A Data Construction Details

Coming soon...

B Omitted Proofs and Derivations

B.1 One-sector model

PROOF OF LEMMA **??** The values, $v(1), v(\omega)$, satisfy the system:

$$\rho v(1) - m_i - \mathbf{1}\{v(\omega) - v(1) > 0\} \frac{(v(\omega) - v(1))^2}{2\alpha_I} + \frac{(v(1))^2}{\zeta} = 0 \tag{1}$$

$$\rho v(\omega) - m_i - \delta (v(1) - v(\omega)) + \frac{v(1)}{\zeta\omega} (v(\omega)) = 0 \tag{2}$$

Then:

$$\rho (v(\omega) - v(1)) = \delta (v(1) - v(\omega)) - \frac{v(1)}{\zeta\omega} (v(\omega)) - \frac{(v(\omega) - v(1))^2}{2\alpha_I} + \frac{v(1)}{\zeta} v(1)$$

$$\left(\rho + \delta + \mathbf{1}\{v(\omega) - v(1) > 0\} \frac{(v(\omega) - v(1))}{2\alpha_I}\right) (v(\omega) - v(1)) = \frac{v(1)}{\zeta} \left(v(1) - \frac{v(\omega)}{\omega}\right)$$

Suppose that $v(\omega) < v(1)$. This implies that the left hand side of the above expression is strictly smaller than 0, while the right hand side is strictly positive since $\omega > 1$. Therefore, it must be that $v(\omega) > v(1)$. If this is the case, the left hand side is strictly positive, and to avoid a contradiction it must be $\omega v(1) > v(\omega)$. Thus, $\omega v(1) > v(\omega) > v(1)$.

Total differentiation of the system, (1)(2), gives:

$$\underbrace{\begin{bmatrix} \rho + \frac{v(\omega) - v(1)}{\alpha_I} + 2\frac{v(1)}{\zeta} & -\frac{v(\omega) - v(1)}{\alpha_I} \\ -\delta + \frac{v(\omega)}{\zeta\omega} & \rho + \delta + \frac{v(1)}{\zeta\omega} \end{bmatrix}}_{\equiv J} \begin{bmatrix} \mathrm{d}v(1) \\ \mathrm{d}v(\omega) \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathrm{d}m_i = 0 \tag{3}$$

The determinant of the Jacobian is:

$$\det J = \left(\rho + x_I + 2\omega x_{e,\omega}\right) \left(\rho + \delta + \frac{v(1)}{\zeta\omega}\right) + x_I \left(x_{e,\omega} - \delta\right) > 0.$$

Note first that the determinant of the Jacobian, J, is strictly positive, setting $\delta = 0$. Solving (3) gives:

$$\begin{bmatrix} \frac{\mathrm{d}v(1)}{\mathrm{d}m_i} \\ \frac{\mathrm{d}v(\omega)}{\mathrm{d}m_i} \end{bmatrix} = \frac{1}{\det J} \begin{bmatrix} \frac{\rho\zeta\omega + v(1)}{\zeta\omega} + \delta & \frac{v(\omega) - v(1)}{\alpha_I} \\ \delta - \frac{v(\omega)}{\zeta\omega} & \rho + \frac{v(\omega) - v(1)}{\alpha_I} + 2\frac{v(1)}{\zeta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Since the first row is strictly positive,

$$\frac{\mathrm{d}v(1)}{\mathrm{d}m_i} > 0.$$

Subtracting line by line gives:

$$\begin{aligned}
\frac{\mathrm{d}v(\omega)}{\mathrm{d}m_i} - \frac{\mathrm{d}v(1)}{\mathrm{d}m_i} &= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta\omega} - \frac{\rho\zeta\omega + v(1)}{\zeta\omega} + \rho + 2\frac{v(1)}{\zeta}\right] \\
&= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta\omega} - \frac{v(1)}{\zeta\omega} + 2\frac{v(1)}{\zeta}\right] \\
&= \frac{1}{\det J}\left[\frac{2\omega v(1) - (v(\omega) + v(1))}{\zeta\omega}\right] > 0
\end{aligned}$$

since $\omega > 1$ and $\omega v(1) > v(\omega)$. It follows that:

$$\frac{\mathrm{d}v(\omega)}{\mathrm{d}m_i} > \frac{\mathrm{d}v(1)}{\mathrm{d}m_i} > 0.$$

Derivation of the Stationary Distribution The law of motion of the distribution across states satisfies:

$$\dot{\mu}_1 = -\left(x_I + x_{e,1}\right)\mu_1 + \delta\mu_\omega + x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}, \tag{4}$$

$$\dot{\mu}_\omega = -\left(x_{e,\omega} + \delta\right)\mu_\omega + x_I\mu_1, \tag{5}$$

$$\dot{\mu}_{e,1} = -\left(x_{e,1} + x_I\right)\mu_{e,1} + x_{e,1}\mu_1 + \delta\mu_{e,\omega}, \tag{6}$$

$$\dot{\mu}_{e,\omega} = -\left(x_{e,\omega} + \delta\right)\mu_{e,\omega} + x_{e,\omega}\mu_\omega + x_I\mu_{e,1}, \tag{7}$$

By equation (5):

$$x_I\mu_1 = \left(x_{e,\omega} + \delta\right)\mu_\omega$$

Since $\mu_1 = 1 - \mu_\omega$, the stationary distribution has:

$$\mu_\omega = \frac{x_I}{x_I + x_{e,\omega} + \delta},$$

$$\mu_1 = \frac{x_{e,\omega} + \delta}{x_I + x_{e,\omega} + \delta},$$

$$\begin{bmatrix} -\delta & x_{e,1} + x_I \\ x_{e,\omega} + \delta & -x_I \end{bmatrix}\begin{bmatrix} \mu_{e,\omega} \\ \mu_{e,1} \end{bmatrix} = \begin{bmatrix} x_{e,1}\mu_1 \\ x_{e,\omega}\mu_\omega \end{bmatrix}. \tag{8}$$

Solving the system (8):

$$\mu_{e,\omega} = \frac{x_I x_{e,1}\mu_1 + \left(x_{e,1} + x_I\right)x_{e,\omega}\mu_\omega}{x_{e,\omega}\left(x_{e,1} + x_I\right) + \delta x_{e,1}},$$

$$\mu_{e,1} = \frac{\left(x_{e,\omega} + \delta\right)x_{e,1}\mu_1 + \delta x_{e,\omega}\mu_\omega}{x_{e,1}\left(x_{e,\omega} + \delta\right) + x_{e,\omega}x_I}$$

By the optimal solution for entrants:

$$x_{e,1} = \omega x_{e,\omega},$$

so (8) is solved for:

$$\begin{aligned}
\mu_{e,\omega} &= \frac{\omega x_I \mu_1 + \left(\omega x_{e,\omega} + x_I\right)\mu_\omega}{\omega\left(x_{e,\omega}+\delta\right)+x_I},\\
\mu_{e,1} &= \frac{\omega\left(x_{e,\omega}+\delta\right)\mu_1 + \delta\mu_\omega}{\omega\left(x_{e,\omega}+\delta\right)+x_I}.
\end{aligned}$$

PROOF OF PROPOSITION **??**. The increase in R&D and growth descend directly from Lemma **??**. Note that the incumbents' share of researchers increases with:

$$\frac{\alpha_I}{\zeta}\frac{x_I}{2}\frac{\mu_1 x_I}{\mu_{e,1}x_{e,1}+\mu_{e,\omega}\omega x_{e,\omega}} = \frac{\mu_1 x_I}{\omega x_{e,\omega}\left[\mu_{e,1}+\mu_{e,\omega}\right]}$$

Now note:

$$\mu_{e,1}+\mu_{e,\omega} = \frac{\omega\left(x_{e,\omega}+\delta\right)\mu_1+\delta\mu_\omega}{\omega\left(x_{e,\omega}+\delta\right)+x_I} + \frac{\omega x_I\mu_1+\left(\omega x_{e,\omega}+x_I\right)\mu_\omega}{\omega\left(x_{e,\omega}+\delta\right)+x_I}$$

with $\delta = 0$:

$$\begin{aligned}
\mu_{e,1}+\mu_{e,\omega} &= \frac{\omega x_{e,\omega}\mu_1}{\omega x_{e,\omega}+x_I} + \frac{\omega x_I\mu_1+\left(\omega x_{e,\omega}+x_I\right)\mu_\omega}{\omega x_{e,\omega}+x_I}\\
&= \frac{\omega x_{e,\omega}\mu_1}{\omega x_{e,\omega}+x_I} + \frac{\omega x_I\mu_1+\left(\omega x_{e,\omega}+x_I\right)\left(1-\mu_1\right)}{\omega x_{e,\omega}+x_I}\\
&= (\omega-1)\,\mu_1\frac{x_I}{\omega x_{e,\omega}+x_I}+1
\end{aligned}$$

Thus the above becomes:

$$\begin{aligned}
\frac{\alpha_I}{\zeta\omega}\frac{x_I}{2}\frac{\mu_1 x_I}{x_{e,\omega}\left[\mu_{e,1}+\mu_{e,\omega}\right]} &= \frac{\alpha_I}{\zeta\omega}\frac{x_I}{2}\frac{\mu_1 x_I}{x_{e,\omega}\left[1+(\omega-1)\,\mu_1\frac{x_I}{\omega x_{e,\omega}+x_I}\right]}\\
&= \frac{\alpha_I}{\zeta\omega}\frac{x_I}{2}\frac{x_I}{\frac{x_{e,\omega}}{\mu_1}+(\omega-1)\,x_{e,\omega}\frac{x_I}{\omega x_{e,\omega}+x_I}}\\
&= \frac{\alpha_I}{\zeta\omega}\frac{x_I}{2}\frac{x_I\left(\omega x_{e,\omega}+x_I\right)}{\left(x_I+x_{e,\omega}\right)\left(\omega x_{e,\omega}+x_I\right)+(\omega-1)\,x_{e,\omega}x_I}\\
&= \frac{\alpha_I}{\zeta\omega}\frac{x_I}{2}\frac{\left(\omega+\frac{x_I}{x_{e,\omega}}\right)}{\left(1+\frac{x_{e,\omega}}{x_I}\right)\left(\omega+\frac{x_I}{x_{e,\omega}}\right)+(\omega-1)}\\
&= \frac{\alpha_I}{\zeta\omega}\frac{x_I}{2}\frac{(\omega+z)}{\left(1+\frac{1}{z}\right)(\omega+z)+(\omega-1)},
\end{aligned}$$

where I define $z \equiv x_I/x_{e,\omega}$.Above, I showed that $\frac{\partial x_I}{\partial m} > 0$. To show that the fraction is also increasing , note that:

$$\frac{\partial\left[\frac{(\omega+z)}{\left(1+\frac{1}{z}\right)(\omega+z)+(\omega-1)}\right]}{\partial z}=\frac{\left(1+\frac{1}{z}\right)(\omega+z)+(\omega-1)-(\omega+z)\left[-\frac{1}{z^2}(\omega+z)+\left(1+\frac{1}{z}\right)\right]}{\left[\left(1+\frac{1}{z}\right)(\omega+z)+(\omega-1)\right]}$$
$$=\frac{\frac{\omega+z}{z^2}(\omega+z)+(\omega-1)}{\left[\left(1+\frac{1}{z}\right)(\omega+z)+(\omega-1)\right]}>0$$

Thus the sign of the fraction only depends on the sign of the derivative of z with respect to m.

By definition:

$$z\equiv\frac{\zeta\omega}{\alpha_I}\left(\frac{v(\omega)}{v(1)}-1\right)m.$$

As shown above, positive research implies that $\omega v(1)>v(\omega)$, or equivalently that $\frac{v(\omega)}{v(1)}<\omega$. This gives

$$z\equiv\frac{x_I}{x_e}<\frac{\zeta\omega(\omega-1)}{\alpha_I}\Rightarrow x_I<\frac{\zeta\omega(\omega-1)}{\alpha_I}x_e.$$

Further:

$$\operatorname{sign}\left(\frac{\partial z}{\partial m}\right)=\operatorname{sign}\left(\frac{\partial\left(v(\omega)/v(1)\right)}{\partial m}\right)=\operatorname{sign}\left(\frac{\partial v(\omega)}{\partial m}v(1)-\frac{\partial v(1)}{\partial m}v(\omega)\right).\tag{9}$$

By Lemma **??**:

$$\begin{bmatrix}\frac{\mathrm{d}v(1)}{\mathrm{d}m_i}\\ \frac{\mathrm{d}v(\omega)}{\mathrm{d}m_i}\end{bmatrix}=\frac{1}{\det J}\begin{bmatrix}\frac{\rho\zeta\omega+v(1)}{\zeta\omega} & \frac{v(\omega)-v(1)}{\alpha_I}\\ -\frac{v(\omega)}{\zeta\omega} & \rho+\frac{v(\omega)-v(1)}{\alpha_I}+2\frac{v(1)}{\zeta}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$
$$=\frac{1}{\det J}\begin{bmatrix}\rho+x_{e,\omega} & x_I\\ -x_{e,\omega} & \rho+x_I+2\omega x_{e,\omega}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

Thus (9) has the same sign as:

$$\left(\rho+x_I+(2\omega-1)x_{e,\omega}\right)v(1)-\left(\rho+x_I+x_{e,\omega}\right)v(\omega)$$

Since there is positive research, it holds:

$$\omega v(1)>v(\omega),$$

therefore a sufficient condition for the ratio z to increase in m is:

$$\left(\rho+x_I+(2\omega-1)x_{e,\omega}\right)>\omega\left(\rho+x_I+x_{e,\omega}\right)$$
$$(\omega-1)x_{e,\omega}>(\omega-1)\left(\rho+x_I\right)$$
$$x_{e,\omega}-x_I>\rho.$$

Note that, by definition of z:

$$\begin{aligned} x_{e,\omega} - x_I &> x_{e,\omega}\left(1 - \zeta\omega\frac{(\omega-1)}{\alpha_I}\right) \\ &= \frac{v(1)}{\zeta\omega}\left(1 - \zeta\omega\frac{(\omega-1)}{\alpha_I}\right) \\ &= v(1)\left(\frac{\alpha_I - \zeta\omega(\omega-1)}{\alpha_I\zeta\omega}\right). \end{aligned}$$

By the definition of the value function:

$$\rho v(1) \geq m - \frac{v(1)^2}{\zeta},$$

with equality only when it is optimal for incumbents not to invest. This gives:

$$v(1) > \frac{-\rho\zeta + \sqrt{(\rho\zeta)^2 + 4m}}{2} > \sqrt{m}$$

Therefore:

$$x_{e,\omega} - x_I > \sqrt{m}\left(\frac{\alpha_I - \zeta\omega(\omega-1)}{\alpha_I\zeta\omega}\right) > \rho,$$

by the main assumption in the statement. The decrease in inventor's productivity follows immediately noting that:

$$\begin{aligned} \frac{g}{L^{RD}} &= \eta\frac{x_{e,\omega}\left(\mu_{e,\omega} + \omega\mu_{e,1}\right)}{\zeta\omega x_{e,\omega}\left(\mu_{e,\omega} + \mu_{e,1}\right) + \alpha_I\frac{x_I^2}{2}\mu_1} \\ &= \eta\frac{x_{e,\omega}\left(\mu_{e,\omega} + \omega\mu_{e,1}\right)}{L_e\left(1 + \frac{L_I}{L_e}\right) + \alpha_I\frac{x_I^2}{2}\mu_1} \\ &= \frac{\eta}{\zeta\omega}\frac{\mu_{e,\omega} + \omega\mu_{e,1}}{\mu_{e,\omega} + \mu_{e,1}}\frac{1}{\left(1 + \frac{L_I}{L_e}\right)} \\ &= \frac{\eta}{\zeta\omega}\frac{\mu_{e,\omega} + \omega\mu_{e,1}}{(\omega-1)\mu_1\frac{x_I}{\omega x_{e,\omega} + x_I} + 1}\frac{1}{\left(1 + \frac{L_I}{L_e}\right)} \end{aligned}$$

Recall:

$$\begin{aligned} \mu_{e,1} + \omega\mu_{e,\omega} &= \frac{\omega x_{e,\omega}\mu_1}{\omega x_{e,\omega} + x_I} + \omega\frac{\omega x_I\mu_1 + \left(\omega x_{e,\omega} + x_I\right)\mu_\omega}{\omega x_{e,\omega} + x_I} \\ &= 1 - \mu_1(\omega-1)\frac{x_{e,\omega} - x_I}{\omega x_{e,\omega} + x_I} \end{aligned}$$

And:

$$\frac{\mu_{e,\omega}+\omega\mu_{e,1}}{\mu_{e,\omega}+\mu_{e,1}} = \frac{\frac{\omega x_{e,\omega}\mu_1}{\omega x_{e,\omega}+x_I}+\omega\frac{\omega x_I\mu_1+(\omega x_{e,\omega}+x_I)\mu_\omega}{\omega x_{e,\omega}+x_I}}{\frac{\omega x_{e,\omega}\mu_1}{\omega x_{e,\omega}+x_I}+\frac{\omega x_I\mu_1+(\omega x_{e,\omega}+x_I)\mu_\omega}{\omega x_{e,\omega}+x_I}}$$

$$= \frac{\omega x_{e,\omega}+\omega\left[\omega x_I+\left(\omega x_{e,\omega}+x_I\right)\frac{\mu_\omega}{\mu_1}\right]}{\omega x_{e,\omega}+\omega x_I+\left(\omega x_{e,\omega}+x_I\right)\frac{\mu_\omega}{\mu_1}}$$

$$= \frac{\omega+\omega\left[\omega\frac{x_I}{x_e}+\left(\omega+\frac{x_I}{x_e}\right)\frac{x_I}{x_e}\right]}{\omega+\omega\frac{x_I}{x_e}+\left(\omega+\frac{x_I}{x_e}\right)\frac{x_I}{x_e}}$$

$$\equiv \omega\frac{1+g\left(\frac{x_I}{x_e}\right)}{\omega+g\left(\frac{x_I}{x_e}\right)}$$

Thus the ratio is decreasing in $g(\cdot)$ which is itself increasing in $x_I/x_{e,\omega}$. By the previous points, increases in markup lower growth by concentrating more resources with incumbents.

B.2 Two-Sector Model

C Additional Results and Robustness

C.1 Using the Raw Number of Inventors instead of Fixed-Effects

This Appendix reports the results for the main analysis presented in Section **??** using the raw number of total inventors instead of the fixed effects from regression (**??**), which might be inconsistently estimated. The following Tables, to be compared with Tables **??** and **??** in the main text, show that the results are qualitatively unchanged. Looking at the scale of the y-axis in panel (a) of Figure 1, it is apparent that the shares of the raw number of inventors are more volatile, and presents larger changes. This is easily explained by the fact that differences in research requirements across patent classes, firms and years are not absorbed as in the effective inventor measure. This greater variability simply results in larger and noisier coefficients, which nevertheless remain positive and significant.

Table 1: Regressions of Change in 4-digit Knowledge Market Share of Total Inventors over Change in HHI Measures, Long-Differences, 1997-2012

	Ch. 4d K.M. Eff. Inv. Share (%)					
	(1)	(2)	(3)	(4)	(5)	(6)
Ch. HHI lower bound	74.172+		74.814+		74.177+	
	(40.957)		(41.208)		(41.047)	
Ch. HHI		71.749**		71.749**		71.583**
		(24.464)		(24.464)		(24.433)
4D Knowledge Market						
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	80	156	80	150	72

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (**??**), when the outcome is the share of total inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Economic Census concentration ratios, or the HHI index reported in the Economic Census. "Full Sample", "Trim Outliers" and "Mahalanobis 5%" refer to the samples described in the main text.

Table 2: Regressions of Change in 4-digit Knowledge Market Share of Total Inventors over Change in HHI Lower Bound, Long-Differences, 1997-2012

(a) Controlling for Change in Log Real Sales

	Ch. 4d K.M. Eff. Inv. Share (%)					
	(1)	(2)	(3)	(4)	(5)	(6)
Ch. HHI lower bound	71.724+	67.160+	72.123+	67.736+	71.772+	68.398+
	(39.265)	(37.176)	(39.530)	(37.504)	(39.316)	(37.717)
Ch. Log Real Sales	1.864*	1.422*	1.852*	1.402+	1.878*	1.443+
	(0.766)	(0.717)	(0.764)	(0.712)	(0.774)	(0.745)
4D Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	156	156	155	150	142

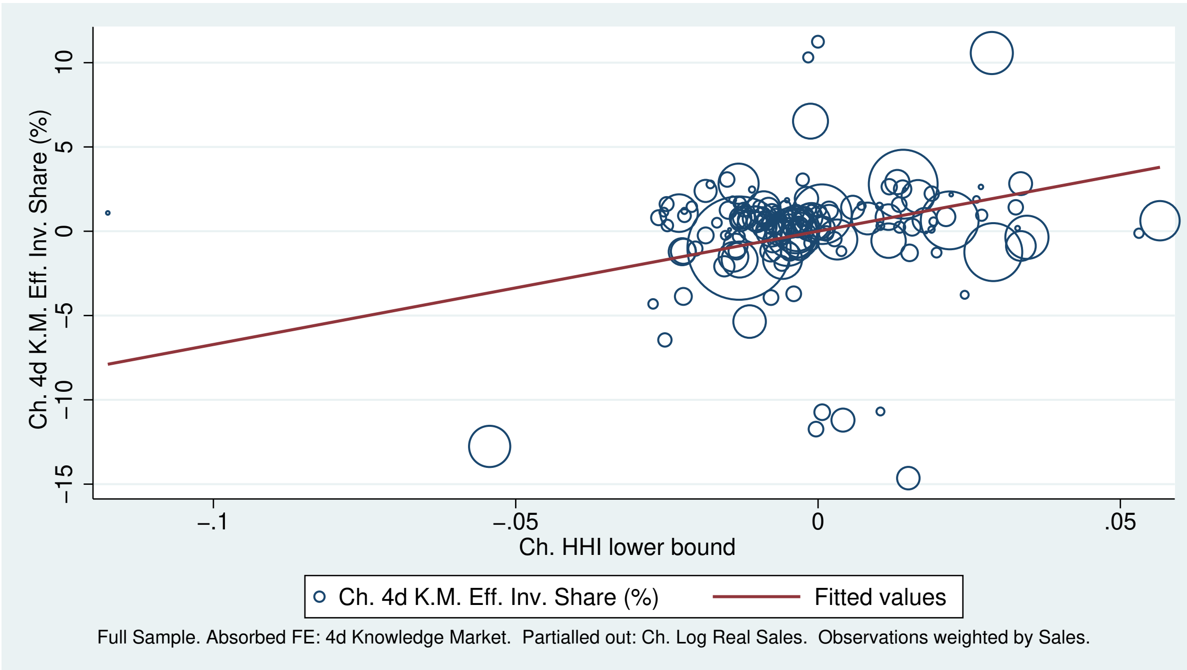
(b) Controlling for Change in Log Real Sales per Company

	Ch. 4d K.M. Eff. Inv. Share (%)					
	(1)	(2)	(3)	(4)	(5)	(6)
Ch. HHI lower bound	35.230**	20.783+	35.230**	20.783+	35.154**	22.854*
	(12.759)	(10.615)	(12.759)	(10.615)	(12.647)	(11.197)
Ch. Log Real Sales per company	0.175	-0.040	0.175	-0.040	0.300	-0.055
	(0.382)	(0.253)	(0.382)	(0.253)	(0.460)	(0.346)
4D Knowledge Market FE		✓		✓		✓
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalanobis 5%
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	81	79	81	79	75	67

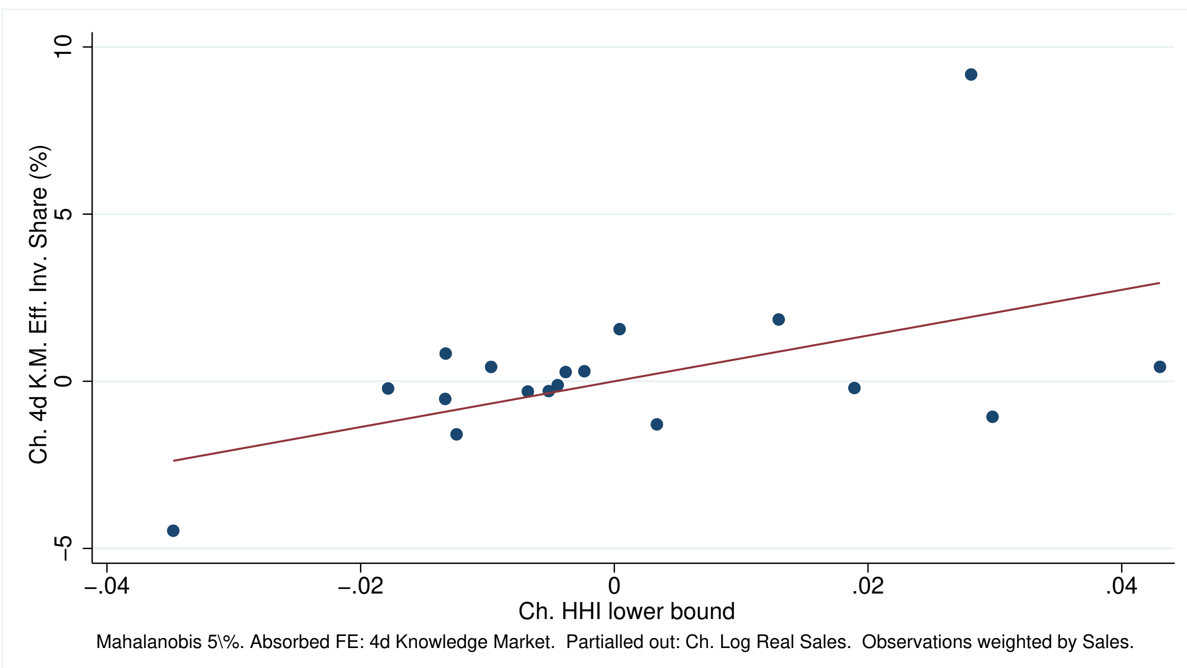
Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (**??**) and (**??**), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. “Full Sample”, “Trim Outliers” and “Mahalanobis 5%” refer to the samples described in the main text.

Figure 1: Residualized Scatter Plots Corresponding to Selected Columns in Table 2, Panel (a)

(a) Raw Scatter Plot, Specification in Column (2)



(b) Binned Scatter Plot, Specification in Column (6)



Note: This figure presents residualized scatter plots of the change in the share of effective inventors of sector p over total inventors in knowledge market k, over the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. The upper panel reports the data corresponding to the full sample, where both variables have been residualized by change in log real sales and knowledge market fixed effects. The size of the markers is proportional to the weight of each observation in the regression, corresponding to total sector sales in 2012. The regression line corresponds to the coefficient on the change in HHI lower bound reported in Column (2) of Table 2. The lower panel presents a binned scatter plot on the sample where the observations with the highest 5% Mahalanobis distance from sample centroid have been removed. Observations are aggregated using sales weights and the regression line results from the specification in Column (6) of Table 2.

C.2 Using the Lerner Index instead of the HHI

Following Grullon et al. (2019), I build the Lerner Index from NBER-CES data for the period 1997-2012 as the ratio:

$$\text{Lerner}_{jt} = \frac{\text{vship}_{jt} - \text{pay}_{jt} - \text{matcost}_{jt} - \text{energy}_{jt}}{\text{vship}_{jt}}, \tag{10}$$

where "vship" is the total value of shipments, "pay" denotes total payrolls, "matcost" and "energy" material and energy costs, respectively, and j denotes a 6- or 4-digit NAICS sector. I build two alternative measures, one using 6-digit NAICS sectors, the original identifier in NBER-CES, and then averaging by sales at the level of 4-digit NAICS, or first aggregating the revenue and cost statistics at the level of 4-digit NAICS. Table 3, shows that the Lerner Index thus constructed is strongly correlated with the HHI measure used in the main analysis. However, the correlation is far from perfect, as suggested by the R^2, suggesting that this estimate of the Lerner Index might be excessively imprecise. Indeed, Table 4 shows that, when using this measure instead of the HHI in the main analysis, the coefficients for the regression of inventors' shares on changes in concentration stay positive, but become smaller and noisier. This suggests the potential presence of attenuation bias, a valid concern due to the fact that the above measure, not based on any structural estimation, can only imperfectly capture markups. Note that this is also due to the fact that the Lerner Index is available only for the manufacturing sectors, which make up about 60% of the sample, so its use lead to dropping a substantial amount of observations. When using fitted values from the regression in Table 3 to extend the measure to more sectors, as well as reducing the volatility of the series for available sectors, the coefficients recover magnitudes and significance close to the baseline presented in **??**.

Table 3: Regressions of Changes in the Lerner Index over Changes in the HHI Lower Bound, Long-Difference, 1997-2012

	Markup Change 1997-2012, 6d Lerner Index	Markup Change 1997-2012, 4d Lerner
	(1)	(2)
HHI Change 1997-2012	1.490***	1.652***
	(0.229)	(0.257)
Observations	258	258
R-squared	.1424476	.139197

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$. "6d Lerner Index" refers to the Lerner Index constructed as in (10)on NAICS 6-digits averaged at the 4-digit NAICS level weighting by the value of shipments; "4d Lerner Index" is computed using 4-digit aggregates for the value of shipments, payroll and costs, summing over the NAICS 6-digit composing each sector.

Table 4: Regressions of Changes in Inventors' Share over Changes in Actual and Fitted Lerner Index, Long-Difference, 1997-2012

	Ch. 4d K.M. Eff. Inv. Share (%)	
	(1)	(2)
Markup Change 1997-2012, 4d Lerner Index	0.556	
	(5.465)	
Fitted Lerner Change		26.736*
		(13.363)
4D Knowledge Market		
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	81	157

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Observations weighted by sales. The markup change 1997-2012 is the long-difference of the Lerner Index described above. "Fitted Lerner change" is the fitted value for the Lerner index based on the estimates in 3, and extended to all available sectors in the main sample.