14.03/14.003 Recitation 6
General Equilibrium

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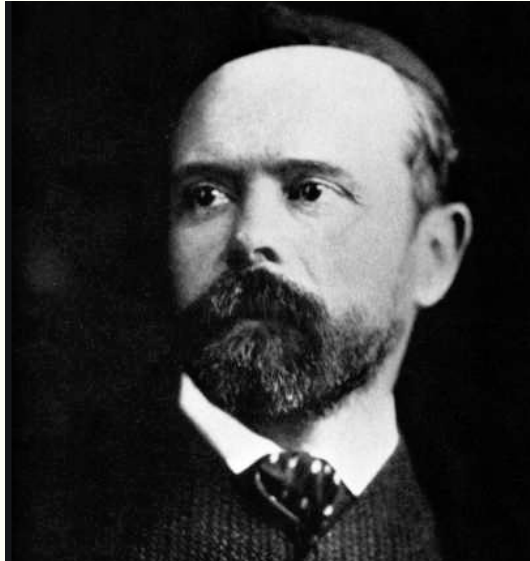
Agenda

- Defining General Equilibrium
- Edgeworth Box
- GE with production in autarky
- Trade and GE

Léon Walras (1834–1910)...

...and his life

- Born in Évreux, France in 1834;
- Failed college admission twice, studies to become an engineer but...
- ...spends time writing two failed novels, that he has to publish with his own money because nobody wants them;
- In 1858, his father persuades him to work on Econ, so he finally drops out of college;
- 1860: He goes to Switzerland to try to advocate full land collectivization, but he fails, so he decides to start a cooperative bank with some friends (among which the grandson of famous French economist Jean-Baptiste Say);
- 1869: his bank fails.
- 1871: He applies to become a professor in Lausanne, arguing that economics has to become more mathematical;
- 1872: publishes his treaty "Elements of Pure Political Economy". Published in 100 copies thanks to the benevolence of some friend in the government.
- 1906: He applies (??!!) to the Nobel Peace prize because he is broke. Too bad Teddy Roosevelt was in the contest that year.

General Equilibrium in Pure Exchange

A General Equilibrium is a set of **prices** $\{p_1, p_2\}$, and **allocations** for consumption $\{\underline{c}_1, \underline{c}_2\}$ such that, given initial endowments of goods $\underline{e}_1, \underline{e}_2$:

❶ Consumers choose consumption $\{\underline{c}_1, \underline{c}_2\}$ subject to their budget constraint for given **prices**;

❷ The good markets clear.

Solving GE

❶ Technology ⇒ PPF;

❷ Preferences ⇒ Indifference curves;

❸ Endowments with 1 and 2 ⇒ price ratio.

Solving GE in Pure Exchange

GE is given by the point **in the Edgeworth box** such that indifference curves are tangent to each other and utility is maximized for given initial endowments, and the price ratio that is tangent to the indifference curves.

Steps:

❶ Solve the consumer problem for given income I for both consumers;

❷ For each consumer, set $I = p_1 e_1 + p_2 e_2$, find c_1, c_2 as a function of price ratio;

❸ Impose market clearing: $c_1^1 + c_1^2 = e_1^1 + e_1^2 = E_1$.[1]

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[1]Superscripts denote the agent's identity, subscripts the goods.

Edgeworth Box: An example

Consider an economy with two goods, x and y, and two agents, A and B, with preferences:

$$U^A(x^A, y^A) = x^A \times y^A, \tag{1}$$

$$U^B(x^B, y^B) = (x^B)^{\frac{2}{3}} \times (y^B)^{\frac{1}{3}}, \tag{2}$$

denote the price ratio $p_y/p_x \equiv p$ (level does not matter), and let the total endowments of goods be $X = 10$, $Y = 10$, and denote individual endowments by e^i_{good} for *good* in $\{x, y\}$ and i in $\{A, B\}$.
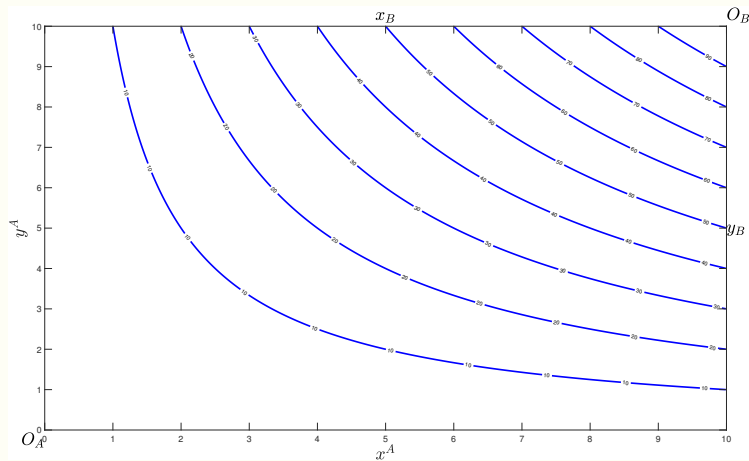
The budget constraint for each agent i is then:

$$x^i + py^i = I^i = e^i_x + pe^i_y \tag{3}$$

The general equilibrium of this economy is an allocation $\{x^A, y^A, x^B, y^B\}$ and a price p, such that $\{x^A, y^A\}$ and $\{x^B, y^B\}$ maximize (1), (2) subject to (3), and markets clear, that is:
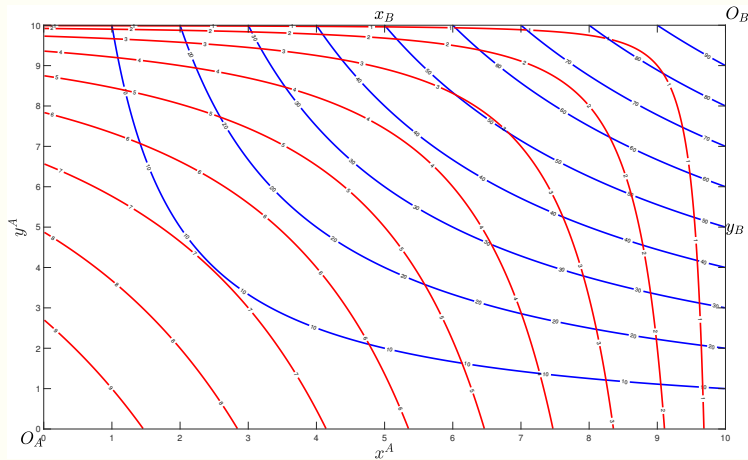
$$x^A + x^B = e^A_x + e^B_x = X, \quad y^A + y^B = e^A_y + e^B_y = Y$$

Edgeworth Box: Indifference curves for A
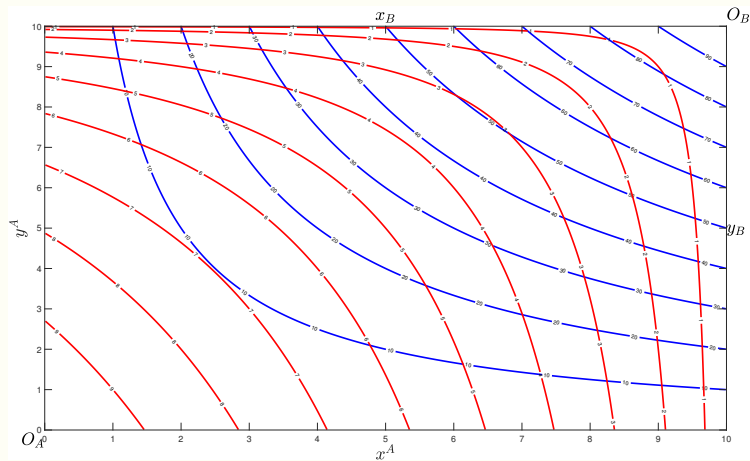
Edgeworth Box: Indifference curves for A and B

Pareto Improvement

Given an initial allocation $\{\underline{x}, \underline{y}\}$ we say that moving to allocation $\{\underline{x'}, \underline{y'}\}$ generates a Pareto-improvement if and only if both $U^A(x', y') \geq U^A(x, y)$ and $U^B(x', y') \geq U^B(x, y)$, with one of the two inequalities being strict (i.e. one agent is strictly better off while the other is not worse off).

Set of Pareto Improving Allocations

What is the contract curve?

- The contract curve identifies the set of all allocations that *can be a general equilibrium of the economy* (i.e. for which there exists a price ratio that makes these point an equilibrium starting from *some* initial allocation).

- In practice, this is the locus of points where the MRS of the two agents are equalized.

- Why should the MRS be equal? Well, the price ratio is the same for all the economy, and agents that satisfy the five axioms set MRS = p.

- In economic logic, if the MRS are not equalized, the "mental valuations" of marginal goods by the two agents are not equalized so there are still beneficial exchanges that can be implemented until the two agents reach agreement on the value of marginal goods.

Finding the contract curve

In the general case, the contract curve might not have an analytical expression, but not here. The MRS of agent A at the optimum is given by:

$$\frac{x^B}{y^B} = 2p, \quad \frac{x^A}{y^A} = p \Rightarrow \frac{x^B}{y^B} = 2\frac{x^A}{y^A}$$

Now, remember that the contract curve requires equilibrium (that is markets clear), so $x^B = X - x^A$ and $y^B = Y - y^A$. Plug in and rearrange and you get the contract curve:

$$y^A = 2\frac{x^A}{(X + x^A)}\, Y.$$

Interpretation: any allocation that satisfies this expression is an equilibrium, supported by:

$$\frac{x^A}{y^A} = p.$$

However, only **one** of these equilibria will arise after exchange for given initial endowments.

Which Equilibrium?

- Geometrically, the final equilibrium is given by the point on the contract curve such that the line that connects this point to the initial endowment (the price ratio) is perpendicular to the contract curve. Why?

- The slope of the contract curve tells me the ratio between y^A and x^A, while the price ratio gives me the opposite ratio, so the two must be reciprocals.

- Economic reasoning: if the price ratio is such that the ratio of consumption of the two goods is not the one required by the contract curve, the two MRS are not equalized, so I am not in an equilibrium because the two agents can carry out beneficial exchanges.

Which Equilibrium?
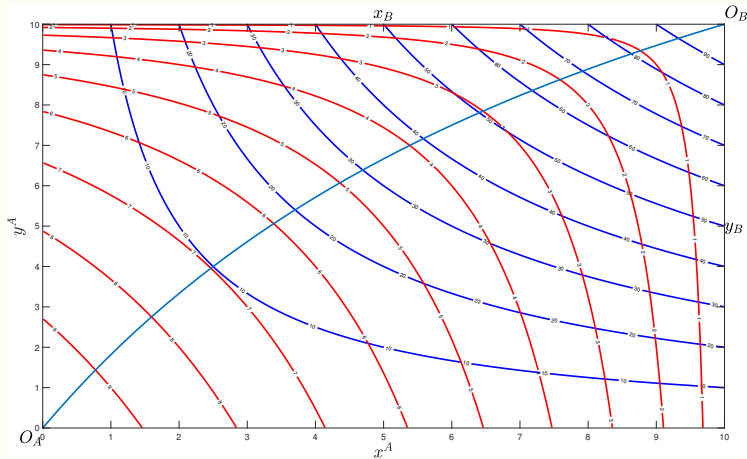
General Equilibrium with Production

A General Equilibrium is a set of **prices** $\{p_1, p_2\}$, and **allocations** for output, labor and consumption, $\left\{\underline{y}_1, \underline{y}_2, \underline{\ell}_1, \underline{\ell}_2, \underline{c}_1, \underline{c}_2\right\}$ such that, given an endowment of labor L:

❶ Producers set output $\left\{\underline{y}_1, \underline{y}_2\right\}$ and choose inputs $\{\underline{\ell}_1, \underline{\ell}_2\}$ to maximize profits given **prices**;

❷ Consumers choose consumption $\{\underline{c}_1, \underline{c}_2\}$ subject to their budget constraint;

❸ The markets for goods and labor clear.

GE with Production: Find the PPF

PPF: the *Production possibility frontier* is the set of all outputs that I can produce given a resource constraint. Example using only labor:

$$y^A = \left(\ell^A\right)^\alpha, \quad y^B = \left(\ell^B\right)^\beta.$$

Say that total labor is 1. Then I have the constraint:

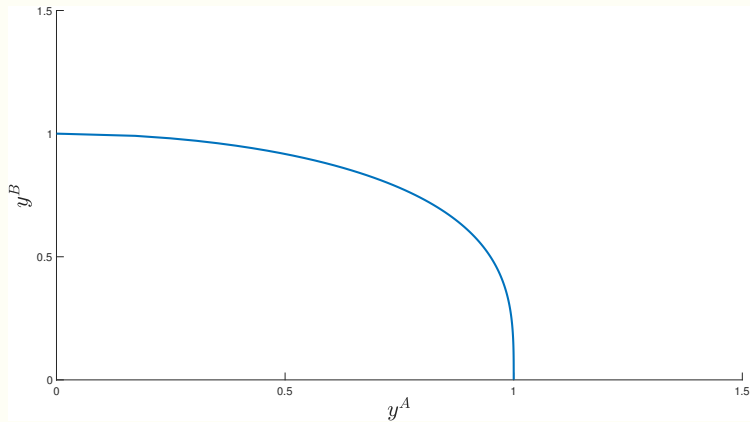$$\ell^A + \ell^B = 1. \tag{4}$$

I can obtain the PPF inverting the production functions for labor and plugging them into this constraint:

$$y^A = \left(\ell^A\right)^\alpha \Rightarrow \ell^A = \left(y^A\right)^{\frac{1}{\alpha}}, \qquad y^B = \left(\ell^B\right)^\beta \Rightarrow \ell^B = \left(y^B\right)^{\frac{1}{\beta}}.$$

Plug into (4) and rearrange:

$$y^B = \left(1 - \left(y^A\right)^{\frac{1}{\alpha}}\right)^\beta.$$

GE with Production: Trade