Competing for Inventors: Market Concentration and the Misallocation of Innovative Talent*

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November 10, 2022

Abstract

The rapid productivity gains achieved by technological innovations in the 20th century have slowed in recent decades. This has come at a time of increased market concentration. In this paper, I explore how dominant companies in concentrated sectors have siphoned off inventors that might have been employed more productively in competitive industries. For the period 1997-2012, I establish that sectors with rising concentration captured a disproportionate share of researchers, while also experiencing a decrease in R&D productivity, signaled by falling forward citations and slowing growth per inventor. These findings imply that inventors became increasingly misallocated, accounting for nearly 30 percent of the decline in output per worker growth over the period. I show that these results arise naturally in a Schumpeterian growth model where monopolistic firms conduct "defensive patenting" to hamper competitors' R&D. A calibration of this model reveals that a growth-maximizing planner should subsidize entrants' R&D in high-concentration sectors.

JEL Codes: O30, O31, O32, O40.

Keywords: Market Concentration, Defensive Innovation, R&D Productivity.

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*I am extremely grateful to my advisors Daron Acemoglu, Martin Beraja, and John Van Reenen for invaluable advice and support. I am also grateful to Michele Fornino for lending indispensable computing power and for his collaboration on a previous version of this project, and Nicholas Zolas, who kindly shared with me the ALP classifications of patents into NAICS sectors of application. I also thank Josh Angrist, David Autor, Pierre Azoulay, Ricardo Caballero, Arnaud Costinot, Joel Flynn, Germán Gutiérrez, Clémence Idoux, John Sturm, Martina Uccioli, Iván Werning, Christian Wolf, and participants in the MIT Macro Lunch, Applied Micro Lunch, and Macro Seminar for useful comments and suggestions. This research was conducted previous to joining the International Monetary Fund. All views expressed in this paper are my own and do not necessarily reflect the position of the IMF, its Executive Board, or IMF management.

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1 Introduction

Research and Development activities, which are key to innovation and growth, have generated increasingly smaller gains in productivity over the last few decades (Bloom et al., 2020; Fernald and Jones, 2014; Gordon, 2016). A prominent explanation: Technological complexity is making it harder to come up with new ideas. A less explored alternative is misallocation of R&D resources (Acemoglu et al., 2018, 2021), a concern that is echoed in policy circles and the press (Metz, 2017; Bass and Brustein, 2020). At issue, for example, is whether dominant high-tech firms are attracting a disproportionate share of highly educated, highly skilled workers at the expense of companies in more competitive sectors. According to TalentSeer (2020), 20% of total Artificial Intelligence experts are employed by just five big-tech companies: Google, Microsoft, Apple, Amazon, and IBM. At the same time, smaller firms in other sectors appear unable to attract this scarce talent. Two natural questions then arise: Is such allocation inefficient? And if so, can inventor misallocation explain the observed fall in R&D productivity?

To answer these questions, I study the broader effect of increasing market concentration on the allocation of inventors across sectors. I start by documenting several novel facts using US Patent and Trademark Office (USPTO) data and concentration measures from the Economic Census over the period 1997-2012. First, I show a positive correlation between increases in a sector's market concentration and the share of inventors it attracts. This correlation might indicate either that firms in high-concentration sectors have drawn more inventors or that they owe their success to large investment in research. To address this reverse causality problem, I adopt an instrumental variable strategy. I use the increase in the number of product-market regulations from Mercatus RegData as an instrument for increased concentration. This specification shows that sectors with increased concentration have increased their share of inventors and not vice versa.

Two additional findings substantiate that high-concentration sectors are using R&D resources inefficiently and consequently depressing aggregate research productivity. First, the quality of patents in sectors with increased concentration declined, as measured by patent forward citations. Second, research productivity, as measured as growth in output per worker per inventor, has decreased in these sectors.[1] Further, I find that dominant

[1]This measure of inventor productivity mirrors the analogous definition of "research productivity" offered by Bloom et al. (2020). In the introduction to their paper, they decompose economic growth into the

firms draw a disproportionate share of their sector's inventors. Based on these three observations, I conclude that incumbent firms are focusing their research efforts on "defensive innovation," that is, projects with the primary aim of warding off potential competition. I provide suggestive evidence for this mechanism for the period 2002-2012, where I document a strong correlation between increases in sectors' inventor shares and patent litigation cases.[2] Quantitatively, the midpoint of my estimates implies that inventor misallocation is responsible for a 0.78pp fall in the average annual growth in output-per-worker growth in the study sample. That translates to 27.3% of the observed decline over the period 1997-2012.

Methodologically, my analysis relies on a novel dataset of "knowledge markets," defined as sets of product markets that share similar inventors. These markets are based on the network of inventor transitions across product categories, identified using USPTO patents classified by their NAICS sectors of application. This approach avoids pooling inventors with unrelated technical expertise, which would bias the response of inventor mobility to sectoral characteristics toward zero.

In the second part of the paper, I develop a model of R&D resource allocation. My objectives are twofold. First, I use the model to explain how inefficient defensive innovations can arise and proliferate as concentration increases. Second, I quantify the R&D productivity effects of increasing concentration and evaluate how R&D subsidies can be best allocated in the presence of defensive innovation. I adopt a Schumpeterian creative-destruction framework in which new entrants conduct productive R&D while incumbents employ inventors in defensive projects. A two-sector general equilibrium model shows that unbalanced changes in concentration across sectors generate a fall in inventor productivity and growth. Indeed, inventors shift to less competitive markets, where defensive projects, which hamper entry and Schumpeterian growth, are more prevalent, and away from competitive sectors where their efforts would be more productive. The theoretical analysis shows that defensive innovation is the key factor behind increased concentration of inventors among incumbents and the fall in R&D productivity.

I calibrate a two-sector version of my model to match moments of the R&D spending

product of the number of researchers and a term capturing research productivity. Following this definition, I compute inventors' productivity as the ratio of the growth in each sector, which I measure using output per worker growth, and the number of inventors employed by that sector.

[2] I have to restrict the analysis to 2002-2012 since the USPTO Patent Number and Case Code File Dataset (Schwartz et al., 2019) only provides data on patent litigation for the period 2003-2016.

distribution in 1997 and growth over the period 1997-2012. This calibration produces a 2.5% fall in output per worker growth from misallocation, close to the 3% lower bound implied by my estimates. I employ this model to study the allocation of cost-neutral R&D subsidies that maximize growth. The model suggests that subsidizing entrants' R&D in more concentrated sectors constitutes the most effective policy, leading to a rise in annual GDP growth of about 0.5pp (a 17% increase from the 2012 benchmark). An R&D subsidy for entrants in both markets produces similar effects. This finding resonates with the fact that defensive innovation is the main inefficiency in the model. Since this friction acts through increased entry barriers, the best way to counter it is to lower entry costs.

The rest of the paper proceeds as follows. In the following section, I survey the related literature and place my study in its context. Section 2 describes my data sources, focusing on the construction of knowledge markets. Section 3 reports my empirical results. Section 4 details the model and conducts the policy analysis. Section 5 concludes.

1.1 Related Literature

My work builds on three main strands of the innovation literature: empirical studies on the effect of competition on innovation; papers on the allocation of R&D and the fall in research productivity; and empirical and theoretical studies of pre-emptive innovation.

In the first strand, Aghion et al. (2005) found that innovation increased at low levels of competition and decreased at high levels, depicting the relationship as an inverted U. Accordingly, this literature has highlighted contrasting effects of competition on R&D activity, focusing mostly on episodes of trade liberalization (see Shu and Steinwender, 2019, for an extensive review). Most papers in this strand identify these effects at the firm-level, which restricts their scope to the effect of competition within product markets. My paper instead adopts a cross-sector view, analyzing the extent to which decreased competition in one market draws away resources from other markets. To do so, I build a novel dataset of "knowledge markets," sets of product markets that share the same inventors. While several papers investigate the mobility of inventors (see, e.g., Azoulay et al., 2017; Moretti and Wilson, 2017), I believe mine to be the first to analyze the effects of market structure on the cross-sector allocation of inventors.

With its focus on competition and innovation, my paper connects to literatures that document increased concentration (Autor et al., Forthcoming; Gutiérrez and Philippon, 2017; Grullon et al., 2019); profits and markups (Barkai, 2020; De Loecker et al., 2020; Eggertsson et al., 2018); the relationship between falling innovation and R&D productivity (Akcigit and Ates, 2019, 2020, 2021; Bloom et al., 2020); and the allocation of R&D within and across sectors (Acemoglu et al., 2018, 2021; Akcigit and Kerr, 2018). My contribution bridges these literatures, explicitly linking changes in the competitive structure to the allocation of R&D resources across more and less concentrated sectors, and their deployment to productive or defensive projects. My findings suggest that increased defensive innovation is a relevant driver of lowered R&D productivity, which acts independently of other channels proposed in the literature, such as incremental innovation and lowered knowledge diffusion (Akcigit and Ates, 2021) and increased technological complexity (Bloom et al., 2020). Indeed, I find no evidence that increased inventor concentration is correlated with incremental innovation or decreased patenting per inventor. Finally, the results in my paper consistent with Vaziri (2021), who shows that stricter antitrust enforcement reduces R&D investments, while increasing productivity and firm entry.

Several papers document the role of pre-emptive innovation in ordinary firm operations (see Guellec et al., 2012, for a review of the evidence), and the high valuation of the resulting patents (Abrams et al., 2013; Czarnitzki et al., 2020; Grimpe and Hussinger, 2008). Most recently, Argente et al. (2020) show that, within product markets, large firms tend to account for the bulk of patenting activity, but are responsible for a smaller share of implemented innovations relative to non-patenting firms. The authors interpret this finding as evidence of defensive innovation, intended to deter competition. My paper builds on this literature showing that increased concentration raises the incentives for defensive innovation, as demonstrated by a fall in forward citations in concentrating sectors. This result connects to Abrams et al. (2013), who study the cross-sectional relation between patent value and forward citations theoretically, showing that high-value patents also tend to receive fewer citations, and rationalize this finding through pre-emptive innovation.

On a theoretical standpoint, I embed defensive patenting as modeled in in (Abrams et al., 2013) into a Schumpeterian growth model, building on the extensive literature inaugurated by Aghion and Howitt (1992). My solution relies on several results derived by Acemoglu and Akcigit (2012). To the best of my knowledge, my paper is the first to analyze the impact of defensive innovation in the context of a general-equilibrium growth model.

The closest precedent to this analysis is Jo (2019), where defensive innovation consists of a refinement of existing product lines, which increases the technological distance of incumbents from entrants in the tradition of Aghion et al. (2001). By contrast, in my framework defensive innovation is specifically aimed at protecting dominant positions and reducing entry as in Abrams et al. (2013). I extend their model to consider the effects of defensive innovation on R&D productivity and overall innovation. My final contribution consists in analyzing the growth-maximizing allocation of R&D subsidies, which has not been previously studied in this context.

2 Data description

In this section, I detail source material and the construction of the dataset used in my empirical analysis. Subsection 2.1 lists the sources of the raw data. Subsection 2.2 focuses on the definition of inventor productivity measures and knowledge markets, which I identify through realized inventor flows across sectors. Subsection 2.3 briefly describes other data construction steps that are discussed in more detail in Appendix A.

2.1 Data Sources

My empirical analysis relies on the variation of concentration across product markets, as defined by 4-digit NAICS sectors, the impact of these shifts on the allocation of inventors with specific competences across these sectors, and the subsequent effect on inventor productivity. I use USPTO patent data to measure inventor productivity and establish the set of product markets that share similar inventors, and US Economic Census data to obtain concentration and productivity growth measures. Finally, I use a dataset of product market regulations, Mercatus RegData 4.0, to conduct an instrumental variable analysis, as well as NBER-CES to estimate the Lerner Index for the calibration of my theoretical model.

My primary source is USPTO patent data from PatentsView. This dataset contains disambiguated patent, inventor, and assignee identifiers, as well as Cooperative Patent Classification (CPC) classes for each of the patents registered from 1975 to 2021. I identify inventor flows across different sectors employing the ALP classification of 1976-2016 patents into NAICS sectors of application developed by Goldschlag et al. (2016). Since this classification is constructed using the PATSTAT dataset, I rely on the crosswalk built by Gianluca Tarasconi to match these two sources.[3] This leaves me with one third of all the patents registered between 1975 and 2021, due to the restriction of the time frame to 1976-2016 and incomplete matching between PATSTAT and PatentsView. I comb patent records for self-citations, truncation-corrected forward citations, and patent generality, following Hall et al. (2001) and Acemoglu et al. (Forthcoming). I restrict my attention to utility patents, as I am interested in patents with a technological content and not just design improvements. Schwartz et al. (2019) provide data on patents enforced in litigation cases between 2003 and 2016. This dataset allows me to analyze the evolution in patent litigation across NAICS 4-digit sectors for the sub-period 2002-2012.

My main source for concentration and sales data is the US Economic Census (EC), which reports sales shares for the top 4, 8, 20, and 50 firms; the Herfindal-Hirschman Index; sales and number of companies in various NAICS 4-digit sectors at a 5-year frequency. I focus on the period between 1997 and 2012 for three reasons. First, as I show below, this period saw substantial increase in the concentration of inventors in specific technology classes. Second, the start of this period coincides with an acceleration of market concentration and markups (see, e.g., De Loecker et al., 2020). Third, 1997 saw the adoption of the NAICS classification, ensuring a consistent definition of product markets throughout the period I analyze. I select the HHI lower bound (Keil, 2017) as a measure of concentration, instead of the Economic Census HHI. This measure allows me to analyze a larger sample of sectors, since it requires only top sales shares in the Economic Census, which are available for a much wider set of industries than the census-computed HHI.[4] The Economic Census also provides sector-level growth in output per worker, which constitutes my main measure of productivity growth. I choose this measure instead of multi-factor productivity since the latter is available for selected sectors, mostly in

[3] See https://patentsview.org/forum/7/topic/143, https://rawpatentdata.blogspot.com/2019/07/patstat-patentsview-concordance-update.html

[4] The expression used to obtain this measure is:

$$\underline{\text{HHI}}_{p,t} = 4\left[\frac{\text{CR4}_{p,t}}{4}\right]^2 + 4\left[\frac{\text{CR8}_{p,t} - \text{CR4}_{p,t}}{4}\right]^2 + 12\left[\frac{\text{CR20}_{p,t} - \text{CR8}_{p,t}}{12}\right]^2 + 30\left[\frac{\text{CR50}_{p,t} - \text{CR20}_{p,t}}{30}\right]^2,$$

where "CR{X}" denotes the concentration ratio, that is the share of sales, of the top X firms. This measure is a lower bound, and coincides with the actual HHI if the sector has less than 50 firms, and sales share are distributed equally in each of the top 0-4, 5-8, 9-20, 21-50 brackets. Keil (2017) reports a correlation of $\underline{\text{HHI}}$ with the actual index of 0.93. Available at https://sites.google.com/site/drjankeil/data.

manufacturing. I deflate sales using NAICS-specific price indices from the Bureau of Labor Statistics. All told, out of a total of 304 NAICS 4-digit sectors, I have assembled 157 business sectors for which I can measure the interrelation between concentration and knowledge markets.

I employ two additional data sources. First, I obtain sector-specific counts of regulations for various NAICS 4-digit sector from the Mercatus RegData 4.0 dataset. I employ them to conduct an instrumental variable analysis, strengthening the causal interpretation of my results.[5] Second, I use NBER-CES data to produce estimates of the Lerner Index following Grullon et al. (2019).

2.2 Effective Inventors and Knowledge Markets

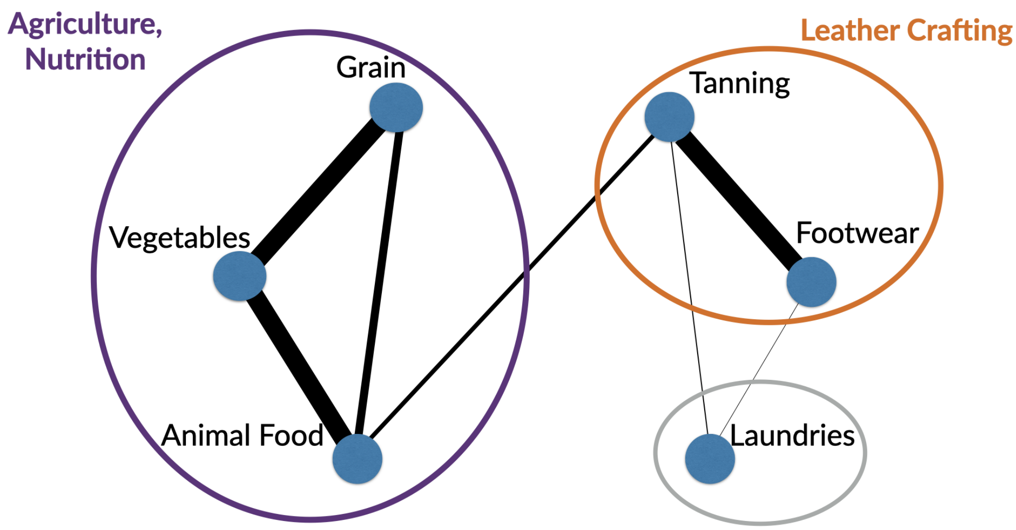
The main aim of this section is grouping product markets that share the same *required knowledge to innovate* and therefore compete for the same R&D inputs, namely inventors. I identify sectors that routinely exchange researchers through the Louvain community-detection algorithm (Blondel et al., 2008).

Figure 1 exemplifies how I construct knowledge markets. Each node in the Figure represents a different NAICS 4-digit sector, and the black lines designate inventor flows, with thickness proportional to the size of the flow. I describe below how flows are computed. I employ a community detection algorithm to group together sectors most closely connected. Figure 1 depicts strong flows among grain, vegetable farming and animal food manufacturing, all of which involve knowledge related to agriculture and nutrition, and separately between footwear and tanning, which both require knowledge of leather crafting. In this illustrative example, my algorithm would identify two knowledge markets, one given by the agriculture and food manufacturing sectors, and the other by leather crafting sectors, leaving the laundry services sector isolated. Based on the strength of these connections, we would expect market conditions in footwear manufacturing to affect the distribution of inventors across the two leather crafting sectors, leaving agriculture and nutrition sectors largely unaffected.

Measuring Inventor Transitions I employ the USPTO patent data classified into 4-digit NAICS sectors by Goldschlag et al. (2016) to construct knowledge markets. Table 1 depicts

[5] Available at https://www.quantgov.org/bulk-download.

Figure 1: Graphical Illustration of Knowledge Markets



Note: This figure displays examples of knowledge markets as sets of product markets with the same required knowledge to innovate. This illustration is based on a subset of transitions and classifications from my data. Additional sectors inside and outside these knowledge markets are excluded for ease of exposition. Nodes represent NAICS sectors 1111 (Oilseed and Grain Farming), 1112 (Vegetable and Melon Farming), 3111 (Animal Food Manufacturing), 3161 (Leather and Hide Tanning and Finishing), 3162 (Footwear Manufacturing), and 8123 (Drycleaning and Laundry Services). Edges represent inventor transitions, with width proportional to the size of undirected inventor flows.

a hypothetical matching of the USPTO dataset with NAICS classifications. Inventors are each assigned a disambiguated ID corresponding to the serial number of their first patent. In this example, inventor 00001-1 and 00001-2 both cooperate on the development of patent US00001. The third column in Table 1 shows the Goldschlag et al. (2016) classification for NAICS 4-digit industries. This classification is not limited to a single sector per patent, and includes multiple sectors in almost all instances. For instance, patent US00001 relates to multiple sectors, while patent US00002 is applicable to just one sector. Importantly, this classification captures the *technological nature* of the patent and the sectors of application of the knowledge required to develop that patent. While other classifications, like the CPC or the USPC, also describe the technological nature of patents, they do not allow a direct match to sectors of application.

I define a transition in two ways. First, I consider transitions *within patents*. This transition occurs between two sectors if an inventor works on a patent that applies to both. The direction of flows does not matter for the definition of knowledge markets,

Table 1: USPTO Data Structure

Patent ID	Inventor ID	Goldschlag et al. (2016) NAICS	Year
US00001	00001-1	1111	1980
US00001	00001-1	1112	1980
US00001	00001-2	1111	1980
US00001	00001-2	1112	1980
US00002	00001-1	3111	1981

Note: This table displays a hypothetical example of the structure of my data. The first two columns report patent and inventor identifiers from PatentsView; the third column reports NAICS 4-digit classifications.

since I am only interested in grouping sectors that exchange researchers. Table 1 depicts two transitions between sectors 1111 and 1112 in 1980. The second type of transition that I consider is *across patents*. This transition occurs when an inventor applies his knowledge to patents in different product markets, such as between sector 1112 and 3111 by inventor 00001-1. Raw transition counts are the basis of my measure of inventor flows.

Weighting Inventor Flows: Effective Inventors I construct two alternative measures to assess the effective flow of inventors across sectors. The first measure weighs each transition equally, computing inventor flows as the raw count of researchers moving across NAICS. The second measure adjusts for the productivity of individual inventors, since raw counts might overstate or understate the importance of each transition, depending on the size of origin and destination sectors, their technological nature, as well as the proficiency of each inventor. I therefore define a measure of "effective inventors" to correct for these and other omitted factors. For each inventor, I estimate the fixed effect, α_i, in the fully-saturated regression

$$\#\text{Patents}_{cfit} = \alpha_i + \gamma_{cft} + \varepsilon_{cfit}, \tag{1}$$

where $\#\text{Patents}_{cfit}$ denotes the number of patents registered in CPC class c; firm (assignee) f; and year t, that include inventor i. In this regression γ_{cft} denotes a of CPC class by firm (assignee) by year fixed effect. I choose to include indicators for one-digit CPC classes, the broadest classification, to identify as many fixed effects as possible. The fixed effect γ_{cft} controls for specific technological features of the patented technology, the firm environment, as well as the year. Further, this specification produces an estimate of inventor productivity that accounts for the number of collaborators on each patent.

Given this specification, I define an *effective inventor* as one unit of the resulting fixed effect α_i, rescaled to take nonnegative values. Since these fixed effects might be inconsistently estimated, I check the robustness of all my results, including the construction of knowledge markets, to the use of the raw count of inventors.

I define *effective inventor flows* between sector j and sector k at time t as:

$$flow_{j\to k,t} = \sum_i \#\{i\text{'s transitions } j \to k \text{ in } t\} \cdot \alpha_i,$$

that is, the sum of transition counts weighted by effective inventors. The *total undirected flow* between two sectors is the sum of inflows and outflows with ends in one of the two sectors over all sample years, t:

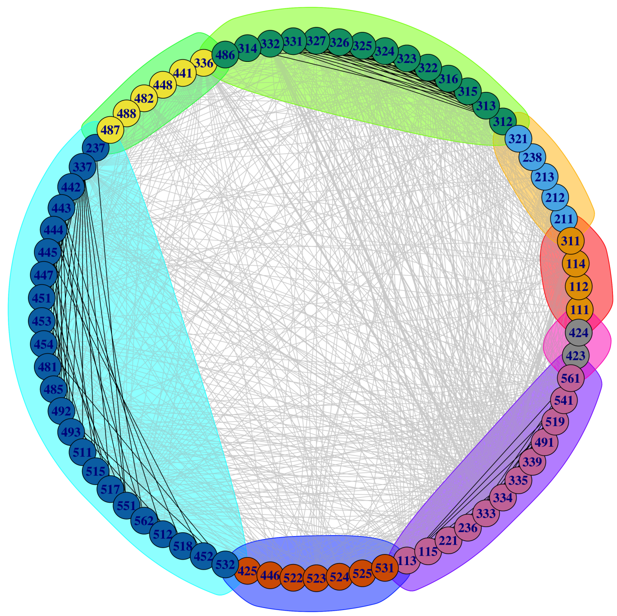
$$flow_{jk} = \sum_t \left(flow_{j\to k,t} + flow_{k\to j,t}\right).$$

This flow measure defines a network of inventor transitions across product markets, where the nodes, j, k, are given by 4-digit NAICS codes and edges are transitions across sectors. Edge weights are a rescaled version of $flow_{jk}$ and measure the strength of the connection between pairs of sectors in the network. Rescaling the flow measure is necessary to remove mechanical effects of sector sizes and to avoid double counting of inventors. More details are in Appendix A.

Community Detection and Resulting Knowledge Markets I use the rescaled undirected flow measure as a network edge weight to identify communities through the Louvain algorithm developed by Blondel et al. (2008). This procedure maximizes the modularity of the network by choosing the number of communities (knowledge markets) and the assignment of nodes (NAICS sectors) to communities. Modularity, a commonly used measure of connectedness of networks, measures the distance between the density of links *within* communities versus *between.*

This procedures produces 10 sets of NAICS 4-digit sectors that share the same inventors and have concentration measures. Applying the community detection algorithm results in knowledge markets that do not overlap: Each NAICS 4-digit sector belongs to one and only one knowledge market. Figure 2 displays the result of my procedure applied to NAICS 3-digit sectors. I report this exercise since the 4-digit equivalent would

Figure 2: Knowledge Markets Obtained from NAICS 3-digit Sectors



Note: This figure displays the network of inventor flows between NAICS 3-digit sectors and the knowledge markets resulting from the Louvain algorithm. Lines denote inventor transitions within (black) and between (gray) knowledge markets, with width proportional to effective undirected inventor flows.

be too dense to depict. However, the knowledge markets identified by the two exercises are qualitatively similar although they are clearly more numerous in the 4-digit case. In this figure, lines denote inventor transitions, with width proportional to the effective undirected inventor flow between sectors. Black lines depict flows within knowledge markets, while gray lines represent transitions between communities.

Three features are worth emphasizing. First, the network is very dense, and transitions across 3-digit as well as 2-digit sectors are pervasive, differing largely in intensity. The approach I propose is therefore far more illuminating than grouping sectors based on broad product markets, which would neglect the linkages across disparate markets, or pooling all sectors together, which would neglect the difference in the strength of inventor flows. Second, the flows between communities appear more numerous than within communities, but this is solely a by-product of the circular layout of the network, whereby nodes mask flows within close communities on the circle. Less than a third of flows occur between communities, as expected since the community detection algorithm maximizes the density of within-community linkages. Third, the classification sensibly groups together sectors that we might expect to share similar knowledge to innovate. Starting from sector 111 and going counter-clockwise, the knowledge markets are as follows. The first market groups sectors involving agricultural production (111, 112 and 114) and food manufacturing (311). The second market, starting with 211, includes oil, gas, and mining. The green cluster at the top of the figure groups several heavy manufacturing industries, such as chemicals, plastics and petroleum products, and pipeline transportation (486). The market in yellow consists largely of transportation services and manufacturing as well as motor vehicle dealers. The large blue cluster captures many retail sectors, as well as data processing, telecom, and broadcasting services. The remaining three markets include insurance and finance (red cluster), computer, electronics, machinery manufacturing and professional services (violet), and wholesalers (gray).

I identify knowledge markets using effective inventors, but I obtain nearly identical results using raw inventor counts; more than 97% of 4-digit NAICS sectors are classified in the same manner using the two measures. That is, 97% of sector pairs belong to the same knowledge market according to both measures.

2.3 Other Constructed Measures and Aggregation at Census Frequency

Patent Citation Measures For each patent classified by Goldschlag et al. (2016), I compute forward citations, a measure of patent generality, and self-citations. I describe self-citation measures together with the related estimation results in the Appendix.

I count forward citations and measure patent generality following Hall et al. (2001). The forward citation measures compute the average number of citations received by each firm's patents, giving an indication of the importance of each patent for future technological developments. As in Hall et al. (2001), I correct for truncation by estimating the empirical CDF of the forward citations lag distribution of patents in the relevant CPC 2-digit technology class, and dividing the overall number of forward citations at the latest available date by the inverse of this CDF. The procedure suggested by Hall et al. (2001) uses only information pertaining to the CPC 2-digit technology class of the cited patent. I also compute an alternative correction that estimates a separate distribution for each citing CPC 2-digit class and sums the corrected forward citations across all citing classes. Patent generality also measures the scope of application of the patent by computing the dispersion of citations received across different CPC classes. The higher the dispersion, the wider the technological applicability of the patent.[6]

Regulation Data Mercatus RegData provides a count of restrictions imposed on a number of NAICS 4-digit product markets, obtained by matching a set of keywords in NAICS descriptions to regulatory texts, and then taking the best match for each document. However, the available data does not include a set of codes due to data quality reasons. Therefore, I process the description of NAICS 4-digit codes and compute the cosine-similarity between all pairs of sectors. I build an estimate of sector-relevant restrictions for missing sectors by taking an average weighted by cosine similarity of sectors included in RegData. I include in the average the five most similar NAICS codes if similarity is larger than .2, and I attribute the regulations of the most similar sector otherwise. I chose this threshold by inspecting the similarity associated to various NAICS pairs, and the assignment of regulations to sectors is not highly sensitive to this choice.

Inventor Distribution Measures I employ the measure of effective inventors constructed as detailed above to compute measures of researchers' concentration within sectors for each year in my sample. Specifically, I use the PatentsView assignee ID to identify firms that employ specific inventors in each sector, and then compute several measures of the concentration of inventors within sectors. I also compute the HHI of inventors across NAICS to document increasing concentration of inventors in specific sectors.

Patent Litigation Cases I match the data on litigation cases compiled by Schwartz et al. (2019) with the data on inventors by NAICS 4-digit. For each sector, I compute the number of litigation cases per patent. These data are available only for the sub-period 2003-2016, which does not allow me to reliably estimate an empirical CDF to correct for truncation. I therefore choose to keep only the litigation cases occurring in the same year as the patent registration, which amounts to assuming that the time profile of cases is constant over time and across sectors.[7] I then average the litigations per patent over the years

[6]The interested reader should consult Acemoglu et al. (Forthcoming) for a detailed discussion, and the related appendix for details on the construction of these measures.

[7]More precisely, this would be the same as using only the contemporaneous patent litigation cases and correcting for truncation dividing by the inverse CDF at period 0, which would scale the estimated coefficient upwards.

2003-2006 and 2013-2016 for the Economic Census waves 2002 and 2012.

Aggregation at Census Frequency Data from the Economic Census are available at five-year intervals for the years 1997-2017, which requires aggregating the other data at the same frequency. Since I am interested in the effect of concentration on the allocation of inventors, I average all variables related to inventors and productivity using the five-year window *starting* in the census year (e.g., 1997- 2001 for 1997), while I use concentration measures for the corresponding census year. In the IV regression I use product restrictions as an instrument for concentration, which is why I average restrictions in the five-year window *ending* in the census year (e.g., 1993-1997 for 1997). Since Goldschlag et al. (2016)'s matching only covers the period up to 2016, I run all specifications in long-differences over the time frame 1997-2012. The only exception is the patent litigation regression, which uses the period 2002-2012.

3 Empirical Analysis

This section presents five main findings that apply to the periods under consideration: (i) effective inventors became more concentrated across economic sectors; (ii) sectors with increased product market concentration attracted a growing share of relevant inventor types; (iii) growth in the share of relevant inventors negatively correlated with inventor productivity, as measured by forward citations as well as average growth in output per worker divided by effective inventors employed; (iv) growth in the share of inventors coincided with increased concentration of inventors *within* sectors; and (v) growth in inventor shares are positively correlated with increases in patent enforcement cases.

Results (i) and (ii) indicate a positive causal link between the growth in product market concentration and the increase in sectors' inventor share. Findings (iii), (iv) and (v) point to misallocation: Inventor concentration in less competitive sectors turns out to be inefficient, as researchers are predominantly employed on projects that do not contribute to the growth of the sector. This work amounts to defensive innovation, as evidenced by the decline in forward citations of patents obtained by these firms, the decrease in growth per inventor, and the increase in patent litigations that accompany the increase in product market concentration. I discuss the causal interpretation of my results through an IV specification in Subsection 3.2.

3.1 Variable Definitions and Main Specification

Key to my analysis are measures of inventor concentration and of R&D productivity. I rely on the definition of effective inventors, α_i, explained in Section 2.2. For each product market p, I define the share of inventors employed by the sector in year t as

$$\text{Inventor Share}_{p,t} \equiv \frac{\sum_{p(i,t)=p} \alpha_i}{\sum_{k(i,t)=k} \alpha_i},$$

where the numerator sums effective inventors in product market p, and the denominator computes total effective inventors in the relevant knowledge market. I also use effective inventors α_i to evaluate the dispersion of inventors across sectors. Results are robust to using raw counts of researchers instead of effective inventors.

When analyzing R&D productivity I focus on two patent-based measures described in Section 2.3—forward citations and patent generality. Further, I compute a more direct measure of the productivity of inventors as the growth in output per worker divided by the number of effective inventors employed by the sector.

In most specifications, the independent variables are measures of concentration and controls for the size of the sector considered. As discussed in Section 2.1, my baseline measure of concentration is the lower bound of the Herfindal-Hirschman Index constructed by Keil (2017), which I label $\underline{\text{HHI}}_{p,t}$. I show below that the Economic Census HHI and its lower bound, when both available, produce equivalent results.

I obtain measures of sales from the Economic Census, which I deflate using BLS NAICS-specific price indexes. I use sales variables for two purposes. First, real sales in 2012 are the weight in my regressions. Second, I use the logarithm real sales as well as a quartic in real sales to control for changes in the size of sectors during my sample period. For the selected subset of sectors that reports the number of companies, I show that my findings are robust to controlling for sales per company, a proxy for the average firm size.

I estimate a sector-level long-difference regression over the period 1997-2012

$$\Delta\text{Share}_{p,\,2012-1997} = f_k \mathbf{1}\left\{p \in k\right\} + \beta\Delta\underline{\text{HHI}}_{p,\,2012-1997} + \gamma'\Delta\text{Size}_{p,\,2012-1997} + \varepsilon_p, \quad (2)$$

where ΔShare denotes the change in the inventors' share of product market p; $f_k \mathbf{1}\left\{p \in k\right\}$ is a dummy variable that takes value 1 if the product market belongs to knowledge market k; $\Delta\underline{\text{HHI}}$ is the change in the HHI lower bound; and ΔSize is a set of controls for the size of sector p. Depending on the specification, ΔSize is the change in log real sales, the change in log real sales per firm, or a quadratic polynomial in real sales changes.

Regressions are weighted by sector sales in 2012 for the findings which rely on Economic Census sector-level measures, and I estimate robust standard errors in all specifications. When looking at patent measures, I employ the same specification as in equation 2, where I replace the outcome variable with the change in patent productivity and the independent variable with the change in inventors' share.[8] In this case, since I do not rely on Economic Census measures, I report unweighted regressions. I discuss the robustness of these findings to using the HHI lower bound and weighting by sales.

3.2 Results

3.2.1 Inventor Concentration across NAICS Sectors has Increased

Figure 3 reports the time series of inventor concentration across NAICS 4-digit industries for the period 1976-2016, for which the Goldschlag et al. (2016) data is available. Panel (a) depicts the share of effective inventors and panel (b) that of raw inventors. I use the HHI index of inventor shares accruing to each sector as a measure of concentration. Both panels display an increasing concentration of inventors beginning in the late 1990s. The increase in inventor concentration is sizable, corresponding to about a 20% increase in the HHI for the effective inventor measure over the period 1997-2012. Based on raw inventor data, the figure rises to 30%. Figure 3 extends the findings in Akcigit and Ates (2019), who document a rising share of patents registered by top firms within sectors, to the cross-industry allocation of inventors.[9] As for the results presented below, the effective inventor measure and the raw inventor count behave similarly, although the figure for effective inventors is less volatile since this measure derives from a regression that residualizes time, firm, and technology class fixed effects.

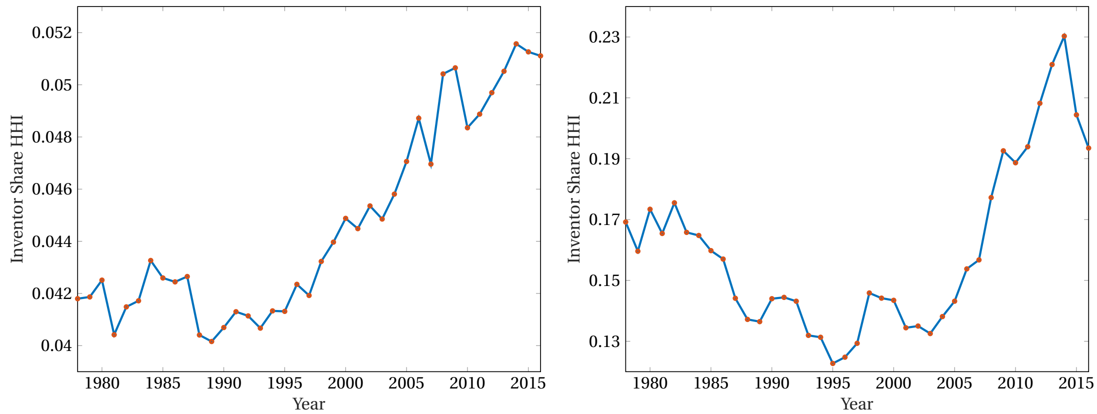
3.2.2 Markets with Growing Concentration Increased Their Inventor Share

In this section, I present three sets of results for each specification, which differ in the estimation sample to account for extreme observations. In regression tables, the "Full

[8] As I will show, the change in the inventor share is highly correlated with the change in the HHI, so this specification essentially amounts to a rescaling of the coefficient that would be obtained using the HHI.

[9] A similar pattern emerges for concentration in specific CPC classes (available on request).

Figure 3: Inventor Concentration, NAICS 4-digit Industries, 1976-2016



(a) Effective Inventors (b) Inventor Count

Note: This figure reports the time series of inventor concentration, as measured by the HHI index of inventor shares across NAICS 4-digit sectors. The left panel reports the series constructed using effective inventors as defined in Section 2.2, the right panel uses instead raw inventor counts. Only the NAICS 4-digit sectors with data for all years are included.

Sample" includes all non-missing observations for the variables considered. The "Trim Outliers" sample drops the most extreme observations for the outcome and the independent variable. I exclude the observations that fall beyond three standard deviations from the sample average of each variable and that are most likely to drive the results estimated using the full sample.[10] The "Mahalanobis" sample excludes the 5% extreme observations based on the Mahalanobis distance of pairs of observations from the data centroid. This procedure is based on the joint distribution of outcomes and independent variables, which may result in a different sample for each regression.

Table 2 presents the results of specification (2) obtained regressing the change in knowledge-market inventor share over either the change in the lower bound of the Herfindal- Hirschman Index or the HHI reported by the Economic Census. The results in Table 2 highlight a strong positive correlation between the change in the HHI and the change in the share of effective inventors accruing to each NAICS sector. These results are unlikely to be driven by the contemporaneous correlation between the two variables, as the share of effective inventors is averaged over the five years *starting* in the Economic Census year, while concentration measures refer to the Economic Census year only.

[10] I justify the choices for each variable in detail in my replication code using the empirical kernel density and detailed tabulations.

Table 2: Regressions of Change in 4-digit Knowledge Market Share over Change in HHI Measures, Long-Differences, 1997-2012

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\underline{HHI}$	27.293*		27.183*		27.326*	
	(11.569)		(11.941)		(11.620)	
Δ HHI		22.399***		22.399***		22.350***
		(6.345)		(6.345)		(6.343)
Sample	Full	Full	Trimmed	Trimmed	Mahalanobis	Mahalanobis
Weight	Sales	Sales	Sales	Sales	Sales	Sales
N	157	80	155	80	150	71

Note: Regressions weighted by sales in 2012; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; N is the number of observations. This table presents the results of specification (2), regressing sector p's share of effective inventors in knowledge market k, over the HHI lower bound for product market p, or the HHI index. "Full", "Trimmed" and "Mahalanobis" refer to the samples described in the main text.

Two important notes on the scale of the variables are in order. First, here and in all following tables and graphs, all variables that refer to shares or growth rates are reported in percentage points. Second, HHI indices are constructed to range between 0 and 1. With regard to the coefficient in Column (1) of Table 2, for example, an increase in one unit of the HHI index leads to an increase in the share of the relevant knowledge market of 27.25 percentage points. In 2012, the HHI lower bound has a sales-weighted average of .03 and a standard deviation of .032. According to Table 2, a standard deviation increase in this measure yields a .87 percentage point increase in the share of inventors accruing to the relevant NAICS sector. In comparison, the sales-weighted average share of inventors in 2012 is 1.18%, with a standard deviation of 1.82%, so the estimated effect of a one standard deviation increase in concentration corresponds to about half a standard deviation increase in the share of inventors. The estimates using the HHI lower bound tend to be noisier as this is a constructed, and therefore imprecise, measure of concentration. However, the number of available observations is much larger than the actual HHI, allowing me to extend my findings to about double the number of sectors.[11]

While suggestive, the correlation presented above neglects two fundamental components. First, it does not include controls for the size of the sectors or firms, which could have a mechanical effect on the share of scientists. Second, it estimates the correlation

[11] Regressions using the Economic Census HHI not reported in the main text or the Appendix are available on request.

both across and within knowledge markets. Table 3 addresses these two limitations by restricting the analysis to within knowledge markets, and controlling for two measures of size. The upper panel uses the change in the logarithm of real sales to measure the change in the size of each sector, while the lower panel includes average sales per firm as a control for average firm size. Indeed, there might be significant barriers to entry in R&D, easier to overcome for larger firms, which would imply a mechanical link between concentration and inventor shares. Since the Economic Census reports the number of companies only for a subset of firms, the sample in the lower panel is smaller. Results are largely unchanged relative to the baseline. All told, these findings imply that sectors with increasing concentration have attracted a rising share of scientists above what would be predicted by their expansion in overall sales and in average firm size. Appendix B.2 establishes the robustness of all the findings in this section to the use of raw inventor counts to compute both inventor shares and knowledge markets.

IV Results More concentration could be the result of increasing technological entry barriers as incumbents hire more R&D inventors. In this scenario, the causality would flow from increased inventor shares to higher concentration. Above, I tried to mitigate this concern using as outcome variable the average share of inventors following the Economic Census years to which the HHI refers. However, reverse causality could still be present if the autocorrelation of inventor shares is sufficiently high. As a consequence, I have calculated 2SLS estimates that instrument the change in the HHI lower bound with changes in product market restrictions, as measured by the Mercatus dataset RegData 4.0. Theoretically, an increase in restrictions should raise barriers to entry in affected product markets, thus leading to higher concentration. As discussed below, such proves to be the case empirically, validating sector-specific restrictions as an instrument for concentration. A violation of the exclusion restriction requires a causal connection between product market regulations and the share of inventors hired by each sector, independent of product market concentration. For example, regulations affecting existing technologies might require more inventors to meet product market restrictions. However, this effect is unlikely to be sufficiently large and persistent to be captured by my measure of inventor shares. Further, the regulations counted in RegData are not exclusively product restrictions, but also include reporting obligations and other legal burdens unrelated to technological components. In addition, while product restrictions might

Table 3: Regressions of Change in 4-digit Knowledge Market Share over Change in HHI Lower Bound, Long-Differences, 1997-2012

(a) Controlling for Change in Log Real Sales

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \underline{HHI}$	26.093*	22.509*	25.904*	22.716*	26.111*	22.554*
	(10.696)	(10.848)	(11.124)	(10.948)	(10.725)	(11.019)
Δ log Sales	0.914**	0.548*	0.881**	0.539*	0.918**	0.562*
	(0.278)	(0.243)	(0.275)	(0.242)	(0.283)	(0.261)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	153	155	152	150	139

(b) Controlling for Change in Log Real Sales per Company

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \underline{HHI}$	35.230**	20.783+	35.230**	20.783+	35.154**	22.854*
	(12.759)	(10.615)	(12.759)	(10.615)	(12.647)	(11.197)
Δ log Size	0.175	-0.040	0.175	-0.040	0.300	-0.055
	(0.382)	(0.253)	(0.382)	(0.253)	(0.460)	(0.346)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	81	79	81	79	75	67

Note: Regressions weighted by sales in 2012; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; checkmarks indicate the inclusion of fixed effects. “Full Sample”, “Trim Outliers” and “Mahalanobis 5%” refer to the samples described in the main text. See notes to Table 2.

Table 4: IV Regressions of Change in 4-digit Knowledge Market Share over Change in HHI Lower Bound, 2SLS Long-Difference, 1997-2012

(a) 2SLS Results

	Δ Inventor Share (pp)	
	(1)	(2)
$\Delta\underline{\text{HHI}}$	32.426+	30.096+
	(16.987)	(15.819)
	[4.850, 99.013]	[4.415, 92.104]
Δ log Sales		0.525*
		(0.247)
Knowledge Market FE	✓	✓
Sample	Full Sample	Mahalanobis 5%
Weight	Sales	Sales
Observations	157	150
First-Stage F	4.656786	4.753009
Anderson-Rubin p-value	.0298009	.0321185

(b) First Stage and Reduced Form

	Δ Inventor Share (pp)	$\Delta\underline{\text{HHI}}$
	(1)	(2)
Δ log Restrictions	0.478*	0.016*
	(0.220)	(0.007)
Δ log Sales	0.539+	-0.000
	(0.274)	(0.005)
Knowledge Market FE	✓	✓
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	153	153

Note: Regressions weighted by sales in 2012; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; checkmarks indicate the inclusion of fixed effects. This table presents the results of specifications (2), regressing changes in the share of effective inventors of sector p over changes in the lower bound of the Herfindal-Hirschman Index for product market p. $\underline{\text{HHI}}$ is instrumented by the change in log-restrictions relevant to the NAICS sector. The lower panel present first-stage and reduced-form relations. Samples are described in the main text.

certainly induce a change in the direction of innovation, there is no a priori reason to believe that the scale of innovation activity should increase. These considerations lead me to believe that the exclusion restriction is not likely to be violated.

The results of the 2SLS estimation are presented in the upper panel of Table 4. The specification is the same as in Column (2) of 3, including both knowledge market and sale change fixed effects. These estimates are statistically indistinguishable from the ones reported in the baseline regression, and equally significant as highlighted by the Anderson-Rubin p-value and the corresponding confidence intervals in square brackets. The latter is the most appropriate measure of significant since the first-stage F clearly indicates that instruments are weak. This is unsurprising since, as detailed above, both the HHI lower bound and the regulation measures are estimated. In particular, I had to impute regulations for a large part of the sample using the cosine-similarity between product market restrictions.[12] The results in the lower panel of Table 4 show that instruments are relevant. The first-stage t-statistic for the regression of the change in the HHI lower bound over log-regulations is 2.07, with a 0.041 p-value. The reduced form regression of inventor shares over log restrictions is equally highly significant, leading the SW underidentification test to rejects the null hypothesis at a 5% confidence level.

Taken together, the results presented in this section establish a causal link between the increase in inventor concentration and the shifts in product market concentration.

3.2.3 Sectors that Attracted More Researchers Saw Increasing Top Firms' Inventor Shares and Falling Patent Forward Citations

While the findings presented so far establish a connection between inventor and product market concentration, they do not establish that changes in the distribution of researchers across sectors are inefficient. It would not be unreasonable, for example, to think that more concentrated sectors saw increased entry as a result of the higher rents captured by incumbents. Table 5 shows that the opposite occurred. Specifically, the share of effective inventors accruing to top inventor-hiring firms increased in the sectors that attracted more inventors over the period considered, relative to firms with fewer inventors in the sector. These outcomes suggest that inventors have increasingly concentrated among

[12] Using only available sectors requires dropping two thirds of the observations. See Appendix A for details on data construction.

Table 5: Regressions of Change in Inventor Distribution Measures over Change in 4-digit Knowledge Market Share, Long-Difference, 1997-2012

	Ch. Inv. 90/50 Quantile Ratio	Δ Top 10%/Bottom 50%
	(1)	(2)
Δ Inventor Share (pp)	0.211+	0.243*
	(0.107)	(0.097)
Δ log Sales	-0.100	0.328
	(0.122)	(0.294)
Knowledge Market FE	✓	✓
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	118	118

Note: Regressions weighted by sales in 2012; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; checkmarks indicate the inclusion of fixed effects. Please refer to notes in Table 3 for further details. Column (1) uses the ratio in the 90 percentile of effective inventors to the median as the outcome variable. Column (2) use the share of inventors accruing to the top 10% over the share accruing to the bottom 50% of the distribution within each NAICS sector.

Table 6: Regressions of Changes in Forward Citation over 4-digit Knowledge Market Share, Long-Differences, 1997-2012

	Δ log Citations/Patent		Δ Patent Generality
	(1)	(2)	(3)
Δ Inventor Share (pp)	-0.197***	-0.227***	-0.004
	(0.044)	(0.051)	(0.004)
Δ log Sales	-0.234*	-0.258+	0.008
	(0.112)	(0.148)	(0.013)
Knowledge Market FE	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample
Observations	153	153	153

Note: Unweighted regressions; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; checkmarks indicate the inclusion of fixed effects. This table present the results of specification (2), regressing the log-change in forward citations and the change in patent generality in sector p over changes in the sector's share of inventors. Column (1) and (2) present the results for forward citations corrected for truncation as described in the text. Column (3) presents results on the patent generality measures. All columns exclude self-citations.

large incumbents, that is, sectors that increased their inventor share also saw a *within-sector* increase in inventor concentration. Throughout this section, I present results using changes in inventor shares to focus directly on the correlation between inventor transitions and their within-sector distribution. I verify that these findings are robust to using the change in the HHI, as should be expected from the strong correlation between these two variables reported above. Results obtained using this independent variable are available upon request.

My next finding suggests that inventor concentration is driven by a rise in defensive innovation, that is R&D aimed at protecting the incumbents' dominant position and raising barriers to entry. Table 6 shows that inventors' concentration in specific sectors went hand in hand with a fall in forward citations for patents, a standard measure of a patent's contribution to further innovations (Hall et al., 2001). The result in Columns (1) and (2) report two different measures of forward citations that differ in how the series are corrected for truncation. As discussed in Section 2.3, the measure in Column (2) uses the procedure delineated by Hall et al. (2001), computing the forward citation lag distribution conditioning on the technology class of the cited patent. Column (2) further conditions on the technology class of citing patents. Column (3) presents the estimates relative to patent generality, a measure of technological scope of application. Regressions are unweighted since they rely only on patent data, but results are robust to weighting by sales. I find a highly significant negative relation between changes in inventor shares and the fall in forward citations. A 1 percentage point increase in the share of inventors leads to a 0.2% reduction in forward citations.

The fall in forward citations is a first indication of the presence of defensive innovation (see, e.g., Guellec et al., 2012). In the next section, I show that these patents also appear to do relatively little to boost productivity, as measured by growth in output per worker.

3.2.4 Markets with Growing Inventor Shares Saw a Fall in Inventor Productivity

Table 7 presents the results of running (2) when the outcome is the average growth in output per worker per effective inventor. I use growth in annual output per worker provided by the Economic Census and average this measure over the five-year window starting in the Economic Census year, and I analogously build a measure of average effective inventors over the same period. Inventor productivity is then defined as average

Table 7: Regressions of Changes in Inventor Productivity over Changes in Inventors' Share and HHI, Long-Difference, 1997-2012

(a) Change in Inventors' Share as Independent Variable

	Δ Growth/Inventor (pp)			
	(1)	(2)	(3)	(4)
Δ Inventor Share (pp)	-0.007**	-0.005*	-0.007**	-0.005*
	(0.002)	(0.002)	(0.002)	(0.002)
Δ log Sales		-0.051*		-0.054*
		(0.021)		(0.021)
Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales
Observations	101	101	96	93

(b) Change in HHI as Independent Variable

	Δ Growth/Inventor (pp)			
	(1)	(2)	(3)	(4)
Δ$\underline{\text{HHI}}$	-0.332**	-0.292*	-0.332**	-0.290*
	(0.113)	(0.123)	(0.114)	(0.126)
Δ log Sales		-0.052*		-0.053*
		(0.021)		(0.022)
Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales
Observations	101	101	98	94

Note: Regressions weighted by sales in 2012; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; checkmarks indicate the inclusion of fixed effects. Inventor productivity is average growth in output per worker over the five years starting in the Economic Census year over total effective inventors in each sector.

output per worker growth divided by average number of effective inventors. Both the outcome and the dependent variable are measured in percentage points. Table 7 reveals a negative and significant correlation between the increase in the number of effective inventors and inventor productivity.

Starting from the upper panel of Table 7, the median change in the share of effective inventors over the period was .014pp, while the measure of effective inventors has a median of 2018.[13] The coefficient for concentration in Column (4) implies a fall of .15pp ($-.005 \times .014$pp $\times$ 2018) in average annual labor productivity growth. This number

[13] Recall that effective inventors in each year are measured as the sum of inventor fixed-effects in each year, and therefore do not represent the simple count of inventors.

decreases to $-.28$pp when considering only sectors with positive growth in labor productivity, which accounted for the bulk of the increase in inventor shares. An alternative back-of-the-envelope computation, using the change in product market concentration to predict the change in inventor shares gives even starker results. Using the coefficient in Column (2) of Table 3(a) and considering a median change in the HHI of 0.002 yields an increase in the share of effective inventors in concentrating sectors of 0.045pp. This implies a fall in average labor productivity implied by misallocation of 0.45pp. While these numbers might appear large considering the entirety of the economy, it is worth noting that my sample includes mainly manufacturing and retail sectors, which experienced a sizable reduction of $2.73pp$ in average annual productivity growth from 1997 to 2012, driven by a steep decline in output per worker growth in manufacturing. Therefore, the mechanism I propose would explain around 15 percent of the observed decrease in output per worker growth in these sectors. The estimates in the lower panel of Table 7, which uses the HHI instead the change in inventor shares as independent variable, imply even larger growth effects. Using the estimates in Column (2), a median HHI change of 0.02 and a median number of effective inventors of 1421 in sectors with growth in inventor shares implies a -0.78pp change in output per worker growth from misallocation, with a confidence interval ranging from -0.13pp to -1.45pp. The midpoint of these estimates would explain 28.6% of the observed fall in output per worker growth over the sample period (-2.73pp), with bounds ranging from 4.8% to 53%.

3.2.5 Markets with Growing Inventors' Share Saw Increased Patent Litigation

I restrict my attention to the period 2002 to 2012, where data on patent litigation cases are available.[14] I employ the specification in (2), now differencing over the horizon 2002-2012, and using as dependent variable the number of litigations per 1000 patents, computed as described in Section 2.3. I weight the regressions by the number of patents registered in each sector in 2012, as sectors with few patents might otherwise drive the results. Column (2) in Table 8 implies that 1pp increase in the share of inventors employed by a sector was associated with a highly significant increase of 3.6 litigation cases per 1000 patents. This compares to the average increase of 1.33 litigations per 1000 patents observed over the period. Alternatively, a one standard deviation increase in the share of inventors (3pp)

[14] My main results are robust to this sample restriction (tables available upon request).

Table 8: Regressions of Changes in Litigations over Changes in Inventor Shares , 2002-2012

	Δ Litigations/(1000 Patents)	
	(1)	(2)
Δ Inventor Share (pp)	3.468***	3.610***
	(0.829)	(0.909)
Δ log Sales		-1.211
		(3.850)
Knowledge Market FE	✓	✓
Sample	Full Sample	Full Sample
Weight	Patents	Patents
Observations	154	154

Note: Regressions weighted by number of patents; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; checkmarks indicate the inclusion of fixed effects.

raises litigations per 1000 patents by 11.8, which corresponds to 0.5 standard deviations. The coefficients are virtually unchanged when size controls are included, outliers are dropped as above, or increases in inventor shares are lagged to mitigate concerns of reverse causality (results are available upon request).

This last set of results further supports the hypothesis that defensive innovation increased in concentrating sectors. To protect their dominant position, firms engage in such R&D to thwart innovation by potential competitors, which manifests through an increase in patent enforcement cases and a reduction in inventors' productivity in sectors with increased concentration.

3.2.6 Discussion of Alternative Channels

The above analysis shows that inventors have accrued to sectors with increased concentration, primarily to incumbent firms, leading to an increase in litigation, a fall in inventors' productivity and forward citations. My interpretation of these empirical findings is that firms in high-concentration sectors have increased their efforts in defensive innovation, meant to secure patents that can later be used to discourage further entry. However, other papers are also consistent with increased concentration of inventors within sectors and decreased R&D productivity. Two competing explanations stand out.

A first explanation may come from technological complexity in the spirit of Bloom et al. (2020). While R&D might in general have become more difficult, my results do not show a decrease in patents per inventor in sectors with increased concentration. On the contrary, given that my inventor measure captures patent per capita, I effectively observe a relative increase in patenting activity in high-concentration sectors, excluding that "ideas getting harder to find" drive my results. Another explanation is increasing incremental, firm-specific R&D, as documented by Akcigit and Ates (2020, 2021). I verify that sectors with increased concentration did not see more self-citations or falling patent generality, which are commonly used measures of the radical nature and applicability of patents.[15] Thus, incremental innovation does not appear to be driving my findings.

4 Model

This section presents a Schumpeterian model based on Abrams et al. (2013), featuring growth through creative destruction by entrants, as well as the possibility for incumbent monopolist of researching a defensive technology that increases research costs for entrants. I first present a single-sector model to clarify the mechanism at play within each sector in the economy and study a constant-growth equilibrium analytically. I then extend the model to two sectors, and I consider a fixed supply of inventors, which shuts down within-sector misallocation occurring independently of inventors' movements across sectors. I show that increasing markups in one of the sectors of the economy lead to a misallocation of inventors towards defensive innovation in the less competitive sector. Finally, I study the optimal allocation of R&D subsidies needed to achieve maximum growth in a calibration of the two-sector model that matches moments of the R&D distribution in 1997. Proofs are reported in Appendix C.

4.1 Single-sector Model

4.1.1 Preferences and production

Consider the following continuous time economy with a single final good, populated by a representative household with preferences over consumption and R&D labor

$$\mathbb{E}_t \int_t^{\infty} \exp\left(-\rho\,(s-t)\right)\left(\ln C_s - \frac{\chi\left(L_s^{RD}\right)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)\mathrm{d}s, \tag{3}$$

[15] See Appendix B.4.

where ϕ is the Frisch labor supply. In addition, the representative household inelastically supplies L units of production labor.[16] The representative household owns a differentiated portfolio of all the firms in the economy, with rate of return r_t, and receives a wages w, w^{RD}, for each unit of production and research labor, respectively. I assume that the economy is closed and that the final good is only used for consumption, $C_t = Y_t$. This structure yields a standard Euler equation and R&D labor supply with Frisch elasticity φ.

The market structure in the model follows Acemoglu and Akcigit (2012) and Abrams et al. (2013). Accordingly, the consumption good in the economy, which I take as numeraire, is the unit-measure aggregate of products

$$\ln Y_t = \int_0^1 \ln y_t(i) \mathrm{d}i. \tag{4}$$

The market for each product $y_t(i)$ consists of an incumbent and a fringe of competitors. In what follows, I focus on a single market, dropping the argument i. Each firm j in the sector produces an undifferentiated good with the linear production technology:

$$c_{j,t} y_{j,t} = l_{j,t},$$

where $c_{j,t}$ denotes the labor requirement to produce a unit of output, and $l_{j,t}$ the production labor employed by the firm. Competitors have labor requirement $c_{e,t} = c_t$, while the incumbent faces a lower unit labor requirement $c_{I,t} = \frac{c_t}{\phi}$, with $\phi > 1$. Profit maximization by the incumbent gives an optimum limit price $p_t = w_t c_{e,t}$, which leads her to capture the entire market and act as a monopolist, charging a markup $\phi > 1$ on the marginal cost. The Cobb-Douglas assumption on the final good implies equilibrium normalized profits,

$$\frac{\Pi_t}{Y_t} \equiv \pi = \left(\frac{\phi - 1}{\phi}\right).$$

4.1.2 Innovation

Incumbents and entrants can conduct innovation activity that reduces their unit costs to

[16] While this assumption is not necessary for the results to hold, it simplifies the analysis considerably. In the following section, I will consider both production and research labor as given by a fixed endowment in the constant growth equilibrium of the economy. In that case, the assumption is equivalent to assuming that both labor endowments grow at a constant rate.

$$c_{I,t+\Delta_t} = \frac{c_{e,t}}{(1+\eta)\phi}, \quad \eta > 1$$

Here, η parametrizes the percentage increase in productivity for the innovating firm, relative to the technology previously operated by the incumbent. Whenever either the incumbent or an entrant realize an innovation, all other firms gain access to a technology with unit costs, $c_{e,t+\Delta_t} = c_{e,t}/(1+\eta)$. These assumptions imply that, if entrants realize an innovation, they outcompete previous incumbents and become the new monopolists. Displaced incumbents join the pool of entrants and from instant $t+\Delta t$ onwards operate the technology $c_{e,t+\Delta_t}$. [17] With this structure, the relative productivity of incumbents to entrants is fixed at ϕ regardless of the number of innovations realized, which allows a recursive formulation of the problem. I therefore drop time indexes in what follows.

Incumbents' and entrants' innovation differ in two respects. First, successful incumbents' R&D produces a *patent wall* of size $\omega > 1$, which decreases the success probability of entrants' innovations. Second, successful entrants' R&D results in an implemented innovation with certainty, while incumbents adopt new technologies with probability $\lambda \in [0,1]$. I introduce this parameter to allow for incremental incumbents' innovations. The lower λ, the lower the expected productivity gains from incumbents' innovations.

Following Acemoglu and Akcigit (2012), innovation investments consist in the choice of an arrival rate of new discoveries x_I, and that R&D costs are increasing and convex in this arrival rate:

$$C\left(x_I; w^{RD}\right) = \alpha_I \frac{x_I^{\gamma}}{\gamma} w^{RD}, \ \gamma > 1,$$

where the term $\alpha_I \frac{x_I^{\gamma}}{\gamma}$ indicates the amount of inventors that the incumbent needs to obtain innovations with a flow probability x_I, and w^{RD} is the wage paid to inventors. For simplicity, incumbents can only erect *one* patent wall of size $\omega > 1$, and cannot invest in further innovation until their patent expires, which occurs at a rate δ.

Under these assumptions, incumbents' values at any given instant are just a function

[17] This amounts to assuming that the incumbent's technology becomes obsolete after displacement or that incumbents scrap the assets needed to operate the innovative technology upon destruction.

of the state of the patent wall in the product market they operate, $\Omega \in \{1, \omega\}$:

$$r V(1) - \dot{V}(1) = \max_{x_I} \left\{ \left(\frac{\phi - 1}{\phi} \right) Y - \alpha_I \frac{x_I^{\gamma}}{\gamma} w^{RD} + x_I \left(V(\omega) - V(1) \right) - x_{e,1} V(1) \right\}, \tag{5}$$

$$r V(\omega) - \dot{V}(\omega) = \left(\frac{\phi - 1}{\phi} \right) Y + \delta \left(V(1) - V(\omega) \right) - x_{e,\omega} V(\omega), \tag{6}$$

where $x_{e,1}$ and $x_{e,\omega}$ denote entrants' innovation intensities, r is the interest rate in the economy, and δ the rate of patent expiration. The first line displays the flow value to incumbents that operate in a market not protected by a patent wall. There, incumbents realize instantaneous profits $\left(\frac{\phi-1}{\phi}\right) Y$, and choose their innovation intensity x_I, taking the researchers' wage w^{RD} and the entrants' innovation intensity $x_{e,1}$ as given. If entrants are successful at rate $x_{e,1}$, incumbents are destroyed. If incumbents' innovation is successful at rate x_I, they obtain the patent wall ω, which grants them the protected value $V(\omega)$. When a patent wall is in place, incumbents realize the same flow profits as in the unprotected state, due to the economy-wide spillovers described above. However, incumbents face a different entrants' innovation intensity, $x_{e,\omega}$, which is in equilibrium lower than $x_{e,1}$ due to the patent wall in place. Incumbents face a flow probability δ that the patent wall is exogenously destroyed, in which case they transition back to the unprotected state. The optimal incumbent's R&D decision reads

$$x_I = \mathbf{1} \left\{ V(\omega) - V(1) > 0 \right\} \left(\frac{V(\omega) - V(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma - 1}}. \tag{7}$$

Following Abrams et al. (2013), I assume that each market has a mass of atomistic entrants, indexed by j, who face innovation costs that feature congestion externalities

$$C\left(x_{e,\Omega,j}; w^{RD}\right) = \zeta \Omega x_{e,\Omega,j} x_{e,\Omega} w^{RD}.$$

In this specification, ζ parametrizes the inventor requirement to obtain a unit aggregate entrants' innovation rate when the market is not protected by patent walls, $\Omega = 1$. Individuals costs are linear in the total entrants' research intensity in the product market, $x_{e,\Omega} \equiv \int_{\mathcal{J}} x_{e,\Omega,j} \mathrm{d} j$. In other terms, individual entry costs increase with the aggregate entry rate. Successful entrants obtain a new unprotected technology, regardless of the state of the market that they target. Free entry gives the entrants' innovation rate as a decreasing function of the market's patent wall, Ω,[18]

$$x_{e,\Omega} = \frac{V(1)}{\zeta \Omega w^{RD}}, \quad \Omega \in \{1, \omega\}. \tag{8}$$

4.1.3 Equilibrium with Constant Growth

The mass of protected and unprotected markets evolve according to:

$$\dot{\mu}_1 = -\left(x_I + x_{e,1}\right)\mu_1 + \delta\mu_\omega + x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}, \tag{9}$$

$$\dot{\mu}_\omega = -\left(x_{e,\omega} + \delta\right)\mu_\omega + x_I\mu_1, \tag{10}$$

where $\mu_{e,\omega}$ and $\mu_{e,1}$ denote the mass of entrants targeting each markets. The interpretation is analogous to Equations (5) and (6). Similarly, the mass of entrants in each product market follows the laws of motion:

$$\dot{\mu}_{e,1} = -\left(x_{e,1} + x_I\right)\mu_{e,1} + x_{e,1}\mu_1 + \delta\mu_{e,\omega}, \tag{11}$$

$$\dot{\mu}_{e,\omega} = -\left(x_{e,\omega} + \delta\right)\mu_{e,\omega} + x_{e,\omega}\mu_\omega + x_I\mu_{e,1}, \tag{12}$$

The model is closed by clearing production and R&D labor markets:

$$L = \int_0^1 l(i)\mathrm{d}i, \tag{13}$$

$$L^{RD} = \zeta\left(\omega x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}\right) + \alpha_I \frac{x_I^\gamma}{\gamma}\mu_1. \tag{14}$$

The definition of the constant growth equilibrium and its properties follow.

Definition. A constant growth equilibrium is a sequence of values $\{V_t(1), V_t(\omega)\}$, wage sequences $\left\{w_t^{RD}, w_t\right\}$, and R&D decisions $\left\{x_{I,t}, x_{e,1,t}, x_{e,\omega,t}\right\}$ such that, given labor endowments, L, L^{RD}: (i) incumbents maximize values (5) and (6), taking entrants' R&D as given; (ii) entrants' R&D satisfies (7) and (8); (iii) the distribution of firms across markets is constant, $\dot{\mu}_1 = \dot{\mu}_\omega = \dot{\mu}_{e,1} = \dot{\mu}_{e,\omega} = 0$ in Equations (9)-(12); (iv) values follow (5) and (6); (v) consumers maximize utility (3) choosing consumption and R&D labor; (vi) labor markets clear according to (13) and (14); (vii) product markets clear, $C_t = Y_t$; and (viii) aggregate output (4) grows at constant rate, $g \equiv \dot{Y}_t / Y_t$.

[18] See Abrams et al. (2013) for more details.

Proposition 1 (Existence and Uniqueness of the Constant Growth Equilibrium)**.** *For any endowment of production labor, L, there exists a unique constant growth equilibrium. Denoting optimal innovation choices as $x_I^\star, x_{e,\omega}^\star, x_{e,1}^\star$, and the stationary distribution as $\left[\mu_1^\star\ \mu_\omega^\star\ \mu_{e,1}^\star\ \mu_{e,\omega}^\star\right]$, the constant growth rate of the economy is given by*

$$g = \eta\left[x_{e,\omega}^\star \mu_{e,\omega}^\star + x_{e,1}^\star \mu_{e,1}^\star + \lambda x_I^\star \mu_1^\star\right],$$

and inventors' productivity reads

$$\frac{g}{L^{RD}} = \eta \frac{x_{e,\omega}^\star \mu_{e,\omega}^\star + x_{e,1}^\star \mu_{e,1}^\star + \lambda x_I^\star \mu_1^\star}{\zeta\left(\omega x_{e,\omega}^\star \mu_{e,\omega}^\star + x_{e,1}^\star \mu_{e,1}^\star\right) + \alpha_I \frac{(x_I^\star)^\gamma}{\gamma} \mu_1^\star}.$$

The expressions in this proposition clarify that an equilibrium increase in the mass of entrants in protected markets, $\mu_{e,\omega}^\star$ leads to a fall in inventors' productivity. While a unit of aggregate research intensity produces the same growth across unprotected and protected markets, the research unit labor requirement is higher than in unprotected markets by a factor of ω, as can be seen comparing the numerator and denominator of inventors' productivity. Since incumbents' research effort acts to raise $\mu_{e,\omega}^\star$, *ceteris paribus* inventors' productivity declines when incumbents employ a larger share of inventors.[19] As the following proposition shows, higher markups unambiguously increase research efforts by incumbents and entrants, as well as the share of R&D labor accruing to incumbents.

Proposition 2 (Effects of Markup Increases on Innovation)**.** *Suppose that defensive research is effective, $\omega > 1$. The constant growth equilibrium features positive incumbents' research, $x_I^\star > 0$; markup increases raise both incumbents' and entrants' research effort*

$$\frac{\partial x_I^\star}{\partial \phi} > 0,\ \frac{\partial x_{e,\omega}^\star}{\partial \phi} > 0\ \frac{\partial x_1^\star}{\partial \phi} > 0.$$

If labor supply is elastic, the incumbents' inventor share increases with markup

$$\frac{\partial \frac{L_I}{L^{RD}}}{\partial \phi} = \frac{\partial\left(\alpha_I \frac{x_1^{\star\gamma}}{\gamma} \mu_1^\star / L^{RD}\right)}{\partial \phi} > 0.$$

[19] In a previous version of this paper, I prove that a sufficient condition for $\mu_{e,\omega}^\star$ to increase with markups is that the research intensity of incumbents is more elastic than entrants', a condition which is verified in all the numerical simulations I explored.

Higher markups raise the value of monopolistic positions, propelling entrants' and incumbents' research effort, and tilting the distribution of inventors toward incumbents. This holds only if defensive R&D is effective, showing that this channel is needed to reproduce the results in Table 5. Note also that when inventors' supply is inelastic, wage increases offset partial-equilibrium increase in labor demand, and the unique equilibrium implies a fixed allocation of inventors across incumbents and entrants.

4.2 Calibration and Policy

I calibrate a two-sector extension of the model, with two objectives. First, I want to analyze misallocation *across* sectors, and show that a realistic calibration can qualitatively reproduce the findings of the empirical analysis. I fix inventor supply, so that markups have no effect on inventors' productivity within sectors independently of cross-sector reallocation.[20] Second, I show that the optimal growth-maximizing policy allocates R&D subsidies to entrants in the less competitive sector.

4.2.1 Model description

The consumption good in the economy is given by the Cobb Douglas aggregate:

$$\ln Y_t = \beta_1 \ln Y_{1,t} + \beta_2 \ln Y_{2,t}, \quad \beta_1 + \beta_2 = 1$$

where $Y_{1,t}$, $Y_{2,t}$ are produced as in Section 4.1.1, and the markup parameter, ϕ, is allowed to vary across the two sectors. The household side of the economy is unchanged relative to the one-sector model, but the supply of inventors is now fixed at $L^{RD} = 100$, allowing a clean interpretation of the results as arising from inventors' mobility across sectors. The rest of the model is as in the previous section, except that now aggregate labor demands are given by the sum of each sectors' demand, and, due to the Cobb-Douglas assumption, the growth rate of the economy is the average sector growth rate weighted by β_i. Appendix C.2 reports detailed derivations.

[20]As discussed above, a rigid inventors' supply excludes reallocation of inventors within sectors in the absence of cross-sector mobility. When aggregate inventors' supply is elastic, increased markups produce more misallocation, since the allocation of inventors becomes less efficient both within and across sectors.

4.2.2 Calibration

I calibrate my model to match features of the R&D distribution and concentration around 1997, which provides conservative parameter choices to analyze the model. Feeding observed changes in markups, inventor productivity falls by about 2.5% over the period 1997-2012, consistent with the lower bounds of my estimates.[21]

The upper part of Table 9 displays parameters calibrated externally. I set the discount rate to 4%, which, together with a 3% growth for my sample in 1997, implies a value for the real interest rate of 7%, in line with the long-run average before 1997. The share of value added of each sector comes from estimates of the Lerner Index in manufacturing, constructed using NBER-CES data as described in Appendix B.5. About half of the sectors (weighted by sales) for which I have data saw an increase in the Lerner Index over the period, which justifies setting $\beta_1 = \beta_2 = 0.5$. Since I rely on the extensive literature estimating markups to set a value of $\phi = 1.08$.[22] As standard in the literature (see e.g., Acemoglu and Akcigit, 2012), I set the curvature of the incumbents' cost function based on Kortum, 1993. I choose the lower bound of these estimates to minimize the asymmetry of innovation costs between incumbents and entrants. More convex incumbents' costs mechanically make their research less effective than entrants. The rate of patent expiration is set to reflect a 20 year patent duration, as established by the Uruguay Round Agreements Act of 1994. Since λ measures how radical are incumbents' innovations relative to entrants', I set $\lambda = 0.785$, consistent with an internal patent share of 21.5% estimated by Akcigit and Kerr (2018). Turning to the value of blocking patents, ω, I rely on estimates by Czarnitzki et al. (2020) and Grimpe and Hussinger (2008), who employ merger data to obtain the effect of pre-emptive patents on the value of acquired firms. Both their estimates imply a unitary elasticity of firm's values to the share of pre-emptive patents. That is, a firm with a patent portfolio composed exclusively of defensive patents is valued on average twice as much as one without pre-emptive patents. The proof of Proposition 1 in the Appendix shows that the percentage increase in firm value when acquiring a defensive patent is at most $\omega - 1$, justifying a choice of $\omega = 2$. I include R&D

[21] Column (4) in the upper panel of Table 7 implies a lower bound for the fall in inventor productivity of about 1.5%, while the lower panel implies a fall in inventor productivity of at least 4.8%. The 2.5% fall in my calibration is in between these two lower bounds.

[22] I follow Akcigit and Ates (2019), who calibrate the same parameter using the midpoint of estimates provided in De Loecker et al. (2020) and Eggertsson et al. (2018).

Table 9: Parameter Values and Sources

Parameter Name	Symbol	Value	Source/Target
Discount rate	ρ	.04	Annual real rate ≈ 7% before 1997
Value Added Share	β	.5	Share of sectors with ↑ Lerner Index
Average Sectors' Markup	ϕ	1.08	Akcigit and Ates (2019)
Innovation Cost Curvature	γ	1/.6	Lower bound in Kortum, 1993
Intensity of Patent Expiration	δ	.05	Uruguay Round Agreements Act (1994)
Share of Implemented Innovations	λ	.785	Akcigit and Kerr, 2018
Value of Blocking Patents	ω	2	Grimpe and Hussinger, 2008
R&D subsidy	$s_I = s_e$	19%	Akcigit et al., 2016
Corporate tax rate	τ	23%	Akcigit et al., 2016
Incumbent Costs	α_I	21.97	Top 10% Firms' Inventor Share, 1997: 30.3%
Entrants' Costs	ζ	4.75	Business R&D Share over GDP, 1997: 1.81%
Innovation Step	η	0.0047	Output per Worker Growth, 1997: 3.03%

subsidies as a percent subsidy on inventors' wages, s, and corporate taxes applied to firms instantaneous profits, τ, parametrized as in Akcigit et al. (2016). The bottom part of Table 9 describes my choices for the remaining parameters, which govern the scale of R&D and the growth rate in the economy. I set the incumbents' and entrants' R&D cost scale, α_I and ζ, to match the share of inventors employed by incumbent firms in 1997 and the R&D business spending as a percent of GDP, as reported by the National Science Foundation. Intuitively, the two cost parameters jointly determine the overall R&D spending in the economy, while their relative value determines the distribution of R&D spending in equilibrium. Given the estimates for α_I and ζ, I set η to match the growth in output per worker for the sectors considered in my analysis in 1997, 3.03%. [23]

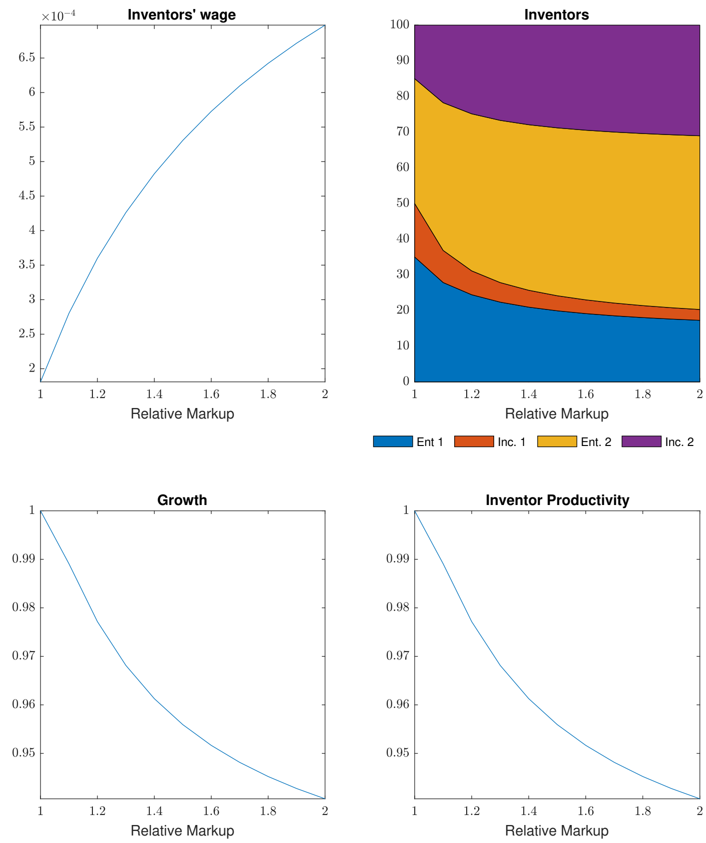
4.2.3 Comparative Statics for a Markup Increase in Sector 2

Figure 4 displays the comparative statics for an increase in markup in sector 2, while leaving the sector 1's markup unchanged. Aggregate quantities are plotted against the markup of sector 2 relative to sector 1 on the x-axis.

An increase in the markup of sector 2 leads to a pronounced reallocation of inventors away from sector 1. In sector 2, incoming inventors are allocated disproportionately to incumbents. Computing the Lerner Index on NBER-CES data as described in Appendix B.5 reveals that the markup gap between more and less concentrated sectors has increased by

[23] Empirical targets are missed by their model analogues by is less than 10^{-6}%.

Figure 4: Comparative Statics for an Increase in Sector 2's Markup



Note: This figure reports the comparative statics for normalized profits, inventors, growth and inventor productivity in the two-sector model. In all figures, the x-axis reports the markup of sector 2 relative to sector 1. The parameters used to produce this figure are reported in Table 9.

about 20% over the period of analysis. This implies a fall in inventors' productivity of 2.5% compared to the benchmark where the two sectors have the same competitive structure. This also results in a 2.5% fall in GDP growth, about 0.075pp. This estimate is in between the lower bound of 0.13pp implied by my estimates in the lower panel of Table 7 and the lower bound of .03pp implied by the upper panel. As discussed in previous sections, a rigid R&D labor supply shuts down inventor misallocation unrelated to sectoral shifts, so this calibration understates the productivity effects of increased concentration. Figure 4 also shows that inventors within sector 1 reallocate toward entrants, as lower entry in this sector reduces the incentives for defensive innovation. Therefore, while the movements of inventors from sector 1 to sector 2 are overall detrimental to growth, they are not for R&D productivity in sector 1. However, overall growth is lower since less R&D resources are available to this sector in equilibrium. Thus, reallocation away from competitive sectors has both costs–a reduction in sectoral growth–and benefits, coming from a more efficient distribution of resources within the sector.

This comparative static exercise also clarifies how defensive innovation undergirds misallocation across sectors. In the absence of defensive innovation, inventors just reallocate across sectors, leaving productivity and growth unaffected.

4.2.4 Growth-Maximizing Policy

I now turn to calculating the combination of R&D subsidies that maximizes growth. In this model, banning defensive innovation leads to a first-best where only entrants conduct R&D and reallocation does not hamper growth. However, such a ban might be impossible to implement, as determining the nature of patents ex-ante might in practice be unfeasible. For this reason, I assume that the planner cannot distinguish productive from unproductive projects, and can only employ using tax instruments.

I start from the 2012 equilibrium of my model economy, where the gap in markups between sectors is 20%, and all firms receive a 19% subsidy to inventors' wages and incur a 23% tax on profits. I study three cost-neutral alternatives, where I constrain the planner to leave the expenditure on R&D subsidies as a percentage of GDP fixed at the 2012 benchmark. In the first scenario, the planner is allowed to distribute subsidies freely and can condition the allocation on both the sector and the identity of the receiving firm (entrant or incumbent). In the other two scenarios, I only allow the planner to act on one of these dimensions at a time. That is, the planner can either control the distribution of funds across sectors, but not between incumbents and entrants, or vice versa.

Table 10 presents the results of this exercise. The 2012 equilibrium is reported in Columns 1 and 2. For reference, the 1997 calibrated model has two identical sectors, which share the stock of inventors equally. Within each sector incumbents have 30.3% of the overall inventors employed, and GDP growth is 3% per annum. In the 2012 baseline, the distribution of inventors is tilted toward the second sector, where markups have increased. This results in a fall in annual GDP growth of .07pp, about 2.5% of the 1997 benchmark. As shown in the graphs above, this new equilibrium sees a larger share of inventors allocated to incumbents in the second sector, which increases its growth relative to sector 1. However, productivity declines because incumbents conduct more defensive

Table 10: Comparison of R&D Policies in the Two-Sector Model

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector		Cost-Neutral Entry	
	Sector 1 (1)	Sector 2 (2)	Sector 1 (3)	Sector 2 (4)	Sector 1 (5)	Sector 2 (6)	Sector 1 (7)	Sector 2 (8)
R&D Subsidies:								
s_I	19%	19%	0%	0%	46.17%	0%	0%	0%
s_e	19%	19%	0%	41.78%	46.17%	0%	29%	29%
Aggregates:								
L_I^{RD}	6.70	24.87	6.37	15.95	10.83	19.51	4.83	18.45
L_e^{RD}	24.41	44.02	23.87	53.81	30.24	39.42	27.41	49.30
L_{TOT}^{RD}	31.11	68.89	30.25	69.75	41.07	58.93	32.25	67.75
Sector Growth	2.12%	3.74%	2.08%	4.78%	2.61%	3.36%	2.45%	4.31%
GDP Growth	2.93%		3.43%		2.99%		3.38%	

Note: The figures reported in this table give the optimal allocation of R&D subsidies and the resulting aggregate outcomes for a planner wishing to maximize aggregate growth in the economy. The column headings refer to the various scenarios described above. “Baseline” refers to the subsidy allocation reflecting the 2012 equilibrium, where subsidies do not condition on sectors or the position of firms within sectors; “Optimal Cost-Neutral” refer to the scenario where the planner is allowed to freely allocate R&D subsidies subject to the constraint that overall R&D subsidy expenditure as a percentage of GDP is held fixed at its 2012 benchmark; “Cost-Neutral Sector” consider a scenario where the planner can choose which sector to allocate funds to, but not which firms within the sector should receive the subsidy; “Cost-Neutral Entry” computes the optimal universal entry subsidy, under the assumption that the planner cannot condition its reception on the sector firms operate in.

innovation. Columns 3 and 4 report the optimal cost-neutral R&D subsidies chosen by a growth-maximizing planner. Somewhat surprisingly, the most efficient allocation of funds is a subsidy to entrants in the more concentrated sector only. Indeed, defensive innovation is inefficient because it makes entrants' R&D less productive. This is the main friction that the growth-maximizing planner wishes to remove. As discussed above, the outflow of inventors from sector 1 increases inventor productivity, as lower R&D by entrants depresses defensive innovation by incumbents. It is therefore undesirable to reallocate inventors to entrants in sector 1, where barriers to entry are now naturally lower. Conversely, the optimal policy acts directly on the higher barriers now present in sector 2, subsidizing inventors' wages for entrants. Consistent with this argument, growth is not maximized when the planner allocates R&D subsidies to a single sector, without condition on the identity of the firm. This scenario is reported in columns 5 and 6. The planner subsidizes the more competitive sector 1, which brings annual growth up to 2.99%, recovering most of the lost ground relative to the 1997 benchmark. However, subsidies now make defensive innovation cheaper for incumbents, as well as more attractive due to increased entry. If a sector-specific subsidy to entrants is politically unfeasible, a viable, effective alternative is a blanket entry subsidy as reported in Columns 7 and 8, which raises growth to 3.38%.

This policy analysis suggests that entry subsidies are the most effective policy to counter the friction introduced by defensive innovation in this model economy. In less competitive sectors, where this friction is most pronounced, R&D entry subsidies increase growth by 0.5pp per annum. A more feasible uniform R&D subsidy to entrants in both sectors produces similar effects. Conversely, sector-specific subsidies to reallocate inventors to the competitive sector are less effective, since they subsidize incumbents' pre-emptive innovation, exacerbating the inefficiency that the planner wishes to contrast.

5 Conclusion and Future Work

In this paper, I propose and document a novel explanation for the observed decline in growth and R&D productivity over the last few decades.

My empirical results show that increasing misallocation of inventors across different product markets accounts for up to 28% of the decline in output per worker growth in the sectors I analyze. This misallocation stems from uneven increases in concentration across product markets that are accompanied by a larger share of inventors accruing to less competitive sectors. I interpret my findings as resulting from an increase in defensive innovation in concentrated sectors. Such R&D activities are conducted primarily to block further entry and are reflected in a decline in patents' forward citations, a larger share of inventors employed by incumbents, and rising patent enforcement cases.

I study defensive innovation in a model of creative destruction, where incumbents can innovate to raise entrants' costs. I calibrate two-sector version of the model to study the growth-maximizing allocation of R&D subsidies across sectors, as well as between incumbents and entrants. My analysis suggests that R&D subsidies to entrants are the most effective policy, directly tackling the friction generated by defensive innovation and potentially increasing growth by 17% of my baseline (0.5pp in absolute terms).

Two directions for future research stand out. The first would investigate the validity of my findings in an international context, as the effects of competition and innovation are often country- and time-dependent. The second would be to conduct a thorough investigation of the evolution, causes, and consequences of pre-emptive innovation.

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Online Appendix (Not for Publication)

A Data Construction Details

A.1 Knowledge Markets

Rescaling Inventor Flows As explained in the main text, the measure of inventor flows aims to capture the strength of the connections between two sectors. I take several steps to ensure that I do not overestimate these connections and to normalize them to account for the size of sending and receiving sectors.

As a first step, I build normalized directed flows for each inventor i in order to avoid double counting. For example, for transitions between sector 1 and 2, I define:

$$\text{flow}_{1\to 2,i,t} \equiv \frac{\sum \mathbf{1}\{i \text{ moves } 1 \to 2 \text{ in } t\}}{\sum_{j,k} \mathbf{1}\left\{i \text{ moves } j \to k \text{ in } t\right\}} \times \alpha_i.$$

This measure attributes a fraction of the effective inventor fixed effect α_i to each transition in proportion to the number of overall inventor i's transitions across sectors in each year. The first term in this formula is precisely the share of transitions from sector 1 to sector 2 relative to overall transitions between any two sectors j and k that inventor i took part in.

Second, I compute total inflows (outflows) for each NAICS 4-digit sector, summing over all years, inventors and origin (destination) sectors. For example, inflows for sector 1are defined as:

$$\text{inflow}_1 = \sum_n \sum_t \sum_i \text{flow}_{n\to 1,i,t},$$

where n denotes origin NAICS sectors, t years, and i inventor identifiers.

Third, I proceed to compute the share of directed flows between each pair of sector as a share of total inflows or outflows. For example, the share of inflows coming from sector 2 and entering sector 1 is defined as:

$$\text{share}_{1\leftarrow 2} = \frac{\sum_t \sum_i \text{flow}_{2\to 1,i,t}}{\text{inflow}_1}.$$

In this example, this measure captures the relative importance of inflows from sector 2 for the overall number of inventors received by sector 1. However, this measure can still overstate flows from large to small sectors, or vice versa. As a result, and since I need undirected flows to apply the Louvain algorithm, I define network edge weights starting from an average of the above shares of inflows and outflows for each sector and taking the minimum between the two measures as follows:

$$W_{12} = W_{21} = \min\left\{\frac{\text{share}_{1\leftarrow 2} + \text{share}_{1\rightarrow 2}}{2}, \frac{\text{share}_{1\leftarrow 2} + \text{share}_{1\rightarrow 2}}{2}\right\},$$

where $W_{12} = W_{21}$ since the final network is undirected.

Modularity Maximization Formula and Algorithm In order to identify knowledge markets from the network constructed above, I employ the Louvain community detection algorithm (Blondel et al., 2008). This algorithm maximizes the modularity of the network, Q, assigning each sector i to one of N *non-overlapping* communities c. Accordingly, the objective function for this problem is given by:

$$\max_{N} \max_{(c_1,\ldots,c_N)} Q \equiv \frac{1}{2\boldsymbol{W}} \sum_{ij} \left[W_{ij} - \frac{\boldsymbol{W}_i \boldsymbol{W}_j}{2\boldsymbol{W}} \right] \mathbf{1}\left\{c_i = c_j\right\},$$

where W_{ij}, weight of the edge connecting node i to j, and bold variables denote other summations for ease of notation. In particular, I define $\boldsymbol{W_i} \equiv \sum_k \sum_i W_{ik}$, as the sum of weights for edges with one end in node i, and the sum of all weights in the network, respectively. The indicator $\mathbf{1}\left\{c_i = c_j\right\}$ takes a value of 1 when nodes i and j belong to the same community. Note that the maximization is carried out both over the number of communities and the assignment of nodes to each community. This measure can be interpreted considering that $\frac{\boldsymbol{W}_i \boldsymbol{W}_j}{2\boldsymbol{W}}$, is the expected number of edges that arise between nodes i and j in a random network. Therefore, modularity maximizes the distance between the density of linkages within communities W_{ij} relative to the overall density of links that would arise randomly.

Since looping over all the permutations of nodes and community is numerically unfeasible, the Louvain algorithm follows an iterative procedure to maximize modularity.

First, it assigns each node to its own community. Then, it repeats iteratively the following three steps:

1. Compute local deviations in modularity from reassigning the node to neighboring communities;

2. Assign nodes to communities following the local improvement granting the highest modularity increase;

3. Redefine a network with new communities as nodes.

These steps are repeated until there is no significant improvement in modularity for further steps.

B Additional Results and Robustness

B.1 Results on Overall Inventor Shares

Table 11 shows estimates using the share of effective inventors of each product market over the total. This amounts to neglecting the fact that inventors flow only across sectors that can employ their skills. In this specification, I find a significant, albeit small, effect of product market concentration on the share of inventors. However, this result only arises when the sample is trimmed to remove outliers. This is not surprising, considering that mismeasuring the labor market for inventors should bias the estimates of inventor mobility towards zero, since many of the sectors would not be routinely connected by inventor flows. Additionally, this result conforms with the findings in Table 3, which show that including knowledge-market fixed effects does not alter the coefficients significantly, suggesting that flows across knowledge markets are indeed negligible. The results presented in this section therefore speak to the importance of accurately delineating labor markets for inventors when assessing their flows across product markets.

B.2 Using the Raw Number of Inventors instead of Fixed-Effects

This Appendix reports the results for the main analysis presented in Section 3.2 using the raw number of total inventors instead of the fixed effects from regression (1), which

Table 11: Regressions of Change in Total Inventors' Share over Change in HHI Lower Bound, Long-Difference, 1997-2012

	Δ Total Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\underline{\text{HHI}}$	0.297	1.692	1.328*	1.532*	0.271	1.889
	(2.007)	(1.956)	(0.649)	(0.696)	(2.038)	(2.023)
Δ log Sales	0.460	0.436	0.133**	0.109*	0.464	0.472
	(0.281)	(0.292)	(0.047)	(0.047)	(0.283)	(0.312)
Knowledge Market FE		✓		✓		✓
Sample	Full	Full	Trimmed	Trimmed	Mahalanobis	Mahalanobis
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	153	147	143	150	139

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. Please refer to notes in Table 3 for further details.

Table 12: Regressions of Change in 4-digit Knowledge Market Share of Total Inventors over Change in HHI Measures, Long-Differences, 1997-2012

	Δ Inventor Share (pp)	
	(1)	(2)
$\Delta\underline{\text{HHI}}$	74.172+	
	(40.957)	
Δ HHI		71.749**
		(24.464)
Knowledge Market FE		
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	157	80

Note: See notes to Table 2.

might be inconsistently estimated. The following Table, to be compared with Table 2 in the main text, shows that the results are qualitatively unchanged, although coefficients are larger, and estimates less precise. This is easily explained by the fact that differences in research requirements across patent classes, firms and years are not absorbed as in the effective inventor measure, making the latter noisier. As In the main text, trimming the sample and introducing the sector and firm size controls does not affect estimates significantly (tables available on request).

B.3 Using a Quartic in Sales as Size Control

This Section displays the results of estimating the specification in Table 3 using the changes in the terms of a fourth-degree polynomial in sales rather than log-sales. This flexible control specification ensures that my main findings do not rely on the specific functional form that I assumed above. Table 13 reports the result of this exercise using both effective inventors (Columns (1) and (2)) and raw inventor counts (Columns (3) and (4)) to compute sector shares. Recall that when using raw inventor counts, knowledge markets are also constructed according to this measure. As clear from a comparison of Columns (1) with (2), and (3) with (4), these two specifications produce statistically undistinguishable results.

Table 13: Regressions of Change in 4-digit Knowledge Market Share of Inventors over Change in HHI Lower Bound, Long-Differences, 1997-2012

	Δ Inventor Share (pp)			
	(1)	(2)	(3)	(4)
$\Delta \underline{\text{HHI}}$	22.509*	24.083*	67.160+	74.769+
	(10.848)	(10.565)	(37.176)	(39.225)
Δ log Sales	0.548*		1.422*	
	(0.243)		(0.717)	
Δ Sales ($ bn)		2.617*		6.382+
		(1.108)		(3.365)
ΔSales^2		-0.749		-1.749
		(0.482)		(1.468)
ΔSales^3		0.081		0.165
		(0.076)		(0.232)
ΔSales^4		-0.003		-0.005
		(0.003)		(0.009)
4D Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales	Sales
Observations	153	153	156	156

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (2), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. “Full Sample” refers to the sample described in the main text.

B.4 Self-Citation Regressions

In this section, I investigate a competing explanation for my findings on output growth. As highlighted by Acemoglu et al. (Forthcoming) and Akcigit and Kerr (2018) among others, large incumbents have a strong incentive to focus on improving their own products at the expense of broadly applicable innovation. This mechanism would also imply that an increase in incumbents' share of R&D resources leads to falling innovation productivity. In order to assess the importance of this channel, and in keeping with the analysis in Akcigit and Kerr (2018), I use the share of self-citations to measure the extent of internal innovation conducted by firms. Table 14 displays the results pertaining to this measure. All columns use as dependent variable the change in excess log self-citations.

To count self-citations, I identify the set of cited patents that belong to the same assignee as the citing patent. I weigh self-citations to account for cited patents that have multiple assignees: one self-citation when the patent has a single assignee; one half if the cited patent has multiple assignees. The share of self-citations is given by the sum of weighted self-citations divided by the number of patents cited by each assignee. I construct five measures to correct self-citations for the assignee's importance in the relevant technology class of cited patents. For each citation made, excess self-citations are defined as $1 - Pr\,(\text{self-citation})$. The measure depends on how the probability of self-citation is computed. For the first three measures, I compute this probability as the assignee's share of total patents in the NAICS code attributed to the citing patent. I employ in turn the share of NAICS patents for the year of citation, the previous five years, and the cumulative share from the beginning of the sample. The other two measures are based on the CPC classification at the group and subgroup levels (the lowest levels of detail in the classification). For this measure, the probability of self-citation is derived for each citation by taking the share of patents by the assignee in the CPC (sub)group and the year corresponding to the cited patents.[24] I aggregate all measures across assignees in the same NAICS 4-digit code using the number of patents in the relevant code by each assignee in each year.

Columns (1) and (2) build excess self-citations correcting for the importance of firms' patents for the CPC group, which reflects the technological classification of the patent. Columns (3) and (4) use the more narrowly defined CPC subgroups for robustness. Coeffi-

[24] This procedure is similar to the approach followed in Akcigit and Kerr's (2018) Appendix C.

Table 14: Regressions of Change in Excess Self-Citations over 4-digit Knowledge Market Share, Long-Differences, 1997-2012

	Δ self-citations			
	(1)	(2)	(3)	(4)
Δ Inventor Share (pp)	0.920	-0.444	0.958+	-0.228
	(0.711)	(1.083)	(0.512)	(0.801)
Δ log Sales	-1.841	-1.954	-1.456	-1.674
	(1.925)	(1.988)	(1.326)	(1.279)
Knowledge Market FE		✓		✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample
Observations	157	153	157	153

Note: Unweighted regressions; robust standard errors in parentheses; symbols denote significance levels $(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001)$; checkmarks indicate the inclusion of fixed effects. This table presents the results of specifications (2), regressing the change in excess self-citations in sector p over the change in the share of inventors.

cients are mostly non-significant and turn negative when knowledge market fixed effects are included. Column (3) displays a marginally significant coefficient. However, this result is not robust to using the HHI as regressor and weighting regressions by sales as in the baseline specification. The findings in this table suggest that incremental innovation does not drive my results.

B.5 Using the Lerner Index instead of the HHI

Following Grullon et al. (2019), I build the Lerner Index from NBER-CES data for the period 1997-2012 as the ratio:

$$\text{Lerner}_{jt} = \frac{\text{vship}_{jt} - \text{pay}_{jt} - \text{matcost}_{jt} - \text{energy}_{jt}}{\text{vship}_{jt}}, \tag{15}$$

where "vship" is the total value of shipments, "pay" denotes total payrolls, "matcost" and "energy" material and energy costs, respectively, and j denotes a 6- or 4-digit NAICS sector. I build two alternative measures, one using 6-digit NAICS sectors, the original identifier in NBER-CES, and then averaging by sales at the level of 4-digit NAICS, or first aggregating the revenue and cost statistics at the level of 4-digit NAICS. Table 15, shows that the Lerner Index thus constructed is strongly correlated with the HHI measure used in the main analysis. However, the correlation is far from perfect, as suggested by the R^2, suggesting that this estimate of the Lerner Index might be excessively imprecise. Indeed, Table 16 shows that, when using this measure instead of the HHI in the main analysis, the coefficients for the regression of inventors' shares on changes in concentration stay positive, but become smaller and noisier. This suggests the potential presence of attenuation bias, a valid concern due to the fact that the above measure, not based on any structural estimation, can only imperfectly capture markups. Note that this is also due to the fact that the Lerner Index is available only for the manufacturing sectors, which make up about 60% of the sample, so its use lead to dropping a substantial amount of observations. When using fitted values from the regression in Table 15 to extend the measure to more sectors, as well as reducing the volatility of the series for available sectors, the coefficients recover magnitudes and significance close to the baseline presented in 3.

Table 15: Regressions of Changes in the Lerner Index over Changes in the HHI Lower Bound, Long-Difference, 1997-2012

	Δ Lerner Index
	(2)
$\Delta \underline{HHI}$	1.652***
	(0.257)
Observations	258
R-squared	.14

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$. "6d Lerner Index" refers to the Lerner Index constructed as in (15)on NAICS 6-digits averaged at the 4-digit NAICS level weighting by the value of shipments; "4d Lerner Index" is computed using 4-digit aggregates for the value of shipments, payroll and costs, summing over the NAICS 6-digit composing each sector.

Table 16: Regressions of Changes in Inventors' Share over Changes in Actual and Fitted Lerner Index, Long-Difference, 1997-2012

	Δ Inventor Share (pp)	
	(1)	(2)
Δ Lerner	0.556	
	(5.465)	
Δ Lerner (Fitted)		26.736*
		(13.363)
Knowledge Market FE		
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	81	157

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Observations weighted by sales. The markup change 1997-2012 is the long-difference of the Lerner Index described above. "Fitted Lerner change" is the fitted value for the Lerner index based on the estimates in 15, and extended to all available sectors in the main sample.

C Omitted Proofs and Derivations

C.1 One-sector model

Proof of Proposition 2. This proof consists of several parts. First, I show that given labor supplies, output, values and wages grow at the same constant rate, so the problem can be solved in a steady state of a normalized model. Second, I show that normalized values, $v(\Omega) \equiv V_t(\Omega)/Y_t$, are uniquely determined, which gives unique research intensities and stationary distribution. Third, I derive the stationary distribution and the expression for growth and inventors' productivity. In what follows I suppress stars to denote equilibrium quantities for ease of notation.

Given an endowment, L, production labor market clearing in each period requires:

$$\int_0^1 l_{i,t}(w)d(i) = L.$$

That is,

$$L = \int_0^1 \frac{c_{i,t}}{\phi} y_{i,t}(w_t)\, d(i) = \frac{1}{\phi}\frac{Y_t}{w_t},$$

where the second equality comes from using the demand for output of product i for

$y_{i,t}(w_t)$. This expression immediately implies that if Y_t grows at a constant rate, so does w_t. Labor market clearing for R&D workers reads:

$$L^{RD} = \zeta \omega x_{e,\omega} \left(\mu_{e,\omega} + \mu_{e,1}\right) + \alpha_I \frac{(x_I)^\gamma}{\gamma} \mu_1.$$

In a constant growth equilibrium (CGE), the distribution is stationary, and since the left hand side is constant, research intensities are also fixed. A contradiction arises otherwise, since the distribution is stationary only if research intensities are fixed by the LOM (43)-(46). Further, R&D labor cannot grow since the growth rate in the economy increases in total R&D labor for any given distribution, as it will be clear below. The fact that research intensities are constant immediately implies, from the optimality of $x_{e,\omega}$, that $V_t(1)$ and w_t^{RD} grow at the same rate. Indeed, from the FOC for entrants' research:

$$0 = \mathrm{d}\log x_{e,\omega,t} = \mathrm{d}\log V_t(1) - \mathrm{d}\log w_t^{RD}.$$

This result in turn implies, combined with the FOC for x_I, that $V_t(\omega)$ also grows at the same constant rate. Now consider the budget constraint of the representative household, combined with product market clearing, $Y_t = C_t$:

$$r_t A_t - \dot{A}_t + w_t^{RD} L^{RD} + w_t L = Y_t,$$

where A_t denote the household's assets, that is all firms in the economy. Therefore the above reads:

$$r_t \left(\mu_1 V_t(1) + \mu_\omega V_t(\omega)\right) - \mu_1 \dot{V}_t(1) - \mu_\omega \dot{V}_t(\omega) + w_t^{RD} L^{RD} + w_t L = Y_t$$

Dividing both sides by $V(1)$, using the Euler equation and rearranging we obtain:

$$\left(g + \rho\right)\left(\mu_1 + \mu_\omega \frac{V_t(\omega)}{V_t(1)}\right) - \mu_1 g_{V_1} - \mu_\omega \frac{V_t(\omega)}{V_t(1)} g_{V_1} + \frac{w_t^{RD}}{V_t(1)} L^{RD} = \frac{Y_t}{V_t(1)} - \frac{w_t}{V_t(1)} L.$$

By what shown above, all terms on the left hand side are constant in t, since research wages and values grows at the same rate and the distribution is stationary. Since Y_t and w_t grow at the same rate positive rate, it must be that $V_t(1)$ also grows at the same rate as Y_t. This proves that $g_{V_1} = g = g_c = g_w = g_{w^{RD}}$.

As a result, in a CGE, it is possible to define normalized constant values, $v(\Omega) \equiv V_t(\Omega)/Y_t$. The system of equations defining the recursive problem in this equilibrium reads:

$$\rho v(1) = \max_{x_I} \left\{ \left(\frac{\phi - 1}{\phi} \right) - \alpha_I \frac{x_I^\gamma}{\gamma} w^{RD} + x_I \left(v(\omega) - v(1) \right) - x_{e,1} v(1) \right\}, \tag{16}$$

$$\rho v(\omega) = \left(\frac{\phi - 1}{\phi} \right) + \delta \left(v(1) - v(\omega) \right) - x_{e,\omega} v(\omega), \tag{17}$$

where the left hand side comes from using the Euler equation:

$$r = g + \rho$$

Which gives

$$r \frac{V_t(\Omega)}{Y_t} - \frac{\dot{V}_t(\Omega)}{Y_t} \frac{Y_t}{\dot{Y}_t} \frac{\dot{Y}_t}{V_t(\Omega)} \frac{V_t(\Omega)}{Y_t} = \left(\rho + g \right) v(\Omega) - g v(\Omega) = \rho v(\Omega).$$

I now move to show that normalized values (16) and (17) are uniquely determined. Given entrants' decisions, and a wage rate w^{RD}, the incumbent's choice of R&D satisfies:

$$x_I = \mathbf{1} \left\{ v(\omega) - v(1) > 0 \right\} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma - 1}}.$$

Entrants taking x_I as given optimally set:

$$x_{e,1} = \mathbf{1} \left\{ v(1) > 0 \right\} \frac{v(1)}{\zeta w^{RD}}, \quad x_{e,\omega} = \mathbf{1} \left\{ v(1) > 0 \right\} \frac{v(1)}{\zeta \omega w^{RD}}.$$

Note that these solutions immediately imply that the normalized value, $v(1)$, is strictly positive. Indeed, $v(1) < 0$ would imply:

$$\rho v(1) = \pi + \mathbf{1} \left\{ v(\omega) - v(1) > 0 \right\} \left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma - 1}} \right) \left(v(\omega) - v(1) \right)$$

where the right hand side is strictly positive. Plugging optimal solutions into the system of equations determining the value functions (39) and (40) gives:

$$\rho v(1) - \pi - \mathbf{1}\{v(\omega) - v(1) > 0\} \left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma - 1}} \right) (v(\omega) - v(1)) + \frac{v(1)^2}{\zeta w^{RD}} = 0 \quad (18)$$

$$\rho v(\omega) - \pi - \delta (v(1) - v(\omega)) + \frac{v(1)}{\zeta w^{RD} \omega} v(\omega) = 0. \quad (19)$$

The second equation gives $v(\omega)$ as the following function of $v(1)$:

$$v(\omega) = \frac{\pi + \delta v(1)}{\rho + \delta + \frac{v(1)}{\zeta w^{RD} \omega}}.$$

Suppose first that $v(\omega) < v(1)$. In this case, the first equation gives:

$$\rho v(1) + \frac{v(1)^2}{\zeta w^{RD}} - \pi = 0.$$

The roots of this equation are:

$$v_{1,2} = \frac{-\rho \pm \sqrt{\rho^2 + 4 \frac{\pi}{\zeta w^{RD}}}}{\frac{2}{\zeta w^{RD}}}.$$

Since the term under the root is strictly positive, only one of these roots is admissible, so the above system is solved for a unique pair $v(1), v(\omega)$. Consider now the case $v(\omega) > v(1)$. It is straightforward to note that $v(\omega) - v(1)$ is decreasing in $v(1)$. This implies that, when rewriting (18) as

$$-\left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma - 1}} \right) (v(\omega) - v(1)) = \pi - \rho v(1) - \frac{v^2(1)}{\zeta w^{RD}}, \quad (20)$$

the left hand side is monotonically increasing in $v(1)$, while the right hand side is monotonically decreasing in $v(1)$. Further, at $v(1) = 0$, the left hand side is strictly negative, while the right hand side equals π, while for $v(1) \to \infty$, the right hand side tends to $+\infty$ while the left hand side decreases towards $-\infty$. As a result, (20) has a unique positive solution.

The uniqueness of $v(1)$ immediately implies unique $v(\omega)$ and R&D choices. Given these R&D choices, the stationary distribution satisfies

$$0 = -\left(x_I + x_{e,1}\right)\mu_1 + \delta\mu_\omega + x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}, \tag{21}$$

$$0 = -\left(x_{e,\omega} + \delta\right)\mu_\omega + x_I\mu_1, \tag{22}$$

$$0 = -\left(x_{e,1} + x_I\right)\mu_{e,1} + x_{e,1}\mu_1 + \delta\mu_{e,\omega}, \tag{23}$$

$$0 = -\left(x_{e,\omega} + \delta\right)\mu_{e,\omega} + x_{e,\omega}\mu_\omega + x_I\mu_{e,1}. \tag{24}$$

By equation (22):

$$x_I\mu_1 = \left(x_{e,\omega} + \delta\right)\mu_\omega$$

Since $\mu_1 = 1 - \mu_\omega$, the stationary distribution has:

$$\mu_\omega = \frac{x_I}{x_I + x_{e,\omega} + \delta},$$

$$\mu_1 = \frac{x_{e,\omega} + \delta}{x_I + x_{e,\omega} + \delta},$$

$$\begin{bmatrix} -\delta & x_{e,1} + x_I \\ x_{e,\omega} + \delta & -x_I \end{bmatrix} \begin{bmatrix} \mu_{e,\omega} \\ \mu_{e,1} \end{bmatrix} = \begin{bmatrix} x_{e,1}\mu_1 \\ x_{e,\omega}\mu_\omega \end{bmatrix}. \tag{25}$$

Since the matrix in (25) is nonsingular, $\mu_{e,\omega}$ and $\mu_{e,1}$ are uniquely determined as:

$$\mu_{e,\omega} = \frac{x_I x_{e,1}\mu_1 + \left(x_{e,1} + x_I\right)x_{e,\omega}\mu_\omega}{x_{e,\omega}\left(x_{e,1} + x_I\right) + \delta x_{e,1}},$$

$$\mu_{e,1} = \frac{\left(x_{e,\omega} + \delta\right)x_{e,1}\mu_1 + \delta x_{e,\omega}\mu_\omega}{x_{e,1}\left(x_{e,\omega} + \delta\right) + x_{e,\omega}x_I}$$

By the optimal solution for entrants:

$$x_{e,1} = \omega x_{e,\omega},$$

so (25) is solved for:

$$\mu_{e,\omega} = \frac{\omega x_I\mu_1 + \left(\omega x_{e,\omega} + x_I\right)\mu_\omega}{\omega\left(x_{e,\omega} + \delta\right) + x_I}, \tag{26}$$

$$\mu_{e,1} = \frac{\omega\left(x_{e,\omega} + \delta\right)\mu_1 + \delta\mu_\omega}{\omega\left(x_{e,\omega} + \delta\right) + x_I}. \tag{27}$$

Thus, the stationary distribution is unique.

It remains to show that equilibrium R&D labor is also unique. To show this, I prove that R&D labor demand is monotonically decreasing in wages and has:

$$\lim_{w^{RD}\to\infty} L^{RD}(w^{RD}) \le 0, \qquad \lim_{w^{RD}\to 0} L^{RD}(w^{RD}) = \infty.$$

Since the converse holds for R&D labor supply is monotonically increasing in wages and ranges between 0 and $+\infty$, this gives a unique intersection of the two schedules. First note that, if labor supply is inelastic, $\phi = 0$, equilibrium R&D labor is constant by definition. Lemma 4 below builds on this observation as well as 3 to prove that research labor demand is indeed monotonically decreasing in the wage.

Lemma 3. *Consider a steady state of the normalized one-sector model, and assume that defensive innovation is effective,* $\omega > 1$. *Then,* $\omega v(1) > v(\omega) > v(1)$. *Around a steady state, and for a fixed wage rate,* w^{RD}, *the normalized values,* $v(1), v(\omega)$, *are increasing in the markup,* ϕ, *and*

$$\frac{\partial v(\omega)}{\partial \phi} > \frac{\partial v(1)}{\partial \phi} > 0.$$

Proof of Lemma 3. Subtracting side by side Equation (18) from (19) gives:

$$\left(\rho + \delta + \mathbf{1}\left\{v(\omega) - v(1) > 0\right\}\left(\frac{\gamma - 1}{\gamma}\left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}}\right)\right)(v(\omega) - v(1)) = \frac{v(1)}{\zeta w^{RD}}\left(v(1) - \frac{v(\omega)}{\omega}\right)$$

Suppose that $v(\omega) < v(1)$. This implies that the left hand side of the above expression is strictly smaller than 0, while $\omega v(1) > v(1) > v(\omega)$, so the right hand side is strictly positive under the assumption $\omega > 1$. Therefore, it must be that $v(\omega) > v(1)$. If this is the case, the left hand side is strictly positive, and to avoid a contradiction it must be $\omega v(1) > v(\omega)$. Thus, $\omega v(1) > v(\omega) > v(1)$, proving the first part of the statement.

Since π is a monotone increasing function of ϕ, I prove the statement for value derivatives with respect to π. Total differentiation of the system of Equations (18) and (19) with respect to π around a CGE gives

$$\underbrace{\begin{bmatrix} \rho + \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} + 2\frac{v(1)}{\zeta} & -\left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} \\ -\delta + \frac{v(\omega)}{\zeta w^{RD}\omega} & \rho + \delta + \frac{v(1)}{\zeta w^{RD}\omega} \end{bmatrix}}_{\equiv J} \begin{bmatrix} \mathrm{d}v(1) \\ \mathrm{d}v(\omega) \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathrm{d}\pi = 0. \tag{28}$$

The determinant of the Jacobian is:

$$\det J = \left(\rho + x_I + 2\omega x_{e,\omega}\right)\left(\rho + \delta + \frac{v(1)}{\zeta\omega}\right) + x_I\left(x_{e,\omega} - \delta\right) > 0.$$

Solving (28) gives:

$$\begin{bmatrix} \frac{dv(1)}{d\pi} \\ \frac{dv(\omega)}{d\pi} \end{bmatrix} = \frac{1}{\det J}\begin{bmatrix} \frac{v(1)}{\zeta w^{RD}\omega} + \rho + \delta & \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} \\ \delta - \frac{v(\omega)}{\zeta w^{RD}\omega} & \rho + \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} + 2\frac{v(1)}{\zeta w^{RD}} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Since the first row is strictly positive,

$$\frac{dv(1)}{d\pi} > 0.$$

Subtracting line by line gives:

$$\begin{aligned} \frac{dv(\omega)}{d\pi} - \frac{dv(1)}{d\pi} &= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta w^{RD}\omega} - \rho + \frac{v(1)}{\zeta w^{RD}\omega} + \rho + 2\frac{v(1)}{\zeta w^{RD}}\right] \\ &= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta w^{RD}\omega} - \frac{v(1)}{\zeta w^{RD}\omega} + 2\frac{v(1)}{\zeta w^{RD}}\right] \\ &= \frac{1}{\det J}\left[\frac{2\omega v(1) - (v(\omega) + v(1))}{\zeta w^{RD}\omega}\right] > 0 \end{aligned} \tag{29}$$

since $\omega > 1$ and $\omega v(1) > v(\omega)$, from what shown above. It follows that:

$$\frac{dv(\omega)}{d\pi} > \frac{dv(1)}{d\pi} > 0.$$

□

Lemma 4. *R&D labor demand is monotonically decreasing in the wage rate w_t^{RD}/Y_t,and:*

$$\lim_{w^{RD}\to\infty} L^{RD}(w^{RD}) \le 0, \qquad \lim_{w^{RD}\to 0} L^{RD}(w^{RD}) = \infty.$$

Proof. Consider the equilibrium with inelastic R&D labor. By the resource constraint in the economy, it holds:

$$\rho\left(\mu_1 v(1) + \mu_\omega v(\omega)\right) + w^{RD}L^{RD} + wL = 1,$$
$$L^{RD} = \frac{\pi}{w^{RD}} - \rho\left(\mu_1\frac{v(1)}{w^{RD}} + \mu_\omega\frac{v(\omega)}{w^{RD}}\right).$$

Since the labor supply is fixed, shifts in the right hand side of this equation identify the elasticity of labor supply to various parameters. Now consider an increase in π to $\pi' > \pi$. In this case, the unique equilibrium requires:

$$\frac{\pi'}{w^{RD\prime}} = \frac{\pi}{w^{RD}}.$$

Indeed, guess that the equilibrium involves no changes in research intensities, and therefore in the stationary distribution. Then:

$$x'_{e,\omega} = x_{e,\omega} \Rightarrow \frac{v'(1)}{\zeta\omega w'^{RD}} = \frac{v(1)}{\zeta\omega w^{RD}},$$

and

$$x'_I = \left(\frac{v'(1) - v'(\omega)}{\alpha_I w'^{RD}}\right)^{\frac{1}{\gamma-1}} = \left(\frac{v(1) - v(\omega)}{\alpha_I w^{RD}}\right)^{\frac{1}{1-\gamma}} = x_I.$$

As a result:

$$\frac{v'(\omega)}{w'^{RD}} = \frac{v(\omega)}{w^{RD}}.$$

Using the expression for $v(\omega)$, and using the fact that the ratio between values and wages is the same in both equilibria, gives:

$$\frac{\pi'}{w'^{RD}} = \frac{\pi}{w^{RD}}.$$

This also ensures that:

$$\rho\frac{v(1)}{w^{RD}} = \rho\frac{v'(1)}{w'^{RD}},$$

as is easily verified plugging the above expression into (16) evaluated at $\left(v(1), w^{RD}\right)$ and $\left(v'(1), w^{RD\prime}\right)$.It remains to show that goods' market clearing holds. Before a markup change we have (in normalized values):

$$\rho\left(\mu_1 v(1) + \mu_\omega v(\omega)\right) + w^{RD} L^{RD} + wL = 1,$$
$$\rho\left(\mu_1 \frac{v(1)}{w^{RD}} + \mu_\omega \frac{v(\omega)}{w^{RD}}\right) + L^{RD} = \frac{1 - wL}{w^{RD}},$$

By what shown above, with an inelastic labor research labor supply, the left hand side has the same value before and after the change in instantaneous profits. Further, the linear production function implies that:

$$wL = \frac{1}{\phi},$$

therefore the right hand side can be written as:

$$\frac{\pi}{w^{RD}},$$

which has the same value in the new equilibrium. Therefore, the unique equilibrium with inelastic labor supply is characterized by a constant ratio $\frac{\pi}{w^{RD}}$. Given that the labor supply is inelastic, L^{RD} in the above expression can be read as the labor demand for R&D:[25]

$$L^{RD,d}\left(w^{RD}\right) = \frac{\pi}{w^{RD}} - \rho\left(\mu_1 \frac{v(1)}{w^{RD}} + \mu_\omega \frac{v(\omega)}{w^{RD}}\right)$$

Now consider an initial equilibrium with $L^{RD,d}(w^{RD}) = L^d$. A change in the wage w^{RD} to $w^{RD\prime} > w^{RD}$ modifies the above expression to:

$$L^{RD,d}\left(w^{RD\prime}\right) = \frac{\pi}{w^{RD\prime}} - \rho\left(\mu_1' \frac{v'(1)}{w^{RD\prime}} + \mu_\omega' \frac{v'(\omega)}{w^{RD\prime}}\right).$$

By what shown above, it must be:

$$\frac{\mathrm{d}\pi}{\pi} = \frac{w^{RD\prime} - w^{RD}}{w^{RD}} > 0$$

[25] Alternatively, the market clearing expression can be rewritten as the accounting identity that instantaneous profits equal the R&D wage bill plus dividends, which gives the demand for R&D labor as the expression reported below.

for $L^{RD,d}$ to be unchanged. Thus, denoting:

$$\pi' = \pi \left(1 + \frac{w^{RD\prime} - w^{RD}}{w^{RD}}\right),$$

the above expression reads:

$$L^{RD,d}\left(w^{RD\prime}\right) = \frac{\pi'}{w^{RD\prime}} + \frac{\pi - \pi'}{w^{RD\prime}} - \rho\left(\mu_1' \frac{v'(1)}{w^{RD\prime}} + \mu_\omega' \frac{v'(\omega)}{w^{RD\prime}}\right).$$

That is:

$$L^{RD,d}\left(w^{RD\prime}\right) = L^{RD,d}\left(w^{RD}\right) + \frac{\pi - \pi'}{w^{RD\prime}} < L^{RD,d}\left(w^{RD}\right).$$

This shows that labor demand is decreasing in the wage. In general, we have:

$$L^{RD,d}\left(w^{RD\prime}\right) = L^{RD,d}\left(w^{RD}\right) + \frac{1}{w^{RD}}\left(\frac{w^{RD}}{w^{RD\prime}} - 1\right)$$

Consider now $w^{RD\prime} \to 0$, in this case we clearly have:

$$L^{RD,d}\left(w^{RD\prime}\right) \to \infty.$$

Conversely, with $w^{RD\prime} \to \infty$:

$$L^{RD,d}\left(w^{RD\prime}\right) \to L^{RD,d}\left(w^{RD}\right) - \frac{1}{w^{RD}} = -\rho\left(\mu_1 \frac{v(1)}{w^{RD}} + \mu_\omega \frac{v(\omega)}{w^{RD}}\right) - \frac{wL}{w^{RD}} < 0.$$

□

By Lemma 4, given an endowment of production labor and an R&D labor supply schedule, the CGE is unique.

To derive the growth rate note that, by the Cobb Douglas assumption on the final good, and given the equilibrium wage rate for production workers, $w = \frac{w_t}{Y_t}$,

$$\begin{aligned}
\log Y_t &= \int_0^1 \log y_t(i)\mathrm{d}i \\
&= \int_0^1 \log\left(\frac{Y_t}{w_t c_t(i)}\right)\mathrm{d}i \\
&= \int_0^1 \log\left(\frac{1}{w c_t(i)}\right)\mathrm{d}i.
\end{aligned}$$

It follows that:

$$g = \log(Y_{t+\Delta t}) - \log(Y_t) = -\int_0^1 \left(\log c_{t+\Delta}(i) - c_t(i)\right) \mathrm{d}i$$
$$= \eta \left[x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1} + \lambda x_I \mu_1\right]$$
$$= \eta \left[x_{e,\omega}\left(\mu_{e,\omega} + \omega\mu_{e,1}\right) + \lambda x_I \mu_1\right].$$

Productivity g/L^{RD} follows directly from total R&D labor demand:

$$\zeta\omega x_{e,\omega}\left(\mu_{e,\omega} + \mu_{e,1}\right) + \alpha_I \frac{(x_I)^\gamma}{\gamma}\mu_1.$$

□

Proof of Proposition 2. The increase in R&D efforts by both incumbents and entrants descend directly from Lemma 3. In what follows, I derive *equilibrium* quantities, that is factoring in wage effects, but I drop stars for ease of notation.

To prove that the share of R&D labor accruing to incumbents increases, note first:

$$\frac{\partial L_I}{\partial \phi} = \alpha_I x_I^{\gamma-1}\mu_1 \frac{\partial x_I}{\partial \phi} + \frac{\alpha_I}{\gamma} x_I^{\gamma-1} \frac{\partial\left(x_I\mu_1\right)}{\partial \phi},$$

where the first term is strictly positive, since I have proven that $\frac{\partial x_I}{\partial \phi} > 0$, and the term, $\frac{\partial(x_I\mu_1)}{\partial \phi}$,denotes the derivative of aggregate incumbents' research intensity with respect to the markup, and is also strictly positive. Indeed:

$$\frac{\partial \mu_1}{\partial \phi} = \frac{\partial\left(\frac{x_{e,\omega}+\delta}{x_{e,\omega}+\delta+x_I}\right)}{\partial \phi} = \left[\frac{\frac{\partial(x_{e,\omega}+\delta)}{\partial \phi} x_I - \left(x_{e,\omega}+\delta\right)\frac{\partial x_I}{\partial \phi}}{\left(x_I + x_{e,\omega} + \delta\right)^2}\right] = \mu_1 \frac{\partial x_I}{\partial \phi} \frac{(\epsilon - 1)}{\left(x_I + x_{e,\omega} + \delta\right)}, \tag{30}$$

where I define the ratio of the elasticity of $x_{e,\omega} + \delta$ and x_I to ϕ as:

$$\epsilon \equiv \frac{\epsilon_e}{\epsilon_I} \equiv \frac{\frac{\partial(x_{e,\omega}+\delta)}{\partial \phi}/x_{e,\omega}}{\frac{\partial x_I}{\partial \phi}/x_I} \in (0,1].$$

therefore:

$$\frac{\partial\left(\mu_1 x_I\right)}{\partial\phi} = \mu_1 \frac{\partial x_I}{\partial\phi}\left[\frac{x_I\left(\epsilon-1\right)}{\left(x_I+x_{e,\omega}+\delta\right)}+1\right]$$
$$= \mu_1 \frac{\partial x_I}{\partial\phi}\left[\frac{x_I\epsilon+x_{e,\omega}+\delta}{\left(x_I+x_{e,\omega}+\delta\right)}\right] > 0,$$

that is, aggregate incumbents' research intensity, $x_I\mu_1$, is increasing in the markup.

□

C.2 Full Description of the Two-Sector Model and Derivations

By the above assumptions, the final good is produced according to:

$$Y = \prod Y_i^{\beta_i}. \tag{31}$$

With the final good as numeraire, the sector's demand schedule is:

$$Y_i = \beta_i \frac{Y}{P_i}. \tag{32}$$

From CD on intermediate goods we also have:

$$P_i Y_i = p_{is} y_{is}, \quad \forall s.$$

In each sector, the price is set at the competitive fringe's marginal cost wc_i, and is identical across subsectors . Thus

$$P_i = p_{is} = wc_i,\ Y_i = \beta_i \frac{Y}{wc_i}. \tag{33}$$

Equilibrium profits are given by:

$$\Pi_i = \left(c_i w - \frac{c_i w}{\phi_i}\right) Y_i = \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y.$$

The monopolist demands production labor:

$$\ell_{is} = \frac{c_i y_{is}}{\phi_i}, \Rightarrow L_i = \int \ell_{is}\mathrm{d}s = Y\frac{\beta_i}{\phi_i w}. \tag{34}$$

Assuming a rigid production labor supply:[26]

$$L^s(w) = L = \frac{Y}{w}\left(\sum \frac{\beta_i}{\phi_i}\right). \tag{35}$$

Which gives:

$$L_i = L\frac{\frac{\beta_i}{\phi_i}}{\sum \frac{\beta_i}{\phi_i}}, Y_i = L\frac{\frac{\beta_i}{c_i}}{\sum \frac{\beta_i}{\phi_i}}. \tag{36}$$

Which gives:

$$Y = L\prod_i \left(\frac{\frac{\beta_i}{c_i}}{\sum \frac{\beta_i}{\phi_i}}\right)^{\beta_i}. \tag{37}$$

Thus, growth is:

$$-\sum \beta_i \Delta \log c_i. \tag{38}$$

Normalized values in each sector are the same as before, with the only difference that they receive a wage w^R, and the above α_I, ζ are replaced by $\zeta w^R, \alpha_I w^R$.

C.2.1 Research Equilibrium in the two-sector model

By the above solutions, the monopolist's values read:

$$\rho V_i(1) = \max_{x_I}\left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y - \alpha_I W^{RD}\frac{x_I^2}{2} + x_I\left(V_i(\omega) - V_i(1)\right) - x_{e,1}V_i(1),$$
$$\rho V_i(\omega_i) = \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y + \delta\left(V(1) - V(\omega)\right) - x_{e,\omega}V_i(\omega).$$

[26]Consider a labor supply with elasticity φ. This gives:

$$\chi w^{\varphi} = \frac{Y}{w}\left(\sum \frac{\beta_i}{\phi_i c_i}\right) \Rightarrow w = \left[\frac{Y}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{\frac{1}{1+\varphi}}$$

Equilibrium labor is then:

$$L^\star = \chi\left[\frac{Y}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{\frac{\varphi}{1+\varphi}}, \frac{Y}{w} = Y^{\frac{\varphi}{1+\varphi}}\left[\frac{1}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{-\frac{1}{1+\varphi}} = L^\star\left(\sum \frac{\beta_i}{\phi_i c_i}\right)^{-1}$$

Which results in the same allocations and outputs as below, with $L^\star$ in place of the fixed L.

And normalized values, $v \equiv V/Y$:

$$\rho v_i(1) = \max_{x_I}\left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i - \alpha_I w^{RD}\frac{x_I^2}{2} + x_I\left(v_i(\omega) - v_i(1)\right) - x_{e,1} v_i(1) \tag{39}$$

$$\rho v_i(\omega_i) = \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i + \delta\left(v(1) - v(\omega)\right) - x_{e,\omega} v_i(\omega), \tag{40}$$

where w^{RD} is the normalized researchers' wage.

Given a normalized wage, each sector demands:

$$x_{e,\omega,i}\left(w^{RD}\right) = \frac{v_i(1)}{w^{RD}\omega\zeta_i}, \tag{41}$$

$$x_{I,i}\left(w^{RD}\right) = \frac{\left(v_i(\omega_i) - v_i(1)\right)}{w^{RD}\alpha_{I,i}}. \tag{42}$$

The stationary distribution within each sector is given by:

$$\mu_{\omega,i}\left(w^{RD}\right) = \frac{x_{I,i}\left(w^{RD}\right)}{x_{e,\omega,i}\left(w^{RD}\right) + \delta_i + x_{I,i}\left(w^{RD}\right)}, \tag{43}$$

$$\mu_{1,i}\left(w^{RD}\right) = \frac{x_{e,\omega,i}\left(w^{RD}\right) + \delta_i}{x_{e,\omega,i}\left(w^{RD}\right) + \delta_i + x_{I,i}\left(w^{RD}\right)}, \tag{44}$$

$$\mu_{e,1,i}\left(w^{RD}\right) = \frac{\omega_i\left(x_{e,\omega,i}\left(w^{RD}\right) + \delta\right)\mu_{1,i} + \delta_i\mu_{\omega,i}}{\left(x_{I,i} + \omega_i\left(x_{e,\omega,i}\left(w^{RD}\right) + \delta_i\right)\right)}, \tag{45}$$

$$\mu_{e,\omega,i}\left(w^{RD}\right) = \frac{\omega_i\mu_{1,i}x_{I,i}\left(w^{RD}\right) - \omega_i\delta_i\mu_{\omega,i}}{\left(x_{I,i} + \omega_i\left(x_{e,\omega,i}\left(w^{RD}\right) + \delta_i\right)\right)} + \mu_{\omega,i}. \tag{46}$$

Sector RD labor demand is given by:

$$L_i^{RD,d}\left(w^{RD}\right) = \mu_{e,\omega,i}\left(w^{RD}\right)\left(\zeta_i\omega_i x_{e,\omega,i}\left(w^{RD}\right)\right) + \mu_{1,e,i}\left(w^{RD}\right)\zeta_i x_{e,1,i}\left(w^{RD}\right) + \mu_{1,i}\left(w^{RD}\right)\alpha_I\frac{x_{I,i}^2\left(w^{RD}\right)}{2}.$$

With an inelastic labor supply fixed to L^{RD}, market clearing for inventors then reads:

$$L^{RD} = \sum_i\left\{\mu_{\omega,i}\left(w^{RD}\right)\left(\zeta_i\omega_i x_{e,\omega,i}\left(w^{RD}\right)\right) + \mu_{1,e,i}\left(w^{RD}\right)\zeta_i x_{e,1,i}\left(w^{RD}\right) + \mu_{1,i}\left(w^{RD}\right)\alpha_I\frac{x_{I,i}^2\left(w^{RD}\right)}{2}\right\}. \tag{47}$$