```
if X <= Y
  then while X <= Y do
         X := X + Y
  else X := X - Y
```

amb l'estat inicial $\{X \mapsto 2,\ Y \mapsto 4\}$

$$\frac{(b,s) \Rightarrow true}{\langle \text{if } b \text{ then } i \text{ else } i_2, s\rangle \to \langle i, s\rangle}$$

```
X:=6;
if X>3 then X:= X-1 else Y:=X
```

(a) Desarrolla la traza de ejecución (mostrando los elementos intermedios semántica small-step con el estado inicial $\{X \mapsto 2\}$.

(1) $\langle X := 6;$ if $X > 3$ then $X := X - 1$ else $Y := X, \{X \mapsto 2\}\rangle \to$
$\langle X := 6, \{X \mapsto 2\}\rangle \to$
$\langle 5, \{X \mapsto 2\}\rangle \Rightarrow 5$
$\langle$skip$, \{X \mapsto 5\}\rangle$
(2) $\langle$skip; if $X > 3$ then $X := X - 1$ else $Y := X, \{X \mapsto 5\}\rangle \to$
(3) $\langle$if $X > 3$ then $X := X - 1$ else $Y := X, \{X \mapsto 5\}\rangle \to$
$\langle X > 3, \{X \mapsto 5\}\rangle \Rightarrow$
$\langle X, \{X \mapsto 5\}\rangle \Rightarrow 5$
$\langle 3, \{X \mapsto 5\}\rangle \Rightarrow 3$
$5 > 3$ is true
$\langle true$
(4) $\langle X := X - 1, \{X \mapsto 5\}\rangle \to$
$\langle X - 1, \{X \mapsto 5\}\rangle \Rightarrow$
$\langle X, \{X \mapsto 5\}\rangle \Rightarrow 5$
$\langle 1, \{X \mapsto 5\}\rangle \Rightarrow 1$
$5 - 1 = 4$ is computed
$\langle 4$
(5) $\langle$skip$, \{X \mapsto 4\}\rangle$

$$\frac{\langle a_1,s\rangle \Rightarrow n_1 \qquad \langle a_2,s\rangle \Rightarrow n_2}{\langle a_1 \le a_2, s\rangle \Rightarrow true} \quad \text{si } n_1 \text{ es menor o igual que } n_2$$

$$\frac{\langle a_1,s\rangle \Rightarrow n_1 \qquad \langle a_2,s\rangle \Rightarrow n_2}{\langle a_1 \le a_2, s\rangle \Rightarrow false} \quad \text{si } n_1 \text{ es major que } n_2$$

- Bucle *while*:

$$\frac{\langle b,s\rangle \Rightarrow false}{\langle \texttt{while } b \texttt{ do } i, s\rangle \to \langle \text{skip}, s\rangle} \qquad \frac{\langle b,s\rangle \Rightarrow true}{\langle \texttt{while } b \texttt{ do } i, s\rangle \to \langle \texttt{while } b \texttt{ do } i, s\rangle}$$

$$\frac{\langle a,s\rangle \Rightarrow n}{\langle x := a, s\rangle \to \langle skip, s[x \mapsto n]\rangle}$$

$\langle \text{if } \overbrace{X <= Y}^{b} \text{ then } \overbrace{\text{while } X <= Y \text{ do } \{X := X+Y\}}^{i} \text{ else } \overbrace{Y := X - Y}^{i_2}, \{X \mapsto 2, Y \mapsto 4\}\rangle \to$ (1)

$\langle X <= Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 2 \le 4 \Rightarrow$ true
$\langle X, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 2$
$\langle Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 4$ $\quad 2 \le 4$

(1) $\to \langle$while $X \le Y$ do $X := X+Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \to$

$\langle X <= Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 2 \le 4 \Rightarrow$ true
$\langle X, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 2$
$\langle Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 4$ $\quad 2 \le 4$

$\to \langle X := X+Y;$ while $X \le Y$ do $X := X+Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \to$ (3)

Seqüència

$$\frac{\langle i_1,s\rangle \to \langle i_1',s'\rangle}{\langle i_1;i_2,s\rangle \to \langle i_1';i_2,s'\rangle}$$

$\langle X := X + Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \to$ (2)

Assignació

$$\frac{\langle a,s\rangle \Rightarrow n}{\langle x := a, s\rangle \to \langle skip, s[x \mapsto n]\rangle}$$

$\langle X, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 2$
$\langle Y, \{X \mapsto 2, Y \mapsto 4\}\rangle \Rightarrow 4$ $\quad 2 + 4 \rightsquigarrow 6$

(2) $\to \langle$skip$, \{X \mapsto 6, Y \mapsto 4\}\rangle$

(3) $\to \langle$skip; while $X \le Y$ do $X := X + Y, \{X \mapsto 6, Y \mapsto 4\}\rangle \to$

$$\overline{\langle skip; i, s\rangle \to \langle i, s\rangle}$$

$\to \langle$while $X \le Y$ do $X := X+Y, \{X \mapsto 6, Y \mapsto 4\}\rangle \to$

$\langle X, \{X \mapsto 6, Y \mapsto 4\}\rangle \Rightarrow 6$
$\langle Y, \{X \mapsto 6, Y \mapsto 4\}\rangle \Rightarrow 4$

$6 \le 4 \rightsquigarrow$ false

Bucle *while*:

$$\frac{\langle b,s\rangle \Rightarrow false}{\langle \texttt{while } b \texttt{ do } i, s\rangle \to \langle \text{skip}, s\rangle}$$

$\to \langle$skip$, \{X \mapsto 6, Y \mapsto 4\}\rangle$ Resultat

Configuració final

↑ no es un punt i coma. Com no podem aplicar cap regla es que hem arribat al final.

$\langle i, s, c\rangle \to$

$\langle X := 3; Y := X + 2, \{\}, 0\rangle \to$

⋮

$\langle Y := X + 2, \{X \mapsto 3\}, 1\rangle \to$

⋮

$\langle$skip$, \{X \mapsto 3, X \mapsto 5\}, 2\rangle$