ECONOMETRICA

VOLUME 53 MAY, 1985 NUMBER 3

A PROFITABLE APPROACH TO LABOR SUPPLY AND COMMODITY DEMANDS OVER THE LIFE-CYCLE

BY MARTIN BROWNING, ANGUS DEATON, AND MARGARET IRISH[1]

The paper presents a general theoretical framework for the analysis of integrated life-cycle models of consumption and family labor supply under uncertainty. Profit functions are used to represent intertemporally additive preferences and to yield convenient characterizations of "constant marginal utility of wealth" or "Frisch" demand functions. Conditions on preferences are derived that allow additive fixed-effect specifications for the Frisch demands. Data from the British Family Expenditure Surveys from 1970–77 are used to derive panel-like information on male labor supply and consumption for several age cohorts over time. These data reproduce standard life-cycle patterns of hours and wages, but more detailed analysis shows that the theory is incapable of offering a satisfactory common explanation of the behavior of hours and wages over both the business cycle and the life cycle. Similarly, although the theory can explain the life-cycle behavior of hours and consumption separately, the same model cannot explain both, essentially because of a failure in symmetry.

INTRODUCTION

OUR OBJECT IN THIS PAPER is to provide a general theoretical framework for the empirical analysis of integrated life-cycle models of consumption and family labor supply. We also use data from the 1970 to 1977 Family Expenditure Surveys of the United Kingdom to estimate life-cycle models of male hours and household consumption. The way in which we combine time-series and cross-section data allows a simultaneous analysis of behavior over both business and life cycles. We find that, although much of our evidence is broadly interpretable in terms of life-cycle theory, the theory is *not* capable of offering a *common* explanation of the business cycle and the life cycle, nor of consumption and hours, even though each can be explained in isolation.

Previous British studies of labor supply have not taken a life-cycle view. The studies referenced in the survey by Greenhalgh and Mayhew [**24**] use either aggregate data from time series and industrial cross-sections or micro data from household surveys and are based on the standard static model of labor supply. Typically, such studies find backward sloping supply curves for prime-age males together with small negative effects on labor supply of assets or unearned income or proxies for them; it is not always clear that the implied substitution elasticity is positive as required by theory. Indeed, in studies using the Family Expenditure Survey, unconstrained regressions tend to produce backward sloping supply

[1] We should like to thank the SSRC for financial support under Grant No. HR7637 "The Economics and Econometrics of Consumer Behavior." We have received helpful comments from Orley Ashenfelter, Tony Atkinson, Cliff Attfield, Gary Becker, David Card, Larry Epstein, Terence Gorman, Jim Heckman, Constantino Lluch, Bob Lucas, John Muellbauer, John Pencavel, Bob Pollak, Nick Stern, and two unusually helpful referees. We also wish to thank the Atkinson/King/Stern project staff for generously allowing us access to their version of the FES tapes.

curves together with zero or positive income effects, clearly contradicting theoretical presupposition; see Atkinson and Stern [**6**] and Deaton [**12**]. Of course, there are severe difficulties in obtaining good data on unearned income in any household survey and there are serious conceptual problems in applying the static model to such data as exist. In a life-cycle context, assets and asset income are not exogenous vaiables but evolve systematically with the labor supply and spending decisions of the household. It is easy to imagine the same household at different points in the life cycle first working long hours with low assets, then working long hours with high assets and yet later working short or no hours with low assets. The only truly exogenous asset variables are inherited assets on the one hand, and asset "surprises" on the other; these are not recorded in standard household surveys. There are similar problems interpreting wage responses in the standard static model. It is reasonable to suppose that a fully anticipated wage increase will have different effects from an unanticipated change since the latter changes the individual's perception of life-time resources while the former does not. Similarly, most households are not surprised when their children attain an age at which their financial demands on their parents becomes very large, so that such events must begin to influence labor supply and savings plans long before they occur. All these phenomena require an explicitly life-cycle perspective as well as a proper integration of labor supply and consumption behavior.

These issues have been recognized in the American literature for some time, although the static model is still the dominant framework of analysis. Mincer's [**40**] model of female participation is explicitly set in a life-cycle background, but modern developments in life-cycle labor supply begin with Heckman's [**28**] Princeton doctoral thesis; see Heckman [**29** and **30**], and with Ghez and Becker [**18**]. Ghez and Becker make much of the important distinction between anticipated wage changes *along* the life-cycle wage profile and unanticipated changes which shift the profile itself. Smith's [**45**] paper is in this tradition but, like Ghez and Becker's work, the analysis is hampered by lack of panel data, so that averages of workers at specified ages (synthetic cohorts) are assumed to represent behavior along a single profile for all workers. Heckman's analysis provides the basis for an appropriate theoretical treatment by showing that the supply functions required for the analysis are neither those holding wealth constant nor those holding utility constant, but those that hold *marginal* utility constant. This is the starting point for much of the analysis in this paper. Marginal utility constant demand functions also turn out to be central in the analysis of intertemporal choice under uncertainty and provide a bridge between the labor supply literature and the "rational expectations" consumption function models of Bewley [**9**] and of Hall [**25**] which trace back ultimately to the intertemporal arbitrage conditions of the finance literature. The final important development is due to MaCurdy (see Heckman [**31**], Heckman and MaCurdy [**32**], and MaCurdy [**37**]). This is the realization that, at least in certain specifications, the essentially unobservable marginal utility is constant over the lifetime of the consumer and so, given panel data, can be treated as a fixed effect in the econometric analysis.

In this paper, our first aim is to develop the full theoretical basis for marginal utility constant demand functions and to relate them to standard concepts in the theory of consumer behavior. In particular, we discuss the relevant duality theory, an appreciation of which gives great advantages, not only in understanding, but also in ease of selecting functional forms and of relating empirical observations to the forms of preferences thereby implied. The key to the analysis turns out to be the life-cycle *profit function*, first discussed (in the context of the Rotterdam model) by Gorman [**22**]. The concept is identical to that used in production theory (see, e.g., McFadden [**38**]), and in the same way it has demand functions as partial derivatives, in this case the marginal utility constant demands, or as we call them here, the *Frisch* demands. In the life-cycle context with uncertainty, the Frisch demands neatly separate anticipated from unanticipated effects, not only of wages on labor supply and participation, but also of commodity prices and demographic structure on both labor supplies and commodity demands. The close link between demands on the one hand and profit and cost functions on the other allows us to use standard techniques from demand analysis to incorporate in a systematic way the presence of children and to predict the effects of the demographic life-cycle on male and female labor supply and household commodity demands.

The first part of the paper takes up these issues in turn. Section 1 is a general discussion of additive preferences and of the characterization of consumer preferences by profit functions; only a summary is given and a fuller analysis can be found in Browning [**10**]. Section 2 applies the results of Section 1 to the life-cycle, derives life-cycle and age-specific profit functions and gives the general results governing labor supplies, participation, and commodity demands in terms of the Frisch demands. Section 3 introduces uncertainty into the intertemporal choice problem and links the Frisch demands to the literature on the consumption function, particularly to Bewley [**9**] and Hall [**25**]. Section 4 contains some simple exercises in comparative statics and dynamics designed to illustrate the power of the model to generate testable hypotheses and to tell "stories" about home economics. For example, under plausible assumptions, anticipated increases in men's wages cause their wives to work *more*. Similarly, the birth of an additional child may cause the husbands of nonparticipating wives to work longer hours but have no effect on the hours of those men whose wives continue to participate in the labor market after the increase in family size.

The second part of the paper is concerned with the selection of appropriate functional forms and with empirical implementation on Family Expenditure Survey data. Section 5 takes up MaCurdy's suggestion of treating the marginal utility as an unobservable fixed effect. We derive the general restrictions on preferences that allow such a formulation and propose from within the class a set of flexible functional forms that permit the testing of a number of important restrictions on behavior. These include separability of husband's and/or wife's leisure from each other and from goods, in addition to the usual symmetry restrictions of demand theory. Section 6 discusses an important device that allows us to use the Family Expenditure Survey to generate what are effectively panel data. In the United Kingdom, we do not have genuine panel data on incomes, hours, and commodity demands. However, we are unusually fortunate in having a *continuous* household survey, the FES, that generates random samples of the population in every year. With complete enumeration, i.e., with census data, we could follow cohorts through time. With a continuous random sample, we can follow cohort *means* through time to the extent that sample means are good estimates of population means. Although we cannot follow individual households through time, we can look at the average behavior of 25 year olds one year, of 26 year olds the next, and so on, thus following actual, not synthetic cohorts. Such data even have certain advantages over panel data, notably the preservation of randomness through the absence of attrition. Section 6 explains exactly how the data were extracted from the FES; the means used in the study although based on nearly 50,000 original observations, make up a relatively small data set which is listed in the Appendix. Section 7 contains the results of an application of our model to both male labor supply and to aggregate household expenditures. The hours data replicate the stylized facts found by others, that for manual and nonmanual workers there is a marked synchronization over the life-cycle between hours worked and discounted wage rates with workers working longest hours when it is most profitable to do so. On such evidence alone, the elasticity of weekly hours to anticipated wage changes is around 0.15, a figure in accord with MaCurdy's [**37**] estimate for the United States. However, more detailed analysis casts considerable doubt on the simple life-cycle explanation. In particular, sensible positive responses between hours and wages are only consistently obtained when year to year changes are separately allowed for by dummy variables. Secondly, the characteristic hump-shaped patterns of both hours and real consumption, though explicable in terms of life-cycle wage variation, can be explained as well as or better by other factors, particularly by the demographic composition of the household. Hence, life-cycle patterns could be interpreted as the response of credit-constrained consumers to the variation in needs accompanying the birth, growth, and departure of children. Finally, a life-cycle interpretation of consumption expenditures requires that consumption and leisure be substitutes, while our estimates of male labor supply imply almost as strongly that leisure and consumption be complements. Overall then, we find a considerable amount of evidence that is contrary to the simple life cycle story.

PART ONE: THEORY

1. ADDITIVE PREFERENCES AND PROFIT FUNCTIONS

Consider first a quite general model of consumer choice with additive preferences. We write this

$$\max_{q_i} \sum_{i=1}^{n} \nu_i(q_i) \quad \text{subject to } p \cdot q = x \tag{1.1}$$

where q_i is the quantity purchased of each of n goods, p_i is the corresponding price, and x is the predetermined expenditure total. If we assume convexity of preferences, all but one of the subutilities must be concave (see Yaari [**46**]); we assume further that *all* the subutility functions ν_i are strictly concave and twice differentiable. We *temporarily* make the assumption of internal solutions; these are characterized by first-order conditions

$$\nu_i'(q_i) = \lambda p_i = p_i/r \tag{1.2}$$

where λ is a Lagrangian multiplier representing the marginal utility of x, given that utility is normalized by taking the explicitly additive form. The quantity r, which plays a key role in what follows, is defined as the reciprocal of λ, i.e., as the marginal cost of utility or better, as the *price of utility*. Since each $\nu_i(\ )$ is strictly concave, $\nu_i'(\ )$ is monotone decreasing so that (1.2) can be inverted to give

$$q_i = f_i(p_i/r) \tag{1.3}$$

for monotone decreasing functions $f_i(\ )$.

Ragnar Frisch [**16**] was one of the first writers to systematically use additive preferences to measure the marginal utility of money, and following Browning [**10**], we refer to the demand functions (1.3) as *Frisch demands*. Under the additivity assumption, Frisch demands characterize quantities purchased in terms of a single quantity, the ratio of the commodity price to the price of utility. Such demands should be distinguished from the usual uncompensated or Marshallian demands that relate quantities to prices and total outlay, as well as from the compensated or Hicksian demands that relate quantities to prices and utility. The Frisch demands can be transformed into Marshallian demands by solving for r in terms of p and x by applying the budget constraint to (1.3), i.e., from

$$\sum p_i f_i(p_i/r) = x, \tag{1.4}$$

or into Hicksian demands by expressing r in terms of u and p through

$$\sum p_i f_i(p_i/r) = c(u, p) \tag{1.5}$$

where $c(u, p)$ is the cost or expenditure function corresponding to the original preferences. The conceptual experiment corresponding to a Frisch demand is one in which consumers are money compensated for a price change until their price of utility returns to its original value. But a more useful and natural interpretation will appear in the life-cycle context.

The analysis clearly extends to "block" additivity or strong separability where the subutility functions are defined over groups of goods rather than single goods. Problem (1.1) becomes

$$\max u = \sum_G \nu_G(q_G) \quad \text{subject to} \quad \sum p_G \cdot q_G = x \tag{1.6}$$

where q_G and p_G represent price and quantity vectors for group G. The solution follows the same lines and the Frisch demands are, for good i in group G,

$$q_{Gi} = f_{Gi}(p_G/r) \tag{1.7}$$

so that demands in the group depend only on prices in the group relative to the price of utility.

Utility is an "output" for the consumer so that Frisch demands can be thought of as relating optimal inputs to the prices of output and the inputs. In production theory such demands arise in the analysis of a profit maximizing firm and this familiar apparatus turns out to be conveniently adaptable to the consumer context. Hence, define the consumer's profit function as the maximum profit attainable from selling utility (to him or herself) at a price r, subject to the technology of utility production, i.e., the utility function, and the prices of the inputs. For a general utility function $\nu(q)$, we write this

$$\pi(r, p)=\max_{u,q}\{ru-p\cdot q;\, u=\nu(q)\}. \tag{1.8}$$

The existence of $\pi(r, p)$ for all $p\geqslant 0$ requires that $\nu(q)$ be strictly concave and co-finite; see Rockafellar [**44**] and Lau [**36**]. Note that, by its definition, $\pi(r, p)$ is convex and linear homogenous in (r, p), increasing in r and decreasing in p (see McFadden [**38**]). An alternative derivation of the profit function that is frequently useful is

$$\pi(r, p)=\max_{u}\{ru-c(u, p)\}. \tag{1.9}$$

In many applications it is easier and more convenient to place structure on the cost function so that (1.9) is useful in deriving the corresponding structure for the profit function.

At a formal level, $\pi(r, p)$ is (minus) the concave conjugate of $r\nu(q)$ with respect to q and the convex conjugate of $c(u, p)$ with respect to u; see Rockafellar [**44**] for a discussion of conjugacy. Since the original functions are the conjugates of their own conjugates, utility and cost functions can be retrieved from the profit function using the two identities

$$c(u, p)=\max_{r}\{ru-\pi(p, r)\}, \tag{1.10}$$

$$r\nu(q)=\min_{p}\{p\cdot q+\pi(p, r)\}. \tag{1.11}$$

At an economic level, the profit function represents consumer preferences as a function of the price of utility and the prices of goods just as, for example, the cost function represents preferences in terms of utility and goods' prices. For the latter, demands can be obtained by differentiation, and the same is true for the profit function. From (1.9), we have immediately

$$-\frac{\partial\pi}{\partial p_i}=\frac{\partial c}{\partial p_i}=q_i=f_i(r, p), \tag{1.12}$$

$$\frac{\partial\pi}{\partial r}=u=f_0(r, p). \tag{1.13}$$

Hence, just as the cost function is the potential function for the Hicksian demands, the profit function is the potential function for the Frisch demands. These relationships allow us to derive the general properties of Frisch demands as well as providing the link between preferences on the one hand and the empirical analysis on the other.

We conclude this section by noting the properties of Frisch demands that we shall require in the subsequent analysis.

(A) Frisch demands are zero degree homogenous in r and p. This follows immediately from the linear homogeneity of $\pi(r, p)$ and the derivative property (1.12). See also (1.3) and (1.7).

(B) Frisch demands have symmetric derivatives, i.e.,

$$\partial f_i/\partial p_j = \partial f_j/\partial p_i. \tag{1.14}$$

The derivative matrix is simply minus the Hessian of the profit function so that symmetry follows by Young's theorem. This symmetry is similar to but not the same as Slutsky symmetry. The relationship between Frisch and Slutsky responses is derived by writing

$$q_i = c_i(\pi_0(r, p), p) \tag{1.15}$$

where c_i is the derivative of $c(u, p)$ with respect to p_i and π_0 that of $\pi(r, p)$ with respect to r. Differentiation with respect to p_j and rearrangement gives the relationship between utility compensated and utility-price compensated derivatives as

$$s_{ij} = f_{ij} + \frac{\partial q_i}{\partial x} \cdot \frac{\partial q_j}{\partial x} \cdot x\breve{\omega}^{-1} \tag{1.16}$$

where s_{ij} is Slutsky substitution, f_{ij} is Frisch substitution, and

$$\breve{\omega} = \partial \ln r/\partial \ln x \tag{1.17}$$

is Frisch's [**17**] income flexibility of the marginal utility of money. Equation (1.16) decomposes substitution effects into "specific" substitution effects (the f_{ij}) and "general" substitution effects (terms coined by Houthakker [**33**]); see also MaCurdy [**37**]. Note that conjugate functions have Hessians which are mutual inverses so that the Frisch substitution matrix is proportional to the inverse of the Hessian of the utility function; see the expressions for f_{ij} in Barten's [**7**] "fundamental matrix equation of demand theory."

(C) Frisch demands slope downwards. By its defintion $\pi(r, p)$ is convex so that its Hessian, where it exists, is positive semi-definite. Since the (f_{ij}) matrix is the negative of this Hessian, it is negative semi-definite. Hence

$$\partial f_i/\partial p_i \leq 0 \tag{1.18}$$

or more generally, for price vectors p^1 and p^0,

$$\{f(r, p^1) - f(r, p^0)\} \cdot \{p^1 - p^0\} \leq 0. \tag{1.19}$$

(D) Additive (or block additive) utility is equivalent to additive (or block additive) profits. It is intuitively clear that additive utility functions allow decentralization of utility production provided each production unit produces output (utility) at the same price, in this case r. Formally, take the strongly separable case (1.6). Then

$$(1.20)\qquad \pi(r,p)=\max_{u,q}\{ru-\sum p_G\cdot q_G;\ u=\sum \nu_G(q_G)\}$$
$$=\max_{u_G,q_G}\{r\sum u_G-\sum p_G\cdot q_G;\ u_G=\nu_G(q_G)\}$$
$$=\sum_G \max_{u_G,q_G}\{ru_G-p_G\cdot q_G;\ u_G=\nu_G(q_G)\}$$
$$=\sum_G \pi_G(r,p_G).$$

Hence, the overall profit function is the sum of the individual profit functions corresponding to each subutility function. Overall profits are the sum of branch profits; input prices are branch specific but the output price is the same for all branches and provides the (only) link between them.

Differentiation of (1.20) yields branch Frisch demands which are a function of group prices and the price of utility alone, i.e., of the form (1.7). The Hessian of (1.20) is block diagonal as is the Hessian of a strongly separable utility function. The effect of additivity is thus to set to zero all cross-branch specific substitution effects. In the next section we shall see how useful this is in the context of intertemporal choice.

2. THE LIFE CYCLE, PROFITS, AND DEMANDS

We begin with the case of perfect certainty and assume that family life-cycle preferences can be represented by the utility function

$$(2.1)\qquad u=\sum_0^L \nu_t(l_{1t}, l_{2t}, q_t)$$

where $t=0,\ldots,L$ indexes age, l_{1t} is leisure of the type 1 worker (husband), l_{2t} is leisure of the type 2 worker (wife), and q_t is a vector of household consumption levels. The period subutility functions $\nu_t(\quad)$ are indexed on age t; this could reflect intertemporal discounting of utility (if such a phenomenon is thought to be sensible), but more importantly the variation with t recognizes the modifying role played by the presence of children and their changing demands over the family life cycle. The intertemporal strong separability that is assumed by (2.1) is a crucial element in all of our analysis and only a limited number of our results hold without it. The fact that additivity is an almost universal assumption in work on intertemporal choice does not suggest that it is innocuous.

For the moment, assume that utility is maximized under perfect certainty in which case the life-cycle budget constraint, discounted back to age 0, can be written

$$(2.2)\qquad \sum_0^L \hat{p}_t\cdot q_t+\sum_0^L \hat{w}_{1t}l_{1t}+\sum_0^L \hat{w}_{2t}l_{2t}=A_0+\sum_0^L(\hat{w}_{1t}T_{1t}+\hat{w}_{2t}T_{2T}).$$

In this expression w_{1t} and w_{2t} are the wages of husband and wife at age t, A_0 is the present discounted value at 0 of nonhuman assets, and T_{1t} and T_{2t} are the age t time endowments for the husband and wife respectively. A caret over a price or wage indicates that the variable is discounted to its present value; for age t, the discount factor is the product of all single period discount factors from 0 to t. Explicit formulae are considered later.

The problem we are concerned with is the maximization of (2.1) subject to (2.2) with l_{1t}, l_{2t} and q_t as instruments. We implicitly assume that the household has already made its fertility and human capital plans; however this has been done, the optimization problem is still correct although the wage rates and the numbers and ages of children cannot be taken as parametric. Allowance for these effects must therefore be made as necessary in the econometric work.

The intertemporal additivity assumption allows decentralization over time (age). Each period of life is regarded as the site for an independent utility factory and lifetime utility is the sum of all the individual plant outputs. The link between periods is the discounted price of lifetime utility, i.e., the reciprocal of the marginal utility of lifetime wealth (or full income). Define then the age t profit function by

$$\text{(2.3)} \qquad \pi_t(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) = \max_{u, l_{1t}, l_{2t}, q_t} \{ru + \hat{w}_{1t}(T_{1t} - l_{1t}) + \hat{w}_{2t}(T_{2t} - l_{2t}) - \hat{p}_t \cdot q_t;\ \nu_t(l_{1t}, l_{2t}, q_t) = u\}.$$

Profits accrue from sales of utility and sales of the two kinds of market labor offset by the costs of inputs. By the arguments of Section 1, we then have

$$\text{(2.4)} \qquad \frac{\partial \pi_t}{\partial \hat{w}_{1t}} = h_{1t} = f_{1t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t),$$

$$\text{(2.5)} \qquad \frac{\partial \pi_t}{\partial \hat{w}_{2t}} = h_{2t} = f_{2t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t),$$

$$\text{(2.6)} \qquad -\frac{\partial \pi_t}{\partial \hat{p}_{ti}} = q_{it} = \phi_{it}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t),$$

where h_{1t} and h_{2t} are the hours of market work supplied by each type of worker, f_{1t} and f_{2t} are the Frisch supply functions for labor, and ϕ_{it}, $i = 1, \ldots, n$ are the Frisch demand functions for commodities.

These equations show immediately the benefit of working with Frisch demands. As emphasized in the work of Heckman and of MaCurdy, labor supplies and commodity demands are a function of immediately observable within period variables such as prices and wages while all the variables from outside the period, many of which are unobservable (future prices, wage rates, and so on), are represented by a single "sufficient statistic" r which, at least under perfect certainty, does not vary from period to period. Such equations are the perfect answer to the difficulties facing the econometrician who attempts to estimate life-cycle models. However, there is still a number of difficulties to be faced both in this section and the next.

Note first that the formulae (2.4) to (2.6) assume internal solutions for the labor supplies and commodity demands. The constraints $q_{ti} > 0$ and $T_{it} > h_{it} > 0$

have been ignored although in practice some are likely to be binding. We illustrate for the case of nonparticipation by the wife, i.e., for $h_{2t}=0$. The same principles apply to the other inequality constraints (although large numbers of regimes are hard to handle). Note first that if h_{1t}, h_{2t} and q_{ti} as given by (2.4) to (2.6) are all nonnegative, then these values are optimal and there is nothing more to be said. Consider then the case where (2.5) yields a value of $f_{2t}<0$. Since the Frisch demands are the inverses of the first order conditions, $f_{2t}<0$ implies that at zero hours, a decrease in home time is worth more than the wage earned, so that $h_{2t}=0$ is optimal. Hence, as might be expected, positive Frisch labor supply functions correspond to positive hours worked while negative Frisch supplies indicate nonparticipation. This is the familiar Tobit specification for censored distributions. Even so, when the wife does not participate, the husband's labor supply and household commodity demands will generally be different since the optimization must allow for the effective ration at $h_{2t}=0$. The analysis of this situation requires a *restricted* profit function for age t in which h_{2t} is set to zero. The details are essentially identical to those given for restricted cost functions in Neary and Roberts [**41**] and Deaton [**11**] and the solution is characterized by the following equations:

$$h_{1t}=f_{1t}(r, \hat{w}_{1t}, \hat{w}_t^*, \hat{p}_t), \tag{2.7}$$

$$0=f_{2t}(r, \hat{w}_{1t}, \hat{w}_t^*, \hat{p}_t), \tag{2.8}$$

$$q_{it}=\phi_{it}(r, \hat{w}_{1t}, \hat{w}_t^*, \hat{p}_t). \tag{2.9}$$

In equation (2.8) $\hat{w}_t^*$ is defined as the wage rate that causes the wife to wish to work zero hours at age t; it is her reservation wage (sometimes shadow or virtual wage). Note that (2.8) yields a unique solution for $\hat{w}_t^*$ since $f_{2t}(\ \ )$ is monotone increasing in $\hat{w}_{2t}$. Husband's labor supply and household consumption demands have the same functional form as before with the wife's reservation wage replacing the actual wage. This has important implications. Consider, for example, a variable that in (2.4)–(2.6) affects only wife's labor supply, for example the presence or absence of an infant. Once the wife ceases to participate, (2.8) becomes relevant and changes in the variable will alter $\hat{w}_t^*$ and hence the husband's labor supply as well as the commodity demands.

From an econometric point of view (2.4)–(2.6) and (2.7)–(2.9) should be regarded as two systems of equations with endogenous switching determined by whether or not f_{2t} is negative. Note that the presence of nonparticipation at some point in the life cycle does not affect the constancy of r; the price of utility is a datum for the life of the family and it is not altered as behavior switches from one set of equations to the other.

3. THE TREATMENT OF UNCERTAINTY

We begin by rewriting the life-cycle Frisch demands under certainty. We take the male labor supply equation (2.4) as representative:

$$h_{1t}=f_{1t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t); \tag{3.1}$$

omitting the others merely saves space. The wage rates and prices in this equation are discounted back to age 0; hence, taking p_{it} to illustrate:

$$(3.2)\qquad \hat{p}_{it} = p_{it}\delta(t,0) = p_{it}\prod_{0}^{t-1}(1+i_\tau)^{-1},$$

where $\delta(t,0)$ is the discount factor to be applied to period t in period 0 and i_τ is the *nominal* rate of interest linking period τ with period $\tau+1$. Since (3.1) is zero degree homogeneous, we can divide through by $\delta(t,0)$ to give

$$(3.3)\qquad h_{1t} = f_{1t}(r_t, w_{1t}, w_{2t}, p_t)$$

where $r_t = r/\delta(t,0)$ is the price of utility in period t, or, more precisely, the price of lifetime utility in terms of period t's money. Because of the discounting, r_t, unlike r, is not constant with age but, by definition, evolves according to

$$(3.4)\qquad r_{t+1} = r_t(1+i_t).$$

This equation guarantees that $r_t\delta(t,0)$, the discounted price of lifetime utility, is the same at all ages (periods). Equations (3.3) and (3.4) taken together are precisely equivalent to the original (3.1) and provide a convenient characterization using only currently dated magnitudes.

Choice under uncertainty is here characterized by expected utility maximization with continuous replanning. Labor supplies and commodity demands at t are thus chosen to maximize

$$(3.5)\qquad \nu_t(l_{1t}, l_{2t}, q_t) + E_t\sum_{t+1}^{L}\nu_k(l_{1k}, l_{2k}, q_k)$$

where $E_t(\cdot)$ is the expectations operator conditional on information available at time t. By taking this form, we assume that the same explicitly additive form of utility can be used to characterize both intertemporal separability and the additivity over states that is implied by the conditional preference axiom of choice under uncertainty. There is no automatic guarantee that this should be so; nevertheless we believe that the characterization embodies the most reasonable interpretation of intertemporal additivity under uncertainty. To summarize the argument, start from the sure-thing principle *without* intertemporal separability, so that preferences can be represented by an additive function $\sum \pi_s G(q_s^1, q_s^2, \ldots, q_s^L)$, with π_s the probability of state s, and q_s^t (temporarily) representing the vector of consumptions and leisures in period t and state s. The crucial question is exactly what is meant by intertemporal additivity in this context. A *minimum* requirement is that, under certainty, utility be additive, or equivalently, that within any given state, utility be additive. In terms of basic preference orderings, we require that for *given s*, the conditional ordering over any pair $\{q_s^t, q_s^{t'}\}$ must be independent of τ for $\tau \neq t$, t'. This implies that utility can be represented by the form $\sum \pi_s F\{\sum \nu_t(q_s^t)\}$. One could reasonably stop at this point, but we feel it is more appropriate to require that preferences be *simultaneously* additive over periods and states. Without restriction on $F\{\ \}$, conditional orderings over, for example, $\{q_s^t, q_{s'}^t\}$ are not independent of q_s^τ and $q_{s'}^\tau$ for *all* $\tau \neq t$. In consequence, my preferences over picnic/sun versus movie/rain tomorrow are not independent of all my future consumption levels in future states that encompass sun or rain tomorrow. We prefer not to characterize such preferences as being intertemporally additive and instead to require simultaneous additivity, so that the conditional ordering of $\{q_s^t, q_{s'}^{t'}\}$ be independent of q_σ^τ for all $\tau \neq t$, t' and $\sigma \neq s$, s'. By the application of Gorman's [**20**] overlapping separability theorem, such simultaneity requires that F be at most an affine transformation and that preferences can be represented by the doubly additive form, as in (3.5). Some of the further complications introduced by recognizing the sequential resolution of uncertainty over time are discussed by Gorman [**23**].

The maximization of (3.5) subject to the life-cycle budget constraint requires the usual recursive substitution from L back to t that is characteristic of stochastic dynamic optimization; for a good exposition see, for example, Epstein [**15**]. Note first that the life-cycle utility function (3.5) still has a two period intertemporally additive structure between "now," period t, and the "expected future," from $t+1$ to L. Consequently, all the previous apparatus of Frisch demands goes through, i.e., we can write exactly as before

$$(3.6) \qquad h_{1t} = f_{1t}(r_t, w_{1t}, w_{2t}, p_t)$$

for the period t male labor supply, conditional on r_t, the price of expected lifetime utility *as perceived at t*. The only difference between certainty and uncertainty is the process controlling the evolution of r_t, and it is this that is derived from the dynamic optimization. To simplify notation, define period τ "full" expenditure as

$$(3.7) \qquad x_\tau = w_{1\tau} l_{2\tau} + w_{2\tau} l_{2\tau} + p_\tau \cdot q_\tau,$$

let $\psi_\tau(x_\tau, w_{1\tau}, w_{2\tau}, p_\tau)$ be the period τ indirect subutility function, $\omega_\tau = w_{1\tau} T_{1\tau} + w_{2\tau} T_{2\tau}$ period τ's endowment, and A_τ assets at the beginning of τ. The evolution of assets is given by

$$(3.8) \qquad A_{\tau+1} = (A_\tau + \omega_\tau - x_\tau)(1 + i_\tau).$$

Let $\psi_\tau^*(A_\tau)$ be the sum of current and expected future utilities as perceived at age τ given assets A_τ inherited from age $\tau - 1$. At the end of life at age L we have

$$(3.9) \qquad \psi_L^*(A_L) = \psi_L(A_L + \omega_L, w_{1L}, w_{2L}, p_L),$$

while for any other $\tau < L$, optimization over period τ full expenditure gives

$$(3.10) \qquad \psi_\tau^*(A_\tau) = \max_x \{\psi_\tau(x, w_{1\tau}, w_{2\tau}, p_\tau) + E_\tau[\psi_{\tau+1}^*((A_\tau + \omega_\tau - x)(1 + i_\tau))]\}.$$

In particular, (3.10) holds for the present, $\tau = t$, so that the first-order condition is

$$(3.11) \qquad \partial\psi_t/\partial x = E_t\{(1 + i_t)\, \partial\psi_{t+1}^*/\partial x\}.$$

But $\partial\psi_t/\partial x$ is simply the marginal (lifetime) utility of period t's money, or the reciprocal of the undiscounted price of utility. Hence, (3.11) becomes

$$E_t\{(1+i_t)r_t/r_{t+1}\}=1. \tag{3.12}$$

This equation is the counterpart of (3.4) in the certainty case; under certainty, (3.12) holds without the expectation. Thus, provided we work with Frisch demand functions, the incorporation of uncertainty is straightforward. The Frisch demands, in undiscounted form, are unchanged, but the price of utility follows a stochastic rather than a deterministic process. Equation (3.11) or (3.12) is the standard stochastic Euler equation of intertemporal equilibrium, familiar from the theory of stock-market prices or from optimal accumulation under uncertainty. It is also the basis for Bewley's [**9**] and Hall's [**25**] versions of the permanent income model.

Recent work by Hansen and Singleton [**27**] has provided an econometric procedure for direct estimation of the Euler equation together with the other first-order conditions (i.e., the Frisch demands). The procedure has also been used by Mankiw, Rotemberg, and Summers [**39**] to study aggregate consumption and labor supply in a paper with similar aims to the current one. Our own approach is to write (3.12) as

$$(1+i_t)/r_{t+1}=1/r_t+\varepsilon_{t+1}, \quad E_t(\varepsilon_{t+1})=0, \tag{3.13}$$

and then to take logarithms and approximate to give

$$\ln r_{t+1}\simeq \ln r_t+\ln(1+i_t)+\eta_{t+1} \tag{3.14}$$

with $\eta_{t+1}=-r_t\varepsilon_{t+1}$ and $E_t(\eta_{t+1})=0$. This technique, unlike Hansen and Singleton's, requires an approximation that clearly removes some of the theoretical sharpness deriving from the rational expectations modelling. The compensating advantage is that the simple structure of the Frisch demands under certainty is preserved, and that the model has the certainty model as a special case when $\eta_{t+1}=0$. In consequence, we shall be able to deal below with the uncertainty case by straightforward differencing and instrumentation of the standard regressions that will represent the model under certainty.

We now have a clear interpretation of life-cycle Frisch demands. With perfect foresight and no uncertainty, consumers track along their predetermined life-cycle trajectories of labor supply and consumption demand. With uncertainty, new information is constantly coming to hand. If the new information leaves r unchanged, or permits (3.12) to hold exactly (not just in expectation), it is as if there had been no new information, and the consumer continues along the predetermined path. The price and wage derivatives of the Frisch demands are derivatives with the utility price constant and so are the derivatives of this predetermined path. It is perhaps not misleading to call them derivatives with respect to *anticipated* changes, although in this context fulfilment of expectations is defined by $r_{t+1}=(1+i_t)r_t$. The great advantage of the Frisch demands is that they separate out the effects of movements along the path (which occur with or without perfect foresight) from those movements of the path itself caused by new information.

There is another important issue that has not been sufficiently emphasized in the literature and which has been brought to our attention by Larry Epstein. Consider equation (3.1), the male labor supply equation under *certainty*. Now it is possible to produce other supply functions under quite different assumptions that look very like (3.1). For example, assume that preferences are *implicitly* additively separable so that the life-cycle cost function takes the form

$$c(u, p_1, p_2, \ldots, p_L, w_1, w_2) = \sum_{t=0}^{L} c_t(u, w_{1t}, w_{2t}, p_t) \tag{3.15}$$

where u is lifetime utility. Such preferences are *not* equivalent to the additively separable preferences of (2.1) and have quite different behavioral consequences; see Gorman [**21**, **22**] and Deaton and Muellbauer [**14**, Chapter 5]. Male labor supply in period t is given by

$$h_{1t} = T_{1t} - \frac{\partial c_t(u, w_{1t}, w_{2t}, p_t)}{\partial w_{1t}} = f_{1t}^*(u, w_{1t}, w_{2t}, p_t). \tag{3.16}$$

Under certainty, u is fixed for the whole life cycle so that, apart from the substitution of u for r, (3.16) and (3.1) are identical. Hence, if we follow MaCurdy's suggestion and treat r in (3.1) as an unobservable fixed effect, we have no means of knowing whether we are estimating Frisch demands as in (3.1) or Hicksian demands as in (3.16) even though the interpretation of the results would be different in the two cases. Other cases can also be generated.

Under uncertainty, it is more difficult to think of alternative sensible models that generate the Frisch demands together with the behavior of the utility price. For instance in the example of (3.15), *inter*temporal preferences are Leontief so that, under uncertainty, an individual who consumed too little early in life would throw away much of his or her later wealth. Indeed, it is difficult to think of any simple characterization of intertemporal choice under uncerainty *without* intertemporal additivity. Even so, the fact that there may exist alternative interpretations of our equations does not threaten their validity. If our life-cycle model is correct, our equations are the appropriate ones to estimate and if they cannot describe the data, then the theory is false. If they do describe the data, the model may be true or something else may be true; this is the normal situation.

4. COMPARATIVE STATICS, DYNAMICS, AND THE ROLE OF CHILDREN

Many of the general characteristics of the class of models discussed here are familiar from the work of Ghez and Becker. Here we look only at those results that are useful later. We also give examples of how one might develop profit functions suitable for empirical implementation. We look first at "cohort effects," differences in behavior predicted for households of different ages at a single moment in time, turning secondly to the analysis of individual life cycles.

Individuals born at different dates face different economic environments throughout their lives. In the models here, these cohort effects show up as differences in r, the price of lifetime utility. In general, we think of younger consumers as being better-off; they typically face higher real wages than did their parents at the same age and they inherit or expect to inherit more assets. Given concave utility, being better-off drives up the price of utility. On the average then, we should expect r to increase as we move from older to younger households. These effects are worth noting formally. They all derive from the budget identity for the Frisch demands, i.e., from

$$\begin{aligned}(4.1)\qquad & \sum_t \hat{w}_{1t} f_{1t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) + \sum_t \hat{w}_{2t} f_{2t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) \\ & + \sum_t p_t \cdot \phi_t(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) = W_0\end{aligned}$$

where W_0 is assets at birth, that is, the present discounted value of present and future time endowments and of future assets to be inherited. This identity implies:

(a) Growth in real wages increases r. It is clear, given zero homogeneity, that this lowers the effective price for goods, thus increasing real consumption throughout the life cycle. The net effects of $\hat{w}_{1t}/r$ and $\hat{w}_{2t}/r$ depend on exactly how wages rise and on the value of assets. Hence, as one would expect, the effect of increasing real wages on labor supply is not theoretically predictable. In this between cohorts context, the analysis is the standard one with offsetting income and substitution effects.

(b) Fully anticipated inflation is neutral if assets at birth are indexed. Otherwise there are real balance effects of the traditional kind.

(c) Increases in inherited assets increase r and so decrease both participation and hours and increase real consumption given the normality of consumption and leisure. We should thus expect older workers to have lower lifetime consumption expenditures because of their lower lifetime real wages and asset levels at birth. If there is real asset accumulation over time we should also expect them to work more lifetime hours than their younger counterparts.

(d) Growth in w_2/w_1 is likely to increase female participation and hours relative to male participation and hours. We can thus expect higher participation rates among younger than among older female workers.

Consider now the evolution of family labor supply and consumption over the life cycle. At the general level note the usual life-cycle model disassociation of income and consumption. Adults work hardest when (discounted) wages are highest, typically somewhat before the peak in lifetime wage rates, and not when they have the greatest need for income. The presence of children affects consumption and the time allocation of the parents only in so far as children's time and goods requirements are typically age specific and are not substitutable across periods. Whether or not the births of children are unanticipated, their subsequent development most surely is so that the general income effects of children are captured by the lifetime utility price. Parents work hard to support their children, but in a world without market imperfections, there is no need to do so at the precise moment when the children are the greatest financial burden. It is much better to earn and save when the wage is highest, or to borrow against that period.

A fairly general example illustrates many of these points. Write the period t profit function in the form

$$\pi_t(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) = \alpha(r, \hat{w}_{1t}, \hat{w}_{2t}, p^*(a_t)) - \beta(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t, a_t) \tag{4.2}$$

where a_t is a vector of demographic characteristics of the household at time t. For example, the vector might comprise two elements, a_1, the number of small children, and a_2 the number of large (older) children. The $\beta(\ \ )$ function represents the *costs* imposed on the parents by the presence of children, in terms of both time and goods. The wage rates $\hat{w}_{1t}$ and $\hat{w}_{2t}$ in β emphasize the children's requirements for parental time; one reasonable specification would make husband's and wife's time substitutes in child care so that, for example, all child care would be assigned to the partner with the lowest market wage after correction for "efficiency" in child care. The prices of goods enter costs through needs for child-related commodities; we might specify that older children are commodity intensive and younger children time intensive. Combining these ideas suggests

$$\beta = \{\theta_1(r)a_1 + \theta_2(r)a_2\} \min(\hat{w}_{1t}, \xi\hat{w}_{2t}) + \mu_1(r, p)a_1 + \mu_2(r, p)a_2 \tag{4.3}$$

for efficiency parameter ξ and with $\theta_1 > \theta_2$ and $\mu_1 < \mu_2$. The presence of r allows the costs of children to vary with the lifetime welfare level of the household. Note that this does *not* imply that child costs are not real costs. The household with high life time resources may feel constrained to buy private education for its children; the fees still come out of the parental budget.

The $\alpha(\ \ )$ function represents the positive side of family life; it is the value of parental pleasures and parental leisure. Here one might expect l_{1t} and l_{2t} to be complementary, at least if the parents enjoy each other's company. If l_{1t} is separable from l_{2t}, the parents enjoy their leisure separately, and the $\alpha(\ \ )$ functions is additive in $\hat{w}_{1t}$ and $\hat{w}_{2t}$ (note that profits are additive if utility is). The $p^*(a_t)$ function indicates that the presence of children alters the effective price faced by parents for adult goods. We have in mind the Barten [**8**] model in which

$$p_i^*(a_t) = p_{it}m_i(a_t) \tag{4.4}$$

for scaling factors $m_i(a_t)$; see Deaton and Muellbauer [**14**, Chapter 7] and Pollak and Wales [**43**] for further discussion. To give a concrete example, the cost of a trip to the cinema is increased by the cost of a babysitter when small children are present. Hence, the presence of children not only has direct effects through the costs $\alpha(\ \ )$ but indirect effects acting through "pseudo" relative price changes on adult consumption patterns. In this sense, the model (4.2) is an application of Gorman's [**22**] formulation of child costs to the life-cycle context.

It is of considerable interest to work through the full set of derivatives for (4.2) and thus to trace out the effects of wage and price change and of family development on labor supply and consumption patterns over the life cycle. Here we limit ourselves to two brief examples.

(a) If the wife is allocated all the child-care duties and still participates in the labor market, the complementarity assumption in $\alpha(\ \ )$ implies, for anticipated wage changes,

$$(4.5) \qquad \partial h_1/\partial w_2 = \partial h_2/\partial w_1 > 0.$$

The equality is by symmetry. An increase in w_1 results in longer hours for the husband; his absence from the home devalues his wife's leisure and she works more hours in the market. The "independent leisures" assumption would set both derivatives to zero. In most studies of female labor supply based on the static model $\partial h_2/\partial w_1$ is found to be negative. But this is an income effect following from the increase in welfare after the wage increase. In the current context, the wage change is anticipated so that there is no income effect.

(b) The reservation wage of the wife, w_t^*, defined by (2.8) varies positively with the number of small children, i.e., $\partial w^*/\partial a_1 > 0$. Hence, an increase in the number of infants will decrease hours of women who continue to participate in the market and will increase the probability of nonparticipation. More interesting is the behavior of the husband's hours. Firstly, there is a direct effect through the Barten prices on goods consumption and thence on labor supply if goods and male leisure are not separable. Ignore this for the moment on the assumption that there is such separability. The direct time cost of the infant falls on the wife and, provided she continues to participate in the labor market, there is no effect on husband's leisure or market hours. Given (4.2) and (4.3) with $\xi w_{2t} < w_{1t}$, the husband's Frisch labor supply function is independent of a_1 (except through the Barten effects). However, once the wife ceases to participate, the effects of a_1 on w^* enter the husband's labor supply function. Given the "loving couple" complementarity assumption, $\partial h_1/\partial a_1 > 0$, i.e., the additional (anticipated) infant causes the husband to work longer hours. Essentially, when the woman cannot adjust her hours in the market, extra infants cause her to adjust her hours in the home. The extra time spent with the children leaves less for her husband who responds by working longer hours. It is clearly not necessary to invoke credit restrictions and the need to feed the extra mouths to explain the finding of greater male labor supply in response to an increase in family size.

PART TWO: IMPLEMENTATION

5. FUNCTIONAL FORMS

Our selection of useful functional forms is partly guided by the usual criteria: (a) that they be flexible up to the first derivatives of the demands and (b), that they allow simple parametric testing of important hypotheses, particularly symmetry and separability restrictions between the types of labor and individual commodities. However, we also have an additional requirement, that it be possible and convenient to treat the price of utility as an unobservable fixed effect, with or without random parts. Fixed effects can most easily be dealt with by differencing, provided that they appear additively in the demand and supply functions, i.e., we require Frisch demands of the form (again using h_1 as an example)

$$t(h_{1t}) = \gamma_1(r) + \eta_1(\hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) \tag{5.1}$$

where t is some monotone parameter-independent transformation (e.g., a logarithm) and $\gamma_1(\ \ )$ and $\eta_1(\ \ )$ are suitable functions. This formulation is also useful for the uncertainty case since $\ln r$ differences to give a sum of an observable and a well-defined stochastic term (see (3.14)) so that $\gamma_1(r)$ will do the same, if not exactly then as an approximation.

In principle, there are a number of choices for the $t(\ \ )$ function. Heckman and MaCurdy [**32**] and MaCurdy [**37**] use logarithms but this has disadvantages. If log hours is the dependent variable, it is hard to analyze participation since the predicted hours can never be zero or negative; effectively such a choice assumes preferences in which hours are essential. Log leisure is better, but leisure is not directly observable. In practice it is measured by subtracting hours worked from available hours (e.g., 168 hours per week) but assigning a value to this is essentially arbitrary and the results obtained are not invariant to the assignment. The same difficulty applies to budget *shares* as dependent variables. In this sort of model, the denominator of the shares is "full income" which, like leisure, requires knowledge of available hours. Hence, the effective choices for the dependent variable are hours or hours multiplied by the wage rate, i.e., earnings. We analyze both.

To ease notation, we temporarily use the vector q to denote all demands and supplies, i.e., commodities and labor supplies. Similarly the vector p is a vector of the two wage rates and the n commodity prices. For *Case 1*, with hours/quantities the dependent variable, we require

$$\frac{\partial q_i}{\partial p_j} = f_{ij}(p) \tag{5.2}$$

to be independent of r. For *Case 2*, with expenditures the dependent variable, we require that

$$\frac{\partial(p_i q_i)}{\partial p_j} = f^*_{ij}(p) \tag{5.3}$$

be independent of r.

Taking *Case 1* first, (5.2) requires, for suitable $\xi_i(\ \ )$ and $\zeta_i(\ \ )$ that

$$\partial\pi/\partial p_i = \xi_i(p) + \zeta_i(r), \tag{5.4}$$

hence

$$\pi(r, p) = a(r) + \xi(p) + \sum \zeta_i(r) p_i. \tag{5.5}$$

Now $\pi_i(r, p)$ is given by (5.4) so $\pi_{i0} = \zeta_i'(r)$ which must be homogeneous of degree -1, i.e., $\pi_{i0} = -\mu_i/r$ for positive constants μ_i. This implies $\zeta_i(r) = -\mu_i \ln r + \eta_i$. Differentiating with respect to r gives $\pi_0 = a'(r) - \mu \cdot p/r$ which is zero degree homogeneous, i.e., $a'(r)=\alpha$, so that the profit function takes the form

$$(5.6)\qquad \pi(r,p)=\alpha r+\xi(p)+\sum(\eta_i-\mu_i\ln r)p_i.$$

Rewrite this as

$$(5.7)\qquad \pi(r,p)=\alpha r+d(p)+\sum\mu_k p_k\ln\left(\frac{p_k}{r}\right)$$

where $d(p)=\xi(p)+\sum\eta_k p_k-\sum\mu_k p_k\ln p_k$. Provided $d(p)$ is chosen to be linear homogenous, $\pi(r,p)$ is linearly homogenous and is the profit function that we want; it represents the most general set of preferences yielding (5.2), i.e., hours and commodity equations that contain r only as an additive effect. The (Frisch) demands corresponding to (5.7) are

$$(5.8)\qquad q_i=-d_i(p)-\mu_i\ln(p_i/r)-\mu_i.$$

Since $\pi_0=u$, we have period utility $u=\alpha-\mu\cdot p/r$, so that substituting for r in (5.8) we have the Hicksian demands corresponding to (5.8):

$$(5.9)\qquad q_i=-d_i(p)-\mu_i\ln(p_i/\mu\cdot p)-\mu_i\{1+\ln(\alpha-u)\}.$$

Since the final term in brackets on the right-hand side is monotone in u and does not contain p, it is clear that (5.9) is a system of demands corresponding to *quasi-homothetic preferences.* Hence, the treatment of r as additive in the hours and quantities demanded implies intraperiod quasi-homotheticity, i.e., that for a single consumer hours and expenditures are linearly related to within period full income.

A similar analysis applied to *Case 2*, with expenditures/earnings as the dependent variable yields, instead of (5.7) for *Case 1*, a profit function

$$(5.10)\qquad \pi(r,p)=\alpha_0^*r+d^*(p)+r\sum\mu_k^*\ln\left(\frac{p_k}{r}\right)$$

with Frisch demands

$$(5.11)\qquad p_iq_i=-p_id_i^*(p)-\mu_i^*r.$$

The Hicksian demands for within period utility u are easily calculated and once again yield quasi-homothetic preferences. Both cases are therefore restricted in this way. Note, however, that, in the context of flexible labor supply, quasi-homotheticity *does not* imply linear Engel curves for goods in terms of either income or total expenditure.

There is little obvious reason to choose one of these forms rather than the other, though the constant marginal propensities to *spend* in (5.11) may be more familiar than the constant marginal propensities to *consume* in (5.9). However, neither formulation is more flexible than the other and arbitrarily, we have chosen to work with *Case 1* and the profit function and Frisch demands given by (5.7) and (5.8). For the linear homogenous $d(p)$ function in (5.7) we choose

$$(5.12)\qquad d(p)=-\sum\eta_k p_k-\sum\sum\theta_{kj}p_k^{1/2}p_j^{1/2}$$

for parameters η_k and $\theta_{kj} = \theta_{jk}$. This choice of $d(p)$ is clearly a second-order flexible functional form.

Substituting into the Frisch demands, reverting to the original notation, and using the forms (3.3) to deal with both the certain and uncertain cases, gives the system (with both partners participating)

$$h_{1t} = \alpha_{1t} + \beta_1 \ln w_{1t} - \theta_{12}\left(\frac{w_{2t}}{w_{1t}}\right)^{1/2} - \sum_{3}^{n+2} \theta_{1j}\left(\frac{p_{jt}}{w_{1t}}\right)^{1/2} - \beta_1 \ln r_t, \tag{5.13}$$

$$h_{2t} = \alpha_{2t} + \beta_2 \ln w_{2t} - \theta_{21}\left(\frac{w_{1t}}{w_{2t}}\right)^{1/2} - \sum_{3}^{n+2} \theta_{2j}\left(\frac{p_{jt}}{w_{2t}}\right)^{1/2} - \beta_2 \ln r_t, \tag{5.14}$$

$$\begin{aligned} q_{it} = \alpha_{it} + \beta_i \ln p_{it} + \theta_{i1}\left(\frac{w_{1t}}{p_{it}}\right)^{1/2} + \theta_{i2}\left(\frac{w_{2t}}{p_{it}}\right)^{1/2} \\ + \sum_{\substack{3\\ j\neq i}}^{n+2} \theta_{ij}\left(\frac{p_{jt}}{p_{it}}\right)^{1/2} - \beta_i \ln r_t \qquad (i = 3, \ldots, n+2), \end{aligned} \tag{5.15}$$

where the α's and β's bear obvious relationships to the η's, μ's, and θ's, and where, for convenience, the commodities have been renumbered from 3 to $n+2$, i.e., male and female labor supply are commodities 1 and 2. The t subscripts on the α's reflect variation in variables other than p, w, and r.

These equations are linear in the parameters and in the $\ln r_t$ and so can be straightforwardly used for estimation and testing. In particular, the following hypotheses are of interest:

(a) *Symmetry*: Frisch symmetry requires $\theta_{ij} = \theta_{ji}$ for all $i, j = 1, 2, \ldots, n+2$.

(b) *Additivity*: Types of leisure and/or goods are additively separable *within* the period if $\theta_{ij} = 0$. Of particular interest is whether or not $\theta_{12} = \theta_{21} = 0$, i.e., whether husband's or wife's leisure is separable. For other purposes, e.g., for many aspects of optimal tax theory, we wish to test separability between specific goods and leisure. In the current context, this can be tested by testing $\theta_{1i} = 0$ and $\theta_{2i} = 0$.

(c) *Intertemporal Substitutability*: Unlike MaCurdy's formulation, elasticities are not parametrized. However β_1 / h_{1t} is the estimated elasticity of current hours with respect to anticipated wage changes and is one of the magnitudes on which we focus.

These functional forms can be modified to account for children and other socio-demographic characteristics in a number of ways. The simplest is to make the α_i's functions of these variables allowing also for an "idiosyncratic" error term. Better would be to explicitly model the effects of children through the Barten-type effective prices and through their time costs, as in the previous section. Participation of the wife can be modelled by analyzing (5.14) as a fixed-effect Tobit as implemented by Heckman and MaCurdy [**32**]. However, (5.14) does not yield an explicit solution for the wife's reservation wage w^* nor therefore does it yield explicit solutions for the male labor supply and commodity demand functions when the wife does not participate. Note, however, that if $\theta_{12} = 0$, the wife's participation status has no effect on her husband's labor supply. In this case, male hours can be analyzed without reference to female participation.

In the rest of this paper, we shall ignore female labor supply and assume it to be additively separable from both goods and male labor supply; we therefore estimate (5.13) together with an aggregate version of (5.15).

Note finally that while it is convenient to work with demand functions with additive fixed-effects such as those discussed, the choice of such forms is not costless. In particular, both profit functions derived here implicitly involve a particular "normalization" of the within period subutility or felicity functions. These essentially determine the allocation of lifetime wealth between periods so that our choice of form results in a complete specification of lifetime preferences.

6. CREATING PANEL DATA

The Family Expenditure Survey is not a panel; individual households are not followed through time. However, the survey is in continuous operation so that it provides a random sample of the population each year (subject to the exclusion of certain groups; see Kemsley, Redpath, and Holmes [**35**]). Currently, we have access to data for the seven tax years (April 5th to April 4th) 1970/1, 1971/2, 1972/3, 1973/4, 1974/5, 1975/6, and 1976/7. Hence although we cannot track individual households, we can track *groups* of households. In particular, if we take age as age of the household head, we can look at the average behavior of, say, 25 year olds in 1970/1, of 26 year olds in 1971/2, ending up with 31 year olds in 1976/7. If we take the first group to be a random sample of all 25 year olds in 1970/1, of 26 year olds in 1971/2, ending up with 31 year olds in 1976/7, then the tracking through the surveys produces a series of random samples from the *same* cohort. Given linear in parameter functional forms such as (5.13)–(5.15), mean cohort behavior reproduces the form of individual behavior and the cohorts can thus effectively be treated as individuals. If the price of lifetime utility is constant for each member of the cohort from one year to the next, then its mean is constant for the cohort as a whole. Hence, the sample mean from the survey will be a consistent estimator of the same quantity from year to year, with a precision determined by the sample design. Similarly, if the (log) utility price follows equation (3.14) for each household, so does its mean. Hence, for all practical purposes, the cohort means can be treated as panel data. Indeed the constant random resampling eliminates the problems caused by attrition in genuine panels and one can evisage very long "panels" created in this way.

Although the empirical analysis proceeds entirely in terms of cohort means, it is important to note how essential are the individual household data. First, the functional forms in the previous section are linear in parameters, not in data, so that to obtain cohort means, it is necessary to obtain the average of log wages or $(w_1/p)^{1/2}$, not the functions of the averages. All such means are straightforwardly obtained from the individual household data. Second, the sample means are subject to sampling errors and the individual data may be used to provide estimates of these errors. Since covariances as well as variances can be obtained,

TABLE I

NUMBER OF HOUSEHOLDS IN EACH COHORT IN EACH YEAR

Cohort no. &	age 1970/1	70/1	71/2	72/3	73/4	74/5	75/6	76/7
				Manual				
1	18–23	—	—	201	244	269	303	313
2	24–28	276	257	269	235	266	278	250
3	29–33	204	247	231	239	259	257	247
4	34–38	258	258	267	248	243	259	237
5	39–43	263	282	264	242	219	265	238
6	44–48	266	310	267	254	271	281	254
7	49–53	230	297	268	248	230	216	205
8	54–58	236	240	248	238	209	191	214
				Nonmanual				
1	18–23	—	—	79	108	130	168	167
2	24–28	105	147	148	156	158	175	163
3	29–33	119	154	141	115	148	143	159
4	34–38	122	136	133	137	160	118	137
5	39–43	123	159	156	132	131	137	116
6	44–48	121	155	143	155	160	134	107
7	49–53	144	116	130	128	104	127	135
8	54–58	90	107	105	113	90	104	78
Totals		2557	2865	3050	2992	3046	3156	3020

we are in a uniquely favorable position to implement errors in variables estimators.

In practice, one year cohorts yield samples that are too small to give accurate estimates of the sample means. Consequently, we use five-year age bands subdivided as to whether the head-of-household is a manual or nonmanual worker. We also limit ourselves to households with heads aged 18–58 in 1970/1 who are then aged 24–64 in 1976/7 so that all are in the normal working span in all of the surveys. The sample is also selected in other ways. We look only at households containing married couples, one of which is listed as a head-of-household. We also eliminate those men who are *not* employees, who are listed as not in work last week, or who have wives listed as self-employed. The elimination of the unemployed is potentially the most serious problem since we are effectively assuming that the unemployed are a random sample of all participants; we plan to extend the analysis to do better than this in further work, for example by treating the unemployed as voluntary nonparticipants. Note that *including* the unemployed in the regressions would not be correct, even if the choice of unemployment is voluntary, since zero hours is a corner solution and must be handled as such. Our exclusion procedure means that our analysis of business cycle effects is confined to variations in hours of those who remain in work; fortunately it is well-known that such variations move parallel with variations in employment; see Pencavel [**42**]. Table I gives the age bands for each cohort in the first year 1970/1, together with the number of households sampled from each cohort in each of the seven years. Ideally, since we are sampling from the same underlying cohorts through time, we would expect to get the same size samples from each survey. In practice, this does not happen for a number of reasons. The cohorts themselves will change somewhat through death, emigration, and immigration. More importantly our sample selection criteria do not act randomly, particularly with age. This appears most dramatically for the first two cohorts, particularly nonmanual workers, where selection on marriage and employment status excludes a higher proportion of younger workers. Hence the observed sample sizes increase with age, by about 50 per cent for manual workers and about 100 per cent for nonmanual workers. Even this understates the problem since we have excluded the first cohort in the first two years in an attempt to limit unrepresentativeness. There is also a suggestion of declining sample sizes in the oldest cohort as it approaches retirement in the last few years. Presumably this is partly a result of early retirement, though the FES response rate is known to decline monotonically with age; see Kemsley [**34**]. Whether or not these sampling effects bias our results will of course depend on the relationship between the selection criteria on the one hand and labor supplies and commodity demands on the other.

We note finally that in constructing cohort samples there is a trade-off between cohort size and the number of cohort means. If we had taken one year cohorts from Table I, there would have been five times as many "observations" although each would have had about one fifth the number of observed households. Smaller cohort size implies less precise sample means so the essential trade off is between the number of observations and the accuracy of each. If errors-in-variables estimators are used, it is possible to optimize on this trade off and thus to determine an optimal cohort size. This turns out to be a substantial research project in its own right and the results are reported elsewhere; see Deaton [**13**]. For the rest of this paper, we shall treat sample cohort means as if they were population cohort means.

7. EMPIRICAL RESULTS FOR MALE LABOR SUPPLY AND FOR CONSUMPTION

In this paper, we deal with male labor supply and aggregate consumption only. Results on female labor supply and on disaggregated commodity demands will be presented elsewhere, but even the limited task here is complex enough. Subsection 7.1 is concerned with preliminaries of data construction for the empirical models. Subsection 7.2 deals with the relationship between male hours and wages, both under perfect certainty and under uncertainty. Finally, subsection 7.3 is concerned with the joint analysis of consumption and male hours.

7.1. *Preliminaries*

We begin from the version of (5.13) in which male leisure and consumption are additively separable from female leisure and in which there is a single aggregate commodity q. Write the demand and supply equations *under certainty* as

$$h_{it}^c = \alpha_{1t} + \beta_1 \ln \hat{w}_{it}^c + \theta_1 \sqrt{\frac{p_t}{w_{it}^c}} - \beta_1 \ln r_i^c, \tag{7.1}$$

$$q_{it}^c = \alpha_{2t} - \beta_2 \ln \hat{p}_{it}^c - \theta_2 \sqrt{\frac{w_{it}^c}{p_t}} + \beta_2 \ln r_i^c, \tag{7.2}$$

where i is the individual household, "born" at time c (the cohort identifier) and observed at time t. The "1" subscript on male hours has been dropped. In theory $\theta_1 = \theta_2$, by symmetry, and we shall be interested to test this. Note that r_i^c is the individual's price of lifetime utility discounted to birth and does *not* have a t subscript. The first step is to average over all i belonging to c which removes the i subscript; (7.1) and (7.2) then hold for cohort means. Note that the means involved are the sample averages of the variables in the equations, not of their components; i.e., the right hand side variables in (7.1) are the means of the logarithms of the discounted wage, of the square root of the price-wage ratio, and so on. To the extent that cohort sample means are error-ridden estimates of the cohort population means, there will be biases in estimation. In particular, it should be noted that samples with abnormally high wages will tend to have abnormally high lifetime utility prices and thus low hours if leisure is normal, so that if $\theta_1 = 0$, treating $\ln r^c$ as time independent will tend to bias downwards the estimates of β_1 if the model is correct.

The second step is to change to a practical method of discounting. As written above, $\hat{w}^c$ and $\hat{p}^c$ are discounted back to the beginning of cohort c's life, i.e., to c, and this date is different for different cohorts. A more convenient procedure is to write $\tilde{w}_t^c = w_t^c \delta(t, T)$ where T is some fixed calendar date (we use January 1974), and w_t^c is the current nominal wage. Clearly $\ln \hat{w}_t^c = \ln \tilde{w}_t^c + \ln \delta(T, c)$, and similarly for prices, so that if we define $\ln \tilde{r}^c = \ln r^c - \ln \delta(T, c)$, (7.1) and (7.2) become

$$h_t^c = \alpha_{1t} + \beta_1 \ln \tilde{w}_t^c + \theta_1 \sqrt{\frac{p_t}{w_t^c}} - \beta_1 \ln \tilde{r}^c, \tag{7.3}$$

$$q_t^c = \alpha_{2t} - \beta_2 \ln \tilde{p}_t - \theta_2 \sqrt{\frac{w_t^c}{p_t}} + \beta_2 \ln \tilde{r}^c. \tag{7.4}$$

Note that $\tilde{p}_t$ has no cohort superscript since it is now a common price discounted to a common date. The quantity $\tilde{r}^c$ does not vary with t. However r^c *must* vary with c since for later cohorts lifetime prices will on average be higher as will real resources if there is economic growth. One *possibility* is that r^c varies with c as r_t varies with t, i.e., that $r^{c+1} = (1 + i_c) r^c$, so that the price of utility increases from cohort to cohort with prices and with the real interest rate. In this case, r^c is independent of c, and the last terms in (7.3) and (7.4) are absorbed into the constant. We shall not impose this restriction however; in the implementation of (7.3) and (7.4) cohort dummies will be included and their significance tested for, while in the intracohort first-differenced forms all time-invariant cohort specific variables are differenced out. Note in particular that fixed date discounting would lead us to expect no coherent pattern in cohort dummies if the model is true.

In the uncertainty case, we use undiscounted Frisch demands from Section 3. Taking the labor supply equation to illustrate, we write

$$h_t^c = \alpha_{1t} + \beta_1 \ln \tilde{w}_t^c + \theta_1 \sqrt{\frac{p_t}{w_t^c}} - \beta_1 \ln r_t^c \tag{7.5}$$

so that, differencing within cohorts and using (3.14), we have

$$\Delta h_t^c = \Delta \alpha_{1t} + \beta_1 \Delta \ln \tilde{w}_t^c + \theta_1 \Delta \sqrt{\frac{p_t}{w_t^c}} - \beta_1 \eta_t \tag{7.6}$$

which is the first difference of (7.3). The corresponding equation for consumption is the first difference of (7.4), together with the term $\beta_2 \eta_t$. Note that the innovation η_t will generally be correlated with the right-hand side variables to the extent that these contain unanticipated components. Instrumental variables are therefore required in this case and natural instruments are available in the shape of quantities known in period t or earlier. Our empirical procedure is therefore a straightforward one. Equation (7.3) and (7.4) represent the model under certainty; their first differences, estimated by instrumental variables, are the appropriate equivalents under uncertainty.

Finally, we modify the equations to allow for the possible effects of variations in household size over the life cycle. This is most conveniently done by treating α_{1t} as a variable to write

$$h_t^c = \alpha_1^0 + \beta_1 \ln \tilde{w}_t^c + \theta_1 \sqrt{\frac{p_t}{w_t^c}} - \beta_1 \ln \tilde{r}^c + \gamma_{11} a_{1t}^c + \gamma_{12} a_{2t}^c + u_{1t}^c, \tag{7.7}$$

$$q_t^c = \alpha_2^0 - \beta_2 \ln \tilde{p}_t - \theta_2 \sqrt{\frac{w_t^c}{p_t}} + \beta_2 \ln \tilde{r}^c + \gamma_{21} a_{1t}^c + \gamma_{22} a_{2t}^c + u_{2t}^c, \tag{7.8}$$

where a_{1t}^c is the cohort mean number of young children (less than 5 years of age) and a_{2t}^c of older children (aged 5–15 in 1970 and 1971 and aged 5–17 from 1972 on). In the experiments in this paper we shall treat a_1 and a_2 as if they were exogenous. This can be objected to on the ground that the timing, spacing, and numbers of births is jointly endogenous over the life cycle with labor supply. While acknowledging this in principle, we believe that the feedbacks from labor supply are likely to be less important for male than for female labor supply. Note also that we work with cohort means, where the patterns of young and old children have relatively more to do with human biology and less with economics than would be the case with individual data. It should also be borne in mind that one of the most obvious alternative hypotheses to the simple life-cycle theory of hours is that, in the absence of access to good capital markets, main earners must work to support their offspring and that they therefore work the longest hours when their needs are greatest.

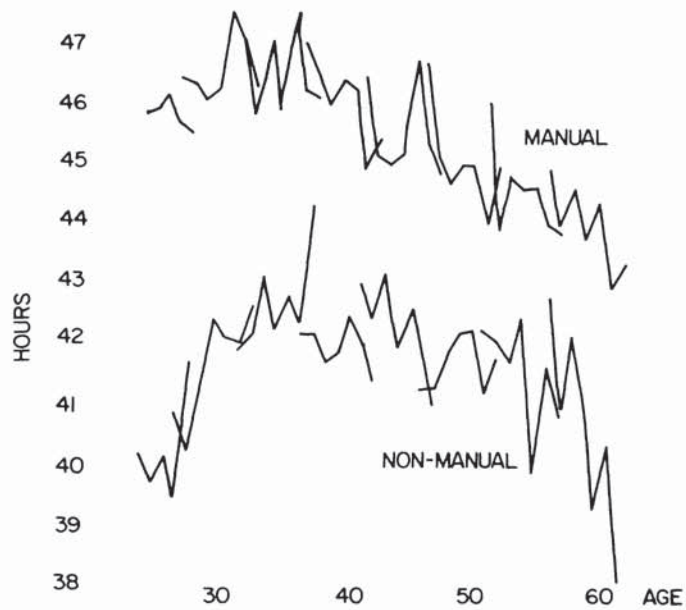

FIGURE 1

The cohort means of hours, discounted wages, prices, price-wage ratios, children, and age of household head are presented in the Appendix, where precise definitions are also given. It should be noted that hours are "normal hours", the definition of which is left to the respondent, while the wage is "normal" income divided by normal hours. (Annual hours are not available in the FES.) The division bias introduced by this construction will tend to impart a downward bias to the hours/wage relationship; this should be offset against the upward bias that results from the inclusion of some overtime in hours and in earnings.

7.2. *Male Labor Supply*

We begin by making the temporary assumption that goods and male labor supply are additive within periods so that $\theta_1 = \theta_2 = 0$. The resulting relationship between h and $\ln w$ is graphed in Figures 1 and 2 which show hours and discounted wages against age for manual and nonmanual workers with each cohort shown separately. As we move from left to right we follow the first cohort as it ages from about 22.5 years in 1972/3 to 26.5 years in 1976/6. The line then breaks to the first observation on the second cohort which in 1970/1 had an average age of close to 26 years. This cohort overlaps with the first cohort for two age observations and we follow them for seven years, breaking off and going back two years in age to pick up the next (third) cohort, and so on. These figures show, in rather noisy form, the traditional life-cycle wage/hours relationships,

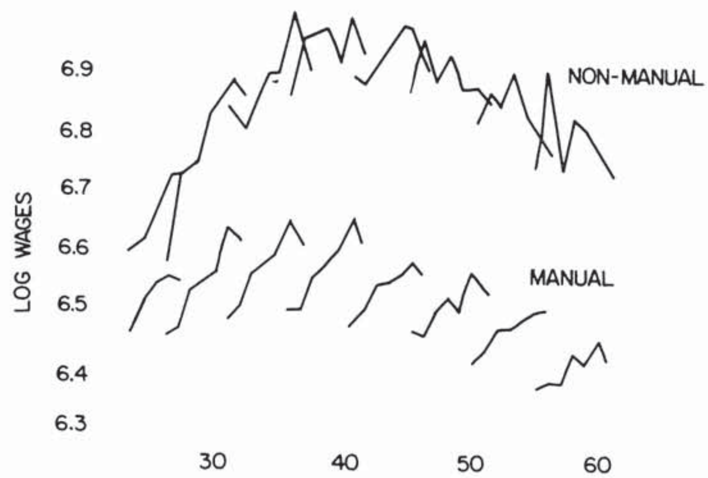

FIGURE 2

and conform to the stylized facts found for the U.S. by Ghez and Becker [**18**] and by Smith [**45**], using cross-sectional data. Manual hours peak early in the life cycle, at age 30–35 and decline fairly steadily thereafter, the total decline being around 3 hours per week. Discounted wages (we use the consol rate) for manual workers also peak early (but somewhat later than do hours) at around 35–40; the total decline is about 14 per cent. For nonmanual workers the pattern is different but there is still a high degree of synchronization between hours and wages over the life cycle. For these workers, hours rise for a longer period reaching a peak around age 35 and declining thereafter; their wages do likewise. But these early life-cycle rises in both hours and wages for nonmanual workers should be treated with particular caution. Our selection criteria exclude such people as students until they enter the labor force. Individuals with high lifetime wage rates will therefore tend to be underrepresented in the earlier years. Even so, the evidence is apparently consistent with the simplest version of life-cycle labor supply. The higher hours by manual workers at all points in the cycle in spite of their lower wages can straightforwardly be ascribed to the domination of income effects between different life cycles, while, as the theory predicts, substitution effects dominate within the cycle.

Table II presents somewhat more formal evidence. Regressions 2.1 and 2.6 confirm the simple correlations over the life cycle between hours and wages for both manual and nonmanual workers. The corresponding (anticipated) intertemporal substitution elasticities, evaluated at the means, are 0.15 and 0.14 respectively, figures close to those calculated by MaCurdy [**37**] for the U.S. However, these results are not very robust. For manual workers, the numbers of young and older children are much more satisfactory predictors of life-cycle labor supply than is the discounted wage; see Figures 3 and 4 below for the behavior of children over the life cycle. The introduction of cohort dummies (the c column)

TABLE II

LABOR SUPPLY REGRESSIONS: LEVELS

	Parameter Estimates				F ratios			
	constant	$\ln \tilde{w}$	a_1	a_2	c	y	R^2	d.w.
				Manual workers				
2.1	−0.08	6.99	—	—	—	—	.213	1.03
		(1.86)						
2.2	59.18	−2.37	2.21	1.02	—	—	.701	1.96
		(1.72)	(0.26)	(0.18)				
2.3	88.97	−7.06	0.29	1.34	0.97	—	.742	1.99
		(2.98)	(1.30)	(0.50)	(7, 43)			
2.4	20.60	3.57	1.71	0.58	—	7.39*	.851	1.74
		(3.57)	(0.34)	(0.29)		(6, 44)		
2.5	80.78	−5.84	0.11	1.35	1.47	7.50*	.883	1.74
		(5.22)	(1.08)	(0.50)	(7, 37)	(6, 37)		
				Nonmanual workers				
2.6	0.98	5.95	—	—	—	—	.278	1.31
		(1.33)						
2.7	20.61	2.96	0.77	0.67	—	—	.384	1.31
		(2.10)	(0.41)	(0.33)				
2.8	47.55	−1.07	1.79	2.61	2.72*	—	.574	1.10
		(2.28)	(1.17)	(0.75)	(7, 43)			
2.9	6.47	5.07	0.82	0.42	—	1.21	.472	1.38
		(2.30)	(0.42)	(0.35)		(6, 44)		
2.10	29.09	1.63	1.10	2.54	2.60*	1.26	.646	1.27
		(2.76)	(1.40)	(0.80)	(7, 37)	(6, 37)		

NOTES: Standard errors are in brackets beneath parameter estimates. Degrees of freedom beneath F ratios: a star indicates significance at a 5 per cent level. Observations are weighted by the square root of cohort size.

has little effect except on the coefficient on a_1; not surprisingly, knowledge of a_1 essentially identifies the cohort and *vice-versa*. However, year dummies are of considerable importance as is to be expected given the clear cyclical effects on both manual hours and wages in Figures 1 and 2. It is clear therefore that the behavior of manual workers' wages and hours over the business cycle is *not*

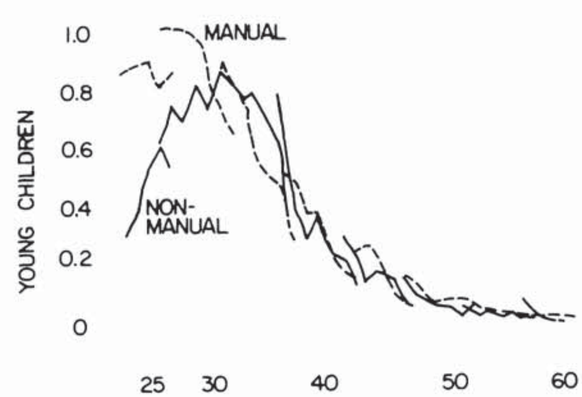

FIGURE 3

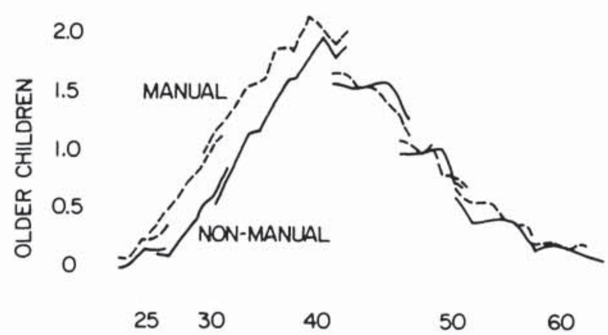

FIGURE 4

explicable by discounted wage variations in a life cycle context, and while the intercohort variation in hours over the life cycle is consistent with the variation in wages, it is much better explained by nonwage variables, such as cohort dummies or, essentially equivalently, by life-cycle variations in household size. The picture for nonmanual workers is similar with some variation in detail. Children and cohorts are somewhat less collinear and business-cycle effects play no role. Regression 2.8 with cohort dummies but no year dummies effectively tells the story; the joint influence of numbers of children and pure cohort dummies leave no significant role for wages in the explanation of hours. For both sets of workers, additional children, particularly older children, exert a consistently positive effect on hours. Such a finding is, of course, explicable from a number of different theoretical viewpoints.

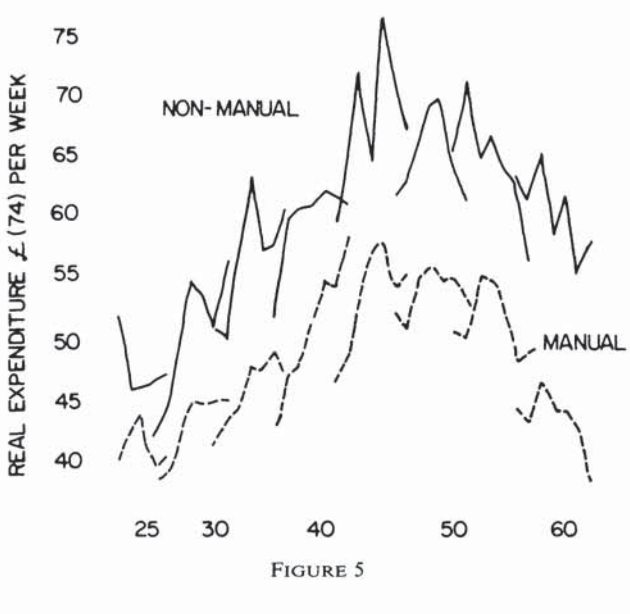

FIGURE 5

TABLE III

POOLED LABOR SUPPLY REGRESSIONS: ALL WORKERS PARAMETER ESTIMATES

	Constant	$\ln \tilde{w}$	a_1	a_2	*NMD*	F ratios c	y	R^2	d.w.
3.1	3.81	6.39 (1.11)	—	—	−5.89 (0.41)	—	—	.812	1.51
3.2	33.3	1.67 (1.30)	1.53 (0.23)	0.77 (0.17)	−4.15 (0.47)	—	—	.879	1.61
3.3	41.9	0.265 (1.27)	3.05 (0.63)	2.04 (0.41)	−3.40 (0.48)	2.72* (7, 96)	—	.901	1.70
3.4	10.6	5.17 (1.38)	1.31 (0.21)	0.44 (0.17)	−5.35 (0.49)	—	6.31* (6, 97)	.910	1.66
3.5	16.5	4.19 (1.54)	1.39 (0.77)	1.47 (0.40)	−4.87 (0.58)	2.41* (7, 90)	4.52* (6, 90)	.924	1.67
3.6	−.306	7.29 (1.29)	—	—	−6.19 (0.44)	6.09* (7, 92)	8.13* (6, 92)	.913	1.58

Table III gives results for pooling manual and nonmanual workers allowing for a shift-term in the intercept only, shown in the table as NMD and taking the value 1 for nonmanual workers and 0 otherwise. The covariance analysis does not reject the restriction implied by pooling except, of course, for the highly significant intercept dummy. These results are rather more favorable for the theory although the year dummies are still significant; behavior over the business cycle is not explicable in terms of life-cycle intertemporal substitution. However, in these pooled regressions, and once year dummies have been allowed for, the wage rate has a significant role to play in explaining variations in hours (lines 3.4 and 3.5). Once again, numbers of children have a significantly positive effect on men's hours. Line 3.6 of the table shows the consequence of deleting the child variables; although the restriction is rejected, the only effect is to increase somewhat the wage elasticity as well as the added effect for being a manual worker.

Table IV reports the results of estimation in first differences, both by ordinary least squares and with the wage variable instrumented using variables theoretically uncorrelated with current innovations. These results are once again not particularly supportive of the theory.

Using OLS, the wage is only significant when negative, and only older children and the year dummies retain their previous roles. Indeed, for manual workers alone (not shown), there is a strong negative relationship within cohorts between changes in hours and changes in wages; in the pooled sample, the sign remains but the significance is lost. With instruments, only the year dummies remain significant, although there is still some evidence of a positive influence for older children. The major feature of the pooled first differences is that there is an essentially random scatter between changes in hours and changes in discounted wages. This could arguably be attributed to our sampling procedure for cohorts; first differencing of sample means may generate an adverse signal to noise ratio (a criticism, interestingly, that is often leveled against household *panel* data). But this is not the whole story. Our data as presented in Figures 1 and 2 and in the

TABLE IV

LABOR SUPPLY INTRA-COHORT FIRST DIFFERENCE REGRESSIONS: ALL WORKERS

OLS	Constant	$\Delta \ln \tilde{w}$	Δa_1	Δa_2	*NMD*	y	R^2	d.w.
4.1	−0.15	−4.13	—	—	0.15	—	.051	1.68
	(0.13)	(2.07)			(0.20)			
4.2	−0.18	−4.37	−0.29	1.58	0.13	—	.103	1.80
	(0.14)	(2.06)	(1.33)	(0.73)	(0.20)			
4.3	−0.02	−1.28	−0.24	1.49	0.15	4.05*	.281	1.71
	(0.25)	(2.23)	(1.28)	(0.70)	(0.18)	(5, 82)		
IE								
4.4	−0.15	−4.08	—	—	0.15	—	.032	1.68
	(0.14)	(2.80)			(0.20)			
4.5	−0.18	−4.53	−0.27	1.58	0.13	—	.085	1.80
	(0.15)	(2.81)	(1.35)	(0.74)	(0.20)			
4.6	0.13	3.77	−0.42	1.42	0.18	2.80*	.273	1.75
	(0.27)	(3.80)	(1.33)	(0.73)	(0.19)	(5, 82)		

NOTE: The $\Delta \ln \tilde{w}$ variable was instrumented using age, age^2, and all one-period lagged prices and wages.

Appendix are consistent with those of other researchers, and such figures have typically been cited as evidence in favor of life-cycle theory. However, Table IV tells us that the figures essentially illustrate a simple positive correlation between hours and wages *across* cohorts. As to behavior *within* cohorts, there is little or no evidence in favor of the theory. Indeed the positive relationship between wages and hours over the life cycle stands in contrast to the essentially negative relationship revealed by year-to-year changes.

7.3. *Labor Supply and Commodity Demands*

The previous subsection imposed the restriction that goods and male labor supply are additive within periods; we now relax the assumption. This allows us to see whether allowing for intratemporal substitution affects the previous negative conclusions concerning male labor supply and whether the behavior of consumption itself is in accord with the theory. Once again, we start from the results under certainty reported in Table V. These reveal a number of interesting relationships, though few would strengthen our beliefs in the life-cycle theory. In interpreting these results it should be borne in mind that, apart from minor variations in interview dates, both the consumption price p_t and the discount factor are the same for all cohorts, varying only from year to year. In consequence, regressions containing $\ln p_t$ cannot also contain all the year dummies, and regressions that contain $\ln \tilde{w}_t$, $\sqrt{(p_t/w_t)}$ and year dummies are identified only by functional form, that is essentially not identified.

Looking first at the hours results and comparing with Table III, note that the term $\sqrt{(p/w)}$ is not significantly different from zero in either (5.1) or (5.2) nor is there any evidence of the required positive intertemporal elasticity with respect to wages. The regressions (5.3) and (5.4), which contain the year dummies, tell

TABLE V

HOURS AND CONSUMPTION: ALL WORKERS: LEVELS

	Constant	$\ln \tilde{w}$	$\sqrt{p/w}$	*Hours*, mean = 43.6 a_1	a_2	*NMD*	c	y	R^2	d.w.
5.1	55.5	−0.62	−5.70	1.41	0.69	−4.43	—	—	.883	1.63
		(1.9)	(3.6)	(0.24)	(0.18)	(0.50)				
5.2	55.1	−1.08	−3.47	2.83	1.96	−3.60	2.75*		.903	1.77
		(1.9)	(3.6)	(0.67)	(0.41)	(0.52)	(7, 95)			
5.3	−66.5	13.6	17.4	1.52	0.57	−4.97	—	5.54*	.913	1.68
		(4.9)	(9.8)	(0.24)	(0.18)	(0.53)		(6, 96)		
5.4	−102	17.2	26.0	1.01	1.49	−4.52	2.92*	5.60*	.929	1.69
		(5.5)	(10.5)	(0.78)	(0.39)	(0.58)	(7, 89)	(6, 89)		

	Constant	$\ln \tilde{p}$	$\sqrt{w/p}$	*Real Expenditures*, mean = 53.3 a_1	a_2	*NMD*	c	R^2	d.w.
5.5	117	−10.0	—	—	—	11.6	—	.427	1.16
		(17)				(1.3)			
5.6	212	−23.6	—	−13.3	4.74	11.6	—	.781	1.28
		(11)		(1.3)	(0.66)	(0.82)			
5.7	156	−16.2	—	−6.01	7.68	12.5	5.62*	.845	1.62
		(9.9)		(3.4)	(1.8)	(0.82)	(7, 96)		
5.8	−133	14.9	94.2	—	—	−3.17	—	.580	1.32
		(15)	(15)			(2.7)			
5.9	−56.6	3.67	102	−14.5	0.40	−5.07	—	.867	1.90
		(8.9)	(13)	(0.99)	(0.74)	(2.2)			
5.10	−41.5	2.46	89.2	−12.3	1.61	−2.76	5.03*	.903	1.61
		(8.2)	(12)	(2.8)	(1.7)	(2.1)	(7, 95)		

a different and apparently more attractive story. There is a significant intertemporal substitution elasticity of around 0.4 (holding the current period ratio p/w constant), and there is a significant cross-price effect. By this, in periods when goods are relatively expensive relative to leisure (i.e., the real wage is low), hours (leisure) are relatively high (low); there is significant (specific) *complementarity* between leisure and goods. However, all this is not hard to disbelieve. As before, the presence of the year dummies absolves the economic variables from explaining the year to year variations in hours. Furthermore, the equation is close to being unidentified and the *net* elasticity with respect to wages, taking together the effects of $\ln \tilde{w}$ and $\sqrt{(p/w)}$ is only 0.007, in conformity with the previous results.

The lower part of Table V gives results for real consumption. Lines 5.5 through 5.7 detail the case where goods and hours are assumed separable. This is the least interesting model since prices, unlike wages, do not follow any pronounced life-cycle pattern. Even so price enters with a negative sign that is close to significance when the child variables are included. Either these or the cohort variables have a consistently important influence as is to be expected from the patterns illustrated in Figures 3 to 5. The largest (negative) coefficient (line 5.6) on $\ln p$ suggests an intertemporal elasticity for consumption of around one-half. However, note the persistent negative sign on a_1. This is not consistent with the notion that children carry with them certain age-specific needs. More likely is the alternative explanation, inconsistent with life cycle theory, that current *family income* should play some role in determining expenditures; a_2 is high when male income is at its peak and a_1 is high when female income is low or nonexistent so that their signs are consistent with income being a relevant omitted variable. Lines 5.8 through 5.10 extend the story. The $\sqrt{(w/p)}$ variable is highly significant and has a *positive* sign, so that, according to the consumption side of the picture, leisure and goods are *substitutes*, not complements as is suggested by the hours results. The introduction of $\sqrt{(w/p)}$ also renders the price term positive and insignificant. The effect of older children is also negated as we should expect if, as argued, a_2 is a proxy for male income which, in turn, is not unrelated to (w/p). Note that while year dummies cannot be included in regressions together with $\ln \tilde{p}$, it is possible to compare the performance of $\ln \tilde{p}$ with a complete set of dummies. Perhaps surprisingly, the year dummies are never jointly significant in these regressions and the specialization required to represent all year effects by the single price term cannot be rejected.

Finally in Table VI, the instrumented first-difference regressions are given for both hours and consumption. Although there are minor differences as compared with the levels, the overall pattern is the same. There is no coherent explanation for hours that works both for life and business cycles. Once year dummies are allowed however, some positive wage effects re-emerge even once intercohort variations have been differenced out, while hours again respond positively to the goods price. Once again, the identification of this equation is dubious. On the consumption side, the price effects are barely significant, but once again, the cross-effects operate in an exactly contrary manner to their operation in the hours equation. This lack of symmetry in both levels and differences is more than an

TABLE VI

HOURS AND CONSUMPTION: INTRA-COHORT FIRST-DIFFERENCES BY INSTRUMENTAL VARIABLES

	Constant	$\Delta \ln \tilde{w}$	$\Delta\sqrt{(p/w)}$	Hours Δa_1	Δa_2	NMD	y	R^2	d.w.
6.1	−0.15	−7.10	−5.76	−0.28	1.52	0.14	—	.096	1.76
		(3.8)	(5.7)	(1.3)	(0.74)	(0.2)			
6.2	−0.02	15.1	24.3	−0.53	1.31	0.12	3.86*	.301	1.73
		(8.0)	(15.2)	(1.3)	(0.71)	(0.2)	(5, 81)		

	Constant	$\Delta \ln \tilde{p}$	$\Delta\sqrt{(w/p)}$	Real Expenditures Δa_1	Δa_2	NMD		R^2	d.w.
6.3	2.42	−133	—	—	—	−0.18		.086	1.61
		(46)				(0.90)			
6.4	1.62	−101	—	−7.11	6.66	−0.23		.186	1.77
		(41)		(5.2)	(3.0)	(0.77)			
6.5	1.05	−62.5	25.0	−8.32	6.83	−0.16		.227	1.95
		(69)	(38)	(5.0)	(2.7)	(0.7)			

intellectual curiosum or an unimportant deficiency of the life-cycle story. The intertemporally additive models used here have quasi-homothetic preferences within periods so that Gorman [**19**] perfect price aggregation is possible. Hence there exists, for each period, a goods/leisure aggregate that has an intertemporal Frisch elasticity just as do its components, hours and goods. Taking lines 6.2 and 6.5 of Table VI to illustrate the point, the elasticities for hours and for goods are approximately 0.4 and 1.1 respectively. But we cannot talk about *the* intertemporal elasticity because, without symmetry, no aggregator function exists.

In defense of the theory, one final point should be allowed. If uncertainty is taken seriously (as it should be) only Table VI contains fully defensible results. These support our contention that the theory is inadequate, but the standard errors are inevitably large. Even with an initial data set containing nearly 50,000 observations, there is not sufficient information for a really convincing test.

CONCLUSIONS

In this paper we have developed the theory of life-cycle labor supply and commodity demands, making particular use of profit functions to represent intertemporally additive preferences. These profit functions are used as "potential functions" for the marginal utility compensated demand functions of Heckman and MaCurdy, here rechristened Frisch demand functions. We show how the profit and Frisch functions can be used to generate empirically tractable functional forms under both certainty and uncertainty, and we derive the general representation theorems for preferences that allow the price of utility to be treated as an additive fixed effect, as suggested originally by MaCurdy.

Our empirical results are based on seven years of the British Family Expenditure Survey aggregated in such a way as to produce what is effectively panel data on cohort means. Such data bridge the gap between micro and macro and allow a simultaneous analysis of year to year changes (the 'business' cycle) and of variations over life cycles. The British data certainly allow us to tell a coherent story of the life cycle; male wages are closely correlated with male hours and consumption peaks at the peak of real wages, as it should if leisure and goods are substitutes. While this broad sketch cannot be challenged, it does not bear up under closer scrutiny. In particular, the short-run variations in hours are not determined by the same factors as are long-term life-cycle variations; if the life-cycle model is correct for hours, it is only so in the long run and workers are somehow forced off their supply curves in the short run (for example, in an implicit contracts story; see Abowd and Card [**1**]). Secondly, the behavior of hours suggests that hours and goods are *substitutes*; this is inconsistent with the consumption story. Thirdly, the estimated consumption functions strongly suggest at least a partial role for household income; the presence of young children is associated with low not high consumption, and indeed the estimated complementarity of goods and hours is also attributable to alternative explanation in which income plays a role. These results seem to us to be consistent with an emerging consensus based on U.S. data from a wide variety of sources. MaCurdy [**37**], using the data from the Michigan panel study on income dynamics (PSID), comes to different conclusions, but our reading of his results suggests only rather weak evidence in favor of the model. Altonji's [**3**] study is based on the same data, and is a careful attempt to control for the undoubted presence of major errors of measurement. He finds (at best) small substitution elasticities with relatively wide confidence intervals. Ham [**26**], like us, finds significant evidence of labor market constraints. As Ashenfelter [**5**] points out, the "raw" data in the PSID is inconsistent with the model; regressions of changes in hours on changes in wages give persistently negative slopes (see also Ashenfelter and Ham [**4**]), while the ratios of mean change in hours to mean change in wages vary widely from year to year, and are as often negative as positive. Of course, sophisticated econometric methodology can "improve" these results, but the confirmation of the hypothesis is hardly transparent in the data. Aggregate time series tests fare no better; see Altonji [**2**], Hansen and Singleton [**27**], and Mankiw, Rotemberg, and Summers [**39**]; typically, estimated intertemporal elasticities have the "wrong" sign. Finally, and perhaps most convincing, are the *experimental* results from SIME/DIME negative income tax experiments quoted in Ashenfelter [**5**, Table 7]. Households "treated" with artificial guarantees and tax rates reduced their hours relative to controls, and those enrolled in the five-year program did so by more than those enrolled in the three-year program. This is consistent with the existence of life-cycle income effects as predicted by the theory. However, in both three and five year programs, there is no continuing evidence of hours reduction beyond the end of the treatment, contradicting the income effects explanation. It is far from clear what theory would explain this evidence, but it is certainly not the standard life-cycle one. All in all, we believe that these studies, together with the evidence of this paper from Britain, cast a great deal of doubt on the simple life-cycle model that is examined in this paper.

McMaster University,
Princeton University
and
University of Bristol

Manuscript received April, 1983; final revision received October, 1984.

APPENDIX

THE DATA

The tables below present the cohort averages for each variable for manual and then for nonmanual men.

Definitions

h_1: Normal weekly hours, FES code A220.

w_1: Normal net weekly wage/salary divided by normal weekly hours all multiplied by the discount factor, δ.

δ: The inverse of the product of monthly yields on consols, i.e.,

$$\delta_t = \left\{ \prod_{i=1}^{t-1} (1 + r_i/12) \right\}^{-1}$$

where r_i is the annual yields on consols in month i.

a_1: Number of children aged under 5 in households, codes A040 and A4041.

a_2: Number of children aged 5-17 (to 1972, 5-15 from 1973), code A042.

age: Age of head of household, code A005.

APPENDIX TABLE I

Manual Males

			Average weekly hours				Table A1	
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	46.4	47.0	46.9	46.5	46.7	46.0	44.9
71-2	—	46.3	45.8	46.4	45.1	45.0	43.9	43.9
72-3	45.8	46.0	47.0	45.9	45.0	44.6	44.8	44.6
73-4	45.9	46.2	45.8	46.4	45.2	45.0	44.6	43.7
74-5	46.0	47.5	47.5	46.3	46.7	45.0	44.8	44.4
75-6	45.6	46.9	46.2	44.9	45.4	44.0	44.0	42.9
76-7	45.4	46.2	46.1	45.4	44.9	44.9	43.8	43.3

			Average ln (discounted wages)				Table A2	
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	6.46	6.49	6.51	6.48	6.47	6.41	6.37
71-2	—	6.47	6.51	6.51	6.51	6.46	6.43	6.38
72-3	6.47	6.54	6.57	6.56	6.55	6.50	6.47	6.38
73-4	6.51	6.55	6.58	6.58	6.55	6.52	6.47	6.43
74-5	6.55	6.57	6.60	6.61	6.56	6.50	6.48	6.41
75-6	6.56	6.64	6.66	6.66	6.59	6.57	6.50	6.46
76-7	6.55	6.62	6.61	6.61	6.56	6.53	6.50	6.42

			Average numbers of small children				Table A3	
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	.99	.87	.50	.33	.15	.07	.02
71-2	—	1.00	.81	.48	.23	.14	.07	.02
72-3	.85	.99	.74	.37	.27	.07	.04	.01
73-4	.88	.95	.57	.38	.19	.07	.04	.02
74-5	.89	.85	.51	.24	.11	.07	.03	.00
75-6	.79	.74	.45	.17	.08	.07	.03	.01
76-7	.85	.64	.28	.15	.05	.05	.02	.00

			Average numbers of older children				Table A4	
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	.32	1.07	1.83	1.65	1.08	.65	.28
71-2	—	.49	1.22	1.85	1.61	1.05	.56	.19
72-3	.08	.62	1.46	1.81	1.50	.92	.54	.17
73-4	.10	.79	1.54	2.17	1.52	1.03	.53	.20
74-5	.22	.88	1.56	2.01	1.41	.78	.34	.16
75-6	.25	1.17	1.81	1.90	1.36	.79	.32	.21
76-7	.38	1.28	1.89	2.00	1.11	.61	.29	.14

Average age — Table A5

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	26.0	31.0	36.1	40.9	45.9	50.8	56.1
71–2	—	27.0	32.0	36.9	41.9	47.1	51.7	57.0
72–3	23.3	28.0	33.1	37.9	43.0	48.2	52.6	58.0
73–4	24.2	29.0	34.0	38.8	43.9	49.1	53.7	59.1
74–5	24.9	30.1	35.1	40.0	45.0	50.0	54.9	59.9
75–6	25.8	30.9	36.0	41.0	45.8	51.0	55.9	61.1
76–7	26.6	32.0	37.0	41.9	47.0	51.7	56.6	61.9

Average Real Expenditure — Table A6

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	37.9	40.6	42.1	45.8	51.4	49.9	43.1
71–2	—	38.1	43.0	46.6	47.7	50.2	49.7	42.4
72–3	39.1	40.9	44.0	47.1	51.1	54.8	55.0	45.7
73–4	41.3	44.8	47.8	51.0	56.5	55.7	54.4	43.5
74–5	43.5	44.3	47.0	54.4	57.6	54.2	52.2	43.2
75–6	38.9	44.7	48.4	53.8	53.8	54.6	47.9	41.8
76–7	39.7	44.7	46.9	58.1	54.8	52.3	48.0	37.4

Average $(\text{Price}/\text{Wage})^{1/2}$ — Table A7

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	1.28	1.26	1.25	1.26	1.27	1.31	1.34
71–2	—	1.26	1.24	1.25	1.24	1.28	1.29	1.33
72–3	1.26	1.21	1.20	1.20	1.20	1.22	1.25	1.31
73–4	1.23	1.20	1.18	1.19	1.20	1.22	1.26	1.28
74–5	1.22	1.21	1.19	1.18	1.21	1.25	1.26	1.31
75–6	1.25	1.21	1.19	1.20	1.24	1.25	1.29	1.32
76–7	1.27	1.22	1.23	1.23	1.25	1.28	1.29	1.35

APPENDIX TABLE II

NONMANUAL MALES

Average weekly hours — Table A8

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	41.0	41.8	42.2	43.0	41.3	42.3	42.8
71–2	—	40.3	42.1	42.2	42.4	41.3	42.1	41.0
72–3	40.2	41.4	43.1	41.7	43.2	41.9	41.7	42.1
73–4	39.7	42.4	42.2	41.9	41.9	42.3	42.4	40.9
74–5	40.2	42.1	42.8	42.5	42.7	42.3	39.9	39.3
75–6	39.4	42.0	42.3	42.0	41.7	41.1	41.7	40.4
76–7	41.8	42.7	44.3	41.4	41.0	41.8	40.8	38.1

Average ln discounted wages — Table A9

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	6.58	6.84	6.86	6.89	6.87	6.81	6.74
71–2	—	6.73	6.81	6.96	6.88	6.95	6.86	6.90
72–3	6.60	6.75	6.86	6.97	6.91	6.88	6.84	6.72
73–4	6.62	6.83	6.90	6.98	6.94	6.93	6.90	6.83
74–5	6.67	6.86	6.90	6.91	6.98	6.87	6.83	6.80
75–6	6.73	6.89	7.00	6.98	6.97	6.87	6.79	6.76
76–7	6.73	6.86	6.90	6.93	6.90	6.85	6.76	6.73

Table A10. Average numbers of small children

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	.61	.87	.75	.28	.14	.03	.06
71–2	—	.73	.79	.39	.21	.08	.03	.01
72–3	.29	.69	.76	.29	.13	.07	.01	.00
73–4	.37	.81	.77	.35	.17	.05	.02	.02
74–5	.52	.71	.69	.25	.13	.04	.01	.01
75–6	.60	.85	.58	.23	.07	.01	.02	.03
76–7	.53	.57	.32	.13	.06	.04	.00	.00

Table A11. Average numbers of older children

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	.13	.51	1.38	1.58	.93	.53	.30
71–2	—	.10	.68	1.58	1.51	.95	.37	.07
72–3	.01	.24	.93	1.58	1.54	1.02	.37	.18
73–4	.04	.35	1.16	1.80	1.46	1.03	.38	.19
74–5	.18	.53	1.15	1.98	1.60	.93	.40	.14
75–6	.12	.62	1.41	1.75	1.47	.67	.35	.13
76–7	.13	.88	1.60	1.88	1.23	.63	.28	.06

Table A12. Average age

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	26.2	30.9	36.2	41.2	46.2	50.8	56.0
71–2	—	26.9	31.9	36.8	42.0	47.1	51.7	56.9
72–3	23.6	28.1	33.0	38.0	43.0	47.8	52.5	57.8
73–4	24.5	29.0	33.9	39.0	44.0	49.0	53.7	58.6
74–5	25.3	30.0	35.0	39.9	45.1	50.1	54.9	59.7
75–6	26.2	30.9	36.3	41.1	45.8	51.1	55.6	60.8
76–7	27.0	32.0	36.9	42.1	47.0	52.1	56.7	61.9

Table A13. Average Real Expenditure

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	41.4	50.2	51.3	59.0	61.2	64.9	62.9
71–2	—	43.9	49.6	59.6	64.2	62.6	71.1	61.1
72–3	51.7	49.5	57.1	60.4	71.5	66.1	64.5	64.9
73–4	45.2	54.6	63.5	60.8	64.1	69.2	66.4	58.2
74–5	45.4	53.5	56.8	62.0	76.2	69.5	63.9	61.6
75–6	46.1	50.2	57.1	61.5	70.8	63.5	62.4	54.9
76–7	46.4	56.1	60.4	60.8	66.7	61.2	55.3	57.1

Table A14. Average $(\text{Price/Wage})^{1/2}$

Year/Cohort	1	2	3	4	5	6	7	8
70–1	—	1.16	1.06	1.05	1.04	1.05	1.09	1.12
71–2	—	1.11	1.08	1.00	1.04	1.01	1.06	1.04
72–3	1.17	1.09	1.04	0.99	1.02	1.04	1.05	1.13
73–4	1.16	1.05	1.03	0.98	1.00	1.02	1.03	1.06
74–5	1.15	1.05	1.03	1.03	0.99	1.05	1.08	1.09
75–6	1.13	1.07	1.02	1.02	1.04	1.09	1.13	1.15
76–7	1.16	1.09	1.07	1.06	1.07	1.11	1.15	1.17

REFERENCES

[1] ABOWD, J. M., AND D. E. CARD: "Intertemporal Substitution in the Presence of Long-term Contracts," Industrial Relations Section, Princeton University, Working Paper # 166, 1983.

[2] ALTONJI, J. G.: "The Intertemporal Substitution Model of Labour Market Fluctuations: An Empirical Analysis," *Review of Economic Studies*, 49(1982), 783–824.

[3] ———: "Intertemporal Substitution in Labor Supply: Evidence from Micro Data," *Journal of Political Economy*, forthcoming.

[4] ASHENFELTER, O., AND J. HAM: "Education, Unemployment, and Earnings," *Journal of Political Economy*, 87(1979), S99-116.

[5] ———: "Macroeconomic Analyses and Microeconomic Analyses of Labor Supply," presented to Carnegie-Rochester Conference, Bal Harbor, Florida, 1983.

[6] ATKINSON, A. B., AND N. STERN: "On Labour Supply and Commodity Demands," in *Essays in the Theory and Measurement of Consumer Behaviour*, ed. by A. S. Deaton. New York: Cambridge University Press, 1981.

[7] BARTEN, A. P.: "Consumer Demand Functions Under Conditions of Almost Additive Preferences," *Econometrica*, 32(1964), 1-38.

[8] ———: "Family Composition, Prices and Expenditure Patterns," in *Econometric Analysis for National Economic Planning*, ed. by P. E. Hart, G. Mills, and J. K. Whitaker. London: Butterworth, 1964.

[9] BEWLEY, T.: "The Permanent Income Hypothesis: a Theoretical Formulation," *Journal of Economic Theory*, 16(1977), 252-292.

[10] BROWNING, M. J.: "Profit Function Representations for Consumer Preferences," Bristol University Discussion Paper No. 82/125, 1982.

[11] DEATON, A. S.: "Theoretical and Empirical Approaches to Consumer Demand under Rationing," in *Essays in the Theory and Measurement of Consumer Behaviour*, ed. by A. S. Deaton. New York: Cambridge University Press, 1981.

[12] ———: "Model Selection Procedures, or Does the Consumption Function Exist?" in *The Reliability of Macroeconomic Models*, ed. by P. Corsi and G. Chow. New York: Wiley, 1982.

[13] ———: "Panel Data from Time Series of Cross-Sections," *Journal of Econometrics*, 1985, forthcoming.

[14] DEATON, A. S., AND J. MUELLBAUER: *Economics and Consumer Behavior*. New York: Cambridge University Press, 1980.

[15] EPSTEIN, L.: "A Disaggregate Analysis of Consumer Choice Under Uncertainty," *Econometrica*, 43(1975), 877-892.

[16] FRISCH, R.: *New Methods of Measuring Marginal Utility*. Tubigen: J. C. B. Mohr, 1932.

[17] ———: "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors," *Econometrica*, 27(1959), 177-196.

[18] GHEZ, G. R., AND G. S. BECKER: *The Allocation of Time and Goods over the Life Cycle*. New York: NBER, 1975.

[19] GORMAN, W. M.: "Separable Utility and Aggregation," *Econometrica*, 27(1959), 469-481.

[20] ———: "The Structure of Utility Functions," *Review of Economic Studies*, 35(1968), 367-390.

[21] ———: "Quasi Separable Preferences, Costs and Technologies," mimeo, University of North Carolina, Chapel Hill, 1970.

[22] ———: "Tricks with Utility Functions," in *Essays in Economic Analysis*, ed. by M. Artis and R. Nobay. Cambridge: Cambridge University Press, 1976.

[23] ———: "Facing an Uncertain Future," I.M.S.S. Technical Report 359, Stanford University, 1982.

[24] GREENHALGH, C., AND K. MAYHEW: "Labour Supply in Great Britain: Theory and Evidence," in *The Economics of the Labour Market*, ed. by Z. Hornstein, J. Grice, and A. Webb. London: HMSO, 1981.

[25] HALL, R. E.: "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86(1978), 971-987.

[26] HAM, J.: "Testing for the Absence of Constraints in a Life-Cycle Model of Labor Supply," mimeo, Institute for Policy Analysis, University of Toronto, 1983.

[27] HANSEN, L. P., AND K. J. SINGLETON: "Generalized Instrumental Variables Estimation of Non-Linear Rational Expectations Models," *Econometrica*, 50(1982), 1269-1286.

[28] HECKMAN, J. J.: "Three Essays on the Supply of Labor and the Demand for Market Goods," Ph.D. Dissertation, Princeton University, 1971.

[29] HECKMAN, J. J.: "Life Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption over the Life Cycle," *American Economic Review*, 64(1974), 188-194.

[30] HECKMAN, J. J.: "A Life-Cycle Model of Earnings, Learning, and Consumption," *Journal of Political Economy*, 84(1976), S11-44.

[31] ———: "A Partial Survey of Recent Research on the Labor Supply of Women," *American Economic Review*, 68(1978), 200-207.

[32] HECKMAN, J. J., AND T. MACURDY: "A Life-Cycle Model of Female Labor Supply," *Review of Economic Studies*, 47(1980), 47-74.

[33] HOUTHAKKER, H. S.: "Additive Preferences," *Econometrica*, 28(1960), 244-256.

[34] KEMSLEY, W. F. F.: "Family Expenditure Survey: a Study of Differential Response Based on a Comparison of the 1971 Sample with the Census," *Statistical News*, 31(1975), 16-21.

[35] KEMSLEY, W. F. F., R. V. REDPATH, AND M. HOLMES: *Family Expenditure Survey Handbook*. London: H.M.S.O., 1980.

[36] LAU, L. J.: "Applications of Profit Functions," in *Production Economics: a Dual Approach to Theory and Applications*, ed. by M. Fuss and D. McFadden. Amsterdam: North-Holland, 1978.

[37] MACURDY, T. E.: "An Empirical Model of Labor Supply in a Life-Cycle Setting," *Journal of Political Economy*, 89(1981), 1059-1085.

[38] MCFADDEN, D.: "Cost, Revenue and Profit Functions," in *Production Economics: a Dual Approach to Theory and Applications*, ed. by M. Fuss and D. McFadden. Amsterdam: North-Holland, 1978.

[39] MANKIW, N. G., J. J. ROTEMBERG, AND L. H. SUMMERS: "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics*, forthcoming.

[40] MINCER, J.: "Labour Force Participation of Married Women: A Study of Labour Supply," in *Aspects of Labor Economics*, ed. by H. G. Lewis. Princeton: Princeton University Press, 1962.

[41] NEARY, J. P., AND K. W. S. ROBERTS: "The Theory of Household Behaviour Under Rationing," *European Economic Review*, 13(1980), 25-42.

[42] PENCAVEL, J.: "Labor Supply of Men: A Survey," in *Handbook of Labor Economics*, ed. by O. Ashenfelter and R. Layard. Amsterdam: North-Holland, forthcoming.

[43] POLLAK, R. A., AND T. J. WALES: "Demographic Variables in Demand Analysis," *Econometrica*, 49(1981), 1533-1551.

[44] ROCKAFELLER, N.: *Convex Analysis*. Princeton: Princeton University Press, 1970.

[45] SMITH, J.: "Family Labor Supply Over the Life Cycle," *Explorations in Research*, 4(1977), 205-276.

[46] YAARI, M.: "A Note on Separability and Quasiconcavity," *Econometrica*, 45(1977), 1183-1186.