

$e ::= x \mid (e\ e) \mid \lambda x^\tau. e \mid (\mathbf{if}\ e\ e\ e) \mid c \mid \#t \mid \#f \mid (\mathbf{cons}\ e\ e) \mid (\mathbf{vec}\ \vec{e}) \mid n$	Expressions
$c ::= \mathbf{add1} \mid = \mid \leq \mid \mathbf{num?} \mid \mathbf{bool?} \mid \mathbf{proc?} \mid \mathbf{cons?} \mid \mathbf{vec?} \mid \mathbf{car} \mid \mathbf{cdr} \mid \mathbf{len} \mid \mathbf{ref}[n]$	Primitive Operations
$pe ::= \mathbf{car} \mid \mathbf{cdr} \mid \mathbf{ref}_n \mid \mathbf{len}$	Path Elements
$\pi ::= \vec{p}\vec{e}$	Paths
$o ::= \emptyset \mid \pi(x)$	Objects
$\phi ::= a_0 o_0 + \dots + a_n o_n \leq a_{n+1}$	Linear Inequalities
$\Phi ::= \vec{\phi}$	System of Linear Inequalities
$\sigma, \tau ::= \top \mid \mathbf{N} \mid \{\diamond : \mathbf{N} \mid \Phi\} \mid \mathbf{T} \mid \mathbf{F} \mid (\bigcup \vec{\tau}) \mid \langle \tau, \tau \rangle \mid [[\vec{\tau}]] \mid x : \sigma \xrightarrow[o]{\psi \mid \psi} \tau$	Types
$\psi ::= \frac{\tau_{\pi(x)}}{\tau_{\pi(x)}} \mid \bar{\tau}_{\pi(x)} \mid \psi \supset \psi \mid \psi \wedge \psi \mid \psi \vee \psi \mid \Phi \mid \mathbf{T} \mid \mathbf{F}$	Propositions
$\Gamma ::= \frac{\psi}{\psi}$	Environments

Note:  $\mathbf{B}$  is equivalent to  $(\bigcup \mathbf{T}\ \mathbf{F})$ , and  $\perp$  is equivalent to  $(\bigcup)$ .

<b>T-NUM</b> $\Gamma \vdash n : \{\diamond : \mathbf{N} \mid \diamond = n\} ; \mathbf{T} \mid \mathbb{F} ; \emptyset$	<b>T-CONST</b> $\Gamma \vdash c : \delta_\tau(c) ; \mathbf{T} \mid \mathbb{F} ; \emptyset$	<b>T-TRUE</b> $\Gamma \vdash \#t : \mathbf{T} ; \mathbf{T} \mid \mathbb{F} ; \emptyset$	<b>T-FALSE</b> $\Gamma \vdash \#f : \mathbf{F} ; \mathbb{F} \mid \mathbf{T} ; \emptyset$
<b>T-VAR</b> $\frac{\Gamma \vdash \tau_x}{\Gamma \vdash x : \tau ; \bar{\mathbf{F}}_x \mid \mathbf{F}_x ; x}$	<b>T-ABS</b> $\frac{\Gamma, \sigma_x \vdash e : \tau ; \psi_+ \mid \psi_- ; o}{\Gamma \vdash \lambda x^\sigma. e : x : \sigma \xrightarrow[o]{\psi_+ \mid \psi_-} \tau ; \mathbf{T} \mid \mathbb{F} ; \emptyset}$	<b>T-APP</b> $\frac{\Gamma \vdash e : x : \sigma \xrightarrow[o_f]{\psi_{f+} \mid \psi_{f-}} \tau ; \psi_+ \mid \psi_- ; o \quad \Gamma \vdash e' : \sigma ; \psi'_+ \mid ; \mid \psi'_- ; o'}{\Gamma \vdash (e\ e') : \tau[o'/x] ; \psi_{f+} \mid \psi_{f-}[o'/x] ; o_f[o'/x]}$	
<b>T-IF</b> $\frac{\Gamma \vdash e_1 : \tau_1 ; \psi_{1+} \mid \psi_{1-} ; o_1 \quad \Gamma \vdash e_2 : \tau ; \psi_{2+} \mid \psi_{2-} ; o \quad \Gamma \vdash e_3 : \tau ; \psi_{3+} \mid \psi_{3-} ; o}{\Gamma \vdash (\mathbf{if}\ e_1\ e_2\ e_3) : \tau ; \psi_{2+} \vee \psi_{3+} \mid \psi_{2-} \vee \psi_{3-} ; o}$	<b>T-LET</b> $\frac{\Gamma \vdash e_0 : \tau ; \psi_{0+} \mid \psi_{0-} ; o_0 \quad \Gamma, \tau_x, \bar{\mathbf{F}}_x \supset \psi_{0+}, \mathbf{F}_x \supset \psi_{0-} \vdash e_1 : \sigma ; \psi_{1+} \mid \psi_{1-} ; o_1}{\Gamma \vdash (\mathbf{let}\ (x\ e_0)\ e_1) : \sigma[o_0/x] ; \psi_{1+} \mid \psi_{1-}[o_0/x] ; o_1[o_0/x]}$	<b>T-CAR</b> $\frac{\Gamma \vdash e : \langle \tau_1, \tau_2 \rangle ; \psi_+ \mid \psi_- ; o \quad \psi_{r+} \mid \psi_{r-} = \bar{\mathbf{F}}_{\mathbf{car}(x)} \mid \mathbf{F}_{\mathbf{car}(x)}[o/x]}{\Gamma \vdash (\mathbf{car}\ e) : \tau_1 ; \psi_{r+} \mid \psi_{r-} ; \mathbf{car}(x)[o/x]}$	
<b>T-CONS</b> $\frac{\Gamma \vdash e_1 : \tau_1 ; \psi_{1+} \mid \psi_{1-} ; o_1 \quad \Gamma \vdash e_2 : \tau_2 ; \psi_{2+} \mid \psi_{2-} ; o_2}{\Gamma \vdash (\mathbf{cons}\ e_1\ e_2) : \langle \tau_1, \tau_2 \rangle ; \mathbf{T} \mid \mathbb{F} ; \emptyset}$	<b>T-VEC</b> $\frac{\Gamma \vdash e_1 : \tau_1 ; \psi_{1+} \mid \psi_{1-} ; o_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n ; \psi_{n+} \mid \psi_{n-} ; o_n}{\Gamma \vdash (\mathbf{vec}\ e_1 \dots e_n) : [[\tau_1 \dots \tau_n]] ; \mathbf{T} \mid \mathbb{F} ; \emptyset}$	<b>T-REF</b> $\frac{\Gamma \vdash e : [[\dots \tau_i \dots]] ; \psi_{i+} \mid \psi_{i-} ; o_i \quad \psi_{r+} \mid \psi_{r-} = \bar{\mathbf{F}}_{\mathbf{ref}_i(x)} \mid \mathbf{F}_{\mathbf{ref}_i(x)}[o/x]}{\Gamma \vdash (\mathbf{ref}[i]\ e) : \tau_i ; \psi_{r+} \mid \psi_{r-} ; \mathbf{ref}_i(x)[o/x]}$	<b>T-SUBSUME</b> $\frac{\Gamma \vdash e : \tau ; \psi_+ \mid \psi_- ; o \quad \Gamma, \psi_+ \vdash \psi'_+ \quad \Gamma, \psi_- \vdash \psi'_- \quad \tau <: \tau' \quad o <: o'}{\Gamma \vdash e : \tau' ; \psi'_+ \mid \psi'_- ; o'}$

<b>S-REFL</b> $\vdash \tau <: \tau$	<b>S-TOP</b> $\vdash \tau <: \top$	<b>S-UNIONSUPER</b> $\frac{\exists \sigma \in \vec{\sigma}, \vdash \tau <: \sigma}{\vdash \tau <: (\bigcup \vec{\sigma})}$	<b>S-UNIONSUB</b> $\frac{\forall \tau \in \vec{\tau}, \vdash \tau <: \sigma}{\vdash (\bigcup \vec{\tau}) <: \sigma}$	<b>S-PAIR</b> $\frac{\vdash \sigma_1 <: \tau_1 \quad \vdash \sigma_2 <: \tau_2}{\vdash \langle \sigma_1, \sigma_2 \rangle <: \langle \tau_1, \tau_2 \rangle}$		
<b>S-FUN</b> $\frac{\vdash \sigma' <: \sigma \quad \vdash \tau <: \tau' \quad \psi_+ \vdash \psi'_+ \quad \psi_- \vdash \psi'_- \quad \vdash o <: o'}{\vdash x:\sigma \xrightarrow[o]{\psi_+ \psi_-} \tau <: x:\sigma' \xrightarrow[o']{\psi'_+ \psi'_-} \tau'}$		<b>S-VEC</b> $\frac{\forall i, \vdash \sigma_i <: \tau_i}{\vdash [[\sigma_o \dots \sigma_i]] <: [[\tau_o \dots \tau_i]]}$	<b>S-INEQ</b> $\frac{\text{overconstrained}(\Phi \bowtie \bar{\Phi}')}{\vdash \{\diamond : \mathbf{N} \mid \Phi\} <: \{\diamond : \mathbf{N} \mid \Phi'\}}$			
<b>S-INEQNUM</b> $\frac{\vdash \mathbf{N} <: \tau}{\vdash \{\diamond : \mathbf{N} \mid \Phi\} <: \tau}$						
<b>L-ATOM</b> $\frac{\psi \in \Gamma}{\Gamma \vdash \psi}$	<b>L-TRUE</b> $\Gamma \vdash \mathbb{T}$	<b>L-FALSE</b> $\frac{\Gamma \vdash \mathbb{F}}{\Gamma \vdash \psi}$	<b>L-ANDI</b> $\frac{\Gamma \vdash \psi_1 \quad \Gamma \vdash \psi_2}{\Gamma \vdash \psi_1 \wedge \psi_2}$	<b>L-ANDE</b> $\frac{\Gamma, \psi_1, \psi_2 \vdash \psi}{\Gamma, \psi_1 \wedge \psi_2 \vdash \psi}$	<b>L-IMPI</b> $\frac{\Gamma, \psi_1 \vdash \psi_2}{\Gamma \vdash \psi_1 \supset \psi_2}$	<b>L-IMPE</b> $\frac{\Gamma \vdash \psi_1 \quad \Gamma \vdash \psi_1 \supset \psi_2}{\Gamma \vdash \psi_2}$
<b>L-ORI</b> $\frac{\Gamma \vdash \psi_1 \text{ or } \Gamma \vdash \psi_2}{\Gamma \vdash \psi_1 \vee \psi_2}$	<b>L-ORE</b> $\frac{\Gamma, \psi_1 \vdash \psi \quad \Gamma, \psi_2 \vdash \psi}{\Gamma, \psi_1 \vee \psi_2 \vdash \psi}$	<b>L-SUB</b> $\frac{\Gamma \vdash \tau_x \quad \vdash \tau <: \sigma}{\Gamma \vdash \sigma_x}$	<b>L-SUBNOT</b> $\frac{\Gamma \vdash \bar{\sigma}_x \quad \vdash \tau <: \sigma}{\Gamma \vdash \bar{\tau}_x}$	<b>L-BOT</b> $\frac{\Gamma \vdash \perp_x}{\Gamma \vdash \psi}$		
<b>L-UPDATE</b> $\frac{\Gamma \vdash \tau_x \quad \Gamma \vdash \nu_x}{\Gamma \vdash \text{update}(\tau, \nu)_x}$	<b>L-INEQ</b> $\frac{\Gamma \vdash \mathbf{N}_x \quad \Gamma \vdash \Phi' \quad \text{overconstrained}(\Phi' \bowtie (\Phi[x/\diamond]))}{\Gamma \vdash \{\diamond : \mathbf{N} \mid \Phi\}_x}$	<b>L-INEQSIMPLE</b> $\frac{\Gamma, \mathbf{N}_x, \Phi[x/*] \vdash \psi}{\Gamma, \{\diamond : \mathbf{N} \mid \Phi\}_x \vdash \psi}$	<b>L-INEQJOIN</b> $\frac{\Gamma, (\Phi_1 \bowtie \Phi_2) \vdash \psi}{\Gamma, \Phi_1, \Phi_2 \vdash \psi}$			