A Data Construction Details

A.1 Knowledge Markets

Rescaling Inventor Flows As explained in the main text, the measure of inventor flows aims to capture the strength of the connections between two sectors. I take several steps to ensure that I do not overestimate these connections and to normalize them to account for the size of sending and receiving sectors.

As a first step, I build normalized directed flows for each inventor i in order to avoid double counting. For example, for transitions between sector 1 and 2, I define:

$$\text{flow}_{1\to 2,i,t} \equiv \frac{\sum \mathbf{1}\{i \text{ moves } 1 \to 2 \text{ in } t\}}{\sum_{j,k} \mathbf{1}\left\{i \text{ moves } j \to k \text{ in } t\right\}} \times \alpha_i.$$

This measure attributes a fraction of the effective inventor fixed effect α_i to each transition in proportion to the number of overall inventor i's transitions across sectors in each year. The first term in this formula is precisely the share of transitions from sector 1 to sector 2 relative to overall transitions between any two sectors j and k that inventor i took part in.

Second, I compute total inflows (outflows) for each NAICS 4-digit sector, summing over all years, inventors and origin (destination) sectors. For example, inflows for sector 1are defined as:

$$\text{inflow}_1 = \sum_n \sum_t \sum_i \text{flow}_{n\to 1,i,t},$$

where n denotes origin NAICS sectors, t years, and i inventor identifiers.

Third, I proceed to compute the share of directed flows between each pair of sector as a share of total inflows or outflows. For example, the share of inflows coming from sector 2 and entering sector 1 is defined as:

$$\text{share}_{1\leftarrow 2} = \frac{\sum_t \sum_i \text{flow}_{2\to 1,i,t}}{\text{inflow}_1}.$$

In this example, this measure captures the relative importance of inflows from sector 2 for the overall number of inventors received by sector 1. However, this measure can still overstate flows from large to small sectors, or vice versa. As a result, and since I need undirected flows to apply the Louvain algorithm, I define network edge weights starting from an average of the above shares of inflows and outflows for each sector and taking the minimum between the two measures as follows:

$$W_{12} = W_{21} = \min\left\{\frac{\text{share}_{1\leftarrow 2} + \text{share}_{1\to 2}}{2}, \frac{\text{share}_{1\leftarrow 2} + \text{share}_{1\to 2}}{2}\right\},$$

where $W_{12} = W_{21}$ since the final network is undirected.

Modularity Maximization Formula and Algorithm In order to identify knowledge markets from the network constructed above, I employ the Louvain community detection algorithm (**?**). This algorithm maximizes the modularity of the network, Q, assigning each sector i to one of N *non-overlapping* communities c. Accordingly, the objective function for this problem is given by:

$$\max_{N} \max_{(c_1,\ldots,c_N)} Q \equiv \frac{1}{2\boldsymbol{W}} \sum_{ij} \left[W_{ij} - \frac{\boldsymbol{W_i} \boldsymbol{W_j}}{2\boldsymbol{W}} \right] \mathbf{1}\left\{c_i = c_j\right\},$$

where W_{ij}, weight of the edge connecting node i to j, and bold variables denote other summations for ease of notation. In particular, I define $\boldsymbol{W_i} \equiv \sum_k \sum_i W_{ik}$, as the sum of weights for edges with one end in node i, and the sum of all weights in the network, respectively. The indicator $\mathbf{1}\left\{c_i = c_j\right\}$ takes a value of 1 when nodes i and j belong to the same community. Note that the maximization is carried out both over the number of communities and the assignment of nodes to each community. This measure can be interpreted considering that $\frac{\boldsymbol{W_i} \boldsymbol{W_j}}{2\boldsymbol{W}}$, is the expected number of edges that arise between nodes i and j in a random network. Therefore, modularity maximizes the distance between the density of linkages within communities W_{ij} relative to the overall density of links that would arise randomly.

Since looping over all the permutations of nodes and community is numerically unfeasible, the Louvain algorithm follows an iterative procedure to maximize modularity. First, it assigns each node to its own community. Then, it repeats iteratively the following three steps:

1. Compute local deviations in modularity from reassigning the node to neighboring communities;
2. Assign nodes to communities following the local improvement granting the highest modularity increase;
3. Redefine a network with new communities as nodes.

These steps are repeated until there is no significant improvement in modularity for further steps.

B Additional Results and Robustness

B.1 Results on Overall Inventor Shares

Table 1 reports the effect of concentration increases on the share of inventors across all knowledge markets. While the correlation is positive and significant when some outliers are removed, this relation is not robust to the inclusion of all observations or the the alternative trimming procedure provided by the Mahalanobis distance. This result is unsurprising in light fo two points discussed in the main text. First, as highlighted in Section **??**, if ordinary flows of inventors across unrelated sectors are small or absent, we should not expect any effect of changes in these sectors' characteristics on the distribution of inventors. Second, the findings reported in Table **??** suggest that cross-knowledge-market flows are not significant, as apparent from a comparison of specifications with and without knowledge-market fixed effects. The results presented in this section therefore speak to the importance of accurately delineating labor markets for inventors when assessing their flows across product markets.

Table 1: Regressions of Change in Total Inventors' Share over Change in HHI Lower Bound, Long-Difference, 1997-2012

	Δ Total Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\underline{\text{HHI}}$	0.297	1.692	1.328*	1.532*	0.271	1.889
	(2.007)	(1.956)	(0.649)	(0.696)	(2.038)	(2.023)
Δ log Sales	0.460	0.436	0.133**	0.109*	0.464	0.472
	(0.281)	(0.292)	(0.047)	(0.047)	(0.283)	(0.312)
Knowledge Market FE		✓		✓		✓
Sample	Full	Full	Trimmed	Trimmed	Mahalanobis	Mahalanobis
Weight	Sales	Sales	Sales	Sales	Sales	Sales
Observations	157	153	147	143	150	139

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p < 0.1,^{*}\ p < 0.05,^{**}\ p < .01,^{***}\ p < .001\right)$; Checkmarks indicate the inclusion of fixed effects. Please refer to notes in Table **??** for further details.

B.2 Using the Raw Number of Inventors instead of Fixed-Effects

This Appendix reports the results for the main analysis presented in Section **??** using the raw number of total inventors instead of the fixed effects from regression (**??**), which might be inconsistently estimated. The following Table, to be compared with Table **??** in the main text, shows that the results are qualitatively unchanged, although coefficients are larger, and estimates less precise. This is easily explained by the fact that differences in research requirements across patent classes, firms and years are not absorbed as in the effective inventor measure, making the latter noisier. As In the main text, trimming the sample and introducing the sector and firm size controls does not affect estimates significantly (tables available on request).

Table 2: Regressions of Change in 4-digit Knowledge Market Share of Total Inventors over Change in HHI Measures, Long-Differences, 1997-2012

	Δ Inventor Share (pp)					
	(1)	(2)	(3)	(4)	(5)	(6
$\Delta\underline{\text{HHI}}$	74.172+		73.706+		74.177+	
	(40.957)		(41.600)		(41.047)	
Δ HHI		71.749**		71.997**		71.58
		(24.464)		(25.060)		(24.4
Knowledge Market FE						
Sample	Full Sample	Full Sample	Trim Outliers	Trim Outliers	Mahalanobis 5%	Mahalan
Weight	Sales	Sales	Sales	Sales	Sales	Sale
Observations	157	80	155	79	150	72

Note: See notes to Table **??**.

B.3 Using a Quartic in Sales as Size Control

This Section displays the results of estimating the specification in Table **??** using the changes in the terms of a fourth-degree polynomial in sales rather than log-sales. This flexible control specification ensures that my main findings do not rely on the specific functional form that I assumed above. Table 3 reports the result of this exercise using both effective inventors (Columns (1) and (2)) and raw inventor counts (Columns (3) and (4)) to compute sector shares. Recall that when using raw inventor counts, knowledge markets are also constructed according to this measure. As clear from a comparison of Columns (1) with (2), and (3) with (4), these two specifications produce statistically undistinguishable results.

B.4 Self-Citation Regressions

In this section, I investigate a competing explanation for my findings on output growth. As highlighted by **?** and **?** among others, large incumbents have a strong incentive to focus on improving their own products at the expense of broadly applicable innovation. This mechanism would also imply that an increase in incumbents' share of R&D resources leads to falling innovation productivity. In order to assess the importance of this channel, and in keeping with the analysis in **?**, I use the share of self-citations to measure the extent of internal innovation conducted by firms. Table 4 displays the results pertaining to this measure. All columns use as dependent variable the change in excess log self-citations as defined in Section **??**. Columns (1) and (2) build excess self-citations correcting for the importance of firms' patents for the CPC group, which reflects the technological classification of the patent. Columns (3) and (4) use the more narrowly defined CPC subgroups for robustness. Coefficients are mostly non-significant and turn negative when knowledge market fixed effects are included. Column (3) displays a marginally significant coefficient. However, this result is not robust to using the HHI as regressor and weighting regressions by sales as in the baseline specification. The findings in this table suggest that incremental innovation does not drive my results.

Table 3: Regressions of Change in 4-digit Knowledge Market Share of Inventors over Change in HHI Lower Bound, Long-Differences, 1997-2012

	Δ Inventor Share (pp)			
	(1)	(2)	(3)	(4)
$\Delta \underline{\text{HHI}}$	22.509*	24.083*	67.160+	74.769+
	(10.848)	(10.565)	(37.176)	(39.225)
Δ log Sales	0.548*		1.422*	
	(0.243)		(0.717)	
Δ Sales ($ bn)		2.617*		6.382+
		(1.108)		(3.365)
ΔSales^2		-0.749		-1.749
		(0.482)		(1.468)
ΔSales^3		0.081		0.165
		(0.076)		(0.232)
ΔSales^4		-0.003		-0.005
		(0.003)		(0.009)
4D Knowledge Market FE	✓	✓	✓	✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample
Weight	Sales	Sales	Sales	Sales
Observations	153	153	156	156

Note: Regressions weighted by sales in 2012; Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Checkmarks indicate the inclusion of fixed effects. This Tables presents the results of specifications (**??**), when the outcome is the share of effective inventors of sector p over total inventors in knowledge market k, and the independent variable is the change in the lower bound of the Herfindal-Hirschman Index for product market p, as implied by Census concentration ratios. "Full Sample" refers to the sample described in the main text.

B.5 Using the Lerner Index instead of the HHI

Following **?**, I build the Lerner Index from NBER-CES data for the period 1997-2012 as the ratio:

$$\text{Lerner}_{jt} = \frac{\text{vship}_{jt} - \text{pay}_{jt} - \text{matcost}_{jt} - \text{energy}_{jt}}{\text{vship}_{jt}}, \tag{1}$$

where "vship" is the total value of shipments, "pay" denotes total payrolls, "matcost" and "energy" material and energy costs, respectively, and j denotes a 6- or 4-digit NAICS sector. I build two alternative measures, one using 6-digit NAICS sectors, the original identifier in NBER-CES, and then averaging by sales at the level of 4-digit NAICS, or first aggregating the revenue and cost statistics at the level of 4-digit NAICS. Table 5, shows that the Lerner Index thus constructed is strongly correlated with the HHI measure used in the main analysis. However, the correlation is far from perfect, as suggested by the R^2, suggesting that this estimate of the Lerner Index might be excessively imprecise. Indeed, Table 6 shows that, when using this measure instead of the HHI in the main analysis, the coefficients for the regression of inventors' shares on changes in concentration stay positive, but become smaller and noisier. This suggests the potential presence of attenuation bias, a valid concern due to the fact that the above measure, not based on any structural estimation, can only imperfectly capture markups.

Table 4: Regressions of Change in Excess Self-Citations over 4-digit Knowledge Market Share, Long-Differences, 1997-2012

	Δ self-citations			
	(1)	(2)	(3)	(4)
Δ Inventor Share (pp)	0.920	-0.444	0.958+	-0.228
	(0.711)	(1.083)	(0.512)	(0.801)
Δ log Sales	-1.841	-1.954	-1.456	-1.674
	(1.925)	(1.988)	(1.326)	(1.279)
Knowledge Market FE		✓		✓
Sample	Full Sample	Full Sample	Full Sample	Full Sample
Observations	157	153	157	153

Note: Unweighted regressions; robust standard errors in parentheses; symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; checkmarks indicate the inclusion of fixed effects. This table presents the results of specifications (**??**), regressing the change in excess self-citations in sector p over the change in the share of inventors.

Note that this is also due to the fact that the Lerner Index is available only for the manufacturing sectors, which make up about 60% of the sample, so its use lead to dropping a substantial amount of observations. When using fitted values from the regression in Table 5 to extend the measure to more sectors, as well as reducing the volatility of the series for available sectors, the coefficients recover magnitudes and significance close to the baseline presented in **??**.

Table 5: Regressions of Changes in the Lerner Index over Changes in the HHI Lower Bound, Long-Difference, 1997-2012

	Markup Change 1997-2012, 6d Lerner Index	Δ Lerner Index
	(1)	(2)
$\Delta \underline{HHI}$	1.490***	1.652***
	(0.229)	(0.257)
Observations	258	258
R-squared	.1424476	.14

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$. "6d Lerner Index" refers to the Lerner Index constructed as in (1)on NAICS 6-digits averaged at the 4-digit NAICS level weighting by the value of shipments; "4d Lerner Index" is computed using 4-digit aggregates for the value of shipments, payroll and costs, summing over the NAICS 6-digit composing each sector.

Table 6: Regressions of Changes in Inventors' Share over Changes in Actual and Fitted Lerner Index, Long-Difference, 1997-2012

	Δ Inventor Share (pp)	
	(1)	(2)
Δ Lerner	0.556	
	(5.465)	
Δ Lerner (Fitted)		26.736*
		(13.363)
Knowledge Market FE		
Sample	Full Sample	Full Sample
Weight	Sales	Sales
Observations	81	157

Note: Robust standard errors in parentheses; Symbols denote significance levels $\left(+\ p<0.1,^{*}\ p<0.05,^{**}\ p<.01,^{***}\ p<.001\right)$; Observations weighted by sales. The markup change 1997-2012 is the long-difference of the Lerner Index described above. "Fitted Lerner change" is the fitted value for the Lerner index based on the estimates in 5, and extended to all available sectors in the main sample.

C Omitted Proofs and Derivations

C.1 One-sector model

Proof of Proposition **??**. This proof consists of several parts. First, I show that given labor supplies, output, values and wages grow at the same constant rate, so the problem can be solved in a steady state of a normalized model. Second, I show that normalized values, $v(\Omega) \equiv V_t(\Omega)/Y_t$, are uniquely determined, which gives unique research intensities and stationary distribution. Third, I derive the stationary distribution and the expression for growth and inventors' productivity. In what follows I suppress stars to denote equilibrium quantities for ease of notation.

Given an endowment, L, production labor market clearing in each period requires:

$$\int_0^1 l_{i,t}(w)d(i) = L.$$

That is,

$$L = \int_0^1 \frac{c_{i,t}}{\phi} y_{i,t}(w_t)\, d(i) = \frac{1}{\phi}\frac{Y_t}{w_t},$$

where the second equality comes from using the demand for output of product i for $y_{i,t}(w_t)$. This expression immediately implies that if Y_t grows at a constant rate, so does w_t. Labor market clearing for R&D workers reads:

$$L^{RD} = \zeta\omega x_{e,\omega}\left(\mu_{e,\omega} + \mu_{e,1}\right) + \alpha_I \frac{(x_I)^\gamma}{\gamma}\mu_1.$$

In a constant growth equilibrium (CGE), the distribution is stationary, and since the left hand side is constant, research intensities are also fixed. A contradiction arises otherwise, since the distribution is stationary only if research intensities are fixed by the LOM (29)-(32). Further, R&D labor cannot grow since the growth rate in the economy increases in total R&D labor for any given distribution, as it will be clear below. The fact that research intensities are constant immediately implies, from the optimality of $x_{e,\omega}$, that $V_t(1)$ and w_t^{RD} grow at the same rate. Indeed, from the FOC for entrants' research:

$$0 = \mathrm{d}\log x_{e,\omega,t} = \mathrm{d}\log V_t(1) - \mathrm{d}\log w_t^{RD}.$$

This result in turn implies, combined with the FOC for x_I, that $V_t(\omega)$ also grows at the same constant rate. Now consider the budget constraint of the representative household, combined with product market clearing, $Y_t = C_t$:

$$r_t A_t - \dot{A}_t + w_t^{RD} L^{RD} + w_t L = Y_t,$$

where A_t denote the household's assets, that is all firms in the economy. Therefore the above reads:

$$r_t \left(\mu_1 V_t(1) + \mu_\omega V_t(\omega)\right) - \mu_1 \dot{V}_t(1) - \mu_\omega \dot{V}_t(\omega) + w_t^{RD} L^{RD} + w_t L = Y_t$$

Dividing both sides by $V(1)$, using the Euler equation and rearranging we obtain:

$$\left(g+\rho\right)\left(\mu_1 + \mu_\omega \frac{V_t(\omega)}{V_t(1)}\right) - \mu_1 g_{V_1} - \mu_\omega \frac{V_t(\omega)}{V_t(1)} g_{V_1} + \frac{w_t^{RD}}{V_t(1)} L^{RD} = \frac{Y_t}{V_t(1)} - \frac{w_t}{V_t(1)} L.$$

By what shown above, all terms on the left hand side are constant in t, since research wages and values grows at the same rate and the distribution is stationary. Since Y_t and w_t grow at the same rate positive rate, it must be that $V_t(1)$ also grows at the same rate as Y_t. This proves that $g_{V_1} = g = g_c = g_w = g_{w^{RD}}$.

As a result, in a CGE, it is possible to define normalized constant values, $v(\Omega) \equiv V_t(\Omega)/Y_t$. The system of equations defining the recursive problem in this equilibrium reads:

$$\rho v(1) = \max_{x_I} \left\{ \left(\frac{\phi-1}{\phi}\right) - \alpha_I \frac{x_I^\gamma}{\gamma} w^{RD} + x_I \left(v(\omega) - v(1)\right) - x_{e,1} v(1) \right\}, \tag{2}$$

$$\rho v(\omega) = \left(\frac{\phi-1}{\phi}\right) + \delta \left(v(1) - v(\omega)\right) - x_{e,\omega} v(\omega), \tag{3}$$

where the left hand side comes from using the Euler equation:

$$r = g + \rho$$

Which gives

$$r \frac{V_t(\Omega)}{Y_t} - \frac{\dot{V}_t(\Omega)}{Y_t} \frac{Y_t}{\dot{Y}_t} \frac{\dot{Y}_t}{V_t(\Omega)} \frac{V_t(\Omega)}{Y_t} = \left(\rho + g\right) v(\Omega) - g v(\Omega) = \rho v(\Omega).$$

I now move to show that normalized values (2) and (3) are uniquely determined. Given entrants' decisions, and a wage rate w^{RD}, the incumbent's choice of R&D satisfies:

$$x_I = \mathbf{1}\{v(\omega) - v(1) > 0\} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma-1}}.$$

Entrants taking x_I as given optimally set:

$$x_{e,1} = \mathbf{1}\{v(1) > 0\} \frac{v(1)}{\zeta w^{RD}}, \quad x_{e,\omega} = \mathbf{1}\{v(1) > 0\} \frac{v(1)}{\zeta \omega w^{RD}}.$$

Note that these solutions immediately imply that the normalized value, $v(1)$, is strictly positive. Indeed, $v(1) < 0$ would imply:

$$\rho v(1) = \pi + \mathbf{1}\{v(\omega) - v(1) > 0\} \left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma-1}} \right) (v(\omega) - v(1))$$

where the right hand side is strictly positive. Plugging optimal solutions into the system of equations determining the value functions (25) and (26) gives:

$$\rho v(1) - \pi - \mathbf{1}\{v(\omega) - v(1) > 0\} \left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma-1}} \right) (v(\omega) - v(1)) + \frac{v(1)^2}{\zeta w^{RD}} = 0 \tag{4}$$

$$\rho v(\omega) - \pi - \delta (v(1) - v(\omega)) + \frac{v(1)}{\zeta w^{RD} \omega} v(\omega) = 0. \tag{5}$$

The second equation gives $v(\omega)$ as the following function of $v(1)$:

$$v(\omega) = \frac{\pi + \delta v(1)}{\rho + \delta + \frac{v(1)}{\zeta w^{RD} \omega}}.$$

Suppose first that $v(\omega) < v(1)$. In this case, the first equation gives:

$$\rho v(1) + \frac{v(1)^2}{\zeta w^{RD}} - \pi = 0.$$

The roots of this equation are:

$$v_{1,2} = \frac{-\rho \pm \sqrt{\rho^2 + 4 \frac{\pi}{\zeta w^{RD}}}}{\frac{2}{\zeta w^{RD}}}.$$

Since the term under the root is strictly positive, only one of these roots is admissible, so the above system is solved for a unique pair $v(1), v(\omega)$. Consider now the case $v(\omega) > v(1)$. It is straightforward to note that $v(\omega) - v(1)$ is decreasing in $v(1)$. This implies that, when rewriting (4) as

$$-\left(\frac{\gamma - 1}{\gamma} \left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}} \right)^{\frac{1}{\gamma-1}} \right) (v(\omega) - v(1)) = \pi - \rho v(1) - \frac{v^2(1)}{\zeta w^{RD}}, \tag{6}$$

the left hand side is monotonically increasing in $v(1)$, while the right hand side is monotonically decreasing in $v(1)$. Further, at $v(1) = 0$, the left hand side is strictly negative, while the right hand side equals π, while for $v(1) \to \infty$, the right hand side tends to $+\infty$ while the left hand side decreases towards $-\infty$. As a result, (6) has a unique positive solution.

The uniqueness of $v(1)$ immediately implies unique $v(\omega)$ and R&D choices. Given these R&D choices, the stationary distribution satisfies

$$0 = -\left(x_I + x_{e,1}\right)\mu_1 + \delta\mu_\omega + x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1}, \tag{7}$$

$$0 = -\left(x_{e,\omega} + \delta\right)\mu_\omega + x_I\mu_1, \tag{8}$$

$$0 = -\left(x_{e,1} + x_I\right)\mu_{e,1} + x_{e,1}\mu_1 + \delta\mu_{e,\omega}, \tag{9}$$

$$0 = -\left(x_{e,\omega} + \delta\right)\mu_{e,\omega} + x_{e,\omega}\mu_\omega + x_I\mu_{e,1}. \tag{10}$$

By equation (8):

$$x_I\mu_1 = \left(x_{e,\omega} + \delta\right)\mu_\omega$$

Since $\mu_1 = 1 - \mu_\omega$, the stationary distribution has:

$$\mu_\omega = \frac{x_I}{x_I + x_{e,\omega} + \delta},$$

$$\mu_1 = \frac{x_{e,\omega} + \delta}{x_I + x_{e,\omega} + \delta},$$

$$\begin{bmatrix} -\delta & x_{e,1} + x_I \\ x_{e,\omega} + \delta & -x_I \end{bmatrix} \begin{bmatrix} \mu_{e,\omega} \\ \mu_{e,1} \end{bmatrix} = \begin{bmatrix} x_{e,1}\mu_1 \\ x_{e,\omega}\mu_\omega \end{bmatrix}. \tag{11}$$

Since the matrix in (11) is nonsingular, $\mu_{e,\omega}$ and $\mu_{e,1}$ are uniquely determined as:

$$\mu_{e,\omega} = \frac{x_I x_{e,1}\mu_1 + \left(x_{e,1} + x_I\right)x_{e,\omega}\mu_\omega}{x_{e,\omega}\left(x_{e,1} + x_I\right) + \delta x_{e,1}},$$

$$\mu_{e,1} = \frac{\left(x_{e,\omega} + \delta\right)x_{e,1}\mu_1 + \delta x_{e,\omega}\mu_\omega}{x_{e,1}\left(x_{e,\omega} + \delta\right) + x_{e,\omega}x_I}$$

By the optimal solution for entrants:

$$x_{e,1} = \omega x_{e,\omega},$$

so (11) is solved for:

$$\mu_{e,\omega} = \frac{\omega x_I\mu_1 + \left(\omega x_{e,\omega} + x_I\right)\mu_\omega}{\omega\left(x_{e,\omega} + \delta\right) + x_I}, \tag{12}$$

$$\mu_{e,1} = \frac{\omega\left(x_{e,\omega} + \delta\right)\mu_1 + \delta\mu_\omega}{\omega\left(x_{e,\omega} + \delta\right) + x_I}. \tag{13}$$

Thus, the stationary distribution is unique.

It remains to show that equilibrium R&D labor is also unique. To show this, I prove that R&D labor demand is monotonically decreasing in wages and has:

$$\lim_{w^{RD}\to\infty} L^{RD}(w^{RD}) \leq 0, \qquad \lim_{w^{RD}\to 0} L^{RD}(w^{RD}) = \infty.$$

Since the converse holds for R&D labor supply is monotonically increasing in wages and ranges between 0 and $+\infty$, this gives a unique intersection of the two schedules. First note that, if labor supply is inelastic, $\phi = 0$, equilibrium R&D labor is constant by definition. Lemma C.2 below builds on this observation as well as C.1 to prove that research labor demand is indeed monotonically decreasing in the wage.

Lemma C.1. *Consider a steady state of the normalized one-sector model, and assume that defensive innovation is effective, $\omega > 1$. Then, $\omega v(1) > v(\omega) > v(1)$. Around a steady state, and for a fixed wage rate, w^{RD}, the normalized values, $v(1), v(\omega)$, are increasing in the markup, ϕ, and*

$$\frac{\partial v(\omega)}{\partial \phi} > \frac{\partial v(1)}{\partial \phi} > 0.$$

Proof of Lemma C.1. Subtracting side by side Equation (4) from (5) gives:

$$\left(\rho + \delta + \mathbf{1}\{v(\omega) - v(1) > 0\}\left(\frac{\gamma - 1}{\gamma}\left(\frac{v(\omega) - v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}}\right)\right)(v(\omega) - v(1)) = \frac{v(1)}{\zeta w^{RD}}\left(v(1) - \frac{v(\omega)}{\omega}\right)$$

Suppose that $v(\omega) < v(1)$. This implies that the left hand side of the above expression is strictly smaller than 0, while $\omega v(1) > v(1) > v(\omega)$, so the right hand side is strictly positive under the assumption $\omega > 1$. Therefore, it must be that $v(\omega) > v(1)$. If this is the case, the left hand side is strictly positive, and to avoid a contradiction it must be $\omega v(1) > v(\omega)$. Thus, $\omega v(1) > v(\omega) > v(1)$, proving the first part of the statement.

Since π is a monotone increasing function of ϕ, I prove the statement for value derivatives with respect to π. Total differentiation of the system of Equations (4) and (5) with respect to π around a CGE gives

$$\underbrace{\begin{bmatrix} \rho + \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} + 2\frac{v(1)}{\zeta} & -\left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} \\ -\delta + \frac{v(\omega)}{\zeta w^{RD}\omega} & \rho + \delta + \frac{v(1)}{\zeta w^{RD}\omega} \end{bmatrix}}_{\equiv J} \begin{bmatrix} \mathrm{d}v(1) \\ \mathrm{d}v(\omega) \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathrm{d}\pi = 0. \tag{14}$$

The determinant of the Jacobian is:

$$\det J = \left(\rho + x_I + 2\omega x_{e,\omega}\right)\left(\rho + \delta + \frac{v(1)}{\zeta\omega}\right) + x_I\left(x_{e,\omega} - \delta\right) > 0.$$

Solving (14) gives:

$$\begin{bmatrix} \frac{\mathrm{d}v(1)}{\mathrm{d}\pi} \\ \frac{\mathrm{d}v(\omega)}{\mathrm{d}\pi} \end{bmatrix} = \frac{1}{\det J} \begin{bmatrix} \frac{v(1)}{\zeta w^{RD}\omega} + \rho + \delta & \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} \\ \delta - \frac{v(\omega)}{\zeta w^{RD}\omega} & \rho + \left(\frac{v(\omega)-v(1)}{\alpha_I w^{RD}}\right)^{\frac{1}{\gamma-1}} + 2\frac{v(1)}{\zeta w^{RD}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Since the first row is strictly positive,

$$\frac{\mathrm{d}v(1)}{\mathrm{d}\pi} > 0.$$

Subtracting line by line gives:

$$\begin{aligned} \frac{\mathrm{d}v(\omega)}{\mathrm{d}\pi} - \frac{\mathrm{d}v(1)}{\mathrm{d}\pi} &= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta w^{RD}\omega} - \rho + \frac{v(1)}{\zeta w^{RD}\omega} + \rho + 2\frac{v(1)}{\zeta w^{RD}}\right] \\ &= \frac{1}{\det J}\left[-\frac{v(\omega)}{\zeta w^{RD}\omega} - \frac{v(1)}{\zeta w^{RD}\omega} + 2\frac{v(1)}{\zeta w^{RD}}\right] \\ &= \frac{1}{\det J}\left[\frac{2\omega v(1) - (v(\omega)+v(1))}{\zeta w^{RD}\omega}\right] > 0 \end{aligned} \tag{15}$$

since $\omega > 1$ and $\omega v(1) > v(\omega)$, from what shown above. It follows that:

$$\frac{\mathrm{d}v(\omega)}{\mathrm{d}\pi} > \frac{\mathrm{d}v(1)}{\mathrm{d}\pi} > 0.$$

□

Lemma C.2. *R&D labor demand is monotonically decreasing in the wage rate w_t^{RD}/Y_t,and:*

$$\lim_{w^{RD}\to\infty} L^{RD}(w^{RD}) \le 0, \qquad \lim_{w^{RD}\to 0} L^{RD}(w^{RD}) = \infty.$$

Proof. Consider the equilibrium with inelastic R&D labor. By the resource constraint in the economy, it holds:

$$\rho\left(\mu_1 v(1) + \mu_\omega v(\omega)\right) + w^{RD}L^{RD} + wL = 1,$$

$$L^{RD} = \frac{\pi}{w^{RD}} - \rho\left(\mu_1\frac{v(1)}{w^{RD}} + \mu_\omega\frac{v(\omega)}{w^{RD}}\right).$$

Since the labor supply is fixed, shifts in the right hand side of this equation identify the elasticity of labor supply to various parameters. Now consider an increase in π to $\pi' > \pi$. In this case, the unique equilibrium requires:

$$\frac{\pi'}{w^{RD\prime}} = \frac{\pi}{w^{RD}}.$$

Indeed, guess that the equilibrium involves no changes in research intensities, and therefore in the stationary distribution. Then:

$$x'_{e,\omega} = x_{e,\omega} \Rightarrow \frac{v'(1)}{\zeta\omega w'^{RD}} = \frac{v(1)}{\zeta\omega w^{RD}},$$

and

$$x_I' = \left(\frac{v'(1) - v'(\omega)}{\alpha_I w'^{RD}}\right)^{\frac{1}{\gamma-1}} = \left(\frac{v(1) - v(\omega)}{\alpha_I w^{RD}}\right)^{\frac{1}{1-\gamma}} = x_I.$$

As a result:

$$\frac{v'(\omega)}{w'^{RD}} = \frac{v(\omega)}{w^{RD}}.$$

Using the expression for $v(\omega)$, and using the fact that the ratio between values and wages is the same in both equilibria, gives:

$$\frac{\pi'}{w'^{RD}} = \frac{\pi}{w^{RD}}.$$

This also ensures that:

$$\rho\frac{v(1)}{w^{RD}} = \rho\frac{v'(1)}{w'^{RD}},$$

as is easily verified plugging the above expression into (2) evaluated at $\left(v(1), w^{RD}\right)$ and $\left(v'(1), w^{RD\prime}\right)$.It remains to show that goods' market clearing holds. Before a markup change we have (in normalized values):

$$\rho\left(\mu_1 v(1) + \mu_\omega v(\omega)\right) + w^{RD}L^{RD} + wL = 1,$$
$$\rho\left(\mu_1\frac{v(1)}{w^{RD}} + \mu_\omega\frac{v(\omega)}{w^{RD}}\right) + L^{RD} = \frac{1 - wL}{w^{RD}},$$

By what shown above, with an inelastic labor research labor supply, the left hand side has the same value before and after the change in instantaneous profits. Further, the linear production function implies that:

$$wL = \frac{1}{\phi},$$

therefore the right hand side can be written as:

$$\frac{\pi}{w^{RD}},$$

which has the same value in the new equilibrium. Therefore, the unique equilibrium with inelastic labor supply is characterized by a constant ratio $\frac{\pi}{w^{RD}}$. Given that the labor supply is inelastic, L^{RD} in the above expression can be read as the labor demand for R&D:[1]

$$L^{RD,d}\left(w^{RD}\right) = \frac{\pi}{w^{RD}} - \rho\left(\mu_1\frac{v(1)}{w^{RD}} + \mu_\omega\frac{v(\omega)}{w^{RD}}\right)$$

Now consider an initial equilibrium with $L^{RD,d}(w^{RD}) = L^d$. A change in the wage w^{RD} to $w^{RD\prime} > w^{RD}$

[1]Alternatively, the market clearing expression can be rewritten as the accounting identity that instantaneous profits equal the R&D wage bill plus dividends, which gives the demand for R&D labor as the expression reported below.

modifies the above expression to:

$$L^{RD,d}\left(w^{RD\prime}\right)=\frac{\pi}{w^{RD\prime}}-\rho\left(\mu_1'\frac{v'(1)}{w^{RD\prime}}+\mu_\omega'\frac{v'(\omega)}{w^{RD\prime}}\right).$$

By what shown above, it must be:

$$\frac{\mathrm{d}\pi}{\pi}=\frac{w^{RD\prime}-w^{RD}}{w^{RD}}>0$$

for $L^{RD,d}$ to be unchanged. Thus, denoting:

$$\pi'=\pi\left(1+\frac{w^{RD\prime}-w^{RD}}{w^{RD}}\right),$$

the above expression reads:

$$L^{RD,d}\left(w^{RD\prime}\right)=\frac{\pi'}{w^{RD\prime}}+\frac{\pi-\pi'}{w^{RD\prime}}-\rho\left(\mu_1'\frac{v'(1)}{w^{RD\prime}}+\mu_\omega'\frac{v'(\omega)}{w^{RD\prime}}\right).$$

That is:

$$L^{RD,d}\left(w^{RD\prime}\right)=L^{RD,d}\left(w^{RD}\right)+\frac{\pi-\pi'}{w^{RD\prime}}<L^{RD,d}\left(w^{RD}\right).$$

This shows that labor demand is decreasing in the wage. In general, we have:

$$L^{RD,d}\left(w^{RD\prime}\right)=L^{RD,d}\left(w^{RD}\right)+\frac{1}{w^{RD}}\left(\frac{w^{RD}}{w^{RD\prime}}-1\right)$$

Consider now $w^{RD\prime}\to 0$, in this case we clearly have:

$$L^{RD,d}\left(w^{RD\prime}\right)\to\infty.$$

Conversely, with $w^{RD\prime}\to\infty$:

$$L^{RD,d}\left(w^{RD\prime}\right)\to L^{RD,d}\left(w^{RD}\right)-\frac{1}{w^{RD}}=-\rho\left(\mu_1\frac{v(1)}{w^{RD}}+\mu_\omega\frac{v(\omega)}{w^{RD}}\right)-\frac{wL}{w^{RD}}<0.$$

□

By Lemma C.2, given an endowment of production labor and an R&D labor supply schedule, the CGE is unique.

To derive the growth rate note that, by the Cobb Douglas assumption on the final good, and given the equilibrium wage rate for production workers, $w = \frac{w_t}{Y_t}$,

$$\begin{aligned}
\log Y_t &= \int_0^1 \log y_t(i)\mathrm{d}i \\
&= \int_0^1 \log\left(\frac{Y_t}{w_t c_t(i)}\right)\mathrm{d}i \\
&= \int_0^1 \log\left(\frac{1}{w c_t(i)}\right)\mathrm{d}i.
\end{aligned}$$

It follows that:

$$\begin{aligned}
g = \log(Y_{t+\Delta t}) - \log(Y_t) &= -\int_0^1 \left(\log c_{t+\Delta}(i) - c_t(i)\right)\mathrm{d}i \\
&= \eta\left[x_{e,\omega}\mu_{e,\omega} + x_{e,1}\mu_{e,1} + \lambda x_I \mu_1\right] \\
&= \eta\left[x_{e,\omega}\left(\mu_{e,\omega} + \omega\mu_{e,1}\right) + \lambda x_I \mu_1\right].
\end{aligned}$$

Productivity g/L^{RD} follows directly from total R&D labor demand:

$$\zeta\omega x_{e,\omega}\left(\mu_{e,\omega} + \mu_{e,1}\right) + \alpha_I \frac{(x_I)^\gamma}{\gamma}\mu_1.$$

□

Proof of Proposition **??**. The increase in R&D efforts by both incumbents and entrants descend directly from Lemma C.1. In what follows, I derive *equilibrium* quantities, that is factoring in wage effects, but I drop stars for ease of notation.

To prove that the share of R&D labor accruing to incumbents increases, note first:

$$\frac{\partial L_I}{\partial\phi} = \alpha_I x_I^{\gamma-1}\mu_1\frac{\partial x_I}{\partial\phi} + \frac{\alpha_I}{\gamma}x_I^{\gamma-1}\frac{\partial\left(x_I\mu_1\right)}{\partial\phi},$$

where the first term is strictly positive, since I have proven that $\frac{\partial x_I}{\partial\phi} > 0$, and the term, $\frac{\partial(x_I\mu_1)}{\partial\phi}$,denotes the derivative of aggregate incumbents' research intensity with respect to the markup, and is also strictly positive. Indeed:

$$\frac{\partial\mu_1}{\partial\phi} = \frac{\partial\left(\frac{x_{e,\omega}+\delta}{x_{e,\omega}+\delta+x_I}\right)}{\partial\phi} = \left[\frac{\frac{\partial(x_{e,\omega}+\delta)}{\partial\phi}x_I - \left(x_{e,\omega}+\delta\right)\frac{\partial x_I}{\partial\phi}}{\left(x_I + x_{e,\omega}+\delta\right)^2}\right] = \mu_1\frac{\partial x_I}{\partial\phi}\frac{(\epsilon-1)}{\left(x_I + x_{e,\omega}+\delta\right)}, \tag{16}$$

where I define the ratio of the elasticity of $x_{e,\omega}+\delta$ and x_I to ϕ as:

$$\epsilon \equiv \frac{\epsilon_e}{\epsilon_I} \equiv \frac{\frac{\partial(x_{e,\omega}+\delta)}{\partial\phi}/x_{e,\omega}}{\frac{\partial x_I}{\partial\phi}/x_I} \in (0,1].$$

therefore:

$$\frac{\partial\left(\mu_1 x_I\right)}{\partial\phi}=\mu_1\frac{\partial x_I}{\partial\phi}\left[\frac{x_I\left(\epsilon-1\right)}{\left(x_I+x_{e,\omega}+\delta\right)}+1\right]$$
$$=\mu_1\frac{\partial x_I}{\partial\phi}\left[\frac{x_I\epsilon+x_{e,\omega}+\delta}{\left(x_I+x_{e,\omega}+\delta\right)}\right]>0,$$

that is, aggregate incumbents' research intensity, $x_I\mu_1$, is increasing in the markup.

□

C.2 Full Description of the Two-Sector Model and Derivations

By the above assumptions, the final good is produced according to:

$$Y=\prod Y_i^{\beta_i}. \tag{17}$$

With the final good as numeraire, the sector's demand schedule is:

$$Y_i=\beta_i\frac{Y}{P_i}. \tag{18}$$

From CD on intermediate goods we also have:

$$P_i Y_i=p_{is}y_{is},\quad \forall s.$$

In each sector, the price is set at the competitive fringe's marginal cost wc_i, and is identical across subsectors . Thus

$$P_i=p_{is}=wc_i,\ Y_i=\beta_i\frac{Y}{wc_i}. \tag{19}$$

Equilibrium profits are given by:

$$\Pi_i=\left(c_i w-\frac{c_i w}{\phi_i}\right)Y_i=\left(\frac{\phi_i-1}{\phi_i}\right)\beta_i Y.$$

The monopolist demands production labor:

$$\ell_{is}=\frac{c_i y_{is}}{\phi_i},\Rightarrow L_i=\int\ell_{is}\mathrm{d}s=Y\frac{\beta_i}{\phi_i w}. \tag{20}$$

Assuming a rigid production labor supply:[2]

$$L^s(w) = L = \frac{Y}{w}\left(\sum \frac{\beta_i}{\phi_i}\right). \tag{21}$$

Which gives:

$$L_i = L\frac{\frac{\beta_i}{\phi_i}}{\sum \frac{\beta_i}{\phi_i}}, Y_i = L\frac{\frac{\beta_i}{c_i}}{\sum \frac{\beta_i}{\phi_i}}. \tag{22}$$

Which gives:

$$Y = L\prod_i \left(\frac{\frac{\beta_i}{c_i}}{\sum \frac{\beta_i}{\phi_i}}\right)^{\beta_i}. \tag{23}$$

Thus, growth is:

$$-\sum \beta_i \Delta \log c_i. \tag{24}$$

Normalized values in each sector are the same as before, with the only difference that they receive a wage w^R, and the above α_I, ζ are replaced by $\zeta w^R, \alpha_I w^R$.

C.2.1 Research Equilibrium in the two-sector model

By the above solutions, the monopolist's values read:

$$\begin{aligned}
\rho V_i(1) &= \max_{x_I}\left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y - \alpha_I W^{RD}\frac{x_I^2}{2} + x_I\left(V_i(\omega) - V_i(1)\right) - x_{e,1}V_i(1),\\
\rho V_i(\omega_i) &= \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i Y + \delta\left(V(1) - V(\omega)\right) - x_{e,\omega}V_i(\omega).
\end{aligned}$$

And normalized values, $v \equiv V/Y$:

$$\rho v_i(1) = \max_{x_I}\left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i - \alpha_I w^{RD}\frac{x_I^2}{2} + x_I\left(v_i(\omega) - v_i(1)\right) - x_{e,1}v_i(1) \tag{25}$$

$$\rho v_i(\omega_i) = \left(\frac{\phi_i - 1}{\phi_i}\right)\beta_i + \delta\left(v(1) - v(\omega)\right) - x_{e,\omega}v_i(\omega), \tag{26}$$

where w^{RD} is the normalized researchers' wage.

---

[2] Consider a labor supply with elasticity φ. This gives:

$$\chi w^{\varphi} = \frac{Y}{w}\left(\sum \frac{\beta_i}{\phi_i c_i}\right) \Rightarrow w = \left[\frac{Y}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{\frac{1}{1+\varphi}}$$

Equilibrium labor is then:

$$L^{\star} = \chi\left[\frac{Y}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{\frac{\varphi}{1+\varphi}}, \frac{Y}{w} = Y^{\frac{\varphi}{1+\varphi}}\left[\frac{1}{\chi}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)\right]^{-\frac{1}{1+\varphi}} = L^{\star}\left(\sum \frac{\beta_i}{\phi_i c_i}\right)^{-1}$$

Which results in the same allocations and outputs as below, with $L^{\star}$ in place of the fixed L.

Given a normalized wage, each sector demands:

$$x_{e,\omega,i}\left(w^{RD}\right) = \frac{v_i(1)}{w^{RD}\omega\zeta_i}, \tag{27}$$

$$x_{I,i}\left(w^{RD}\right) = \frac{\left(v_i(\omega_i) - v_i(1)\right)}{w^{RD}\alpha_{I,i}}. \tag{28}$$

The stationary distribution within each sector is given by:

$$\mu_{\omega,i}\left(w^{RD}\right) = \frac{x_{I,i}\left(w^{RD}\right)}{x_{e,\omega,i}\left(w^{RD}\right) + \delta_i + x_{I,i}\left(w^{RD}\right)}, \tag{29}$$

$$\mu_{1,i}\left(w^{RD}\right) = \frac{x_{e,\omega,i}\left(w^{RD}\right) + \delta_i}{x_{e,\omega,i}\left(w^{RD}\right) + \delta_i + x_{I,i}\left(w^{RD}\right)}, \tag{30}$$

$$\mu_{e,1,i}\left(w^{RD}\right) = \frac{\omega_i\left(x_{e,\omega,i}\left(w^{RD}\right) + \delta\right)\mu_{1,i} + \delta_i\mu_{\omega,i}}{\left(x_{I,i} + \omega_i\left(x_{e,\omega,i}\left(w^{RD}\right) + \delta_i\right)\right)}, \tag{31}$$

$$\mu_{e,\omega,i}\left(w^{RD}\right) = \frac{\omega_i\mu_{1,i}x_{I,i}\left(w^{RD}\right) - \omega_i\delta_i\mu_{\omega,i}}{\left(x_{I,i} + \omega_i\left(x_{e,\omega,i}\left(w^{RD}\right) + \delta_i\right)\right)} + \mu_{\omega,i}. \tag{32}$$

Sector RD labor demand is given by:

$$L_i^{RD,d}\left(w^{RD}\right) = \mu_{e,\omega,i}\left(w^{RD}\right)\left(\zeta_i\omega_i x_{e,\omega,i}\left(w^{RD}\right)\right) + \mu_{1,e,i}\left(w^{RD}\right)\zeta_i x_{e,1,i}\left(w^{RD}\right) + \mu_{1,i}\left(w^{RD}\right)\alpha_I\frac{x_{I,i}^2\left(w^{RD}\right)}{2}.$$

With an inelastic labor supply fixed to L^{RD}, market clearing for inventors then reads:

$$L^{RD} = \sum_i\left\{\mu_{\omega,i}\left(w^{RD}\right)\left(\zeta_i\omega_i x_{e,\omega,i}\left(w^{RD}\right)\right) + \mu_{1,e,i}\left(w^{RD}\right)\zeta_i x_{e,1,i}\left(w^{RD}\right) + \mu_{1,i}\left(w^{RD}\right)\alpha_I\frac{x_{I,i}^2\left(w^{RD}\right)}{2}\right\}. \tag{33}$$