14.02 – Fall 2018
Recitation 6

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Announcements

- Problem set posted on Black Friday, due following **Friday 11/30**.
- It will be **long** so plan to start it before Thursday night.

The Solow Model
Equations to know
Application to Problem set 7.2

The "meat" of the Solow model

- The main message of the Solow model is that capital accumulation on its own cannot sustain economic growth in the long run;
- It show this by highlighting that the aggregate production function has *decreasing returns on capital*;
- The models adding population and technology growth just change the definition for capital to reach the same result. That is:
  - Accumulation of capital *per capita* cannot sustain growth;
  - Accumulation of capital *per effective worker* cannot sustain growth
- **Bottom line:** something else must drive the growth in GDP per capita . . . Technology!
- By-products of the framework: Golden rule savings rate that maximizes consumption, a lot of comparative statics/dynamics...

Capital accumulation

In order to reach the conclusion of the model, need to focus on capital accumulation. We always start from the following equation:

$$K_{t+1} = (1-\delta)K_t + sF(K_t, X_t)$$

where X_t is a measure of the effective labor in production. In class we saw:

1. $X_t = N$ for all t: constant population Solow model;
2. $X_t = N_t$ with $N_{t+1} = (1+n)N_t$: constant population growth Solow model;
3. $X_t = A_t N_t$, with both A_t, N_t growing.

In all models, define a measure of capital per effective worker and show that it will converge to a steady state.
Note: *standardized* capital K_t/X_t converges but capital keeps growing at the rate X_t grows.

The convergence to the steady state

Why does standardized capital converge? Because the production function has decreasing returns to capital, while depreciation is fixed. As capital increases:

- We increase depreciation by δ;
- We increase production by a lesser and lesser amount, ultimately smaller than δ (and 0 as $K_t \to \infty$);
- At some point, not enough resources produced in the economy to replenish capital, we cannot support it in the log run.

The steady state of an appropriately defined measure of capital is precisely the amount of capital that a society can sustain.

Note: as the economy is growing, steady state is a proper terminology only for standardized variables (that do not grow in the long run). Non-standardized variables are instead said to be on a *balanced growth path*.

The convergence to the steady state in formulas

Let us look at the more general model where both A_t, N_t grow. Then, our measure of standardized capital is $k_t = \frac{K_t}{A_t N_t}$. Whereby:

$$K_t = A_t N_t k_t$$

So using capital accumulation:

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = k_{t+1} = (1-\delta)k_t \frac{A_t N_t}{A_{t+1}N_{t+1}} + sf(k_t)\frac{A_t N_t}{A_{t+1}N_{t+1}}$$

Where we use the fact that $F(K_t, A_t N_t)$ is CRS and define $f(k_t) \equiv F(k_t, 1)$. Replacing the growth rates:

$$k_{t+1} = (1-\delta)k_t \frac{1}{(1+g)(1+n)} + sf(k_t)\frac{1}{(1+g)(1+n)}$$

The convergence to the steady state in formulas II

We had obtained:

$$k_{t+1} = (1-\delta)k_t \frac{1}{(1+g)(1+n)} + sf(k_t)\frac{1}{(1+g)(1+n)}$$

Again $f()$ has decreasing returns in k_t so the same logic as with non-standardized capital applies. There will be a steady state, find it setting $k_t = \bar{k}$. As g, n are small, replace $(1+g)(1+n)$ with its approximation $1+g+n$. Thus:

$$\bar{k}\frac{\delta+g+n}{1+g+n} = sf(\bar{k})\frac{1}{(1+g)(1+n)}$$

Now let: $g(\bar{k}) \equiv \bar{k}/f(\bar{k})$. Then:

$$\bar{k} = g^{-1}\left(\frac{s}{\delta+g+n}\right)$$

Application to PS 7: $g = n = 0$

the economy's production function is given by:

$$Y = AK^{\alpha}N^{1-\alpha}$$

Assume that $A = 1$, and $\alpha = \frac{1}{3}$. Population N is constant.

1. Is this production function characterized by constant returns to scale?
2. Are there decreasing returns to capital?
3. Give an expression for capital per worker in the steady state.
4. Give an expression for output per worker.

The Golden Rule for savings

Defined as the savings rate that maximizes consumption per capita in the steady state of the standardized economy. Give that savings are exogenous we always have that consumption per capita is:

$$c = (1 - s)y$$

In the simple case where $F = \sqrt{K}\sqrt{AN}$ we get:

$$c = (1 - s)\frac{s}{\delta + g + n}$$

This is maximized for $s = 0.5$. Important to keep in mind:

- c increases with s if the initial savings rate is below the golden rule;
- c decreases with s if the initial savings rate is at/above the golden rule;

What rate are things growing at in SS?

Simple but helpful trick: take $d\log$'s. This uses the approximation that if a variable X_t is growing at g_x then

$$d\log(X_t) = \log(X_t) - \log(X_{t-1}) = \log(1+g_x) \approx g_x$$

if g_x is small. Then can use properties of the logarithms to unpack terms. Important example:

$$d\log(y_t) = d\log\left(\frac{Y_t}{A_t N_t}\right) = d\log(Y_t) - d\log(A_t) - d\log(N_t)$$

So we have:

$$d\log(y_t) = g_Y - (g+n)$$

since $y_t = f(k_t)$ is fixed, then we immediately see that $g_Y = g + n$. If we want the growth in output per capita, just unpack log differently (see blackboard).

What rate are things growing at outside SS?

In this case, we no longer have that $d\log(y_t) = 0$, since the capital per effective worker is increasing (if we are below the steady state). In this case:

$$d\log(y_t) = d\log f(k_t)$$

If e.g. $f(k_t) = k_t^\alpha$, then we have:

$$d\log(y_t) = \alpha g_K$$

So, in this case generally

$$g_Y = g + n + \alpha g_K$$

Note:

1. $g_K = 0$ at steady state (and therefore in the long-run of the economy);
2. $g_K > 0$ if $k_t < \bar{k}$. Very useful for comparative statics!

Final Bottom lines

We have obtained that if we have power CRS production function:

$$g_Y = g + n + \alpha g_K$$

And decreasing returns in the long-run will force $g_K = 0$. It follows that:

- Factors affecting the level of $\bar{k}$ (e.g. changes in s, δ) will *never affect growth in the long run*...
- ...but they *can* affect growth in the *short run* (e.g. if δ increases and we are suddenly above S.S.). See PS 7.2
- However the effect on growth, they will always affect **levels** ($\bar{y} = f(\bar{k})$).