COCI 2009/2010 Task AUTORI

4th round, February 13th 2010.

1 second / 32 MB / 30 points

Great scientific discoveries are often named by the last names of scientists that made them. For example, the most popular asymmetric cryptography system, RSA was discovered by Rivest, Shamir and Adleman. Another notable example is the Knuth-Morris-Pratt algorithm, named by Knuth, Morris and Pratt.

Scientific papers reference earlier works a lot and it's not uncommon for one document to use two different naming conventions: the **short variation** (e.g. KMP) using only the first letters of authors last names and the **long variation** (e.g. Knuth-Morris-Pratt) using complete last names separated by hyphens.

We find mixing two conventions in one paper to be aesthetically unpleasing and would like you to write a program that will transform **long variations into short**.

#### **INPUT**

The first and only line of input will contain at most 100 characters, uppercase and lowercase letters of the English alphabet and hyphen ('-' ASCII 45). The first character will always be an uppercase letter. Hyphens will **always** be followed by an uppercase letter. All other characters will be lowercase letters.

## **OUTPUT**

The first and only line of output should contain the appropriate short variation.

Input:	Input:	Input:
Knuth-Morris-Pratt	Mirko-Slavko	Pasko-Patak
Output:	Output:	Output:
KMP	MS	PP

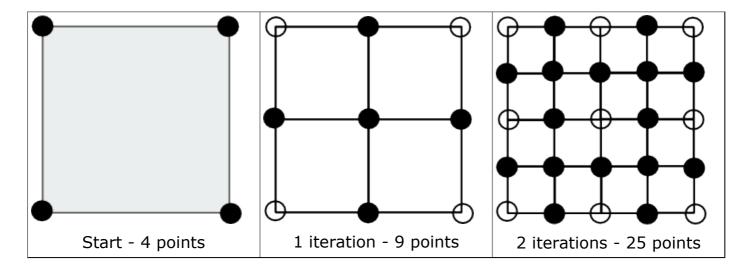
1 second / 32 MB / 50 points

Mirko and Slavko are filming a movie adaptation of the popular SF novel "Chicks in space 13". The script requires them to present a lot of different worlds so they decided to film the entire movie in front of a green screen and add CGI backgrounds later. Mirko heard that the best way to generate artificial terrain is to use **midpoint displacement algorithm**.

To start the algorithm, Mirko selects 4 points forming a perfect square. He then performs the following steps:

- 1. On each side of the square, he adds a new point in the exact middle of the side. The height of this new point is the average height of the two points on that side.
- 2. In the exact center of the square he adds a new point whose height is the average height of all 4 square vertices, plus a small random value.

After those two steps are performed, he now has 4 new squares. He performs the same steps on the newly created squares again and again until he is pleased with the results. The following diagram illustrates 2 iterations of the algorithm.



Mirko noticed that some of the points belong to more than one square. In order to decrease memory consumption, he stores calculates and stores such points **only once**. He now wonders how many points in total will he need to store in memory after **N** iterations.

#### **INPUT**

The first and only line of input contains one integer  $\mathbf{N}$  (1  $\leq$   $\mathbf{N}$   $\leq$  15), number of iterations.

# **OUTPUT**

The first and only line of output should contain one number, the number of points stored after  ${\bf N}$  iterations.

Input:	Input:	Input:
1	2	5
Output:	Output:	Output:
9	25	1089

COCI 2009/2010 Task IKS

4th round, February 13th 2010.

1 second / 32 MB / 70 points

Mirkos great grandmother Katica is an avid mathematician. She likes to torment Mirko with math games. This time she wrote down a sequence of numbers on a piece of paper and told Mirko he may do the following:

 Choose any two numbers in the sequence (let's call them A i B) and a prime number X such that A is divisible by X. After that, Mirko erases A and writes \( \frac{A}{X} \) in its place. In the end he erases B and writes \( B \times X \) in its place.

Mirko may perform this operation as many times he wants. His goal is to obtain the maximum possible score, because he gets candy from his great grandmother if he does so. The score for one sequence is the **greatest common divisor of all the numbers in the sequence**.

He is not very good at this, and he likes his candy so he has asked you to help him. Write a program that will calculate the maximum possible score. Since you are such a nice guy, you should also print the **smallest number** of times Mirko must perform the operation to obtain the maximum possible score.

#### **INPUT**

The first line of input contains one integer  $\mathbf{N}$ , (1  $\leq$   $\mathbf{N}$   $\leq$  100), number of elements in the starting sequence.

The second line of input contains **N** positive integers smaller than or equal to **1 000 000**, the sequence Katica gave to Mirko.

#### **OUTPUT**

The one and only line of output should contain two integers. The first integer is the maximal possible score Mirko can obtain. The second integer is the smallest number of operations Mirko needs to perform to obtain it.

Input:	Input:	Input:
3	3	5
4 4 1	8 24 9	4 5 6 7 8
Output:	Output:	Output:
2 1	12 3	2 2

1 second / 32 MB / 100 points

Matija needs to paint his old fence. The fence is made from **N** planks, each 1 cm in width and varying in height. To do this easy and fast, he bought himself a Super Paint Roller Deluxe. The paint roller is **X** cm wide. The Super Paint Roller Deluxe model comes with a catch, however. Matija must at **all times touch the planks with full width of the roller**, otherwise paint drops all around and stains everything. Also, the roller must always be parallel to the ground to prevent leakage. This means that in order for Matija to use the roller safely, he needs to select **X** planks, and paint them from bottom to the top of the lowest plank in one swoop. Then he selects some other **X** planks, paints them and so on.

This leaves parts of some planks unpainted. Matija will have to paint such parts with a toothbrush. This is obviously quite tedious so he asked you to help him **paint as much as possible** using the Super Paint Roller Deluxe. Since there is more than one way to do this he is also interested in the painting that requires the **minimal number of swoops**.

#### **INPUT**

The first line of input contains two integers **N** ( $1 \le N \le 1000000$ ), number of planks, and **X** ( $1 \le X \le 100000$ ), width of the Super Paint Roller. Width of the Super Paint Roller will not exceed the width of the fence.

The second line of input contains  $\bf N$  positive integers, smaller than 1 000 000, heights of planks in the fence.

#### **OUTPUT**

The first line of output should contain the **smallest possible area** Matija will have to paint manually.

The second line of output should contain the **smallest number of swoops** needed.

#### **SCORING**

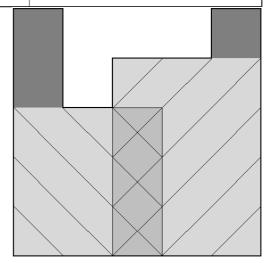
If only one of the output numbers is correct, you will receive 50% of points for that test case. You must always follow the output format to the letter, even if you do not calculate both numbers. In such case, you may output any integer in place of the number you did not calculate.

#### **SAMPLE TESTS**

Input:	Input:	Input:
5 3 4 4 5	10 3 3 3 3 3 3 3 3 3 3 3	7 4 1 2 3 4 3 2 1
Output:	Output:	Output:
3	0	4
2	4	4

# 1. sample description:

Matija needs two swoops with his roller - one to paint planks 1, 2 and 3 to the height of 3 cm, the other to paint planks 3, 4 and 5 to the height of 4 cm. Note that  $3 \text{ cm}^2$  ( $2 \text{ cm}^2$  on plank 1 and  $1 \text{ cm}^2$  on plank 5) are left unpainted. Also,  $3 \text{ cm}^2$  on plank 3 are painted over twice, but that's OK.



0.3 seconds / 32 MB / 120 points

Luka found an interesting tape in the chem lab. The tape is divided into  $\bf N$  segments of equal length, and can easily be bent between two segment, but only by exactly 180 degrees.

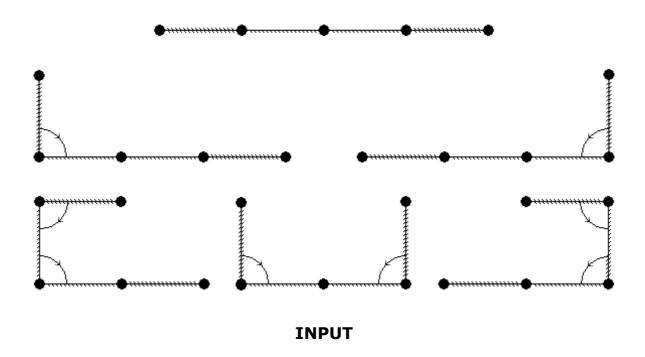
One side of the tape is completely covered with a very volatile chemical. If the chemical comes in contact with itself, it reaches critical mass and explodes.

The other side of the tape is not completely covered yet. Only the first **A** segments and last **B** segments are covered, with the exact same chemical.

Write a program that will calculate the number of different ways Luka can bend the tape so that it does not explode. He can bend the tape more than once and **two ways** are different if there is at least one bevel between segments that is not bent in one and is bent in the other

Since the solution can be huge, print the result modulo 10301.

The following example illustrates all 6 possible ways for N=4, A=1 and B=1. For clarity, the tape is only bent 90 degrees on the illustration. Luka would actually bend it 180 degrees.



The first and only line of input contains three natural numbers **N**, **A** and **B** (**A**>0, **B**>0, **A**+**B**  $\leq$  **N**  $\leq$  1000), total number of segments, number of covered segments from the left and from the right respectively.

# **OUTPUT**

The first and only line of output should contain the number of possible ways to bend the tape modulo  ${\bf 10301}$ .

Input:	Input:	Input:
4 1 1	5 2 2	6 1 2
Output:	Output:	Output:
6	1	7

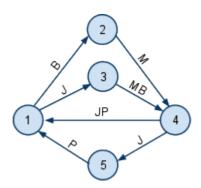
1 second / 32 MB / 130 points

Ana has a couple of classmates coming over for crêpes (known as palačinke in Croatian). They are coming in T minutes, and Ana just found out that she has neither one of the four required ingredients (flour, milk, eggs and jam). She hops into her car and drives around her neighbourhood buying supplies.

Her neighbourhood contains  $\mathbf{N}$  crossroads numbered 1 to  $\mathbf{N}$ , and  $\mathbf{M}$  one way roads connecting them. Ana starts on crossroad 1. On each road there is exactly one shop, selling some, maybe all, ingredients.

Ana needs 1 minute to drive down any given road if she does not stop in the shop, or 2 minutes if she does. She needs to obtain all ingredients and drive back to crossroad one in time. She likes to compare shop prices so she may enter a shop even if she already has all ingredients.

Consider the following example with 5 crossroads and 7 roads.



Ana can make the ingredient run in 5 different ways, as shown in the table below.

1. minute	2. minute	3. minute	4. minute	5. minute	6. minute	7. minute
1→3	3→sh	op→4	4→sh	op→1		
1→sh	op→2	2→sh	op→4	4→sh	op→1	
1→sh	op→3	3→sh	op→4	4→sh	op→1	
1→sh	op→3	3→shop→4		4→5	→5 5→shop→1	
1→3	3→sh	op→4 4→sho		op→5	5→sh	op→1

Write a program that will calculate the number of different ways Ana can buy the ingredients and return home in **T** minutes or less. Since this number can be very large, output it modulo 5557.

#### **INPUT**

The first line contains two integers **N** and **M** ( $1 \le N \le 25$ ,  $1 \le M \le 500$ ), number of crossroads and roads.

Each of the next  $\mathbf{M}$  lines contains two different integers  $\mathbf{u}$  and  $\mathbf{v}$  and a string  $\mathbf{s}$ , separated by exactly one space. They describe a road connecting crossroads  $\mathbf{u}$  and  $\mathbf{v}$ , and the shop located on the road selling ingredients  $\mathbf{s}$ .

The string **s** will contain between 1 and 4 uppercase characters. Character 'B' for flour, 'J' for eggs, 'M' for milk and 'P' for jam.

There are at most two direct roads between two crossroads, and only if they are in opposite directions.

The last line contains one integer  $\mathbf{T}$  (1  $\leq$   $\mathbf{T}$   $\leq$  1 000 000 000), time until Anas friends arrive, in minutes.

#### **OUTPUT**

The first and only line of output should contain the number of different ways Ana can buy the ingredients, modulo **5557**.

Input:	Input:	Input:
3 3	3 4	5 7
1 2 BMJ	1 2 в	1 2 в
2 3 MJP	2 1 P	2 4 M
3 1 JPB	1 3 Ј	1 3 Ј
5	3 1 M	3 4 MB
	8	4 1 JP
		4 5 J
		5 1 P
		7
Output:	Output:	Output:
3	2	5

Young Mirko threw matches all over the floor of his room.

His mom did not like that and ordered him to put all the matches in a box. Mirko soon noticed that not all of the matches on the floor fit in the box, so he decided to take the matches that don't fit and throw them in the neighbour's garbage, where his mom (hopefully) won't find them.

Help Mirko determine which of the matches fit in the box his mom gave him. A match fits in the box if its entire length can lie on the bottom of the box. Mirko examines the matches one by one.

#### Input

The first line of input contains an integer N ( $1 \le N \le 50$ ), the number of matches on the floor, and two integers W and H, the dimensions of the box ( $1 \le W \le 100$ ,  $1 \le H \le 100$ ).

Each of the following N lines contains a single integer between 1 and 1000 (inclusive), the length of one match.

## Output

For each match, in the order they were given in the input, output on a separate line "DA" if the match fits in the box or "NE" if it does not.

5 3 4 3 21 21 4 20 5 6 output	input	input
	3 1 5	
		output
output NE DA	output	
DA	)A	
DA	)A	
DA	PΑ	
NE	1E	
NE	1E	

#### 2. SKENER

Having solved the match issue, Mirko faced another challenging problem. His mom ordered him to read an article about the newest couples on the Croatian show-biz scene in the "Moja Tajna" magazine. The article is written in a very small font size which Mirko cannot read. Luckily, Mirko has a scanner in the closet which will enlarge the article for him.

The article is a matrix of characters with R rows and C columns. The characters are letters of the English alphabet, digits and the character '.' (period). Mirko's scanner supports two parameters, ZR and ZC. It substitutes each character it scans with a matrix of ZR rows and ZC columns, where all entries are equal to the scanned character.

Mirko realized his scanner's software no longer works so he needs your help.

## Input

The first row contains four integers, R, C, ZR and ZC. R and C are between 1 and 50, while ZR and ZC are between 1 and 5.

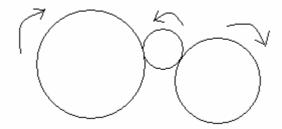
Each of the following R rows contains C characters each, representing the article from "Moja Tajna".

## Output

The output should be a matrix of characters with R•ZR rows and C•ZC columns, the enlarged article.

input	input
3 3 1 2	3 3 2 1
.X.	.x.
X.X	x.x .x.
.X.	.x.
output	output
xx	.x.
xxxx	.x.
xx	x.x
	x.x
	.x.
	.x.

After an exhausting morning, Mirko fell asleep. His brother Stanko, however, just awoke and, like his brother, is all about excitement. Stanko found N rings of varying radiuses in the garage. He arranged them on the floor so that each ring (except the first and last) touches the ones before and after it.



He started turning the first ring and noticed that the other rings turned as well; some faster, some slower!

Thrilled with his discovery, he decided to count how many times the other rings turn while the first ring turns once. He gave up after noticing that this number is not always an integer and not knowing what to do.

Write a program that determines how many times each ring turns while the first turns once.

# Input

The first line of input contains an integer N ( $3 \le N \le 100$ ), the number of rings.

The next line contains N integers between 1 and 1000, the radiuses of Stanko's rings, in the order they are laid out on the floor.

## Output

The output must contain N-1 lines. For every ring other than the first, in the order they are given in the input, output a fraction A/B, meaning that the respective ring turns A/B times while the first ring turns once.

The fractions must be in reduced form (the numerator and denominator must not have a common divisor larger than 1).

input	input	input
3 8 4 2	4 12 3 8 4	4 300 1 1 300
output	output	output

#### 4. ZBRKA

Consider a sequence of N integers where each integer between 1 and N appears exactly once.

A pair of numbers in the sequence is **confused** if the number that comes earlier in the sequence is larger than the later number.

The **confusion** of the sequence is the number of confused pairs in it. For example, the confusion of the sequence (1, 4, 3, 2) is 3 because there are 3 confused pairs: (4, 3), (4, 2) and (3, 2).

Write a program that calculates the number of sequences of length N whose confusion is exactly C.

# Input

The first and only line of input contains two integers, N ( $1 \le N \le 1000$ ) and C ( $0 \le C \le 10000$ ).

## Output

Output the number of sequences modulo 1000000007.

input	input	input
10 1	4 3	9 13
output	output	output
9	6	17957

## 5. JOGURT

A complete binary tree is made of nodes arranged in a hierarchic structure. One of the nodes is the root node, said to be at level 0. The root node has two child nodes, which are at level 1. Each of those has two children at level 2 etc.

In general, a complete binary tree with N levels has 2<sup>N</sup>-1 nodes, each of which has two child nodes, except those at level L-1.

A number can be written into each node. Write the numbers 1 to  $2^N-1$  into a complete binary tree with N levels so that, for each node at level D, the absolute value of the difference of the sum of all numbers in the left subtree and the sum of all numbers in the right subtree is  $2^D$ .

For example, the sum of the left subtree of the root node must differ from the sum of the right subtree by 1. The sums of the left and right subtrees of a node at level 1 must differ by 2.

Each number must be used exactly once. The solution need not be unique.

#### Input

The first and only line of input contains the integer N ( $1 \le N \le 15$ ), the number of levels in the tree.

## Output

Output the 2<sup>N</sup>-1 separated by spaces on a single line, the binary tree in the preorder traversal. The preorder traversal first outputs the number in the root node, then outputs the left subtree (again in the preorder traversal), then the right subtree.

input	input
2	3
output	output
3 1 2	3 1 7 5 6 2 4

It's exam time in Mirko's village. Everyone wants to pass the exam with as little effort as possible, which is not easy. Mirko realized that it would be best for him to find someone who knows more than him and learn from them. Everyone followed and now everyone is looking for someone to learn from.

We can model how well a student is prepared for the exam with two integers, A and B. The number A represents how well a student understands the subject, while the number B is proportional to the quantity of their knowledge.

As the head of the village, Mirko decided that a student will ask another student for help only if that student has **both numbers greater than or equal to** the first student's numbers (no student will ask someone who doesn't understand the subject as well as themselves or who knows less).

Additionally, students will try to minimize the difference in knowledge quantity (so that students don't bother those that are way better). If this choice is not unique, they will try to minimize the difference in understanding.

Mirko's village has recently become a very popular suburb and new students keep moving in (in time for the exam). With Mirko's strict rules, they get confused about Mirko's rules and don't know where to go). They decided to ask a programmer from a neighbouring village for help.

#### Input

The first line of input contains an integer N ( $1 \le N \le 200~000$ ), the number of queries and arrivals in the village. Each of the following N lines contains either:

- "D A B", a student has moved in whose knowledge is A and B
- "P i", the i-th student to move in wants to know whom to ask for help

The numbers A and B are between 1 and 2·10<sup>9</sup>. No two students have both numbers equal.

# Output

For each query ("P i" line), output which student the i-th student should ask for help. The students are numbered in the order they moved into the village (starting from 1). If a student cannot be helped, output "NE".

input	input	input
6 D 3 1 D 2 2 D 1 3 P 1 P 2 P 3	6 D 8 8 D 2 4 D 5 6 P 2 D 6 2 P 4	7 D 5 2 D 5 3 P 1 D 7 1 D 8 7 P 3 P 2
output	output	
NE NE NE	3 1	output 2 4
		4