Competing for Inventors: Market Concentration and the Misallocation of Innovative Talent*

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Abstract

Inventors are a scarce resource, whose skill sets can apply to R&D in disparate product markets. Motivated by this fact and by the fall in R&D productivity that characterized recent decades, I explore the impact of product market competition on the misallocation of inventors and growth. First, I delineate the boundaries of "knowledge markets", employing USPTO patent data to group NAICS sectors that employ inventors with similar skills. Second, I analyze the relation between 4-digit NAICS sectors' market concentration and the share of inventors employed in R&D projects relevant to these sectors. Four findings emerge from the analysis. First, the last thirty years saw a sizable increase in the concentration of inventors across NAICS sectors of application. Second, over the period 1997-2012, increases in sector-level concentration are positively correlated with the share of inventor markets captured by each sector. An IV analysis based on the increase in sector-specific regulations suggests a causal interpretation of this result. Third, higher shares of relevant inventors are positively correlated with rising concentration of inventors at the top of the innovating firms distribution, and negatively with forward citations. Fourth, concentrating sectors have seen a decrease in R&D productivity, as measured by growth in output per worker per inventor. A back-of-the-envelope computation suggest that the increased concentration of inventors in less competitive sectors can account for about 27% of the overall decrease in output per worker growth over the period 1997-2012 (a fall of 0.78% in absolute value). I interpret my findings through a Schumpeterian model of creative destruction, where incumbents can conduct defensive patenting. In the model, unbalanced increases in markups shift inventors towards less competitive sectors, and towards monopolistic firms carrying out R&D projects with the aim to erect barriers to entry. This shift reduces equilibrium inventors' productivity and growth. I calibrate a two-sector version of this model to match moments of the inventor distribution and R&D spending in the US and study the optimal allocation of R&D subsidies for a planner wishing to maximize growth. I show that subsidies to entrants in highly concentrated sectors constitute the most effective policy, raising output by 0.5% in absolute value in the calibrated model (17% of the baseline in percentage terms).

JEL Codes: O30, O31, O32, O40.

Keywords: Market Concentration, Defensive Innovation, R&D Productivity.

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1 Introduction

Research and Development activities are key to innovation and growth. Yet, as documented by a growing literature, R&D productivity and growth have been falling over the last few decades (Bloom et al., 2020; Fernald and Jones, 2014; Gordon, 2016). A prominent explanation of these trends is that "ideas are getting harder to find" due to increased technological complexity. A less explored alternative sees instead growing inefficiency in the allocation of R&D inputs as the root cause of this phenomenon (Acemoglu et al., 2018, 2021), a concern that is echoed in the policy circles as well as the press (Metz, 2017; Bass and Brustein, 2020). Central in the current policy debate is the concern that dominant big-tech firms are inefficiently appropriating a large share of highly-educated, highly-skilled workers, subtracting this crucial resource from more competitive and dynamic sectors. A stark example of this phenomenon is the high concentration of AI expert workforce. TalentSeer (2020) estimates that up to 20% of total AI experts are employed by just five companies: Google, Microsoft, Apple, Amazon and IBM. At the same time, smaller firms in other economic sectors appear unable to attract this scarce talent, raising concerns over the potential misallocation of innovators.

This paper studies the broader effect of increasing concentration on the allocation of inventors across sectors, and shows that such concerns are justified. Using USPTO patent data and concentration measures from the Economic Census over the period 1997-2012, I demonstrate that sectors where concentration has increased have indeed attracted an increasing share of inventors. This relation has a causal interpretation, according to an IV specification where I use the increase in the number of regulations from Mercatus RegData as an instrument for increased concentration. I establish three additional facts that suggest that this allocation is inefficient. First, researchers accrued mostly to incumbent firms in concentrated sectors. Second, the quality of patents in concentrating sectors has fallen, as measured by patent forward citations. Third, inventors' productivity, that I measure as growth in output per worker per inventor, has fallen in these sectors. These findings suggest that additional inventors have accrued to incumbents who employed them on "defensive innovation", that is, projects with a low growth footprint conducted with the primary aim of preventing further entry and sheltering existing dominant positions. Quantitatively, my findings imply that inventor misallocation implies a fall of up to 0.78pp in output-per-worker growth in the sectors I study (27.3% of the overall observed reduction over the period 1997-2012).

Methodologically, my analysis relies on a novel dataset of "knowledge markets," defined as sets of product markets that share similar inventors. These inventor markets originate from the network generated by transitions of individual inventors across product categories, identified using USPTO patent data, where patents are classified according to their main NAICS sector of applications. This classification is crucial to avoid pooling inventors and sectors that do not share the same technical expertise. If this were the case, the response of inventor mobility to changes in sectoral characteristics would be biased toward zero, as many sectors identified as potential outlets for each inventor would in facts not be such.

In order to interpret these results, I build a Schumpeterian model where, in addition to productive R&D conducted by new entrants, incumbents can engage in defensive projects to increase entry cost faced by potential competitors. A two-sector general equilibrium model shows that unbalanced changes in concentration across sectors generate a fall in inventor productivity and growth. Indeed, inventors are misallocated towards less competitive markets where defensive projects—which hamper entry and Schumpeterian growth—are more prevalent, and away from more competitive sectors where growth is more sustained. The theoretical analysis shows that defensive innovation is a crucial mechanism to generate the increased concentration of inventors among incumbents that I find in the data, as well as the fall in R&D productivity, which are both absent in a framework abstracting from these features.

I calibrate a two-sector version of my model to match moments of the R&D spending distribution in 1997 and growth over the period 1997-2012. This calibration implies productivity losses from misallocation that are close to the lower bound implied by my estimates. In the context of this model, I study the allocation of cost-neutral R&D subsidies that maximizes growth. Subsidizing entrants' R&D in more concentrated sectors constitutes the most effective policy, leading to a rise in annual GDP growth of about 0.5pp (17% relative to my benchmark). Similar results obtain if entry is uniformly subsidized across the two sectors. This finding resonates with the fact that defensive innovation constitutes the main friction hampering efficiency in the model. Since this friction acts through an increase in entry barriers, the most effective way to counter it consists in alleviating entry costs.

The rest of the paper proceeds as follows. In the following section, I survey the related literature place my study in its context. Section 2 describes my data sources, focusing in particular on the construction of knowledge markets. Section 3 reports the results of my empirical analysis. Section 4 details my theoretical framework, and conducts the policy analysis. Section 5 concludes and provides directions for future work.

1.1 Related Literature

My work connects to several strand in the empirical and theoretical literature analyzing the effect of product market competition on innovation. As detailed below, I contribute to this literature through four substantial deviations. First, I analyze the effect of competition on the allocation of R&D *across sectors,* while previous work has focused on either a within-sector or economy-wide perspective. Second, I explicitly analyze the allocation of R&D inputs across sectors, and I do so identifying the boundaries of markets for inventors rather than relying on R&D spending or similar measures of innovative effort as in previous work. Third, I connect changes in the competitive structure to the documented trends on falling R&D productivity. Fourth, I study theoretically the impact of pre-emptive innovation in a multi-sector model on R&D productivity, and the growth-maximizing policy in this context. Both the focus on R&D productivity and the policy analysis are novel to the theoretical literature analyzing defensive innovation.

First an foremost, my paper relates to the vast literature estimating the empirical effect of competition on innovation. Aghion et al. (2005) famously documented an inverted-U relationship between competition and innovation, whereby this relation is increasing at low levels of competition, and decreases at high levels. Accordingly, papers in this literature have highlighted contrasting effects of competition on overall R&D activity, focusing mostly on episodes of trade liberalization (see Shu and Steinwender, 2019, for an extensive review). Most papers in this trend identify these effects at the firm-level, which restricts their scope to the effect of competition within product markets. My paper instead adopts a cross-sector view, analyzing the effect of increased concentration in some sectors on the *share of available R&D resources* they appropriate. In order to do so, I build a novel dataset of "knowledge markets", sets of product markets that share the same inventors. While several papers investigate the mobility of inventors (see, e.g., Azoulay et al., 2017; Moretti and Wilson, 2017), my paper is to the best of my knowledge the first to analyze the effects of market structure on inventors' transitions across sectors.

Due to my focus on competition and innovation, my paper naturally connects to the literatures that document increased concentration (Autor et al., Forthcoming; Gutiérrez and Philippon, 2017; Grullon et al., 2019), profits and markups (Barkai, 2020; De Loecker et al., 2020; Eggertsson et al., 2018), as well as the growing literature documenting a fall in innovation and R&D productivity (Akcigit and Ates, 2019, 2020, 2021; Bloom et al., 2020), and its connection to the allocation of R&D within and across sectors (Acemoglu et al., 2018, 2021; Akcigit and Kerr, 2018). My contribution bridges these literatures, explicitly linking changes in the competitive structure to the allocation of R&D resources across more and less concentrated sectors, as well as their deployment to productive or defensive projects.

Several papers document the role of pre-emptive innovation in ordinary firm operations (see Guellec et al., 2012, for a review of the evidence), and the high valuation of the resulting patents (Abrams et al., 2013; Czarnitzki et al., 2020; Grimpe and Hussinger, 2008). Most recently, Argente et al. (2020) show that, within product markets, large firms tend to account for the bulk of patenting activity, but are responsible for a smaller share of implemented innovations relative to non-patenting firms. The authors interpret this finding as evidence of defensive innovation, which provides high value to patents of entrenched firms who can thus shelter their dominant position from competitors. My paper builds on this literature showing that increased concentration raises the incentives for defensive innovation, as demonstrated by a fall in forward citations in concentrating sectors. This result connects to the findings and the model Abrams et al. (2013), who study the cross-sectional relation between patent value and forward citations theoretically, showing that high-value patents also tend to receive less citations, and rationalize this result through pre-emptive innovation.

On a theoretical standpoint, I embed defensive patenting as modeled in (Abrams et al., 2013) into a Schumpeterian growth model, building on the extensive literature inaugurated by Aghion and Howitt (1992). In particular, my solution relies on several results derived by Acemoglu and Akcigit (2012). To the best of my knowledge, my paper is the first to analyze the impact of defensive innovation in the context of general-equilibrium growth model. The closest precedent to this analysis is Jo (2019), who builds on Akcigit and Kerr (2018), and defines their incremental innovation, that is innovation conducted on existing product lines available to incumbents, as "defensive". Therefore, in his framework, defensive innovation corresponds to incremental innovation aimed at "escaping competition" from entrants by increasing the technological distance of incumbents from entrants, in the tradition of Aghion et al. (2001). By contrast, in my framework defensive innovation is specifically aimed at protecting dominant positions and reduce entry as in Abrams et al. (2013), who however do not consider the effects of defensive innovation on R&D productivity and overall innovation. My final contribution consists in analyzing the growth-maximizing allocation of R&D subsidies, which has not been previously studied in a framework featuring defensive innovation.

2 Data description

In this section, I describe my main data sources and the procedure to build the dataset employed in my empirical analysis. Subsection 2.1 lists the sources of the raw data. Subsection 2.2 focuses on the definition of inventor productivity measures and knowledge markets, which I identify through realized inventor flows across sectors. Subsection 2.3 briefly describes other data construction steps that are discussed in more detail in Appendix A.

2.1 Data Sources

My empirical analysis relies on the variation of concentration across product markets, as defined by 4-digit NAICS sectors, and the impact of these shifts on the allocation of inventors with specific competences across these sectors, and their effect on inventors' productivity. I use USPTO patent data to measure inventor productivity and establish the set of product markets that share similar inventors, and US Economic Census data to obtain concentration and productivity growth measures. Finally, I also use a dataset of product market regulations, Mercatus RegData 4.0, to conduct an instrumental variable analysis, as well as NBER-CES that I employ to obtain estimates of the Lerner Index that I employ in the calibration of my theoretical model.

My primary source is given by USPTO patent data from PatentsView. This dataset contains disambiguated patent, inventor and assignee identifiers, as well as Cooperative Patent Classification (CPC) classes for each of the patents deposited in the period 1975-2021. I employ these data to construct inventor flows across different sectors, which I build employing the ALP classification of 1976-2016 patents into NAICS sectors of application developed by Goldschlag et al. (2016). Since this classification is constructed using the PATSTAT dataset, I rely on the crosswalk built by Gianluca Tarasconi to match these two sources.[1] This leaves me with around one third of all the patents registered between 1975 and 2021, due to the restriction of the time frame to 1976-2016 as well as an incomplete match between PATSTAT and PatentsView. Patent data also provides my with citation data, which I use to build self-citations as well as truncation-corrected forward citations and patent generality, following the procedure in Hall et al. (2001) and Acemoglu et al. (Forthcoming). I restrict my attention to utility patents, since I am interested in patents with a technological content, and not just design improvements.

My main data source for concentration and sales data is given by the US Economic Census (EC), which reports sales shares for the top 4, 8, 20, and 50 firms, Herfindal-Hirschman Index, sales and number of companies in various NAICS 4-digit sectors at a 5 year frequency. I restrict my attention to the period between 1997 and 2012 for three main reasons. First, as I show below, this period saw a substantial increase in the concentration of inventors in specific technology classes. Second, the start of this period coincides with an acceleration in the growth of market concentration and markups (see, e.g., De Loecker et al., 2020). Third, 1997 marks the adoption of the NAICS classification, ensuring a consistent definition of product markets throughout the period I analyze, without the need to rely on crosswalk with the pre-existing SIC classification. As my baseline concentration measure, I rely on the HHI lower bound constructed by Keil (2017).[2] This choice is dictated by the fact that the Economic Census reports the HHI only for a subset of industries, which would severely limit my sample. The method proposed by Keil (2017) obviates to this issue by constructing the implied lower bound of the HHI implied by top sales shares reported in the Economic Census, which are available for a much wider set of industries than the HHI.[3] While my estimates are robust to using the EC-reported HHI, this choice allows me to obtain more power for my findings as well as generalize them.

The Economic Census also provides me with sector-level growth in output per worker, which constitutes my main measure of productivity growth in the sectors I analyze. I choose this measure instead of multi-factor productivity since the latter is available only for a limited set of sectors, mostly in manufacturing. I deflate sales using NAICS-specific price indices from the BLS.

I employ two additional data sources in the empirical analysis and in the calibration of my model. First, I obtain sector-specific counts of regulations for various NAICS 4-digit sector from the Mercatus RegData 4.0 dataset, which I employ to conduct an instrumental variable analysis, strengthening the

[1] See https://patentsview.org/forum/7/topic/143, https://rawpatentdata.blogspot.com/2019/07/patstat-patentsview-concordance-update.html

[2] Available at https://sites.google.com/site/drjankeil/data.

[3] As detailed in Keil (2017) this measure is very strongly correlated with the HHI reported by the Economic Census when this is available, with a correlation of around 0.93.

causal interpretation of my results.[4] Second, I use NBER-CES data to produce estimates of the Lerner Index following Grullon et al. (2019).

Matching my various data sources results in a dataset of 157 NAICS 4-digit sectors over the period 1997-2012, out of a total of 304 business sectors, for which I have data on concentration and that I merge to my estimates of knowledge markets.

2.2 Effective Inventors and Knowledge Markets

In order to identify the effect of product market concentration on the allocation of inventors, I first have to establish which set of sectors different categories of inventors can actually choose from. Indeed, pooling all sectors together would inevitably attenuate the response of the inventor distribution to changes in the features of individual product markets, as this design would correspond to the assumption that all inventors could potentially be employed by any sector. Since inventors have limited employment opportunities due to their specific skills set, this assumption would incorrectly suggest that, in a majority of cases, sector characteristics have no impact on the allocation of researchers. That is, the mismeasurement of labor market for inventors would produce an attenuation bias in my coefficient of interest.

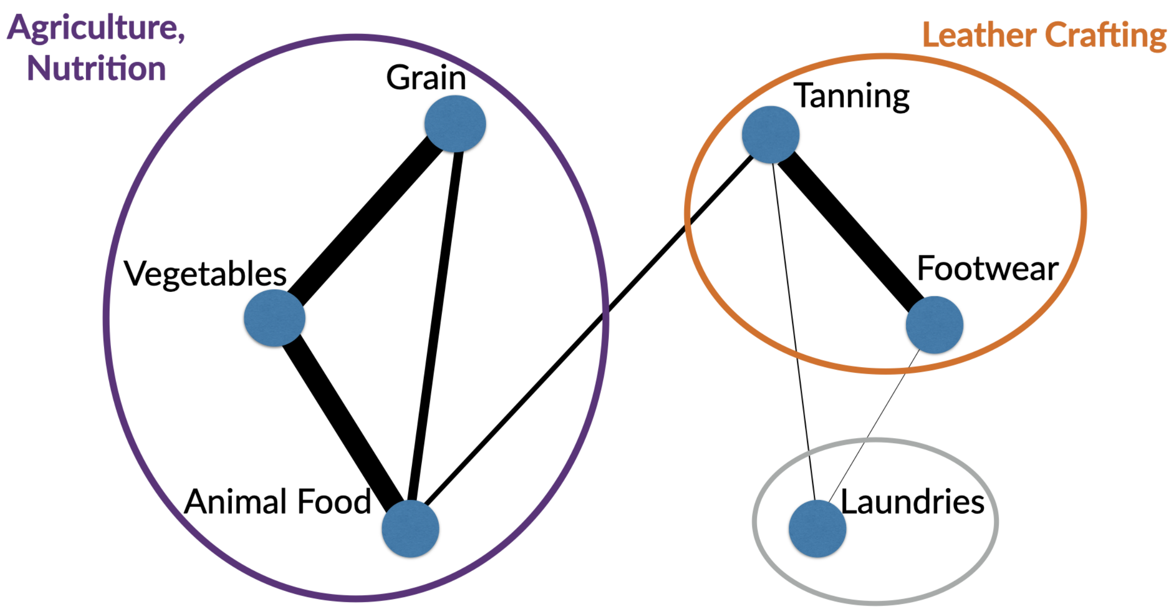
The main aim of this section is correctly grouping product markets that share the same *required knowledge to innovate*, and therefore compete for the same R&D inputs, namely inventors. In practice, I construct transition of inventors across different product markets, and use the resulting network of flows to identify sectors that routinely exchange researchers through a Louvain community-detection algorithm (Blondel et al., 2008).

Figure 1 presents a graphical illustration that clarifies my definition of knowledge markets as well as the procedure I follow to construct them. Each node in the Figure represents a different NAICS 4-digit sector, which is connected to other sectors by inventor flows, indicated with black lines. The width of each line represents the strength of these flows. The procedure described in detail in the rest of this section, shows how I define these flows and measure their strength. After obtaining these weighted flows, I employ a community detection algorithm to group together sectors that are most closely connected to each other. In the sectors that I selected for this illustration, as one might expect, there are strong flows between grain, vegetable farming and animal food manufacturing, all of which involve knowledge related to agriculture and nutrition, and separately between footwear and leather tanning, which both require knowledge of leather crafting. In this case, my algorithm would identify two knowledge markets, one given by the agriculture and food manufacturing sectors, and one given by leather crafting sectors, leaving the laundry services sector isolated.

This example makes clear the importance of identifying separate knowledge market to analyze the

[4] Available at https://www.quantgov.org/bulk-download.

Figure 1: Graphical Illustration of Knowledge Markets



Note: This Figure provides a graphical illustration of the definition of knowledge markets as sets of product markets sharing the same required knowledge to innovate. This illustration is based on transitions of inventors across product markets observed in my data and classified in the same knowledge markets, although many other sectors in these markets are excluded for the sake of exposition. In the figure, nodes represent NAICS sectors 1111 (Oilseed and Grain Farming), 1112 (Vegetable and Melon Farming), 3111 (Animal Food Manufacturing), 3161 (Leather and Hide Tanning and Finishing), 3162 (Footwear Manufacturing), and 8123 (Drycleaning and Laundry Services). The edges connecting nodes represent inventor transitions across sectors, while the width of these edges represents the strength of the connection between the two sectors as measured by undirected inventor flows.

effects of the evolution of product markets on the allocation of inventors. In this figure, where there are routinely almost no links of the laundry sectors to the others, we would not observe a response of the inventor distribution to changes in the concentration of these sectors, since the other inventors in the economy specialize on different skills set and would therefore not be able to transition there. Analogously, we would expect a higher profitability in footwear manufacturing to attract inventors away from leather tanning, but not from vegetable farming.

Measuring Inventor Transitions The first step to construct knowledge markets consists in defining transitions of inventors across sectors. I employ the USPTO patent data classified into 4-digit NAICS sectors by Goldschlag et al. (2016) to do so. Table 1 exemplifies the structure of the USPTO dataset matched with this NAICS classification. In this dataset, each patent is listed together with all the inventors listed in the registered patents. Each inventor is assigned a disambiguated ID corresponding to the serial number of the first patent the inventor appears. In this example, inventor 00001-1 and 00001-2 both cooperate on the development of patent US00001. The third column in Table 1 shows an example of the Goldschlag et al. (2016) classification for NAICS 4-digit industries. The authors conduct

Table 1: USPTO Data Structure

Patent ID	Inventor ID	Goldschlag et al. (2016) NAICS	Year
US00001	00001-1	1111	1980
US00001	00001-1	1112	1980
US00001	00001-2	1111	1980
US00001	00001-2	1112	1980
US00002	00001-1	3111	1981

Note: This Table reports an example of the data structure employed to build knowledge markets. The columns "Patent ID" and "Inventor ID" represent disambiguated patent and inventor identifiers as classified by USPTO PatentsView Data. The column "Goldschlag et al. (2016) NAICS" reports an example of patent classification into NAICS 4-digit sectors. The data reported in this table have pure expository aim, and do not represent actual observations in the dataset.

a text analysis to classify each patent into the various NAICS sectors of application of that patent. As the table illustrates, this classification is not limited to a single sector per patent, and includes multiple sectors in almost all instances. Importantly, this classification captures the *technological nature* of the patent and the sectors of application of the knowledge required to develop that patent. While other classifications, like the CPC or the USPC, also describe the technological nature of patents, they do not allow a direct match to sectors of application without arbitrarily assigning sectors to each of these classes.

Given this data structure, I define a transition in two ways. First, I consider inventor transitions *within patents*. That is, I consider that an inventor transition occurs between two sectors if an inventor works on a patent that applies to both of these sectors. The direction of flows does not matter for the definition of knowledge markets, since I am interested in grouping sectors which exchange researchers and not in the specific direction of these exchanges. Therefore in the case of Table 1, I would say that there have been two transitions between sectors 1111 and 1112 in 1980. Another type of transition that I consider is *across patents*. This transition occurs when an inventor works on different patents that apply to different product markets. An example of such transition in Table 1, is the transition between sector 1112 and 3111 by inventor 00001-1. The raw count of transitions of inventors across sectors in each year constitutes the basis of my measure of inventor flows.

Weighting Inventor Flows: Effective Inventors After identifying transitions, I proceed to weigh them by two alternative measures in order to quantify the flow of inventors across sectors. The first measure simply weigh each transition equally, computing inventor flows as the raw count of researchers moving across NAICS. The second measure adjusts for the productivity of individual inventors, since raw counts might overstate or understate the importance of each transition, depending on the size of origin and destination sectors, their technological nature, as well as the ability of each inventor. I therefore define a measure of "effective inventors" that aims to correct for these and other omitted factors. For each inventor, I estimate the fixed effect, α_i, in the fully-saturated regression,

$$\#\text{Patents}_{cfit} = \alpha_i + \gamma_{cft} + \varepsilon_{cfit}, \tag{1}$$

where $\#\text{Patents}_{cfit}$ denotes the number of patents registered in CPC class, c, firm (assignee), f, and year t, that include inventor i. In this regression γ_{cft} denotes a of CPC class by firm (assignee) by year fixed effect. I choose to include indicators for CPC classes at one digit, the broadest classification, in order to identify as many fixed effects as possible. The inclusion of CPC, firm, and year controls corrects for specific technological features of the patented technology, the firm environment, as well as the specific year. Further, this specification produces an estimate of inventor productivity that accounts for the number of collaborators on each patent. Clearly, these fixed effects might be inconsistently estimated, and for this reason I check the robustness of all my results, including the construction of knowledge markets, to the use of the raw count of inventors rather than the inventor's productivity captured by the fixed effect, α_i. Given this specification, I define an *effective inventor* as one unit of the resulting fixed effects, rescaled to take nonnegative values.

Armed with the results of this estimate, I define *effective inventor flows* between sector j and sector k at time t as:

$$flow_{j\to k,t} = \sum_i \#\{i\text{'s transitions } j \to k \text{ in } t\} \cdot \alpha_i,$$

that is, the sum of transition counts weighted by effective inventors. The total undirected flow between two sectors is then given by the sum of inflows and outflows with ends in one of the two sectors:

$$flow_{jk} = \sum_t \left(flow_{j\to k,t} + flow_{k\to j,t}\right).$$

This flow measure defines a network of inventor transitions across product markets, where the nodes, j, k, are given by 4-digit NAICS codes, edges are given by transitions across sectors, and edge weights are defined as a rescaled version of $flow_{jk}$. I use these edge weights as a measure of the strength of the connection between pairs of sectors in the network. Rescaling the flow measure is necessary in order to exclude effects of sector size as well as to avoid double counting of inventors. I describe how I rescale this series in Appendix A.

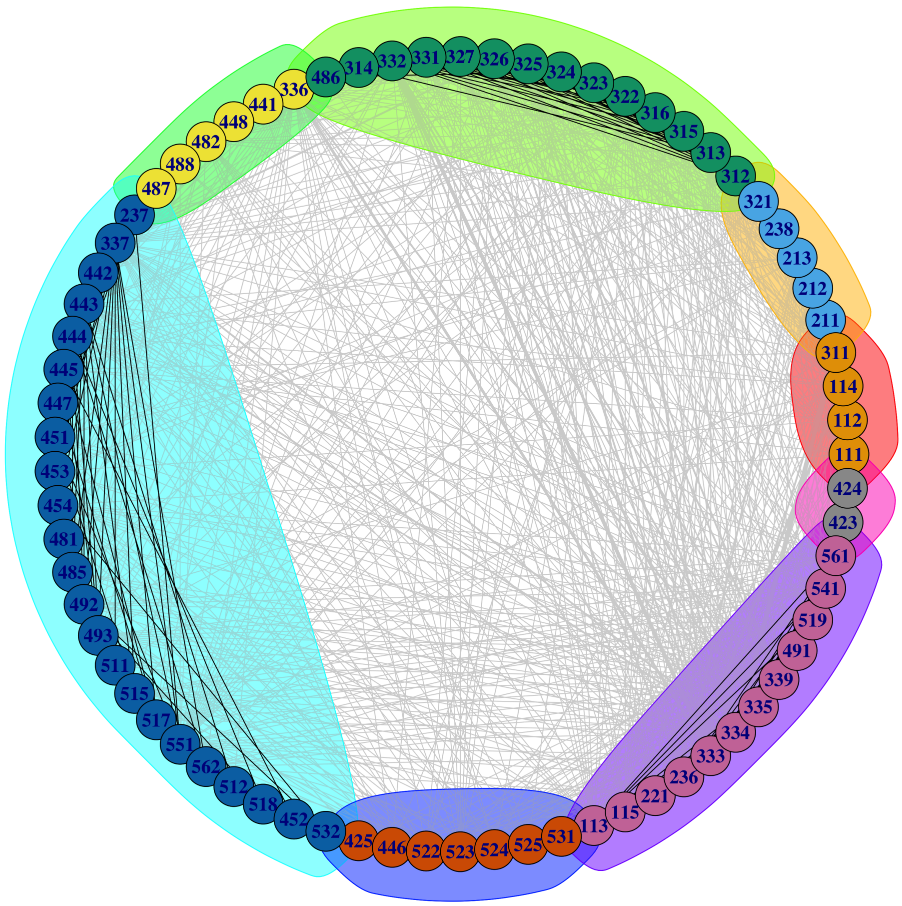
Community Detection and Resulting Knowledge Markets I used rescaled the rescaled undirected flow measure as a network edge weight to identify communities through the Louvain algorithm developed by Blondel et al. (2008). This procedure maximizes the modularity of the network choosing the number of communities (knowledge market) and the assignment of nodes (NAICS sectors) to communities. Modularity, a commonly used measure of connectedness of networks, measures the distance between the density of links *within* communities versus *between*.

This procedures produces 10 non-singleton sets of NAICS 4-digit sectors that share the same inventors and have non-missing concentration measures. By construction of the community detection algorithm, these knowledge markets do not overlap, so each NAICS 4-digit sector belongs to one and only one knowledge market. Figure 2 displays the result of my procedure applied to NAICS 3-digit sectors. I report this exercise since the 4-digit equivalent would be unreadable. However, the knowledge markets identified by the two exercises are qualitatively similar although they are clearly more numerous in the 4-digit case. In this figure, lines denote inventor transitions, with width proportional to the effective undirected inventor flow between sectors. Nodes represent the NAICS 3-digit sectors reported on each node. Black lines depict flows within knowledge markets, while gray lines represent transitions between communities.

Three features are worth emphasizing. First, the network is very dense, and transitions across 3-digit as well as 2-digit sectors are pervasive, and differ largely in intensity. This points to the relevance of this classification and its stark difference from what could be obtained by grouping sectors based on broad product markets, which would neglect the linkages across disparate markets, as well as pooling all sectors together, which would neglect the difference in the strength of inventor flows. Second, while the flows between communities might seem more numerous than within communities, this is solely a by-product of the circular layout of the network, whereby flows within close communities on the circle are masked by the nodes. When applying the algorithm to 4-digit sectors, I find that less than a third of flows occur between communities, as expected since the community detection algorithm maximizes the density of within-community linkages. Third, and perhaps most importantly, the classification that I obtain is sensible, grouping together sectors that we might expect to share similar knowledge to innovate. Starting from sector 111 and going counter-clockwise, the knowledge markets in the Figure can be described as follows. The first market, including sector 111, groups sectors involving agricultural production (111, 112 and 114) as well as food manufacturing (311). The second market, starting with 211, includes oil, gas, and mining. The green cluster at the top of the figure groups several heavy manufacturing industries, like chemicals plastics and petroleum products, as well as pipeline transportation (486). The market in yellow is concerned mostly with transportation services and manufacturing as well as motor vehicle dealers. The large blue cluster collects a large number of retail sectors, as well as data processing, telecom and broadcasting services. The remaining three markets include insurance and finance (red cluster), computer, electronics, and machinery manufacturing and professional services (violet), and wholesalers (gray).

Knowledge markets are identified using my measure of effective inventors, but the algorithm produces nearly identical results when using raw inventor counts; over 97% of 4-digit NAICS pairs of sectors are classified in the same manner using the two measures. That is, 97% of sector pairs belong to the same knowledge market according to both measures.

Figure 2: Knowledge Markets Obtained from NAICS 3-digit Sectors



Note: This Figure displays the network of inventor flows between NAICS 3-digit sectors and the knowledge markets resulting from the application of the Louvain community detection algorithm. Lines denote inventor transitions, with width proportional to the effective undirected inventor flow between sectors. Nodes represent the NAICS 3-digit sectors reported on each node. Black lines depict flows within knowledge markets, while gray lines represent transitions between communities.

2.3 Other Constructed Measures and Aggregation at Census Frequency

Patent Citation Measures For each patent classified by Goldschlag et al. (2016), I count the set of cited patents that belong to the citing patent's assignee. In the case of cited patents with multiple assignees, I consider half a count if the assignee is among them. The share of self-citation is given by this count divided by total citations. I construct five measures to correct self-citations for the assignee's importance in the relevant technology class of cited patents. For each citation made, excess self-citations are defined as $1 - Pr$ (self-citation). The various measures differ on how the probability of self-citation is computed. For the first three measures, I compute this probability as the assignee's share of total patents in the NAICS code attributed to the citing patent. In employ in turn the share of NAICS patents in the year, the previous five years, and the cumulative share from the beginning of the sample. The other two measures are based on the CPC classification at the group and subgroup levels (the lowest levels of detail in the classification). For this measure, the probability of self-citation is constructed for each citation by taking the share of patents by the assignee in the CPC (sub)group and year corresponding to the cited patents.[5]
Finally, I aggregate all measures across assignees in the same NAICS 4-digit code using the number of patents in the relevant NAICS code by each assignee in each year.

I also construct truncation-corrected two forward citation measures and a patent generality measures following the definitions and the procedures described in Hall et al. (2001). The forward citation measures compute the average number of citations received by each firm's patents, giving an indication of the importance of each patent for future technological developments. The correction for truncation is conducted estimating the empirical CDF of the forward citations lag distribution of patents in the relevant CPC 2-digit technology class. The correction is then carried out by dividing the overall amount of forward citations at the latest available date by the inverse of the CDF thus obtained. The procedure suggested by Hall et al. (2001) uses only information pertaining to the CPC 2-digit technology class of the cited patent. In addition to this measure, I also conduct an alternative correction that estimates a separate distribution for each citing CPC 2-digit class and sums the corrected forward citations across all citing classes. Patent generality also measures the technological impact of patents, but rather than focusing on citations it examines the scope of application of the patent. In particular, it computes a measure of the dispersion of citation received across different CPC classes. The higher is this dispersion, the wider is the technological applicability of the patent.[6]

Regulation Data Mercatus RegData provides a count of restrictions imposed on a number of NAICS 4d-digit product markets, obtained by matching a set of keywords in NAICS descriptions to regulatory

[5] This procedure is similar to the approach followed in Akcigit and Kerr's (2018) Appendix C.

[6] The interested reader should consult Acemoglu et al. (Forthcoming) for a detailed discussion, and the related appendix for details on the construction of these measures.

texts, and then taking the best match for each document. However, the available data does not include a set of codes for data quality reasons.
Therefore, I process the description of NAICS 4d codes and compute the cosine-similarity between all pairs of sectors. I build an estimate of sector-relevant restrictions for missing sectors by taking an average weighted by cosine similarity of sectors included in RegData. In particular, I include in the average the five most similar NAICS codes if similarity is larger than .2, and I attribute the regulations of the most similar sector otherwise. I chose this threshold by inspecting the similarity associated to various NAICS pairs, and the assignment of regulations to sectors is not highly sensitive to this choice.

Inventor Distribution Measures I employ the measure of effective inventors constructed as detailed above to compute measures of researchers' concentration within sectors for each year in my sample. Specifically, I use the PatentsView assignee ID to identify firms that employ specific inventors in each sector, and then compute several measures of the concentration of inventors within sectors. I focus in particular on the top 10% and bottom 50% share of inventors, as well as other commonly employed measures of dispersion like the ratio of the 90^{th} quantile to the median. I also compute the Gini coefficient of inventors across CPC classes and NAICS 4-digit, assigning effective inventors to the relevant technology class or NAICS sector, to document the trend in increasing concentration of inventors in specific patent classes and sectors.

Aggregation at Census Frequency Data from the Economic Census are available at five-year frequency for the years 1997-2017, which requires aggregating the other data at the same frequency. Since I am interested in the effect of concentration on the allocation of inventors, I average all variables related to inventors and productivity using the five-year window *starting* in the census year (e.g., 1997-2001 for 1997), while I use concentration measures for the related census year. In the IV regression I use product restrictions as an instrument for concentration, which motivates me to average restrictions in the five-year window *ending* in the census year (e.g., 1993-1997 for 1997). Since Goldschlag et al. (2016)'s matching only covers the period up to 2016, I run all specifications in long-differences over the time frame 1997-2012.

3 Empirical Analysis

This section presents four main findings that apply to the period 1997-2012: (i) effective inventors have become more concentrated across economic sectors; (ii) sectors with increased concentration have attracted a growing share of relevant inventor types; (iii) growth in the share of relevant inventors is negatively correlated with inventor productivity, as measured by forward citations as well as average growth in output per worker divided by effective inventors employed; and (iv) growth in the share of relevant inventors is positively correlated with the share of self-citations and excess self-citations, as well as concentration of inventors at the top within sectors.

Results (i) and (ii) are indicative of a growing concentration of inventors, and establish a positive causal link between the growth in product market concentration and the increase in sectors' inventor share. Findings (iii) and (iv) provide evidence in favor of misallocation. Inventors' concentration in less competitive sectors turns out to be inefficient, since there researchers are predominantly employed on projects that do not contribute to the growth of the sector, which I interpret as defensive innovation. This interpretation is supported by the concentration of inventors among larger firms, the decline in forward citations received by patents obtained by these firms, and the decrease in growth per inventor that accompanies the increase in product market concentration.

The rest of this section proceeds as follows. The first subsection presents my empirical framework and variable definition. Remaining sections present in order results (i)-(iv) above. I discuss the causal interpretation of my results an IV specification in Subsection 3.2.

3.1 Variable Definitions and Main Specification

The main outcomes of interest throughout the analysis refer to measures of inventors concentration or measures of R&D productivity. For the former, I rely on the definition of effective inventors, α_i, provided in Section 2.2, that is productivity-adjusted inventors. For each product market, p, I define the share of inventors employed by the sector in year t as:

$$\text{Inventor Share}_{p,t} \equiv \frac{\sum_{p(i,t)=p} \alpha_i}{\sum_{k(i,t)=k} \alpha_i},$$

where the sum at the numerator is taken over all effective inventors that are mentioned in patents registered in product market p, while the denominator computes the total effective inventors that belong to the knowledge market. Effective inventors, α_i, are also the measure I use to evaluate the dispersion of inventors across sectors and technology classes. My results are robust to computing the inventor share using raw counts of researchers instead of effective inventors.

When analyzing R&D productivity I focus on the three patent-based measures described in Section 2.3, that is, forward citations, share of self-citations, and patent generality. Further, I compute a more direct measure of the productivity of inventors given by the growth in output per worker divided by the number of effective inventors employed by the sector.

In most specifications, the independent variables are measures of concentration and controls for the size of the sector considered. As discussed in Section 2.1, my baseline measure of concentration is the lower bound of the Herfindal-Hirschman Index constructed by Keil (2017) using top sales share reported by the Economic Census for each sector.[7] I label this variable $\underline{\text{HHI}}_{p,t}$, where the line below stands for the lower bound. This choice is motivated by the small number of sectors in my sample that have an HHI index reported by the Economic Census (about 80). Using the lower bound I can extend my analysis to a total of 157 sectors. Due to the high correlation between the two variables, the results are robust to using the Economic Census HHI, as shown in robustness exercises.

I obtain sales variable from the Economic Census, which I deflate using BLS NAICS-specific price indexes. I use sales variables for two purposes. First, real sales in 2012 as the weight in my regressions. Second, I use the logarithm real sales as well as a quartic in real sales in order to control for changes in the size of sectors during my sample period. For the selected subset of sectors that reports the number of establishments, I also explore the robustness of my findings to controlling for sales per establishment.

Given these definitions, my main specification is a sector-level long-difference regression over the period 1997-2012:

$$\Delta \text{Share}_{p,\,2012-1997} = f_k \mathbf{1}\left\{p \in k\right\} \;+\; \beta \Delta \underline{\text{HHI}}_{p,\,2012-1997} \;+\; \gamma' \Delta \text{Size}_{p,\,2012-1997} \;+\; \varepsilon_p, \tag{2}$$

where ΔSharedenotes the change in the inventors' share of product market p, $f_k \mathbf{1}\left\{p \in k\right\}$ is a dummy variable that takes value 1 if the product market, p, belongs to knowledge, k, $\Delta\underline{\text{HHI}}$ is the change in the HHI lower bound, and ΔSize is a set of controls for the size of sector p, which are given by either the change in the logarithm of real sales or real sales per firm, or the change in the terms of a quadratic polynomial in the real sales of the sector.

Regressions are weighted by sector sales in 2012 for the findings which rely on Economic Census sector-level measures, and I estimate robust standard errors in all specifications. When looking at patent measures, I employ the same specification as 2, where I replace the outcome variable with the change in patent productivity and the dependent variable with the change in inventors' share. In this case, since I do not rely on Economic Census measures, I report unweighted regressions.[8] I also discuss the robustness of these findings to adopting the same specification as 2, using the HHI lower bound and weighting by sales.

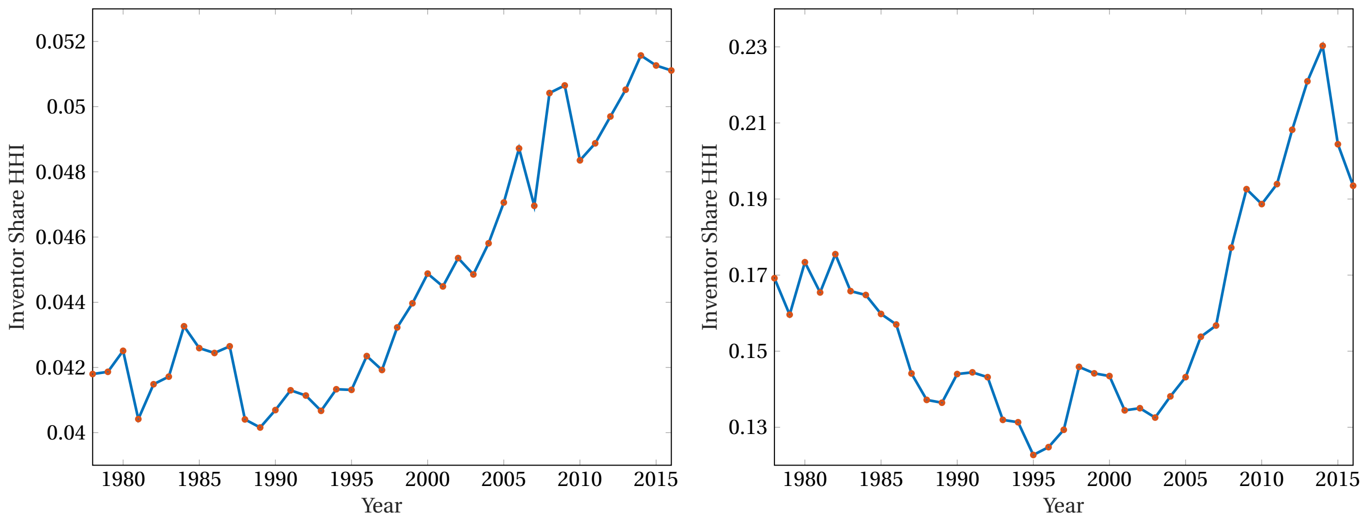
[7] The expression used to obtain this measure is:

$$\underline{\text{HHI}}_{p,t} = 4\left[\frac{\text{CR4}_{p,t}}{4}\right]^2 + 4\left[\frac{\text{CR8}_{p,t} - \text{CR4}_{p,t}}{4}\right]^2 + 12\left[\frac{\text{CR20}_{p,t} - \text{CR8}_{p,t}}{12}\right]^2 + 30\left[\frac{\text{CR50}_{p,t} - \text{CR20}_{p,t}}{30}\right]^2,$$

where "CR{X}" denotes the concentration ratio, that is the share of sales, of the top X firms. This measure is a lower bound, and coincides with the actual HHI if the sector has less than 50 firms, and sales share are distributed equally in each of the top 0-4, 5-8, 9-20, 21-50 brackets. Keil (2017) reports a correlation of $\underline{\text{HHI}}$ with the actual index of 0.93.

[8] As I will show, the change in the inventor share is highly correlated with the change in the HHI, so this specification essentially amounts to a rescaling of the coefficient that would be obtained using the HHI.

Figure 3: Herfindal-Hirschman Index of Effective Inventors across NAICS 4-digit Industries, 1976-2016



(a) Effective Inventors

(b) Inventor Count

Note: This Figure reports the time series of inventor concentration, as measured by the HHI index of inventor shares across NAICS 4-digit sectors. The left panel reports the series constructed using effective inventors as defined in Section 2.2, the right panel uses instead raw inventor counts. Only the NAICS 4-digit sectors with data for all years are included.

3.2 Results

3.2.1 Inventor Concentration across NAICS Sectors has Increased

Figure 3 reports the time series of inventor concentration across NAICS 4-digit industries for the period 1976-2016, where the Goldschlag et al. (2016) series is available. I construct these series using the share of effective inventors (in panel (a)) or raw inventor counts (panel (b)). I use the HHI index of inventor shares accruing to each sector as a measure of concentration. Both panels display an increasing trend in the concentration of inventors starting from the late 1990s. These patterns align closely with the trends reported in Akcigit and Ates (2019), which document a rising share of patents registered by top firms within sectors. Figure 3 extends this findings to the cross-industry allocation of inventors. Quantitatively, the increase in inventor concentration is sizable, corresponding to about a 20% increase in the HHI for the effective inventor measure over the period 1997-2012. Using raw inventor counts increases this number to 30%.

As for the other results presented below, the effective inventor measure and the raw inventor count behave similarly, albeit the series for raw counts is more volatile and exhibits larger changes. This difference in volatility stems from the regression method implied to obtain effective inventors, which accounts for several features through time, firm, and technology class fixed effects.

3.2.2 Markets with Growing Concentration Increased Their Inventor Share

In this section, I present three sets of results for each specification, which differ in the estimation sample to account for extreme observations. In regression tables, “Full Sample” refers to the sample of observations with non-missing observations for all the variables included. I propose two sample selections to rule out that outliers drive the baseline results. “Trim Outliers” refer to a sample which trims the most extreme observations for the outcome and the independent variable separately. I trim the observations that fall beyond three standard deviations from the sample average of each variable, and that are most likely to drive the results estimated using the full sample.[9] “Mahalanobis 5%” denotes the sample where I trim the 5% extreme observations based on the Mahalanobis distance of pairs of observations from the data centroid. Since this procedure is based on the joint distribution of the outcome and independent variable, the sample thus obtained varies in each regression.

Table 2 presents the results of regression (2) where the outcome variable is the change in knowledge-market inventor share, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index discussed above, or the index as reported by the Economic Census. The results in Table 2 highlight a strongly significant positive correlation between the change in the HHI and the change in the share of effective inventors accruing to each NAICS sector. Note that this regression is only partially driven by the contemporaneous correlation between the two variables. As discussed above, the share of effective inventors is average over the five years *starting* in the Economic Census year, while the concentration measures refer to the Economic Census year only.

Two important notes on the scale of the variables are in order. First, here and in all following tables and graphs, all variables which refer to shares or growth rates are reported in percentage points for ease of interpretation. Therefore, for example the coefficient in Column (1) of Table 2 should be interpreted as saying that an increase in one unit of the HHI index leads to an increase in the share of the relevant knowledge market of 27.25 pp. Second, HHI indices are instead constructed to range between 0 and 1. In particular, the HHI lower bound has sales-weighted an average of about .03, and a standard deviation of .032 in 2012. According to Table 2, a standard deviation increase in this measure is associated to a 0.87 pp increase in the share of inventors accruing to the relevant NAICS sector. In comparison, the sales-weighted average share of inventors in 2012 is 1.18%, with a standard deviation of 1.82%, so the estimated effect of a one standard deviation increase in concentration corresponds to about half a standard deviation increase in the share of inventors in the relevant market. Clearly, the estimates using the HHI lower bound tend to be noisier as this is a constructed, and therefore imprecise, measure of concentration. However, the number of available observations is much larger than the actual HHI, allowing me to extend my findings to a larger number of sectors (about double as can be seen from the

[9] I justify the choices for each variable in detail in my replication code using the empirical kernel density and detailed tabulations.

number of observations in each specification).[10]

While suggestive, the correlation presented above is far from causal, as it neglects two fundamental components. First, it does not include controls for the size of the sectors or firms, which could have a confounding and mechanical effect on the share of scientists accruing to a specific sector. Second, it estimates the correlation both across and within knowledge markets. In Table 3, I address these two limitations by restricting the analysis within knowledge markets, and controlling for two measures of size. In the upper panel of Table 3, I use the change in the logarithm of real sales as a measure of the size of each sector, while in the lower panel I present the results when average sales per firm are included as a control. The inclusion of sales per firm is motivated by the fact that there might be significant barriers to entry to R&D, easier to overcome for larger firms, mechanically linking concentration and inventor hiring. Since the Economic Census reports the number of companies only for a subset of firms, the sample used in the lower panel is smaller than the upper panel. The results in Table 3 confirm the positive relation between the change in inventor shares and concentration, and are largely unchanged relative to the estimates in Table 2, suggesting that the correlation does not arise mechanically from factors related to firm or sector size. In particular, these findings imply that sectors with increasing concentration have attracted a rising share of scientists above what would be implied by their expansion in overall sales as well as average firm size.

Figure 4 depicts graphically the residualized observations underlying the estimated coefficients in Columns (2) and (6) of Table 3, Panel (a). The upper panel portrays changes of knowledge-market inventor shares over the change in the HHI lower bound, after partialling out fixed effects for the relevant knowledge market and changes in log real sales, with the marker size proportional to the regression weight. Although the sample displays some observations that appear extreme, the bulk of observations—and especially weighted observations—falls on the regression lines, mitigating the concerns that a few outliers might drive the results. In any event, I explore the robustness of the results to the exclusion of non-residualized observations, both manually and defining extreme observations based on the Mahalanobis distance. Importantly, this exercise reveals that the observations that appear extreme in the residualized scatter are not unusual when considering the marginal or joint distribution of non-residualized outcome and independent variable. The bottom panel of Figure 4 reports the binned scatter plot corresponding to the sample where the 5% extreme observations according to the Mahalanobis distance have been removed, and confirms that the positive relation between concentration and inventor shares is not driven by a few extreme observations. In particular, the corresponding regression results in Table 3(a), Column (6), show that the estimated coefficient is significant at a 5% confidence level. The results presented in this section are robust to using the raw number of inventors to compute the share of researchers captured by each product market.

Appendix Table 11 shows estimates using the share of effective inventors of each product market

[10] Regressions using the Economic Census HHI not reported in the main text or the Appendix are available on request.

over the total. This amounts to neglecting the fact that inventors flow only across sectors that can employ their skills. In this specification, I find a significant, albeit small, effect of product market concentration on the share of inventors. However, this result only arises when the sample is trimmed to remove outliers. This is unsurprising in light of the discussion in Section 2.2, whereby mismeasuring the labor market for inventors should bias the estimates of inventor mobility towards zero, since many of the sectors would actually not be routinely connected by inventor flows. Finally, this result also conforms with the findings in Table 3, which show that including knowledge-market fixed effects does not alter the coefficients significantly, suggesting that flows across knowledge markets are indeed negligible.

Appendix establishes the robustness of all the findings in this section to the use of raw inventor counts rather than effective inventors to compute both inventor shares and knowledge markets.

IV Results I now present IV results that suggest that the relation between concentration and inventor shares is causal. Indeed, more concentration might just be the result of an increase in technological entry barriers, established by incumbents through an increase in their R&D inventors. In this scenario, the causality would flow from increased inventor shares to higher concentration. Above, I tried to mitigate this concern using the the average share of inventors following the Economic Census years to which the HHI refers as my outcome variable. However, reverse causality could still be present if the autocorrelation of inventor shares is sufficiently high. This motivates me to produce 2SLS estimates that instrument the change in the HHI lower bound with changes in product market restrictions, as measured by the Mercatus dataset RegData 4.0. Theoretically, an increase in restrictions should raise barriers to entry in affected product markets, thus leading to higher concentration. As discussed below, this proves to be the case empirically, making a case for the validity of restrictions as an instrument for concentration. A violation of the exclusion restriction requires a causal connection between product market regulations and the share of inventors hired by each sector, which acts independently of product market concentration. A possibility in this sense is the increase in the number of inventors required to fulfill product market restrictions, if such regulations specifically affect technologies currently in use in the industry. However, this effect should be both large and persistent to be captured by my measure of inventor shares. Further, while RegData certainly include product restrictions, there are also a number of regulatory burdens that are not related to technological components, like reporting obligations and other legal burdens. In addition, while product restrictions might certainly induce a change in the direction of innovation, there is no a priori reason to believe that the scale of innovation activity should also increase. These considerations lead me to believe that the exclusion restriction is not highly likely to be violated.

The results of the 2SLS estimation are presented in the upper panel of Table 4. The specification is the same as in Column (2) of 3, including both knowledge market and sale change fixed effects. The

Table 2: Regressions of Change in 4-digit Knowledge Market Share over Change in HHI Measures, Long-Differences, 1997-2012

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\underline{HHI}$	27.293*		27.183*		27.326*	
	(11.569)		(11.941)		(11.620)	
Δ HHI		22.399***		22.399***		22.350***
		(6.345)		(6.345)		(6.343)
Knowledge Market FE						
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	80	155	80	150	71

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (2), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Economic Census concentration ratios, or the HHI index reported in the Economic Census. "Full Sample", "Trim Outliers" and "Mahalanobis 5%" refer to the samples described in the main text.

Table 3: Regressions of Change in 4-digit Knowledge Market Share over Change in HHI Lower Bound, Long-Differences, 1997-2012

(a) Controlling for Change in Log Real Sales

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \underline{HHI}$	26.093*	22.509*	25.904*	22.716*	26.111*	22.554*
	(10.696)	(10.848)	(11.124)	(10.948)	(10.725)	(11.019)
Δ log Sales	0.914**	0.548*	0.881**	0.539*	0.918**	0.562*
	(0.278)	(0.243)	(0.275)	(0.242)	(0.283)	(0.261)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	153	155	152	150	139

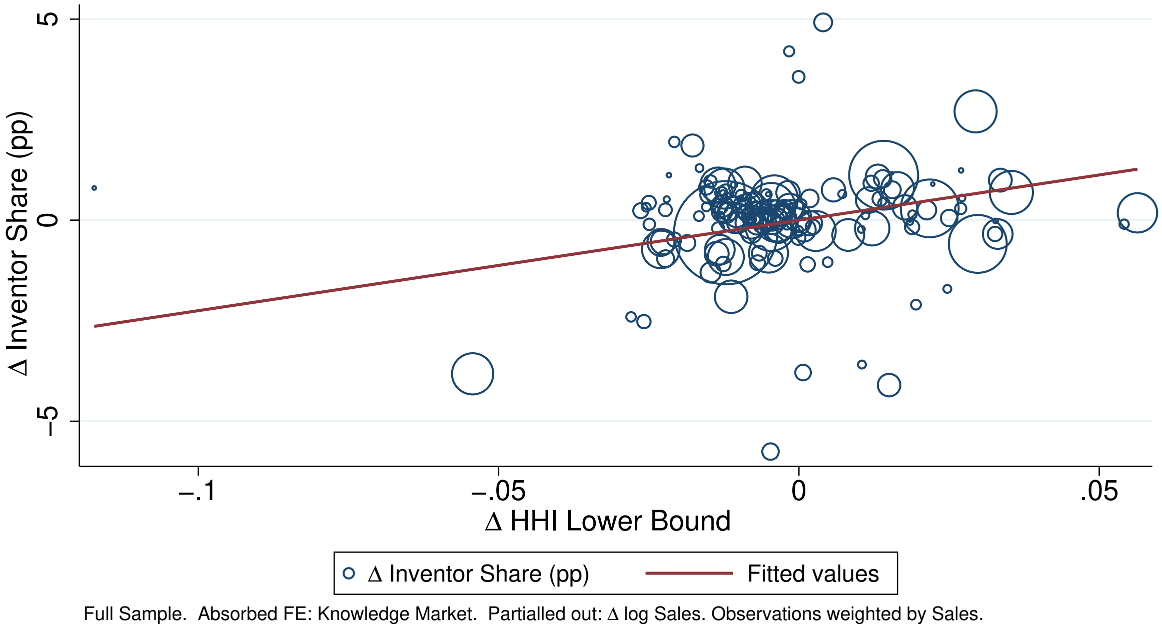
(b) Controlling for Change in Log Real Sales per Company

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \underline{HHI}$	35.230**	20.783+	35.230**	20.783+	35.154**	22.854*
	(12.759)	(10.615)	(12.759)	(10.615)	(12.647)	(11.197)
Δ log Size	0.175	-0.040	0.175	-0.040	0.300	-0.055
	(0.382)	(0.253)	(0.382)	(0.253)	(0.460)	(0.346)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	81	79	81	79	75	67

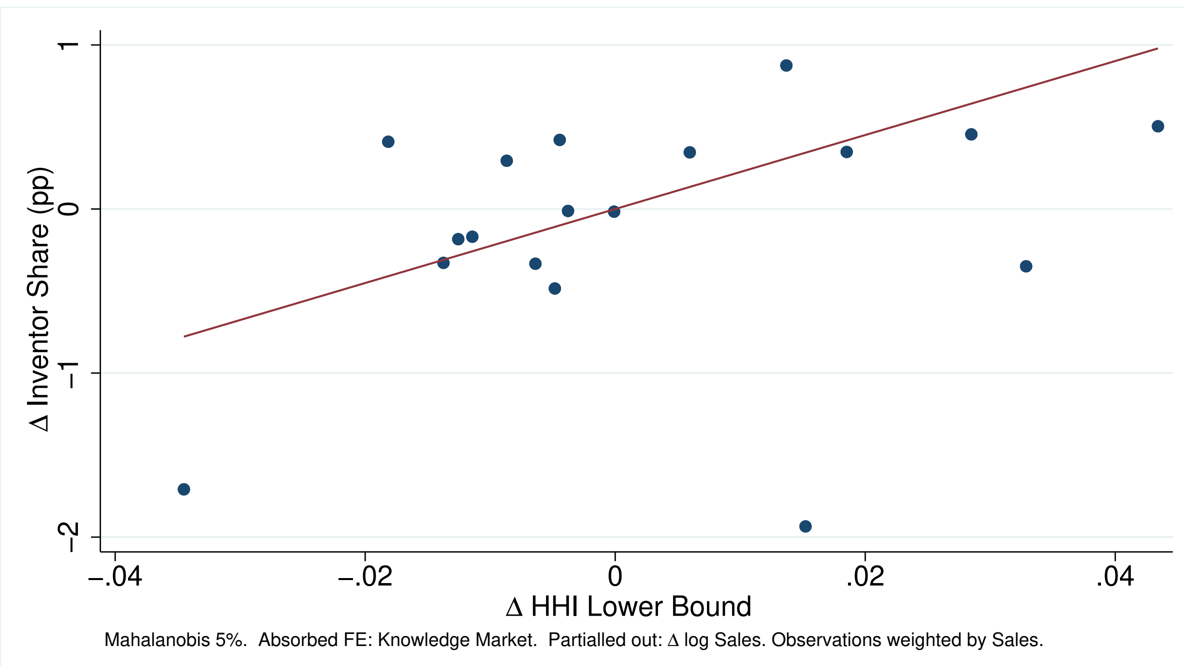
Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (2), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. "Full Sample", "Trim Outliers" and "Mahalanobis 5%" refer to the samples described in the main text.

Figure 4: Residualized Scatter Plots Corresponding to Selected Columns in Table 3, Panel (a)

(a) Raw Scatter Plot, Specification in Column (2)



(b) Binned Scatter Plot, Specification in Column (6)



Note: This figure presents residualized scatter plots of the change in the share of effective inventors of sector p over total inventors in knowledge market k, over the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. The upper panel reports the data for the full sample, where both variables are residualized by change in log real sales and knowledge market fixed effects. The size of the markers is proportional to the weight of each observation in the regression (sector sales in 2012). The regression line uses the coefficient on the change in HHI lower bound in Column (2) of Table 3. The lower panel presents a binned scatter plot removing the observations with the highest 5% Mahalanobis distance from the sample centroid. Observations are aggregated using sales weights and the regression line is from Column (6) of Table 3.

2SLS estimates confirm the significance of concentration changes for the increase in knowledge market inventor shares. The magnitudes of estimated coefficients are statistically indistinguishable from the ones reported in the baseline regression. The first-stage F clearly indicates that instruments are weak. This is unsurprising since, as detailed above, both the HHI lower bound and the regulation measures are constructed and therefore imprecise. In particular, I had to impute regulations for a large part of the sample using the cosine-similarity between product market restrictions.[11] However, instruments are not irrelevant. The results in the lower panel of Table 4 imply that the first-stage t-statistic for the regression of the change in the HHI lower bound over log-regulations is 2.07, which corresponds to a p-value of 0.041. The reduced form regression of inventor share over log restriction change is equally highly significant. Accordingly, the SW underidentification test rejects the null hypothesis at a 5% confidence level. Given the weakness of the instruments, I also report the Anderson-Rubin p-value, which confirms that the coefficient is 5% significant, and the corresponding confidence intervals in brackets.

Taken together, the results presented in this section establish a causal link between the increase in inventor concentration and the shifts in product market concentration across NAICS 4-digit sectors.

3.2.3 Sectors that Attracted More Researchers Saw Increasing Top Firms' Inventor Shares and Falling Patent Forward Citations

While the findings presented so far establish a connection between inventor and product market concentration, they do not establish that changes in the distribution of researchers across sectors are inefficient. In particular, it would not be unreasonable to think that more concentrated sectors saw increased entry as a result of the higher rents captured by incumbents. Table 5 shows that the opposite is occurring. Specifically, the share of effective inventors accruing to top inventor-hiring firms has increased in the sectors that attracted more inventors over the period considered, relative to firms with less inventors in the sector. This finding is consistent across a variety of measures displayed in Columns (1) to (6). These findings suggest that inventors have increasingly concentrated among large incumbents, that is, sectors that increased their inventor share also saw a *within-sector* increase in inventor concentration.

Throughout this section, I present results using changes in inventor shares to focus directly on the correlation between inventor transitions and their within-sector distribution. Unless otherwise noted, these findings are robust to using the change in the HHI rather than the inventor share, as should be expected from the strong correlation between these two variables reported in previous tables. For this section, and other patent-based measure, I present robustness results employing this alternative specification in Appendix B.4.

[11] Using only available sectors requires dropping two thirds of the observations. See Appendix A for details on data construction.

Table 4: IV Regressions of Change in 4-digit Knowledge Market Share over Change in HHI Lower Bound, 2SLS Long-Difference, 1997-2012

(a) 2SLS Results

	Δ Inventor Share (pp)	
	(1)	(2)
$\Delta\underline{\text{HHI}}$	32.426+	30.096+
	(16.987)	(15.819)
	[4.850, 99.013]	[4.415, 92.104]
Δ log Sales		0.525*
		(0.247)
		[0.525, 0.525]
Knowledge Market FE	✓	✓
Sample	Full Sample	Mahalanobis 5%
Weight	Sales	Sales
Observations	157	150
First-Stage F	4.65	4.75
Anderson-Rubin p-value	.0298	.0321

(b) First Stage and Reduced Form

	Δ Inventor Share (pp)	$\Delta\underline{\text{HHI}}$
	(1)	(2)
Δ log Restrictions	0.478*	0.016*
	(0.220)	(0.007)
Δ log Sales	0.539+	-0.000
	(0.274)	(0.005)
Knowledge Market FE	✓	✓
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	153	153

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. These Tables presents the results of specifications (2), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Economic Census concentration ratios, instrumented by the change in log-restrictions relevant to the NAICS sector. The lower panel present first-stage and reduced-form relations. “Full Sample” and “Mahalanobis 5%” refer to the samples described in the main text.

My next finding suggests that this trend might be driven by a rise in defensive innovation, that is R&D aimed at sheltering the incumbents' dominant position and raising barriers to entry. Table 6 shows that inventors' concentration in specific sectors has gone hand in hand with a fall in patents' forward citations in these sectors, a standard measure of the impact of patents and their role in paving the way for further innovations (Hall et al., 2001). The result in Columns (1) and (2) report two different measures of forward citations, that differ in how the series are corrected for truncation. As discussed in Section 2.3, one measure (Column (2)) uses the procedure delineated by Hall et al. (2001), computing the forward citation lag distribution conditioning on the technology class of the cited patent, while the other (Column (1)) also conditions on the technology class of citing patents. Column (3) presents the estimates relative to patent generality, another measure of patent impact, which increases with the scope of application of the patent. The regressions in this table are unweighted since they rely only on patent data, but results are robust to using the HHI as a regressor and weighting by sales. I present results for the full sample, as well as restricting to the middle range of changes in inventor shares, which contains more than 90% of the observations. In both samples, I find a highly significant negative relation between changes in inventor shares and the fall in forward citations. The coefficients imply a high semi-elasticity of self citations to changes in the inventor shares, whereby a 1pp increase in the share of inventors leads to an average 0.2-0.5% reduction in forward citations. When dropping extreme observations, I also find a significant decrease in the generality of the patents, indicating that concentrating sectors produce less widely applicable patents. This effect is however not robust to estimating the regression using the HHI as the independent variable.

The fall in forward citations is a first indication of the presence of defensive innovation, aimed primarily at barring entrants from developing new technologies (see, e.g., Guellec et al., 2012). In the next section, I show that these patents also appear to produce limited benefits in terms of ensuing productivity growth, as measured by output per worker growth. This provides further support to my interpretation.

Before moving to the results on productivity, I investigate a competing explanation for my findings on output growth. As highlighted by Acemoglu et al. (Forthcoming) and Akcigit and Kerr (2018) among others, large incumbents have a strong incentive to focus on improving their own products at the expense of broadly applicable innovation. In the words of the authors, internal and incremental innovation prevails on more radical, external innovations. This mechanism would also imply that an increase in incumbents' share of R&D resources leads to falling innovation productivity. In order to asses the importance of this channel, and in keeping with the analysis in Akcigit and Kerr (2018), I use the share of self-citations to measure the extent of internal innovation conducted by firms. Table 7 displays the results pertaining to this measure. All columns use as dependent variable the change in excess log self-citations as defined in Section 2.3. Columns (1) and (2) build excess self-citations correcting for the importance of firms' patents for the CPC group, which reflects the technological classification of the patent. Columns (3) and (4) use the more narrowly-defined CPC subgroups for robustness. Coefficients are mostly non-significant, and turn negative when knowledge market fixed effects are included. Column (3) displays a marginally significant coefficient. However, this result is not robust to using the HHI as regressor and weighting regressions by sales as in the baseline specification. The findings in this table suggest that incremental innovation does changes significantly following changes in sector concentration, reducing the scope for this alternative explanation.

Table 5: Regressions of Change in Inventor Distribution Measures over Change in 4-digit Knowledge Market Share, Long-Difference, 1997-2012

	Δ 90/50 Quantile Ratio	Δ Top 10%/Bottom 50%	Δ Top-50/Bottom-50 Share Ratio	Δ Top 10%	Δ Bottom 50%
	(1)	(2)	(3)	(4)	(5)
Δ Inventor Share (pp)	0.211+	0.243*	0.314+	0.018**	-0.008*
	(0.107)	(0.097)	(0.184)	(0.006)	(0.004)
Δ log Sales	-0.100	0.328	0.147	0.026	0.005
	(0.122)	(0.294)	(0.316)	(0.020)	(0.007)
Knowledge Market FE	✓	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales	Sales	Sales
Observations	118	118	118	118	118

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. Please refer to notes in Table 3 for further details. Column (1) uses the ratio in the 90 percentile of effective inventors to the median as the outcome variable. Columns (2) and (3) instead present the share ratio, that is the share of effective inventors accruing to the top 10 or 50% relative to the share accruing to the bottom 50% of the distribution within each NAICS sector.

Table 6: Regressions of Changes in Forward Citation over 4-digit Knowledge Market Share, Long-Differences, 1997-2012

(a) Full sample

	Δ log Citations/Patent (CPC)	Δ log Citations/Patent (Total)	Δ Patent Generality
	(1)	(2)	(3)
Δ Inventor Share (pp)	-0.197***	-0.227***	-0.004
	(0.044)	(0.051)	(0.004)
Δ log Sales	-0.234*	-0.258+	0.008
	(0.112)	(0.148)	(0.013)
Knowledge Market FE	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample
Weight			
Observations	153	153	153

(b) Full sample, restricting to the middle range of the change in inventor shares (−2% to +2%)

	Δ log Citations/Patent (CPC)	Δ log Citations/Patent (Total)	Δ Patent Generality
	(1)	(2)	(3)
Δ Inventor Share (pp)	-0.545***	-0.618***	-0.025*
	(0.113)	(0.137)	(0.012)
Δ log Sales	-0.232*	-0.255+	0.008
	(0.109)	(0.146)	(0.012)
Knowledge Market FE	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample
Weight			
Observations	144	144	144

Note: Unweighted regressions; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. These Tables presents the results of specification (2), when the outcome is the log-change in forward citations and the change in patent generality in sector p over the change in the share of inventors employed in sector p. Column (1) and (2) presents the results when forward citations are extrapolated the procedure Hall et al. (2000) to avoid truncation bias. A specific cite-lag distribution over 35 years is estimated for each pair of cited and citing CPC2-codes. Column (1) employs the extrapolation scheme by each pair of CPC2 cited and citing sector. Column (2) applies the extrapolation scheme to total citations received by each cited patent. Column (3) presents results on the patent generality measures. All columns exclude self-citations. Upper panel: full sample; Bottom panel: excluding sectors with absolute increase in the inventor share above 2%.

Table 7: Regressions of Change in Excess Self-Citations over 4-digit Knowledge Market Share, Long-Differences, 1997-2012

	Δ CPC group self-citations		Δ CPC subgroup self-citations	
	(1)	(2)	(3)	(4)
Δ Inventor Share (pp)	0.920	-0.444	0.958+	-0.228
	(0.711)	(1.083)	(0.512)	(0.801)
Δ log Sales	-1.841	-1.954	-1.456	-1.674
	(1.925)	(1.988)	(1.326)	(1.279)
Knowledge Market FE		✓		✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample
Weight				
Observations	157	153	157	153

Note: Unweighted regressions; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (2), when the outcome is the change in excess self-citations in sector p over the change in the share of inventors employed in sector p.

3.2.4 Markets with Growing Inventor Shares Experienced a Fall in Inventor Productivity

Table 8 presents the results of running regression (2) when the outcome is the average growth in output per worker per effective inventor. I use growth in annual output per worker provided by the Economic Census and average this measure over the five-year window starting in the Economic Census year, and I proceed analogously to build a measure of average effective inventors over the same period. Inventor productivity is then defined as average output per worker growth divided by average effective inventors. Both the outcome and the dependent variable are measured in percentage points. Table 8 reveals a negative and significant correlation between the increase in the effective inventors' change and inventor productivity, regardless of the independent variable employed and the sample restriction adopted.

The magnitude of estimated coefficients can be grasped considering the scale of the variables and their changes over the sample period. Starting from the upper panel of Table 8, the median change in the share of effective inventors over the period was .014pp, while the measure of effective inventors has a median of 2018.[12] Using the coefficient in Column (5) to predict the median annual change in labor productivity growth implied by rising inventor concentration amounts to a fall of .15pp $(-.005 \times .014pp \times 2018)$. This number increases to . 28pp when using the statistics relative to sectors with positive growth in labor productivity only, which accounted for the bulk of the increase in inventor shares. An alternative back-of-the-envelope computation, using the change in product market concentration to predict the change in inventor shares gives even starker results. Using the coefficient in Column (2) of Table 3(a), and considering a median change in the HHI of 0.002 yields an increase in the share of effective inventors in concentrating sectors of 0.045pp, which implies a fall

[12] Recall that effective inventors in each year are measured as the sum of inventor fixed-effects in each year, and therefore do not represent the simple count of inventors.

in average labor productivity implied by misallocation of 0.45pp. While these numbers might appear sizable considering the entirety of the economy, it is worth noting that the sample I have data for includes mainly manufacturing and retail sectors, which experienced a sizable reduction of about 2.8pp in average annual productivity growth from 1997-2012, driven by a steep decline in output per worker growth in manufacturing. Therefore, the mechanism I propose would explain around 15% of the observed decrease in output per worker growth in these sectors.

The estimates in the lower panel of Table 8, which uses the HHI instead the change in inventor shares as independent variable, imply even larger growth effects. Using the estimates in Column (2), a median HHI change of 0.02 and a median number of effective inventors of 1421 in sectors with growth in inventor shares implies a −0.78pp change in output per worker growth from misallocation, with a confidence interval ranging from −0.13 to −1.45pp. The midpoint of these estimates would explain 27% of the observed fall in output per worker growth over the sample period, with bounds ranging from around 5% to about 50%.

This last set of results provides further support to the hypothesis that defensive innovation increased in concentrating sectors. Indeed, a central feature of this type of R&D activity is that it does not produce substantial growth, since it is aimed at hampering innovation projects by potential entrants. In this sense, patents are registered in order to prevent others from doing so and implementing inventions that would threaten the incumbents' dominant position. This interpretation also resonates with the finding in Argente et al. (2020), who note that incumbent firms tend to register a large number of patents, but account for a small share of overall innovations; a finding they also interpret as evidence of defensive innovation.

Table 8: Regressions of Changes in Inventor Productivity over Changes in Inventors' Share and HHI, Long-Difference, 1997-2012

(a) Change in Inventors' Share as Independent Variable

	Δ Growth/Inventor (pp)			
	(1)	(2)	(3)	(4)
Δ Inventor Share (pp)	-0.007**	-0.005*	-0.007**	-0.005*
	(0.002)	(0.002)	(0.002)	(0.002)
Δ log Sales		-0.051*		-0.054*
		(0.021)		(0.021)
Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales
Observations	101	101	96	93

(b) Change in HHI as Independent Variable

	Δ Growth/Inventor (pp)			
	(1)	(2)	(3)	(4)
Δ<u>HHI</u>	-0.332**	-0.292*	-0.332**	-0.290*
	(0.113)	(0.123)	(0.114)	(0.126)
Δ log Sales		-0.052*		-0.053*
		(0.021)		(0.022)
Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales
Observations	101	101	98	94

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. Please refer to notes in Table 3 for further details. Inventor productivity is measured as the average growth in output per worker over the five years starting in the Economic Census year over the total number of effective inventors in each sector. The upper panel presents estimates when the independent variable is the change in the share of inventors accruing to a sector, while the bottom panel uses the change in the lower bound of the HHI index.

4 Model

This section presents a Schumpeterian model based on Abrams et al., featuring growth through creative destruction by entrants, as well as the possibility for incumbent monopolist of researching a defensive technology that increases research costs for entrants. I first present a single-sector model to clarify the mechanism at play within each sector in the economy and study the properties of a constant-growth equilibrium analytically. I derive sufficient conditions under which markup increases lower R&D productivity, and show that this occur only if the distribution of inventors shifts in favor of incumbent firms carrying out defensive innovation.[13] I also show that in this model, when the supply of inventors

[13] In the empirical analysis, I used the HHI as a measure of concentration and market power. Appendix B.5 shows that the Lerner Index from NBER-CES, a standard measure of markups, is strongly correlated with the HHI in my sample, justifying the reduced-form mapping I adopt in this section.

is rigid, inventors' productivity is unaffected by markup changes. Then, I move to consider a two-sector model, where each sector is identical to the single-sector model, and the supply of inventors is perfectly rigid, which shuts down within-sector misallocation occurring independently of inventors' movements across sectors. I show that increasing markups in one of the two sectors of the economy lead to a misallocation of R&D resources towards defensive innovation in the less competitive sector. Finally, I study the optimal allocation of R&D subsidies needed to achieve maximum growth in a calibration of the two-sector model that matches moments of the R&D distribution in 1997, the starting year for my empirical analysis.

All omitted proofs are reported in Appendix C.

4.1 Single-sector Model

4.1.1 Preferences and production

Consider the following continuous time economy with a single final good. There is a representative household with King-Plosser-Rebelo preferences over consumption and R&D labor:

$$\mathbb{E}_t \int_t^{\infty} \exp\left(-\rho\,(s-t)\right) \left(\ln C_s - \frac{\chi \left(L_s^{RD}\right)^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right) \mathrm{d}s, \tag{3}$$

where ϕ is the Frisch labor supply. In addition, the representative household inelastically supplies L units of production labor.[14] The representative household owns a differentiated portfolio of all the firms in the economy, with rate of return r_t, and receives a wages w, w^{RD}, for each unit of production and research labor, respectively. I assume that the economy is closed and that the final good is only used for consumption, $C_t = Y_t$. The above utility function yields the Euler equation:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho,$$

as well as a R&D labor supply:

$$L_t^{RD} = \left(\frac{w_t^{RD}}{\chi C_t} \right)^{\phi}.$$

The consumption good in the economy in each instant is a Cobb-Douglas aggregate of a measure-one continuum of products:

$$\ln Y_t = \int_0^1 \ln y_t(i) \mathrm{d}i \tag{4}$$

The consumption good in the economy is taken as the numeraire. The market for each product $y_t(i)$

[14] While this assumption is not necessary for the results to hold, it simplifies the analysis considerably. In the following section, I will consider both production and research labor as given by a fixed endowment in the constant growth equilibrium of the economy. In that case, the assumption is equivalent to assuming that both labor endowments grow at a constant rate.

consists of an incumbent and a fringe of competitors. In what follows, I focus on a single market, dropping the argument i. Each agent j in the sector has the linear production technology:

$$y_{j,t} = c_{j,t} l_{j,t},$$

where $c_{j,t}$ denotes the labor requirement for agent j to produce a unit of output, and $l_{j,t}$ denotes the production labor employed by the firm. The incumbent and competitors produce undifferentiated goods, and differ in their labor requirement. Competitors have labor requirement, $c_{e,t} = c_t$, while the incumbent faces a lower unit labor requirement $c_{I,t} = \frac{c_t}{\phi}$, with $\phi > 1$. The incumbent maximizes profits by choosing a price p_t for her product. Profit maximization gives an optimum limit price $p_t = w_t c_t$, which leads her to capture the entire market realizing profits:

$$\Pi_t = \left(c_{e,t} - c_{I,t}\right) w_t y_t = \left(\frac{\phi - 1}{\phi}\right) c_t w_t y_t.$$

Therefore, the incumbent acts as a monopolist, charging a markup $\phi > 1$ on its marginal cost. By the Cobb-Douglas assumption on the final good, the demand facing each product line is:

$$y_t = \frac{Y_t}{p_t} = \frac{Y_t}{w_t c_t}.$$

Therefore, equilibrium normalized profits read:

$$\frac{\Pi_t}{Y_t} \equiv \pi = \left(\frac{\phi - 1}{\phi}\right).$$

4.1.2 Innovation

Both incumbents and entrants can conduct innovation activity that, if successful, reduces their unit costs to

$$c_{I,t+\Delta_t} = \frac{c_{e,t}}{\left(1+\eta\right)\phi}, \quad \eta > 1$$

Here, η parametrizes the percentage increase in productivity for the innovating firm, relative to the technology previously operated by the incumbent. I assume that there are spillovers from realized innovations as follows. Whenever either the incumbent or an entrant realize an innovation, all other firms gain access to a technology with unit costs

$$c_{e,t+\Delta_t} = \frac{c_{e,t}}{\left(1+\eta\right)}.$$

These assumptions imply that, if entrants realize an innovation, they outcompete previous incumbents, and become the new monopolists. Displaced incumbents join the pool of entrants and from instant $t+\Delta t$ onwards operate the technology $c_{e,t+\Delta_t}$. This amounts to assuming that the incumbent's technology becomes obsolete after displacement.[15] Technically, this innovation structure allows to avoid keeping track of the number of realized innovations. Indeed, in each product market the relative productivity of incumbents relative to entrants is fixed at ϕ, making it possible to formulate the choice of innovation as a recursive problem.

Incumbents' and entrants' innovation differ in two respects. First, successful incumbents' R&D produces a *patent wall* of size $\omega > 1$, which decreases the success probability of entrants' innovations. Second, successful entrants' R&D results in an implemented innovation with certainty, while incumbents adopt new technologies with probability, $\lambda \in [0,1]$. This parameter can be interpreted in two ways. The first interpretation is that λ captures the probability that the newly discovered technology is compatible with the incumbents' current production techniques. The second interpretation is related to the radical nature of incumbents' innovations. A low value for λ reduces the expected productivity improvement from an innovation by the incumbent. In other words, the lower λ, the more incremental are incumbents' innovations. Under these assumptions, incumbents *always* obtain a patent wall, but they only implement their innovations with probability λ.[16]

Following Acemoglu and Akcigit (2012), I assume that innovation investments consist in the choice of an arrival rate of new discoveries, x_I, and that R&D costs are increasing and convex in this arrival rate. In particular, I specify incumbents' R&D costs as:

$$C\left(x_I; w^{RD}\right) = \alpha_I \frac{x_I^{\gamma}}{\gamma} w^{RD},\ \gamma > 1,$$

where the term, $\alpha_I \frac{x_I^{\gamma}}{\gamma}$, indicates the amount of inventors that the incumbent needs to obtain innovations with a flow probability, x_I,and w^{RD} is the wage paid to inventors. For simplicity, I assume that incumbents can only have one available innovation at a time. That is, incumbents can only erect *one* patent wall of size $\omega > 1$, and cannot invest in further innovation until this wall is destroyed at a rate, δ, which captures the rate of patent expiration.

Given these assumptions incumbents' values at any given instant are just a function of the state of the patent wall in the product market they operate, $\Omega \in \{1, \omega\}$. Given the recursive nature of the

[15] An alternative interpretation of this assumption is that incumbents are forced to scrap the assets needed to operate the innovative technology upon destruction, so that the technology is no longer available to them from the following instant onwards.

[16] This specification connects to the empirical evidence in Argente et al. (2020), who show that large incumbent firms tend to produce a large amount of patents, but implement a small number of product innovations.

problem, I drop time indexes in what follows. Incumbents' value functions read:

$$r V(1) - \dot{V}(1) = \max_{x_I} \left\{ \left(\frac{\phi - 1}{\phi} \right) Y - \alpha_I \frac{x_I^{\gamma}}{\gamma} w^{RD} + x_I \left(V(\omega) - V(1) \right) - x_{e,1} V(1) \right\}, \tag{5}$$

$$r V(\omega) - \dot{V}(\omega) = \left(\frac{\phi - 1}{\phi} \right) Y + \delta \left(V(1) - V(\omega) \right) - x_{e,\omega} V(\omega), \tag{6}$$

where $x_{e,1}$, $x_{e,\omega}$ denote entrants' innovation intensities, r is the interest rate in the economy to be determined in equilibrium, and δ the rate of patent expiration. The first line displays the flow value to incumbents that operate in a market not protected by a patent wall. There, incumbents realize instantaneous profits, $\left(\frac{\phi-1}{\phi}\right) Y$, and choose their innovation intensity, x_I, taking the researchers' wage, w^{RD}, as well as the entrants' innovation intensity $x_{e,1}$, as given. If entrants are successful at rate $x_{e,1}$, incumbents are destroyed. If incumbents' innovation is successful at rate, x_I, they obtain the patent wall ω, which grants them the protected value, $V(\omega)$. When a patent wall is in place, incumbents realize the same flow profits as in the unprotected state, since economy-wide spillovers imply that incumbents are unable to reap profits from implemented innovations. However, incumbents face a different entrants' innovation intensity, $x_{e,\omega}$, which is lower than $x_{e,1}$ due to the patent wall in place, as I will show below. Finally, incumbents in state ω face a flow probability δ that the patent wall is exogenously destroyed, in which case they transition back to the unprotected state. Under these assumptions, the optimal incumbent's R&D decision is given by:

$$x_I = \mathbf{1} \left\{ V(\omega) - V(1) > 0 \right\} \left(\frac{V(\omega) - V(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma - 1}}. \tag{7}$$

Following Abrams et al., I assume that each market has a mass of atomistic entrants, indexed by j, who face innovation costs that feature congestion externalities:

$$C\left(x_{e,\Omega,j}; w^{RD}\right) = \zeta \Omega x_{e,\Omega,j} x_{e,\Omega} w^{RD}.$$

In this specification, ζ parametrizes the inventor requirement to obtain a unit aggregate entrants' innovation rate when the market is not protected by patent walls, $\Omega = 1$. Individuals costs are linear in the total entrants' research intensity in the product market, $x_{e,\Omega} \equiv \int_{\mathcal{J}} x_{e,\Omega,j} \mathrm{d}j$. In other terms, individual entry costs increase with the aggregate entry rate. I assume that successful entrants obtain a new unprotected technology, regardless of the state of the market that they target, capturing the fact that entrants do not obtain the patents deposited by the previous incumbents, instead implementing entirely new production techniques.

Under the above assumptions, the free entry condition for entrants targeting a market with patent wall Ω reads:

$$\max_{x_{e,\Omega,i}} \left\{ x_{e,\Omega,i} V(1) - \zeta \Omega x_{e,\Omega,i} x_{e,\Omega} w^{RD} \right\}.$$

This condition pins down the entry rate for each product market with patent wall Ω as:

$$x_{e,\Omega} = \frac{V(1)}{\zeta \Omega w^{RD}}, \quad \Omega \in \{1, \omega\}. \tag{8}$$

This expression clarifies the effect of defensive innovation in this model. The size of the patent wall ω represents the factor decrease in the entry rate when a defensive innovation is successful.

4.1.3 Equilibrium with Constant Growth

The laws of motion of product markets across protected and unprotected states is given by:

$$\dot{\mu}_1 = -\left(x_I + x_{e,1}\right)\mu_1 + \delta \mu_\omega + x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}, \tag{9}$$

$$\dot{\mu}_\omega = -\left(x_{e,\omega} + \delta\right)\mu_\omega + x_I \mu_1, \tag{10}$$

where $\mu_{e,\omega}$ and $\mu_{e,1}$ denote the mass of entrants targeting protected and unprotected markets, respectively. Equation (9) states that the mass of unprotected products decreases when incumbents successfully develop an innovation at flow probability, x_I, or entrants displace existing incumbents at rate, $x_{e,1}$. Products instead become unprotected if existing patent walls depreciate, or successful entrants become new monopolists. Conversely, Equation (23) shows that protected markets loose mass whenever entrants displace existing protected incumbents or defensive patents depreciate, and gain mass when incumbents in unprotected markets develop defensive innovations.

The mass of entrants in the two types of product markets is determined in equilibrium following the laws of motion:

$$\dot{\mu}_{e,1} = -\left(x_{e,1} + x_I\right)\mu_{e,1} + x_{e,1}\mu_1 + \delta \mu_{e,\omega}, \tag{11}$$

$$\dot{\mu}_{e,\omega} = -\left(x_{e,\omega} + \delta\right)\mu_{e,\omega} + x_{e,\omega}\mu_\omega + x_I \mu_{e,1}. \tag{12}$$

Here, the first line states that the pool of entrants in unprotected markets looses mass if entrants successfully develop innovations at rate, $x_{e,1}$, or incumbents make markets protected at rate, x_I. The pool of entrants in markets with $\Omega = 1$ instead grows when incumbents get displaced by successful entrants, and rejoin the ranks of outsiders, or previously protected markets loose their defensive walls. Equation (12) is obtained analogously. Given research intensities, clearing in the labor markets for production and research labor require:

$$L = \int_0^1 l(i)\mathrm{d}i, \tag{13}$$

$$L^{RD} = \zeta\left(\omega x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}\right) + \alpha_I \frac{x_I^\gamma}{\gamma}\mu_1. \tag{14}$$

A constant growth equilibrium of this economy is defined as follows.

Definition 4.1 (Constant Growth Equilibrium)**.** A constant growth equilibrium is a sequence of values $\{V_t(1), V_t(\omega)\}$, production workers' and inventors' wage sequences $\{w_t^{RD}, w_t\}$, and incumbents' and entrants' R&D decisions $\{x_{I,t}, x_{e,1,t}, x_{e,\omega,t}\}$ such that, given an endowment of production labor, L, L^{RD}: (i) incumbents maximize values (5) and (6), taking entrants R&D decisions as given, (ii) entrants' R&D decisions satisfy (7) and (8) taking $V_t(1)$ as given, (iii) the distribution of incumbents and entrants across protected and unprotected markets is constant, $\dot{\mu}_1 = \dot{\mu}_\omega = \dot{\mu}_{e,1} = \dot{\mu}_{e,\omega} = 0$ in Equations (9)-(12), (iv) values in each instant are determined by (5) and (6), (v) consumers maximize utility (3) choosing consumption and R&D labor optimally, (vi) labor markets clear according to (13) and (14), (vii) product markets clear, $C_t = Y_t$, and (vi) aggregate output (4) grows at the constant rate, $g \equiv \dot{Y}_t / Y_t$.

The following proposition summarizes the properties of the constant growth equilibrium.

Proposition 4.2 (Existence and Uniqueness of the Constant Growth Equilibrium)**.** *For any endowments of production labor, L, there exists a unique constant growth equilibrium. Denoting optimal incumbents' and entrants' choices as $x_I^\star, x_{e,\omega}^\star, x_{e,1}^\star$, and the masses of incumbents and entrants across states as $\mu_1^\star, \mu_\omega^\star, \mu_{e,1}^\star, \mu_{e,\omega}^\star$, the constant growth rate of the economy is given by:*

$$g = \eta\left[x_{e,\omega}^\star\mu_{e,\omega}^\star + x_{e,1}^\star\mu_{e,1}^\star + \lambda x_I^\star\mu_1^\star\right],$$

and inventors' productivity is given by:

$$\frac{g}{L^{RD}} = \eta\frac{x_{e,\omega}^\star\mu_{e,\omega}^\star + x_{e,1}^\star\mu_{e,1}^\star + \lambda x_I^\star\mu_1^\star}{\zeta\left(\omega x_{e,\omega}^\star\mu_{e,\omega}^\star + x_{e,1}^\star\mu_{e,1}^\star\right) + \alpha_I\frac{(x_I^\star)^\gamma}{\gamma}\mu_1^\star}.$$

The expression for inventors' productivity can be used to obtain a first intuition of the mechanism through which higher markups lead to decreased productivity in this model. First, note that a unit of total entrants' research intensity in either protected and unprotected markets produces the same growth. Indeed, by the expression for growth, it is clear that a unit increase in either $x_{e,\omega}^\star\mu_{e,\omega}^\star$ or $x_{e,1}^\star\mu_{e,1}^\star$ gives a growth of η. However, the unit labor requirement of research intensity in protected markets is larger than in unprotected markets as a result of defensive patents. This is evident from the first term at the denominator, that shows that a unit of total research in protected markets requires $\zeta\omega$

inventors, compared with just ζ in unprotected markets. These facts immediately imply that any force that pushes entrants towards protected markets will lower their inventor productivity. One such force is an increase in incumbents' research intensity that outstrips entrants', which acts to increase $\mu_{e,\omega}$, the mass of entrants active in protected markets. As I show formally below, an increase in the markup raises the value of monopolistic positions, which pushes up both incumbents' and entrants' research intensities, $x_I, x_{e,\omega}$. This results in an increase in $\mu_{e,\omega}$ only if incumbents' R&D intensity is more elastic than entrants'.

The following proposition states the main result in this section, showing that higher markups unambiguously increase research efforts by incumbents and entrants, and the share of R&D labor accruing to incumbents. The proposition also states the main condition for this result to lead to a fall in overall inventors' productivity in terms of sufficient statistics. In particular, the incumbents' R&D elasticity to markups must be larger than entrants'. This condition is satisfied in the data, as the estimates in Table 5, which show that incumbents increase their share of inventors following increases in markups.

Proposition 4.3 (Effects of Markup Increases on Innovation)**.** *Suppose that defensive research is effective, $\omega > 1$. The constant growth equilibrium features positive incumbents' research, $x_I^\star > 0$, and markup increases raise both incumbents' and entrants' research effort:*

$$\frac{\partial x_I^\star}{\partial \phi} > 0,\ \frac{\partial x_{e,\omega}^\star}{\partial \phi} > 0\ \frac{\partial x_{e,1}^\star}{\partial \phi} > 0,$$

and the share of R&D labor employed by incumbents increases with markup:

$$\frac{\partial \frac{L_I}{L^{RD}}}{\partial \phi} = \frac{\partial \left(\alpha_I \frac{x_I^\gamma}{\gamma} \mu_1 / L^{RD} \right)}{\partial \phi} > 0.$$

Moreover, if (i) $\lambda = 0$; (ii) inventor supply is not fully inelastic; and (iii) the model parameters are such that equilibrium incumbents' research effort is more elastic than entrants',

$$\frac{\partial x_I^\star}{\partial \phi} \frac{\phi}{x_I^\star} > \frac{\partial x_{e,\omega}^\star}{\partial \phi} \frac{\phi}{x_{e,\omega}^\star},$$

it further holds:

$$\frac{\partial \left(g / L^{RD} \right)}{\partial \phi} < 0.$$

That is, an increase in the markup, ϕ, lowers equilibrium inventors' productivity.

The intuition behind this proposition is that increases in the markup raise the value of monopolistic positions, thus raising both entrants' and incumbents' research effort. In addition, this leads to an overall increase in inventors employed by incumbents. Importantly, this holds only if defensive R&D is effective, showing the importance of this channel to reproduce the empirical findings in Table 5. However, this finding is not sufficient to generate a fall in inventors' productivity. This is because the overall change in inventors' productivity can be decomposed as:

$$\mathrm{d}\left(\frac{g}{L^{RD}}\right) = \mathrm{d}\left(\frac{L_e}{L^{RD}}P_e\right) + \mathrm{d}\left(\frac{L_I}{L^{RD}}P_I\right),$$

where P_e, P_I denote entrants' and incumbents' productivity, respectively. As long as incumbents' productivity is lower than entrants, a reallocation of inventors towards dominant firm leads to a decrease in inventors productivity. However, this effect can in principle be offset by an increase in entrants' productivity. The sufficient conditions in Proposition 4.3 ensure that this is not the case.[17] In particular, if incumbents' research effort is more elastic than entrants', an increase in markups reduces the overall share of unprotected markets, making entry overall more difficult, and reducing entrants' productivity as well. This reasoning also clarifies why the condition is sufficient. Indeed, a fall in inventors' productivity occurs as long as the increase in creative-destruction growth by entrants is not high enough relative to the increase in defensive innovation by entrants. As noted above, this condition is satisfied in the data, and always appear to be in numerical simulations, regardless of specific assumptions on parameter values. In particular, the assumption $\lambda = 0$ is not necessary to generate the result, but I make to obtain clear analytical results in the proof to this Proposition.

It is important to stress that the decline in productivity requires a strictly positive elasticity of inventors' supply. As I show in Lemma C.2 in the Appendix, *aggregate* inventor labor demand is increasing in markups. As a result, when inventors' supply is fully rigid, wage effects need to fully offset the increase in labor demand, and by the uniqueness of the CGE, it the allocation of inventors across incumbents and entrants is fixed. Indeed, aggregate labor demand is uniquely pinned down by research intensities through the stationary distribution.

Figure 5 provides a graphical illustration of the comparative statics for an increase in markup in a calibration of the one-sector model, which follows the strategy described in detail in the following Section.[18] An increase in the markup raises profits, thereby increasing the value of dominant positions, and propelling both incumbents' and entrants' research. Importantly, the incumbents' inventor demand is more elastic to increased profits, which raises the overall proportion of researchers employed in defensive projects. Although overall growth increases as a result of higher aggregate R&D, the productivity of inventors fall, as they get increasingly allocated to incumbents. Two observations are in order. First, with a fixed supply of inventors, equilibrium allocations do not change with increased

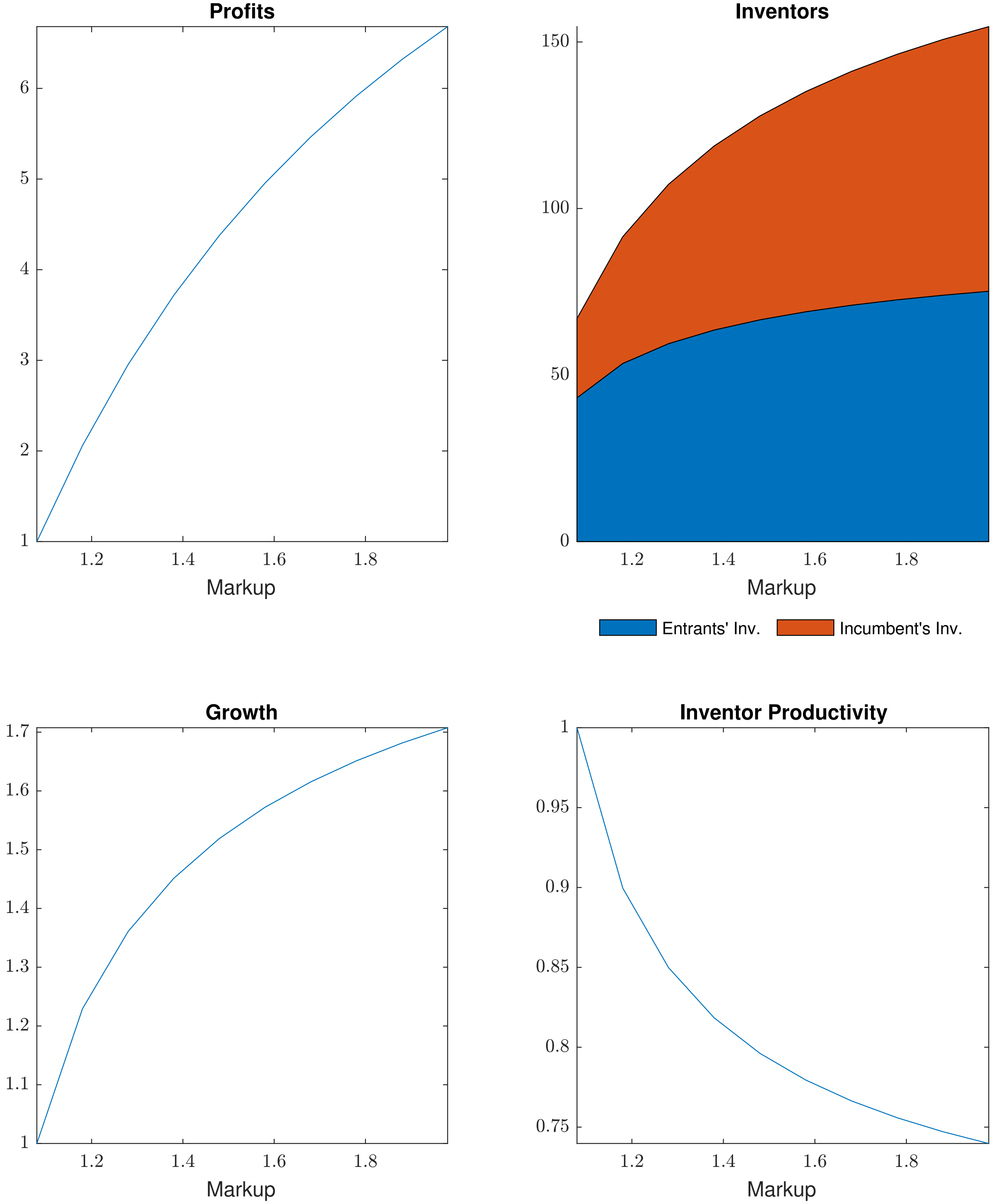
[17] Corollary C.3 in the Appendix provides parametric sufficient conditions for the case with quadratic costs, $\gamma = 2$, which is analytically tractable.

[18] The parameters of the calibration are chosen to match moments of the R&D expenditure distribution as in Section 4.2, and the complete set of parameters is reported in Appendix.

profits, leaving R&D productivity unaffected. Second, in the absence of defensive innovation, the increase in profits would just result in increased entry and growth, leaving productivity constant.[19]

[19]In the absence of defensive innovation, the whole mass of firms operates in unprotected markets and there is no incumbent research. From the expression for growth in Proposition 4.3 inventor productivity is constant at η/ζ.

Figure 5: Comparative Statics for Changes in the Markup in the Single-Sector Model



Note: This figure reports the comparative statics for normalized profits, inventors, growth and inventor productivity in the single-sector model. All variables are expressed in units relative to the equilibrium with $\phi = 1.08$. The calibration follows the same strategy as in Section 4.2.2, assuming that the economy is only composed by a single sector. I set the elasticity of inventors' supply at $\varphi = 1$.

4.2 Calibration and Policy

In this section, I calibrate a two-sector extension of the model presented in the previous Section, with two main objectives. First, I want to analyze misallocation *across* sectors, and show that, under a realistic calibration, this extension can qualitatively reproduce the main findings of the empirical analysis. I focus on the benchmark with *fixed* inventor supply where, by the results in the previous section, markups have no effect on inventors' productivity within sectors. This choice excludes that my findings are driven by within-sector misallocation alone. Second, I wish to study the distribution of R&D subsidies across firms that maximizes aggregate growth. The policy analysis reveals that a planner interested in maximizing growth chooses to subsidize entry in less competitive sectors, rather than reallocating inventors back to more competitive sectors. This result arises for two reasons. First, while misallocation arises only as a result of flows across sectors, it acts through misallocation within the sector that becomes more concentrated, where incumbents' defensive research increases more than entrants' productive R&D. Second, the outflow of inventors reduces defensive innovation more competitive sectors. As a result, reallocating inventors back to competitive sectors would then come at the cost of inventor productivity in these sectors, while tackling the misallocation within concentrated sectors only indirectly.

4.2.1 Model description

The consumption good in the economy is given by the Cobb Douglas aggregate:

$$\ln Y_t = \beta_1 \ln Y_{1,t} + \beta_2 \ln Y_{2,t}, \quad \beta_1 + \beta_2 = 1$$

where $Y_{1,t}$, $Y_{2,t}$ are produced as in Section 4.1.1, and the markup parameter, ϕ, is allowed to vary across the two sectors.

The household side of the economy is unchanged relative to the one-sector model. In this section, however, the supply of inventors is assumed to be fully rigid and given by $L^{RD} = 100$. This assumption is motivated by two considerations. First, a fixed inventor supply captures an aggregate scarcity of inventors, allowing me to focus exclusively on their allocation across sectors. Second, as discussed above, in this benchmark markup changes have no effect on misallocation if inventors are not allowed to move across sectors. Therefore, the results below isolate the effect of inventors' transitions across sectors on misallocation.

Given the presence of two markets, labor market clearing for production workers and inventors is now given by:

$$L = \sum_{i=1}^{2} \int_0^1 l_{i,j,t}\left(w^{RD}\right) \mathrm{d}j, \tag{15}$$

$$L^{RD} = \sum_{i=1}^{2} L_{i,t}^{RD}\left(w^{RD}\right), \tag{16}$$

where the subscript i denotes sectors and j the product markets in each sector. I define a constant growth equilibrium as in the previous section. Given the Cobb-Douglas assumption on the final good, growth is now given by:

$$\Delta \ln Y_t = \sum_{i=1} \beta_i g_i,$$

where g_i denotes the sector-specific output growth obtained as in Proposition 4.3. Appendix C.2 reports the derivations for the two-sector model and a complete description of the equations characterizing the equilibrium.

4.2.2 Calibration

I calibrate my model in order to match features of the R&D distribution and concentration around 1997, the starting year for my analysis. The aim of my calibration is to provide a qualitative description of the features of the model under realistic parameter choices. For this reason, I do not attempt to match the fall in growth implied by the model within or across sectors. As I will show below, my calibration implies a fall in inventors' productivity of about 2% over the period 1997-2012, about half of the lower bound implied by my empirical estimates. The calibration is therefore very conservative relative to my empirical estimates, which provides a useful benchmark to establish a lower bound of the effects of switching to an optimal R&D subsidy policy in the following section.

Table 9b displays my choices for parameters calibrated externally. I set the discount rate to 4%, which, together with a 3% growth for my sample in 1997, implies a value for the real interest rate of 7%, in line with the long-run average before 1997. I obtain a value for β, the share of value added of each sector, from estimates of the Lerner Index in manufacturing that I obtain from the NBER-CES as described in Appendix B.5. According to these estimates, about half of the sectors (weighted by sales) for which I have these data saw an increase in the Lerner Index over the period. This suggests $\beta_1 = \beta_2 = 0.5$. Since I only have the Lerner Index for about half of the sectors, I rely on the extensive literature estimating markups to set a value of $\phi = 1.08$. In particular, I follow Akcigit and Ates (2019), who calibrate the same parameter using the midpoint of estimates provided in De Loecker et al. (2020) and Eggertsson et al. (2018). As standard in the literature (see e.g., Acemoglu and Akcigit, 2012), I set the curvature of the incumbents' cost function relying on the estimates by Kortum, 1993. I choose the lower bound of these estimates in order to minimize the asymmetry of innovation costs between incumbents and entrants, as more convex incumbents' costs mechanically make incumbents' research, even when productive, less effective than entrants. The rate of patent expiration comes directly from the legislative framework in the US, as established by the Uruguay Round Agreements Act of 1994. Since λ measures how radical are incumbents' innovations relative to entrants', I set $\lambda = 0.785$, the complement of the internal patent share of 21.5% estimated by Akcigit and Kerr (2018). Turning to the value of blocking patents, parametrized by ω in my model, I rely on estimates by Czarnitzki et al. (2020) and Grimpe and Hussinger (2008), who employ merger data to obtain the effect of pre-emptive patents on the value of acquired firms. Both their estimates imply an elasticity of firm's values to the share of patents with pre-emptive value of more than one. This implies that a firm with a patent portfolio composed exclusively of defensive patents is valued on average twice as much as one with only patents that have no pre-emptive value. This suggest a value of $\omega = 2$. As shown in the proof of Proposition 4.2, my model gives an elasticity of firms' value of at most $\omega - 1$, therefore $\omega = 2$ effectively caps this elasticity to 1. I also include R&D subsidies, modeled as a percent subsidy on inventors' wages, s, and corporate taxes applied to firms instantaneous profits, τ. I set these two parameters following the values reported by Akcigit et al. (2016).

Table 9b describes my choices for the remaining parameters, that govern the scale of R&D and the growth rate in the economy. Specifically, I set the incumbents' and entrants' R&D cost scale, α_I and ζ, in order to match the share of inventors employed by incumbent firms in 1997 and the R&D business spending as a percent of GDP, as reported by the National Science Foundation. Intuitively, the two cost parameters jointly determine the overall R&D spending in the economy, while their relative value determines the distribution of R&D spending in equilibrium. Given the estimates for α_I and ζ, I set η to match the growth in output per worker for the sectors considered in my analysis in 1997, 3.03%. All targets are matched almost exactly.[20]

4.2.3 Comparative Statics in General Equilibrium

Figure 6 displays the comparative statics for an increase in markup in sector 2, while leaving the other sector's markup unchanged. The graphs compares the aggregates of interest across different constant growth equilibria, and each figure reports the markup of sector 2 relative to sector 1 on the x-axis.

An increase in the relative markup of sector 2 leads to a pronounced reallocation of inventors away from sector 1. In sector 2, incoming inventors are allocated disproportionately to incumbents, who expand their share of researchers relative to entrants. This leads to a decrease in overall inventor productivity of about Computing the Lerner Index on NBER-CES data as described in Appendix B.5 reveals that the markup gap between more concentrated and less concentrated sectors has increased by about 20% over the period of interest. This implies a fall in inventors' productivity of about 2% compared to the benchmark where the two sectors have the same competitive structure. Since the

[20] The average percentage point deviation of moments in the model from their empirical targets is less than 10^{-6}.

Table 9: Parameter Values and Sources

(a) Parameters Calibrated Externally

Parameter Name	Symbol	Value	Source/Target
Discount rate	ρ	.04	Annual real interest rate $\approx$ 7% before 1997
Value Added Share	β	.5	Share of sectors with $\uparrow$ Lerner Index
Average Sectors' Markup	ϕ	1.08	De Loecker et al., 2020 and Eggertsson et al., 2018
Innovation Cost Curvature	γ	1/.6	Lower bound of estimates in Kortum, 1993
Intensity of Patent Expiration	δ	.05	Uruguay Round Agreements Act (1994)
Share of Implemented Innovations	λ	.785	Internal patent share of 21.5% (Akcigit and Kerr, 2018)
Value of Blocking Patents	ω	2	Czarnitzki et al., 2020; Grimpe and Hussinger, 2008
R&D subsidy	$s_I = s_e$	19%	Akcigit et al., 2016
Corporate tax rate	τ	23%	Akcigit et al., 2016

(b) Parameters Calibrated Internally

Parameter Name	Symbol	Value	Target
Incumbent Costs	α_I	21.97	Top 10% Firms' Inventor Share, 1997: 30.3%
Entrants' Costs	ζ	4.75	Business R&D Share over GDP, 1997: 1.81%
Innovation Step	η	0.0047	Output per Worker Growth, 1997: 3.03%

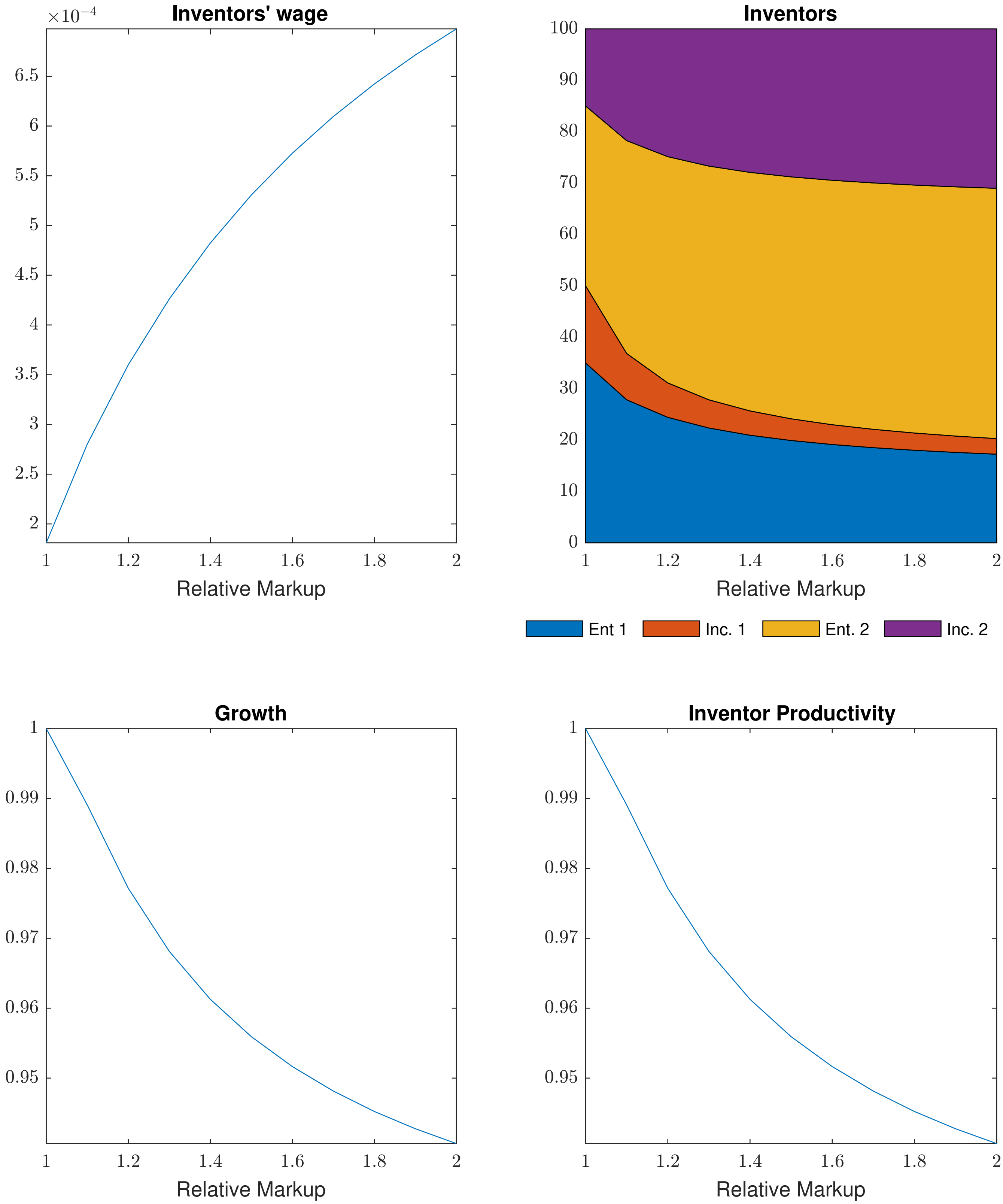
supply of inventors is fixed at $L^{RD} = 100$, this results in a 2.5% fall in GDP growth, about 0.075pp. This estimate is close to the lower bound of 0.13pp implied by my estimates in Table 8. As discussed above, assuming an inelastic labor supply mutes the response of inventors' productivity to increases in the markup, so it is reasonable to expect the model in this section to understate the productivity effects of increased concentration. However, this benchmark is desirable since it shuts down productivity effects unrelated to reallocation.

Figure 7 shows the changes occurring in each sector that correspond to the aggregates reproduced in Figure 6. In this figure all variables are normalized by their value in the initial equilibrium with $\phi_1 = \phi_2 = 1.08$. Starting from the upper-left panel, increasing markups raise profits in sector 2 relative to sector 1. This shift translates into an increase in inventors in sector 2 relative to sector 1. The upper-right panel show that incumbents' inventor demand is more elastic in response to increases in the markup, which raises top firms' share in sector 2. The increase in the equilibrium inventors' wage leads to a reallocation within sector 1 as well, where entrants gain inventors relative to incumbents. As a result of these shifts, growth increases in sector 2, and falls in sector 1, while inventors' productivity follows the opposite pattern.

The reallocation of inventors within sector 1 is a feature of the model that will be important to interpret the policy results in the following section. The specular behavior of sectors 1 and 2 reveals the crucial role of the overall R&D activity in shaping inventor productivity. In particular, the incentives to conduct defensive innovation increase when there is a larger number of inventors employed in the sector. Therefore, while the movements of inventors from sector 1 to sector 2 are overall detrimental to growth, they are not for R&D productivity in sector 1. Since incumbents there face a lower risk of being displaced by entrants, they reduce their efforts in defensive innovation, which increases R&D productivity in sector 1. However, overall growth is lower since less R&D resources are available to this sector in equilibrium. Overall, the results in this section suggest that inventor reallocation away from competitive sectors has both costs, resulting from a reduction in sectoral growth, and benefits, coming from a more efficient distribution of resources within this sector.

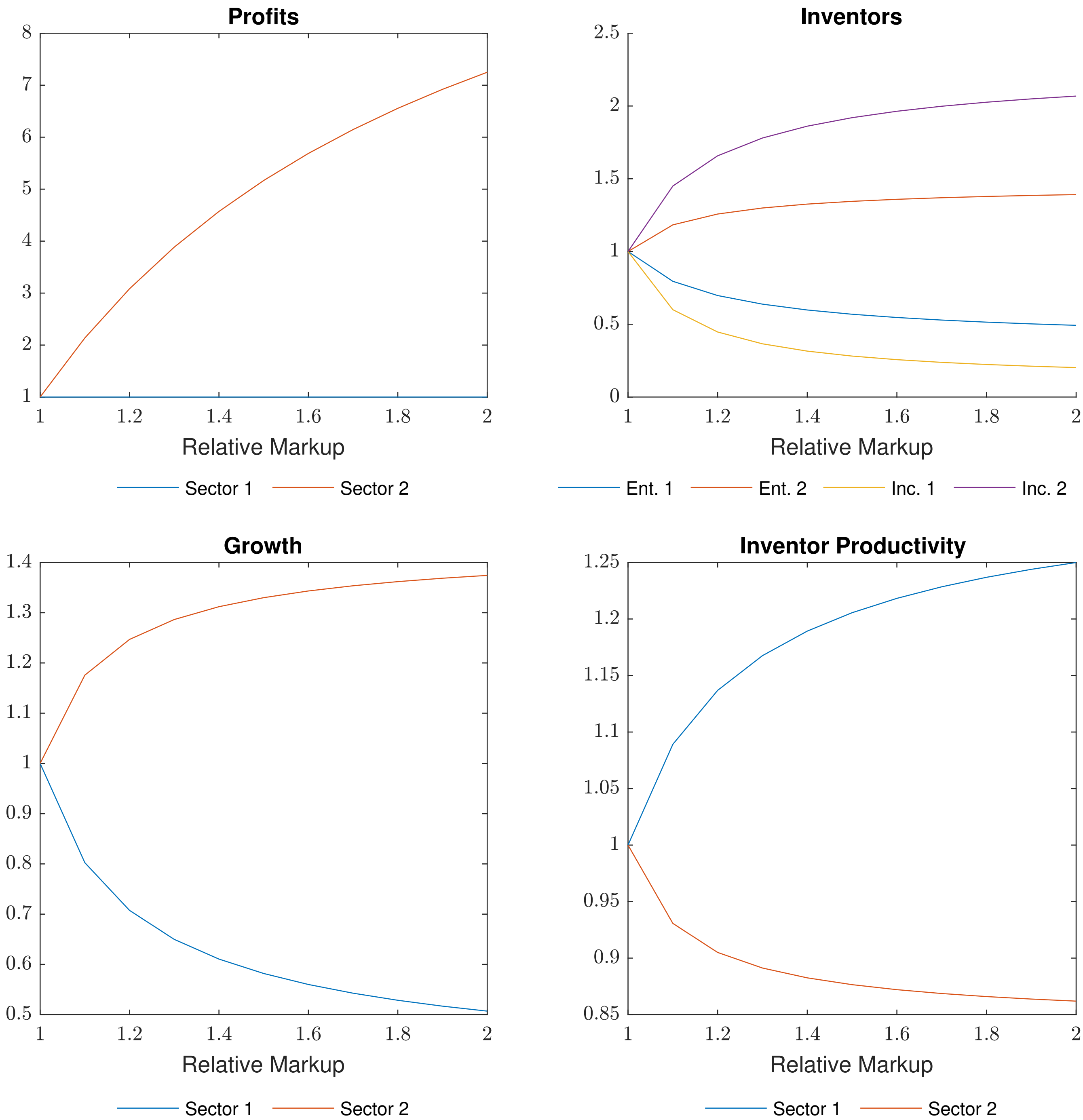
This comparative static exercise also clarifies that defensive innovation is crucial to generate misallocation across sectors. In the absence of defensive innovation, just reallocate across sectors, leaving productivity unaffected, and as a result overall growth is also unchanged, since the Cobb-Douglas assumption with $\beta = 0.5$ gives aggregate growth as the simple average of growth in the two sectors.

Figure 6: Comparative Statics in Sector 2's Markup Relative to Sector 1 in the Two-Sector Model, Economy Aggregates



Note: This figure reports the comparative statics for normalized profits, inventors, growth and inventor productivity in the two-sector model. In all figures, the x-axis reports the markup of sector 2 relative to sector 1. The parameters used to produce this Figure are reported in Tables 9a and 9b.

Figure 7: Comparative Statics in Sector 2's Markup Relative to Sector 1 in the Two-Sector Model, Sector-level Aggregates



Note: This figure reports the comparative statics for normalized profits, inventors, growth and inventor productivity in the two-sector model. In all figures, the x-axis reports the markup of sector 2 relative to sector 1. The parameters used to produce this Figure are reported in Tables 9a and 9b.

4.2.4 Growth-Maximizing Policy

I now turn to solve numerically for the combination of R&D subsidies that maximizes growth. I assume that the planner wishes to maximize growth under a set of different constraint on the instruments available. In particular, I assume that the planner cannot alter the nature of innovation, that is, the planner cannot distinguish productive from unproductive projects, and cannot forbid depositing patents that have no productive value. In this model, eliminating protection for incumbents would lead to a first-best where only entrants conduct R&D, and reallocation could only be beneficial to growth, as discussed in the previous section.

I start from the 2012 equilibrium of my model economy, where the gap in markups between sector 2 and sector 1 is 20%, and all firms receive a 19% subsidy to inventors' wages and a 23% tax on profits. I then evaluate numerically three cost-neutral alternatives relative to this benchmark, that is, I constrain the planner to leave the expenditure on R&D subsidies as a percentage of GDP fixed at the 2012 benchmark. In the first scenario, the planner is allowed to distribute subsidies freely, and can condition their reception on both the state of the market (protected or unprotected) and the identity of the receiving firm (entrant or incumbent). In the other two scenarios, I only allow the planner to act on one of these dimensions at a time. That is, the planner can either control the cross-sector distribution of funds, but not the allocation across incumbents and entrants, or vice versa.

The results of this exercise are presented in Table 10. The 2012 equilibrium is reported in Columns 1 and 2. For reference, the 1997 calibrated model has two identical sectors, that share the stock of inventors equally, and within each sector incumbents have 30.3% of the overall inventors employed, and GDP growth is 3% per annum, as reported in Table 9b. In the 2012 baseline, the distribution of inventors is tilted towards the second sector, where markups have increased, which results in a fall in annual GDP growth of .07pp, about 2.5% of the 1997 benchmark. As shown in the graphs above, this new equilibrium sees a larger share of inventors allocated to incumbents in the second sectors, which increases its growth relative to its more competitive counterpart, and sees a decline in productivity resulting from higher defensive innovation. Column 3 and 4 report the optimal cost-neutral R&D subsidies chosen by a growth-maximizing planner. Somewhat surprisingly, the most efficient allocation of funds turns out to be an entry subsidy to entrants in the more concentrated sector only. However, this result can be easily rationalized through the findings in Figure 7. As discussed above, the outflow of inventors from sector 1 increases inventor productivity due to reduced defensive innovation. It is therefore more efficient to leave entry unchanged in sector 1, and act directly on the inventors' misallocation occurring in sector 2. This intuition is confirmed by the results for the scenario in which the planner can only allocate R&D subsidy to one of the sector, but cannot condition on the identity of the firm. In this case, subsidies are in part used to increase incumbents' defensive innovation, which is also made more attractive by the overall increase in entry. While this policy does allow the planner to recover most of the lost ground relative to the 1997 equilibrium, bringing growth up to 2.99%, it is still not as effective as alternative cost-equivalent policies. In particular, an alternative that might be easily implementable is a blanket entry subsidy as reported in Columns 6 and 7, which achieves a growth in annual output of 3.38%, which is above the starting 1997 equilibrium.

To conclude, the policy analysis suggests that entry subsidies are the most effective policy to contrast the friction introduced by defensive innovation in this model economy. In particular, it is best to subsidize entrants in less competitive sectors, where this friction is most pronounced, which increases growth by 0.5pp per annum. A more feasible uniform R&D subsidy to entrants produces quantitatively similar effects. Conversely, sector-specific subsidies to reallocate inventors to more competitive sectors are less effective, since they get channeled to incumbents who employ them to conduct pre-emptive innovation, precisely the source of inefficiency that the planner wishes to contrast.

5 Conclusion and Future Work

In this paper, I have proposed and documented a novel explanation for the observed decline in growth and R&D productivity over the last few decades.

My empirical results showed that increasing misallocation of inventors across different product markets can account for up to 27% of the observed decline in output per worker growth in the sectors I analyze. This misallocation resulted from uneven increases in concentration across product markets, which were accompanied by an increase in the share of inventors accruing to less competitive sectors. I interpret my findings as resulting from an increase in defensive innovation in concentrated sectors, defined as R&D activities conducted with the primary aim of blocking further entry, which manifest in the data as a fall in patents' forward citations and an increased share of inventors employed by the largest incumbents in these sectors.

The theoretical analysis examined the effects of defensive innovation in a Schumpeterian model of creative destruction, where incumbents can conduct defensive innovation to raise new entrants' costs. In the model, pre-emptive innovation is crucial to generate misallocation across sectors, which would not occur in the absence of this mechanism. Having established the importance of this channel, I employed a calibrated two-sector version of the model to study the growth-maximizing allocation of R&D subsidies across sectors, as well as between incumbents and entrants. My analysis suggests that R&D subsidies provided to entrants constitute the most effective policy, directly tackling the friction generated by defensive innovation, and can increase growth by up to 17% of my baseline (0.5pp in absolute terms).

Two main directions for future research stand out. First, it would be important to investigate the validity of my findings to an international context. Indeed, many findings in the literature of competition and innovation depend strongly on the country and the period analyzed. Second, my

Table 10: Comparison of R&D Policies in the Two-Sector Model

	Baseline		Optimal Cost-Neutral		Cost-Neutral Sector		Cost-Neutral Entry	
	Sector 1 (1)	Sector 2 (2)	Sector 1 (3)	Sector 2 (4)	Sector 1 (5)	Sector 2 (6)	Sector 1 (7)	Sector 2 (8)
R&D Subsidies:								
s_I	19%	19%	0%	0%	46.17%	0%	0%	0%
s_e	19%	19%	0%	41.78%	46.17%	0%	29%	29%
Aggregates:								
L_I^{RD}	6.70	24.87	6.37	15.95	10.83	19.51	4.83	18.45
L_e^{RD}	24.41	44.02	23.87	53.81	30.24	39.42	27.41	49.30
L_{TOT}^{RD}	31.11	68.89	30.25	69.75	41.07	58.93	32.25	67.75
Sector Growth	2.12%	3.74%	2.08%	4.78%	2.61%	3.36%	2.45%	4.31%
GDP Growth	2.93%		3.43%		2.99%		3.38%	

Note: The figures reported in this Table give the optimal allocation of R&D subsidies and the resulting aggregate outcomes for a planner wishing to maximize aggregate growth in the economy. The column headings refer to the various scenarios described above. “Baseline” refers to the subsidy allocation reflecting the 2012 equilibrium, where subsidies do not condition on sectors or the position of firms within sectors; “Optimal Cost-Neutral” refer to the scenario where the planner is allowed to freely allocate R&D subsidies subject to the constraint that overall R&D subsidy expenditure as a percentage of GDP is held fixed at its 2012 benchmark; “Cost-Neutral Sector” consider a scenario where the planner can choose which sector to allocate funds to, but not which firms within the sector should receive the subsidy; “Cost-Neutral Entry” computes the optimal universal entry subsidy, under the assumption that the planner cannot condition its reception on the sector firms operate in.

paper highlights the role of pre-emptive innovation as a major driver of the fall in R&D productivity. However, this topic is still relatively under-explored, and a thorough investigation of the evolution, causes and consequences of this phenomenon seems highly relevant in view of my results.

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A Data Construction Details

A.1 Knowledge Markets

Rescaling Inventor Flows As explained in the main text, the measure of inventor flows aims to capture the strength of the connections between two sectors. I take several steps to ensure that I do not overestimate these connections and to normalize them to account for the size of sending and receiving sectors.

As a first step, I build normalized directed flows for each inventor i in order to avoid double counting. For example, for transitions between sector 1 and 2, I define:

$$\text{flow}_{1\to 2,i,t} \equiv \frac{\sum \mathbf{1}\{i \text{ moves } 1 \to 2 \text{ in } t\}}{\sum_{j,k} \mathbf{1}\left\{i \text{ moves } j \to k \text{ in } t\right\}} \times \alpha_i.$$

This measure attributes a fraction of the effective inventor fixed effect α_i to each transition in proportion to the number of overall inventor i's transitions across sectors in each year. The first term in this formula is precisely the share of transitions from sector 1 to sector 2 relative to overall transitions between any two sectors j and k that inventor i took part in.

Second, I compute total inflows (outflows) for each NAICS 4-digit sector, summing over all years, inventors and origin (destination) sectors. For example, inflows for sector 1are defined as:

$$\text{inflow}_1 = \sum_n \sum_t \sum_i \text{flow}_{n\to 1,i,t},$$

where n denotes origin NAICS sectors, t years, and i inventor identifiers.

Third, I proceed to compute the share of directed flows between each pair of sector as a share of total inflows or outflows. For example, the share of inflows coming from sector 2 and entering sector 1 is defined as:

$$\text{share}_{1\leftarrow 2} = \frac{\sum_t \sum_i \text{flow}_{2\to 1,i,t}}{\text{inflow}_1}.$$

In this example, this measure captures the relative importance of inflows from sector 2 for the overall number of inventors received by sector 1. However, this measure can still overstate flows from large to small sectors, or vice versa. As a result, and since I need undirected flows to apply the Louvain algorithm, I define network edge weights starting from an average of the above shares of inflows and outflows for each sector and taking the minimum between the two measures as follows:

$$W_{12} = W_{21} = \min\left\{\frac{\text{share}_{1\leftarrow 2} + \text{share}_{1\rightarrow 2}}{2}, \frac{\text{share}_{1\leftarrow 2} + \text{share}_{1\rightarrow 2}}{2}\right\},$$

where $W_{12} = W_{21}$ since the final network is undirected.

Modularity Maximization Formula and Algorithm In order to identify knowledge markets from the network constructed above, I employ the Louvain community detection algorithm (Blondel et al., 2008). This algorithm maximizes the modularity of the network, Q, assigning each sector i to one of N *non-overlapping* communities c. Accordingly, the objective function for this problem is given by:

$$\max_{N} \max_{(c_1,\ldots,c_N)} Q \equiv \frac{1}{2\boldsymbol{W}} \sum_{ij} \left[W_{ij} - \frac{\boldsymbol{W}_i \boldsymbol{W}_j}{2\boldsymbol{W}} \right] \mathbf{1}\left\{c_i = c_j\right\},$$

where W_{ij}, weight of the edge connecting node i to j, and bold variables denote other summations for ease of notation. In particular, I define $\boldsymbol{W_i} \equiv \sum_k \sum_i W_{ik}$, as the sum of weights for edges with one end in node i, and the sum of all weights in the network, respectively. The indicator $\mathbf{1}\left\{c_i = c_j\right\}$ takes a value of 1 when nodes i and j belong to the same community. Note that the maximization is carried out both over the number of communities and the assignment of nodes to each community. This measure can be interpreted considering that $\frac{\boldsymbol{W}_i \boldsymbol{W}_j}{2\boldsymbol{W}}$, is the expected number of edges that arise between nodes i and j in a random network. Therefore, modularity maximizes the distance between the density of linkages within communities W_{ij} relative to the overall density of links that would arise randomly.

Since looping over all the permutations of nodes and community is numerically unfeasible, the Louvain algorithm follows an iterative procedure to maximize modularity. First, it assigns each node to its own community. Then, it repeats iteratively the following three steps:

1. Compute local deviations in modularity from reassigning the node to neighboring communities;
2. Assign nodes to communities following the local improvement granting the highest modularity increase;
3. Redefine a network with new communities as nodes.

These steps are repeated until there is no significant improvement in modularity for further steps.

B Additional Results and Robustness

B.1 Results on Overall Inventor Shares

Table 11 reports the effect of concentration increases on the share of inventors across all knowledge markets. While the correlation is positive and significant when some outliers are removed, this relation is not robust to the inclusion of all observations or the the alternative trimming procedure provided by the Mahalanobis distance. This result is unsurprising in light fo two points discussed in the main text. First, as highlighted in Section 2.2, if ordinary flows of inventors across unrelated sectors are small or absent, we should not expect any effect of changes in these sectors' characteristics on the distribution of inventors. Second, the findings reported in Table 3 suggest that cross-knowledge-market flows are not significant, as apparent from a comparison of specifications with and without knowledge-market fixed effects. The results presented in this section therefore speak to the importance of accurately delineating labor markets for inventors when assessing their flows across product markets.

Table 11: Regressions of Change in Total Inventors' Share over Change in HHI Lower Bound, Long-Difference, 1997-2012

	Ch. Total Eff. Inv. Share (%)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \underline{HHI}$	0.297	1.692	1.328*	1.532*	0.271	1.889
	(2.007)	(1.956)	(0.649)	(0.696)	(2.038)	(2.023)
Δ log Sales	0.460	0.436	0.133**	0.109*	0.464	0.472
	(0.281)	(0.292)	(0.047)	(0.047)	(0.283)	(0.312)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	153	147	143	150	139

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. Please refer to notes in Table 3 for further details.

B.2 Using the Raw Number of Inventors instead of Fixed-Effects

This Appendix reports the results for the main analysis presented in Section 3.2 using the raw number of total inventors instead of the fixed effects from regression (1), which might be inconsistently estimated. The following Tables, to be compared with Tables 2 and 3 in the main text, show that the results are qualitatively unchanged. Looking at the scale of the y-axis in panel (a) of Figure 8, it is apparent that the shares of the raw number of inventors are more volatile, and presents larger changes. This is easily explained by the fact that differences in research requirements across patent classes, firms and years are not absorbed as in the effective inventor measure. This greater variability simply results in larger and noisier coefficients, which nevertheless remain positive and significant.

Table 12: Regressions of Change in 4-digit Knowledge Market Share of Total Inventors over Change in HHI Measures, Long-Differences, 1997-2012

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\underline{HHI}$	74.172+		73.706+		74.177+	
	(40.957)		(41.600)		(41.047)	
Δ HHI		71.749**		71.997**		71.583**
		(24.464)		(25.060)		(24.433)
Knowledge Market FE						
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	80	155	79	150	72

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (2), when the outcome is the share of total inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Economic Census concentration ratios, or the HHI index reported in the Economic Census. “Full Sample”, “Trim Outliers” and “Mahalanobis 5%” refer to the samples described in the main text.

Table 13: Regressions of Change in 4-digit Knowledge Market Share of Total Inventors over Change in HHI Lower Bound, Long-Differences, 1997-2012

(a) Controlling for Change in Log Real Sales

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\underline{HHI}$	71.724+	67.160+	71.308+	67.860+	71.772+	68.398+
	(39.265)	(37.176)	(40.036)	(37.518)	(39.316)	(37.717)
Δ log Sales	1.864*	1.422*	1.688*	1.381+	1.878*	1.443+
	(0.766)	(0.717)	(0.736)	(0.711)	(0.774)	(0.745)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	156	155	154	150	142

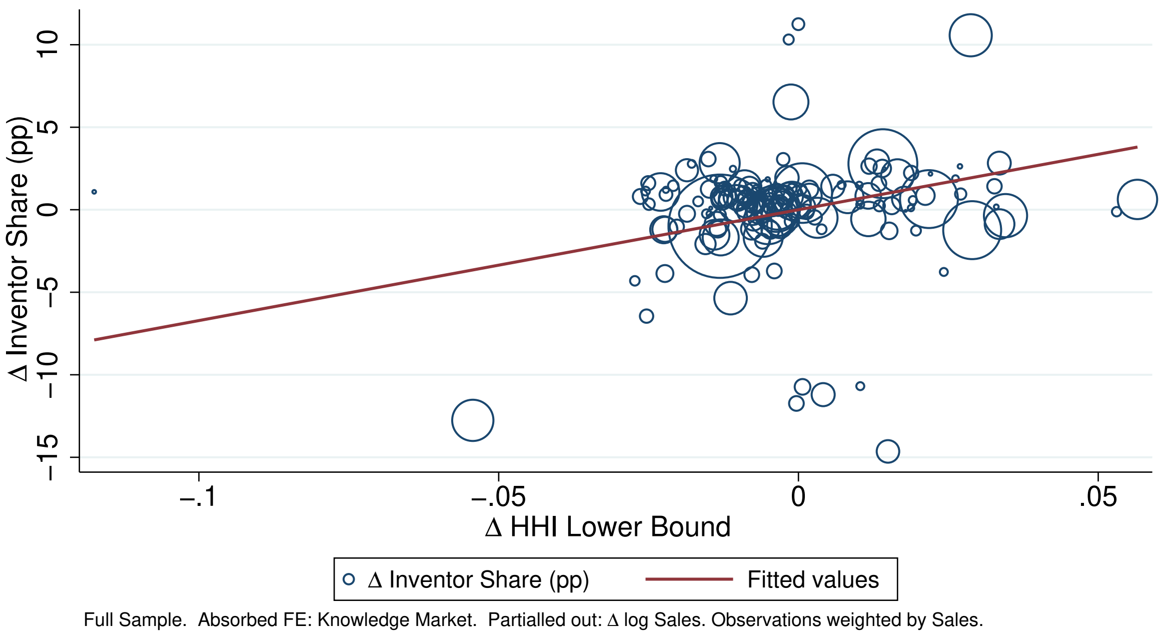
(b) Controlling for Change in Log Real Sales per Company

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\underline{HHI}$	104.562*	81.339+	103.402+	82.040+	104.355*	82.964+
	(51.534)	(43.722)	(52.824)	(43.556)	(51.356)	(46.147)
Δ log Size	0.571	-0.277	0.196	-0.515	0.571	-0.656
	(1.013)	(0.809)	(0.920)	(0.793)	(1.048)	(1.049)
Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	81	80	80	79	76	69

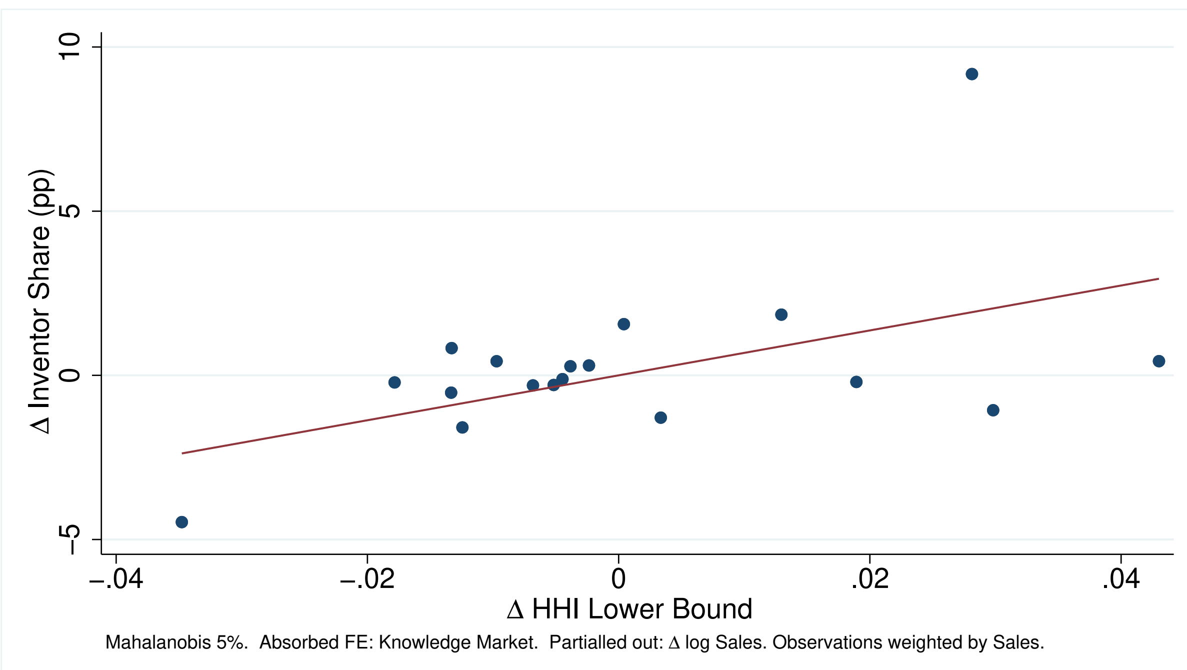
Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (2) and (**??**), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. "Full Sample", "Trim Outliers" and "Mahalanobis 5%" refer to the samples described in the main text.

Figure 8: Residualized Scatter Plots Corresponding to Selected Columns in Table 13, Panel (a)

(a) Raw Scatter Plot, Specification in Column (2)



(b) Binned Scatter Plot, Specification in Column (6)



Note: This figure presents residualized scatter plots of the change in the share of effective inventors of sector p over total inventors in knowledge market k, over the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. The upper panel reports the data corresponding to the full sample, where both variables have been residualized by change in log real sales and knowledge market fixed effects. The size of the markers is proportional to the weight of each observation in the regression, corresponding to total sector sales in 2012. The regression line corresponds to the coefficient on the change in HHI lower bound reported in Column (2) of Table 13. The lower panel presents a binned scatter plot on the sample where the observations with the highest 5% Mahalanobis distance from sample centroid have been removed. Observations are aggregated using sales weights and the regression line results from the specification in Column (6) of Table 13.

B.3 Using a Quartic in Sales as Size Control

This Section displays the results of estimating the specification in Table 3 using the changes in the terms of a fourth-degree polynomial in sales rather than log-sales. This flexible control specification ensures that my main findings do not rely on the specific functional form that I assumed above. Table 14 reports the result of this exercise using both effective inventors (Columns (1) and (2)) and raw inventor counts (Columns (3) and (4)) to compute sector shares. Recall that when using raw inventor counts, knowledge markets are also constructed according to this measure. As clear from a comparison of Columns (1) with (2), and (3) with (4), these two specifications produce statistically undistinguishable results.

Table 14: Regressions of Change in 4-digit Knowledge Market Share of Inventors over Change in HHI Lower Bound, Long-Differences, 1997-2012

	Δ Inventor Share (pp)			
	(1)	(2)	(3)	(4)
$\Delta\underline{\text{HHI}}$	22.509*	24.083*	67.160+	74.769+
	(10.848)	(10.565)	(37.176)	(39.225)
Δ log Sales	0.548*		1.422*	
	(0.243)		(0.717)	
Δ Sales ($ bn)		2.617*		6.382+
		(1.108)		(3.365)
ΔSales^2		-0.749		-1.749
		(0.482)		(1.468)
ΔSales^3		0.081		0.165
		(0.076)		(0.232)
ΔSales^4		-0.003		-0.005
		(0.003)		(0.009)
4D Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales	Sales
Observations	153	153	156	156

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (2), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. "Full Sample" refers to the sample described in the main text.

B.4 Using the HHI as the Independent Variable in Patent Quality Regressions

This Section reports the results analogous to Tables 5 and 6 in the main text, using the HHI lower bound as the independent variable rather than the share of inventors. As should be expected from the high correlation between the two variables, the results are qualitatively similar to those reported there, with some distinctions. First, Table 15 confirms that there has a been a large and significant increase in the concentration of inventors among top firms in product market with increasing HHI. Differently from the main specification, this seems to be driven primarily by a fall in the share of the bottom half innovative firms. However, this does not counter the interpretation provided in the main text that increased concentration tends to reduce entry, which manifests in these regressions through a fall in the share of inventors employed by smaller, and presumably younger, firms. Table 16 shows the robustness of my findings on forward citations to the use of the HHI as well as weighting the regressions by 2012 sales, although the generality coefficient appears non-significant, as discussed in the main text.

B.5 Using the Lerner Index instead of the HHI

Following Grullon et al. (2019), I build the Lerner Index from NBER-CES data for the period 1997-2012 as the ratio:

$$\text{Lerner}_{jt} = \frac{\text{vship}_{jt} - \text{pay}_{jt} - \text{matcost}_{jt} - \text{energy}_{jt}}{\text{vship}_{jt}}, \tag{17}$$

where “vship” is the total value of shipments, “pay” denotes total payrolls, “matcost” and “energy” material and energy costs, respectively, and j denotes a 6- or 4-digit NAICS sector. I build two alternative measures, one using 6-digit NAICS sectors, the original identifier in NBER-CES, and then averaging by sales at the level of 4-digit NAICS, or first aggregating the revenue and cost statistics at the level of 4-digit NAICS. Table 17, shows that the Lerner Index thus constructed is strongly correlated with the HHI measure used in the main analysis. However, the correlation is far from perfect, as suggested by the R^2, suggesting that this estimate of the Lerner Index might be excessively imprecise. Indeed, Table 18 shows that, when using this measure instead of the HHI in the main analysis, the coefficients for the regression of inventors’ shares on changes in concentration stay positive, but become smaller and noisier. This suggests the potential presence of attenuation bias, a valid concern due to the fact that the above measure, not based on any structural estimation, can only imperfectly capture markups. Note that this is also due to the fact that the Lerner Index is available only for the manufacturing sectors, which make up about 60% of the sample, so its use lead to dropping a substantial amount of observations. When using fitted values from the regression in Table 17 to extend the measure to more sectors, as well as reducing the volatility of the series for available sectors, the coefficients recover magnitudes and significance close to the baseline presented in 3.

Table 15: Regressions of Change in Inventor Distribution Measures over Change in 4-digit Knowledge Market Share, Long-Difference, 1997-2012

	Δ 90/50 Quantile Ratio	Δ Top 10%/Bottom 50%	Δ Top-50/Bottom-50 Share Ratio	Δ Top 10%	Δ Bottom 50%
	(1)	(2)	(3)	(4)	(5)
ΔHHI	15.426*	1.793	10.566	-0.085	-0.409*
	(6.848)	(5.797)	(8.078)	(0.539)	(0.188)
Δ log Sales	0.048	0.464	0.340	0.036	-0.000
	(0.154)	(0.349)	(0.407)	(0.022)	(0.008)
4D Knowledge Market FE	✓	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales	Sales	Sales
Observations	118	118	118	118	118

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. Please refer to notes in Table 3 for further details. Column (1) uses the ratio in the 90 percentile of effective inventors to the median as the outcome variable. Columns (2) and (3) instead present the share ratio, that is the share of effective inventors accruing to the top 10 or 50% relative to the share accruing to the bottom 50% of the distribution within each NAICS sector.

Table 16: Regressions of Changes in Forward Citation over HHI Changes, Long-Differences, 1997-2012

(a) Full sample

	Δ log Citations/Patent (CPC)	Δ log Citations/Patent (Total)	Δ Patent Generality
	(1)	(2)	(3)
$\Delta\underline{\text{HHI}}$	-11.133**	-12.524**	-0.335
	(3.730)	(4.324)	(0.431)
Δ log Sales	-0.454*	-0.523*	-0.019
	(0.201)	(0.257)	(0.022)
Knowledge Market FE	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales
Observations	153	153	153

(b) Full sample, restricting to the middle range of the change in inventor shares (−2% to +2%)

	Δ log Citations/Patent (CPC)	Δ log Citations/Patent (Total)	Δ Patent Generality
	(1)	(2)	(3)
$\Delta\underline{\text{HHI}}$	-10.646**	-13.052**	-0.624
	(4.018)	(4.979)	(0.473)
Δ log Sales	-0.467*	-0.554*	-0.022
	(0.214)	(0.273)	(0.023)
Knowledge Market FE	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales
Observations	144	144	144

Note: Unweighted regressions; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specification (**??**), when the outcome is the log-change in forward citations and the change in patent generality in sector p over the change in the share of inventors employed in sector p. Column (1) and (2) presents the results when forward citations are extrapolated the procedure Hall et al. (2000) to avoid truncation bias. A specific cite-lag distribution over 35 years is estimated for each pair of cited and citing CPC2-codes. Column (1) employs the extrapolation scheme by each pair of CPC2 cited and citing sector. Column (2) applies the extrapolation scheme to total citations received by each cited patent. Column (3) presents results on the patent generality measures. All columns exclude self-citations. Upper panel: full sample; Bottom panel: excluding sectors with absolute increase in the inventor share above 2%.

Table 17: Regressions of Changes in the Lerner Index over Changes in the HHI Lower Bound, Long-Difference, 1997-2012

	Δ Lerner Index
	(2)
$\Delta \underline{HHI}$	1.652***
	(0.257)
Observations	258
R-squared	.14

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$. "6d Lerner Index" refers to the Lerner Index constructed as in (17)on NAICS 6-digits averaged at the 4-digit NAICS level weighting by the value of shipments; "4d Lerner Index" is computed using 4-digit aggregates for the value of shipments, payroll and costs, summing over the NAICS 6-digit composing each sector.

Table 18: Regressions of Changes in Inventors' Share over Changes in Actual and Fitted Lerner Index, Long-Difference, 1997-2012

	Δ Inventor Share (pp)	
	(1)	(2)
Δ Lerner	0.556	
	(5.465)	
Δ Lerner (Fitted)		26.736*
		(13.363)
Knowledge Market FE		
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	81	157

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Observations weighted by sales. The markup change 1997-2012 is the long-difference of the Lerner Index described above. "Fitted Lerner change" is the fitted value for the Lerner index based on the estimates in 17, and extended to all available sectors in the main sample.

C Omitted Proofs and Derivations

C.1 One-sector model

Proof of Proposition 4.3. This proof consists of several parts. First, I show that given labor supplies, output, values and wages grow at the same constant rate, so the problem can be solved in a steady state of a normalized model. Second, show that normalized values, $v(\Omega) \equiv V_t(\Omega)/Y_t$, are uniquely determined, which gives unique research intensities and stationary distribution. Third, I derive the stationary distribution and the expression for growth and inventors' productivity. In what follows I suppress stars to denote equilibrium quantities for ease of notation.

Given an endowment, L, production labor market clearing in each period requires:

$$\int_0^1 l_{i,t}(w)d(i) = L.$$

That is,

$$L = \int_0^1 \frac{c_{i,t}}{\phi} y_{i,t}(w_t)\, d(i) = \frac{1}{\phi}\frac{Y_t}{w_t},$$

where the second equality comes from using the demand for output of product i for $y_{i,t}(w_t)$. This expression immediately implies that if Y_t grows at a constant rate, so does w_t. Labor market clearing for R&D workers reads:

$$L^{RD} = \zeta\omega x_{e,\omega}\left(\mu_{e,\omega} + \mu_{e,1}\right) + \alpha_I \frac{(x_I)^\gamma}{\gamma}\mu_1.$$

In a constant growth equilibrium (CGE), the distribution is stationary, and since the left hand side is constant, research intensities are also fixed. A contradiction arises otherwise, since the distribution is stationary only if research intensities are fixed by the LOM (46)-(49). Further, R&D labor cannot grow since the growth rate in the economy increases in total R&D labor for any given distribution, as it will be clear below. The fact that research intensities are constant immediately implies, from the optimality of $x_{e,\omega}$, that $V_t(1)$ and w_t^{RD} grow at the same rate. Indeed, from the FOC for entrants' research:

$$0 = \mathrm{d}\log x_{e,\omega,t} = \mathrm{d}\log V_t(1) - \mathrm{d}\log w_t^{RD}.$$

This result in turn implies, combined with the FOC for x_I, that $V_t(\omega)$ also grows at the same constant rate. Now consider the budget constraint of the representative household, combined with product market clearing, $Y_t = C_t$:

$$r_t A_t - \dot{A}_t + w_t^{RD} L^{RD} + w_t L = Y_t,$$

where A_t denote the household's assets, that is all firms in the economy. Therefore the above reads:

$$r_t\left(\mu_1 V_t(1) + \mu_\omega V_t(\omega)\right) - \mu_1 \dot{V}_t(1) - \mu_\omega \dot{V}_t(\omega) + w_t^{RD} L^{RD} + w_t L = Y_t$$

Dividing both sides by $V(1)$, using the Euler equation and rearranging we obtain:

$$\left(g+\rho\right)\left(\mu_1 + \mu_\omega \frac{V_t(\omega)}{V_t(1)}\right) - \mu_1 g_{V_1} - \mu_\omega \frac{V_t(\omega)}{V_t(1)} g_{V_1} + \frac{w_t^{RD}}{V_t(1)} L^{RD} = \frac{Y_t}{V_t(1)} - \frac{w_t}{V_t(1)} L.$$

By what shown above, all terms on the left hand side are constant in t, since research wages and values grows at the same rate and the distribution is stationary. Since Y_t and w_t grow at the same rate positive rate, it must be that $V_t(1)$ also grows at the same rate as Y_t. This proves that $g_{V_1} = g = g_c = g_w = g_{w^{RD}}$.

As a result, in a CGE, it is possible to define normalized constant values, $v(\Omega) \equiv V_t(\Omega)/Y_t$. The system of equations defining the recursive problem in this equilibrium reads:

$$\rho\, v(1) = \max_{x_I} \left\{ \left(\frac{\phi - 1}{\phi}\right) - \alpha_I \frac{x_I^{\gamma}}{\gamma} w^{RD} + x_I \left(v(\omega) - v(1)\right) - x_{e,1}\, v(1) \right\}, \tag{18}$$

$$\rho\, v(\omega) = \left(\frac{\phi - 1}{\phi}\right) + \delta \left(v(1) - v(\omega)\right) - x_{e,\omega}\, v(\omega), \tag{19}$$

where the left hand side comes from using the Euler equation:

$$r = g + \rho$$

Which gives

$$r \frac{V_t(\Omega)}{Y_t} - \frac{\dot{V}_t(\Omega)}{Y_t} \frac{Y_t}{\dot{Y}_t} \frac{\dot{Y}_t}{V_t(\Omega)} \frac{V_t(\Omega)}{Y_t} = \left(\rho + g\right) v(\Omega) - g\, v(\Omega) = \rho\, v(\Omega).$$

I now move to show that normalized values (18) and (19) are uniquely determined. Given entrants' decisions, and a wage rate w^{RD}, the incumbent's choice of R&D satisfies:

$$x_I = \mathbf{1}\left\{v(\omega) - v(1) > 0\right\} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma - 1}}.$$

Entrants taking x_I as given optimally set:

$$x_{e,1} = \mathbf{1}\left\{v(1) > 0\right\} \frac{v(1)}{\zeta w^{RD}}, \quad x_{e,\omega} = \mathbf{1}\left\{v(1) > 0\right\} \frac{v(1)}{\zeta \omega w^{RD}}.$$

Note that these solutions immediately imply that the normalized value, $v(1)$, is strictly positive. Indeed, $v(1) < 0$ would imply:

$$\rho\, v(1) = \pi + \mathbf{1}\left\{v(\omega) - v(1) > 0\right\} \left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma - 1}}\right) \left(v(\omega) - v(1)\right)$$

where the right hand side is strictly positive. Plugging optimal solutions into the system of equations determining the value functions (42) and (43) gives:

$$\rho\, v(1) - \pi - \mathbf{1}\left\{v(\omega) - v(1) > 0\right\} \left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma - 1}}\right) \left(v(\omega) - v(1)\right) + \frac{v(1)^2}{\zeta w^{RD}} = 0 \tag{20}$$

$$\rho\, v(\omega) - \pi - \delta \left(v(1) - v(\omega)\right) + \frac{v(1)}{\zeta w^{RD} \omega} v(\omega) = 0. \tag{21}$$

The second equation gives $v(\omega)$ as the following function of $v(1)$:

$$v(\omega) = \frac{\pi + \delta\, v(1)}{\rho + \delta + \frac{v(1)}{\zeta w^{RD} \omega}}.$$

Suppose first that $v(\omega) < v(1)$. In this case, the first equation gives:

$$\rho v(1) + \frac{v(1)^2}{\zeta w^{RD}} - \pi = 0.$$

The roots of this equation are:

$$v_{1,2} = \frac{-\rho \pm \sqrt{\rho^2 + 4\frac{\pi}{\zeta w^{RD}}}}{\frac{2}{\zeta w^{RD}}}.$$

Since the term under the root is strictly positive, only one of these roots is admissible, so the above system is solved for a unique pair $v(1), v(\omega)$. Consider now the case $v(\omega) > v(1)$. It is straightforward to note that $v(\omega) - v(1)$ is decreasing in $v(1)$. This implies that, when rewriting (20) as

$$-\left(\frac{\gamma-1}{\gamma}\left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}}\right)(v(\omega)-v(1)) = \pi - \rho v(1) - \frac{v^2(1)}{\zeta w^{RD}}, \tag{22}$$

the left hand side is monotonically increasing in $v(1)$, while the right hand side is monotonically decreasing in $v(1)$. Further, at $v(1) = 0$, the left hand side is strictly negative, while the right hand side equals π, while for $v(1) \to \infty$, the right hand side tends to $+\infty$ while the left hand side decreases towards $-\infty$. As a result, (22) has a unique positive solution.

The uniqueness of $v(1)$ immediately implies unique $v(\omega)$ and R&D choices. Given these R&D choices, the stationary distribution satisfies

$$0 = -\left(x_I + x_{e,1}\right)\mu_1 + \delta\mu_\omega + x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}, \tag{23}$$

$$0 = -\left(x_{e,\omega} + \delta\right)\mu_\omega + x_I\mu_1, \tag{24}$$

$$0 = -\left(x_{e,1} + x_I\right)\mu_{e,1} + x_{e,1}\mu_1 + \delta\mu_{e,\omega}, \tag{25}$$

$$0 = -\left(x_{e,\omega} + \delta\right)\mu_{e,\omega} + x_{e,\omega}\mu_\omega + x_I\mu_{e,1}. \tag{26}$$

By equation (24):

$$x_I\mu_1 = \left(x_{e,\omega} + \delta\right)\mu_\omega$$

Since $\mu_1 = 1 - \mu_\omega$, the stationary distribution has:

$$\mu_\omega = \frac{x_I}{x_I + x_{e,\omega} + \delta},$$

$$\mu_1 = \frac{x_{e,\omega} + \delta}{x_I + x_{e,\omega} + \delta},$$

$$\begin{bmatrix} -\delta & x_{e,1} + x_I \\ x_{e,\omega} + \delta & -x_I \end{bmatrix}\begin{bmatrix} \mu_{e,\omega} \\ \mu_{e,1} \end{bmatrix} = \begin{bmatrix} x_{e,1}\mu_1 \\ x_{e,\omega}\mu_\omega \end{bmatrix}. \tag{27}$$

Since the matrix in (27) is nonsingular, $\mu_{e,\omega}$ and $\mu_{e,1}$ are uniquely determined as:

$$\mu_{e,\omega} = \frac{x_I x_{e,1}\mu_1 + \left(x_{e,1} + x_I\right) x_{e,\omega}\mu_\omega}{x_{e,\omega}\left(x_{e,1} + x_I\right) + \delta x_{e,1}},$$
$$\mu_{e,1} = \frac{\left(x_{e,\omega} + \delta\right) x_{e,1}\mu_1 + \delta x_{e,\omega}\mu_\omega}{x_{e,1}\left(x_{e,\omega} + \delta\right) + x_{e,\omega} x_I}$$

By the optimal solution for entrants:

$$x_{e,1} = \omega x_{e,\omega},$$

so (27) is solved for:

$$\mu_{e,\omega} = \frac{\omega x_I \mu_1 + \left(\omega x_{e,\omega} + x_I\right)\mu_\omega}{\omega\left(x_{e,\omega} + \delta\right) + x_I}, \tag{28}$$
$$\mu_{e,1} = \frac{\omega\left(x_{e,\omega} + \delta\right)\mu_1 + \delta\mu_\omega}{\omega\left(x_{e,\omega} + \delta\right) + x_I}. \tag{29}$$

Thus, the stationary distribution is unique.

It remains to show that equilibrium R&D labor is also unique. To show this, I prove that R&D labor demand is monotonically decreasing in wages and has:

$$\lim_{w^{RD}\to\infty} L^{RD}(w^{RD}) \leq 0, \qquad \lim_{w^{RD}\to 0} L^{RD}(w^{RD}) = \infty.$$

Since the converse holds for R&D labor supply is monotonically increasing in wages and ranges between 0 and $+\infty$, this gives a unique intersection of the two schedules. First note that, if labor supply is inelastic, $\phi = 0$, equilibrium R&D labor is constant by definition. Lemma C.2 below builds on this observation as well as C.1 to prove that research labor demand is indeed monotonically decreasing in the wage.

Lemma C.1. *Consider a steady state of the normalized one-sector model, and assume that defensive innovation is effective,* $\omega > 1$*. Then,* $\omega v(1) > v(\omega) > v(1)$*. Around a steady state, and for a fixed wage rate,* w^{RD}*, the normalized values,* $v(1), v(\omega)$*, are increasing in the markup,* ϕ*, and*

$$\frac{\partial v(\omega)}{\partial \phi} > \frac{\partial v(1)}{\partial \phi} > 0.$$

Proof of Lemma C.1. Subtracting side by side Equation (20) from (21) gives:

$$\left(\rho + \delta + \mathbf{1}\left\{v(\omega) - v(1) > 0\right\}\left(\frac{\gamma - 1}{\gamma}\left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}}\right)\right)(v(\omega) - v(1)) = \frac{v(1)}{\zeta w^{RD}}\left(v(1) - \frac{v(\omega)}{\omega}\right)$$

Suppose that $v(\omega) < v(1)$. This implies that the left hand side of the above expression is strictly smaller than 0, while $\omega v(1) > v(1) > v(\omega)$, so the right hand side is strictly positive under the assumption $\omega > 1$. Therefore, it must be that $v(\omega) > v(1)$. If this is the case, the left hand side is strictly positive, and to avoid a contradiction it must be $\omega v(1) > v(\omega)$. Thus, $\omega v(1) > v(\omega) > v(1)$, proving the first part of the statement.

Since π is a monotone increasing function of ϕ, I prove the statement for value derivatives with respect to π. Total differentiation of the system of Equations (20) and (21) with respect to π around a CGE gives

$$\underbrace{\begin{bmatrix} \rho + \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} + 2\frac{v(1)}{\zeta} & -\left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} \\ -\delta + \frac{v(\omega)}{\zeta w^{RD}\omega} & \rho + \delta + \frac{v(1)}{\zeta w^{RD}\omega} \end{bmatrix}}_{\equiv J} \begin{bmatrix} \mathrm{d}v(1) \\ \mathrm{d}v(\omega) \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathrm{d}\pi = 0. \tag{30}$$

The determinant of the Jacobian is:

$$\det J = \left(\rho + x_I + 2\omega x_{e,\omega}\right)\left(\rho + \delta + \frac{v(1)}{\zeta\omega}\right) + x_I\left(x_{e,\omega} - \delta\right) > 0.$$

Solving (30) gives:

$$\begin{bmatrix} \frac{\mathrm{d}v(1)}{\mathrm{d}\pi} \\ \frac{\mathrm{d}v(\omega)}{\mathrm{d}\pi} \end{bmatrix} = \frac{1}{\det J} \begin{bmatrix} \frac{v(1)}{\zeta w^{RD}\omega} + \rho + \delta & \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} \\ \delta - \frac{v(\omega)}{\zeta w^{RD}\omega} & \rho + \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} + 2\frac{v(1)}{\zeta w^{RD}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Since the first row is strictly positive,

$$\frac{\mathrm{d}v(1)}{\mathrm{d}\pi} > 0.$$

Subtracting line by line gives:

$$\begin{aligned} \frac{\mathrm{d}v(\omega)}{\mathrm{d}\pi} - \frac{\mathrm{d}v(1)}{\mathrm{d}\pi} &= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta w^{RD}\omega} - \rho + \frac{v(1)}{\zeta w^{RD}\omega} + \rho + 2\frac{v(1)}{\zeta w^{RD}}\right] \\ &= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta w^{RD}\omega} - \frac{v(1)}{\zeta w^{RD}\omega} + 2\frac{v(1)}{\zeta w^{RD}}\right] \\ &= \frac{1}{\det J}\left[\frac{2\omega v(1) - (v(\omega) + v(1))}{\zeta w^{RD}\omega}\right] > 0 \end{aligned} \tag{31}$$

since $\omega > 1$ and $\omega v(1) > v(\omega)$, from what shown above. It follows that:

$$\frac{\mathrm{d}v(\omega)}{\mathrm{d}\pi} > \frac{\mathrm{d}v(1)}{\mathrm{d}\pi} > 0.$$

□

Lemma C.2. *R&D labor demand is monotonically decreasing in the wage rate* w_t^{RD}/Y_t*,and:*

$$\lim_{w^{RD}\to\infty} L^{RD}(w^{RD}) \leq 0, \qquad \lim_{w^{RD}\to 0} L^{RD}(w^{RD}) = \infty.$$

Proof. Consider the equilibrium with inelastic R&D labor. By the resource constraint in the economy, it holds:

$$\rho\left(\mu_1 v(1) + \mu_\omega v(\omega)\right) + w^{RD}L^{RD} + wL = 1,$$
$$L^{RD} = \frac{\pi}{w^{RD}} - \rho\left(\mu_1 \frac{v(1)}{w^{RD}} + \mu_\omega \frac{v(\omega)}{w^{RD}}\right).$$

Since the labor supply is fixed, shifts in the right hand side of this equation identify the elasticity of labor supply to various parameters. Now consider an increase in π to $\pi' > \pi$. In this case, the unique equilibrium requires:

$$\frac{\pi'}{w^{RD\prime}} = \frac{\pi}{w^{RD}}.$$

Indeed, guess that the equilibrium involves no changes in research intensities, and therefore in the stationary distribution. Then:

$$x'_{e,\omega} = x_{e,\omega} \Rightarrow \frac{v'(1)}{\zeta\omega w'^{RD}} = \frac{v(1)}{\zeta\omega w^{RD}},$$

and

$$x'_I = \left(\frac{v'(1) - v'(\omega)}{\alpha_I w'^{RD}}\right)^{\frac{1}{\gamma-1}} = \left(\frac{v(1) - v(\omega)}{\alpha_I w^{RD}}\right)^{\frac{1}{1-\gamma}} = x_I.$$

As a result:

$$\frac{v'(\omega)}{w'^{RD}} = \frac{v(\omega)}{w^{RD}}.$$

Using the expression for $v(\omega)$, and using the fact that the ratio between values and wages is the same in both equilibria, gives:

$$\frac{\pi'}{w'^{RD}} = \frac{\pi}{w^{RD}}.$$

This also ensures that:

$$\rho\frac{v(1)}{w^{RD}} = \rho\frac{v'(1)}{w'^{RD}},$$

as is easily verified plugging the above expression into (18) evaluated at $\left(v(1), w^{RD}\right)$ and $\left(v'(1), w^{RD\prime}\right)$.It remains to show that goods' market clearing holds. Before a markup change we have (in normalized values):

$$\rho\left(\mu_1 v(1) + \mu_\omega v(\omega)\right) + w^{RD}L^{RD} + wL = 1,$$
$$\rho\left(\mu_1 \frac{v(1)}{w^{RD}} + \mu_\omega \frac{v(\omega)}{w^{RD}}\right) + L^{RD} = \frac{1 - wL}{w^{RD}},$$

By what shown above, with an inelastic labor research labor supply, the left hand side has the same value before and after the change in instantaneous profits. Further, the linear production function implies that:

$$wL = \frac{1}{\phi},$$

therefore the right hand side can be written as:

$$\frac{\pi}{w^{RD}},$$

which has the same value in the new equilibrium. Therefore, the unique equilibrium with inelastic labor supply is characterized by a constant ratio $\frac{\pi}{w^{RD}}$. Given that the labor supply is inelastic, L^{RD} in the above expression can be read as the labor demand for R&D:[21]

$$L^{RD,d}\left(w^{RD}\right) = \frac{\pi}{w^{RD}} - \rho\left(\mu_1 \frac{v(1)}{w^{RD}} + \mu_\omega \frac{v(\omega)}{w^{RD}}\right)$$

Now consider an initial equilibrium with $L^{RD,d}(w^{RD}) = L^d$. A change in the wage w^{RD} to $w^{RD\prime} > w^{RD}$ modifies the above expression to:

$$L^{RD,d}\left(w^{RD\prime}\right) = \frac{\pi}{w^{RD\prime}} - \rho\left(\mu_1' \frac{v'(1)}{w^{RD\prime}} + \mu_\omega' \frac{v'(\omega)}{w^{RD\prime}}\right).$$

By what shown above, it must be:

$$\frac{\mathrm{d}\pi}{\pi} = \frac{w^{RD\prime} - w^{RD}}{w^{RD}} > 0$$

for $L^{RD,d}$ to be unchanged. Thus, denoting:

$$\pi' = \pi\left(1 + \frac{w^{RD\prime} - w^{RD}}{w^{RD}}\right),$$

the above expression reads:

$$L^{RD,d}\left(w^{RD\prime}\right) = \frac{\pi'}{w^{RD\prime}} + \frac{\pi - \pi'}{w^{RD\prime}} - \rho\left(\mu_1' \frac{v'(1)}{w^{RD\prime}} + \mu_\omega' \frac{v'(\omega)}{w^{RD\prime}}\right).$$

That is:

$$L^{RD,d}\left(w^{RD\prime}\right) = L^{RD,d}\left(w^{RD}\right) + \frac{\pi - \pi'}{w^{RD\prime}} < L^{RD,d}\left(w^{RD}\right).$$

This shows that labor demand is decreasing in the wage. In general, we have:

$$L^{RD,d}\left(w^{RD\prime}\right) = L^{RD,d}\left(w^{RD}\right) + \frac{1}{w^{RD}}\left(\frac{w^{RD}}{w^{RD\prime}} - 1\right)$$

[21] Alternatively, the market clearing expression can be rewritten as the accounting identity that instantaneous profits equal the R&D wage bill plus dividends, which gives the demand for R&D labor as the expression reported below.

Consider now $w^{RD\prime} \to 0$, in this case we clearly have:

$$L^{RD,d}\left(w^{RD\prime}\right) \to \infty.$$

Conversely, with $w^{RD\prime} \to \infty$:

$$L^{RD,d}\left(w^{RD\prime}\right) \to L^{RD,d}\left(w^{RD}\right) - \frac{1}{w^{RD}} = -\rho\left(\mu_1 \frac{v(1)}{w^{RD}} + \mu_\omega \frac{v(\omega)}{w^{RD}}\right) - \frac{wL}{w^{RD}} < 0.$$

$\square$

By Lemma C.2, given an endowment of production labor and an R&D labor supply schedule, the CGE is unique.

To derive the growth rate note that, by the Cobb Douglas assumption on the final good, and given the equilibrium wage rate for production workers, $w = \frac{w_t}{Y_t}$,

$$\begin{aligned}
\log Y_t &= \int_0^1 \log y_t(i) \mathrm{d}i \\
&= \int_0^1 \log\left(\frac{Y_t}{w_t c_t(i)}\right) \mathrm{d}i \\
&= \int_0^1 \log\left(\frac{1}{w c_t(i)}\right) \mathrm{d}i.
\end{aligned}$$

It follows that:

$$\begin{aligned}
g = \log(Y_{t+\Delta t}) - \log(Y_t) &= -\int_0^1 \left(\log c_{t+\Delta}(i) - c_t(i)\right) \mathrm{d}i \\
&= \eta\left[x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1} + \lambda x_I \mu_1\right] \\
&= \eta\left[x_{e,\omega}\left(\mu_{e,\omega} + \omega\mu_{e,1}\right) + \lambda x_I \mu_1\right].
\end{aligned}$$

Productivity g/L^{RD} follows directly from total R&D labor demand:

$$\zeta\omega x_{e,\omega}\left(\mu_{e,\omega} + \mu_{e,1}\right) + \alpha_I \frac{(x_I)^\gamma}{\gamma}\mu_1.$$

$\square$

Proof of Proposition 4.3. The increase in R&D efforts by both incumbents and entrants descend directly from Lemma C.1. In what follows, I derive *equilibrium* quantities, that is factoring in wage effects, but I drop stars for ease of notation.

To prove that the share of R&D labor accruing to incumbents increases, note first:

$$\frac{\partial L_I}{\partial \phi} = \alpha_I x_I^{\gamma-1} \mu_1 \frac{\partial x_I}{\partial \phi} + \frac{\alpha_I}{\gamma} x_I^{\gamma-1} \frac{\partial \left(x_I \mu_1\right)}{\partial \phi},$$

where the first term is strictly positive, since I have prover that $\frac{\partial x_I}{\partial \phi} > 0$, and the term, $\frac{\partial (x_I \mu_1)}{\partial \phi}$,denotes the derivative of aggregate incumbents' research intensity with respect to the markup, and is also strictly positive. Indeed:

$$\frac{\partial \mu_1}{\partial \phi} = \frac{\partial \left(\frac{x_{e,\omega}+\delta}{x_{e,\omega}+\delta+x_I}\right)}{\partial \phi} = \left[\frac{\frac{\partial (x_{e,\omega}+\delta)}{\partial \phi} x_I - \left(x_{e,\omega}+\delta\right)\frac{\partial x_I}{\partial \phi}}{\left(x_I + x_{e,\omega}+\delta\right)^2}\right] = \mu_1 \frac{\partial x_I}{\partial \phi} \frac{(\epsilon - 1)}{\left(x_I + x_{e,\omega}+\delta\right)}, \tag{32}$$

where I define the ratio of the elasticity of $x_{e,\omega}+\delta$ and x_I to ϕ as:

$$\epsilon \equiv \frac{\epsilon_e}{\epsilon_I} \equiv \frac{\frac{\partial (x_{e,\omega}+\delta)}{\partial \phi}/x_{e,\omega}}{\frac{\partial x_I}{\partial \phi}/x_I} \in (0,1].$$

therefore:

$$\begin{aligned}
\frac{\partial \left(\mu_1 x_I\right)}{\partial \phi} &= \mu_1 \frac{\partial x_I}{\partial \phi}\left[\frac{x_I(\epsilon-1)}{\left(x_I + x_{e,\omega}+\delta\right)} + 1\right] \\
&= \mu_1 \frac{\partial x_I}{\partial \phi}\left[\frac{x_I \epsilon + x_{e,\omega}+\delta}{\left(x_I + x_{e,\omega}+\delta\right)}\right] > 0.
\end{aligned}$$

This proves that the aggregate incumbents' research intensity, $x_I \mu_1$, is increasing in the markup. By (32), μ_1 decreases with ϕ if and only if $\epsilon < 1$, that is, x_I is more elastic than $x_{e,\omega}$ to changes in the markup. Therefore, I now proceed to show that, when $\lambda = 0$, productivity is unambiguously decreasing in ϕ if the mass of unprotected markets, μ_1, falls with ϕ. With $\lambda = 0$, inventors' productivity reads:

$$\begin{aligned}
\frac{g}{L^{RD}} &= \eta \frac{x_{e,\omega}\left(\mu_{e,\omega} + \omega \mu_{e,1}\right)}{L_e + L_I}, \\
&= \eta \frac{x_{e,\omega}\left(\mu_{e,\omega} + \omega \mu_{e,1}\right)}{L_e\left(1 + \frac{L_I}{L_e}\right)} \\
&= \frac{\eta}{\zeta\omega} \underbrace{\frac{\mu_{e,\omega} + \omega \mu_{e,1}}{\mu_{e,\omega} + \mu_{e,1}}}_{\equiv R} \frac{1}{\left(1 + \frac{L_I}{L_e}\right)},
\end{aligned}$$

where L_e denotes entrants' R&D labor, $\zeta\omega x_{e,\omega}\left(\mu_{e,1} + \mu_{e,\omega}\right)$, and L_I denotes incumbents' inventors, $\frac{\alpha_I}{\gamma} x_I^{\gamma}$. By what I have shown above, L_I/L_e increases with ϕ, so the second term is decreasing in the markup. The statement is the verified if the first ratio, R, is also decreasing in ϕ. Dividing numerator and denominator in R by $\mu_{e,\omega}$,we have that:

$$R = \frac{1+\omega\frac{\mu_{e,1}}{\mu_{e,\omega}}}{1+\frac{\mu_{e,1}}{\mu_{e,\omega}}}.$$

Since $\omega > 1$, R increases in the ratio of entrants in unprotected versus protected markets, as intuitive. Now define this ratio writes, using the stationary distribution of entrants in (28) and (29), and after some algebra:

$$\frac{\mu_{e,1}}{\mu_{e,\omega}} = \frac{\omega\left[\frac{\mu_1}{1-\mu_1}\right]^2}{\omega\frac{\mu_1}{1-\mu_1}\left(\frac{2x_{e,\omega}+\delta}{x_{e,\omega}+\delta}\right)+1} + \frac{\delta}{\omega\left(x_{e,\omega}+\delta\right)+\omega x_{e,\omega}+x_I},$$

where the second term is always decreasing in ϕ since research intensities are increasing in ϕ. Provided that μ_1 is decreasing in ϕ, it is also straightforward to show that the first term is decreasing if μ_1 decreases.[22]

This proves that if the mass of unprotected markets, μ_1, decreases with markups, R&D productivity also falls. By (32), μ_1 decreases with ϕ if and only if $\epsilon < 1$, that is, x_I is more elastic than $x_{e,\omega}$,

$$\frac{\partial x_I}{\partial \phi}\frac{\phi}{x_I} > \frac{\partial x_{e,\omega}}{\partial \phi}\frac{\phi}{x_{e,\omega}},$$

proving the statement.[23] □

Corollary C.3. *If costs are quadratic, $\gamma = 2$, there is no depreciation, $\delta = 0$, and the supply of inventors is perfectly elastic, a sufficient condition for productivity to decrease with markups is given by:*

$$\sqrt{\zeta\frac{\phi-1}{\phi}}\left(\frac{\alpha_I-\zeta\omega\left(\omega-1\right)}{\alpha_I\zeta\omega}\right) > \rho.$$

Proof. In the quadratic case, optimal incumbents' research intensity reads:

$$x_I = \frac{v(\omega)-v(1)}{\alpha_I w^{RD}}$$

[22] Let $z \equiv \frac{\mu_1}{1-\mu_1}$, $t \equiv \left(\frac{2x_{e,\omega}+\delta}{x_{e,\omega}+\delta}\right)$, and let primes denote derivatives with respect to ϕ. Then:

$$\partial\left[\frac{\omega z^2}{\omega z t+1}\right] = \frac{2\omega z z' + 2\omega^2 z^2 z' t - \omega^2 z^2 z' t - \omega z^2 t'}{(\omega z t+1)^2} = \frac{\omega^2 z^2 z' t + 2\omega z z' - \omega z^2 t'}{(\omega z t+1)^2} < 0$$

if $z' < 0$. Indeed $t' > 0$ since $x_{e,\omega}$ increases in ϕ.

[23] In particular, this condition holds if, at given wages, the elasticity of incumbents' demand for research intensity is larger than entrants', and

$$(\omega-1) \in \left[2, \frac{1}{\gamma-1}\right].$$

In this case, it is possible to show that incumbents' demand for research intensity is less wage elastic than entrants', so equilibrium wage effects do not overturn demand effects on the ratio $x_I/x_{e,\omega}$.

Therefore:

$$\partial\left[\frac{x_I}{x_{e,\omega}}\right] = \frac{\zeta\omega}{\alpha_1}\partial\left[\frac{v(\omega)}{v(1)} - 1\right].$$

Therefore the elasticity of x_I to ϕ is larger than that of $x_{e,\omega}$ if and only if:

$$\operatorname{sign}\left(\frac{\partial\left(v(\omega)/v(1)\right)}{\partial m}\right) = \operatorname{sign}\left(\frac{\partial v(\omega)}{\partial m}v(1) - \frac{\partial v(1)}{\partial m}v(\omega)\right) > 0. \tag{33}$$

By Lemma C.1 applied to the case $\gamma = 2$:

$$\begin{aligned}\begin{bmatrix}\frac{\mathrm{d}v(1)}{\mathrm{d}\pi}\\ \frac{\mathrm{d}v(\omega)}{\mathrm{d}\pi}\end{bmatrix} &= \frac{1}{\det J}\begin{bmatrix}\frac{\rho\zeta\omega+v(1)}{\zeta\omega} & \frac{v(\omega)-v(1)}{\alpha_I}\\ -\frac{v(\omega)}{\zeta\omega} & \rho+\frac{v(\omega)-v(1)}{\alpha_I}+2\frac{v(1)}{\zeta}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\\ &= \frac{1}{\det J}\begin{bmatrix}\rho+x_{e,\omega} & x_I\\ -x_{e,\omega} & \rho+x_I+2\omega x_{e,\omega}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\end{aligned}$$

Thus Equation (33) has the same sign as:

$$\left(\rho + x_I + (2\omega-1)\,x_{e,\omega}\right)v(1) - \left(\rho + x_I + x_{e,\omega}\right)v(\omega).$$

With $\omega > 1$, by Lemma C.1 it holds:

$$\omega v(1) > v(\omega).$$

Therefore a sufficient condition for the ratio z to increase in m is:

$$\begin{aligned}\left(\rho + x_I + (2\omega-1)\,x_{e,\omega}\right) &> \omega\left(\rho + x_I + x_{e,\omega}\right)\\ (\omega-1)\,x_{e,\omega} &> (\omega-1)\left(\rho + x_I\right)\\ x_{e,\omega} - x_I &> \rho.\end{aligned}$$

Using once again, $\omega v(1) > v(\omega)$, it is possible to write

$$\begin{aligned}x_{e,\omega} - x_I &> x_{e,\omega}\left(1-\zeta\omega\frac{(\omega-1)}{\alpha_I}\right)\\ &= \frac{v(1)}{\zeta\omega}\left(1-\zeta\omega\frac{(\omega-1)}{\alpha_I}\right)\\ &= v(1)\left(\frac{\alpha_I-\zeta\omega\,(\omega-1)}{\alpha_I\zeta\omega}\right).\end{aligned}$$

Finally, by definition of the value function:

$$\rho v(1) \geq m - \frac{v(1)^2}{\zeta},$$

with equality only when it is optimal for incumbents not to invest. Solving gives:

$$v(1) > \frac{-\rho\zeta + \sqrt{(\rho\zeta)^2 + 4\zeta m}}{2} > \sqrt{\zeta m}$$

Therefore:

$$x_{e,\omega} - x_I > \sqrt{\zeta m}\left(\frac{\alpha_I - \zeta\omega(\omega-1)}{\alpha_I \zeta\omega}\right) > \rho,$$

proving that the statement gives a sufficient condition for the elasticity of x_I to be larger than $x_{e,\omega}$. By Proposition 4.3, it follows that when this condition is satisfied, increases in markup lower growth. □

C.2 Full Description of the Two-Sector Model and Derivations

By the above assumptions, the final good is produced according to:

$$Y = \prod Y_i^{\beta_i}. \tag{34}$$

With the final good as numeraire, the sector's demand schedule is:

$$Y_i = \beta_i \frac{Y}{P_i}. \tag{35}$$

From CD on intermediate goods we also have:

$$P_i Y_i = p_{is} y_{is}, \quad \forall s.$$

In each sector, the price is set at the competitive fringe's marginal cost wc_i, and is identical across subsectors . Thus

$$P_i = p_{is} = wc_i, \; Y_i = \beta_i \frac{Y}{wc_i}. \tag{36}$$

Equilibrium profits are given by:

$$\Pi_i = \left(c_i w - \frac{c_i w}{\phi_i}\right) Y_i = \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y.$$

The monopolist demands production labor:

$$\ell_{is} = \frac{c_i y_{is}}{\phi_i}, \Rightarrow L_i = \int \ell_{is} \mathrm{d}s = Y\frac{\beta_i}{\phi_i w}. \tag{37}$$

Assuming a rigid production labor supply:[24]

$$L^s(w) = L = \frac{Y}{w}\left(\sum \frac{\beta_i}{\phi_i}\right). \tag{38}$$

Which gives:

$$L_i = L\frac{\frac{\beta_i}{\phi_i}}{\sum \frac{\beta_i}{\phi_i}}, Y_i = L\frac{\frac{\beta_i}{c_i}}{\sum \frac{\beta_i}{\phi_i}}. \tag{39}$$

Which gives:

$$Y = L\prod_i \left(\frac{\frac{\beta_i}{c_i}}{\sum \frac{\beta_i}{\phi_i}}\right)^{\beta_i}. \tag{40}$$

Thus, growth is:

$$-\sum \beta_i \Delta \log c_i. \tag{41}$$

Normalized values in each sector are the same as before, with the only difference that they receive a wage w^R, and the above α_I, ζ are replaced by $\zeta w^R, \alpha_I w^R$.

C.2.1 Research Equilibrium in the two-sector model

By the above solutions, the monopolist's values read:

$$\rho V_i(1) = \max_{x_I}\left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y - \alpha_I W^{RD}\frac{x_I^2}{2} + x_I\left(V_i(\omega) - V_i(1)\right) - x_{e,1}V_i(1),$$
$$\rho V_i(\omega_i) = \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y + \delta\left(V(1) - V(\omega)\right) - x_{e,\omega}V_i(\omega).$$

And normalized values, $v \equiv V/Y$:

$$\rho v_i(1) = \max_{x_I}\left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i - \alpha_I w^{RD}\frac{x_I^2}{2} + x_I\left(v_i(\omega) - v_i(1)\right) - x_{e,1}v_i(1) \tag{42}$$

$$\rho v_i(\omega_i) = \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i + \delta\left(v(1) - v(\omega)\right) - x_{e,\omega}v_i(\omega), \tag{43}$$

[24] Consider a labor supply with elasticity φ. This gives:

$$\chi w^{\varphi} = \frac{Y}{w}\left(\sum \frac{\beta_i}{\phi_i c_i}\right) \Rightarrow w = \left[\frac{Y}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{\frac{1}{1+\varphi}}$$

Equilibrium labor is then:

$$L^\star = \chi\left[\frac{Y}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{\frac{\varphi}{1+\varphi}}, \frac{Y}{w} = Y^{\frac{\varphi}{1+\varphi}}\left[\frac{1}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{-\frac{1}{1+\varphi}} = L^\star\left(\sum \frac{\beta_i}{\phi_i c_i}\right)^{-1}$$

Which results in the same allocations and outputs as below, with $L^\star$ in place of the fixed L.

where w^{RD} is the normalized researchers' wage.

Given a normalized wage, each sector demands:

$$x_{e,\omega,i}\left(w^{RD}\right)=\frac{v_i(1)}{w^{RD}\omega\zeta_i}, \tag{44}$$

$$x_{I,i}\left(w^{RD}\right)=\frac{\left(v_i(\omega_i)-v_i(1)\right)}{w^{RD}\alpha_{I,i}}. \tag{45}$$

The stationary distribution within each sector is given by:

$$\mu_{\omega,i}\left(w^{RD}\right)=\frac{x_{I,i}\left(w^{RD}\right)}{x_{e,\omega,i}\left(w^{RD}\right)+\delta_i+x_{I,i}\left(w^{RD}\right)}, \tag{46}$$

$$\mu_{1,i}\left(w^{RD}\right)=\frac{x_{e,\omega,i}\left(w^{RD}\right)+\delta_i}{x_{e,\omega,i}\left(w^{RD}\right)+\delta_i+x_{I,i}\left(w^{RD}\right)}, \tag{47}$$

$$\mu_{e,1,i}\left(w^{RD}\right)=\frac{\omega_i\left(x_{e,\omega,i}\left(w^{RD}\right)+\delta\right)\mu_{1,i}+\delta_i\mu_{\omega,i}}{\left(x_{I,i}+\omega_i\left(x_{e,\omega,i}\left(w^{RD}\right)+\delta_i\right)\right)}, \tag{48}$$

$$\mu_{e,\omega,i}\left(w^{RD}\right)=\frac{\omega_i\mu_{1,i}x_{I,i}\left(w^{RD}\right)-\omega_i\delta_i\mu_{\omega,i}}{\left(x_{I,i}+\omega_i\left(x_{e,\omega,i}\left(w^{RD}\right)+\delta_i\right)\right)}+\mu_{\omega,i}. \tag{49}$$

Sector RD labor demand is given by:

$$L_i^{RD,d}\left(w^{RD}\right)=\mu_{e,\omega,i}\left(w^{RD}\right)\left(\zeta_i\omega_i x_{e,\omega,i}\left(w^{RD}\right)\right)+\mu_{1,e,i}\left(w^{RD}\right)\zeta_i x_{e,1,i}\left(w^{RD}\right)+\mu_{1,i}\left(w^{RD}\right)\alpha_I\frac{x_{I,i}^2\left(w^{RD}\right)}{2}.$$

With an inelastic labor supply fixed to L^{RD}, market clearing for inventors then reads:

$$L^{RD}=\sum_i\left\{\mu_{\omega,i}\left(w^{RD}\right)\left(\zeta_i\omega_i x_{e,\omega,i}\left(w^{RD}\right)\right)+\mu_{1,e,i}\left(w^{RD}\right)\zeta_i x_{e,1,i}\left(w^{RD}\right)+\mu_{1,i}\left(w^{RD}\right)\alpha_I\frac{x_{I,i}^2\left(w^{RD}\right)}{2}\right\}. \tag{50}$$