The exponential function

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June 24, 2022

1 Introduction

The real exponential function is define as function, which takes values from $\mathbb{R} \to \mathbb{R}$. It can be defined by the taylor serie:

$$\exp(x) = \sum_{i=0}^{\infty} \frac{x^i}{i!} \tag{$\star$}$$

Looking at equation ($\star$) we can conclude that the nummber in the base $\exp(x) = e^x$ is given by

$$e = \exp(1) = \sum_{i=0}^{\infty} \frac{1^i}{i!} = \sum_{i=0}^{\infty} \frac{1}{i!}. \tag{1}$$

The exponential function is also define for a complex number, where is appears in Euler's formula

$$\exp(i \cdot x) = \cos(x) + i \cdot \sin(x) \tag{2}$$

1.1 Other definition of the expexponential function

Other way to define the exponential function is as a solution to the differential equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y \tag{3}$$

with the initial condition of $y(0) = 1$. Another way tp define the exponential function is as a limit when n goes to infinity of

$$\exp(x) = \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n \tag{4}$$

2 Numerical approach to exponential function

Our "quick-and-dirty" implementation of the exponential function, is taking the have calculated the taylor serie (equation ($\star$)) to $n = 10$ and uses it for our calculation. If we receive a negative number, x, we will call our function recursive with $\frac{1}{\exp(x)}$ since $a^{-b} = \frac{1}{a^b}$. To make sure our function is precise, we will call the function recursively with $\left(\exp\left(\frac{x}{2}\right)\right)^2$ if the function is called with x value greater than $\frac{1}{8}$. At figure 2 we can see the result of our numerical approach to exponential function alongside with the actual values, which can be found in table 1

3 Evaluation of our numerical approach

As we see on figure 2 our approch is close to the value, and if we use our to calculate

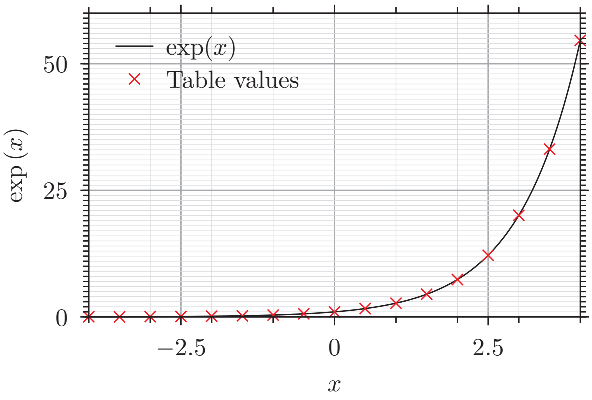

Figure 1: A plot of the "quick-and-dirty" implementation of the exponential function in the interval $[-4, 4]$. Alongs with the plots of points of the actual exponential function

Table 1: Values of x and $\exp(x)$ calculated with System.Math.Exp in C#.

x	$\exp(x)$
-4	0.0183156388887342
-3.5	0.0301973834223185
-3	0.0497870683678639
-2.5	0.0820849986238988
-2	0.135335283236613
-1.5	0.22313016014843
-1	0.367879441171442
-0.5	0.606530659712633
0	1
0.5	1.64872127070013
1	2.71828182845905
1.5	4.48168907033806
2	7.38905609893065
2.5	12.1824939607035
3	20.0855369231877
3.5	33.1154519586923
4	54.5981500331442

our approch to eulers number as

$$\begin{aligned} e_{\text{approach}} &= \sum_{i=0}^{10} \frac{1}{i!} \\ &\approx 2.71828180 \end{aligned} \tag{5}$$

Comparing equation (5) to our tabel value of $\exp(1)$ from table 1 is on the 8^{th} decimal our approach begins to vary from eulers number in equation (1).