments:
  - ◇ corporate tax, fixed and flat  $\tau = 23\%$
  - ◇ sector- and position-specific R&D subsidies,  $s_{I,i}, s_{e,i}$
  - ◇ balanced budget

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  - ◇ balanced budget
- ▶ Three scenarios versus “current” system (flat taxes and subsidies):



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  - ◇ balanced budget
- ▶ Three scenarios versus “current” system (flat taxes and subsidies):
  - ◇ Cost-Neutral sector- and position-specific subsidies

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  - ◇ balanced budget
- ▶ Three scenarios versus “current” system (flat taxes and subsidies):
  - ◇ Cost-Neutral sector- and position-specific subsidies
  - ◇ Cost-Neutral sector-specific subsidies

# Growth-Maximizing Policy

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  - ◇ sector- and position-specific R&D subsidies,  $s_{I,i}, s_{e,i}$
  - ◇ balanced budget
- ▶ Three scenarios versus “current” system (flat taxes and subsidies):
  - ◇ Cost-Neutral sector- and position-specific subsidies
  - ◇ Cost-Neutral sector-specific subsidies
  - ◇ Cost-Neutral position-specific subsidies

# Policy Results

	Baseline	Optimal Cost-Neutral	Cost-Neutral Sector	Cost-Neutral Entry
	Sector 1	Sector 2		
	(1)	(2)		
<i>R&amp;D Subsidies:</i>				
$s_I$	19%	19%		
$s_e$	19%	19%		
<i>Aggregates:</i>				
$L_I^{RD}$	8.66	22.20		
$L_e^{RD}$	27.36	41.78		
$L_{TOT}^{RD}$	36.02	63.98		
Sector Growth	1.91%	2.93%		
GDP Growth	2.42%			

# Policy Results

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector	Cost-Neutral Entry
	Sector 1	Sector 2	Sector 1	Sector 2		
	(1)	(2)	(3)	(4)		
<i>R&amp;D Subsidies:</i>						
$s_I$	19%	19%	0%	0%		
$s_e$	19%	19%	0%	42.42%		
<i>Aggregates:</i>						
$L_I^{RD}$	8.66	22.20	8.01	13.89		
$L_e^{RD}$	27.36	41.78	26.44	51.66		
$L_{TOT}^{RD}$	36.02	63.98	34.45	65.55		
Sector Growth	1.91%	2.93%	1.85%	3.61%		
GDP Growth	2.42%		2.73%			

# Policy Results

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector	Cost-Neutral Entry
	Sector 1	Sector 2	Sector 1	Sector 2		
	(1)	(2)	(3)	(4)		
<i>R&amp;D Subsidies:</i>						
$s_I$	19%	19%	0%	0%		
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GDP Growth	2.42%		2.73%			

# Policy Results

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector		Cost-Neutral Entry
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	
	(1)	(2)	(3)	(4)	(5)	(6)	
<i>R&amp;D Subsidies:</i>							
$s_I$	19%	19%	0%	0%	41.90%	0%	
$s_e$	19%	19%	0%	42.42%	41.90%	0%	
<i>Aggregates:</i>							
$L_I^{RD}$	8.66	22.20	8.01	13.89	12.65	17.45	
$L_e^{RD}$	27.36	41.78	26.44	51.66	32.42	37.47	
$L_{TOT}^{RD}$	36.02	63.98	34.45	65.55	45.07	54.93	
Sector Growth	1.91%	2.93%	1.85%	3.61%	2.27%	2.62%	
GDP Growth	2.42%		2.73%		2.45%		

# Policy Results

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector		Cost-Neutral Entry	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>R&amp;D Subsidies:</i>								
$s_I$	19%	19%	0%	0%	41.90%	0%	0%	0%
$s_e$	19%	19%	0%	42.42%	41.90%	0%	28.25%	28.25%
<i>Aggregates:</i>								
$L_I^{RD}$	8.66	22.20	8.01	13.89	12.65	17.45	6.26	16.41
$L_e^{RD}$	27.36	41.78	26.44	51.66	32.42	37.47	30.61	46.72
$L_{TOT}^{RD}$	36.02	63.98	34.45	65.55	45.07	54.93	36.87	63.13
Sector Growth	1.91%	2.93%	1.85%	3.61%	2.27%	2.62%	2.14%	3.26%
GDP Growth	2.42%		2.73%		2.45%		2.7%	



# Policy Results

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector		Cost-Neutral Entry	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>R&amp;D Subsidies:</i>								
$s_I$	19%	19%	0%	0%	41.90%	0%	0%	0%
$s_e$	19%	19%	0%	42.42%	41.90%	0%	28.25%	28.25%
<i>Aggregates:</i>								
$L_I^{RD}$	8.66	22.20	8.01	13.89	12.65	17.45	6.26	16.41
$L_e^{RD}$	27.36	41.78	26.44	51.66	32.42	37.47	30.61	46.72
$L_{TOT}^{RD}$	36.02	63.98	34.45	65.55	45.07	54.93	36.87	63.13
Sector Growth	1.91%	2.93%	1.85%	3.61%	2.27%	2.62%	2.14%	3.26%
GDP Growth	2.42%		2.73%		2.45%		2.7%	

# Policy Results

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector		Cost-Neutral Entry	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>R&amp;D Subsidies:</i>								
$s_I$	19%	19%	0%	0%	41.90%	0%	0%	0%
$s_e$	19%	19%	0%	42.42%	41.90%	0%	28.25%	28.25%
<i>Aggregates:</i>								
$L_I^{RD}$	8.66	22.20	8.01	13.89	12.65	17.45	6.26	16.41
$L_e^{RD}$	27.36	41.78	26.44	51.66	32.42	37.47	30.61	46.72
$L_{TOT}^{RD}$	36.02	63.98	34.45	65.55	45.07	54.93	36.87	63.13
Sector Growth	1.91%	2.93%	1.85%	3.61%	2.27%	2.62%	2.14%	3.26%
GDP Growth	2.42%		2.73%		2.45%		2.7%	

⇒ **Subsidizing entry** is the most effective policy: .28-.31pp higher growth

# Conclusion

- ▶ Concentrating sectors attracted inventors away from competitive
- ▶ Incumbents increasingly deployed inventors to defensive projects
- ▶ Misallocation can explain 28.6% of 1997-2012 fall in annual output/worker growth
- ▶ Key model mechanism: *defensive innovation*
- ▶ Policy: entrant subsidies in *less competitive sectors*
- ▶ Cost-neutral entry subsidies give .28pp higher annual growth

Deals

## Big Tech Swallows Most of the Hot AI Startups

An acquisition spree by Apple, Amazon, Facebook, Google and Microsoft eliminated potential rivals and concentrated brain power in this critical field.

**The New York Times**

<https://www.nytimes.com/2017/10/22/technology/artificial-intelligence-experts-salaries.html>

## Tech Giants Are Paying Huge Salaries for Scarce A.I. Talent

# Total Inventor Flows

[▶ Back](#)

- ▶ Strength of connection between two sectors
- ▶ Build directed flows for each inventor  $i$  (avoid double counting):

$$\text{flow}_{1 \rightarrow 2, i, t} \equiv \frac{\sum \mathbf{1}\{i \text{ moves } 1 \rightarrow 2 \text{ in } t\}}{\sum_{j, k} \mathbf{1}\{i \text{ moves } j \rightarrow k \text{ in } t\}} \times \alpha_i$$

- ▶ Compute total outflows and inflows for each NAICS 4-digit sector:

$$\text{inflow}_{\text{NAICS}} = \sum_n \sum_t \sum_i \text{flow}_{n \rightarrow \text{NAICS}, i, t},$$

# Network Weights [▶ Back](#)

- ▶ Compute share of inflows and outflows, e.g.:

$$\text{share}_{1 \leftarrow 2} = \frac{\sum_t \sum_i \text{flow}_{2 \rightarrow 1, i, t}}{\text{inflow}_1}$$

- ▶ Define weight:

$$W_{12} = W_{21} = \min \left\{ \frac{\text{share}_{1 \leftarrow 2} + \text{share}_{1 \rightarrow 2}}{2}, \frac{\text{share}_{2 \leftarrow 1} + \text{share}_{2 \rightarrow 1}}{2} \right\}$$

- ▶ Average tends to overstate flows from small sectors to large

# Problem: Maximize Modularity

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- Assigns sectors  $i$  to  $N$  *non-overlapping* communities  $c_i$  to maximize modularity

$$\max_N \max_{(c_1, \dots, c_N)} Q \equiv \frac{1}{2W} \sum_{ij} \left[ W_{ij} - \frac{W_i W_j}{2W} \right] \mathbf{1} \{c_i = c_j\},$$

- ◇  $W_{ij}$ , weight of edge connecting node  $i$  to  $j$
- ◇  $W_i = \sum_k W_{ik}$ , sum of weights for edges with one end in node  $i$ ,  $W$  sum of all weights in the graph
- ◇  $\frac{W_i W_j}{2W}$  is the expected number of weighted edges between nodes  $i$  and  $j$

# Louvain Algorithm

[▶ Back](#)

- ▶ Louvain method (Blodel et al., 2008). Assign each node to its own community. Then, repeat iteratively:
  1. Compute local deviations in modularity from reassigning the node to neighboring communities
  2. Move node in highest modularity direction
  3. Redefine a network with new communities as nodes



# HHI Lower Bound

[▶ Back](#)

▶ Keil, 2017:

$$\begin{aligned}\underline{\text{HHI}}_{p,t} = & 4 \left[ \frac{\text{Top-4 Share}_{p,t}}{4} \right]^2 + 4 \left[ \frac{\text{Top-8 Share}_{p,t} - \text{Top-4 Share}_{p,t}}{4} \right]^2 \\ & + 12 \left[ \frac{\text{Top-20 Share}_{p,t} - \text{Top-8 Share}_{p,t}}{12} \right]^2 \\ & + 30 \left[ \frac{\text{Top-50 Share}_{p,t} - \text{Top-20 Share}_{p,t}}{30} \right]^2\end{aligned}$$

▶ Coincides with actual if:

- ◇ Sector has 50 or less firms
- ◇ Shares are distributed equally between firms in brackets 0-4, 5-8, 9-20, 21-50

▶ Correlation with actual HHI is .93

## IV Analysis: Regulation measure [▶ Back](#)

- ▶ Regulation measure from Mercatus RegData 4.0
- ▶ Counts of regulation affecting NAICS 4d using text analysis
- ▶ Extended to all sectors with HHI using cosine-similarity between sector descriptions
- ▶ For all pairs NAICS 4-d sectors:
  - ◇ Build cosine similarity between descriptions
- ▶ For each NAICS 4-d without missing data:
  - ◇ Rank 5 most similar sectors with regulation data
  - ◇ Attribute regulations of top 5 most similar sectors, weighted by cos. similarity
  - ◇ If highest cos-similarity is smaller than .2, use only most similar sector.

## IV Regression: Reduced Form and First Stage

	$\Delta$ Inventor Share (pp)	$\Delta$ HHI
	(1)	(2)
$\Delta \log$ Restrictions	0.478*	0.016*
	(0.220)	(0.007)
$\Delta \log$ Sales	0.539+	-0.000
	(0.274)	(0.005)
Knowledge Market FE	✓	✓
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	153	153

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

## IV Regression: 2SLS Results

Ch. 4d K.M. Eff. Inv. Share (%)		
	(1)	(2)
Ch. HHI lower bound	30.560+ (15.904)	30.096+ (15.819)
Ch. Log Real Sales	0.544* (0.244)	0.525* (0.247)
4D Knowledge Market FE	✓	✓
Sample	Full Sample	Mahalanobis 5%
Weight	Sales	Sales
Observations	157	150
First-Stage F	4.587229	4.753009
Anderson-Rubin p-value	.0281448	.0321185

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# No Controls [▶ Back](#)

	$\Delta$ Inventor Share (pp)	
	(1)	(2)
$\Delta \text{HHI}$	27.293*	
	(11.569)	
$\Delta \text{HHI}$		22.399***
		(6.345)
Knowledge Market FE		
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	157	80

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# Main Specification: Actual HHI

[▶ Back](#)

	$\Delta$ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta$ HHI	27.293*		27.183*		27.326*	
	(11.569)		(11.941)		(11.620)	
$\Delta$ HHI		22.399***		22.399***		22.350***
		(6.345)		(6.345)		(6.343)
Knowledge Market						
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	80	155	80	150	71

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# Main Specification: Robustness to Outliers [▶ Back](#)

	$\Delta$ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta$ HHI	26.093*	22.509*	25.904*	22.716*	26.111*	22.554*
	(10.696)	(10.848)	(11.124)	(10.948)	(10.725)	(11.019)
$\Delta$ log Sales	0.914**	0.548*	0.881**	0.539*	0.918**	0.562*
	(0.278)	(0.243)	(0.275)	(0.242)	(0.283)	(0.261)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	153	155	152	150	139

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# Robustness to Individual Firm Size [▶ Back](#)

	$\Delta$ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta$ HHI	35.230** (12.759)	20.783+ (10.615)	35.230** (12.759)	20.783+ (10.615)	35.154** (12.647)	22.854* (11.197)
$\Delta$ log Size	0.175 (0.382)	-0.040 (0.253)	0.175 (0.382)	-0.040 (0.253)	0.300 (0.460)	-0.055 (0.346)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	81	79	81	79	75	67

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$



# Robustness to Raw Inventors ► Back

	$\Delta$ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta$ HHI	71.724+	67.160+	72.123+	67.736+	71.772+	68.398+
	(39.265)	(37.176)	(39.530)	(37.504)	(39.316)	(37.717)
$\Delta$ log Sales	1.864*	1.422*	1.852*	1.402+	1.878*	1.443+
	(0.766)	(0.717)	(0.764)	(0.712)	(0.774)	(0.745)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	156	156	155	150	142

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# Within-Sector Distribution [▶ Back](#)

	$\Delta$ Top 10%/Bottom 50%	$\Delta$ Top 10%	$\Delta$ Bottom 50%
	(2)	(4)	(5)
$\Delta$ HHI	0.243*	0.018**	-0.008*
	(0.097)	(0.006)	(0.004)
$\Delta$ log Sales	0.328	0.026	0.005
	(0.294)	(0.020)	(0.007)
Knowledge Market FE	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales
Observations	118	118	118

Robust standard errors in parentheses, +  $p < 0.1$ ,\*  $p < 0.05$ ,\*\*  $p < .01$ ,\*\*\*  $p < .001$

# Self-Citations [▶ Back](#)

	$\Delta$ CPC group self-citations		$\Delta$ CPC subgroup self-citations	
	(1)	(2)	(3)	(4)
$\Delta$ Inventor Share (pp)	5.540** (1.783)	5.244* (2.469)	4.561*** (1.211)	4.110* (1.600)
$\Delta$ log Sales	-2.217 (1.879)	-2.099 (1.976)	-1.780 (1.287)	-1.780 (1.265)
Knowledge Market FE		✓		✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample
Weight				
Observations	145	144	145	144

Results for  $\Delta \text{Share}_p \in [-2, 2]$ . Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# Forward Citations [▶ Back](#)

	$\Delta \log \text{ citations/patent}$	$\Delta \text{ patent generality}$
	(1)	(2)
$\Delta \text{ Inventor Share (pp)}$	-0.545*** (0.113)	-0.025* (0.012)
$\Delta \log \text{ Sales}$	-0.232* (0.109)	0.008 (0.012)
4D Knowledge Market FE	✓	✓
Sample	Full Sample	Full Sample
Weight		
Observations	144	144

Results for  $\Delta \text{Share}_p \in [-2, 2]$ . Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# Robustness for Inventor Productivity [▶ Back](#)

	$\Delta$ Growth/Inventor (pp)			
	(1)	(2)	(3)	(4)
$\Delta$ HHI	-0.332** (0.113)	-0.292* (0.123)	-0.332** (0.114)	-0.290* (0.126)
$\Delta$ log Sales		-0.052* (0.021)		-0.053* (0.022)
Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales
Observations	101	101	98	94

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

## Lerner Index v. Concentration in Sample [▶ Back](#)

$\Delta$ Lerner Index	
(2)	
$\Delta \underline{\text{HHI}}$	1.652*** (0.257)
Observations	258
R-squared	.14

Robust standard errors in parentheses, +  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

# Constant-Growth Equilibrium [▶ Back](#)

## Definition

A constant-growth equilibrium is a set of prices  $p(i)$ , investment intensities  $x_I, x_{e,\omega}, x_{e,1}$ , and normalized values  $v(\omega), v(1)$ , such that, given a wage for production and R&D workers, incumbent and entrants optimally choose research intensities, the stationary distribution satisfies:

$$0 = -(x_I + x_{e,1}) \mu_1 + \delta \mu_\omega + x_{e,\omega} \mu_{e,\omega} + x_{e,1} \mu_{e,1},$$

$$0 = -(x_{e,\omega} + \delta) \mu_\omega + x_I \mu_1,$$

$$0 = -(x_{e,1} + x_I) \mu_{e,1} + x_{e,1} \mu_1 + \delta \mu_{e,\omega},$$

$$0 = -(x_{e,\omega} + \delta) \mu_{e,\omega} + x_{e,\omega} \mu_\omega + x_I \mu_{e,1}.$$

and output grows at a constant rate,  $g$ .

# Stationary Distribution [▶ Back](#)

LOM:

$$\dot{\mu}_1 = -(\chi_I + \chi_{e,1}) \mu_1 + \delta \mu_\omega + \chi_{e,\omega} \mu_{e,\omega} + \chi_{e,1} \mu_{e,1},$$

$$\dot{\mu}_\omega = -(\chi_{e,\omega} + \delta) \mu_\omega + \chi_I \mu_1,$$

$$\dot{\mu}_{e,1} = -(\chi_{e,1} + \chi_I) \mu_{e,1} + \chi_{e,1} \mu_1 + \delta \mu_{e,\omega},$$

$$\dot{\mu}_{e,\omega} = -(\chi_{e,\omega} + \delta) \mu_{e,\omega} + \chi_{e,\omega} \mu_\omega + \chi_I \mu_{e,1}.$$

$$\mu_\omega = \frac{\chi_I}{\chi_I + \chi_{e,\omega} + \delta},$$

$$\mu_1 = \frac{\chi_{e,\omega} + \delta}{\chi_I + \chi_{e,\omega} + \delta},$$

$$\mu_{e,\omega} = \frac{\omega \chi_I \mu_1 + (\omega \chi_{e,\omega} + \chi_I) \mu_\omega}{\omega (\chi_{e,\omega} + \delta) + \chi_I},$$

$$\mu_{e,1} = \frac{\omega (\chi_{e,\omega} + \delta) \mu_1 + \delta \mu_\omega}{\omega (\chi_{e,\omega} + \delta) + \chi_I}.$$