14.03/14.003 Recitation 10
Final Review

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Agenda

- Externalities
- Expected Utility and Insurance
- Signaling Games

Externalities: Basic Concepts

- Individual actions create an effect to agents that *external* to the decision;
- *Positive* if the effect increases others' utility, *negative* otherwise;
- Policy criterion: we use additive welfare, sum of all agents' utilities;
- Policy issue: if agents decide *in isolation*, the supply of the good in question is sub-optimal:
  - Negative externality: excess supply. Example: pollution; an individual firm might pollute so much that the individual gains in terms of profits are outweighed by *collective losses*;
  - Positive externality: insufficient supply. Example: R&D; an individual firms' research can benefit other firms. *Collective benefits* are larger than private in isolation.
- Remedies:
  - Coase theorem: define property rights, but need *costless bargaining*;
  - Impose quotas: always feasible, but need a lot of info;
  - Pigouvian taxation: tax negative ext. goods, subsidize positive ext. goods.

Pollution example (negative externality)

A firm produces x polluting goods, maximizing profits:

$-\tau^x \cdot x$

$$\Pi(x) = x - b \cdot x^2 \Rightarrow x^* = \frac{1}{2b}$$

$(1-\tau^x) - 2bx = 0$

$$\boxed{x^* = \frac{(1-\tau^x)}{2b}}$$

$b, c > 0$

However, for the community there is the following loss from pollution:

$$C(x) = -c \cdot x^2.$$

$x^* = x^{pl}$

Social (utilitarian/additive) welfare, maximized by social planner:

$$W(x) = \Pi(x) + C(x) = x - b \cdot x^2 - c \cdot x^2. \Rightarrow \boxed{x^{pl} = \frac{1}{2(b+c)}}.$$

$\frac{1-\tau^x}{2b} = \frac{1}{2(b+c)}$

$\tau^x = \tau^x = 1 - \frac{2b}{2(b+c)}$

$\Rightarrow \boxed{\tau^x = \frac{2c}{2(b+c)}} = cx^*$

There is over-supply of polluting goods since $x^{pl} < x^*$.

Solving Externalities

The aim is to make the firm *internalize* the negative effects. Three ways to go about it:

1. Coase theorem: give property rights to the community, firm has to pay p^x to the community in order to produce each unit of x. If actually no bargaining costs (one consumer) community sets:

$$\underbrace{U(x) = C(x) + p^x x = 0 \rightarrow p^x = cx.}$$

2. Impose a cap: the planner directly sets cap $\underline{\bar{x} = x^{\text{pl}}}$; $x^* = \bar{x}$

3. Pigouvian tax: impose a tax τ^x per unit of x such that $\Pi(x) - \tau^x \cdot x = W(x)$. Solution: $\tau^x = cx$ (like in Coase case).

$cx^* = \frac{2c}{2(b+c)} = \tau^x$

General Steps:

- Set up individual problem (like $\Pi(x)$ in example) and optimal quantities x^*;
- Set up the planner problem. Write down *social* welfare, $W(x)$ as sum of individual utilities of *entire society*, solve for quantity that maximizes *social welfare*. This is the planner's solution x^{pl};
- Find *policy instrument* to make $x^* = x^{\text{pl}}$. With this instrument, the individual function of whoever is choosing x must coincide with social welfare $W(x)$.

Expected Utility: Basic Concepts

- Agents now evaluate different risky scenarios, choosing *lotteries* instead of goods;
- Recall: lotteries L are a set of *probabilities* with associated payoffs, e.g.:

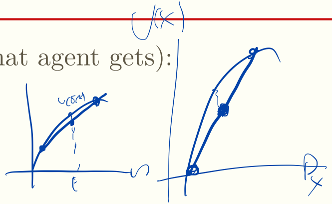
$$L = \begin{cases} x_1 \text{ w.p. } p_1 \\ x_2 \text{ w.p. } p_2 \end{cases} .$$

- VnM theorem tells us that—subject to assumptions—agents evaluate lotteries using the *expected utility of outcome x*:

$$U(L) = \sum_{j=1}^{n} p_j u(x_j) \equiv E[u(x)].$$

Insurance

Define the *expected outcome* as the expectation of x (average x that agent gets):



$$E(x) = \sum_{j=1}^{n} p_j x_j.$$

This yields utility $\boxed{u(E(x))}$. We can then define the *certainty equivalent* corresponding to the lottery, as $CE(L)$ such that:

$$u(CE(L)) = E[u(x)].$$

This gives how much the agent is *willing to pay* in order to avoid risk and be indifferent to the situation with risk. Three cases with associated convexities (comes from Jensen's inequality):

- $CE(L) < E[x]$: risk-averse IFF $E[u(x)] < u(E[x])$ IFF $u(x)$ is concave; AVOID RISK
- $CE(L) = E[x]$: risk-neutral IFF $E[u(x)] = u(E[x])$ IFF $u(x)$ is linear;
- $CE(L) > E[x]$: risk-loving IFF $E[u(x)] > u(E[x])$ IFF $u(x)$ is convex.

Risk-averse individuals will want to pay (reduce their expected wealth) to avoid risk, generating *demand for insurance*.

How much insurance?

General form: agents pay a premium p per unit of money received in a certain state and a fixed fee F to buy into insurance. Both are paid *regardless of the state that occurs.* Suppose there are two states, good, and bad (where insurance pays). Utility without insurance (w.p of good event π^G):

$$E(u(x)) = \pi^G u(x^G) + (1-\pi^G)u(x^B).$$

G: GOOD

B: BAD

$$\mathbb{1}(I>0) = \begin{cases} 1 & \text{if } I>0 \\ 0 & \text{o.w.} \end{cases}$$

Utility with insurance I:

$$E(u(x)) = \max_{I\geq 0} \pi^G u\left[x^G - p\cdot I - F\cdot \mathbf{1}(I>0)\right] + (1-\pi^G)u\left[x^B - p\cdot I + I - F\cdot \mathbf{1}(I>0)\right].$$

Split into two parts:

1. Assuming I take positive insurance and pay F, what I would I set? Maximize expected utility in I and find solution I^*.
2. If I found the $I^*>0$, is the expected utility with insurance and fee F at least as large as the utility without? If yes, I will purchase insurance $I^*>0$, otherwise no insurance.

Example

$$x = \begin{cases} 1 \text{ w.p. } p \\ 2 \text{ w.p. } (1-p) \end{cases}.$$

p premium

Insurance available at cost $c = .75$ per unit insured and fee F, utility is $u(x) = \sqrt{(x)}$. Suppose I buy $x^* > 0$

$$E[u(x)] = \max_{x>0} p\sqrt{(1+(1-c)x-F)} + (1-p)\sqrt{(2-cx-F)}.$$

Suppose insurance is *actuarially fair*, that is insurance company does not make profits in expectation:

$$p(1-c) = (1-p)c \Rightarrow \frac{c}{1-c} = \frac{p}{1-p} \Rightarrow c = p.$$

$$-\pi(1-p)x + p(1-\pi)x = 0$$

Then, FOC:

$$p(1-p)\sqrt{(1+(1-p)x-F)} = p(1-p)\sqrt{(2-px-F)} \Rightarrow 1+(1-p)x-F = 2-px-F$$

Which implies $x^* = 1$, **full insurance**.

Example cont.

We found:

$$x^* = 1.$$

Regardless of F. Now we have to figure out whether the agent will purchase it. Note that, with full insurance, the agent now gets a fixed wealth $2-p-F$ regardless of the state! Purchase will then occur if and only if:

$$2-p-F \geq CE \text{ s.t. } u(CE) = E[u(x)].$$

Find CE:

$$\sqrt{CE} = p\sqrt{1} + (1-p)\sqrt{2} \Rightarrow CE = \left(p + (1-p)\sqrt{2}\right)^2$$

So we obtain:

$$F \leq 2-p-CE = 2-p-\left(p+(1-p)\sqrt{2}\right)^2$$

In words: the agent is willing to pay all the expected wealth in excess of the certainty equivalent!

Insurance Recap

- If insurance is *actuarially fair* (no profits from *premium part* of the insurance), a risk averse agent will buy **full insurance**, conditional on fee not being too high.
- If full insurance is purchased, wealth is fixed at some level x^{ins}.
- Insurance can charge a fee $F = x^{\text{ins}} - CE$.
- Intuition: by definition, the agent is indifference between the lottery *without insurance* and receiving CE w.p. 1.
- Thus, the insurance can capture all the surplus that the agent is getting from the insurance contract, by giving the agent their certainty equivalent on average.

Games: Nash Equilibrium

Basic concept of equilibrium: we are in an equilibrium if no agent would be better off changing their action, given what the other agent does. Simple example:

Agent 1/Agent 2	Action 1	Action 2
Action 1	0,0	1,-1
Action 2	-1,1	1, 1

(Agent 1, Agent 2)

Choose an action without knowing what the other agent does, so I have to define a policy *conditional on each of the actions*. Suppose I am agent 1.

- If Agent 2 chooses Action 1, I would want to choose Action 1;
- If Agent 2 chooses Action 2, I would want to choose Action 1 again.

In light of my choices, what will agent 2 do? She knows that:

- If she chooses Action 1, she gets 0;
- If she chooses Action 2, she gets -1.

Thus, what is an equilibrium? It is the pair (Action 1, Action 1). Why? Nobody wants to deviate from their policies.

Signaling Games

- Same concept of equilibrium: everybody is choosing their *optimal response* to what everybody else is doing (no-deviation principle);
- But actions are now *a signal acquisition* for the *sender* and a *screening rule* for the receiver;
- Education example: *senders* are the types, who can acquire the costly education signal; *receivers* are the employers who have to choose a *wage rule* to maximize their profits given the signal they observe.
- Equilibrium in education:
  1. amount of education is consistent *with the screening policy of the employer*: no agent wants a different amount of education than what they are acquiring;
  2. policy of the employer is consistent with *the signals acquired by the market*: if employer is giving a different wage according to the signal, she does not want to switch to a flat wage, and vice versa.

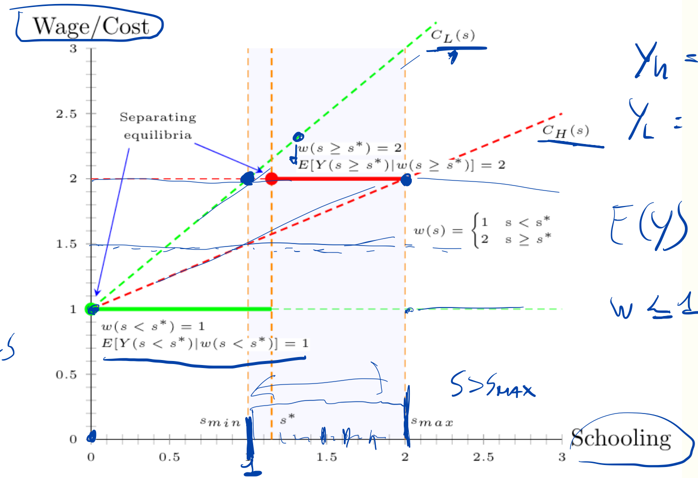
Equilibrium in Signaling Games

- Two *types* of equilibria:
  1. Pooling: everyone does the same thing;
  2. Separating: agents choose signals that *separate* them into groups, so at least one agent does something differently from the others.
- Pooling in education: employer must not attach value to the education that workers would find convenient to attain (we will discuss this below). Simplest case:
  1. Employer offers wage = average productivity in population. Outside option is getting 0 profits;
  2. Nobody acquires education because it is costly and not valued;
  3. Given that nobody acquires 0 education, the employer will hire random workers and on average make
  $$\text{profits} = \text{average productivity} - \text{wage} = 0.$$
  4. This makes sense, the employer makes profits equal to the outside option. Since nobody acquires education, she does not want to offer a wage that depends on education.

Separating: all you need is this graph

Recall: λ is the share of agents with productivity $\boxed{Y=2,}$ others have $Y=1$.

Figure 1 : Potential Separating Equilibria with $\lambda = 0.5$



$Y_H = 2$

$Y_L = 1$

$E(Y) = 1/2$

$w \leq 1.5$ for all s

$C_L(0) = 1$

$C_L(s) = 1+s$

$= s$

$S > S_{MAX}$

Market for lemons

Idea is the same as before, however now the signal is the **price** and it is *costless to acquire*, so the only equilibrium is a **pooling equilibrium**:

- Buyer offer the same price to all sellers;
- Sellers charge the same price.

Equilibrium has to satisfy two conditions:

1. Demand equals supply at equilibrium price;
2. Price is such that consumer gets the average value of good supplied and sellers actually want to supply goods with that average quality.

Separating alternative: Full disclosure, i.e. everyone reveals their quality. Only if:

- the signal is credible;
- the signal is costless.

If either fails, you will have at least some extent of pooling.

P $\quad Q_L(P)$ $\quad Q_H(P)$

$$\Rightarrow \bar{V} = \frac{Q_L(P)}{Q_L(P)+Q_H(P)} \cdot V_L + \frac{Q_H(P)}{Q_L(P)+Q_H(P)} \cdot V_H$$

$$\begin{cases} D(P) = Q(P) = Q_L(P)+Q_H(P) \\ P = \bar{V} \end{cases}$$

Market Unraveling

With this term, we indicate a case where:

- *a pooling equilibrium cannot be supported by any price*;
- therefore, only bad-quality good will be sold in equilibrium.

Example: quality can be 1 or 0, with equal probability. Price if both are supplied: .5.
A zero-quality good has supply

$$Q_0(p) = p - .2 \rightarrow Q(.5) = .3,$$

while a one-quality good has supply

$$Q_1(p) = p - .5. \rightarrow Q_1(.5) = 0.$$

But then the only price that makes sense is 0 (only zero-quality goods are supplied). The market unravels completely (in this case to the point of not existing, nobody sells or buys anything).

Econometric Methods

- Instrumental variables (Feyrer, 2009; Autor et al., 2013).
  - Idea: solve endogeneity issues by using a variable Z that does not affect the outcome Y directly, but only shifts the endogenous variable interest X;
  - Assumption: Z only affects Y *through its effect on* X, and *no other channels*.
- Regression discontinuity (Tyler et al., 2000), assumption is very similar to an instrumental variable:
  - Idea: solve endogeneity issues by using a *running* variable Z that, around a specific cutoff $\bar{Z}$ (GED score), does not affect the outcome Y directly, but only shifts the endogenous variable interest X (GED acquired);
  - Assumption 1: if $\bar{Z}$ was not chosen to be a cutoff, units with Z in a close neighborhood of the cutoff would be indistinguishable in their outcomes Y and other covariates (no other effect of $Z > \bar{Z}$ on Y other than change in X);
  - Assumption 2: units cannot self-select on either side of the cutoff (also no other effect of $Z > \bar{Z}$ on Y other than change in X; if people could choose, the outcomes Y would likely change discretely around the cutoff).

Cool visualizations (website is old but gold):
`http://www.nickchk.com/causalgraphs.html`