14.661 Recitation 1: The Task Framework, or Modeling Wage Inequality

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Wage Trends in Search of a Model

Troubling inequalities:

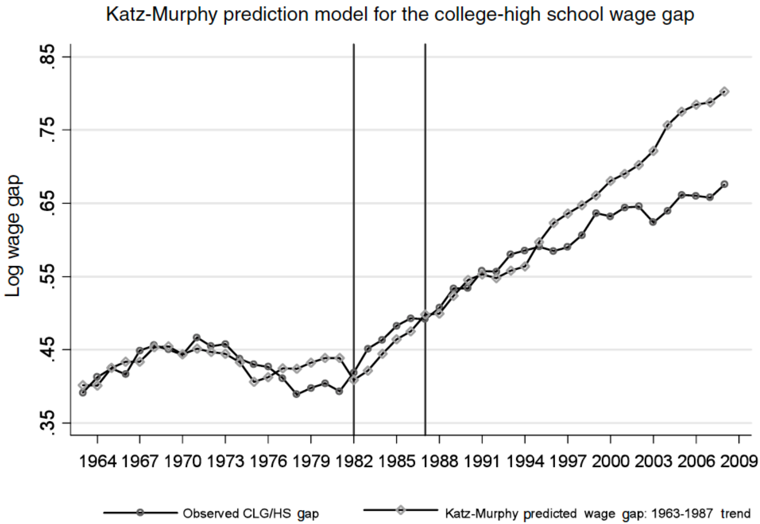
1. Increased *wage premium* between college and the rest;
2. *Fall* in real wages of low-skilled workers;
3. Falling labor share;
4. *Hollowing-out* of the wage and employment distribution;
5. Changes in skill allocation across occupations;
6. Machines replacing workers.

How do you explain all these changes together? And how do you *model* them?

Objective and Roadmap

- Build the *most tractable model* with features we want:
- Competitive labor markets (workers get their marginal product);
- Changes explained by *technology only*, and endogenous response;
- Easy aggregation.
- Roadmap:
  - Review of facts
  - CES model
  - Task Framework (Acemoglu and Autor, HoLE 2011)
- Byproduct: Review CES and GE, very useful in "life"

Fact 1: Increase in College Wage Premium



Source: Acemoglu and Autor (HoLE, 2011)

Fact 2: Lower Wages for Low-Skilled and "Fanning Out"

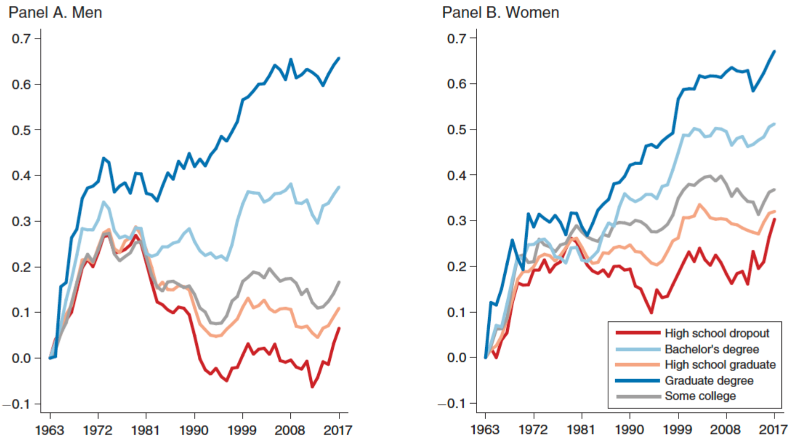


FIGURE 1. CUMULATIVE CHANGE IN REAL WEEKLY EARNINGS OF WORKING-AGE ADULTS AGES 18–64, 1963–2017

Source: Autor (AEA P&P, 2019)

Fact 3: Occupational Employment Polarization

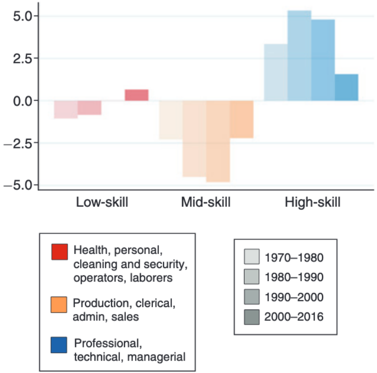


Figure 4. Changes in Occupational Employment Shares among Working-Age Adults, 1970–2016

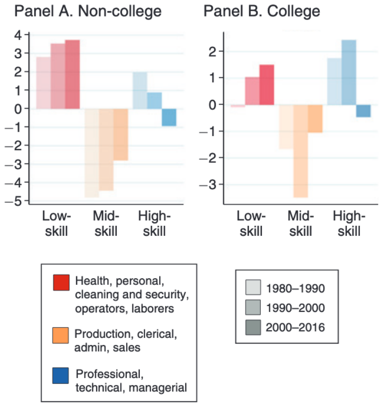
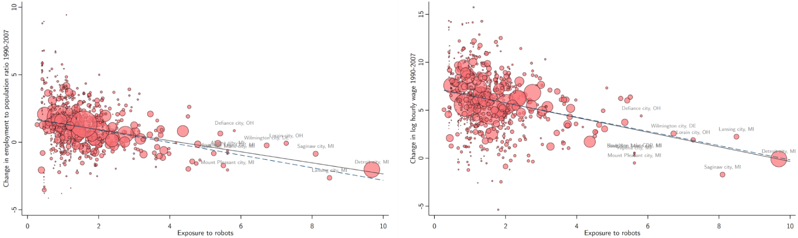


Figure 5. Changes in Occupational Employment Shares among Working-Age Adults, 1970–2016

Source: Autor (AEA P&P, 2019)

Fact 4: Automation, Some Workers are Replaced...



Source: Acemoglu and Restrepo (2018)

Katz and Murphy (1992): CES

Increasing wage premium is easy to explain.

- Assume output aggregates low-skilled and high-skilled labor:

$$Y = \left[(A_L L_L)^{\frac{\sigma-1}{\sigma}} + (A_H L_H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- $\sigma > 0$: $\sigma \in (0,1)$ gross complements; $\sigma > 1$ gross substitutes
- Competitive markets:

$$w_s = Y^{\frac{1}{\sigma}} L_s^{-\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}} \Rightarrow \frac{w_H}{w_L} = \left(\frac{A_H}{A_L} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{L_H}{L_L} \right)^{-\frac{1}{\sigma}}$$

- "Race between education and technology" (Goldin and Katz, 2008), estimates of $\sigma \in (1,2)$ imply:
  - Technology increased, biased towards high-skilled $\left(\frac{A_H}{A_L} \uparrow\uparrow\uparrow \right)$;
  - Supply of highly-educated could not keep up $\left(\frac{L_H}{L_L} \uparrow \right)$.

Issue: It is a Factor-Augmenting Model!

- Technology is modeled as increase in productivity of a group, which raises overall productivity
- How to see that:

$$w_s = Y^{\frac{1}{\sigma}} L_s^{-\frac{1}{\sigma}} A_s^{\frac{\sigma-1}{\sigma}},$$

  $\Rightarrow$ Wage increases when Y increases, regardless of s or whether A_s increases
- Further, suppose equilibrium labor supply is:

$$L_s = \chi w_s^{\varphi}, \ \varphi > 0$$

  $\Rightarrow$ Either groups' employment increases when technology improves!!
- Thus framework fails to capture:

1. Worker substitution by other factors (offshoring, automation)
2. Falling real wages for low skilled (would only work if relative supply of low-skilled increased, FALSE).

Solution: David and Daron's Task Framework

- In words:
  - Each good is produced through a *collection of tasks* (can be thought of as intermediate services)
  - Each type of workers has comparative advantages in producing some of them, which gives a split of tasks across skill groups
  - Factor-augmenting technology now shifts the task allocation
    - Increases the marginal product of workers with growing assigned tasks
    - And reduces it for workers who lose tasks
    - Great to model substitution!

Overview

- In math:
  - The collection of tasks is a continuum measure-one, aggregate Cobb-Douglas:
$$\log Y = \int_0^1 \log y(i) \mathrm{d} i$$
  - Each task is produced linearly by different factors:
$$y(i) = \sum_{j \in \mathcal{J} = \{L,M,H,K\}} A_j \alpha_j(i) \ell_j(i)$$
    where $\alpha_j(i)$ is a schedule of productivities giving a comp.adv. structure
  - Linearity and schedules α_j imply the existence of a cutoff, that partitions tasks across factors j
  - Increasing A_j will shift the task assignment.
- Now, to the details!

Task Demand

Derivation that the aggregate is Cobb-Douglas:

$$\max_{y(i)} \ \exp\left\{\int_0^1 \log y(i)\mathrm{d}i\right\}$$
$$\text{s.t.} \int_0^1 p(i)y(i)\mathrm{d}i = Y$$

(Imposed final good is numeraire $P = 1$) gives:

$$\frac{Y}{y(i)} = \lambda p(i) \ \forall i, \Rightarrow p(i)y(i) = p(s)y(s) = Y$$

Equal expenditure on all goods!

Factor Demands

Task-level profit-max:

$$\max_{\{\ell_j\}} p \left(\sum_{j \in \mathcal{J}} A_j \alpha_j \ell_j \right) - \sum_{j \in \mathcal{J}} w_j \ell_j$$

Implies:

1. Use only ℓ_j s.t. $\frac{pA_j\alpha_j}{w_j} = \max_{j \in \mathcal{J}} \left\{ \frac{pA_j\alpha_j}{w_j} \right\}$;
2. j workers can move freely across tasks i and comp. labor markets:
$$w_j = p(i)A_j\alpha_j(i), \ \forall i \text{ s.t.} \ell_j(i) > 0.$$
3. Use task demand from Cobb-Douglas:
$$p(i)y(i) = p(s)y(s)$$
$$p(i)\left(A_j\alpha_j(i)\ell_j(i)\right) = p(s)\left(A_j\alpha_j(s)\ell_j(s)\right)$$
$$\ell_j(i) = \ell_j(s)$$

Thresholds I

Consider the three skill groups and assume:

$$\frac{\alpha_M(i)}{\alpha_L(i)}, \frac{\alpha_H(i)}{\alpha_M(i)}$$

increasing in i. That is H has comp. adv. on high i, L on low i.

- All i s.t. $i < I_L$ performed by L
- Threshold given by "arbitrage condition":

$$p\left(I_L\right) A_L \alpha_L\left(I_L\right) \ell_L = p\left(I_L\right) A_M \alpha_M\left(I_L\right) \ell_M$$
$$\frac{A_L \alpha_L\left(I_L\right)}{A_M \alpha_M\left(I_L\right)} = \frac{\ell_M}{\ell_L},$$

- Analogously for I_H s.t. H is used for all $i > I_H$.

Equilibrium Thresholds

Above we found that ℓ_j is the same for all tasks performed by j.
With total supply of each type, L_j :

$$L_L = \int_0^{I_L} \ell_L \mathrm{d}i \Rightarrow \ell_L = \frac{L_L}{I_L},$$
$$L_M = \int_{I_L}^{I_H} \ell_M \mathrm{d}i \Rightarrow \ell_M = \frac{L_M}{I_H - I_L},$$
$$L_H = \int_{I_H}^{1} \ell_M \mathrm{d}i \Rightarrow \ell_H = \frac{L_H}{1 - I_H}.$$

Equilibrium Thresholds II

In equilibrium, the NA conditions found before are, e.g. for L:

$$\underbrace{\frac{\alpha_L(I_L)}{\alpha_M(I_L)}}_{\text{decr. in } I_L} = \frac{A_M L_M}{A_L L_L} \underbrace{\frac{I_L}{I_H - I_L}}_{\text{incr. in } I_L},$$

Crucial result:

- The increase in the relative *effective supply* $A_j L_j$ of a factor shifts the task assignment in favor of that factor

Prices

Now define price "indexes for" the goods produced by each skill, e.g.

$$P_L = p\left(i\right)\alpha_M\left(i\right), \forall i < I_L$$

NA gives (just multiply $p(I_L)$ at the threshold):

$$\frac{P_M}{P_L} = \left(\frac{A_M L_M}{I_H - I_L}\right)^{-1}\left(\frac{A_L L_L}{I_L}\right)$$

Relative price of the good sold by a skill group increases in assigned tasks.

Wages, Finally!

Wages are simply:

$$w_s = A_s P_s$$

for all skill levels. Relative wages:

$$\frac{w_L}{w_M} = \frac{A_L P_L}{A_M P_M} = \frac{L_M}{L_L} \left(\frac{I_L}{I_H - I_L} \right),$$

depends positively on relative tasks, negatively on relative labor. Intuition from labor market clearing

- Measure of assigned tasks gives demand for that skill group,
- Total labor is supply.

Results on Wages

Various great CS:

1. Increase in A_H increases relative high-skilled wages, as in CES
2. Machines or other factors that get more productive can reduce the labor share
3. Increase in A_H can create wage polarization (middle deprived of tasks), or some routine tech. competing with L_M
4. Increase in A_H (or A_K with capital) can reduce low-skilled wages
5. Increase in high-skilled relative supply reallocates tasks

In sum, technical change can *lower wages!*