NetXPTO - LinkPlanner

22 de Novembro de 2017

Conteúdo

Capítulo 1

Introduction

LinkPlanner is devoted to the simulation of point-to-point links.

Capítulo 2

Simulator Structure

LinkPlanner is a signals open-source simulator.
The major entity is the system.
A system comprises a set of blocks.
The blocks interact with each other through signals.

2.1 System

2.2 Blocks

2.3 Signals

List of available signals:

- Signal

Capítulo 3

Development Cycle

The NetXPTO-LinkPlanner has been developed by several people using git as a version control system. The NetXPTO-LinkPlanner repository is located in the GitHub site http://github.com/netxpto/linkplanner. The more updated functional version of the software is in the branch master. Master should be considered a functional beta version of the software. Periodically new releases are delivered from the master branch under the branch name Release<Year><Month><Day>. The integration of the work of all people is performed by Armando Nolasco Pinto in the branch Develop. Each developer has is how branch with his/her name.

Capítulo 4

Visualizer

visualizer

Capítulo 5

Case Studies

5.1 QPSK Transmitter

2017-08-25, Review, Armando Nolasco Pinto

This system simulates a QPSK transmitter. A schematic representation of this system is shown in figure 5.1.

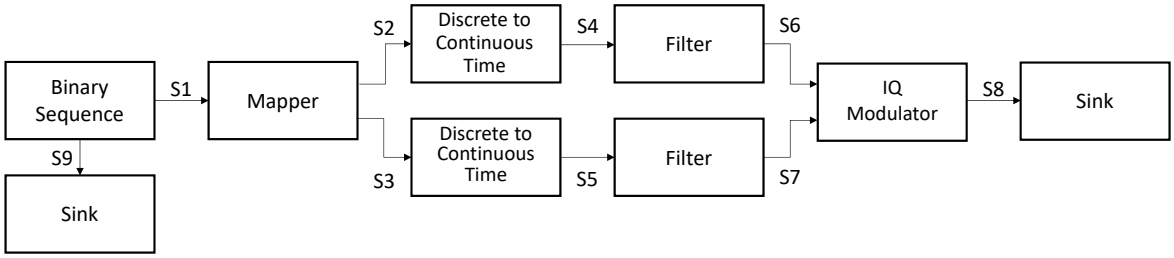


Figura 5.1: QPSK transmitter block diagram.

System Input Parameters

Parameter: *sourceMode*

Description: Specifies the operation mode of the binary source.

Accepted Values: PseudoRandom, Random, DeterministicAppendZeros, DeterministicCyclic.

Parameter: *patternLength*

Description: Specifies the pattern length used my the source in the PseudoRandom mode.

Accepted Values: Integer between 1 and 32.

Parameter: *bitStream*

Description: Specifies the bit stream generated by the source in the DeterministicCyclic and DeterministicAppendZeros mode.

Accepted Values: "XXX..", where X is 0 or 1.

Parameter: *bitPeriod*

Description: Specifies the bit period, i.e. the inverse of the bit-rate.

Accepted Values: Any positive real value.

Parameter: *iqAmplitudes*

Description: Specifies the IQ amplitudes.

Accepted Values: Any four par of real values, for instance { { 1,1 },{ -1,1 },{ -1,-1 },{ 1,-1 } }, the first value correspond to the "00", the second to the "01", the third to the "10"and the forth to the "11".

Parameter: *numberOfBits*

Description: Specifies the number of bits generated by the binary source.

Accepted Values: Any positive integer value.

Parameter: *numberOfSamplesPerSymbol*

Description: Specifies the number of samples per symbol.

Accepted Values: Any positive integer value.

Parameter: *rollOffFactor*

Description: Specifies the roll off factor in the raised-cosine filter.

Accepted Values: A real value between 0 and 1.

Parameter: *impulseResponseTimeLength*

Description: Specifies the impulse response window time width in symbol periods.

Accepted Values: Any positive integer value.

»»»> Romil

5.2 BPSK Transmission System

Student Name	:	Daniel Pereira (2017/09/01 - 2017/11/16)
Goal	:	Estimate the BER in a Binary Phase Shift Keying optical transmission system with additive white Gaussian noise. Comparison with theoretical results.
Directory	:	sdf/bpsk_system

Binary Phase Shift Keying (BPSK) is the simplest form of Phase Shift Keying (PSK), in which binary information is encoded into a two state constellation with the states being separated by a phase shift of π (see Figure 5.2).

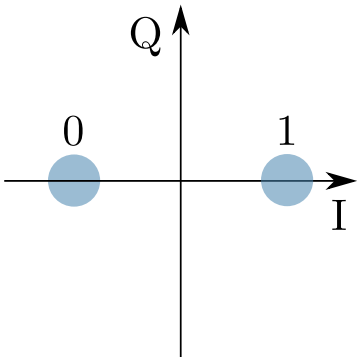


Figura 5.2: BPSK symbol constellation.

White noise is a random signal with equal intensity at all frequencies, having a constant power spectral density. White noise is said to be Gaussian (WGN) if its samples follow a normal distribution with zero mean and a certain variance σ^2. For WGN its spectral density equals its variance. For the purpose of this work, additive WGN is used to model thermal noise at the receivers.

The purpose of this system is to simulate BPSK transmission in back-to-back configuration with additive WGN at the receiver and to perform an accurate estimation of the BER and validate the estimation using theoretical values.

5.2.1 Theoretical Analysis

The output of the system with added gaussian noise follows a normal distribution, whose first probabilistic moment can be readily obtained by knowledge of the optical power of the received signal and local oscillator,

$$m_i = 2\sqrt{P_L P_S} G_{ele} \cos(\Delta\theta_i), \tag{5.1}$$

where P_L and P_S are the optical powers, in watts, of the local oscillator and signal, respectively, G_{ele} is the gain of the trans-impedance amplifier in the coherent receiver and

$\Delta\theta_i$ is the phase difference between the local oscillator and the signal, for BPSK this takes the values π and 0, in which case (5.1) can be reduced to,

$$m_i = (-1)^{i+1} 2\sqrt{P_L P_S} G_{ele},\ i = 0,\ 1. \tag{5.2}$$

The second moment is directly chosen by inputting the spectral density of the noise σ^2, and thus is known *a priori*.

Both probabilist moments being known, the probability distribution of measurement results is given by a simple normal distribution,

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m_i)^2}{2\sigma^2}}. \tag{5.3}$$

The BER is calculated in the following manner,

$$BER = \frac{1}{2}\int_0^{+\infty} f(x|\Delta\theta = \pi)\mathrm{d}x + \frac{1}{2}\int_{-\infty}^{0} f(x|\Delta\theta = 0)\mathrm{d}x, \tag{5.4}$$

given the symmetry of the system, this can be simplified to,

$$BER = \int_0^{+\infty} f(x|\Delta\theta = \pi)\mathrm{d}x = \frac{1}{2}\mathrm{erfc}\left(\frac{-m_0}{\sqrt{2}\sigma}\right) \tag{5.5}$$

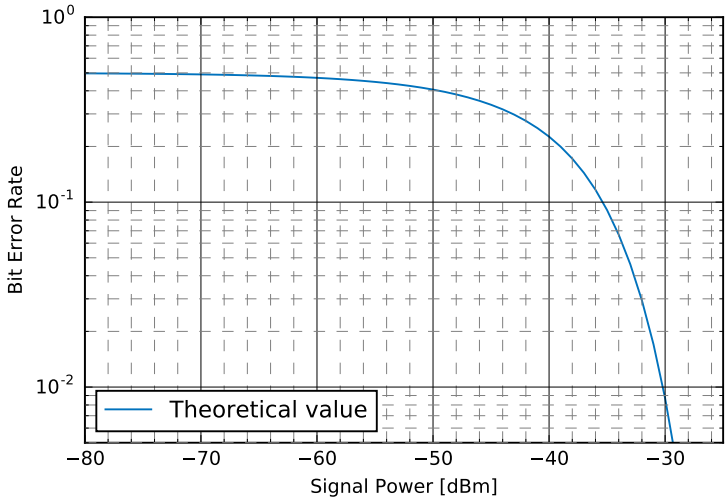


Figura 5.3: Bit Error Rate in function of the signal power in dBm at a constant local oscillator power level of 0 dBm.

5.2.2 Simulation Analysis

A diagram of the system being simulated is presented in the Figure 5.4. A random binary sequence is generated and encoded in an optical signal using BPSK modulation. The decoding of the optical signal is accomplished by an homodyne receiver, which combines the

signal with a local oscillator. The received binary signal is compared with the transmitted binary signal in order to estimate the Bit Error Rate (BER). The simulation is repeated for multiple signal power levels, each corresponding BER is recorded and plotted against the expectation value.

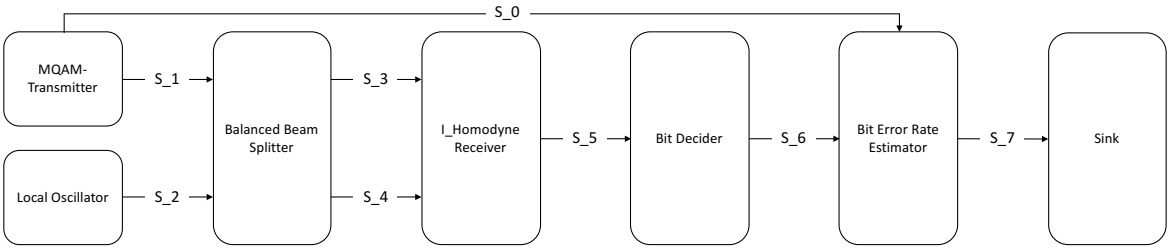


Figura 5.4: Overview of the BPSK system being simulated.

Required files

Header Files		
File	**Comments**	**Status**
add.h		✓
balanced_beam_splitter.h		✓
binary_source.h		✓
bit_decider.h		✓
bit_error_rate.h		✓
discrete_to_continuous_time.h		✓
netxpto.h		✓
m_qam_mapper.h		✓
m_qam_transmitter.h		✓
local_oscillator.h		✓
i_homodyne_reciever.h		✓
ideal_amplififer.h		✓
iq_modulator.h		✓
photodiode.h		✓
pulse_shaper.h		✓
sampler.h		✓
sink.h		✓
super_block_interface.h		✓
white_noise.h		✓

Source Files		
File	**Comments**	**Status**
add.cpp		✓
balanced_beam_splitter.cpp		✓
binary_source.cpp		✓
bit_decider.cpp		✓
bit_error_rate.cpp		✓
discrete_to_continuous_time.cpp		✓
netxpto.cpp		✓
m_qam_mapper.cpp		✓
m_qam_transmitter.cpp		✓
local_oscillator.cpp		✓
i_homodyne_reciever.cpp		✓
ideal_amplififer.cpp		✓
iq_modulator.cpp		✓
photodiode.cpp		✓
pulse_shaper.cpp		✓
sampler.cpp		✓
sink.cpp		✓
super_block_interface.cpp		✓
white_noise.cpp		✓

System Input Parameters

This system takes into account the following input parameters:

System Input Parameters		
Parameter	**Default Value**	**Comments**
numberOfBitsGenerated	40000	
bitPeriod	20×10^{-12}	
samplesPerSymbol	16	
pLength	5	
iqAmplitudesValues	$\{\{-1,\ 0\}\ ,\ \{1,\ 0\}\ \}$	
outOpticalPower_dBm	Variable	Value varied from -75 dBm to -25 dBm with intervals of 5 dBm
loOutOpticalPower_dBm	0	
localOscillatorPhase	0	
transferMatrix	$\{\{\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \frac{-1}{\sqrt{2}}\}\ \}$	
responsivity	1	
amplification	10^3	
noiseSpectralDensity	$5 \times 10^{-4}\sqrt{2}\ \mathrm{V}^2$	
confidence	0.95	
midReportSize	0	

Inputs

This system takes no inputs.

Outputs

This system outputs the following objects:

Parameter: Signals:

Description: Initial Binary String; (S_0)

Description: Optical Signal with coded Binary String; (S_1)

Description: Local Oscillator Optical Signal; (S_2)

Description: Beam Splitter Outputs; (S_3, S_4)

Description: Homodyne Detector Electrical Output; (S_5)

Description: Decoded Binary String; (S_6)

Description: BER result String; (S_7)

Parameter: Other:

Description: Bit Error Rate report in the form of a .txt file. (BER.txt)

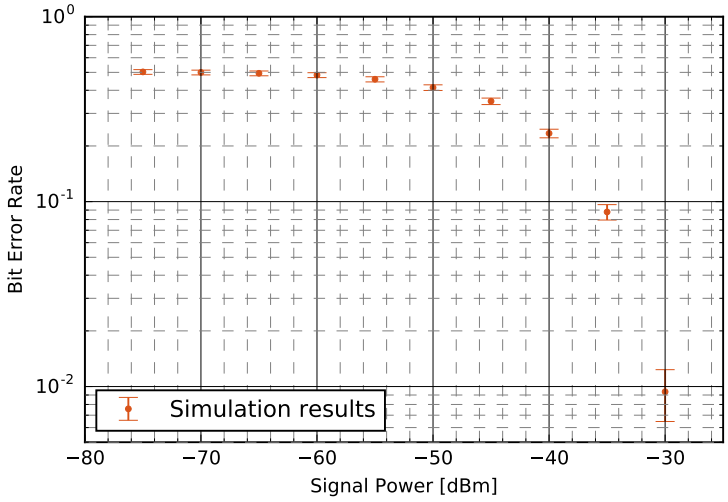


Figura 5.5: Bit Error Rate in function of the signal power in dBm at a constant local oscillator power level of 0 dBm.

5.2.3 Comparative Analysis

The following results show the dependence of the error rate with the signal power assuming a constant Local Oscillator power of $0\ dBm$, the signal power was evaluated at levels between -70 and -25 dBm, in steps of 5 dBm between each. The simulation results are presented in orange with the computed lower and upper bounds, while the expected value, obtained from (5.5), is presented as a full blue line. A close agreement is observed between the simulation results and the expected value. The noise spectral density was set at $5 \times 10^{-4}\sqrt{2}\ \mathrm{V}^2$ [1].

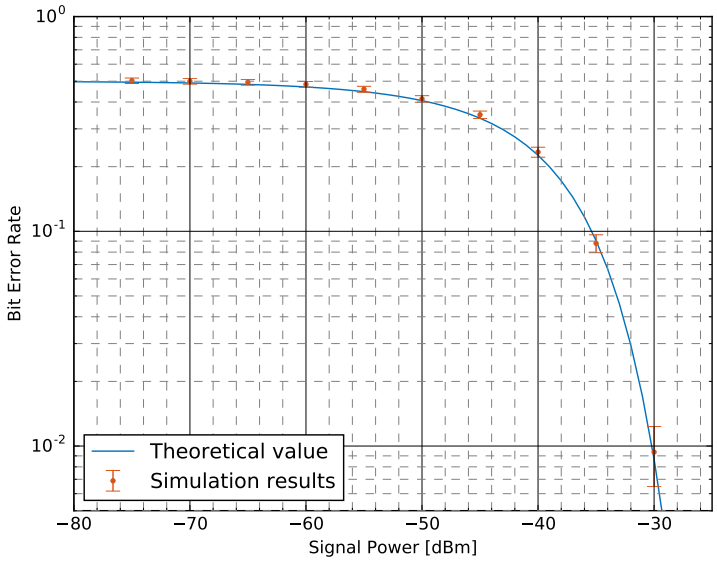


Figura 5.6: Bit Error Rate in function of the signal power in dBm at a constant local oscillator power level of 0 dBm. Theoretical values are presented as a full blue line while the simulated results are presented as a errorbar plot in orange, with the upper and lower bound computed in accordance with the method described in 6.2

Bibliografia

[1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.

[2] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.

[3] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.

[4] Min Xu, Run-hua Shi, Zhen-yu Luo, and Zhen-wan Peng. Nearest private query based on quantum oblivious key distribution. *Quantum Information Processing*, 16(12):286, Oct 2017.

[5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

5.3 M-QAM Transmission System

Student Name	:	Ana Luisa Carvalho
Goal	:	M-QAM system implementation with BER measurement and comparison with theoritical values.
Directory	:	sdf/m_qam_system

5.3.1 Introduction

M-QAM, which stands for Quadrature Amplitude Modulation, is a modulation scheme that takes advantage of two carriers (usually sinusoidal waves) with a phase difference of $\frac{\pi}{2}$. The resultant output consists of a signal with both amplitude and phase variations. The two carriers, refered to as I (In-phase) and Q (Quadrature), can be represented as

$$I = A\cos(\phi) \tag{5.6}$$
$$Q = A\sin(\phi) \tag{5.7}$$

which means that any sinusoidal wave can be decomposed in its I and Q components:

$$A\cos(\omega\, t + \phi) = A\left(\cos(\omega\, t)\cos(\phi) - \sin(\omega\, t)\sin(\phi)\right) \tag{5.8}$$
$$= I\cos(\omega\, t) - Q\sin(\omega\, t), \tag{5.9}$$

where we have used the expression for the cossine of a sum and the definitions of I and Q.

For M$= 4$ the symbol constellation is shown figure **??**.

M can take several values: $2, 4, 16, 32, ...$ The first two correspond to BPSK and QPSK modulation, respectively.

5.3.2 Bit Error Rate for 4-QAM with Additive White Gaussian Noise (AWGN)

When demodulating a signal it is necessary to associate the received signal to the corresponding signal. The existence of noise in the channel means that we can only compute the probability that a given signal corresponds to a certain carrier and that's why we need to define the Bit Error Rate (BER). Using

$$P_i f(s|c_i) > P_j f(s|c_j), \qquad i \neq j \tag{5.10}$$

where $f(s|c_i)$ stands for the probability of detecting the signal s given that c_i was emmited. This inequallity can be rewritten in the following way

$$P(c_i|s) > P(c_j|s) \tag{5.11}$$

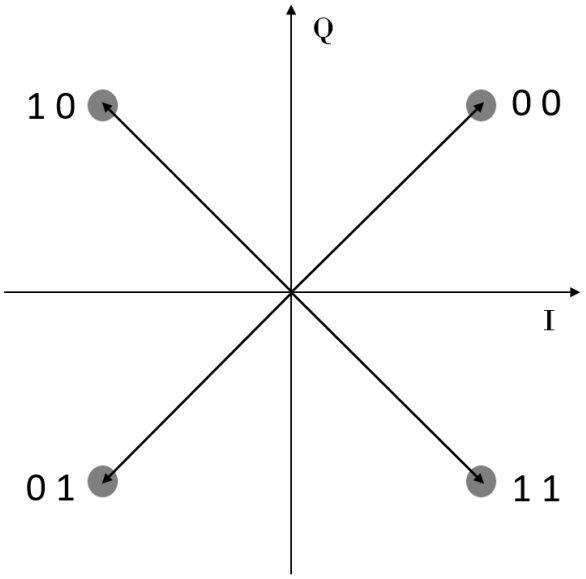


Figura 5.7: 4-QAM symbol constellation

where $P(c_i|s)$ and $P(c_j|s)$ are called *a posteriori* probabilities and represent the probability that c_i or c_j were transmitted given that s was received. In terms of the systems this simply means that we should select the signal most likely to have been transmitted.

In the case of additive white gaussian noise the f function is simply given by

$$f(s|c_i) = \frac{e^{-x^2/n_0}}{(\pi n_0)^{N/2}} \tag{5.12}$$

where x is the Euclidean distance in the I-Q plane between the signal received and carrier i and N is the number of noise samples.

When using 4-QAM modulation all points are at an equal distance from the origin (in the I-Q plane) so they all have the same energy given by

$$E = \frac{d^2}{2} \tag{5.13}$$

where d is the side of the square formed bye the constellation points.

The probability that a given signal is identified correctly is given by

$$P_c = r^2 \tag{5.14}$$

where $n_0/2$ is the noise variance for AWGN and

$$r = \int_{-d/2}^{\infty} \frac{e^{-x^2/n_0}}{\sqrt{\pi n_0}} dx. \tag{5.15}$$

The error probability, P_e, given by $1 - P_c$ is given by

$$P_e = erfc\sqrt{\frac{E}{2n_0}}. \tag{5.16}$$

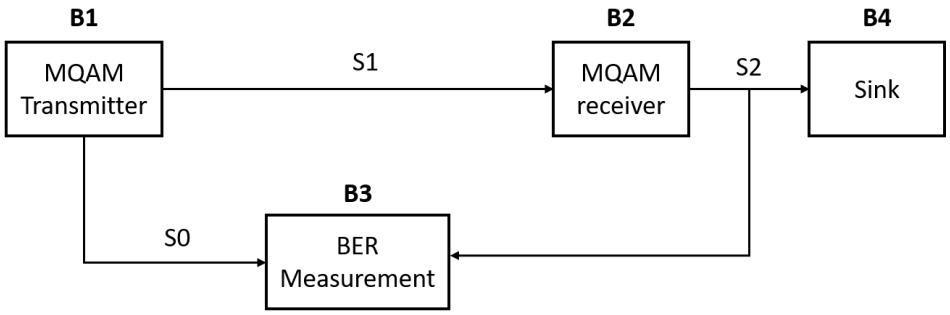


Figura 5.8: Schematic representation of the MQAM system.

5.3.3 Simulation setup

The M-QAM system transmission system is a complex block of code that simulates the modulation, transmission and demodulation of an optical signal using M-QAM modulation. It is composed of four blocks: a transmitter, a receiver, a sink and a block that performs a Bit Error Rate (BER) measurement.

Current state: The system currently being implement is a QPSK system (M=4).

Future work: Extend this block to include other values of M.

5.3.4 Functional description

The schematic representation of the system is presented in figure 5.8.

A complete description of the M-QAM transmitter and M-QAM receiver blocks can be found in the *Library* chapter of this document as well as a detailed description of the independent blocks that compose these blocks.

The M-QAM transmitter is a complex block that generates one or two optical signals by enconding a binary string using M-QAM modulation. It also outputs a binary signal that is used to perform a BER measurement.

The M-QAM receiver is a homodyne receiver. It is a complex block that accepts one input optical signal and outputs a binary signal. It performs the M-QAM demodulation of the input signal by combining the optical signal with a local oscillator.

The demodulated optical signal is compared to the one produced by the transmitter in order to estimate the Bit Error Rate (BER).

The files corresponding to each of the system's blocks are summarized in table **??**. Along with the library and corresponding source files these allow for the full operation of the M-QAM system described here.

Tabela 5.1: Main system files

System blocks	**cpp file**	**include file**
Main	m_qam_system_sdf.cpp	—
M-QAM transmitter	m_qam_transmitter.cpp	m_qam_transmitter.h
M-QAM receiver	homodyne_receiver.cpp	homodyne_receiver.h
Sink	sink.cpp	sink.h
BER estimator	bit_error_rate.cpp	bit_error_rate.h

5.3.5 Input Parameters

5.3.6 Output Parameters

As output this block

5.3.7 BER measurement

Tabela 5.2: Input parameters

Parameter	**Type**	**Description**
numberOfBitsGenerated	t_integer	Determines the number of bits to be generated by the binary source
samplesPerSymbol	t_integer	
prbsPatternLength	int	Determines the length of the pseudorandom sequence pattern (used only when the binary source is operated in *PseudoRandom* mode)
bitPeriod	t_real	Temporal interval occupied by one bit
rollOffFactor	t_real	
signalOutputPower_dBm	t_real	Determines the power of the output optical signal in dBm
numberOfBitsReceived	int	
iqAmplitudeValues	vector<t_iqValues>	Determines the constellation used to encode the signal in IQ space
symbolPeriod	double	Given by bitPeriod/samplesPerSymbol
localOscillatorPower_dBm	t_real	Power of the local oscillator
responsivity	t_real	Responsivity of the photodiodes (1 corresponds to having all optical power transformed into electrical current)
amplification	t_real	??
noiseAmplitude	t_real	??
samplesToSkip	t_integer	Number of samples to be skipped by the *sampler* block
confidence	t_real	Determines the confidence limits for the BER estimation
midReportSize	t_integer	
bufferLength	t_integer	Corresponds to the number of samples that can be processed in each run of the system

5.4 Quantum Noise

Student Name	:	Diamantino Silva
Starting Date	:	October 19, 2017
Goal	:	Simulation of quantum noise in double homodyne detection.
Directory	:	sdf/quantum_noise

Quantum noise is an intrinsic property of light, a manifestation of the vaccum field fluctuations [**?**]. Contrarily to the majority of noise sources, that are overcomed by better equipment, quantum noise has a lower bound, given by the Heisenberg uncertanty principle, which cannot be broken. This propertie can be useful in some areas, such in quantum cryptography, were many protocols depend on it to ensure their security.
The objective of this work is to develop a numerical model for the quantum noise in a double homodyne detection and to validate the numerical model with experimental results.

5.4.1 Theoretical description

We start by defining number states $|n\rangle$ (or Fock states), which correspond to states with perfectly fixed number of photons [**?**]. Associated to those states are two operators, the creation $\hat{a}^\dagger$ and annihilation $\hat{a}$ operators, which in a simple way, remove or add one photon from a given number state [**?**]. Their action is defined as

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle \quad (5.17)\,, \qquad \hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle \quad (5.18)\,, \qquad \hat{n}\,|n\rangle = n\,|n\rangle \quad (5.19)$$

in which $\hat{n} = \hat{a}^\dagger\hat{a}$ is the number operator. Therefore, number states are eigenvectors of the number operator.

Coherent states have properties that closely resemble classical electromagnetic waves, and are generated by single-mode lasers well above the threshold. [**?**] We can defined them, using number states in the following manner

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle \tag{5.20}$$

in which the complex number α is the sole parameter that characterizes it. In fact, if we calculate the expected number of photons with $\langle\alpha|\,\hat{n}\,|\alpha\rangle$ we will obtain $|\alpha|^2$. The coherent state is an eigenstate of the annihilation operator, $\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$.

Using the creation and annihilation operators, we can define two quadrature operators [**?**]

$$\hat{X} = \frac{1}{2}\left(\hat{a}^\dagger + \hat{a}\right) \quad (5.21)\,, \qquad \hat{Y} = \frac{i}{2}\left(\hat{a}^\dagger - \hat{a}\right) \quad (5.22)$$

The expected value of these two operators, using a coherent state $|\alpha\rangle$ are

$$\langle \hat{X} \rangle = \text{Re}(\alpha) \quad (5.23)\,, \qquad \langle \hat{Y} \rangle = \text{Im}(\alpha) \quad (5.24)$$

We see that the expected value of these operators give us the real and imaginary part of α. Now, we can obtain the uncertainty of these operators, using:

$$\text{Var}(\hat{X}) = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2 \tag{5.25}$$

For each of these quadrature operators the variance will be

$$\text{Var}(\hat{X}) = \text{Var}(\hat{Y}) = \frac{1}{4} \tag{5.26}$$

This result show us that for both quadratures, the variance of measurement is the same and independent of the value of α.

Homodyne detection

The measurent of a quadrature of an input signal (S) is made by using the balanced homodyne detection technique, which measures the phase difference between the input signal and a local oscillator (LO). The measurement of quadrature are made relative to a reference phase of the LO, such that if the measurement is made in-phase with this reference, the value will be proportional to the $\hat{X}$ quadrature of the signal. If the phase of the LO is has an offset of $\pi/2$ relative to the reference, the output will be proportional to the $\hat{Y}$ quadrature of the signal.

Experimentally, the balanced homodyne detection requires a local oscillator with the same frequency as the input signal, but with a much larger amplitude. These two signals are combined using a 50:50 beam splitter, from were two beams emerge, which are then converted to currents using photodides. Finally, the two currents are subtracted, resulting in an output current proportional to a quadrature of the input signal [**?**].
A phase of the local oscillator can be defined as the reference phase. A phase offset equal to 0 or $\pi/2$ will give an output proportional to the signal's in-phase component or to the quadrature component, respectively. Therefore, the $\hat{X}$ operator will correspond to the in-phase component and $\hat{Y}$ operator correspond to quadrature component

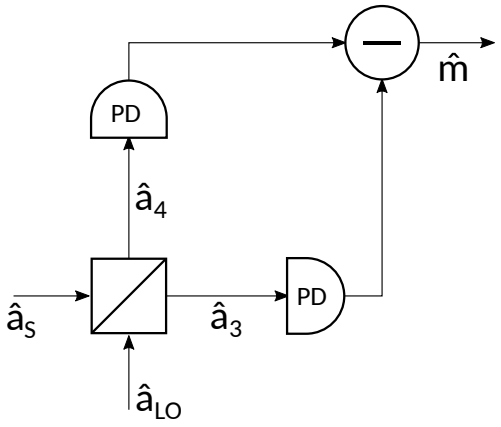


Figura 5.9: Balanced homodyne detection.

In the lab and in our simulations, a more complex system is used, the double balanced homodyne detection, which allows the simultaneous measurement of the $\hat{X}$ and $\hat{Y}$ components. The signal is divided in two beam with half the power of the original. One of the beams is used in a balanced homodyne detection with a local oscillator. The other beam is used in another balanced homodyne detection, but using a local oscillator with a phase difference $\pi/2$ relative to the first one.

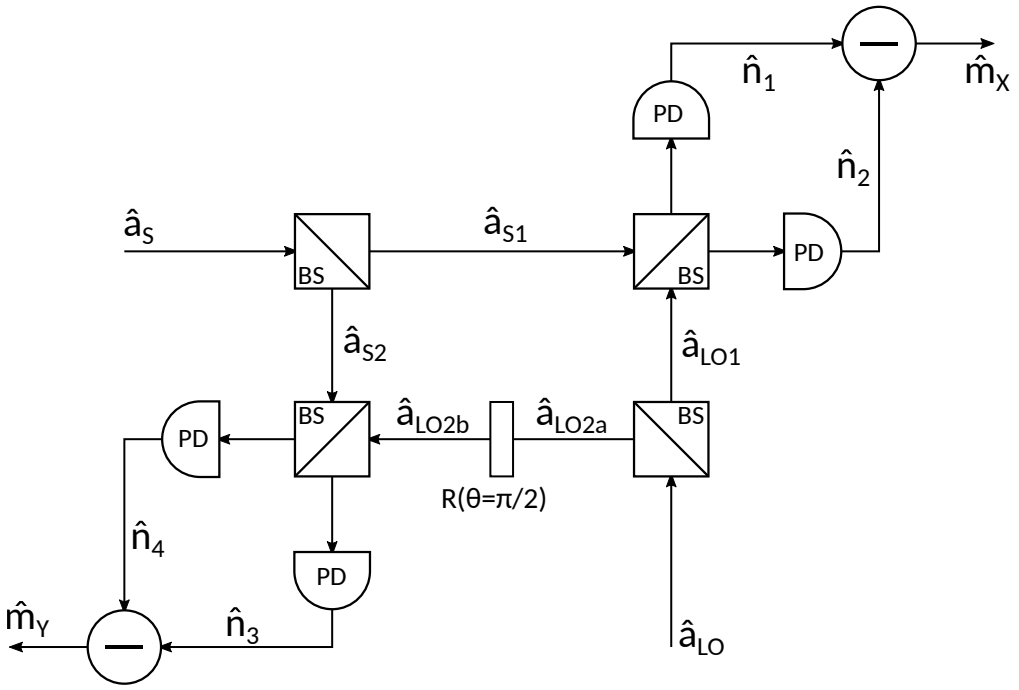


Figura 5.10: Balanced double homodyne detection.

Noise sources in homodyne detection

The detection of light using photodiodes is subjected to various sources of noise. One of these sources is the electrical field itself. The interaction of the signal with the vaccuum field adds quantum noise to the detection. Another source of noise comes from the detection system, such as photodiodes and other electrical circuits, originating various kinds of noise, such as thermal noise, dark noise and amplifier noise [**?**]. In the following sections, we will focus on two noise sources, quantum noise and thermal noise.

Quantum Noise

In order to grasp this effect, the quantum mechanical description of balanced homodyne detection will be used, employing quantum operators to describe the effect of each component in the system (fig. **??**). We start with the operators $\hat{a}_S$ and $\hat{a}_{LO}$ corresponding to the annihilation operator for the signal and local oscillator, which are the inputs in a beam divisor. The outputs will be $\hat{a}_3$ and $\hat{a}_4$. Using a balanced beam splitter, we can write the output as

$$\hat{a}_3 = \frac{1}{\sqrt{2}}\left(\hat{a}_S + \hat{a}_{LO}\right) \quad (5.27)\,, \qquad \hat{a}_4 = \frac{1}{\sqrt{2}}\left(\hat{a}_S - \hat{a}_{LO}\right) \quad (5.28)$$

The final output of a homodyne measurement will be proportional to the difference between the photocurrents in arm 3 and 4. Then

$$I_{34} = I_3 - I_4 \sim \langle \hat{n}_3 - \hat{n}_4 \rangle \tag{5.29}$$

We can define an operator that describes the difference of number of photons in arm 3 and arm 4:

$$\hat{m} = \hat{a}_3^\dagger \hat{a}_3 - \hat{a}_4^\dagger \hat{a}_4 \tag{5.30}$$

If we assume that the local oscillator produces the the coherent state $|\beta\rangle$, then the expected value of this measurement will be

$$\langle m \rangle = 2|\alpha||\beta|\cos(\theta_\alpha - \theta_\beta) \quad (5.31)\,, \qquad \text{Var}(m) = |\alpha|^2 + |\beta|^2 \quad (5.32)$$

The local oscillator normally has a greater power than the signal , then $|\alpha| \ll |\beta|$. If we use as unit, $2|\beta|$, then these two quantities can be simplified to

$$\langle m \rangle = |\alpha|\cos(\theta_\alpha - \theta_\beta) \quad (5.33)\,, \qquad \text{Var}(m) \approx \frac{1}{4} \quad (5.34)$$

[?]
Has we have seen previously, in order to measure two quadratures simultaneously, we can use double balanced homodyne detection. For each quadrature, the input signal now has half the power, so $|\alpha| \to |\alpha/\sqrt{2}|$. If we use a local oscillator that produces states $|\beta\rangle$, then we can divide it in two beams in state $|\beta/\sqrt{2}\rangle$ and $|i\beta/\sqrt{2}\rangle$ which will be used in each homodyne detection. In this setting, the expected values for each quadrature, X and Y, (in normalized values of $\sqrt{2}|\beta|$) are

$$\langle m_X \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \cos(\theta_\alpha - \theta_\beta) \quad (5.35)\,, \qquad \text{Var}(m_X) \approx \frac{1}{4} \quad (5.36)$$

$$\langle m_Y \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \sin(\theta_\alpha - \theta_\beta) \quad (5.37)\,, \qquad \text{Var}(m_Y) \approx \frac{1}{4} \quad (5.38)$$

Therefore the measurement of each quadrature will have half the amplitude, but the same variance.

Thermal noise

Thermal noise is generated by electrons in response to temperature. It's contribution to the resulting current can be described by the following equation [**?**]

$$\langle (\Delta i_T)^2 \rangle = 4K_B T_0 B / R_L \tag{5.39}$$

in which K_B it's Boltzmann's constant, T_0 is the absolute temperature, B is the bandwidth and R_L is the receiver load impedance. The B value is imposed by default or chosen when the measurements are made, but the R_L value is dependent in the internal setup of the various components of the detection system. Nevertheless, for simulation purposes, we can just introduce an experimental value.

5.4.2 Simulation setup

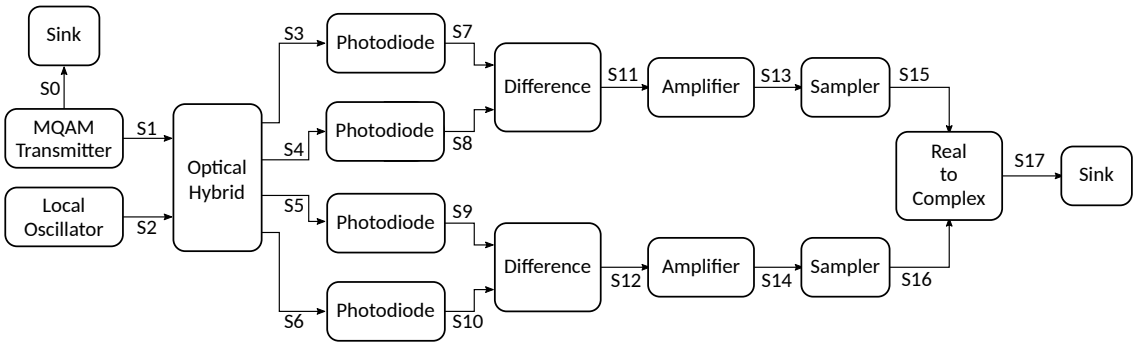


Figura 5.11: Overview of the simulated optical system.

List of signals used in the simulation:

Signal name	Signal type	Status
S0	Binary	check
S1	OpticalSignal	check
S2	OpticalSignal	check
S3	OpticalSignal	check
S4	OpticalSignal	check
S5	OpticalSignal	check
S6	OpticalSignal	check
S7	TimeContinuousAmplitudeContinuousReal	check
S8	TimeContinuousAmplitudeContinuousReal	check
S9	TimeContinuousAmplitudeContinuousReal	check
S10	TimeContinuousAmplitudeContinuousReal	check
S11	TimeContinuousAmplitudeContinuousReal	check
S12	TimeContinuousAmplitudeContinuousReal	check
S13	TimeContinuousAmplitudeContinuousReal	check
S14	TimeContinuousAmplitudeContinuousReal	check
S15	TimeDiscreteAmplitudeContinuousReal	check
S16	TimeDiscreteAmplitudeContinuousReal	check
S17	OpticalSignal	check

This system takes into account the following input parameters:

System Parameters	Default value	Description
localOscillatorPower1	2.0505×10^{-8}W	Sets the optical power, in units of W, of the local oscillator inside the MQAM
localOscillatorPower2	2.0505×10^{-8}W	Sets the optical power, in units of W, of the local oscillator used for Bob's measurements
localOscillatorPhase	0 rad	Sets the initial phase of the local oscillator used in the detection
responsivity	1 A/W	Sets the responsivity of the photodiodes used in the homodyne detectors
iqAmplitudeValues	$\{\{1,1\},\{-1,1\},$ $\{-1,-1\},\{1,-1\}\}$	Sets the amplitude of the states used in the MQAM

The simulation setup is represented in figure 5.11. The starting point is the MQAM, which generates random states from the constelation given by the variable `iqAmplitudeValues`. The output from the generator is received in the Optical Hybrid where it is mixed with a local oscillator, outputing two optical signal pairs. Each pair is converted to currents by two photodiodes, and the same currents are subtracted from

each other, originating another current proportional to one of the quadratures of the input state. The other pair suffers the same process, but the resulting subtraction current will be proportional to another quadrature, dephased by $\pi/2$ relative to the other quadrature.

Required files

Header Files

File	**Description**	**Status**
netxpto.h	Generic purpose simulator definitions.	check
m_qam_transmitter.h	Outputs a QPSK modulated optical signal.	check
local_oscillator.h	Generates continuous coherent signal.	check
optical_hybrid.h	Mixes the two input signals into four outputs.	check
photodiode.h	Converts an optical signal to a current.	check
difference.h	Ouputs the difference between two input signals.	check
ideal_amplifier.h	Performs a perfect amplification of the input sinal	check
sampler.h	Samples the input signal.	check
real_to_complex.h	Combines two real input signals into a complex signal	check
sink.h	Closes any unused signals.	check

Source Files

File	**Description**	**Status**
netxpto.cpp	Generic purpose simulator definitions.	check
m_qam_transmitter.cpp	Outputs a QPSK modulated optical signal.	check
local_oscillator.cpp	Generates continuous coherent signal.	check
optical_hybrid.cpp	Mixes the two input signals into four outputs.	check
photodiode.h	Converts an optical signal to a current.	check
difference.h	Ouputs the difference between two input signals.	check
ideal_amplifier.h	Performs a perfect amplification of the input sinal	check
sampler.cpp	Samples the input signal.	check
real_to_complex.cpp	Combines two real input signals into a complex signal	check
sink.cpp	Empties the signal buffer.	check

Simulation Results

To test the simulated implementation, a series of states $\{|\phi_i\rangle\}$ were generated and detected, resulting in a series of measurements $\{(x_i, y_i)\}$. The simulation result is presented in figure 5.12:

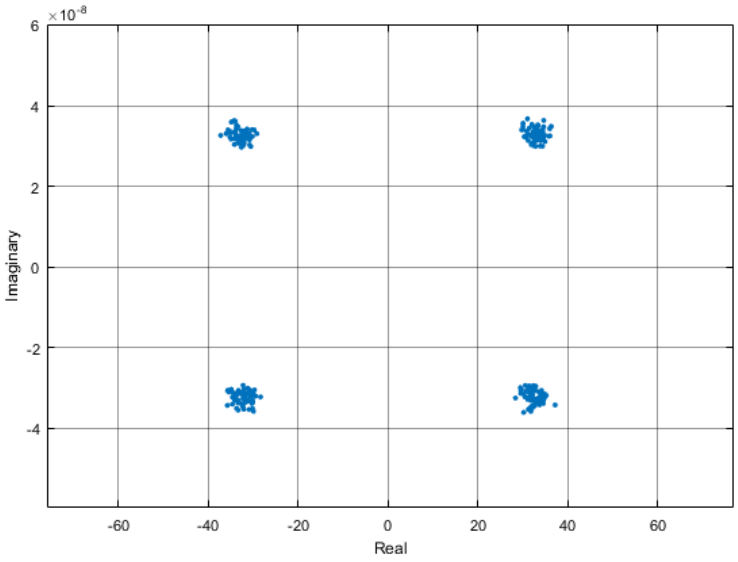


Figura 5.12: Simulation of a constelation of 4 states (n = 100)

We see that the measurements made groups in certain regions. Each of this groups is centered in the expected value $(\langle \mathrm{X} \rangle, \langle \mathrm{Y} \rangle)$ of one the generated states. Also, they show some variance, which was tested for various expected number of photons, $\langle n \rangle$, resulting in figure 5.13:

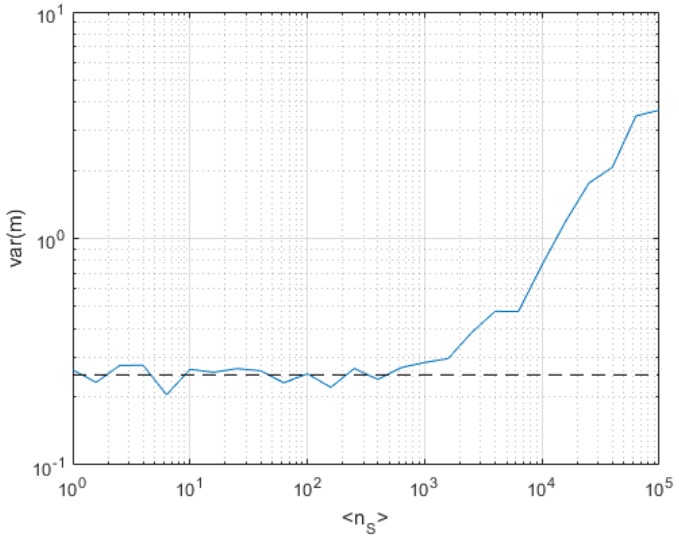


Figura 5.13: Simulation of the variance of m.
Local oscillator expected number of photons: 10^4

It was expected that the variance should independent of the input's signal number of photons. Plot 5.13 shows that for low values of n_S, the simulation is in accordance with the theoretical prevision, with Var(X) = Var(Y) = $\frac{1}{4}$. For large values of n_S, when the number of photons is about the same has the local oscillator, the quantum noise variance starts to grow proportionally to n_S, in accordance with the non approximated calculation of quantum noise (eq. **??**).

Noise Variance with LO power Simulation
The following plot shows the behavior of current noise variance $\langle(\Delta i)^2\rangle$ with local oscilator power, P_{LO}:

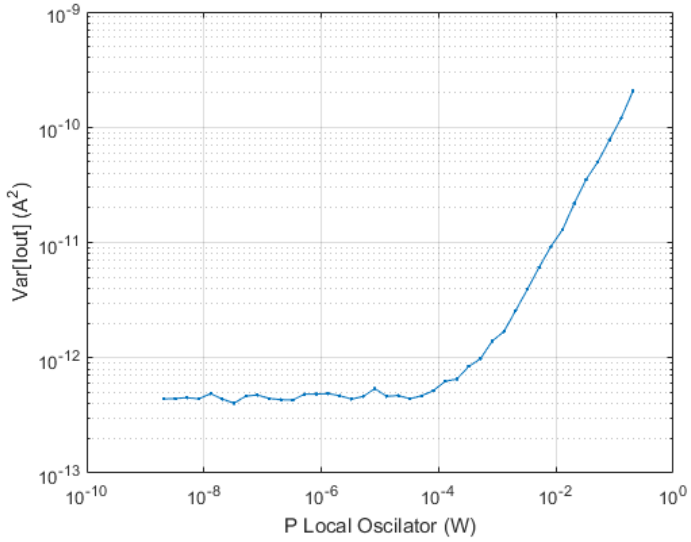


Figura 5.14: Output current variance in function of LO power.

We see that for low LO power, the dominant noise is the thermal contribution, but for higher power, quantum noise dominates, growing proportionally to P_{LO}. This in accordance with equation **??**

5.4.3 Experimental setup

The main objective of this experimental setup is to obtain the relation of the magnitude of quantum noise with the power of the inputs of a homodyne detection.
This experiment will be divided in two stages: a first stage for homodyne detection, measuring the noise in a single quadrature and a second stage measureing noise in a double homodyne detection where two quadratures will be measured simultaneously.
Our experiment will follow some settings from the paper [**?**], such as the timing between pulses and the pulse width.
To keep the experiment simple and avoid extra sources of noise, a simple setup was devised, as shown in fig. 5.15:

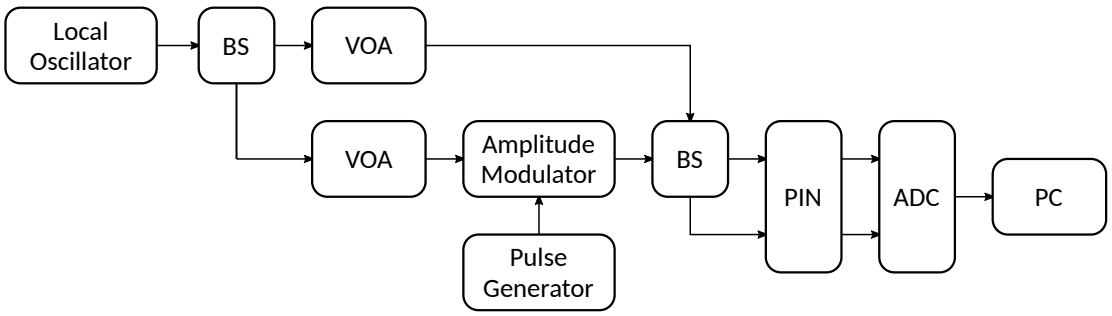


Figura 5.15: Experimental setup

Material list

Device	**Description**
Local Oscillator	Yenista OSICS Band C/AG
BS	Beam Splitter
Pulse Generator	HP 8116A Pulse Generator
Amplitude Modulator	Mach Zehnder SDL OC 48
VOA	Eigenlicht Power Meter 420
VOA	Thorlabs VOA 45-APC
PIN	Thorlabs PDB 450C
ADC	Picoscope 6403D

A single laser is splitted and used as the source for the signal (S) and the local oscillator (LO). The signal beam is pulsed and highly attenuated. The local oscillator is also attenuaded, but not pulsed. The signal and local oscillator interfere in a Beam Splitter originating two beams which are then converted to voltages in the PIN. These voltages are read in the ADC and collect in the computer. In the post processing phase, the quantum noise is measured by applying a difference between the two beams and measuring it's variance.
The second stage of the experiment will be very similar to the first one, in which the signal and local oscillator branches will be divided. One of the new branches of the local oscilator will suffer a phase delay of $\pi/2$, in order to measure the quadrature component of the incoming signal.

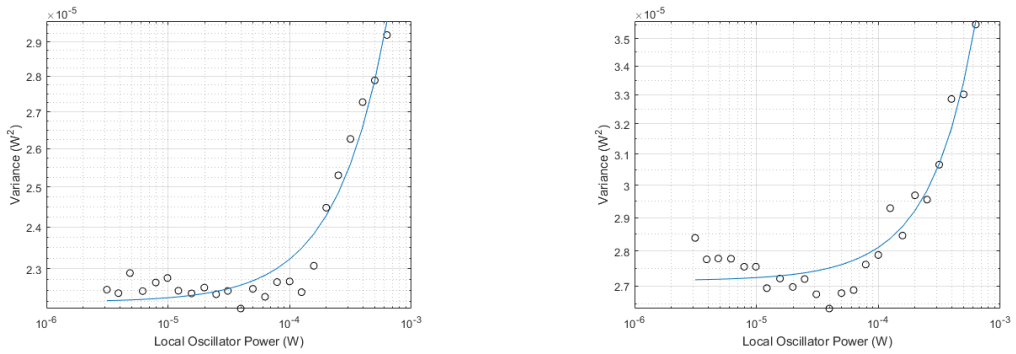
Data analysis

For each combination of the expected number of photons present in the signal and the local oscillator, we will obtain the variance in the plateau of the pulse. The final variance is simply the mean of the variances of all pulses.

5.4.4 Comparative analysis

WARNING – OLD DATA –

Given the theoretical, simulated and experimental frameworks, we will now compare the results given by them.



(a) X quadrature (b) Y quadrature

Figura 5.16: Noise variance dependency with local oscilator power for two different quadratures. Experimental vs fitted data.

Figures $5.16a$ and $5.16b$ show measurements of total noise for two different quadratures. For low power of LO, the noise variance flutuates around a constant value. For high power of LO, $(P_{LO} > 10^{-4}W)$, the variance of noise shows an increasing trend roughly proportional to P_{LO}^2. The polynomial fittings confirm this trend, showing a degree 2 coefficient much larger than the degree 1 coefficient

$$\text{Var}_X = 2.22{\times}10^{-5} + 9.6{\times}10^{-3}P_{LO} + 3.40P_{LO}^2 \tag{5.40}$$

$$\text{Var}_Y = 2.71{\times}10^{-5} + 8.9{\times}10^{-3}P_{LO} + 7.25P_{LO}^2 \tag{5.41}$$

The expected growth should be proportional to P_{LO}, but the RIN noise, originated by the electric apparatus, which grows quadratically with the power, is dominating the noise amplitude for large P_{LO}.
We see that both the simulation and experimental data display a similar behaviour, but the quadratic growth of noise for large P_{LO} was not predicted in the simulations.

5.4.5 Known problems

Bibliografia

[1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.

[2] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.

[3] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.

[4] Min Xu, Run-hua Shi, Zhen-yu Luo, and Zhen-wan Peng. Nearest private query based on quantum oblivious key distribution. *Quantum Information Processing*, 16(12):286, Oct 2017.

[5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

5.5 Continuous Variable QKD Transmission System

Student Name	:	Daniel Pereira (2017/05/01 -)
Goal	:	Simulation and experimental validation of a CV-QKD transmission system.
Directory	:	sdf/cv_system

The aim of Continuous Variable Quantum Key Distribution (CV-QKD) is to encode information in observables whose measurements take continuous values.

The purpose of this study is to analyse a CV-QKD transmission system in which the information is sent in the two orthogonal quadratures of a coherent state.

5.5.1 Theoretical Analysis

The security of QKD is a complex topic to tackle, given the difficulty of proving the non-existence of an attack that cracks a protocol's security. This topic is usually approached by analysing various eavesdropping strategies and evaluating their effects on the key rate referenced in (**??**). Information between Alice and Bob needs to be shared with current existing technology, so their shared information is always classical and Shannon's formalism is enough to describe it. However, Eve has no such restriction, so the information she has on Bob's results needs to be described according to the quantum properties of the system.

5.5.2 Simulation Analysis

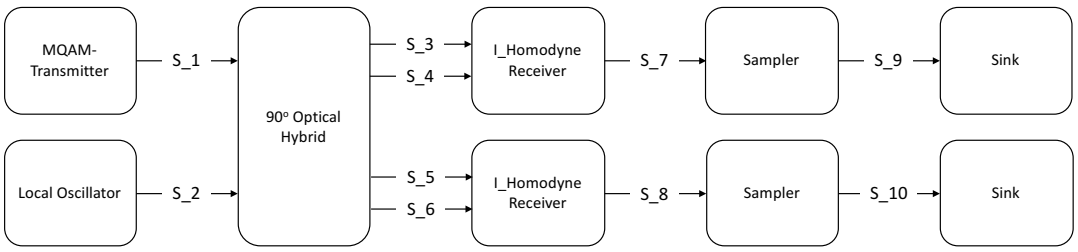


Figura 5.17: Overview of the CV-QKD system being simulated.

Required files

Header Files		
File	**Comments**	**Status**
add.h		✓
binary_source.h		✓
discrete_to_continuous_time.h		✓
i_homodyne_reciever.h	Working, but bootstrapped.	✓
ideal_amplififer.h		✓
iq_modulator.h		✓
local_oscillator.h		✓
m_qam_mapper.h		✓
m_qam_transmitter.h		✓
netxpto.h		✓
optical_hybrid.h		✓
photodiode.h	Currently being rewritten.	
pulse_shaper.h		✓
sampler.h		✓
sink.h		✓
super_block_interface.h		✓
white_noise.h		✓

Source Files		
File	**Comments**	**Status**
add.cpp		✓
binary_source.cpp		✓
discrete_to_continuous_time.cpp		✓
i_homodyne_reciever.cpp	Working, but bootstrapped.	✓
ideal_amplififer.cpp		✓
iq_modulator.cpp		✓
local_oscillator.cpp		✓
m_qam_mapper.cpp		✓
m_qam_transmitter.cpp		✓
netxpto.cpp		✓
optical_hybrid.cpp		✓
photodiode.cpp	Currently being rewritten.	
pulse_shaper.cpp		✓
sampler.cpp		✓
sink.cpp		✓
super_block_interface.cpp		✓
white_noise.cpp		✓

System Input Parameters

This system takes into account the following input parameters:

System Input Parameters		
Parameter	**Default Value**	**Comments**
numberOfBitsGenerated	40000	
bitPeriod	20×10^{-12}	
samplesPerSymbol	16	
pLength	5	
iqAmplitudesValues	$\{\{-1,\ 0\},\ \{1,\ 0\}\}$	
outOpticalPower_dBm	Variable	Value varied for presented study
loOutOpticalPower_dBm	0	
localOscillatorPhase	0	
transferMatrix	$\{\{\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \frac{-1}{\sqrt{2}}\}\}$	
responsivity	1	
amplification	10^3	
noiseSpectralDensity	$5 \times 10^{-4}\sqrt{2}\,\mathrm{V}^2$	
confidence	0.95	
midReportSize	0	

Inputs

This system takes no inputs.

Outputs

This system outputs the following objects:

Parameter: Signals:

Description: Optical Signal with coded Binary String; (S_1)

Description: Local Oscillator Optical Signal; (S_2)

Description: 90º Optical Hybrid Outputs; (S_3, S_4, S_5, S_6)

Description: Homodyne Detectors' Electrical Output; (S_7, S_8)

Description: Sampled Signals; (S_9, S_{10})

5.5.3 Comparative Analysis

Bibliografia

[1] Thorlabs. *Thorlabs Balance Amplified Photodetectors: PDB4xx Series Operation Manual*, 2014.

[2] Tie-Ming Liu, Lie-hui Jiang, Hong-qi He, Ji-zhong Li, and Xian Yu. *Researching on Cryptographic Algorithm Recognition Based on Static Characteristic-Code*, pages 140–147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.

[3] Gilles Brassard and Louis Salvail. *Secret-Key Reconciliation by Public Discussion*, pages 410–423. Springer Berlin Heidelberg, Berlin, Heidelberg, 1994.

[4] Min Xu, Run-hua Shi, Zhen-yu Luo, and Zhen-wan Peng. Nearest private query based on quantum oblivious key distribution. *Quantum Information Processing*, 16(12):286, Oct 2017.

[5] Álvaro J Almeida, Nelson J Muga, Nuno A Silva, João M Prata, Paulo S André, and Armando N Pinto. Continuous control of random polarization rotations for quantum communications. *Journal of Lightwave Technology*, 34(16):3914–3922, 2016.

5.6 Kramers-Kronig Transceiver with Stokes PolDemux

Student Name	:	Romil Patel
Starting Date	:	August 16, 2017
Goal	:	Develop a simplified structure (low cost) for a coherent transceiver, that can be used in coherent PON, inter-data center connections, or metropolitan networks (optical path lengths < 100 km). We are going to explore a Kramers-Kronig transceiver with Stokes based PolDemux.
Directory	:	LinkPlanner\doc\tex\sdf\simplified_coherent_receiver

Coherent optical transmission schemes are spectrally efficient since they allow the encoding of information in both quadrature of sinusoid signal. However, the cost of coherent receiver becomes a major obstacle in the case of short-reach links applications like PON, inter-data-center communications and metropolitan network. In order to make the transceiver applicable in short-reach links, an architecture has been proposed which combines the advantages of coherent transmission and cost effectiveness of direct detection. The working principle of the proposed transceiver is based on the Kramers-Kronig (KK) relationship. The KK transceiver scheme allows digital compensation of propagation impairment because both amplitude and phase of the electrical field can be retrieved at the receiver.

5.6.1 Theoretical Analysis

The Kramers-Kronig relations are bidirectional mathematical relations, connecting the real and imaginary parts of any complex function that is analytic in the upper half-plane. For instance, a signal $x(t) = x_r(t) + ix_i(t)$ satisfies the Kramers-Kronig relationship if,

$$x_r(t) = -\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{x_i(t')}{t - t'} dt'$$

$$x_i(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{x_r(t')}{t - t'} dt'$$

This relationship imposes that the real and the imaginary parts of the signal are related to each other though Hilbert transform. Therefore, if we have the real part of the signal then the imaginary part can be calculated by its Hilbert transform.
For a signal that satisfies the Kramers-Kronig relationship, the real and imaginary part can be obtained only from the module. The following questions would give a comprehensive overview of Kramers-Kroning relation and the detailed mathematical calculation which depicts how phase can be extracted from the amplitude information.

1. What is Hilbert transform?

If we consider a filter $H(\omega)$, described in Figure 5.18, that has a unity magnitude response for all frequencies and the phase response is $-\pi/2$ for all positive frequencies and $\pi/2$ for negative frequencies. The transfer function of this filter is given by

$$H(\omega) = -isgn(\omega) \tag{5.42}$$

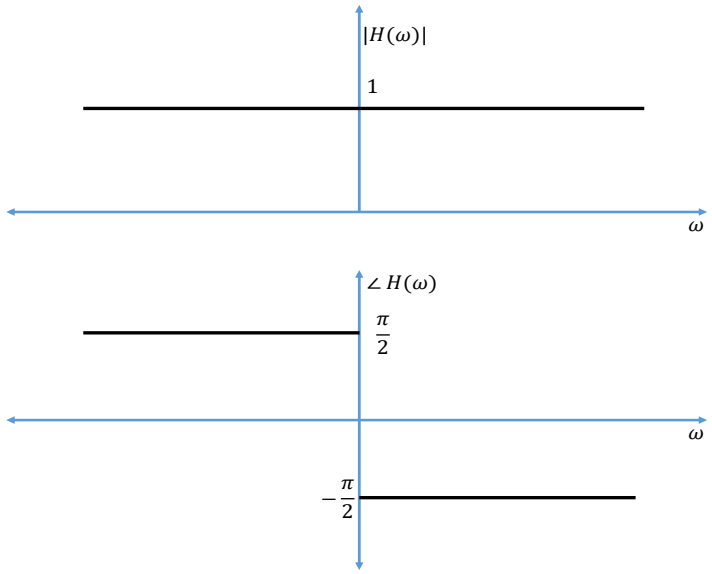


Figura 5.18: Magnitude and phase of Hilbert transform filter

The inpulse response of this filter can be given as,

$$\begin{aligned} h(t) &= \mathcal{F}^{-1}[H(i\omega)] \\ &= -i\mathcal{F}^{-1}[sgn(\omega)] \\ &= -i\left(\frac{i}{\pi t}\right) \\ &= \frac{1}{\pi t} \end{aligned} \tag{5.43}$$

When this filter driven by an arbitrary signal $s(t)$, the filter produces the output as,

$$\begin{aligned} \hat{s}(t) &= s(t) * h(t) \\ &= \int_{-\infty}^{\infty} \frac{s(u)}{\pi(t-u)} du \end{aligned} \tag{5.44}$$

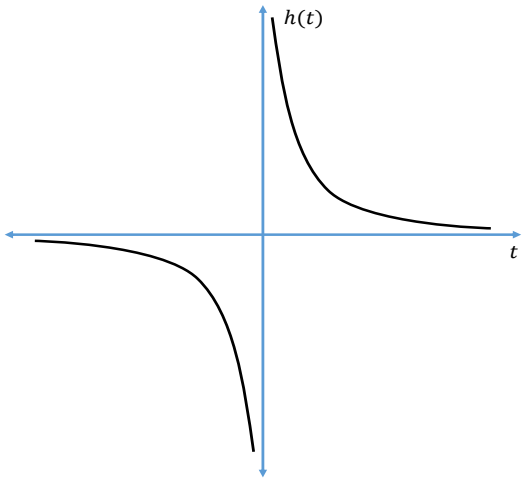


Figura 5.19: Impulse response $h(t)$ of Hilbert transform filter

The function $\hat{s}(t)$ is called the Hilbert transform if $s(t)$. Note that

$$\mathcal{F}[\hat{s}(t)] = H(\omega)S(\omega) = -isgn(\omega)S(\omega) \tag{5.45}$$

In conclusion, if we convolve any time domain signal with $\frac{1}{\pi t}$ then it will give us Hilbert transformed signal in time domain. Similarly, from the convolution property of the Fourier transform, if we multiply $-isgn(\omega)$ with any frequency domain signal $S(\omega)$ then it'll give us Hilbert transformed signal in frequency domain.

2. What is analytical signal?

An analytic signal is a complex-valued signal that has no negative frequency components, and its real and imaginary parts are related to each other by the Hilbert transform.

$$s_a(t) = s(t) + i\hat{s}(t) \tag{5.46}$$

where, $s_a(t)$ is an analytical signal and $\hat{s}(t)$ is the Hilbert transform of the signal $s(t)$. Such analytical signal can be used to generate Single Sideband Signal (SSB) signal.

3. What is a SSB signal and how it can be generated?

By definition, the SSB is the signal which contains either upper sideband or lower sideband and hence it reduces the spectral occupancy by half.
This section will represent the brief idea of generating SSB signal using Hilbert transform method. To understand this, we may express signal $s(t)$ as a summation of the two complex-valued functions.

$$s(t) = \frac{1}{2}[s(t) + i\hat{s}(t)] + \frac{1}{2}[s(t) - i\hat{s}(t)] \tag{5.47}$$

From Equation 5.46,

$$s(t) = s_a(t) + is_a^*(t) \tag{5.48}$$

Such representation of $s_a(t)$ and $s_a^*(t)$ divide the signal into non-negative frequency component and non-positive frequency component respectively. Considering only non-negative frequency $s_a(t)$ part, we can write it as

$$\frac{1}{2}S_a(f) = \begin{cases} S(f) & \text{for } f > 0 \\ 0 & \text{for } f < 0 \end{cases} \tag{5.49}$$

where $S_a(f)$ and $S(f)$ are the Fourier transform of $t_a(t)$ and $s(t)$ respectively. The frequency translated version of $S_a(f-f_0)$ contains only one side (positive) of $S(f)$ and hence it is called single sideband signal $s_{ssb}(t)$,

$$F^{-1}\{S_a(f-f_0)\} = s_a(t)e^{i2\pi f_0 t} = s_{ssb}(t) + i\hat{s}_{ssb}(t) \tag{5.50}$$

Therefore, from the Euler's formula,

$$\begin{aligned} s_{ssb}(t) &= Re\{s_a(t)e^{i2\pi f_0 t}\} \\ &= Re\{[s(t) + i\hat{s}(t)][cos(2\pi f_0 t) + isin(2\pi f_0 t)]\} \\ &= s(t)cos(2\pi f_0 t) - \hat{s}(t)sin(2\pi f_0 t) \end{aligned} \tag{5.51}$$

This Equation 5.51 displays the mathematical modeling of the upper sideband SSB signal. Similarly, we can generate lower sideband SSB signal by,

$$s_{ssb}(t) = s(t)cos(2\pi f_0 t) + \hat{s}(t)sin(2\pi f_0 t) \tag{5.52}$$

4. What is minimum phase signal?

A necessary and sufficient condition for a complex signal $A(t)$ to be minimum phase is that the curve described in a complex plane by $A(t)$ when $t \to -\infty$ to $t \to \infty$ **does not encircle the origin**. A minimum-phase signal has an useful property that the natural logarithm of the magnitude of the frequency response is related to the phase angle of the frequency response by the Hilbert transform.
For instance, if we consider a complex data-carrying signal whose spectrum is contained between $-B/2$ and $B/2$, and consider a SSB signal of the form,

$$x(t) = A + A_s(t)exp(-i\pi Bt) \tag{5.53}$$

where A is a constant. Here, $x(t)$ is minimum phase if and only if the winding number of its trajectory into complex plane is zero. The condition $|A| > |A_s(t)|$ is sufficient for guaranteeing minimum phase property [3].

5. How we can use these signals and profit from them?

This section represents the justification that why we need to use these signals into our proposed transceiver system.

Analytical Signal:
If we denote an analytic signal $A_s(t)$ as,

$$A_s(t) = A_{s,r}(t) + iA_{s,i}(t) \tag{5.54}$$

then in the equation 5.54, the real and imaginary parts $A_{s,r}(t)$ and $A_{s,i}(t)$ are related through the Kramers-Kronig relation with each other. An intuitive way to analyze the relation is based on expressing its Fourier transform $A_s(\omega)$ as follows,

$$A_s(\omega) = \frac{1}{2}[1 + sgn(\omega)]A_s(\omega) \tag{5.55}$$

The equation 5.55 follows the SSB signal condition $A_s(\omega) = 0$ for $\omega < 0$. Further, simplification0n of the signal can be summarized as follows:

$$\begin{aligned} A_s(\omega) &= \frac{1}{2}[1 + sgn(\omega)]A_s(\omega) \\ &= \frac{1}{2}A_s(\omega) + \frac{1}{2}sgn(\omega)A_s(\omega) \end{aligned} \tag{5.56}$$

Taking inverse Fourier transform of the equation 5.56,

$$\begin{aligned} A_s(t) &= IFT\{A_s(\omega)\} \\ &= \frac{1}{2}A_s(t) + \underline{\frac{1}{2}[IFT\{sgn(\omega)\} \circledast A_s(t)]} \end{aligned} \tag{5.57}$$

The underlined term in Equation 5.57 displays that multiplication in frequency domain converted into the convolution in the time domain. Further, IFT of the function $sgn(\omega)$ given as $(-i/\pi t)$. As a consequences, we can further simplify our equation as,

$$\begin{aligned} A_s(t) &= \frac{1}{2}A_s(t) + \frac{1}{2}\left[\frac{i}{\pi t} \circledast A_s(t)\right] \\ \frac{A_s(t)}{2} &= \frac{1}{2}\left[\frac{i}{\pi t} \circledast A_s(t)\right] \\ A_s(t) &= i\left[\frac{1}{\pi t} \circledast A_s(t)\right] \\ A_s(t) &= \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_s(t')}{t - t'} dt' \end{aligned} \tag{5.58}$$

Using Equation 5.54 into Equation 5.58,

$$A_{s,r}(t) + iA_{s,i}(t) = \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_s(t')}{t - t'} dt' \tag{5.59}$$

Therefore,

$$\begin{aligned} A_{s,r}(t) + iA_{s,i}(t) &= \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,r}(t') + iA_{s,r}(t')}{t - t'} dt' \\ A_{s,r}(t) + iA_{s,i}(t) &= -\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,i}(t')}{t - t'} dt' + \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,r}(t')}{t - t'} dt' \end{aligned} \tag{5.60}$$

which leads to,

$$\begin{aligned} A_{s,r}(t) &= -\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,i}(t')}{t - t'} dt' \\ A_{s,i}(t) &= \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{A_{s,r}(t')}{t - t'} dt' \end{aligned} \tag{5.61}$$

Minimum Phase signal:
Given function $A(t) = A_s(t) + \bar{A}$ never encircles the origin for $t \in (-\infty, \infty)$. if we define,

$$G(t) = log\left[\frac{A(t)}{\bar{A}}\right] \tag{5.62}$$

then $G(\omega)$, the spectrum of $G(t)$, is such that $G(\omega) = 0$ for $\omega < 0$. Under the hypothesis stated by Equation 5.59, we can write $G(t)$ as,

$$G(t) = \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{G(t')}{t - t'} dt' \tag{5.63}$$

From the equations 5.62 and 5.63,

$$log|A(t)| - log|\bar{A}| + i[\phi(t) - \bar{\phi}] = \frac{i}{\pi} p.v. \int_{-\infty}^{\infty} \frac{log|A(t)| - log|\bar{A}|}{t - t'} dt' + \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{\phi(t) - \bar{\phi}}{t - t'} dt' \tag{5.64}$$

Comparing Imaginary part of Equation 5.64,

$$\phi(t) - \bar{\phi} = +\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{log|A(t)| - log|\bar{A}|}{t - t'} dt' \tag{5.65}$$

In equation 5.65, $\frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{log|\bar{A}|}{t-t'} dt' = 0$ which leads to,

.

$$\begin{aligned} \phi(t) &= \bar{\phi} + \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{log|A(t)|}{t - t'} dt' \\ \phi(t) &= \bar{\phi} + \frac{1}{2\pi} p.v. \int_{-\infty}^{\infty} \frac{log|A(t)|^2}{t - t'} dt' \end{aligned} \tag{5.66}$$

5.6.2 Simulation Analysis

Transmitter setup

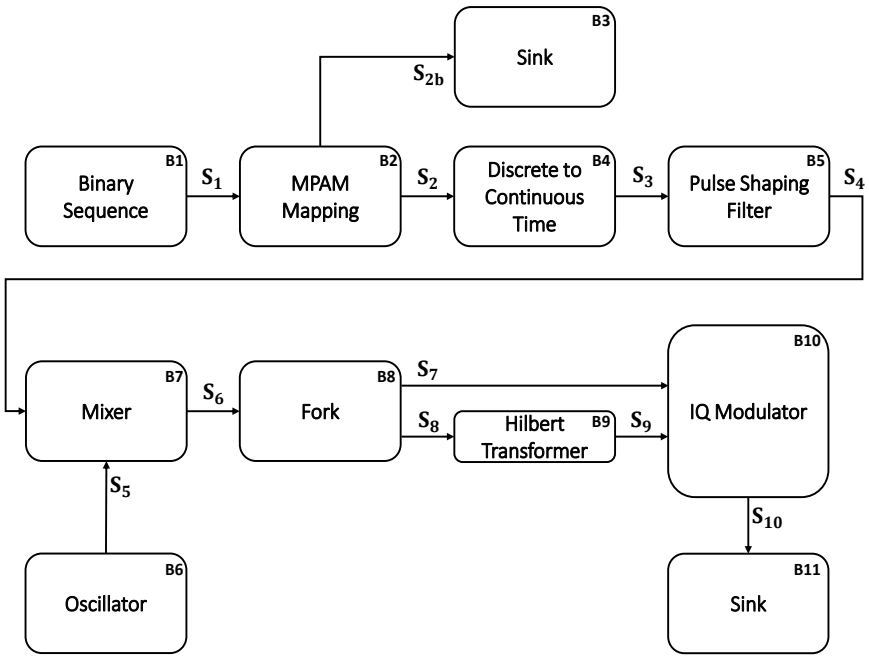


Figura 5.20: Transmitter simulation setup

System input parameters:

Parameter	Default Value	Description
sourceMode	PseudoRandom	
patternLength	5	
bitPeriod	1/1.25e9	
iqAmplitudes	{{0,0},{1,0},{2,0},{3,0}}	
numberOfBits	1000	
numberOfSamplesPerSymbol	8	
filterType	RaisedCosine	
rollOffFactor	0.3	
impulseResponseTimeLength	16	
outputOpticalPower	1e-3	

Transmitter setup description:

Header Files		
File name	**Comments**	**Status**
binary_source.h		✓
m_qam_mapper.h	DONE!	✓
discrete_to_continuous_time.h		✓
pulse_shaper.h		✓
RF_Oscillator.h	DONE!	✓
mixer.h	DONE!	✓
fork.h	DONE!	✓
hilbert_transform.h	DONE!	✓
iq_modulator.h		✓
sink.h		✓
netxpto.h		✓

Source Files		
File name	**Comments**	**Status**
binary_source.cpp		✓
m_qam_mapper.cpp	DONE!	✓
discrete_to_continuous_time.cpp		✓
pulse_shaper.cpp		✓
RF_Oscillator.cpp	DONE!	✓
mixer.cpp	DONE!	✓
fork.cpp	DONE!	✓
hilbert_transform.cpp		
iq_modulator.cpp		✓
sink.cpp		✓
netxpto.cpp		✓
kramers_kronig_transceiver_sdf.cpp		✓

Simulation results:

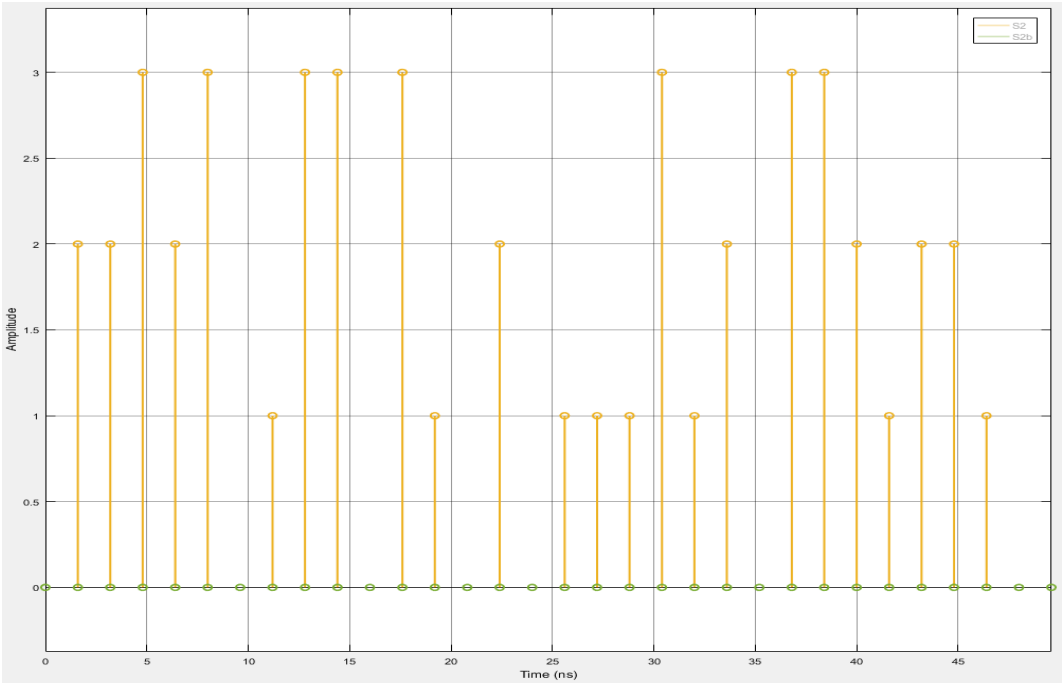


Figura 5.21: S2 and S2b

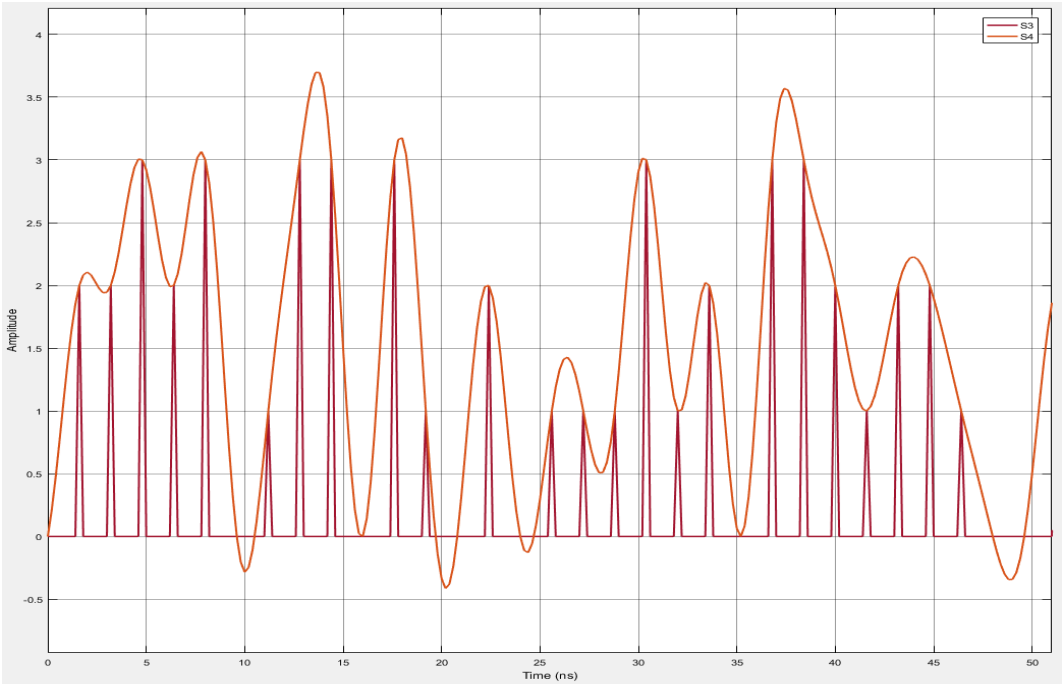


Figura 5.22: S3 and S4

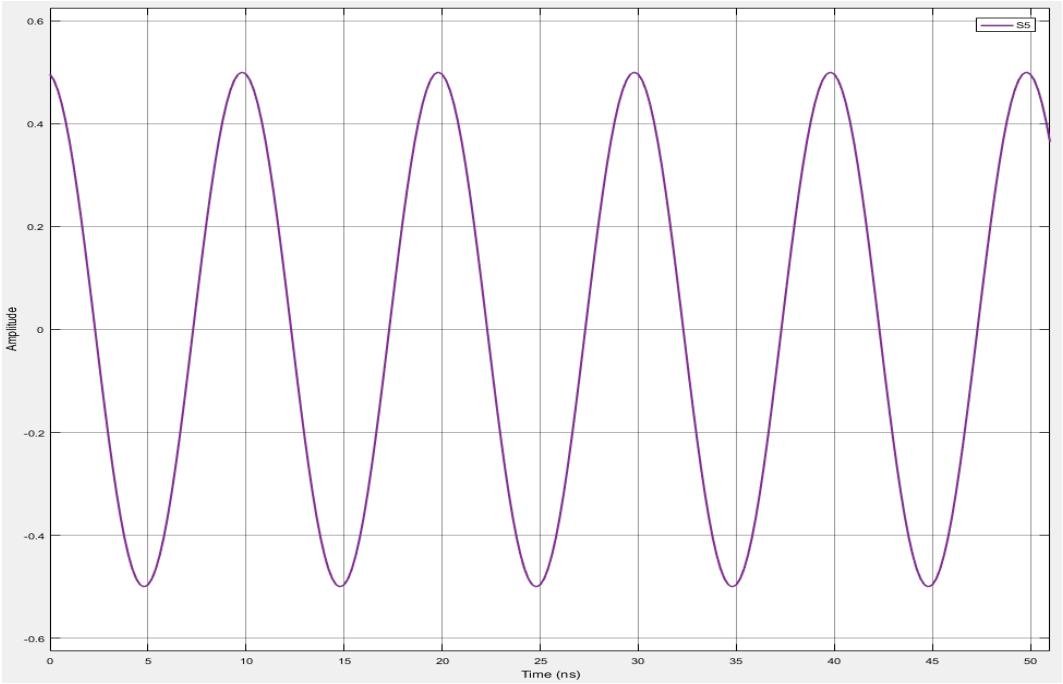


Figura 5.23: S5

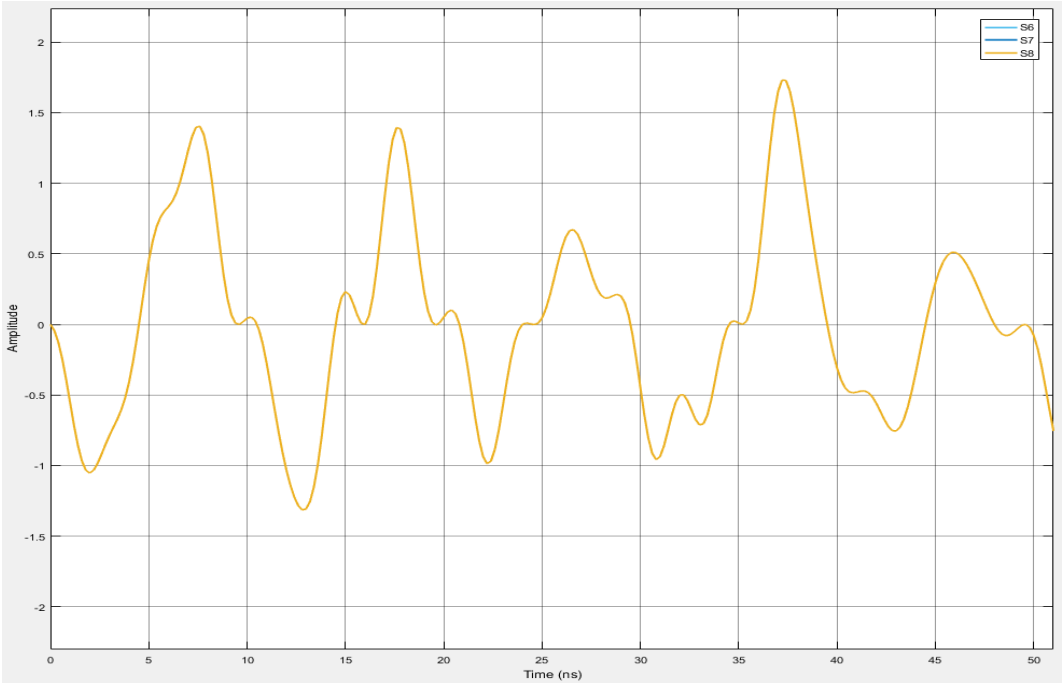


Figura 5.24: S6, S7 and S8

Receiver setup

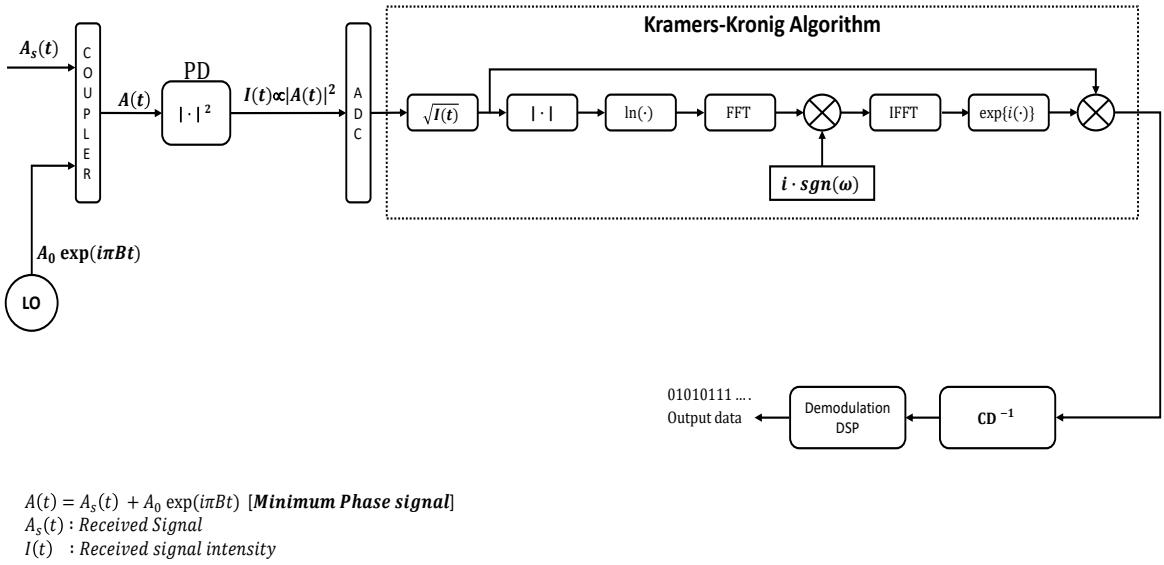


Figura 5.25: Receiver simulation setup

5.6.3 Experimental Analysis

As shown in the Figure 5.26, at the transmitter end, analytical signal generated with the help of netxpto and applied to the AWG. Waveform generated by AWG applied dual polarization IQ modulator which generates SSB signal in optical domain. SSB optical signal generated by both the IQ modulator is then combined using polarization beam combiner (PBC) and launched into the optical fiber.

At the receiver end, the PDM received signal first spitted by a polarization beam splitter (PBS). Each polarization is combined with an LO tapped from the transmit laser. The laser's wavelength and power are set to ensure that the received signal should satisfy the minimum phase condition. After direct-detection, Kramers-Kronig algorithm is performed on each polarization separately to recover full complex signal. After compensation of the chromatic dispersion, stokes parameter based poldemux algrithm can be applied to recover PDM signals.

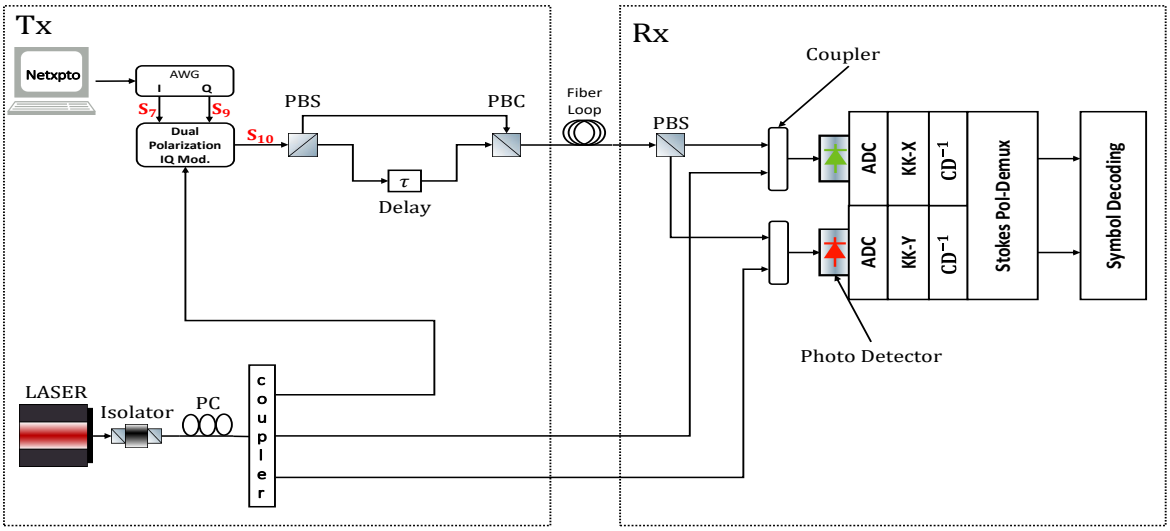


Figura 5.26: PDM Kramers-Kronig receiver experimental setup

Status of equipment

Equipment name	Description	Status
LASER		✓
PC		✓
Coupler		✓
AWG		✓
Dual polarization IQ mod		✓
PBS		✓
PLC		✓
Delay		✓
Fiber loop		✓
Single photodetector	APD+TIA Optical Receiver: - Maximum bit rate: 10 Gb/s - Sensitivity: -26 dBm	✓

5.6.4 Comparative Analysis

5.6.5 Know Problems

Problem type	Description	Note
Simulator	Require a generalized block for FFT and IFFT	

Bibliografia

[1] Antonio Mecozzi, Cristian Antonelli, and Mark Shtaif. *Kramers-Kronig Coherent Receiver*. Optica, vol.3, no.11, 2016, p.1220., doi:10.1364/optica.3.001220.

[2] Antonio Mecozzi. *Retrieving the full optical response from amplitude data by Hilbert transform*. Opt. Comm. 282, 4183-4187.

[3] Antonio Mecozzi. *A necessary and sufficient condition for minimum phase and implication of phase retrieval*. arXiv:1606.04861.

APPENDICES

Appendix A : SSB with graphical explanation

This section describes the SSB signal generation using Hilbert transformation method (Phase Shift Method). Consider a message signal $m(t)$ with its frequency domain spectrum $M(F)$ as shown in Figure5.27. From the Figure 5.27, we can see that both the side are scaled by factor '1' which means it represents the original signal.

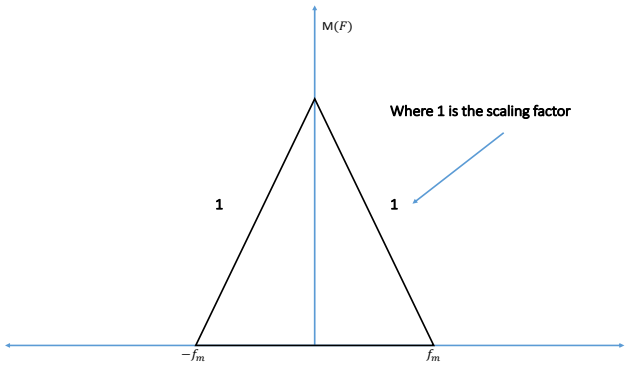


Figura 5.27: Original baseband signal

Now let's consider the modulated signal $x(t)$ given as,

$$x(t) = m(t)cos(2\pi f_c t) \tag{5.67}$$

Frequency domain representation of the equation 5.67 can be given as,

$$X(F) = \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c) \tag{5.68}$$

Here in equation 5.68, we can observe that each side band are scaled by $\frac{1}{2}$ on the frequency spectrum. Figure displays the frequency domain representation of the modulated signal $X(F)$.

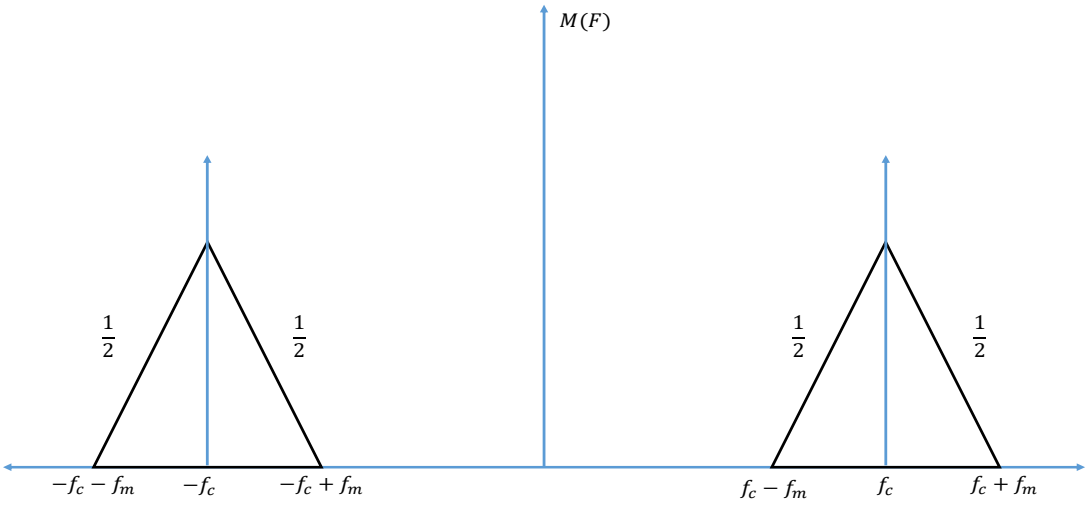


Figura 5.28: Original modulated signal

Next, we will discuss something more interesting which is called as Hilbert transform of the original message signal $m(t)$. As we discussed earlier, in the frequency domain, the Hilbert transformed signal $\hat{M}(f)$ can be achieved by multiplying the Fourier transformed signal $M(F)$ with $[-isgn(F)]$. Suppose we modulate the Hilbert transformed message signal $\hat{m}(t)$

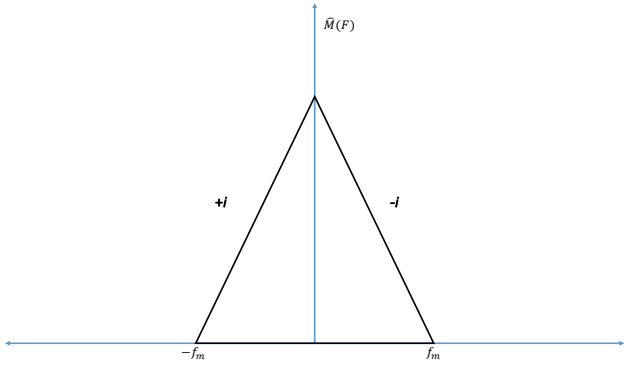


Figura 5.29: Hilbert transformed modulated signal

with the $sin(2\pi f_c t)$ (quadrature phase carrier), then we get the following results:

$$\begin{aligned}\hat{m}(t)sin(2\pi f_c t) &= \hat{m}(t)\frac{e^{i2\pi f_c t} - e^{-i2\pi f_c t}}{2} \\ &= \hat{m}(t)\frac{e^{i2\pi f_c t}}{2} - \hat{m}(t)\frac{e^{-i2\pi f_c t}}{2} \\ &= \frac{\hat{M}(f - f_c)}{2i} - \frac{\hat{M}(f + f_c)}{2i} \\ &= \frac{-i}{2}\hat{M}(f - f_c) + \frac{-i}{2}\hat{M}(f + f_c)\end{aligned} \tag{5.69}$$

The detailed explanation of the equation 5.69 has been given in the Figure 5.30 and 5.31. Figure 5.30 displays the spectrum of the $\hat{M}(f + f_c)$ and $\hat{M}(f - f_c)$ for the positive and negative frequencies respectively. The final equation resolution of equation displays that both positive and negative side of the spectrum multiplied with $\frac{i}{2}$ and $\frac{-i}{2}$ respectively. Finally the spectrum of the signal $\hat{m}(t)sin(2\pi f_c t)$ can be given as Figure 5.31.

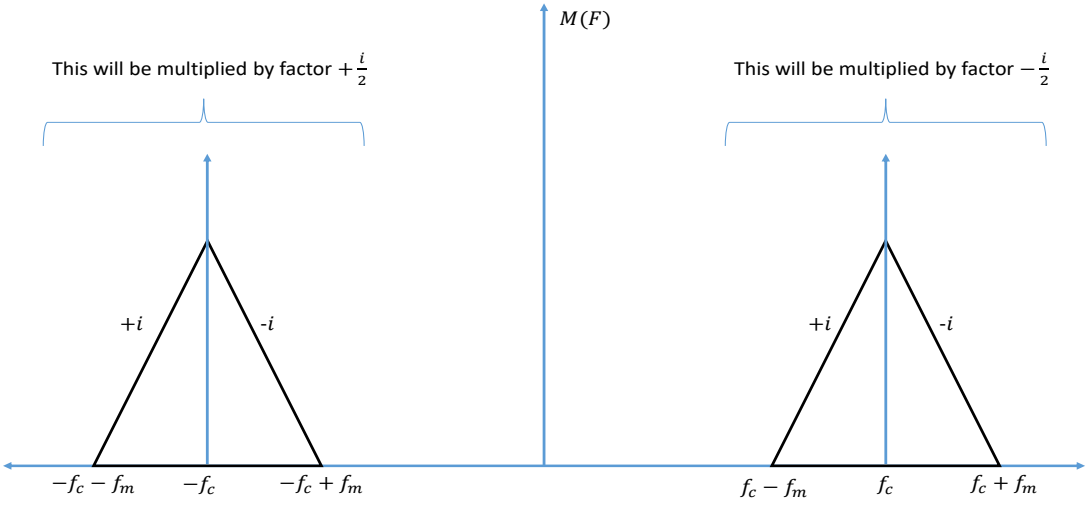


Figura 5.30: Hilbert transformed modulated signal

Further, summation of the two signals $m(t)cos(2\pi f_c t)$ and $\hat{m}(t)sin(2\pi f_c t)$ will generate the upper sideband SSB signal as follows,

$$u(t) = m(t)cos(2\pi f_c t) - \hat{m}(t)sin(2\pi f_c t) \tag{5.70}$$

From the above discussion, the spectrum of the Equation 5.70 can be given by the Figure 5.32. Similarly, for the lower sideband SSB can be generated by Equation,

$$u(t) = m(t)cos(2\pi f_c t) + \hat{m}(t)sin(2\pi f_c t) \tag{5.71}$$

Appendix B : Kramers-Kronig scheme

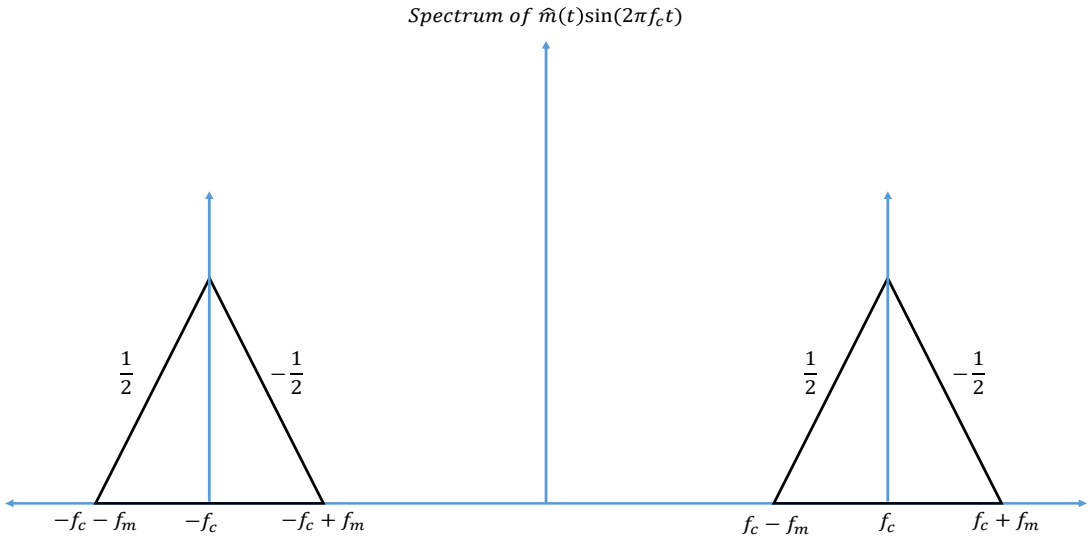


Figura 5.31: Hilbert transformed modulated signal

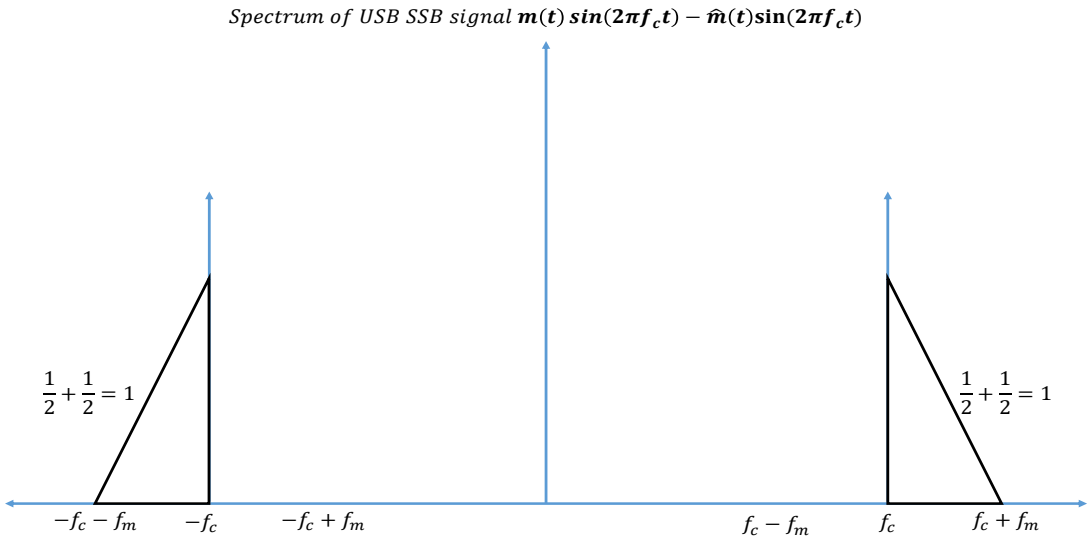


Figura 5.32: SSB signal spectrum

If we consider the complex envelope of the incoming electric field by $A_s(t)$ confined within the optical bandwidth denoted by B. The LO assumed to be a continuous wave (CW) signal whose amplitude is A_0 whose frequency coincides with the left edge of the information-carrying signal spectrum. Here, we assumed that A_0 is real-valued and positive, which is equivalent to referring all phase value to that of LO.

The complex envelope of the field striking upon the photo-diode can be given as,

$$A(t) = A_s(t) + A_0 exp(i\pi Bt) \tag{5.72}$$

The photo current I produced by the photo-diode is proportional to the field intensity $I = |A(t)|^2$, here proportionality constant considered as 1 for the sake of simplicity. If A_0 is large enough to ensure that the signal $A(t)exp(-i\pi Bt) = A_0 + A_s(t)exp(-i\pi Bt)$ is minimum phase. The discussed hypothesis can be used to reconstruct the signal $E_s(t)$ as follows[1]:

$$A_s(t) = \{\sqrt{I(t)}exp[i\phi_E(t)] - A_0\}exp(i\pi Bt) \tag{5.73}$$

$$\phi_A(t) = \frac{1}{2\pi} p.v. \int_{-\infty}^{\infty} dt' \frac{log[|I(t')|]}{t - t'} \tag{5.74}$$

5.7 Quantum Oblivious Key Distribution with Discrete Variables

Student Name	:	Mariana Ramos
Starting Date	:	September 18, 2017
Goal	:	Quantum oblivious key distribution (QOKD) implementation with discrete variables.
Directory	:	sdf/ot_with_discrete_variables.

Oblivious Transfer (OT) is a fundamental primitive in multi-party computation. The one-out-of-two OT consists in a communication protocol between Alice and Bob. At the beginning of the protocol Alice has two messages m_1 and m_2 and Bob wants to know one of them, m_b, without Alice knowing which one, i.e. without Alice knowing b, and Alice wants to keep the other message private, i.e. without Bob knowing $m_{\bar{b}}$. therefore two conditions must be fulfilled:

1. The protocol must be concealing, i.e at the beginning of the protocol Bob does not know nothing about Alice's messages, while at the end of the protocol Bob will learn the message m_b chosen by him.

2. The protocol is oblivious, i.e Alice cannot learn anything about Bob's choice, bit b, and Bob cannot learning nothing about the other message $m_{\bar{b}}$.

In order to implement OT between two parties (Alice and Bob) they must be able to exchange continuously oblivious keys, i.e a QOKD system must exist between them.

5.7.1 Theoretical Description

Quantum Oblivious Key Distribution System (QOKD)

In this section we are going to describe the Quantum Oblivious Key Distribution system (QOKD). The QOKD system enables two parties (Alice and Bob) to share a set of keys. These keys have the particularity of being half right and half wrong. Only Bob knows which are right and wrong keys.

Considering a discrete variables implementation, both Alice and Bob agree with the following correspondence, where $+$ corresponds to *Rectilinear Basis* and $\times$ corresponds to *Diagonal Basis*,

Basis	
0	$+$
1	$\times$

Alice and Bob also agree with the bit correspondence for each direction for each basis. For *Rectilinear basis*, "+",

	Basis "+"
0	$\rightarrow (0°)$
1	$\uparrow (90°)$

and for *Diagonal Basis*, "$\times$",

	Basis "$\times$"
0	$\searrow (-45°)$
1	$\nearrow (45°)$

1. The first step is to establish for both Alice and Bob the block length l. In this case, lets assume $l = 16$. Alice randomly generate a bit sequence with length l. Therefore, she must define two sets randomly: S_{A1} which contains the basis values; and S_{A2}, which contains the key values.

   In that case, lets assume she generates the following sets $S_{A1'}$ and $S_{A2'}$:

   $$S_{A1'} = \{0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1\},$$

   $$S_{A2'} = \{1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1\}.$$

2. Next, Alice sends to Bob throughout a quantum channel l photons encoded using the basis defined in $S_{A1'}$ and according to the key bits defined in $S_{A2'}$.

   Therefore, in the current example, Alice sends the following photons,

   $$\begin{aligned} S_{AB} &= \{\uparrow, \uparrow, \nearrow, \searrow, \searrow, \rightarrow, \rightarrow, \searrow, \nearrow, \uparrow, \rightarrow, \searrow, \nearrow, \searrow, \uparrow, \nearrow\} \\ &= \{90°, 90°, 45°, -45°, -45°, 0°, 0°, -45°, 45°, 90°, 0°, -45°, 45°, -45°, 90°, 45°\}. \end{aligned}$$

3. Bob also randomly generates $l = 16$ bits, which are going to define his measurement basis, $S_{B1'}$. Lets assume,

   $$\begin{aligned} S_{B1'} &= \{0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 0, 1\} \\ &= \{+, \times, \times, +, +, \times, +, \times, \times, +, \times, \times +, +, +, \times\}. \end{aligned}$$

   Bob will get l results:

   $$S_{B2'} = \{1, -, \underline{0}, 0, -, 1, \underline{1}, -, 1, -, 1, 0, 1, 1, \underline{0}, 1\}.$$

   The "$-$" corresponds to no clicks in Bob's detector, due to attenuation. The underlined values are bits which were measured with a correct basis but an error has occurred due to imperfections in the quantum communication system.

4. Bob is going to send a "-1" or a hash value to Alice for each measurement that he performed, thereby being "-1" the measurements which correspond to no clicks. In this case, we are going to assume that the hash value is calculated using the *SHA-256* algorithm [2]. In detail, Bob has two sets $S_{B1'}$ and $S_{B2'}$ and he is going to generate the set S_{BH1} with l values ("-1" or hash values calculated for each position of $S_{B1'}$ with the correspondent position of $S_{B2'}$). Therefore, Bob will send to Alice the following set:

$$S_{BH1} = \{S_1, -1, S_2, S_3, -1, S_4, S_5, -1, S_6, -1, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}\}.$$

5. Since Alice has received the confirmation of measurement from Bob, i.e after Alice has received S_{BH1}, she sends throughout a classical channel the basis which she has used to codify the photons updated with the information about the no received photons,

$$S_{A1'} = \{0, -1, 1, 1, -1, 0, 0, -1, 1, -1, 0, 1, 1, 1, 0, 1\}$$

.

Due to attenuation, the previous sets are reduced to the length 12 and they shall be replaced by the following:

$$S_{A1} = \{0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1\},$$

$$S_{A2} = \{1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1\},$$

$$S_{B1} = \{0, 1, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1\},$$

$$S_{B2} = \{1, \underline{0}, 0, 1, \underline{1}, 1, 1, 0, 1, 1, \underline{0}, 1\}$$

Note that S_{B2} still has errors.

6. In order to know which photons were measured correctly, Bob does the operation $S_{B3} = S_{B1} \oplus S_{A1}$. In the current example,

S_{B1}	0	1	0	1	0	1	1	1	0	0	0	1
S_{A1}	0	1	1	0	0	1	0	1	1	1	0	1
$\oplus$	1	1	0	0	1	1	0	1	0	0	1	1

In this way, Bob gets

$$S_{B3} = \{1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1\}.$$

When Bob uses the right basis he gets the values correctly, apart from possible errors in transmission, when he uses the wrong basis he just guess the value. The values "1" correspond to the values he measured correctly and "0" to the values he just guessed. Thus, Bob is building two sets of keys, one with correct basis measurements values and other with the wrong basis measurement values that he just guessed.

Thus, Bob has two pair of sets, one for the right basis,

$$S_{B_{rp}} = \{1, 2, 5, 6, 8, 11, 12\},$$
$$S_{B_{rb}} = \{1, 0, 1, 1, 0, 0, 1\},$$

where $S_{B_{rp}}$ is the set of positions and SB_{rb} is the set of bit values he measured for each position. The other pair is for photons he measured with the wrong basis and then he just guessed the values,

$$S_{B_{wp}} = \{3, 4, 7, 9, 10\},$$
$$S_{B_{wb}} = \{0, 1, 1, 1, 1\},$$

where $S_{B_{wp}}$ is the set of positions and $S_{B_{wb}}$ is the set of bit values he measured for each position.

Nevertheless, due to errors in transmission, some bits in $S_{B_{rb}}$ may be not right.

At this point, in order to test Bob's honesty and to estimate the *QBER* of the channel, Alice is going to ask Bob to open some pairs of the Bob's sets. The definition of the protocol to test Bob's honesty is still an open issue. However, depending on the *QBER* estimated by her, Alice must have a parameter to set the number of right position she wants to open, i.e she must open a minimum number of right position in order to guarantee a minimum *QBER*. This will increase the security of the protocol. Alice chooses some positions to open and tells Bob which positions she wants to open. Bob sends to Alice the pairs she chose and then these pairs are eliminated from them sets. Lets assume she asked to open the positions 10, 11 and 12. If she concludes Bob is not being honest, she stops the protocol and they must start it again. Otherwise, the protocol continues. Lets assume Alice has verified these pairs using the hash function committed by Bob and concluded that he is being honest. Therefore, she sends to Bob the *QBER* estimated by her.

Now, Bob has the previous sets replaced by the following,

$$S_{B_{rp}} = \{1, 2, 5, 6, 8\}$$
$$S_{B_{rb}} = \{1, 0, 1, 1, 0\}$$
$$S_{B_{wp}} = \{3, 4, 7, 9\}$$
$$S_{B_{wb}} = \{0, 1, 1, 1\}$$

Bob is going to use a modified version of *Cascade algorithm* to correct the errors due transmission.

Modified version of Cascade Algorithm

The Cascade algorithm is often used with a key set where all values are supposed right. In this case, Bob has two pairs of sets, one with the position and bit values of photon he measured with the correct basis and other with position and bit values of photon

he measured with the wrong basis. He only needs to apply the Cascade algorithm in the set that he measured the photons correctly [3]. However, he must apply a modified version of the Cascade in the other set in order to keep in secret from Alice which set corresponds to right and which set corresponds to wrong measurements.

Bob randomly generates a bit value. If he gets 0, he will send to Alice the set $\{S_{B_{rp}}, S_{B_{wp}}\}$. Otherwise, if he gets 1 he will send the set $\{S_{B_{wp}}, S_{B_{rp}}\}$. This guarantee that Alice does not know which is the right or wrong set. Lets assume this random bit is "0"and he sends $\{S_{B_{rp}}, S_{B_{wp}}\}$.

(a) Bob starts by applying the normal cascade to the set $S_{B_{rb}}$. After both know the error estimative Bob determine if the error rate is above the fail threshold. If it is truth they must start the procedure again. Lets assume the estimated error rate is acceptable. Bob and Alice use a random permutation which is represented in figure 5.33 for a larger number of bits (agreed at the beginning) by applying it to the shifted keys, in order to guarantee the spread out of the error bits randomly and to separate consecutive errors from each other.

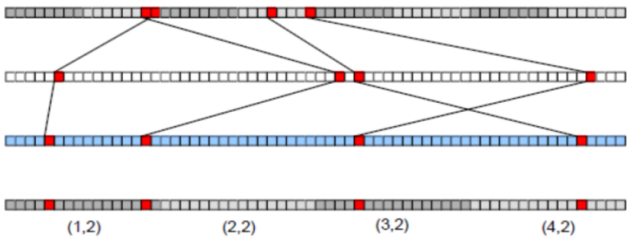


Figura 5.33: Cascade Algorithm - permutation

(b) Bob and Alice divide all the shifted key bits into blocks of N bits depending on the estimated error rate in order to have one or no error per block. In general, the sets of keys are too large and it is easier to explain the algorithm based on a larger number of bits. Therefore, figure 5.34 represents the typical cascade initial steps. However, in this case, the set to be corrected only has five bits, therefore they divide the set in two sub-blocks, one with 3 bits and other with 2 bits.

(c) They use a classical channel to compare the block parities. For blocks with different parities, an odd number of errors must exist, otherwise an even number of errors would mask each other. Thus, the block in which the parities disagree is divided in half into two smaller blocks of length $\frac{N}{2}$, and another parity check is performed on the first sub-block, as one can see in figure 5.35. As it was referred above, there is at least one error in one sub-block being the error location revealed by the parity of one sub-block. In other words, if the parity of the first sub-block passes, the error will be in the second sub-block. The sub-block with error will be sub-divided until the error is found.

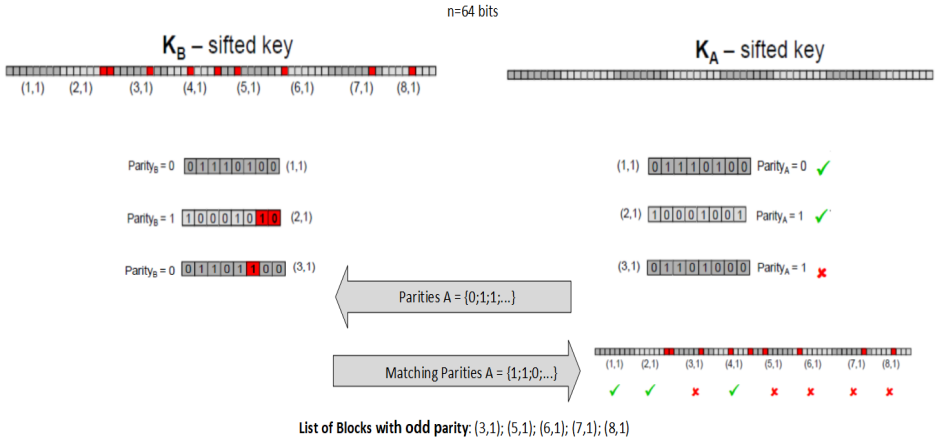


Figura 5.34: Cascade Algorithm

Figura 5.35: Cascade Algorithm - example of error correction

(d) When the error is corrected, the last bit of the block is discard in order to prevent the gain of additional information by Bob.
In this case, lets assume the set of right positions was corrected with the algorithm described above and it will be replaced by the following:

$$S_{B_{rp}} = \{1, 2, 5, 6, 8\}$$
$$S_{B_{rb}} = \{1, 1, 0, 1, 0\}$$

In order to test Alice's honesty, Bob must verify if the *QBER* sent by Alice is a realistic value. If it is not he stops the protocol and they must start again. Otherwise, the protocol continues.

After that, Bob needs to apply the Fake Cascade to the set $S_{B_{wb}}$. The main goal of this

step is to convince Alice she is performing the real Cascade but she is not.

(a) First of all, based on the positions contained in $S_{B_{wb}}$, Bob must build an array with the correspondent bits in a random order and informs Alice the order of positions. In order to best explain this version of the algorithm, lets assume a larger set of bits.

Bob sends to Alice throughout a classical channel the new positions order as if it were the permutation step represented in figure 5.33 in real Cascade algorithm.

(b) Assuming each of them has a set with 32 bits randomly organized by Bob, they divide the supposed shifted keys in blocks with N bits according to the estimated error rate. As the *QBER* is the same as for real cascade, Bob will assume the same number of errors, even if he starts for this modified version he can know the number of errors from *QBER* estimated by Alice.

(c) Bob and Alice use a classical channel to compare the block parities. Alice sends to Bob her parity list. Based on Alice's parity list, Bob sends a block list with odd parities, i.e the blocks position in which parity supposed disagree. This list is randomly built based on the number of errors considered by Bob, i.e if he considered five errors from *QBER* estimative, he will distributed them randomly and after that he will fill the remaining spaces with even parities. Bob sends to Alice the set with the list of odd parities, i.e the list of sub-sets he has different parities than Alice.

(d) The blocks with errors will be consecutively divided until they found the supposed errors. Since we have assumed there were five errors, this is the number of errors that Alice must supposedly correct.

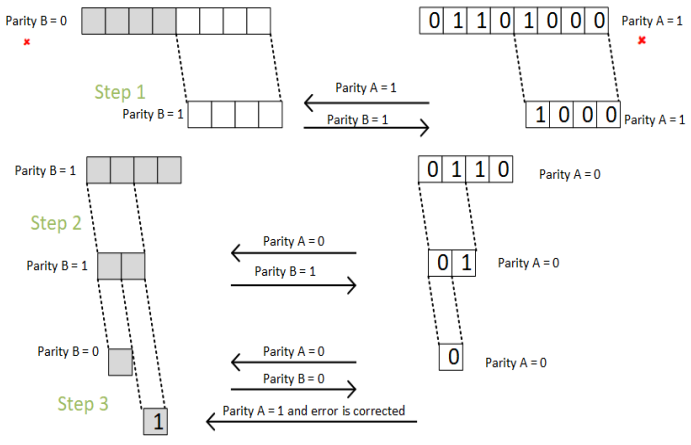


Figura 5.36: Fake cascade - example of error correction

Lets assume one of the blocks with error and analyse figure 5.36. Bob starts with a set filled with random bits, therefore we do not need to know which bits are. Alice starts by dividing her set in half with two blocks with N bits.

Step 1: Bob chooses one of the to blocks and informs Alice she must send the parity of this block. Lets assume he chose sub-block 2. She sends the parity and Bob is going to send his parity, which after know Alice's block parity he send the opposite parity. As referred in normal Cascade, there must be one error or no error in each block. Thus, since the parities disagree, the error must be in seconde block. They start the procedure to correct it.

Step 2: Bob divides again the sub-block in half with $\frac{N}{2}$ bits and asks Alice for the parity of the first sub-block. She sends her parity equals to 0 and Bob sends to her the opposite parity again.

Step 3: They divide the sub-block in half again and Bob asks Alice for the parity of the first bit. Alice sends to him the parity equals to her. As the error is not in the first be, it must be in the second, therefore Bob is able to correct this bit with the information sent by Alice.

Note that Bob make his choice of which half analyse first using a random bit generator result. If he got "0" he starts with the first half of the sub-block, otherwise, if he got "1", he starts with the second half. In addition, they must discard the last bit of each block and sub-block in which fake Cascade were applied in order to guarantee that Bob does not gain additional information.

In this case, after apply the fake Cascade to $S_{B_{wb}}$, lets assume,

$$S_{B_{wp}} = \{3, 4, 7, 9\}$$
$$S_{B_{wb}} = \{0, 1, 1, 0\}$$

If Bob starts by applying the fake Cascade, he must test Alice's honesty at the beginning of the real Cascade application, based on the number of errors he has. If he thinks that the *QBER* sent by Alice is unrealistic, he stops the protocol at this point.

7. When Alice sends to Bob a photons set, they are building a set of pairs (array positions and bit values which correspond to measured photons at Bob's side and to the key bit with the photon was encoded at Alice's side). The main goal is to guarantee that Bob has the same number of right and wrong pairs. In addition, they must know information about t (represented in figure 5.37) which corresponds to the points where the previous condition is verified.

   Since Bob has sent to Alice the information about the smallest set, in this example, Alice know that there are four pairs of wrong positions and five pairs of right positions. Alice must destroy one of the right pairs by asking Bob to open it. Therefore, at $t = 8$ both know that there are the same number of right and wrong pairs thereby being the main goal guaranteed.

   As we can see in figure 5.37, unlike Bob, Alice does not know which positions corresponds to right or wrong measurements performed by Bob. They have been building these sets during all protocol.

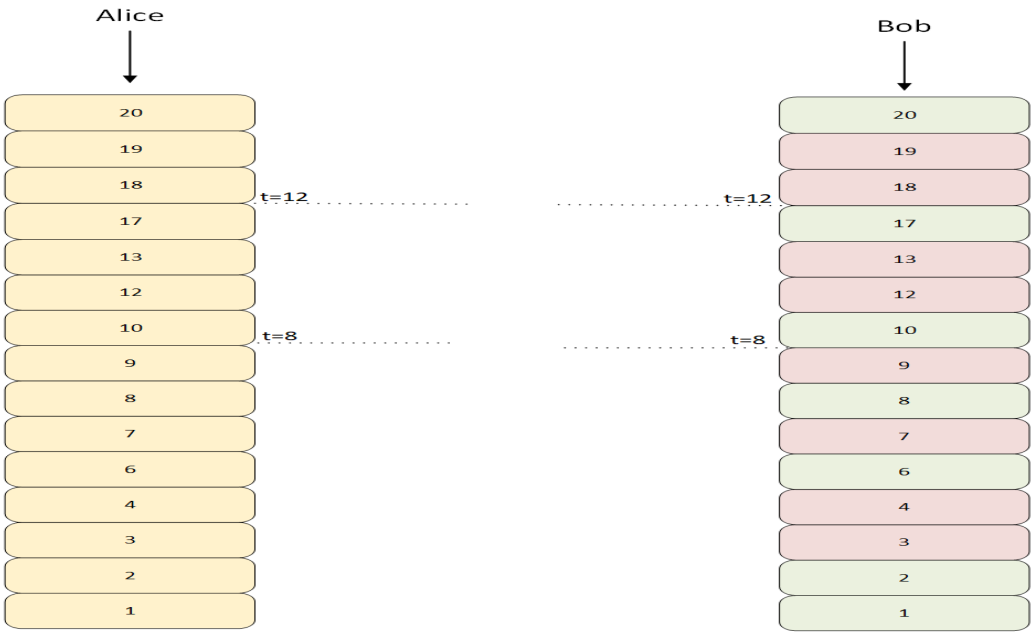


Figura 5.37: Alice and Bob key sets.

1-out-of-2 Oblivious Transfer Protocol with QOKD system

At this time, we are going to describe the oblivious transfer protocol with detail. As it was referred at the beginning, Alice sends two messages to Bob and he wants to know one of them. Alice does not know which message Bob wants and Bob only know the message he wants, i.e he does not know anything about the other message. Furthermore, only Alice knows information about messages m_0 and m_1. In this case, lets assume the following two messages with size $s = 4$, $m_0 = \{0011\}$ and $m_1 = \{0001\}$. Alice must guarantee $t = s \times 2$. In order to do that, she must destroy the remaining pairs. In this case, there is no need to do that because they have a set for $t = 8$ with the same number of wrong and right pairs.

1. Bob defines two new sub-sets, I_0 and I_1. I_0 is a set of values with photons array positions which Bob just guessed the measurement since he did not measure them with the same basis as Alice, I_1 is a set of values with photons array positions which Bob measured correctly since he used the same basis as Alice used to encoded them. The position of the pairs of each right and wrong message are in the keys sets that they have been building during the protocol.

   In this example, the message size is 4. Since, at this time $t = 10$ and we have 5 right pairs and 5 wrong pairs, Alice ask to Bob to open one right pair and one wrong pair in order to both have exactly the message's size number of right and wrong pairs. Lets assume that Alice opened two pairs, position 15 which is a wrong measurement and position 10 which is a right measurement. We have now $t = 8$.

Next, Bob defines two sub-sets with size $s = 4$:

$$I_0 = \{3, 4, 7, 9\},$$

and

$$I_1 = \{1, 2, 6, 8\},$$

where I_0 is the sequence of positions in which Bob was wrong about basis measurement and I_1 is the sequence of positions in which Bob was right about basis measurement. Bob sends to Alice the set S_b

Thus, if Bob wants to know m_0 he must send to Alice throughout a classical channel the set $S_0 = \{I_1, I_0\}$, otherwise if he wants to know m_1 he must send to Alice throughout a classical channel the set $S_1 = \{I_0, I_1\}$.

2. Alice is sure about Bob's honesty, since she knows he only has 4 right basis to measure the photons. In addition, Alice cannot know which message Bob chose because she did not know the order that he sent the sets.

3. Lets assume Bob sent $S_0 = \{I_1, I_0\}$. Alice defines two encryption keys K_0 and K_1 using the values in positions defined by Bob in the set sent by him. In this example, lets assume:

$$K_0 = \{1, 1, 1, 0\}$$

$$K_1 = \{0, 0, 0, 1\}.$$

Alice does the following operations:

$$m = \{m_0 \oplus K_0, m_1 \oplus K_1\}.$$

m_0	0	0	1	1
K_0	1	1	1	0
$\oplus$	1	1	0	1

m_1	0	0	0	1
K_1	0	0	0	1
$\oplus$	0	0	0	0

Adding the two results, m will be:

$$m = \{1, 1, 0, 1, 0, 0, 0, 0\}.$$

After that, Alice sends to Bob the encrypted message m through a classical channel.

4. When Bob receives the message m, in the same way as Alice, Bob uses S_{B1}, values of positions given by I_1 and I_0 and does the decrypted operation. In this case, he does following operation:

m	1	1	0	1	0	0	0	0
	1	1	1	0	0	1	1	0
$\oplus$	0	0	1	1	0	1	1	0

The first four bits corresponds to message 1 and he received $\{0, 0, 1, 1\}$, which is the right message m_0 and $\{0, 1, 1, 0\}$ which is a wrong message for m_1.

Nearest Private Query

The Nearest Private Query is another example of QOKD application [4]. In this case, there are also two parties: a user (who we called Bob) and a data owner (who we called Alice). Bob has an input secrete parameter x and Alice has a private data set A.

Lets assume Bob's secret input parameter$x = 8$ and Alice's data set $A = \{1, 2, 3, 6, 7, 10, 11, 14\}$.

Bob wants to know which element (x_i) is the closest to x in Alice data set A, without revealing his secrete x. Alice does not know which is the Bob's secret parameter, and Bob does not know any information about Alice's set except the closest element x_i to his x.

Step 1 Alice generates a new set with $N = 2^n$ elements, $D(j)$ for $j = 0, 1, ...N - 1$, in which $D(j) = x_l$ being $x(l)$ the closest element to j in A. n is the number of bits that Alice needs to represent each element of her data set.

j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$D(j)$	1	1	2	3	3	6	6	7	7	10	10	11	11	14	14	14
	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1
	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	1	0	1	1	0	0	1	1	0	0	1	1	0	0	0

Step 2 Bob and Alice have a set of keys K_i^* and K_i, respectively, with 64 elements where 60 are bits resulted from wrong basis measurement and 4 resulted from correct basis measurement. This allows Alice to know that Bob is being honest. They start from QOKD with symmetric keys and then Alice is being asking Bob to destroy pairs of keys until they have a set with the characteristics above.

Step 3 Bob sends to Alice the set S with all positions referred random measurements except the position bits at $j = 8$.

Step 4 Alice encoded all bits with the corresponding bit keys at the positions sent by Bob, and then she sends the encoded message to Bob.

Step 5 Bob receives the encoded message and apply an operation XOR based on the correspondent bits to the positions the he sent two Alice above.

Step 6 At the end, Bob gets the message he wanted, the closest element to 8, which corresponds to message $0, 1, 1, 1$ or 7 and he remains know nothing about the other elements of Alice's data set. In the same way, Alice does not know which element Bob wants to know.

QKD from QOKD

All Quantum Key Distribution systems can be obtained from the QOKD system presented in this report. The Quantum Oblivious Key Distribution system allows to obtain symmetric secret keys and symmetric or asymmetric oblivious keys.

In fact, since Alice and Bob has the same set of keys and at some tab Alice knows that there are the same number of right and random bits, she can obtain any combination from this set by asking Bob to destroy some pairs.

QOKD has a big advantage over other QKD systems because of its versatility. However, the biggest disadvantage is related with a large number of photons consumption which can became the communication rate slower.

5.7.2 Simulation Analysis

First of all, the protocol will be simulated and then a experimental setup will be built in the laboratory.

The main goal of this simulation is to demonstrate that Bob was able to learn correctly message m_b and he does not know the message $m_{\bar{b}}$.

As one may see in figure 5.45 this simulation will have three top level blocks. Two of them are Alice and Bob and they are connected through two classical channels and one quantum channel. In addition, a third block will be performed in order to calculate the *Mutual Information*. The mutual information (MI) between Alice and Bob is defined in terms of their join distribution.

1. In figure 5.39 one can observe a block diagram of the simulation at Alice's side. As it is shown in the figure, Alice must have one block for random number generation which is responsible for basis generation to polarize the photons, and for key random generation in order to have a random state to encode each photon. Furthermore, she has a Processor block for all logical operations: array analysis, hash function results validation, random number generation requests, and others. This block also receives the start information, i.e. message size s and messages m_0 and m_1, as well as information from Bob, i.e sets I_0 and I_1, hash function results, and others. In addition it is responsible for set the initial length l of the first array of photons which will send to Bob. This block also must be responsible for send classical information to Bob. Finally, Processor block will also send a real continuous time signal to single photon generator, in order to generate photons according to this signal, and finally this block

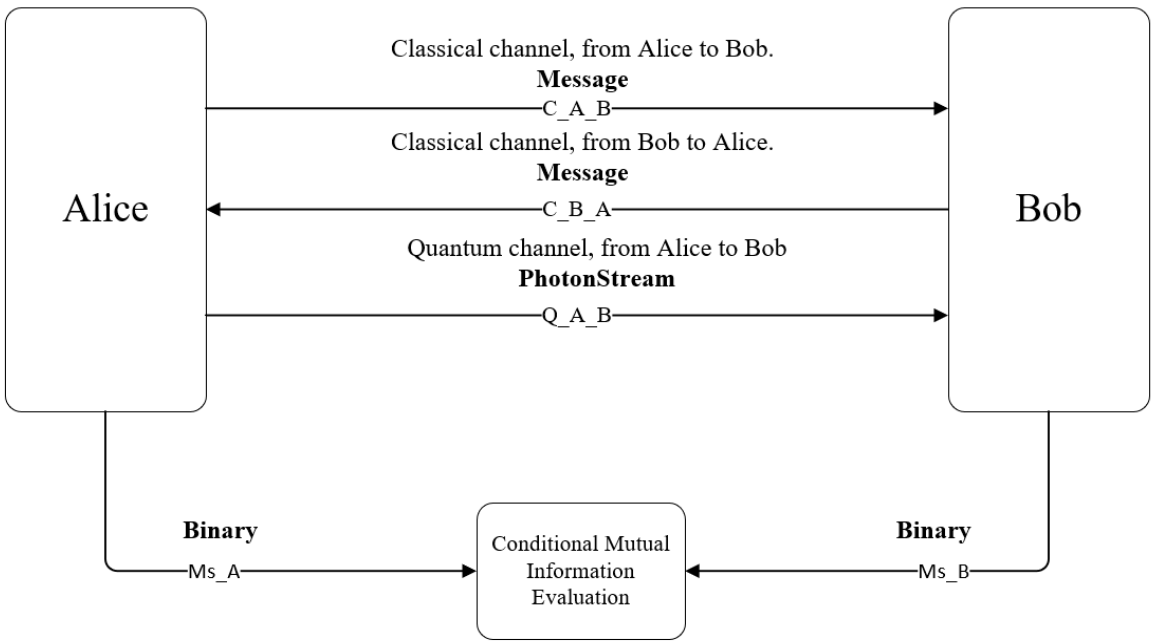


Figura 5.38: Simulation diagram at a top level

also sends to polarizer a real discrete signal in order to inform the polarizer which basis it should use. Therefore, she has two more blocks for quantum tasks: the single photon generator and the polarizer block which is responsible to encode the photons generated from the previous block and send them throughout a quantum channel from Alice to Bob.

Finally, Alice's processor has an output to Mutual Information top level block, Ms_A.

2. In figure 5.40 one can observe a block diagram of the simulation at Bob's side. From this side, Bob has one block for Random Number Generation which is responsible for randomly generate basis values which Bob will use to measure the photons sent by Alice throughout the quantum channel. Furthermore, this Block will generate the random bits that Bob needs in Modified Version of Cascade Algorithm. Like Alice, Bob has a Processor block responsible for all logical tasks, i.e Hash function generation, analysing functions, requests for random number generator block, etc. Additionally, it receives information from Alice throughout a classical channel and a quantum channel but it sends information to Alice only throughout a classical channel. Furthermore, Bob has one more block for single photon detection which receives from processor block a real discrete time signal, in order to obtain the basis it should use to measure the photons.

   Finally, Bob's processor has an output to Mutual Information top level block, Ms_B.

3. Mutual Information calculation

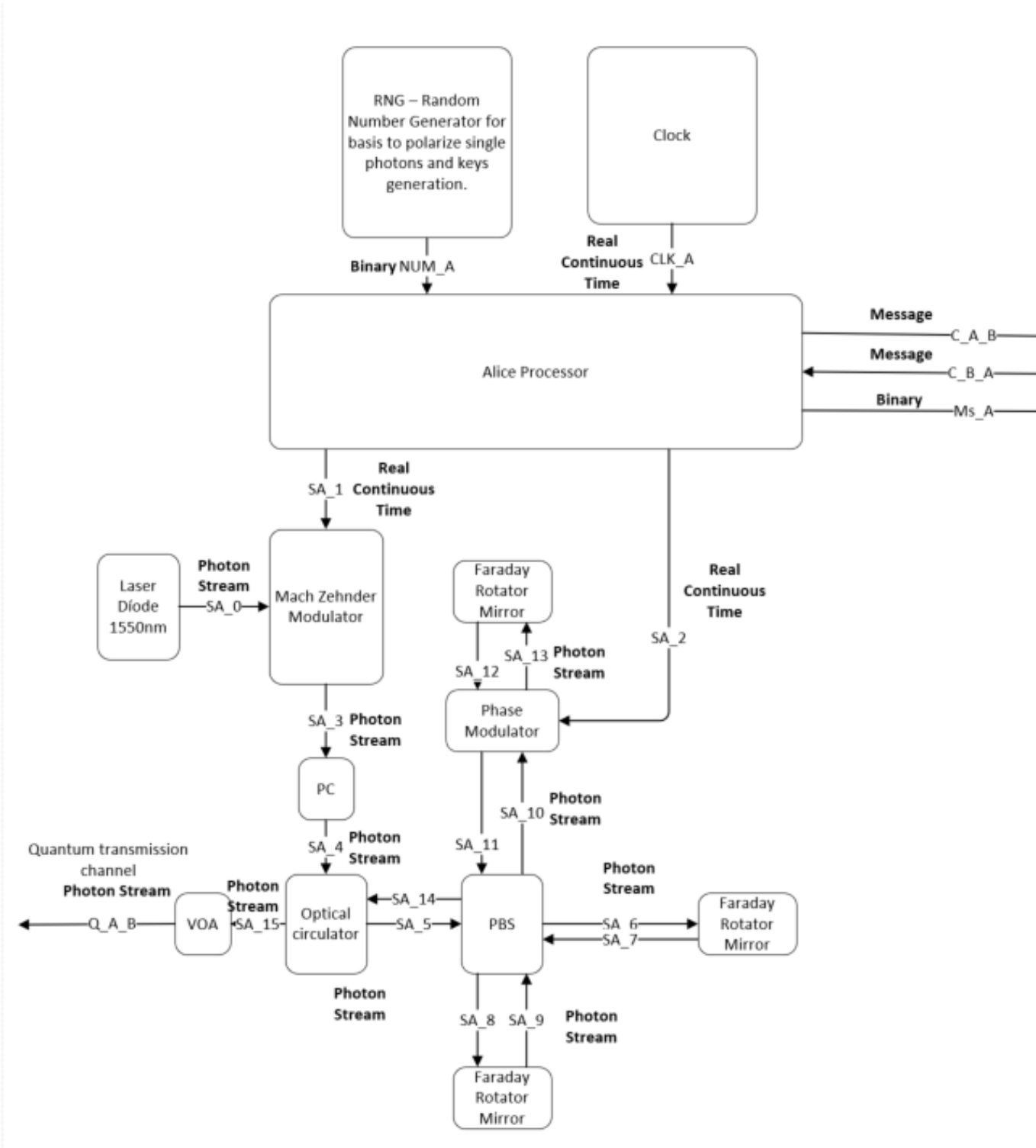


Figura 5.39: Simulation diagram - Alice's side

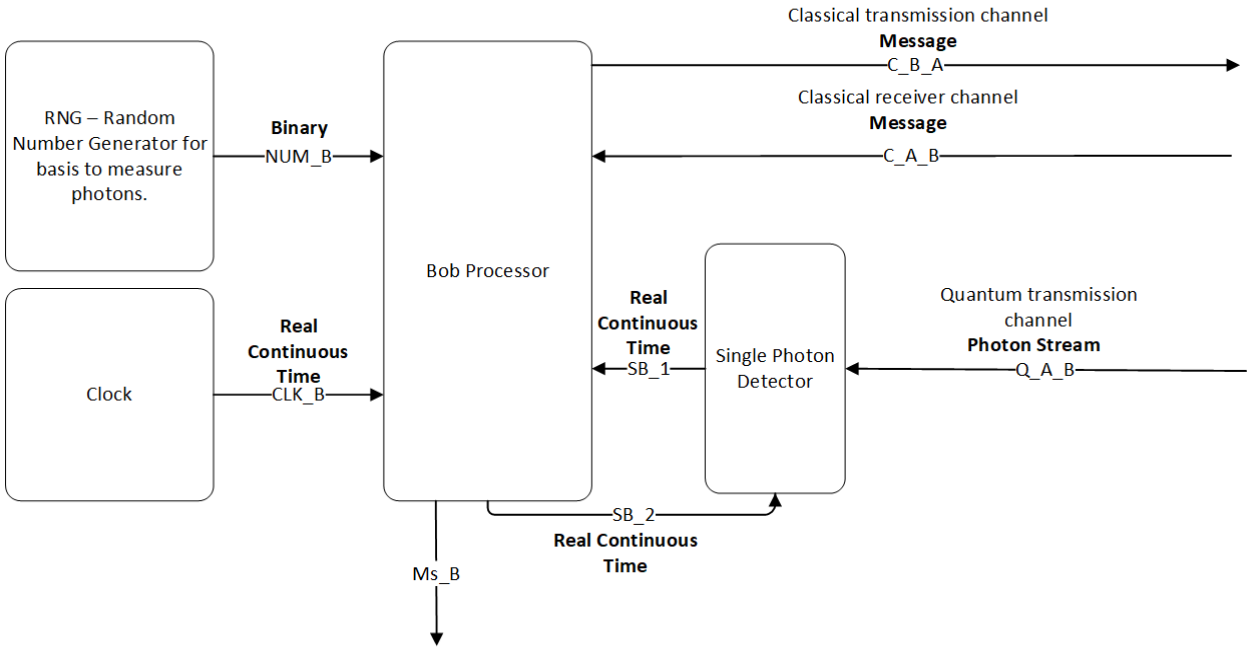


Figura 5.40: Simulation diagram - Bob's side

Tabela 5.3: System Signals

Signal name	Signal type	Status
NUM_A	Binary signal	
NUM_B	Binary signal	
CLK_A	Real continuous Time	
CLK_B	Real continuous Time	
C_A_B	Message	
C_B_A	Message	
SA_1	Real Continuous Time	
SA_2	Real Continuous Time	
SA_3	Real Discrete Time	
SA_3	Real Continuous Time	
Q_A_1	Photon Stream	
Q_A_2	Photon Stream	
Q_A_B	Photon Stream	
Ms_A	Binary	
Ms_B	Binary	
S_B1	Real continuous Time	
S_B2	Real continuous Time	

Tabela 5.4: System input parameters

Parameter	Default Value	Description
messageSize	4	Size of the message Alice must send to Bob.
blockLenght	16	Block length.
symbolRate	100K	

Tabela 5.5: Header Files

File name	Description	Status
random_number_generator.h		
single_photons_generator.h		
single_photons_detector.h		
encorder.h		
decoder.h		
message_toSend.h		
message_toReceive.h		
mutual_information.h		
cascade_truth.h		
cascade_fake.h		
Sha256.h		

Tabela 5.6: Source Files

File name	Description	Status
random_number_generator.c		
single_photons_generator.c		
single_photons_detector.c		
encorder.c		
decoder.c		
message_toSend.c		
message_toReceive.c		
mutual_information.c		
QOKD_main.c		
cascade_truth.c		
cascade_fake.c		
Sha256.c		

5.7.3 Experimental Setup

In figure 5.41 are presented the experimental setup to be performed in the lab. The main goal is to build an experimental setup in with Alice and Bob communicate through two classical

channels and one quantum channel that will have only one direction (Alice to Bob).

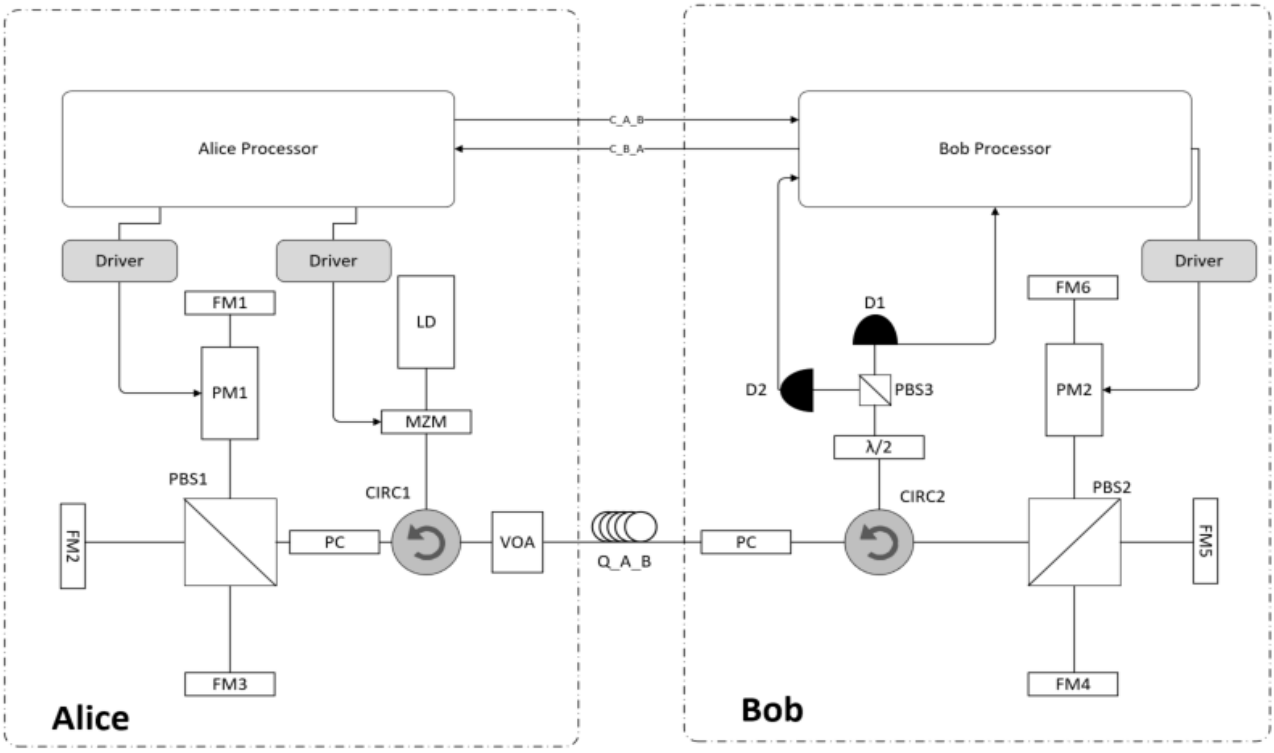


Figura 5.41: QOKD Experimental setup

The laser **LD** emits light with a wavelength of 1550 nm and then the light passes throughout a Mach-Zenhder Modulator (**MZM**) in order to have pulsed light. The light is polarized with a polarizer controller and gets a linear polarization of $45°$. When the light reaches the polarization beam splitter, **PBS1**, single photon pulses in $45°$ linearly polarization state

$$|in\rangle = |45\rangle = \frac{\sqrt{2}}{2}(|H\rangle + |V\rangle)$$

and each pulse is divided in PBS1 into two orthogonal components P_x and P_y. P_x is directly transmitted in **FM2** direction and it is reflected by it with its stated rotated, which means it has a new direction $|V\rangle$. This way, when it reaches again **PBS1** is reflected to **FM3** direction and it happens the same being the P_x component transmitted through the **PBS1** in **FM1** direction passing through the phase modulator and suffers a certain phase shift. Relatively to vertical component which reaches the **PBS1** and was reflected to **FM1** direction and passes through the phase modulator,**PM**, that applies a phase shift to it, than it follows to the **FM1** and is reflected rotated by $90°$, meaning that it follows to **FM3** and it is reflected at its original state. At the end, the two components recombine in **PSB1** and the single photon pulse follows its path with a polarization state defined by the phase shift applied in **PM**.

In table 5.7 is present the phases shifts that must be applied at Phase Modulator in order to get the polarized photons chose by Alice.

In table 5.8 are described all components needed to build the experimental setup.

Tabela 5.7: Different Alice's output polarization states based on shift phases applied in Phase Modulator.

Alice:(ϕ_1,ϕ_2)	**Output Polarization**
($\frac{\pi}{2}$,0)	Vertical
($\frac{\pi}{2}$,$\frac{3\pi}{2}$)	Horizontal
(0,0)	$-45°$
(0,$\frac{\pi}{2}$)	$45°$

Tabela 5.8: List of material

Material Name	**Quantity**	**Status**
Laser semiconductor 1550nm	1	✓
Manual polarization controller	5	
Faraday Mirror (FM)	6	
Mach-Zehnder Modulator	1	✓2.56GHz
Single Photon Detector	2	✓
Phase modulator	2	
Four-port polarization beam splitter	2	
Three-port polarization beam splitter	1	
Half-wave plate	1	
Optical circulator	2	
Variable Optical Attenuator	1	✓
Computer	1	

5.7.4 Comparative Analysis

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5.8 BB84 with Discrete Variables

Students Name	:	Mariana Ramos and Kevin Filipe
Starting Date	:	November 7, 2017
Goal	:	BB84 implementation with discrete variables.

BB84 is a key distribution protocol which involves three parties, Alice, Bob and Eve. Alice and Bob exchange information between each other by using a quantum channel and a classical channel. The main goal is continuously build keys only known by Alice and Bob, and guarantee that eavesdropper, Eve, does not gain any information about the keys.

5.8.1 Theoretical Description

BB84 protocol was created by Charles Bennett and Gilles Brassard in 1984. This was the first created Quantum Key Distribution (QKD) protocol. A basic model is depicted in figure 5.42. It involves two parties sharing keys through a quantum channel to decipher the classical channel data with an eavesdropper, Eve, between them.

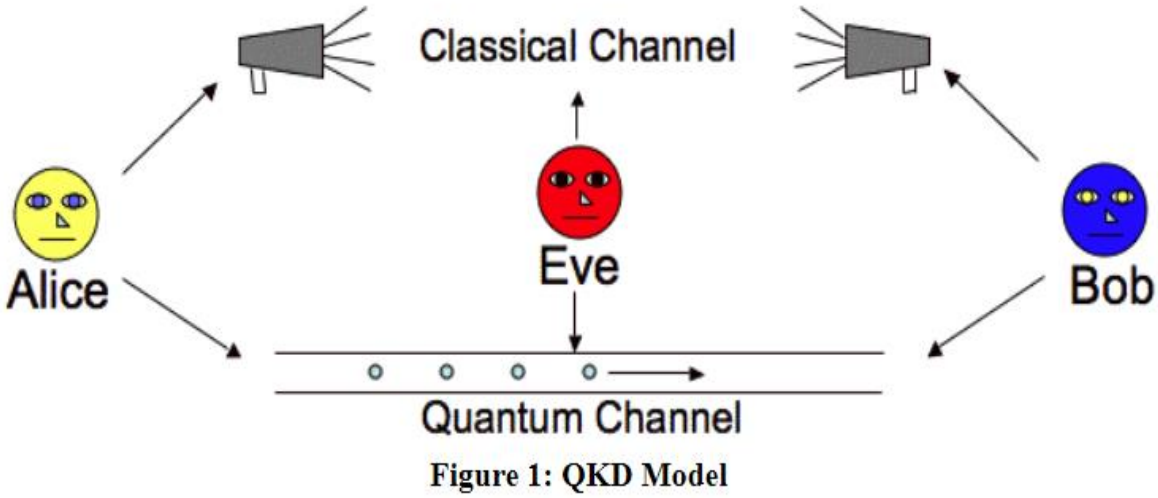

Figura 5.42: Basic QKD Model.

BB84 protocol encodes bits into photon state polarization. This either can have an angle of 0 or 45 degrees, to represent bit 0, or an angle of 90 or 135 degrees, to represent bit 1, in rectilinear or diagonal basis, respectively. Figure 5.43 shows this bit encoding and basis.

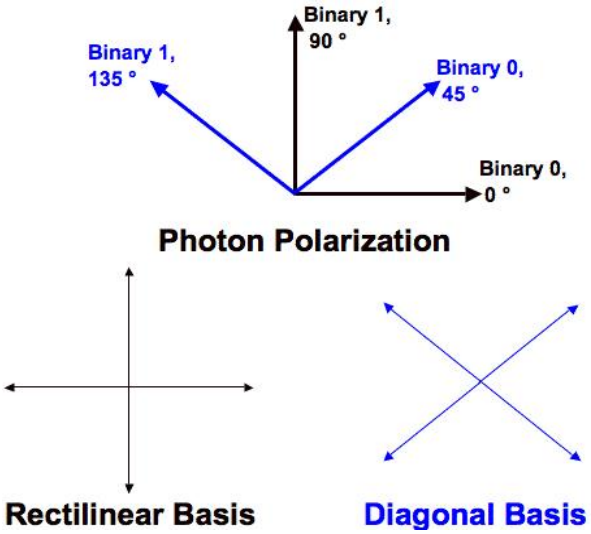

Figura 5.43: Bit Encoding and polarization basis.

The BB82 protocol is divided in different phases. For the first phase, by using the quantum communication channel,

1. Alice chooses a random bit string and polarization, for each bit.
2. For each bit, a photon is transmitted, with an associated polarization, to Bob.
3. Bob receives, measure photon polarization by using a random basis.
4. If Bob chooses a matched polarization basis compared to the one encoded in the photon, then he can correctly deduce the right bit, otherwise deduced bit is randomly read.

The second phase, uses the classical communication channel:

1. Bob notifies Alice about what basis he used to deduce each bit.
2. Alice announces, to Bob, if the deduced basis is correct for each photon, and so, discards the bits measured with different one.
3. If no errors occurred the secret key should be the same for Bob and Alice.

This two phases are depicted in figure 5.44.

Alice's bit	0	1	1	0	1	0	0	1
Alice's basis	+	+	X	+	X	X	X	+
Alice's polarization	↑	→	↖	↑	↖	↗	↗	→
Bob's basis	+	X	X	X	+	X	+	+
Bob's measurement	↑	↗	↖	↗	→	↗	→	→
Public discussion								
Shared Secret key	0		1			0		1

Figura 5.44: Bit Encoding and polarization basis.

In the presence of Eve, error will be introduced, because for Eve to determine the key, she will have to measure the photons sent between Alice and send them to Bob. By doing so, errors will be introduced since its impossible to replicate a particle with an unknown state. Eve will try to guess and these guesses will increase the QBER. –/–/–/–/

Basis	
0	$+$
1	$\times$

	Basis "+"
0	$\rightarrow (0°)$
1	$\uparrow (90°)$

	Basis "×"
0	$\searrow (-45°)$
1	$\nearrow (45°)$

1. Alice randomly generate a bit sequence with length ks being, in this case, $k = 2$ and $s = 4$ as it was defined at the beginning. Therefore, she must define two sets randomly: S_{A1} which contains the basis values; and S_{A2}, which contains the key values.

   In that case, lets assume she gets the following sets S_{A1} and S_{A2}:

   $$S_{A1} = \{0, 1, 1, 0, 0, 1, 0, 1\},$$

   $$S_{A2} = \{1, 1, 0, 0, 0, 1, 0, 0\}.$$

2. Next, Alice sends to Bob throughout a quantum channel ks photons encrypted using the basis defined in S_{A1} and according to the keys defined in S_{A2}.

   In the current example, Alice sends the photons, throughout a quantum channel, according to the following,

   $$S_{AB} = \{\uparrow, \nearrow, \searrow, \rightarrow, \rightarrow, \nearrow, \rightarrow, \searrow\}.$$

$$S_{AB} = \{90°, 45°, -45°, 0°, 0°, 45°, 0°, -45°\}.$$

3. Bob also randomly generates ks bits, which are going to define his measurement basis, S_{B1}. He will measure the photons sent by Alice. Lets assume:

   $$S_{B1} = \{0, 1, 0, 1, 0, 1, 1, 1\}.$$

   When Bob receives photons from Alice, he measures them using the basis defined in S_{B1}. In the current example, S_{B1} corresponds to the following set:

   $$\{+, \times, +, \times, +, \times, \times, \times\}.$$

   Bob will get ks results:

   $$S_{B1'} = \{1, 1, 0, 1, 0, 1, 1, 0\}.$$

4. Bob will send a *Hash Function* result HASH1 to Alice. This value will do Bob's commitment with the measurements done. In this case, this *Hash Function* is calculated from *SHA-256* algorithm for each pair (Basis from S_{B1} and measured value from $S_{B1'}$), i.e Bob sends to Alice sk pairs as his commitment. In this case, Bob sends eight pairs encoded using a *Hash Function* which is also send to Alice. From that moment on Bob cannot change his commitment neither the basis which he uses to measure the photons sent by Alice.

5. Once Alice has received the confirmation of measurement from Bob, she sends throughout a classical channel the basis which she has used to codify the photons, which in this case we assumed $S_{A1} = \{0, 1, 1, 0, 0, 1, 0, 1\}$.

6. In order to know which photons were measured correctly, Bob does the operation $S_{B2} = S_{B1} \oplus S_{A1}$. In the current example the operation will be:

S_{B1}	0	1	0	1	0	1	1	1
S_{A1}	0	1	1	0	0	1	0	1
$\oplus$	1	1	0	0	1	1	0	1

   In this way, Bob gets

   $$S_{B2} = \{1, 1, 0, 0, 1, 1, 0, 1\}.$$

   When Bob uses the right basis he gets the values correctly, when he uses the wrong basis he just guess the value. The values "1" correspond to the values he measured correctly and "0" to the values he just guessed.

   Next, Bob sends to Alice, through a classical channel, information about the minimum number between "ones" and "zeros", i.e

   $$n = min(\#0, \#1) = 3,$$

where $\#0$ represents the number of zeros in S_{B2} and $\#1$ the number of ones in S_{B2}. At this time, Alice must be able to know if Bob is being honest or not. Therefore, she will open Bob's commitment from *step 4* and she verify if the number n sent by Bob is according with the commitment values sent by him. In other words, she opens a number of pairs committed by Bob which is known from the beginning.

7. If $n < s$, being s the message's size, Alice and Bob will repeat the steps from 1 to 7. In this case, $n = 3$ which is smaller than $s = 4$. Therefore, Alice and Bob repeat the steps from 1 to 7 in order to enlarge Bob's sets S_{B1} and S_{B2} as well as Alice's sets S_{A1} and S_{A2}.

8. Lets assume :

$$S_{B1} = \{1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1\}.$$

At Alice's side the new sets S_{A1}, which contains the basis values, and S_{A2}, which contains the key values, will be the following:

$$S_{A1} = \{0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0\},$$
$$S_{A2} = \{1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1\}.$$

Finally, for $S_{B2} = S_{B1} \oplus S_{A1}$ Bob gets the following sequence:

$$S_{B2} = \{1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 1\}.$$

Note that the sets were enlarge in the second iteration.

9. At this time, Bob sends again to Alice, through a classical channel, the minimum number between "ones" and "zeros", $n = min(\#0, \#1)$. In this case, n is equal to 7 which is the number of zeros.

10. Alice checks if $n > s$ and acknowledge to Bob that she already knows that $n > s$. In this case, $n = 7$ and $s = 4$ being $n > s$ a valid condition.

11. Next, Bob defines two new sub-sets, I_0 and I_1. I_0 is a set of values with photons array positions which Bob just guessed the measurement since he did not measure them with the same basis as Alice, I_1 is a set of values with photons array positions which Bob measured correctly since he used the same basis as Alice used to encoded them.

In this example, Bob defines two sub-sets with size $s = 4$:

$$I_0 = \{3, 4, 7, 11\},$$

and

$$I_1 = \{2, 5, 6, 13\},$$

where I_0 is the sequence of positions in which Bob was wrong about basis measurement and I_1 is the sequence of positions in which Bob was right about basis measurement. Bob sends to Alice the set S_b

Thus, if Bob wants to know m_0 he must send to Alice throughout a classical channel the set $S_0 = \{I_1, I_0\}$, otherwise if he wants to know m_1 he must send to Alice throughout a classical channel the set $S_1 = \{I_0, I_1\}$.

12. With both the received set S_b and the hash function value HASH1, Alice must be able to prove that Bob has being honest.

13. Lets assume Bob sent $S_0 = \{I_1, I_0\}$. Alice defines two encryption keys K_0 and K_1 using the values in positions defined by Bob in the set sent by him. In this example, lets assume:

$$K_0 = \{1, 0, 1, 0\}$$
$$K_1 = \{0, 0, 0, 1\}.$$

Alice does the following operations:

$$m = \{m_0 \oplus K_0, m_1 \oplus K_1\}.$$

m_0	0	0	1	1
K_0	1	0	1	0
$\oplus$	1	0	0	1

m_1	0	0	0	1
K_1	0	0	0	1
$\oplus$	0	0	0	0

Adding the two results, m will be:

$$m = \{1, 0, 0, 1, 0, 0, 0, 0\}.$$

After that, Alice sends to Bob the encrypted message m through a classical channel.

14. When Bob receives the message m, in the same way as Alice, Bob uses S_{B1}, values of positions given by I_1 and I_0 and does the decrypted operation. In this case, he does following operation:

m	1	0	0	1	0	0	0	0
	1	0	1	0	0	1	1	0
$\oplus$	0	0	1	1	0	1	1	0

The first four bits corresponds to message 1 and he received $\{0, 0, 1, 1\}$, which is the right message m_0 and $\{0, 1, 1, 0\}$ which is a wrong message for m_1.

5.8.2 Simulation Setup

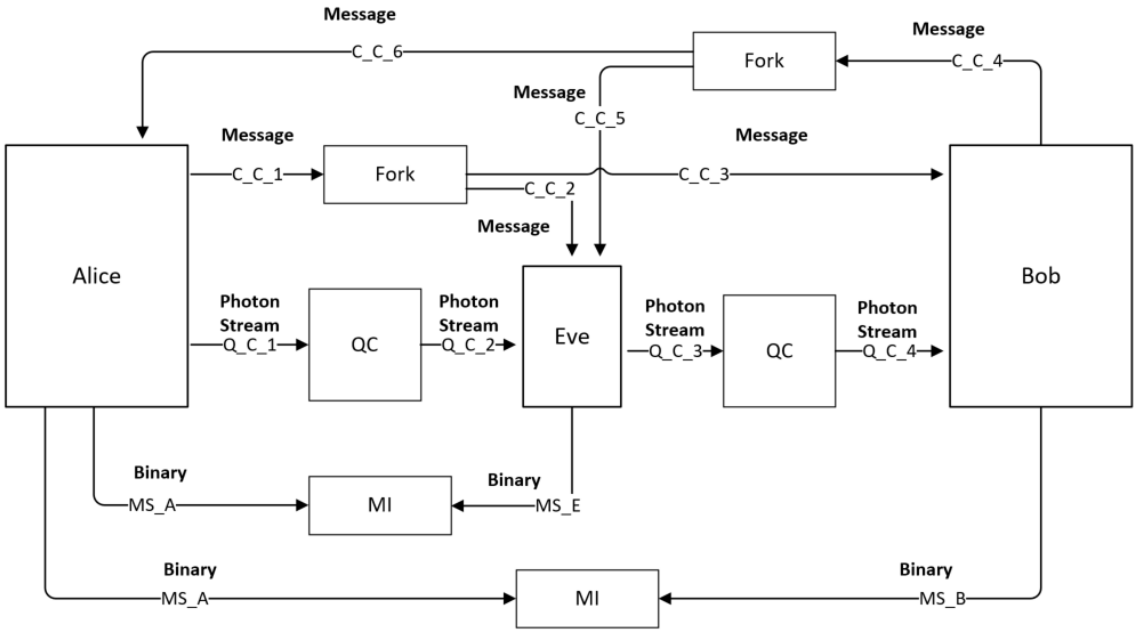

Figura 5.45: Simulation diagram at a top level

Figure 5.45 presents the top level diagram of our simulation. The setup contains three parties, Alice, Eve and Bob where the communication between them is done throughout two classical and one quantum channel. In the middle of the classical channel there is a Fork's diagram which has one input and two outputs. In the case of the classical channel **C_C_4** which has the information sent by Bob, the fork's block enables Alice and Eve have access to it. In the quantum communication, the information sent by Alice can be intercepted by Eve and changed by her, or can follow directly to Bob as we can see later in figure 5.48. Furthermore, for mutual information calculation there must be two blocks **MI**, one to calculate the mutual information between Alice and Eve, and other to calculate the mutual information between Alice and Bob.

In figure 5.46 one can observe a block diagram of the simulation at Alice's side. As it is shown in the figure, Alice must have one block for random number generation which is responsible for basis generation to polarize the photons, and for key random generation in order to have a random state to encode each photon. Furthermore, she has a Processor block for all logical operations: array analysis, random number generation requests, and others. This block also receives the information from Bob after it has passed through a fork's block. In addition, it is responsible for set the initial length l of the first array of photons which will send to Bob. This block also must be responsible for send classical information to Bob. Finally, Processor block will also send a real continuous time signal to single photon generator, in order to generate photons according to this signal, and finally this block also

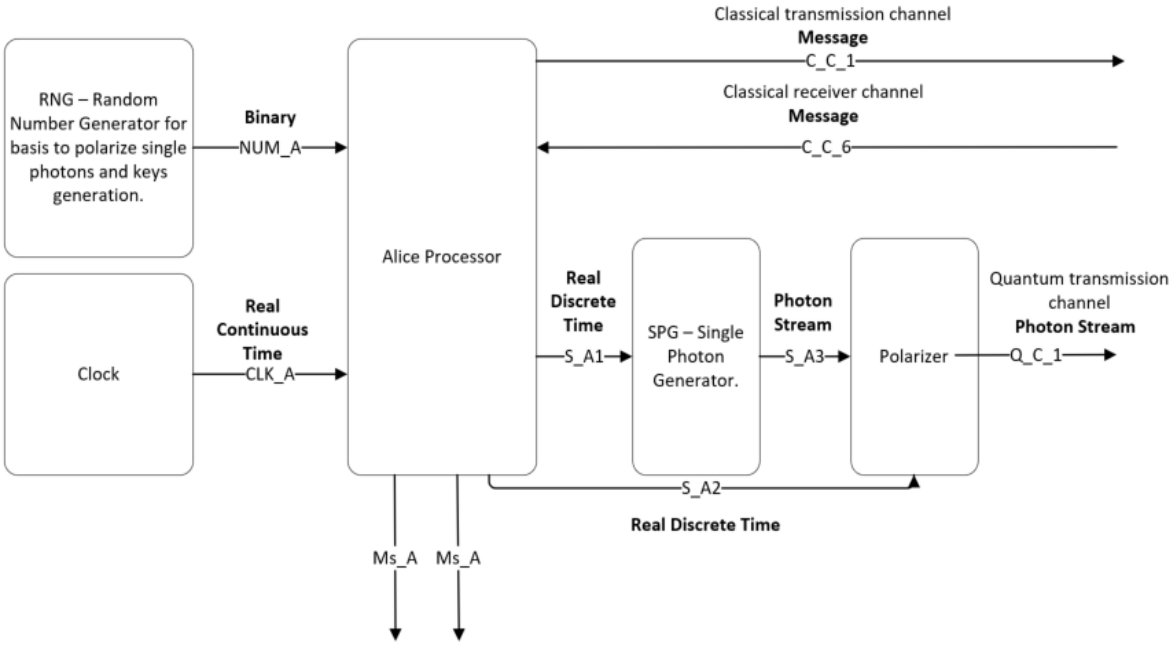

Figura 5.46: Simulation diagram at Alice's side

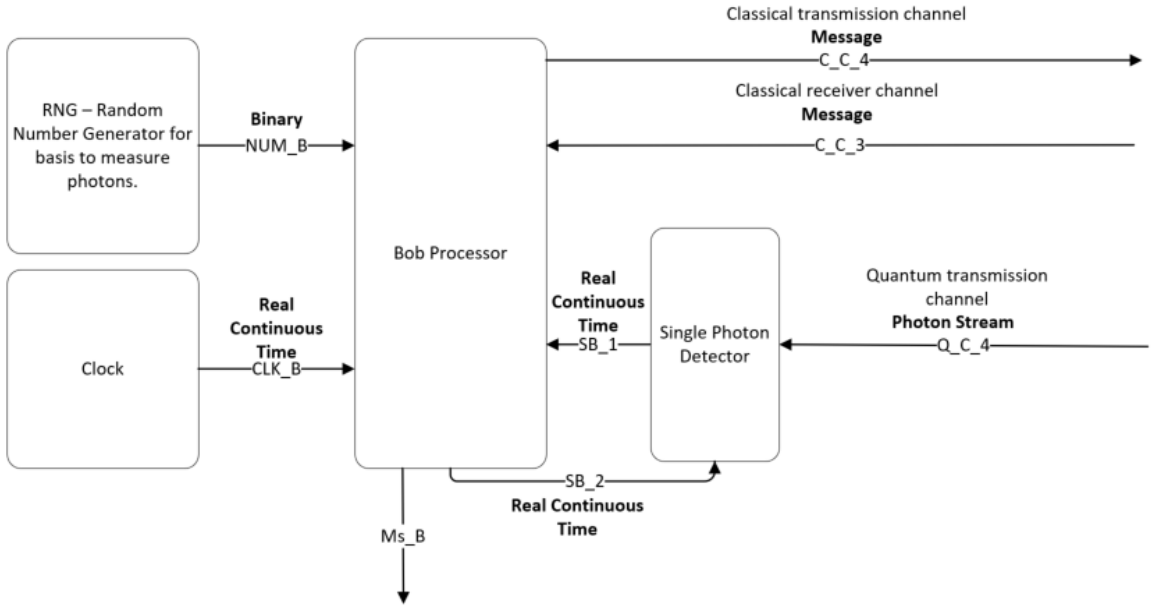

Figura 5.47: Simulation diagram at Bob's side

sends to the polarizer a real discrete signal in order to inform the polarizer which basis it should use. Therefore, she has two more blocks for quantum tasks: the single photon generator and the polarizer block which is responsible to encode the photons generated

from the previous block and send them throughout a quantum channel from Alice to Bob.

Finally, Alice's processor has an output to Mutual Information top level block, Ms_A.

In figure 5.47 one can observe a block diagram of the simulation at Bob's side. From this side, Bob has one block for Random Number Generation which is responsible for randomly generate basis values which Bob will use to measure the photons sent by Alice throughout the quantum channel. Like Alice, Bob has a Processor block responsible for all logical tasks, analysing functions, requests for random number generator block, etc. Additionally, it receives information from Alice throughout a classical channel after passed through a fork's block and a quantum channel. However, Bob only sends information to Alice throughout a classical channel. Furthermore, Bob has one more block for single photon detection which receives from processor block a real discrete time signal, in order to obtain the basis it should use to measure the photons.

Finally, Bob's processor has an output to Mutual Information top level block, Ms_B.

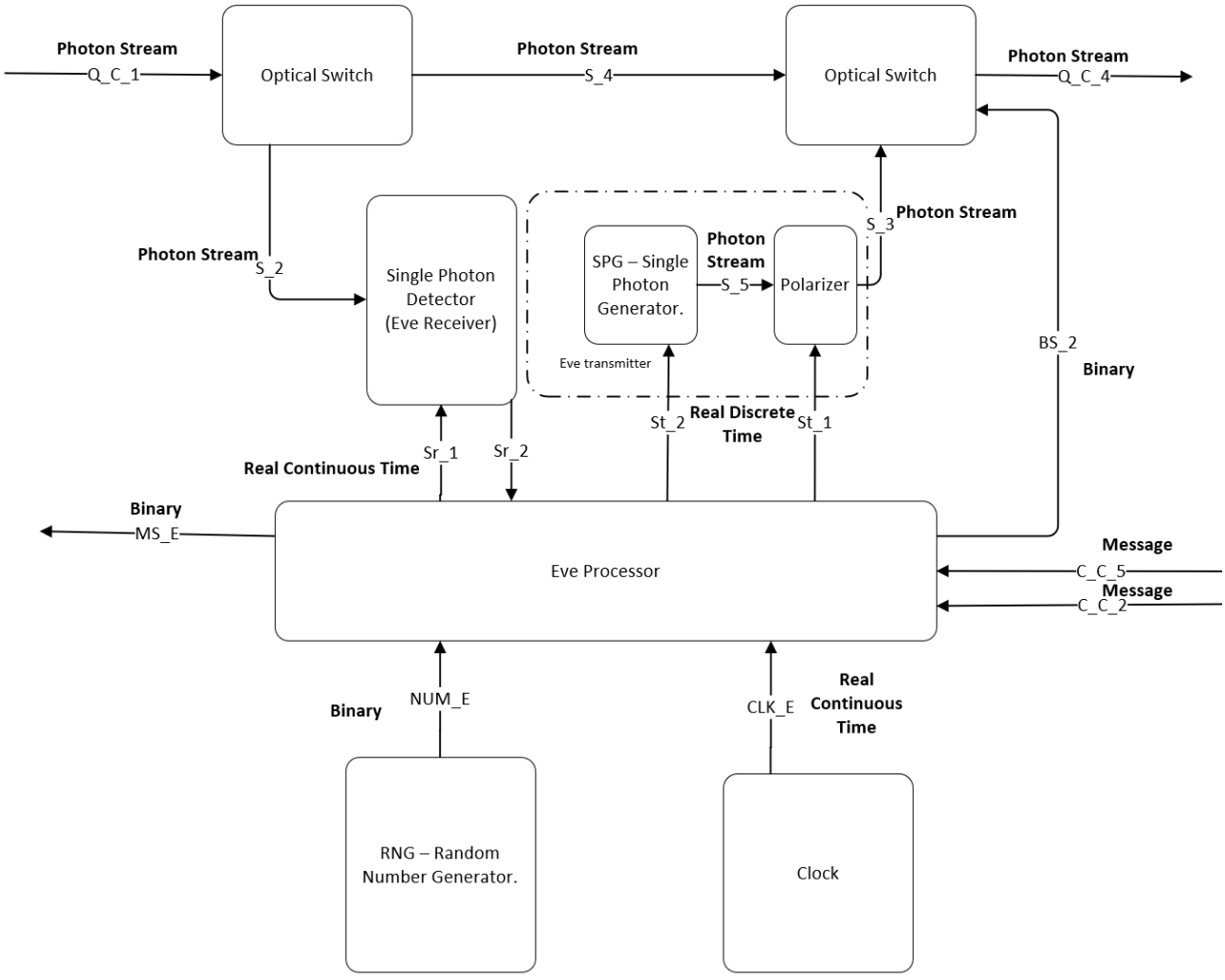

Figura 5.48: Simulation diagram at Eve's side

Figure 5.48 presents the Eve's side diagram. Eve's processor has two receiver classical signals, one from Alice (**C_C_2**) and other from Bob (**C_C_5**). About quantum channel, Eve received a quantum message from Alice through the channel **Q_C_1** and depends on her decision the photon can follows directly to Bob or the photon's state can be changed by her. In this case, the photon is received by a block similar to Bob's diagram 5.47 and this block sends a message to Eve's processor in order to reveal the measurement result. After that, Eve's processor sends a message to Alice's diagram similar to figure 5.46 and this block is responsible for encode the photon in a new state. Now, the changed photon is sent to Bob.

In addition, Eve's diagram has one more output Ms_E which is a message sent to the mutual information block as an input parameter.

Tabela 5.9: System Signals

Signal name	**Signal type**	**Status**
C_C_1 ... C_C_6	Message	
Q_C_1 .. Q_C_4	Photon Stream	
Ms_A, Ms_B, Ms_E	Binary	
NUM_A , NUM_B, NUM_E	Binary	
CLK_A, CLK_B, CLK_E	Real continuous time	
SB_1, SB_2, Sr_1, Sr_2	Real continuous time	
SA_1, SA_2, St_1, St_2	Real discrete time	
SA_3	Photon Stream	
S_2, S_3, S_4, S_5	Photon Stream	
BS_1, BS_2	Binary	

Table 5.12 presents the system signals as well as them type.

Tabela 5.10: System Input Parameters

Parameter	**Default Value**	**Description**
SymbolRate	100K	
NumberOfBits	Number of photons that Alice sends to Bob	

Tabela 5.11: Header Files

File name	Description	Status
binary_source.h		
single_photon_source.h		
single_photon_detector.h		
optical_switch.h		Missing
polarization_beam_splitter.h		
mutual_information.h		Missing
bit_error_rate.h		
clock.h		
fiber.h		
qrng_decision_circuit.h		
message_to_send.h		Missing
message_to_receive.h		Missing
netxpto.h		

Tabela 5.12: Source Files

File name	Description	Status
binary_source.cpp		
single_photon_sourcer.cpp		
single_photon_detector.cpp		
optical_switch.cpp		Missing
polarization_beam_splitter.cpp		
mutual_information.cpp		Missing
bit_error_rate.cpp		
clock.cpp		
fiber.cpp		
qrng_decision_circuit.cpp		
message_to_send.cpp		Missing
message_to_receive.cpp		Missing
netxpto.cpp		
bb84_sdf.cpp		

5.9 Radio Over Fiber Transmission System

Student Name	:	Celestino Martins
Starting Date	:	September 25, 2017
Goal	:	Simulation of Radio over fiber Transmission considering the uplink of base station cooperation systems.

Radio over fiber (RoF) technology comprises the transmission over fiber technology, where radio signal is modulated onto optical carrier and transmitted over an optical fiber link to provide a simple antenna front ends with increased capacity and broadband wireless services. In this network a central processing units (CPU) is connected to numerous base stations (BSs) via optic fibers. That means, RoF networks use optic fiber links to distribute radio frequency (RF) signals between the CPU and BSs. The downlink RF signals are distributed from a CPU to many BSs through the fibres, while the uplink signals received at BSs are sent back to the CPU for any signal processing. Figure 5.49 shows a general RoF architecture, where the wireless signals are transported over the optical fiber between a CPU and a set of base stations before being radiated through the air. RoF transmission systems are usually classified into two main categories, depending on the frequency range of the radio signal to be transported: i) RF-over-Fiber; ii) intermediate frequency (IF)-over-Fiber.

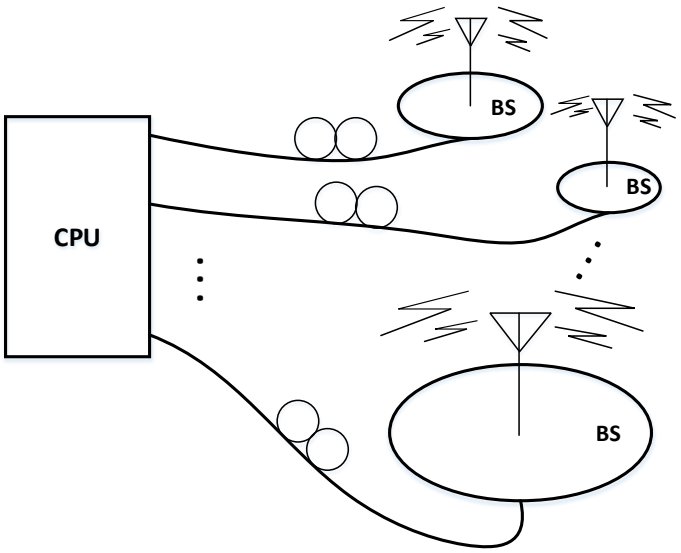

Figura 5.49: Schematic showing the concept of a centralized CPU architecture for future integrated optical wireless networks based on RoF.

(i) In RF-over-Fiber architecture, a data-carrying RF (Radio Frequency) signal with a high frequency (usually greater than 10 GHz) is imposed on a lightwave signal before being transported over the optical link. Therefore, wireless signals are optically distributed to base stations directly at high frequencies and converted from the optical to electrical

domain at the base stations before being amplified and radiated by an antenna. As a result, no frequency up/down conversion is required at the various base stations, thereby resulting in simple and rather cost-effective implementation is enabled at the base stations.

(ii) In IF-over-Fiber architecture, an IF (Intermediate Frequency) radio signal with a lower frequency (less than 10 GHz) is used for modulating light before being transported over the optical link. Therefore, before radiation through the air, the signal must be up-converted to RF at the base station.

In addition, the RoF technology can be implemented as analog RoF or digital RoF:

(i) In analog RoF technology, the analog signal is transmitted over the optical fiber, being either RF signal, IF signal or baseband BB signal. In the optical transmitter, the RF/IF/BB signal is modulated onto the optical carrier by either using direct or external modulation of the laser. In this case, the signal distribution through RoF has the advantage of simplified BS design, however it is susceptible to fiber chromatic dispersion and nonlinearity generated by optical devices.

(ii) In the digitized RoF the wireless carrier RF signal is first digitized prior to transport over the optical link. The digitalization of an RF signal produces a sampled digital signal in a serial form that can be directly modulated on an optical carrier, transmitted over the fiber optic link, and then detected like any other digital information. Modulation of the digital signal onto an optical carrier minimizes the nonlinear effects originating from the optical-to-electrical conversion function presented on analog RoF. In order to use not so high sample rates at the ADC/DAC components generally the bandpass sampling technique i applied to the RF signal.

5.9.1 Theoretical Analysis

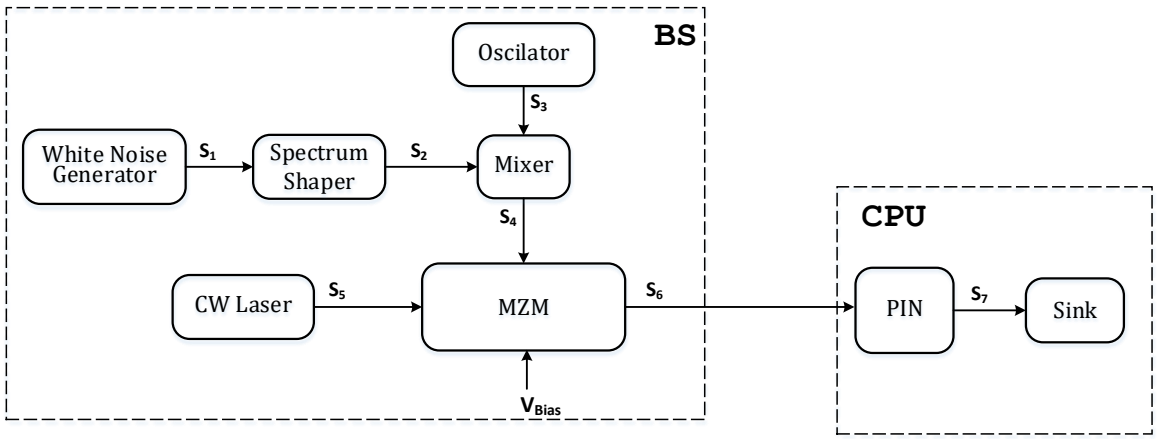

Figura 5.50: Simulation setup for the uplink of RoF transmission system;

Figure 5.50 depicts the simulation setup for an uplink of RoF, providing the connection between BS to CPU. At BS, we model the RF signal received from a mobile terminal, as zero mean complex gaussian (CG) signal with a given bandwidth imposed by the low pass filter, Filter-1. The generated baseband signal is then up-converted to RF carrier frequency by utilizing an oscillator and a mixer. In this simulation we consider the RF carrier frequency between 2 to 5 GHz, according to the 5G technologies specifications. The generated RF passband signal modulates optical carrier by utilizing a laser and a single driver mach-zehnder modulator (MZM). The optical signal is then transmitted to CPU using optical fiber. In CPU, the optical signal is detected by a PIN, amplified and followed with an electrical filter. After these operations, digital signal processing techniques can be applied to recover the transmitted signal.

Tabela 5.13: System Input Parameters.

Parameter	Default Value	Description
sourceMode		
symbolPeriod		
samplePeriod		
numberOfSamplesPerSymbol		
filterType1		
rollOffFactor		
filterType2		
outputOpticalPower		
outputOpticalWavelength		
rfCenterFrequency		
fiberAttenuation		

Tabela 5.14: Header Files for RoF Transmission System.

File Name	Description	Status
complex_gaussian_signal.h		
pulse_shaper.h		✓
local_oscillator.h		✓
mixer.h		
cw_laser.h		
iq_modulator.h		✓
fiber.h		
pin.h		
amplifier.h		
filter_rx.h		
sink.h		✓
netxpto.h		✓

Tabela 5.15: Source Files for RoF Transmission System.

File Name	Description	Status
complex_gaussian_signal.cpp		
pulse_shaper.cpp		✓
local_oscillator.cpp		✓
mixer.cpp		
cw_laser.cpp		
iq_modulator.cpp		✓
fiber.cpp		
pin.cpp		
amplifier.cpp		
filter_rx.cpp		
sink.cpp		✓
netxpto.cpp		✓

5.9.2 Experimental

Capítulo 6

Library

6.1 Add

Input Parameters

This block takes no parameters.

Functional Description

This block accepts two signals and outputs one signal built from a sum of the two inputs. The input and output signals must be of the same type, if this is not the case the block returns an error.

Input Signals

Number: 2
Type: Real, Complex or Complex_XY signal (ContinuousTimeContinuousAmplitude)

Output Signals

Number: 1
Type: Real, Complex or Complex_XY signal (ContinuousTimeContinuousAmplitude)

6.2 Bit Error Rate

Header File	:	bit_error_rate.h
Source File	:	bit_error_rate.cpp
Version	:	20171810 (Responsible: Daniel Pereira)

Input Parameters

Name	Type	Default Value
Confidence	double	0.95
MidReportSize	integer	0
LowestMinorant	double	1×10^{-10}

Input Signals

Number: 2

Type: Binary (DiscreteTimeDiscreteAmplitude)

Output Signals

Number: 1

Type: Binary (DiscreteTimeDiscreteAmplitude)

Functional Description

This block accepts two binary strings and outputs a binary string, outputting a 1 if the two input samples are equal to each other and 0 if not. This block also outputs *.txt* files with a report of the estimated Bit Error Rate (BER), $\widehat{\text{BER}}$ as well as the estimated confidence bounds for a given probability α.

The block allows for mid-reports to be generated, the number of bits between reports is customizable, if it is set to 0 then the block will only output the final report.

Theoretical Description

The $\widehat{\text{BER}}$ is obtained by counting both the total number received bits, N_T, and the number of coincidences, K, and calculating their relative ratio:

$$\widehat{\text{BER}} = 1 - \frac{K}{N_T}. \tag{6.1}$$

The upper and lower bounds, BER_{UB} and BER_{LB} respectively, are calculated using the Clopper-Pearson confidence interval, which returns the following simplified expression for

$N_T > 40$ [5]:

$$\text{BER}_{\text{UB}} = \widehat{\text{BER}} + \frac{1}{\sqrt{N_T}} z_{\alpha/2} \sqrt{\widehat{\text{BER}}(1 - \widehat{\text{BER}})} + \frac{1}{3N_T} \left[2 \left(\frac{1}{2} - \widehat{\text{BER}} \right) z_{\alpha/2}^2 + (2 - \widehat{\text{BER}}) \right] \quad (6.2)$$

$$\text{BER}_{\text{LB}} = \widehat{\text{BER}} - \frac{1}{\sqrt{N_T}} z_{\alpha/2} \sqrt{\widehat{\text{BER}}(1 - \widehat{\text{BER}})} + \frac{1}{3N_T} \left[2 \left(\frac{1}{2} - \widehat{\text{BER}} \right) z_{\alpha/2}^2 - (1 + \widehat{\text{BER}}) \right], \quad (6.3)$$

where $z_{\alpha/2}$ is the $100\left(1 - \frac{\alpha}{2}\right)$th percentile of a standard normal distribution.

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6.3 Binary source

This block generates a sequence of binary values (1 or 0) and it can work in four different modes:

1. Random
2. PseudoRandom
3. DeterministicCyclic
4. DeterministicAppendZeros

This blocks doesn't accept any input signal. It produces any number of output signals.

Input Parameters

Parameter: mode{PseudoRandom}
(Random, PseudoRandom, DeterministicCyclic, DeterministicAppendZeros)

Parameter: probabilityOfZero{0.5}
(real $\in$ [0,1])

Parameter: patternLength{7}
(integer $\in$ [1,32])

Parameter: bitStream{"0100011101010101"}
(string of 0's and 1's)

Parameter: numberOfBits{-1}
(long int)

Parameter: bitPeriod{1.0/100e9}
(double)

Methods

BinarySource(vector⟨Signal *⟩ &InputSig, vector⟨Signal *⟩ &OutputSig) :Block(InputSig, OutputSig){};

void initialize(void);

bool runBlock(void);

void setMode(BinarySourceMode m) BinarySourceMode const getMode(void)

void setProbabilityOfZero(double pZero)

double const getProbabilityOfZero(void)

void setBitStream(string bStream)

string const getBitStream(void)

void setNumberOfBits(long int nOfBits)

long int const getNumberOfBits(void)

void setPatternLength(int pLength)

int const getPatternLength(void)

void setBitPeriod(double bPeriod)

double const getBitPeriod(void)

Functional description

The *mode* parameter allows the user to select between one of the four operation modes of the binary source.

Random Mode Generates a 0 with probability *probabilityOfZero* and a 1 with probability 1-*probabilityOfZero*.

Pseudorandom Mode Generates a pseudorandom sequence with period $2^{patternLength} - 1$.

DeterministicCyclic Mode Generates the sequence of 0's and 1's specified by *bitStream* and then repeats it.

DeterministicAppendZeros Mode Generates the sequence of 0's and 1's specified by *bitStream* and then it fills the rest of the buffer space with zeros.

Input Signals

Number: 0

Type: Binary (DiscreteTimeDiscreteAmplitude)

Output Signals

Number: 1 or more

Type: Binary (DiscreteTimeDiscreteAmplitude)

Examples

Random Mode

PseudoRandom Mode As an example consider a pseudorandom sequence with *patternLength*=3 which contains a total of 7 ($2^3 - 1$) bits. In this sequence it is possible to find every combination of 0's and 1's that compose a 3 bit long subsequence with the exception of 000. For this example the possible subsequences are 010, 110, 101, 100, 111, 001 and 100 (they appear in figure 6.1 numbered in this order). Some of these require wrap.

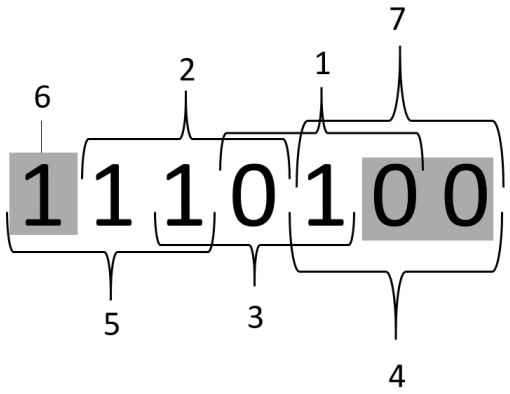

Figura 6.1: Example of a pseudorandom sequence with a pattern length equal to 3.

DeterministicCyclic Mode As an example take the *bit stream* '0100011101010101'. The generated binary signal is displayed in.

DeterministicAppendZeros Mode Take as an example the *bit stream* '0100011101010101'. The generated binary signal is displayed in 6.2.

Sugestions for future improvement

Implement an input signal that can work as trigger.

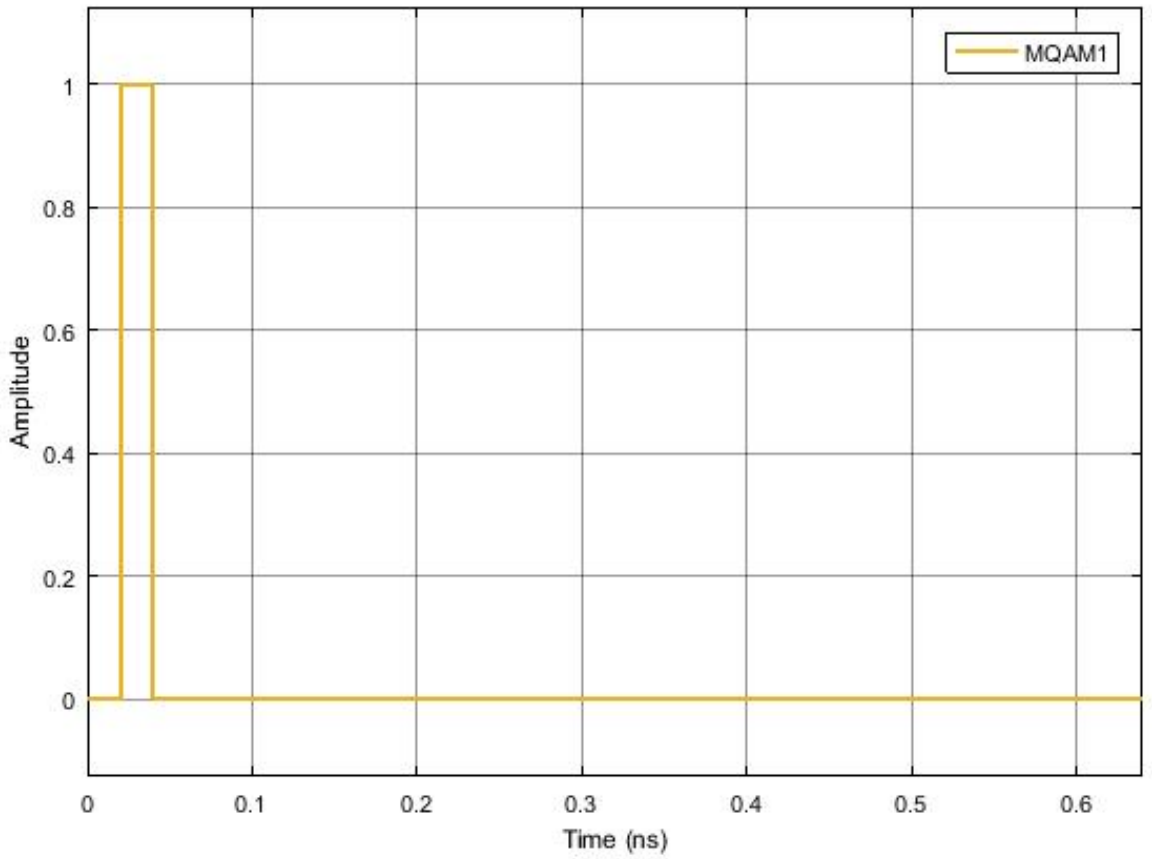

Figura 6.2: Binary signal generated by the block operating in the *Deterministic Append Zeros* mode with a binary sequence 01000...

6.4 Bit Decider

Input Parameters

Parameter: setPosReferenceValue **Parameter:** setNegReferenceValue

Functional Description

This block accepts one real discrete signal and outputs a binary string, outputting a 1 if the input sample is above the predetermined reference level and 0 if it is below another reference value The reference values are defined by the values of *PosReferenceValue* and *NegReferenceValue*.

Input Signals

Number: 1
Type: Real signal (DiscreteTimeContinuousAmplitude)

Output Signals

Number: 1
Type: Binary (DiscreteTimeDiscreteAmplitude)

6.5 Clock

This block doesn't accept any input signal. It outputs one signal that corresponds to a sequence of Dirac's delta functions with a user defined *period*.

Input Parameters

Parameter: period{ 0.0 };

Parameter: samplingPeriod{ 0.0 };

Methods

Clock()

Clock(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig, OutputSig)

void initialize(void)

bool runBlock(void)

void setClockPeriod(double per)

void setSamplingPeriod(double sPeriod)

Functional description

Input Signals

Number: 0

Output Signals

Number: 1

Type: Sequence of Dirac's delta functions. (TimeContinuousAmplitudeContinuousReal)

Examples

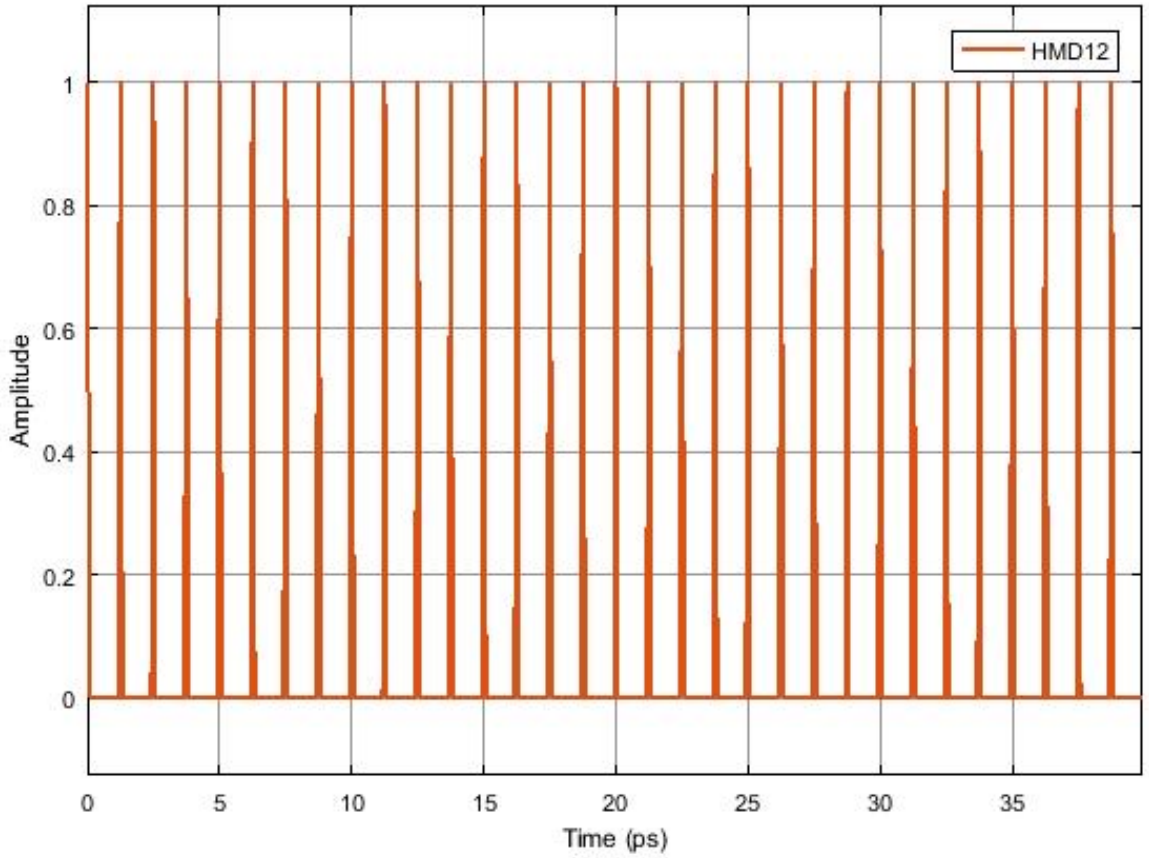

Figura 6.3: Example of the output signal of the clock

Sugestions for future improvement

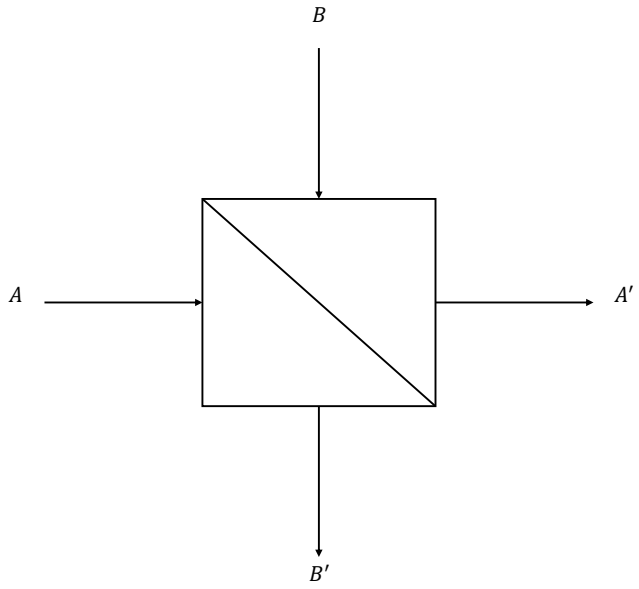

Figura 6.4: 2x2 coupler

6.6 Coupler 2 by 2

In general, the matrix representing 2x2 coupler can be summarized in the following way,

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} T & iR \\ iR & T \end{bmatrix} . \begin{bmatrix} A \\ B \end{bmatrix} \tag{6.4}$$

Where, A and B represent inputs to the 2x2 coupler and A′ and B′ represent output of the 2x2 coupler. Parameters T and R represent transmitted and reflected part respectively which can be quantified in the following form,

$$T = \sqrt{1 - \eta_R} \tag{6.5}$$

$$R = \sqrt{\eta_R} \tag{6.6}$$

Where, value of the $\sqrt{\eta_R}$ lies in the range of $0 \leq \sqrt{\eta_R} \leq 1$.

It is worth to mention that if we put $\eta_R = 1/2$ then it leads to a special case of "Balanced Beam splitter"which equally distribute the input power into both output ports.

6.7 Decoder

This block accepts a complex electrical signal and outputs a sequence of binary values (0's and 1's). Each point of the input signal corresponds to a pair of bits.

Input Parameters

Parameter: `t_integer` m{ 4 }

Parameter: vector<`t_complex`> iqAmplitudes{ { 1.0, 1.0 },{ -1.0, 1.0 },{ -1.0, -1.0 },{ 1.0, -1.0 } };

Methods

Decoder()

Decoder(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig, OutputSig)

void initialize(void)

bool runBlock(void)

void setM(int mValue)

void getM()

void setIqAmplitudes(vector<`t_iqValues`> iqAmplitudesValues)

vector<`t_iqValues`>getIqAmplitudes()

Functional description

This block makes the correspondence between a complex electrical signal and pair of binary values using a predetermined constellation.

To do so it computes the distance in the complex plane between each value of the input signal and each value of the *iqAmplitudes* vector selecting only the shortest one. It then converts the point in the IQ plane to a pair of bits making the correspondence between the input signal and a pair of bits.

Input Signals

Number: 1

Type: Electrical complex (TimeContinuousAmplitudeContinuousReal)

Output Signals

Number: 1

Type: Binary

Examples

As an example take an input signal with positive real and imaginary parts. It would correspond to the first point of the *iqAmplitudes* vector and therefore it would be associated to the pair of bits 00.

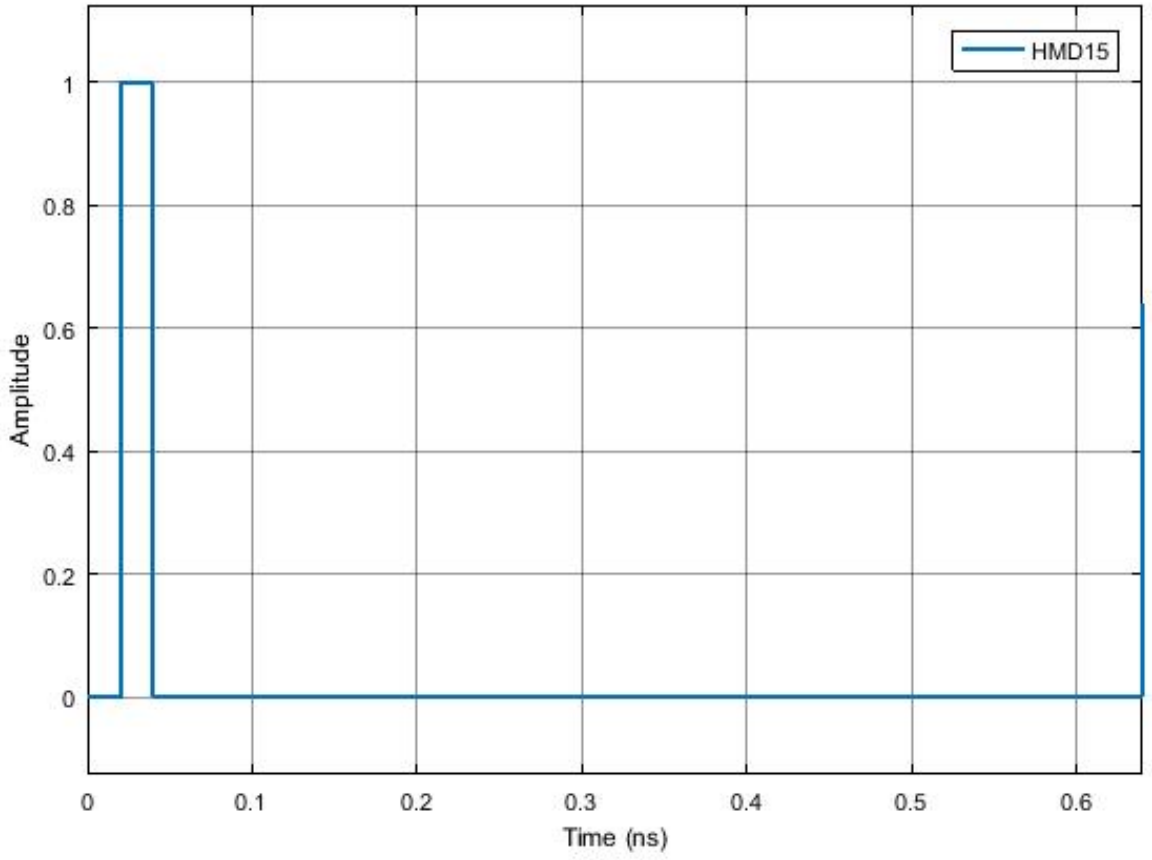

Figura 6.5: Example of the output signal of the decoder for a binary sequence 01. As expected it reproduces the initial bit stream

Sugestions for future improvement

6.8 Discrete to continuous time

This block converts a signal discrete in time to a signal continuous in time. It accepts one input signal that is a sequence of 1's and -1's and it produces one output signal that is a sequence of Dirac delta functions.

Input Parameters

Parameter: numberOfSamplesPerSymbol{8}
(int)

Methods

DiscreteToContinuousTime(vector<Signal *> &inputSignals, vector<Signal *> &outputSignals) :Block(inputSignals, outputSignals){};

void initialize(void);

bool runBlock(void);

void setNumberOfSamplesPerSymbol(int nSamplesPerSymbol)

int const getNumberOfSamplesPerSymbol(void)

Functional Description

This block reads the input signal buffer value, puts it in the output signal buffer and it fills the rest of the space available for that symbol with zeros. The space available in the buffer for each symbol is given by the parameter *numberOfSamplesPerSymbol*.

Input Signals

Number : 1

Type : Sequence of 1's and -1's. (DiscreteTimeDiscreteAmplitude)

Output Signals

Number : 1

Type : Sequence of Dirac delta functions (ContinuousTimeDiscreteAmplitude)

Example

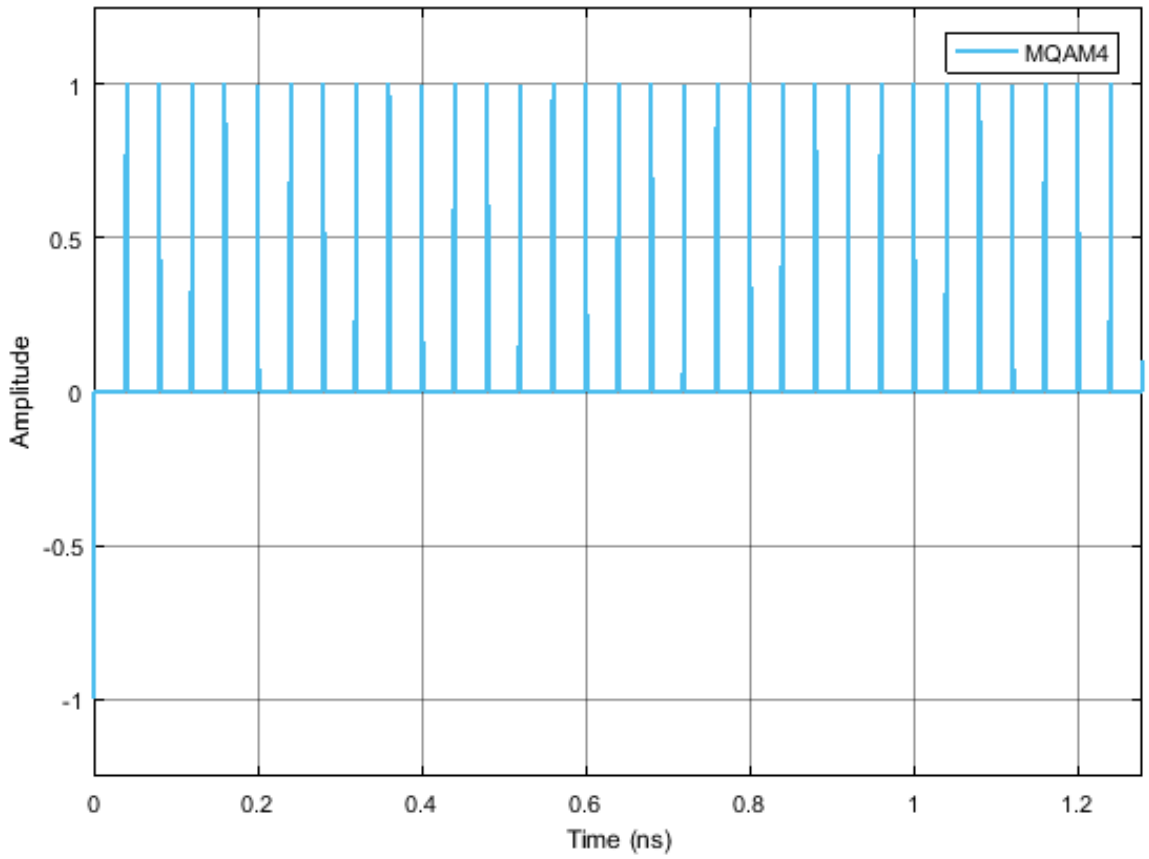

Figura 6.6: Example of the type of signal generated by this block for a binary sequence 0100...

6.9 MQAM Homodyne receiver

Warning: *homodyne_receiver* is not recommended. Use *m_qam_homodyne_receiver* instead.

This block of code simulates the reception and demodulation of an optical signal (which is the input signal of the system) outputing a binary signal. A simplified schematic representation of this block is shown in figure 6.7.

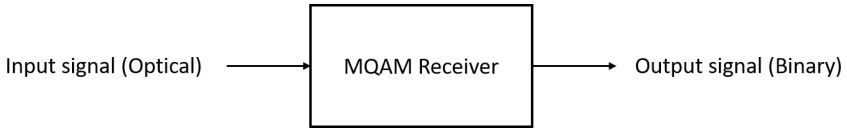

Figura 6.7: Basic configuration of the MQAM receiver

Functional description

This block accepts one optical input signal and outputs one binary signal that corresponds to the M-QAM demodulation of the input signal. It is a complex block (as it can be seen from figure 6.8) of code made up of several simpler blocks whose description can be found in the *lib* repository.

In can also be seen from figure 6.8 that there's an extra internal (generated inside the homodyne receiver block) input signal generated by the *Clock*. This block is used to provide the sampling frequency to the *Sampler*.

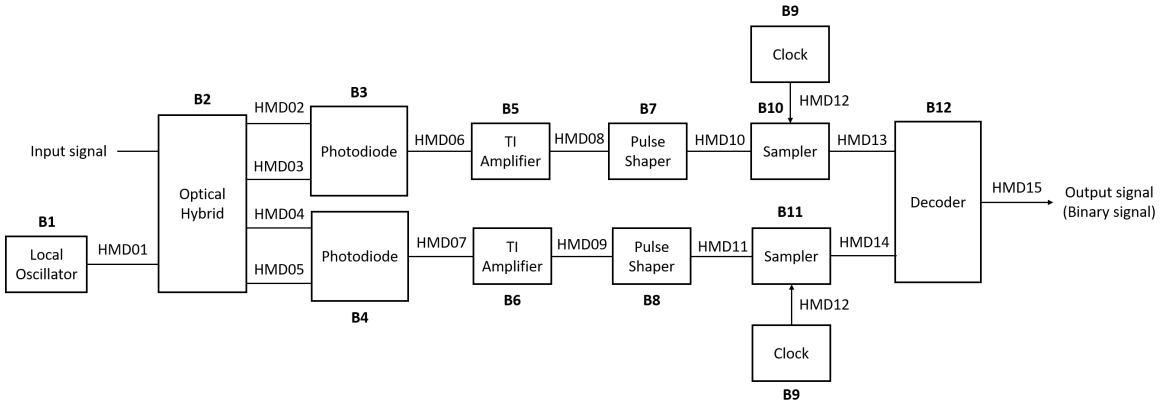

Figura 6.8: Schematic representation of the block homodyne receiver.

Input parameters

This block has some input parameters that can be manipulated by the user in order oto change the basic configuration of the receiver. Each parameter has associated a function that allows for its change. In the following table (table 6.2) the input parameters and corresponding functions are summarized.

Input parameters	**Function**	Type	**Accepted values**
IQ amplitudes	setIqAmplitudes	Vector of coordinate points in the I-Q plane	**Example** for a 4-qam mapping: { { 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 } }
Local oscillator power (in dBm)	setLocalOscillatorOpticalPower_dBm	double(t_real)	Any double greater than zero
Local oscillator phase	setLocalOscillatorPhase	double(t_real)	Any double greater than zero
Responsivity of the photodiodes	setResponsivity	double(t_real)	$\in$ [0,1]
Amplification (of the TI amplifier)	setAmplification	double(t_real)	Positive real number
Noise amplitude (introduced by the TI amplifier)	setNoiseAmplitude	double(t_real)	Real number greater than zero
Samples to skipe	setSamplesToSkip	int(t_integer)	
Save internal signals	setSaveInternalSignals	bool	True or False
Sampling period	setSamplingPeriod	double	Given by *symbolPeriod/samplesPerSy*

Tabela 6.1: List of input parameters of the block MQAM receiver

Methods

HomodyneReceiver(vector<Signal *> &inputSignal, vector<Signal *> &outputSignal) (**constructor**)

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues)

vector<t_iqValues> const getIqAmplitudes(void)

void setLocalOscillatorSamplingPeriod(double sPeriod)

void setLocalOscillatorOpticalPower(double opticalPower)

void setLocalOscillatorOpticalPower_dBm(double opticalPower_dBm)

void setLocalOscillatorPhase(double lOscillatorPhase)

void setLocalOscillatorOpticalWavelength(double lOscillatorWavelength)

void setSamplingPeriod(double sPeriod)

void setResponsivity(t_real Responsivity)

void setAmplification(t_real Amplification)

void setNoiseAmplitude(t_real NoiseAmplitude)

void setImpulseResponseTimeLength(int impResponseTimeLength)

void setFilterType(PulseShaperFilter fType)

void setRollOffFactor(double rOffFactor)

void setClockPeriod(double per)

void setSamplesToSkip(int sToSkip)

Input Signals

Number: 1

Type: Optical signal

Output Signals

Number: 1

Type: Binary signal

Example

Sugestions for future improvement

6.10 IQ modulator

This blocks accepts one inupt signal continuous in both time and amplitude and it can produce either one or two output signals. It generates an optical signal and it can also generate a binary signal.

Input Parameters

Parameter: outputOpticalPower{1e-3}
(double)

Parameter: outputOpticalWavelength{1550e-9}
(double)

Parameter: outputOpticalFrequency{speed_of_light/outputOpticalWavelength}
(double)

Methods

IqModulator(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig, OutputSig){};

void initialize(void);

bool runBlock(void);

void setOutputOpticalPower(double outOpticalPower)

void setOutputOpticalPower_dBm(double outOpticalPower_dBm)

void setOutputOpticalWavelength(double outOpticalWavelength)

void setOutputOpticalFrequency(double outOpticalFrequency)

Functional Description

This block takes the two parts of the signal: in phase and in amplitude and it combines them to produce a complex signal that contains information about the amplitude and the phase.

This complex signal is multiplied by $\frac{1}{2}\sqrt{outputOpticalPower}$ in order to reintroduce the information about the energy (or power) of the signal. This signal corresponds to an optical signal and it can be a scalar or have two polarizations along perpendicular axis. It is the signal that is transmited to the receptor.

The binary signal is sent to the Bit Error Rate (BER) meaurement block.

Input Signals

Number : 2

Type : Sequence of impulses modulated by the filter (ContinuousTimeContiousAmplitude))

Output Signals

Number : 1 or 2

Type : Complex signal (optical) (ContinuousTimeContinuousAmplitude) and binary signal (DiscreteTimeDiscreteAmplitude)

Example

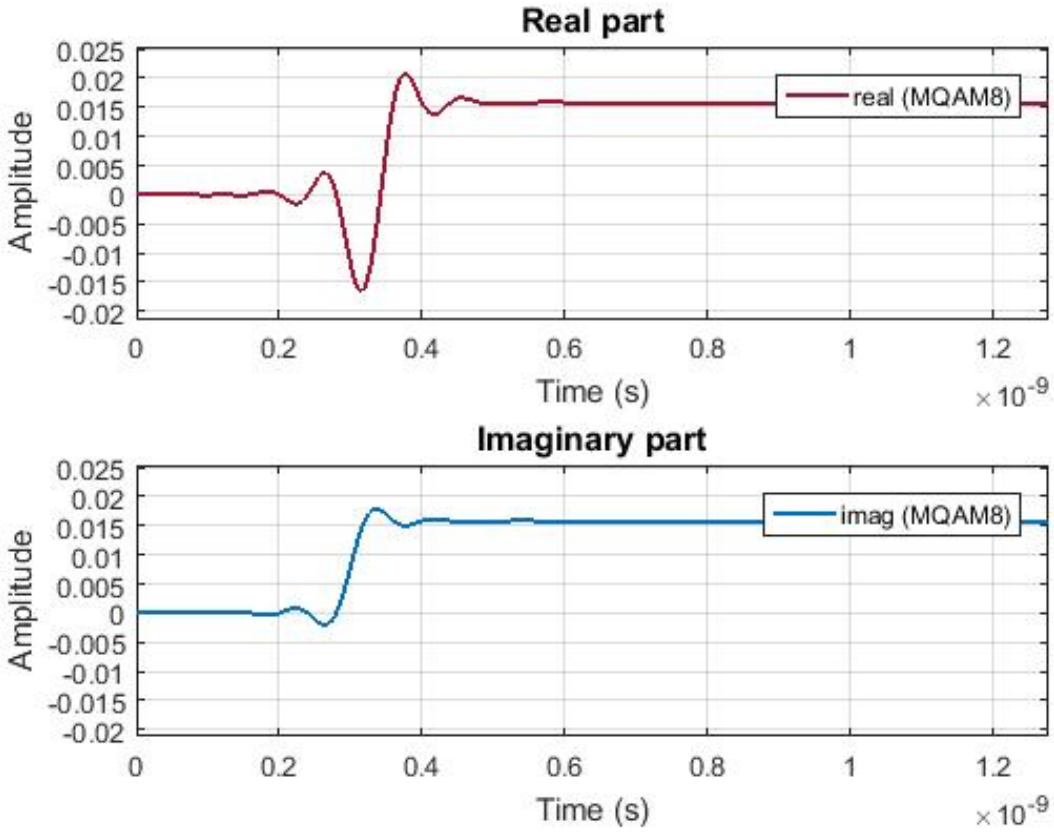

Figura 6.9: Example of a signal generated by this block for the initial binary signal 0100...

6.11 Local Oscillator

This block simulates a local oscillator with constant power and initial phase. It produces one output complex signal and it doesn't accept input signals.

Input Parameters

Parameter: opticalPower{ 1e-3 }

Parameter: wavelength{ 1550e-9 }

Parameter: frequency{ SPEED_OF_LIGHT / wavelength }

Parameter: phase{ 0 }

Parameter: samplingPeriod{ 0.0 }

Methods

LocalOscillator()

LocalOscillator(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig, OutputSig){};

void initialize(void);

bool runBlock(void);

void setSamplingPeriod(double sPeriod);

void setOpticalPower(double oPower);

void setOpticalPower_dBm(double oPower_dBm);

void setWavelength(double wlength);

void setPhase(double lOscillatorPhase);

Functional description

This block generates a complex signal with a specified phase given by the input parameter *phase*.

Input Signals

Number: 0

Output Signals

Number: 1

Type: Optical signal

Examples

Sugestions for future improvement

6.12 MQAM mapper

This block does the mapping of the binary signal using a *m*-QAM modulation. It accepts one input signal of the binary type and it produces two output signals which are a sequence of 1's and -1's.

Input Parameters

Parameter: m{4}
(m should be of the form 2^n with n integer)

Parameter: iqAmplitudes{{ 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 }}

Methods

MQamMapper(vector<Signal *> &InputSig, vector<Signal *> &OutputSig) :Block(InputSig, OutputSig) {};

void initialize(void);

bool runBlock(void);

void setM(int mValue);

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues);

Functional Description

In the case of m=4 this block atributes to each pair of bits a point in the I-Q space. The constellation used is defined by the *iqAmplitudes* vector. The constellation used in this case is ilustrated in figure 6.10.

Input Signals

Number : 1

Type : Binary (DiscreteTimeDiscreteAmplitude)

Output Signals

Number : 2

Type : Sequence of 1's and -1's (DiscreteTimeDiscreteAmplitude)

Example

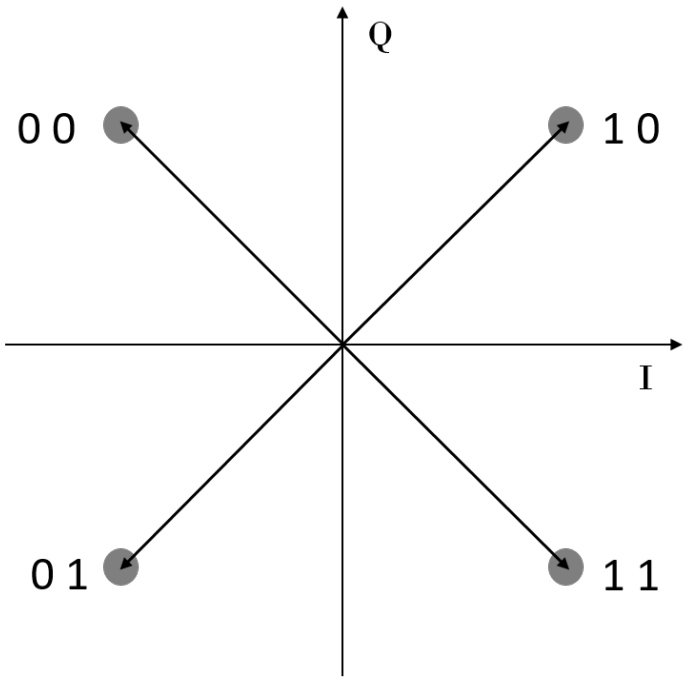

Figura 6.10: Constellation used to map the signal for m=4

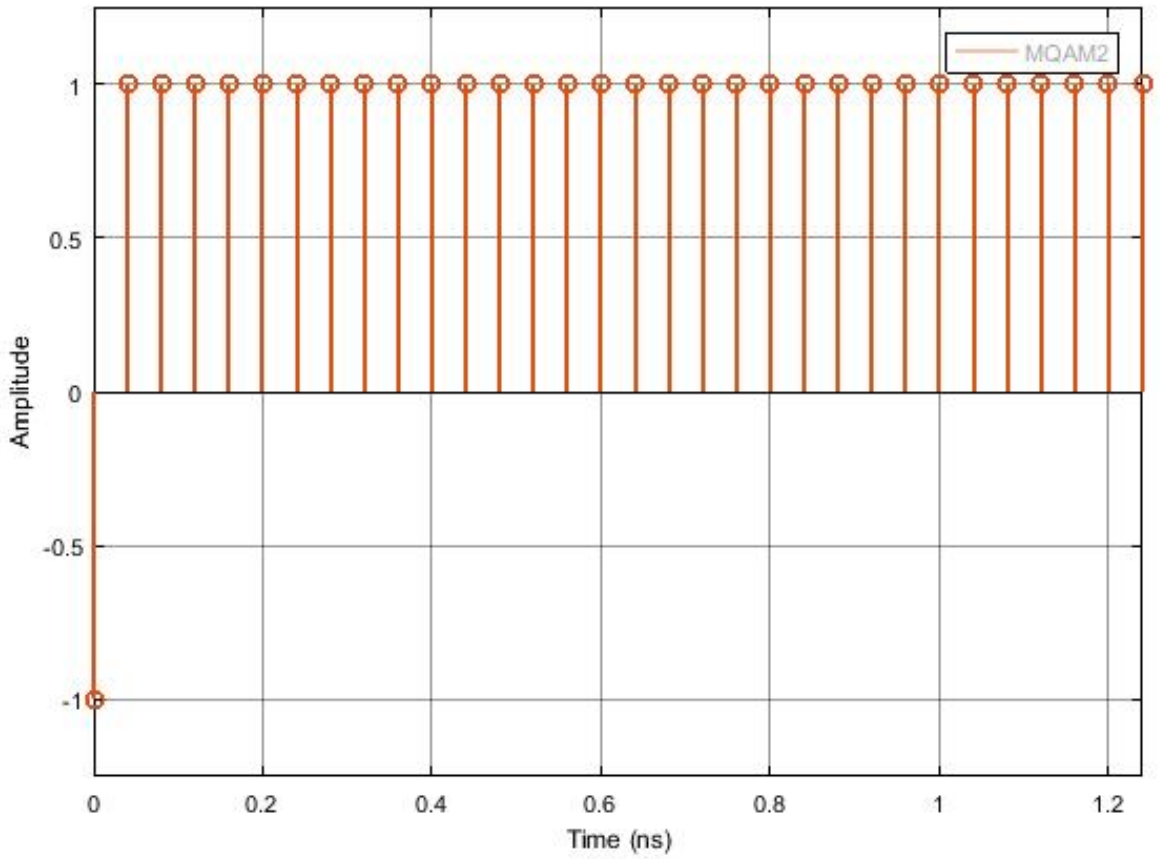

Figura 6.11: Example of the type of signal generated by this block for the initial binary signal 0100...

6.13 MQAM transmitter

This block generates a MQAM optical signal. It can also output the binary sequence. A schematic representation of this block is shown in figure 6.12.

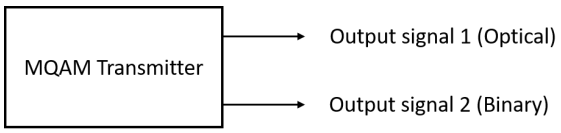

Figura 6.12: Basic configuration of the MQAM transmitter

Functional description

This block generates an optical signal (output signal 1 in figure 6.13). The binary signal generated in the internal block Binary Source (block B1 in figure 6.13) can be used to perform a Bit Error Rate (BER) measurement and in that sense it works as an extra output signal (output signal 2 in figure 6.13).

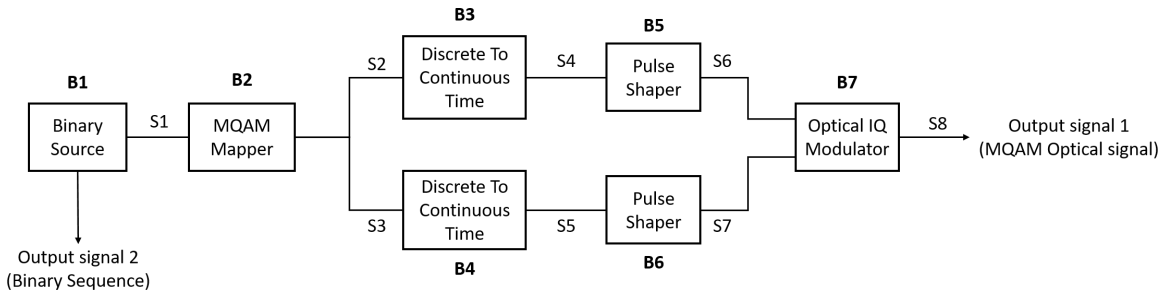

Figura 6.13: Schematic representation of the block MQAM transmitter.

Input parameters

This block has a special set of functions that allow the user to change the basic configuration of the transmitter. The list of input parameters, functions used to change them and the values that each one can take are summarized in table 6.2.

Input parameters	**Function**	Type	**Accepted values**
Mode	setMode()	string	PseudoRandom Random DeterministicAppendZeros DeterministicCyclic
Number of bits generated	setNumberOfBits()	int	Any integer
Pattern length	setPatternLength()	int	Real number greater than zero
Number of bits	setNumberOfBits()	long	Integer number greater than zero
Number of samples per symbol	setNumberOfSamplesPerSymbol()	int	Integer number of the type 2^n with n also integer
Roll of factor	setRollOfFactor()	double	$\in$ [0,1]
IQ amplitudes	setIqAmplitudes()	Vector of coordinate points in the I-Q plane	**Example** for a 4-qam mapping: { { 1.0, 1.0 }, { -1.0, 1.0 }, { -1.0, -1.0 }, { 1.0, -1.0 } }
Output optical power	setOutputOpticalPower()	int	Real number greater than zero
Save internal signals	setSaveInternalSignals()	bool	True or False

Tabela 6.2: List of input parameters of the block MQAM transmitter

Methods

MQamTransmitter(vector<Signal *> &inputSignal, vector<Signal *> &outputSignal); (**constructor**)

void set(int opt);

void setMode(BinarySourceMode m)

BinarySourceMode const getMode(void)

void setProbabilityOfZero(double pZero)

double const getProbabilityOfZero(void)

void setBitStream(string bStream)

string const getBitStream(void)

void setNumberOfBits(long int nOfBits)

long int const getNumberOfBits(void)

void setPatternLength(int pLength)

int const getPatternLength(void)

void setBitPeriod(double bPeriod)

double const getBitPeriod(void)

void setM(int mValue) int const getM(void)

void setIqAmplitudes(vector<t_iqValues> iqAmplitudesValues)

vector<t_iqValues> const getIqAmplitudes(void)

void setNumberOfSamplesPerSymbol(int n)

int const getNumberOfSamplesPerSymbol(void)

void setRollOffFactor(double rOffFactor)

double const getRollOffFactor(void)

void setSeeBeginningOfImpulseResponse(bool sBeginningOfImpulseResponse)

double const getSeeBeginningOfImpulseResponse(void)

void setOutputOpticalPower(t_real outOpticalPower)

t_real const getOutputOpticalPower(void)

void setOutputOpticalPower_dBm(t_real outOpticalPower_dBm)

t_real const getOutputOpticalPower_dBm(void)

Output Signals

Number: 1 optical and 1 binary (optional)

Type: Optical signal

Example

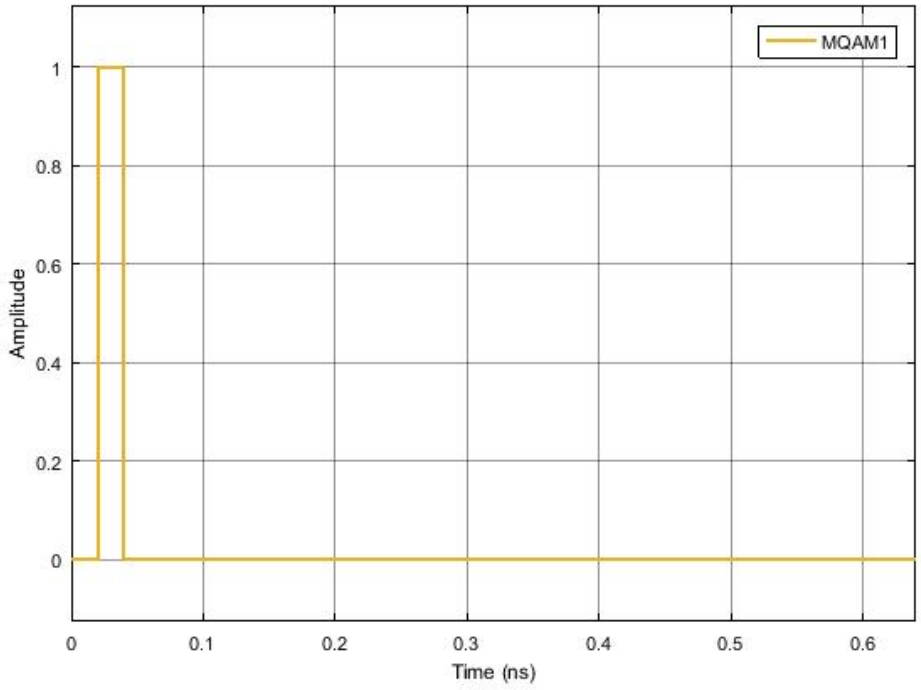

Figura 6.14: Example of the binary sequence generated by this block for a sequence 0100...

Sugestions for future improvement

Add to the system another block similar to this one in order to generate two optical signals with perpendicular polarizations. This would allow to combine the two optical signals and generate an optical signal with any type of polarization.

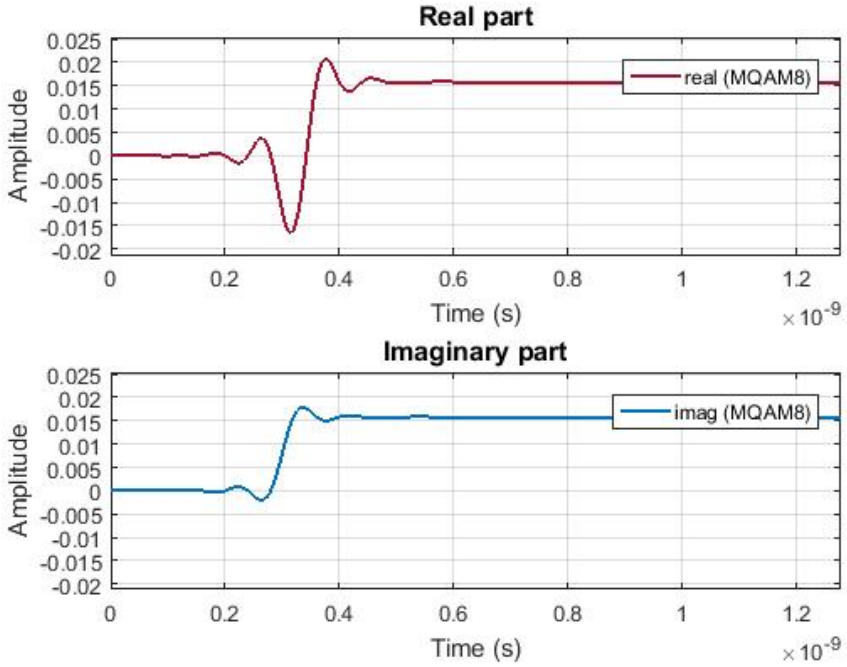

Figura 6.15: Example of the output optical signal generated by this block for a sequence 0100...

6.14 Fork

Header File	:	fork.h
Source File	:	fork.cpp
Version	:	201711119 (Responsible: Romil Patel)

Input Parameters

—- NA —-

Input Signals

Number: 1

Type: t_real (TimeContinuousAmplitudeContinuousReal)

Output Signals

Number: 2

Type: t_real (TimeContinuousAmplitudeContinuousReal)

Functional Description

This block accepts a time domain signal and outputs two replicas of the input signal.

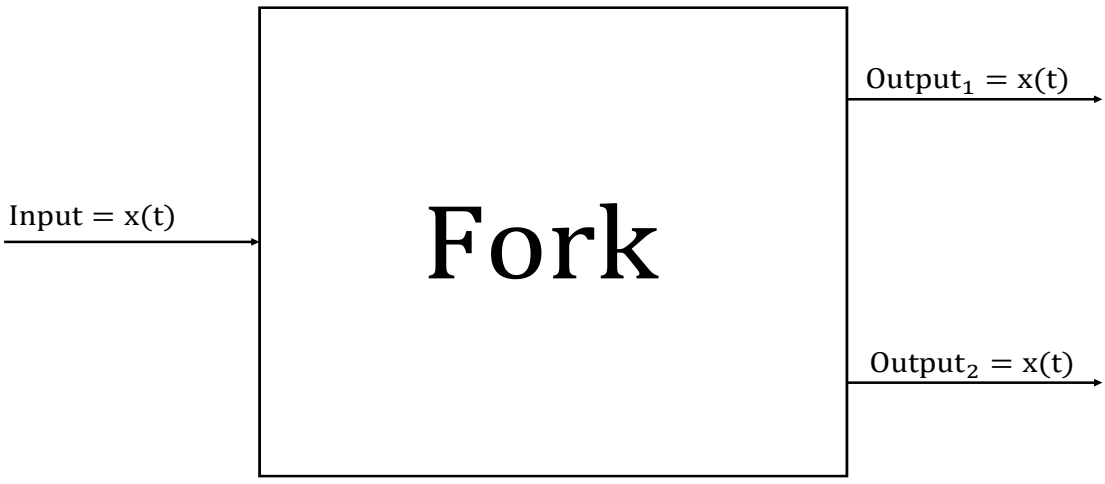

Figura 6.16: Fork

Capítulo 7

Mathlab Tools

7.1 Generation of AWG Compatible Signals

Student Name	:	Francisco Marques dos Santos
Starting Date	:	September 1, 2017
Goal	:	Convert simulation signals into waveform files compatible with the laboratory's Arbitrary Waveform Generator
Directory	:	mtools

This section shows how to convert a simulation signal into an AWG compatible waveform file through the use of a matlab function called sgnToWfm. This allows the application of simulated signals into real world systems.

7.1.1 sgnToWfm

[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm(fname_sgn, nReadr, fname_wfm);

Inputs

fname_sgn: Input filename of the signal (*.sgn) you want to convert. It must be a real signal (Type: TimeContinuousAmplitudeContinuousReal).

nReadr: Number of symbols you want to extract from the signal.

fname_wfm: Name that will be given to the waveform file.

Outputs

A waveform file will be created in the Matlab current folder. It will also return six variables in the workspace which are:

data: A vector with the signal data.

symbolPeriod: Equal to the symbol period of the corresponding signal.

samplingPeriod: Sampling period of the signal.

type: A string with the name of the signal type.

numberOfSymbols: Number of symbols retrieved from the signal.

samplingRate: Sampling rate of the signal.

Functional Description

This matlab function generates a *.wfm file given an input signal file (*.sgn). The waveform file is compatible with the laboratory's Arbitrary Waveform Generator (Tekatronix AWG70002A). In order to recreate it appropriately, the signal must be real, not exceed $8 * 10^9$ samples and have a sampling rate equal or bellow 16 GS/s.

This function can be called with one, two or three arguments:

Using one argument:

[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm('S6.sgn');

This creates a waveform file with the same name as the *.sgn file and uses all of the samples it contains.

Using two arguments:

[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm('S6.sgn',256);

This creates a waveform file with the same name as the signal file name and the number of samples used equals nReadr x samplesPerSymbol. The samplesPerSymbol constant is defined in the *.sgn file.

Using three arguments:

[data, symbolPeriod, samplingPeriod, type, numberOfSymbols, samplingRate] = sgnToWfm('S6.sgn',256,'myWaveform.wfm');

This creates a waveform file with the name "myWaveform"and the number of samples used equals nReadr x samplesPerSymbol. The samplesPerSymbol constant is defined in the *.sgn file.

7.1.2 Loading a signal to the Tekatronix AWG70002A

The AWG we will be using is the Tekatronix AWG70002A which has the following key specifications:

Sampling rate up to 16 GS/s: This is the most important characteristic because it determines the maximum sampling rate that your signal can have. It must not be over 16 GS/s or else the AWG will not be able to recreate it appropriately.

8 GSample waveform memory: This determines how many data points your signal can have.

After making sure this specifications are respected you can create your waveform using the function. When you load your waveform, the AWG will output it and repeat it constantly until you stop playing it.

1. Using the function sgnToWfm: Start up Matlab and change your current folder to mtools and add the signals folder that you want to convert to the Matlab search path. Use the function accordingly, putting as the input parameter the signal file name you want to convert.

2. AWG sampling rate: After calling the function there should be waveform file in the mtools folder, as well as a variable called samplingRate in the Matlab workspace. Make sure this is equal or bellow the maximum sampling frequency of the AWG (16 GS/s), or else the waveform can not be equal to the original signal. If it is higher you have to adjust the parameters in the simulation in order to decrease the sampling frequency of the signal(i.e. decreasing the bit period or reducing the samples per symbol).

3. Loading the waveform file to the AWG: Copy the waveform file to your pen drive and connect it to the AWG. With the software of the awg open,go to browse for waveform on the channel you want to use, and select the waveform file you created (Figure 7.1).

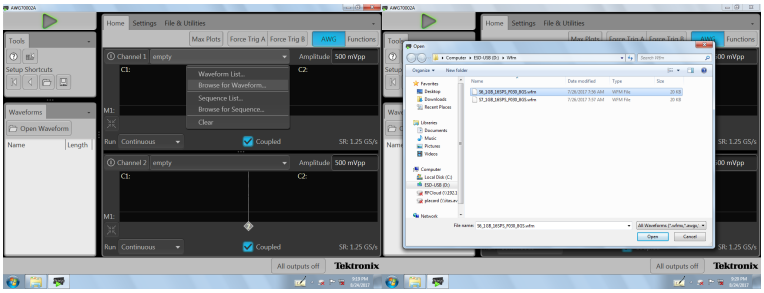

Figura 7.1: Selecting your waveform in the AWG

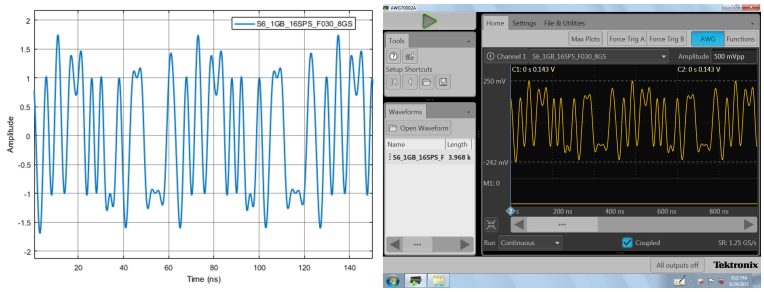

Figura 7.2: Comparison between the waveform in the AWG and the original signal before configuring the sampling rate

Now you should have the waveform displayed on the screen. Although it has the same shape, the waveform might not match the signal timing wise due to an incorrect sampling rate configured in the AWG. In this example (Figure 7.2), the original signal has a sample

rate of 8 GS/s and the AWG is configured to 1.25 GS/s. Therefore it must be changed to the correct value. To do this go to the settings tab, clock settings, and change the sampling rate to be equal to the one of the original signal, 8 GS/s (Figure 7.3). Compare the waveform in

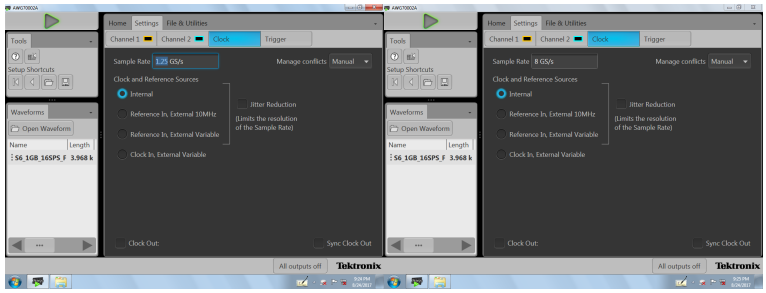

Figura 7.3: Configuring the right sampling rate

the AWG with the original signal, they should be identical (Figure 7.4).

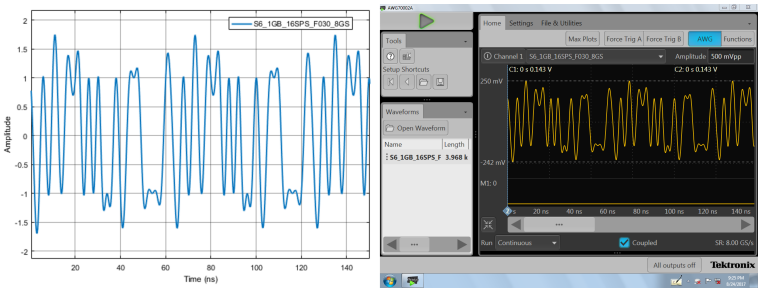

Figura 7.4: Comparison between the waveform in the AWG and the original signal after configuring the sampling rate

4. Generate the signal: Output the wave by enabling the channel you want and clicking on the play button.

Capítulo 8

Algorithms

8.1 Overlap-Save Method

Linear filtering can be easily implemented in time-domain resorting to the use of finite impulse response (FIR) digital filters and convolution property as,

$$y(n) = \sum_{k=0}^{R-1} x(n-k)h\left(k\right), \tag{8.1}$$

where $x(n)$ is the input signal, $h(k)$ is the FIR filter coefficients, R is the length of FIR filter coefficients and $y(n)$ represents the filtered output signal. Analysing this equation we can note that, for a block signal of length R, the required number of operations for the direct implementation of equation (8.2) evolves with R^2, $\mathcal{O}(R^2)$. This limitation imposed the emergence of algorithm, where the linear convolution is calculated faster than the directly implementing of (8.2). In this sense, it is used the computation of linear filtering in frequency domain resorting to the use of fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) algorithms as However, for long input sequence, the direct implementation of frequency domain filtering in real-time is limited by the limited memory of the digital processors. Hence, the filtering in frequency-domain is implemented by sectioning or block the input signal, such that the practical implementation of FFT and IFFT is feasible. In order to implement the non-cyclic convolution with the finite-length of cyclic convolution that the FFT provides, overlap-save and overlap-add method are use, enabling that the complexity evolves in log scale $\mathcal{O}(N \log_2 N)$. The general method is to split the input signal into manageable blocks, then apply the FFT to to perform the linear convolution and at the end recombine the output blocks such that it is avoided the wrap-around errors due to the circular convolution imposed by FFT.

The overlap-save method can be computed in the following steps:

1. Step 1:

Parameter: Determine the length R of impulse response, $h(n)$;

2. Step 2:

Parameter: Define the size of FFT and IFFT operation, N;

3. Step 3:

Parameter: Determine the length of block L to section the input sequence $x(n)$, considering that $N = L + R - 1$;

4. Step 4:

Parameter: Pad $L - 1$ zeros to the impulse response $h(n)$ to obtain the length N;

5. Step 5:

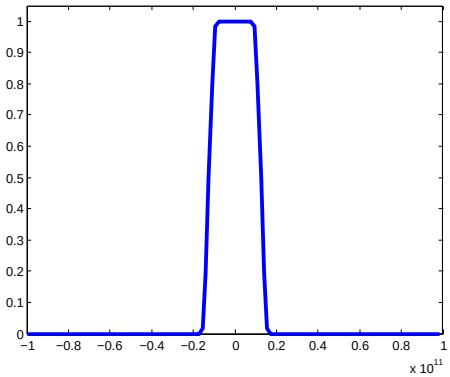

Figura 8.1: Frequency response of raised-cosine filter.

Parameter: Make the segments of the input sequences of length L, $x_i(n)$, where index i correspond to the i^{th} block. Overlap $R-1$ samples of the previous block at the beginning of the segmented block to obtain a block of length N. In the first block, it is added $R-1$ null samples;

6. Step 6:

Parameter: Compute the circular convolution of segmented input sequence $x_i(n)$ and $h(n)$ described as,

$$y_i(n) = x_i(n) \circledast h(n). \tag{8.2}$$

This is obtained in the following steps:

Description: Compute the FFT of x_i and h both with length N;

Description: Compute the multiplication of $X_i(f)$ and the transfer function $H(f)$;

Description: Compute the IFFT of the multiplication result to obtain the time-domain block signal, y_i;

7. Step 7:

Parameter: Discarded $R-1$ initial samples from the y_i, and save only the error-free $N-R-1$ samples in the output record.

In the Figure 8.4 it is illustrated an example of overlap-save method.

8.1.1 Frequency Response of Filter

The frequency response of filter can be directly defined by using the frequency-domain formula, or it can be equivalently calculated from the FFT of impulse response of the filter. In this sense, we present an example of FIR filter (*raised-cosine filter*) to illustrate these two cases.

Frequency-domain Formula

The frequency-domain description of raised-cosine filter can be given as,

$$H(f) = \begin{cases} 1, & |f| \leq \frac{1-\beta}{2T} \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}\left[|f| - \frac{1-\beta}{2T}\right]\right)\right], & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$

where, f is the frequency, $0 \leq \beta \leq 1$ corresponds to the roll-off factor and T is the reciprocal of the symbol-rate. A plot of direct implementation of frequency response of raised-cosine filter is presented in Figure 8.1, for a roll-off factor of 0.3. The frequency response, $H(f)$, calculated for N frequency bins, which can be defined as,

$$f = -\frac{fwindowTF}{2} : \frac{fwindowTF}{N} : \left(\frac{fwindowTF}{2} - \frac{fwindowTF}{N}\right), \tag{8.3}$$

where $fwindowTF$ is the sampling frequency. This, imposes that the length of $H(f)$ is N as expected for the N-point FFT and the transfer function multiplication.

FFT of Impulse Response

Alternatively to the frequency-domain formula, we can obtain the frequency response of filter by calculating the FFT of its impulse response. In the case of raised-cosine filter, the impulse response is given as,

$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T}\frac{\cos(\pi t\beta/T)}{1-(2\beta t/T)^2}, \tag{8.4}$$

where t is the time. Figure 8.2 shows a plot of impulse response of raised-cosine filter for a roll-off factor of 0.3. Before calculating the FFT of the impulse response we must zero-pad the

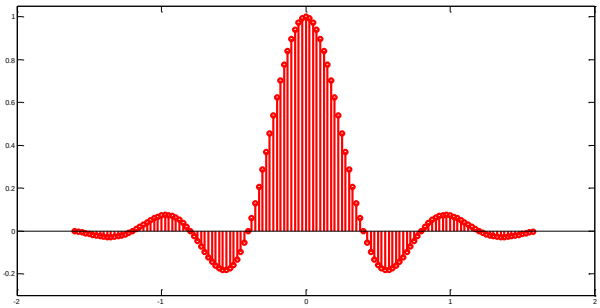

Figura 8.2: Impulsee response of raised-cosine filter.

impulse response, which has the length R, to the length N. In this case N is the FFT length, which is efficiently defined as power of 2. The zero-padding process can be performed by appending $L-1$ zeros at the end of impulse response, as shown in the Figure 8.3.

In both cases, the frequency response of the filter will be limited to the frequency interval $fwindowTF$, $[-\frac{fwindowTF}{2}, \frac{fwindowTF}{2}]$, and this range show us the N frequency components

Figura 8.3: Zero-padding of impulse response $h(n)$.

obtained from FFT. The minimum frequency bin is $-\frac{fwindowTF}{2}$ and the maximum bin is $\frac{fwindowTF}{2}$, in which $fwindowTF$ corresponds to the sampling frequency. We can note that the spectral width of $H(f)$ is $fwindowTF$, which is the inverse of sampling period, dt. It is also important to note that, for a given sampling frequency, the frequency resolution, Δf, of $H(f)$ depends on the parameter N and it increases with N, $\Delta f = \frac{fwindowTF}{N}$.

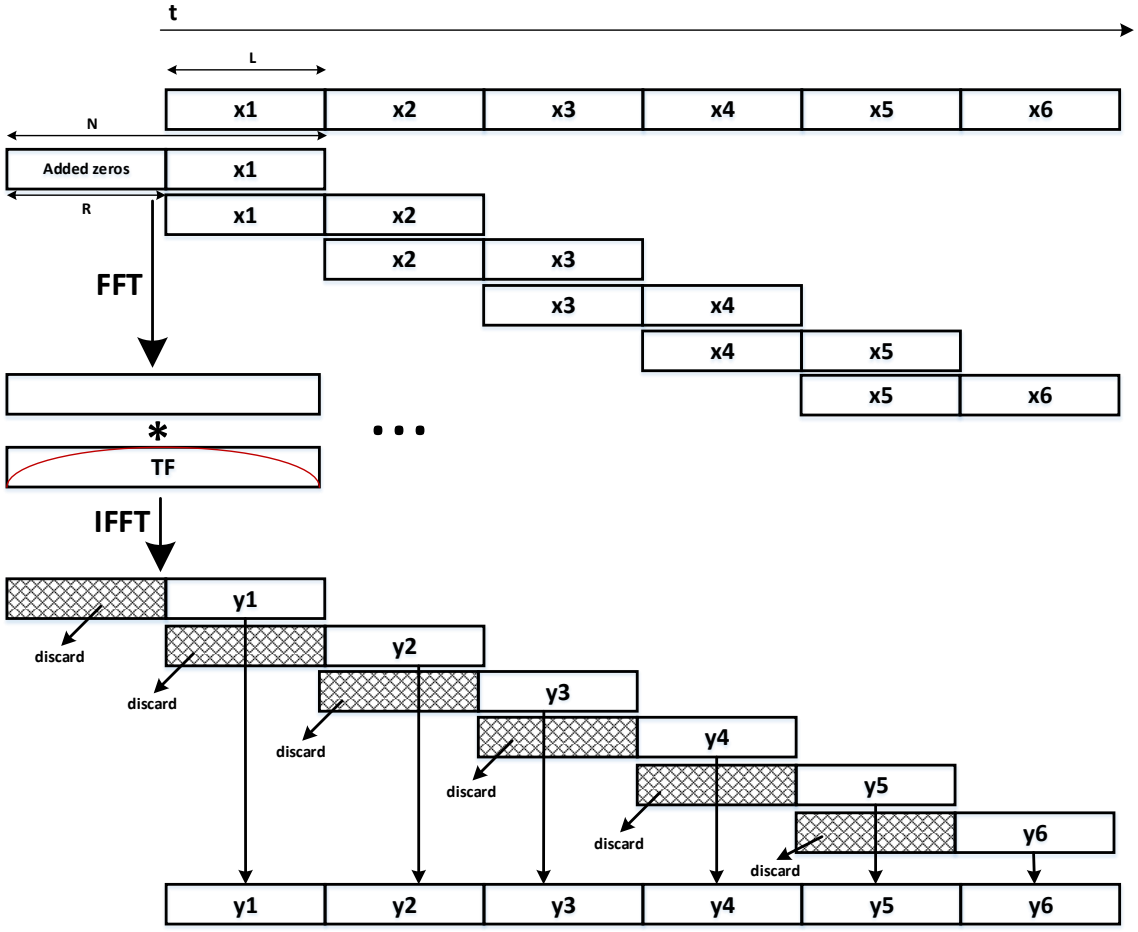

Figura 8.4: Illustration of Overlap-save method.

Bibliografia

[1] Blahut, R.E. *Fast Algorithms for Digital Signal Processing*, Addison-Wesley, Reading, MA, 1985.

[2] Steven W. Smith. *The Scientist and Engineer's Guide to Digital Signal Processing*. California Technical Publishing, San Diego, CA, USA, 1997.

8.2 FFT

This is the generalized FFT algorithm which facilitates time domain to frequency domain conversion for both real and complex signal. In case of real signal, we have to manipulate imaginary part to be zero for the execution of algorithm. The figure 8.8 displays top level architecture of the FFT algorithm. If the length of the input signal is 2^N, then it'll execute Radix-2 algorithm otherwise it'll execute Bluestein algorithm.

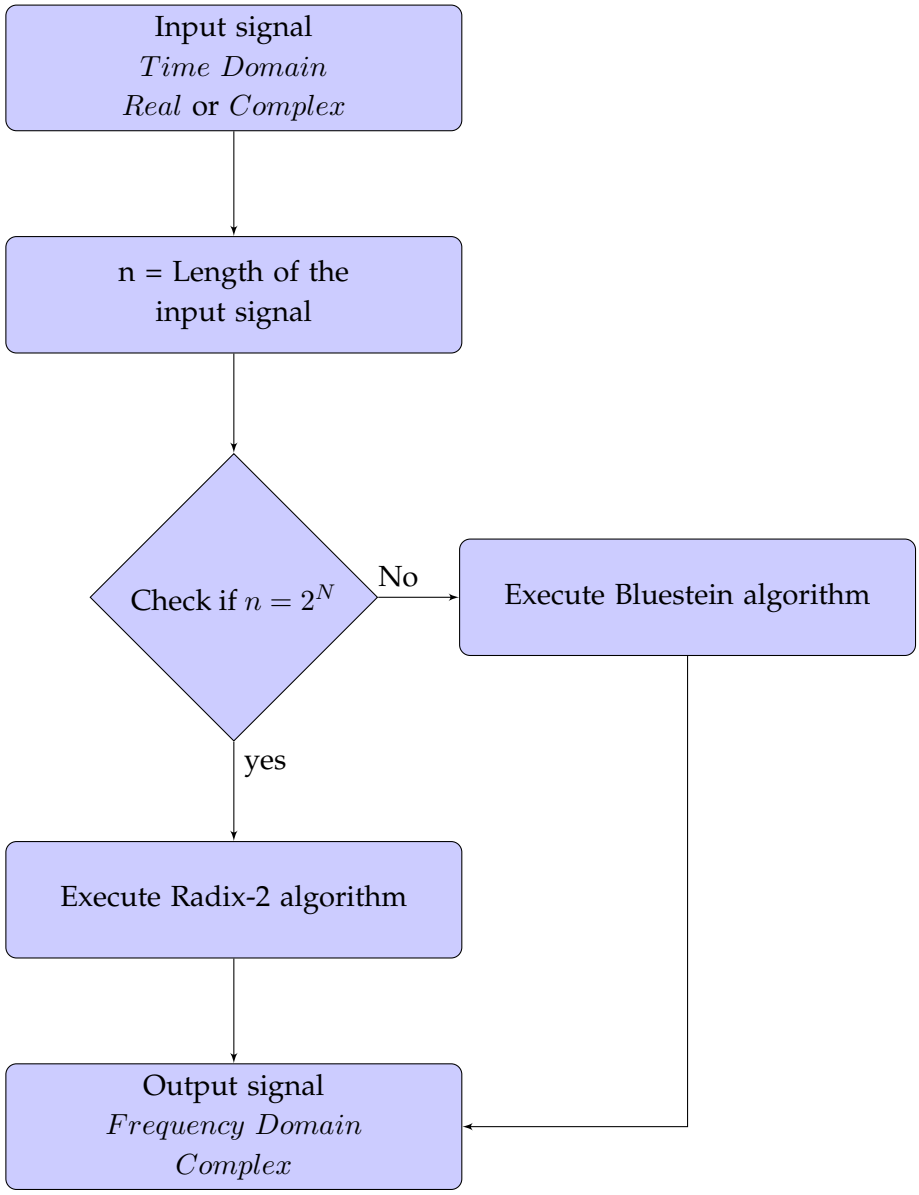

Figura 8.5: Top level architecture of the FFT algorithm

Radix-2 algorithm

The architecture of the algorithm is to accept time domain complex signal and generate frequency domain complex output signal. In the case of real input signal, first we have to convert it into the complex form by adding zero to the imaginary part.

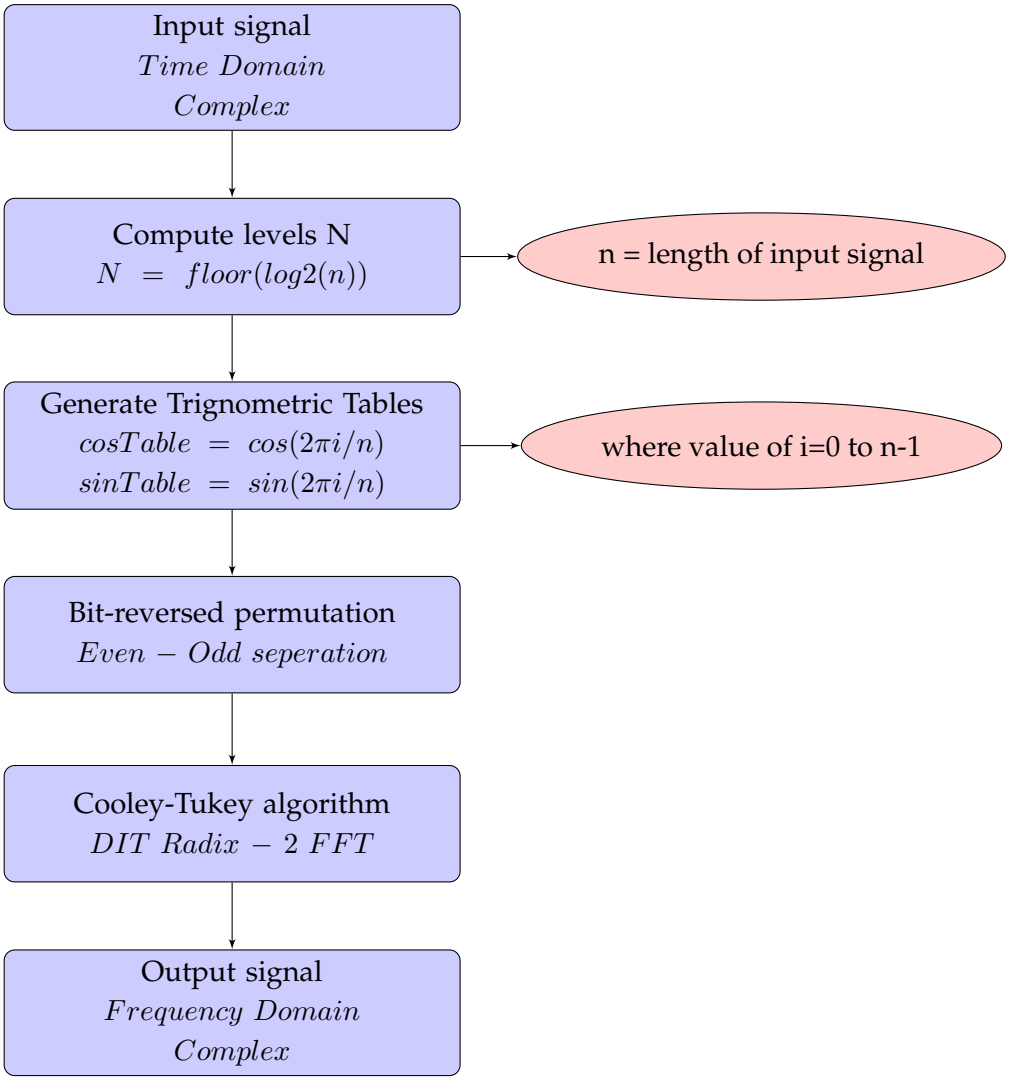

Figura 8.6: Flowchart of Cooly-Tukey DIT Radix-2 algorithm

Bluestein algorithm

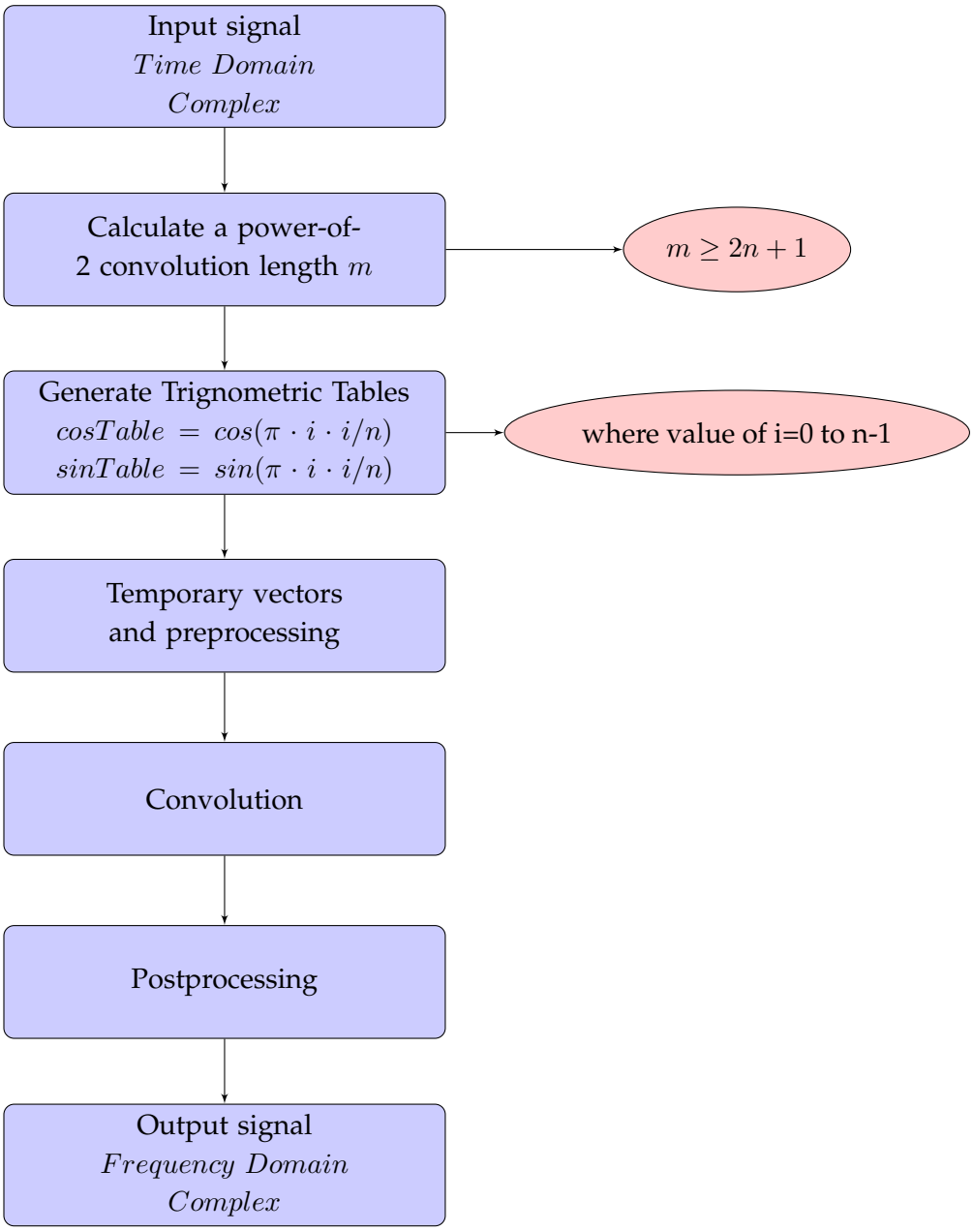

Figura 8.7: Flowchart of Bluestein algorithm

8.3 IFFT

This is the generalized IFFT algorithm which facilitates frequency domain to time domain conversion. First, the frequency domain input signal separated into real and imaginary part. The implementation of the IFFT can be carried out by swaping the position of the real and imaginary part of the FFT functions. In other words, the function $FFT(imaginary, real)$ calculates the IFFT of the frequency domain signal applied at the input. Following that, the output of the IFFT function normalized to get exact time domain signal.

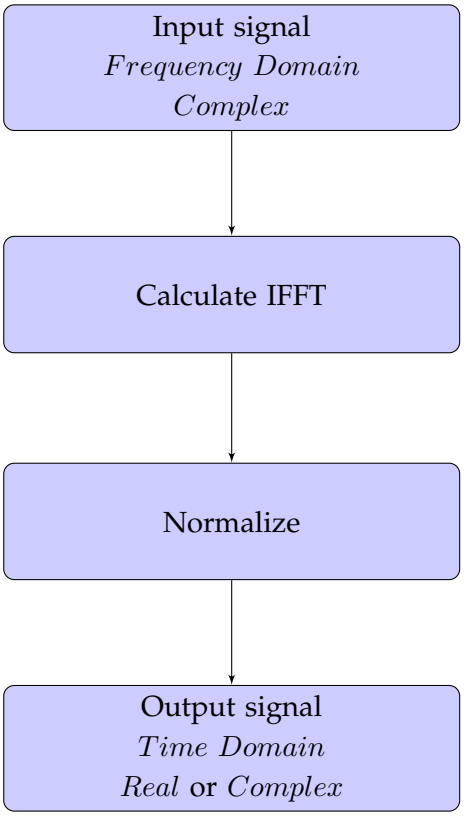

Figura 8.8: Top level architecture of the IFFT algorithm