

Bifactor Models

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The purpose of this exercise was to test the factor structure of a measure of habit development created for a study on behavior changes during the COVID-19 pandemic. The measure had 9 items and the response options were on a scale of 1-5.

Response scale: *1, not at all true of me, to 5, very true of me*

Since the outbreak,

- newhabit01 = I have established a work or schoolwork routine
- newhabit02 = I have created a new schedule and have kept it somewhat consistently
- newhabit03 = I have restarted doing my usual tasks without realizing I'm doing them
- newhabit04 = I have established new regular routines (e.g., morning, bedtime routines)
- newhabit05 = I have found ways to accomplish the same activities I used to get done before the outbreak
- newhabit06 = I have found ways to include new tasks or obligations in my schedule
- newhabit07 = *I have created or assigned* a specific *space* to complete my work and school activities
- newhabit08 = *I have created or assigned* a specific *place* to exercise or engage in my usual physical activities
- newhabit09 = *I have assigned* new *times* to complete my activities (e.g., work, study)

One-Factor CFA

We first fit a one-factor CFA to examine whether this was a one-factor model with no correlated residual covariances. From taking a peek at the scale items, you can imagine that it is unlikely that residuals are uncorrelated, as many items share the same stem (e.g., I have created...)

We first specified the model so that all indicators were freely estimated. This means we had to fix the variance of the factor to 1 for identification;

```
modelcfa <- 'f1 =~ NA*newhabit01 + newhabit02 + newhabit03 + newhabit04 + newhabit05 +  
              newhabit06 + newhabit07 + newhabit08 + newhabit09  
  
              f1~~1*f1'
```

Then we fit the model with MLR as the estimator and FIML to account for missing data;

```
fitcfa <- cfa(modelcfa,  
              data = habit,  
              missing = "fiml",  
              estimator = "mlr")
```

- The lavaan warning is telling that two cases had no data on any of the variables. These two were excluded from the analysis.

And requested the model summary with standardized estimates, fit indices, and r-square.

```
summary(fitcfa,
        standardized = TRUE,
        fit.measures = TRUE,
        rsquare = TRUE)
```

```
## lavaan 0.6-20.2265 ended normally after 29 iterations
##
##      Estimator              ML
##      Optimization method    NLMINB
##      Number of model parameters      27
##
##                               Used      Total
##      Number of observations         430         432
##      Number of missing patterns         40
##
## Model Test User Model:
##                               Standard      Scaled
##      Test Statistic          175.549      123.819
##      Degrees of freedom           27           27
##      P-value (Chi-square)         0.000         0.000
##      Scaling correction factor              1.418
##      Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##      Test statistic          1554.087      1043.454
##      Degrees of freedom           36           36
##      P-value                    0.000         0.000
##      Scaling correction factor              1.489
##
## User Model versus Baseline Model:
##
##      Comparative Fit Index (CFI)          0.902         0.904
##      Tucker-Lewis Index (TLI)             0.870         0.872
##
##      Robust Comparative Fit Index (CFI)              0.902
##      Robust Tucker-Lewis Index (TLI)                 0.869
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)          -5273.233      -5273.233
##      Scaling correction factor              1.091
##      for the MLR correction
##      Loglikelihood unrestricted model (H1)    -5185.458      -5185.458
##      Scaling correction factor              1.254
##      for the MLR correction
##
##      Akaike (AIC)          10600.465      10600.465
##      Bayesian (BIC)         10710.187      10710.187
##      Sample-size adjusted Bayesian (SABIC)    10624.505      10624.505
##
## Root Mean Square Error of Approximation:
```

```

##
## RMSEA                                0.113      0.091
## 90 Percent confidence interval - lower 0.097      0.078
## 90 Percent confidence interval - upper 0.129      0.105
## P-value H_0: RMSEA <= 0.050          0.000      0.000
## P-value H_0: RMSEA >= 0.080          1.000      0.918
##
## Robust RMSEA                                0.117
## 90 Percent confidence interval - lower 0.097
## 90 Percent confidence interval - upper 0.139
## P-value H_0: Robust RMSEA <= 0.050    0.000
## P-value H_0: Robust RMSEA >= 0.080    0.998
##
## Standardized Root Mean Square Residual:
##
## SRMR                                0.052      0.052
##
## Parameter Estimates:
##
## Standard errors                        Sandwich
## Information bread                      Observed
## Observed information based on          Hessian
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## f1 =~
## newhabit01    0.876   0.061  14.378  0.000   0.876   0.712
## newhabit02    1.025   0.044  23.078  0.000   1.025   0.795
## newhabit03    0.749   0.060  12.473  0.000   0.749   0.617
## newhabit04    0.877   0.054  16.216  0.000   0.877   0.679
## newhabit05    0.822   0.052  15.773  0.000   0.822   0.696
## newhabit06    0.732   0.057  12.874  0.000   0.732   0.658
## newhabit07    0.832   0.063  13.301  0.000   0.832   0.652
## newhabit08    0.839   0.068  12.328  0.000   0.839   0.603
## newhabit09    0.873   0.056  15.491  0.000   0.873   0.677
##
## Intercepts:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .newhabit01    3.402   0.062  55.255  0.000   3.402   2.766
## .newhabit02    3.227   0.063  51.283  0.000   3.227   2.504
## .newhabit03    2.908   0.059  49.072  0.000   2.908   2.393
## .newhabit04    3.083   0.063  49.063  0.000   3.083   2.387
## .newhabit05    3.264   0.057  56.883  0.000   3.264   2.765
## .newhabit06    3.532   0.055  64.570  0.000   3.532   3.174
## .newhabit07    3.517   0.065  54.300  0.000   3.517   2.757
## .newhabit08    3.277   0.070  46.760  0.000   3.277   2.353
## .newhabit09    3.208   0.064  50.337  0.000   3.208   2.489
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## f1          1.000
## .newhabit01    0.747   0.087   8.552  0.000   0.747   0.493
## .newhabit02    0.611   0.066   9.313  0.000   0.611   0.368
## .newhabit03    0.915   0.091  10.063  0.000   0.915   0.620

```

```
##      .newhabit04      0.899    0.088    10.176    0.000    0.899    0.539
##      .newhabit05      0.718    0.066    10.943    0.000    0.718    0.515
##      .newhabit06      0.702    0.071     9.842    0.000    0.702    0.567
##      .newhabit07      0.935    0.090    10.389    0.000    0.935    0.575
##      .newhabit08      1.235    0.108    11.470    0.000    1.235    0.637
##      .newhabit09      0.899    0.086    10.488    0.000    0.899    0.541
##
## R-Square:
##           Estimate
##      newhabit01      0.507
##      newhabit02      0.632
##      newhabit03      0.380
##      newhabit04      0.461
##      newhabit05      0.485
##      newhabit06      0.433
##      newhabit07      0.425
##      newhabit08      0.363
##      newhabit09      0.459
```

We can also request the residual covariance matrix to see the residual covariances.

```
lavInspect(fitcfa, what = "resid") # Look at the residual covariances (estimated)
```

```
## $cov
##           nw hb01 nw hb02 nw hb03 nw hb04 nw hb05 nw hb06 nw hb07 nw hb08 nw hb09
## newhabit01  0.003
## newhabit02  0.141 -0.006
## newhabit03  0.090  0.041 -0.001
## newhabit04 -0.099  0.124  0.039  0.002
## newhabit05  0.053 -0.051  0.023 -0.055 -0.001
## newhabit06 -0.090 -0.069 -0.064  0.032  0.161  0.001
## newhabit07 -0.035 -0.112 -0.092 -0.074 -0.031  0.045 -0.003
## newhabit08 -0.080 -0.109 -0.003 -0.119 -0.032 -0.010  0.235  0.006
## newhabit09 -0.118 -0.067 -0.087  0.020 -0.056 -0.014  0.179  0.319 -0.001
##
## $mean
## newhabit01 newhabit02 newhabit03 newhabit04 newhabit05 newhabit06 newhabit07 newhabit08
##           0.001      0.002      0.001      -0.005      0.002      0.000      0.003      -0.003
## newhabit09
##           0.004
```

Fit isn't good and you can see from the residual covariance matrix that items that share a question stem (or the same wording) are more strongly covarying (e.g., newhabit8 and newhabit9).

```
modindices(fitcfa, sort = TRUE)
```

```
##           lhs op      rhs      mi      epc sepc.lv sepc.all sepc.nox
## 65 newhabit08 ~~ newhabit09 42.933  0.401  0.401    0.381    0.381
## 30 newhabit01 ~~ newhabit02 26.302  0.231  0.231    0.342    0.342
## 56 newhabit05 ~~ newhabit06 23.547  0.197  0.197    0.278    0.278
## 63 newhabit07 ~~ newhabit08 19.530  0.280  0.280    0.261    0.261
## 64 newhabit07 ~~ newhabit09 19.512  0.238  0.238    0.259    0.259
```

```

## 39 newhabit02 ~~ newhabit04 17.799 0.198 0.198 0.267 0.267
## 42 newhabit02 ~~ newhabit07 10.436 -0.157 -0.157 -0.208 -0.208
## 37 newhabit01 ~~ newhabit09 9.779 -0.155 -0.155 -0.189 -0.189
## 43 newhabit02 ~~ newhabit08 8.815 -0.163 -0.163 -0.187 -0.187
## 54 newhabit04 ~~ newhabit08 6.945 -0.160 -0.160 -0.152 -0.152
## 34 newhabit01 ~~ newhabit06 6.469 -0.111 -0.111 -0.153 -0.153
## 31 newhabit01 ~~ newhabit03 5.247 0.111 0.111 0.134 0.134
## 32 newhabit01 ~~ newhabit04 5.203 -0.112 -0.112 -0.137 -0.137
## 48 newhabit03 ~~ newhabit07 4.758 -0.116 -0.116 -0.125 -0.125
## 50 newhabit03 ~~ newhabit09 4.638 -0.111 -0.111 -0.122 -0.122
## 44 newhabit02 ~~ newhabit09 4.465 -0.100 -0.100 -0.136 -0.136
## 36 newhabit01 ~~ newhabit08 4.097 -0.118 -0.118 -0.122 -0.122
## 47 newhabit03 ~~ newhabit06 3.540 -0.084 -0.084 -0.104 -0.104
## 41 newhabit02 ~~ newhabit06 2.981 -0.071 -0.071 -0.109 -0.109
## 51 newhabit04 ~~ newhabit05 2.906 -0.079 -0.079 -0.098 -0.098
## 40 newhabit02 ~~ newhabit05 2.795 -0.071 -0.071 -0.107 -0.107
## 33 newhabit01 ~~ newhabit05 2.748 0.074 0.074 0.101 0.101
## 59 newhabit05 ~~ newhabit09 2.555 -0.075 -0.075 -0.094 -0.094
## 53 newhabit04 ~~ newhabit07 2.453 -0.084 -0.084 -0.092 -0.092
## 38 newhabit02 ~~ newhabit03 2.116 0.066 0.066 0.089 0.089
## 60 newhabit06 ~~ newhabit07 1.699 0.062 0.062 0.076 0.076
## 58 newhabit05 ~~ newhabit08 1.491 -0.067 -0.067 -0.071 -0.071
## 45 newhabit03 ~~ newhabit04 1.296 0.057 0.057 0.063 0.063
## 35 newhabit01 ~~ newhabit07 0.844 -0.046 -0.046 -0.056 -0.056
## 52 newhabit04 ~~ newhabit06 0.791 0.040 0.040 0.051 0.051
## 46 newhabit03 ~~ newhabit05 0.480 0.031 0.031 0.039 0.039
## 57 newhabit05 ~~ newhabit07 0.358 -0.029 -0.029 -0.035 -0.035
## 55 newhabit04 ~~ newhabit09 0.220 0.024 0.024 0.027 0.027
## 62 newhabit06 ~~ newhabit09 0.136 -0.017 -0.017 -0.021 -0.021
## 61 newhabit06 ~~ newhabit08 0.118 -0.018 -0.018 -0.020 -0.020
## 49 newhabit03 ~~ newhabit08 0.113 -0.020 -0.020 -0.019 -0.019

```

```
# The sort argument allows us to sort the indices from largest to smallest
```

If we look at the modification indices, these covariances are indeed associated with the highest MIs, which indicates that freeing these residual covariances would improve model fit.

Scale unidimensionality using omega

We can also request the omega coefficient. This function called `reliability()` is from the *semTools* package. It provides several measures of reliability, including alpha, and three types of omega. The first and second omega estimates are calculated so that the denominator equals the model-implied variance of the total scores. The third omega is calculated so that the denominator equals the observed, sample variance of scores. To the extent that the model is a good fitting one, the second and third estimates of omega should be very similar (because the model-implied and observed matrices should be similar).

Omega is interpreted as the proportion of total-score variance that is due to - or explained by - a single, or general, factor. In other words, it tells whether an overall score is worth interpreting despite any multidimensionality of the construct. As noted in Reise et al. (Chapter 18), omega is influenced by the number of items: more items, higher omega because scores better reflect the general construct.

Omega is high and equals .88.

```
reliability(fitcfa) # Omega
```

```
##           f1
## alpha  0.8828567
## omega  0.8835894
## omega2 0.8835894
## omega3 0.8826630
## avevar 0.4597160
```

Two-Factor EFA

To explore whether variables with similar stems cluster together, we can run a two-factor EFA.

We specify the CFA model as we did previously in the class:

```
# these are exploratory blocks (you can give them any name), you'll need to specify them
efa_model <- 'efa("block1")*F1 =~ newhabit01 + newhabit02 + newhabit03 + newhabit04 +
              newhabit05 + newhabit06 + newhabit07 + newhabit08 +
              newhabit09
              efa("block1")*F2 =~ newhabit01 + newhabit02 + newhabit03 + newhabit04 +
              newhabit05 + newhabit06 + newhabit07 + newhabit08 +
              newhabit09'
```

We fit it.

```
# Estimate the Model
efa_f1 <-
  sem(model = efa_model,
      data = habit,
      rotation = "oblimin",
      estimator = "MLR"
  )
```

And request the output.

```
summary(efa_f1,
  fit.measures = TRUE,
  standardized = TRUE,
  rsquare = TRUE)
```

```
## lavaan 0.6-20.2265 ended normally after 1 iteration
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters          28
##      Row rank of the constraints matrix          2
##
##      Rotation method              OBLIMIN OBLIQUE
##      Oblimin gamma                      0
##      Rotation algorithm (rstarts)      GPA (30)
##      Standardized metric              TRUE
```

##	Row weights	None	
##			
##		Used	Total
##	Number of observations	317	432
##			
##	Model Test User Model:		
##		Standard	Scaled
##	Test Statistic	53.735	41.744
##	Degrees of freedom	19	19
##	P-value (Chi-square)	0.000	0.002
##	Scaling correction factor		1.287
##	Yuan-Bentler correction (Mplus variant)		
##			
##	Model Test Baseline Model:		
##			
##	Test statistic	1391.843	951.686
##	Degrees of freedom	36	36
##	P-value	0.000	0.000
##	Scaling correction factor		1.463
##			
##	User Model versus Baseline Model:		
##			
##	Comparative Fit Index (CFI)	0.974	0.975
##	Tucker-Lewis Index (TLI)	0.951	0.953
##			
##	Robust Comparative Fit Index (CFI)		0.978
##	Robust Tucker-Lewis Index (TLI)		0.959
##			
##	Loglikelihood and Information Criteria:		
##			
##	Loglikelihood user model (H0)	-3987.159	-3987.159
##	Scaling correction factor		1.285
##	for the MLR correction		
##	Loglikelihood unrestricted model (H1)	-3960.291	-3960.291
##	Scaling correction factor		1.286
##	for the MLR correction		
##			
##	Akaike (AIC)	8026.318	8026.318
##	Bayesian (BIC)	8124.049	8124.049
##	Sample-size adjusted Bayesian (SABIC)	8041.583	8041.583
##			
##	Root Mean Square Error of Approximation:		
##			
##	RMSEA	0.076	0.061
##	90 Percent confidence interval - lower	0.052	0.039
##	90 Percent confidence interval - upper	0.100	0.084
##	P-value H_0: RMSEA <= 0.050	0.036	0.184
##	P-value H_0: RMSEA >= 0.080	0.416	0.089
##			
##	Robust RMSEA		0.070
##	90 Percent confidence interval - lower		0.041
##	90 Percent confidence interval - upper		0.099
##	P-value H_0: Robust RMSEA <= 0.050		0.120
##	P-value H_0: Robust RMSEA >= 0.080		0.300

```

##
## Standardized Root Mean Square Residual:
##
##      SRMR                      0.027          0.027
##
## Parameter Estimates:
##
##      Standard errors          Sandwich
##      Information bread        Observed
##      Observed information based on    Hessian
##
## Latent Variables:
##      Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##      F1 =~ block1
##      newhabit01      1.023    0.087   11.765    0.000    1.023    0.837
##      newhabit02      1.122    0.083   13.557    0.000    1.122    0.862
##      newhabit03      0.868    0.097    8.910    0.000    0.868    0.718
##      newhabit04      0.753    0.128    5.890    0.000    0.753    0.603
##      newhabit05      0.654    0.124    5.254    0.000    0.654    0.565
##      newhabit06      0.399    0.151    2.649    0.008    0.399    0.364
##      newhabit07      0.121    0.130    0.927    0.354    0.121    0.095
##      newhabit08     -0.003    0.094   -0.030    0.976   -0.003   -0.002
##      newhabit09     -0.029    0.070   -0.409    0.683   -0.029   -0.023
##      F2 =~ block1
##      newhabit01     -0.052    0.077   -0.678    0.498   -0.052   -0.043
##      newhabit02     -0.010    0.076   -0.131    0.896   -0.010   -0.008
##      newhabit03     -0.086    0.096   -0.899    0.369   -0.086   -0.071
##      newhabit04      0.186    0.123    1.513    0.130    0.186    0.149
##      newhabit05      0.263    0.123    2.128    0.033    0.263    0.227
##      newhabit06      0.364    0.159    2.292    0.022    0.364    0.332
##      newhabit07      0.824    0.139    5.931    0.000    0.824    0.645
##      newhabit08      1.028    0.107    9.596    0.000    1.028    0.742
##      newhabit09      1.042    0.100   10.413    0.000    1.042    0.824
##
## Covariances:
##      Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##      F1 ~~
##      F2          0.713    0.045   15.993    0.000    0.713    0.713
##
## Variances:
##      Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##      .newhabit01    0.520    0.089    5.851    0.000    0.520    0.349
##      .newhabit02    0.451    0.071    6.376    0.000    0.451    0.266
##      .newhabit03    0.806    0.102    7.870    0.000    0.806    0.552
##      .newhabit04    0.756    0.098    7.731    0.000    0.756    0.486
##      .newhabit05    0.599    0.059   10.102    0.000    0.599    0.447
##      .newhabit06    0.703    0.081    8.694    0.000    0.703    0.585
##      .newhabit07    0.794    0.097    8.173    0.000    0.794    0.487
##      .newhabit08    0.869    0.113    7.716    0.000    0.869    0.452
##      .newhabit09    0.554    0.125    4.433    0.000    0.554    0.347
##      F1            1.000
##      F2            1.000
##
## R-Square:

```


##		Estimate
##	newhabit01	0.651
##	newhabit02	0.734
##	newhabit03	0.448
##	newhabit04	0.514
##	newhabit05	0.553
##	newhabit06	0.415
##	newhabit07	0.513
##	newhabit08	0.548
##	newhabit09	0.653

The pattern of loadings is consistent with question wording: items 7-9 are loading on one factor and items 1-6 on another.

If we check the items again, it appears that items 1-6 are assessing structuring one's schedules or routines, and items 7-9 are assessing whether someone has assigned new times and spaces for their activities. Both of these (routines, context stability) are features of habits.

Methods factor CFA

To deal with these three items that seem to share some method variance, potentially due to the wording, we could add a method factor to the model. To make sure your model is empirically identified and conceptually sensible, the correlation between the method factor (f2) and the conceptual factor (f1) must be 0 - otherwise, we assume they share something in common, but that shared variance is already being captured by f1.

```
methodfactor <- 'f1 =~ NA*newhabit01 + newhabit02 + newhabit03 + newhabit04 + newhabit05 +
                newhabit06 + newhabit07 + newhabit08 + newhabit09
                f2 =~ NA*newhabit07 + newhabit08 + newhabit09

                f1~~1*f1
                f2~~1*f2
                f1~~0*f2
                '
```

```
methodfactorfit <- sem(methodfactor,
                        data = habit,
                        missing = "fiml",
                        estimator = "mlr")
```

```
summary(methodfactorfit,
         standardized = TRUE,
         fit.measures = TRUE,
         rsquare = TRUE)
```

```
## lavaan 0.6-20.2265 ended normally after 37 iterations
##
##      Estimator              ML
##      Optimization method    NLMINB
##      Number of model parameters    30
##
##                               Used      Total
##      Number of observations    430      432
```

```

##      Number of missing patterns                40
##
## Model Test User Model:
##                                     Standard      Scaled
##      Test Statistic                90.250      64.939
##      Degrees of freedom              24         24
##      P-value (Chi-square)           0.000      0.000
##      Scaling correction factor              1.390
##      Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##      Test statistic                1554.087      1043.454
##      Degrees of freedom              36         36
##      P-value                        0.000      0.000
##      Scaling correction factor              1.489
##
## User Model versus Baseline Model:
##
##      Comparative Fit Index (CFI)                0.956      0.959
##      Tucker-Lewis Index (TLI)                  0.935      0.939
##
##      Robust Comparative Fit Index (CFI)              0.958
##      Robust Tucker-Lewis Index (TLI)                0.937
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)                -5230.583      -5230.583
##      Scaling correction factor              1.146
##      for the MLR correction
##      Loglikelihood unrestricted model (H1)          -5185.458      -5185.458
##      Scaling correction factor              1.254
##      for the MLR correction
##
##      Akaike (AIC)                10521.166      10521.166
##      Bayesian (BIC)                10643.079      10643.079
##      Sample-size adjusted Bayesian (SABIC)          10547.877      10547.877
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                0.080      0.063
##      90 Percent confidence interval - lower          0.063      0.048
##      90 Percent confidence interval - upper          0.098      0.079
##      P-value H_0: RMSEA <= 0.050                0.003      0.081
##      P-value H_0: RMSEA >= 0.080                0.526      0.038
##
##      Robust RMSEA              0.081
##      90 Percent confidence interval - lower          0.058
##      90 Percent confidence interval - upper          0.105
##      P-value H_0: Robust RMSEA <= 0.050          0.015
##      P-value H_0: Robust RMSEA >= 0.080          0.566
##
## Standardized Root Mean Square Residual:
##

```

```

##      SRMR                                0.035      0.035
##
## Parameter Estimates:
##
##      Standard errors                                Sandwich
##      Information bread                                Observed
##      Observed information based on                                Hessian
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      f1 =~
##      newhabit01      0.900   0.059  15.217   0.000   0.900   0.731
##      newhabit02      1.058   0.043  24.824   0.000   1.058   0.821
##      newhabit03      0.763   0.059  12.832   0.000   0.763   0.628
##      newhabit04      0.890   0.055  16.187   0.000   0.890   0.689
##      newhabit05      0.826   0.053  15.507   0.000   0.826   0.700
##      newhabit06      0.727   0.058  12.477   0.000   0.727   0.653
##      newhabit07      0.755   0.066  11.505   0.000   0.755   0.593
##      newhabit08      0.749   0.068  11.052   0.000   0.749   0.537
##      newhabit09      0.794   0.058  13.762   0.000   0.794   0.616
##      f2 =~
##      newhabit07      0.494   0.098   5.061   0.000   0.494   0.387
##      newhabit08      0.734   0.103   7.099   0.000   0.734   0.527
##      newhabit09      0.625   0.103   6.076   0.000   0.625   0.485
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      f1 ~~
##      f2      0.000
##
## Intercepts:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      .newhabit01      3.404   0.062  55.345   0.000   3.404   2.767
##      .newhabit02      3.228   0.063  51.324   0.000   3.228   2.505
##      .newhabit03      2.907   0.059  49.060   0.000   2.907   2.393
##      .newhabit04      3.083   0.063  49.072   0.000   3.083   2.387
##      .newhabit05      3.263   0.057  56.855   0.000   3.263   2.763
##      .newhabit06      3.531   0.055  64.502   0.000   3.531   3.173
##      .newhabit07      3.518   0.064  54.623   0.000   3.518   2.761
##      .newhabit08      3.275   0.070  46.729   0.000   3.275   2.349
##      .newhabit09      3.214   0.064  50.556   0.000   3.214   2.495
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      f1      1.000
##      f2      1.000
##      .newhabit01      0.704   0.086   8.204   0.000   0.704   0.465
##      .newhabit02      0.541   0.062   8.739   0.000   0.541   0.326
##      .newhabit03      0.894   0.091   9.854   0.000   0.894   0.606
##      .newhabit04      0.876   0.090   9.701   0.000   0.876   0.525
##      .newhabit05      0.712   0.067  10.576   0.000   0.712   0.511
##      .newhabit06      0.711   0.073   9.785   0.000   0.711   0.574
##      .newhabit07      0.809   0.093   8.746   0.000   0.809   0.498
##      .newhabit08      0.845   0.143   5.921   0.000   0.845   0.434

```

```
##      .newhabit09      0.639    0.128    4.991    0.000    0.639    0.385
##
## R-Square:
##              Estimate
##      newhabit01      0.535
##      newhabit02      0.674
##      newhabit03      0.394
##      newhabit04      0.475
##      newhabit05      0.489
##      newhabit06      0.426
##      newhabit07      0.502
##      newhabit08      0.566
##      newhabit09      0.615
```

```
reliability(methodfactorfit)
```

```
##              f1              f2
## alpha  0.8828567 0.7874316
## omega  0.8921357 0.5995169
## omega2 0.8456186 0.3118791
## omega3 0.8451412 0.3118590
## avevar      NA      NA
```

Two-Correlated Factors CFA

Alternatively, we could estimate a two correlated-factors CFA, where items 07, 08, and 09 load onto a second factor. This would be consistent with most measurement models with two correlated subscales.

However, it's important to remember that this process masks the overall construct hierarchy (e.g., a single construct with two subscales that capture meaningful variance specific to the subscale) as well as potentially meaningful underlying patterns of relationships between indicators. These misspecifications can lead to inflated correlations between the two factors.

```
twofactor <- 'f1 =~ NA*newhabit01 + newhabit02 + newhabit03 + newhabit04 + newhabit05 +
              newhabit06
              f2 =~ NA*newhabit07 + newhabit08 + newhabit09

              f1~~1*f1
              f2~~1*f2
              '
```

```
twofactorfit <- cfa(twofactor,
  data = habit,
  missing = "fiml",
  estimator = "mlr")
```

```
summary(twofactorfit,
  standardized = TRUE,
  fit.measures = TRUE,
  rsquare = TRUE)
```

```

## lavaan 0.6-20.2265 ended normally after 34 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters      28
##
##                               Used      Total
##      Number of observations          430      432
##      Number of missing patterns      40
##
## Model Test User Model:
##                               Standard      Scaled
##      Test Statistic                 93.640      67.429
##      Degrees of freedom                26        26
##      P-value (Chi-square)             0.000      0.000
##      Scaling correction factor         1.389
##      Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##      Test statistic                 1554.087      1043.454
##      Degrees of freedom                36        36
##      P-value                         0.000      0.000
##      Scaling correction factor         1.489
##
## User Model versus Baseline Model:
##
##      Comparative Fit Index (CFI)      0.955      0.959
##      Tucker-Lewis Index (TLI)        0.938      0.943
##
##      Robust Comparative Fit Index (CFI)      0.958
##      Robust Tucker-Lewis Index (TLI)        0.941
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)      -5232.278      -5232.278
##      Scaling correction factor           1.129
##      for the MLR correction
##      Loglikelihood unrestricted model (H1) -5185.458      -5185.458
##      Scaling correction factor           1.254
##      for the MLR correction
##
##      Akaike (AIC)                     10520.556      10520.556
##      Bayesian (BIC)                     10634.342      10634.342
##      Sample-size adjusted Bayesian (SABIC) 10545.486      10545.486
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                             0.078      0.061
##      90 Percent confidence interval - lower 0.061      0.046
##      90 Percent confidence interval - upper 0.095      0.076
##      P-value H_0: RMSEA <= 0.050         0.004      0.112
##      P-value H_0: RMSEA >= 0.080         0.435      0.019
##

```

```

## Robust RMSEA 0.079
## 90 Percent confidence interval - lower 0.056
## 90 Percent confidence interval - upper 0.102
## P-value H_0: Robust RMSEA <= 0.050 0.020
## P-value H_0: Robust RMSEA >= 0.080 0.484
##
## Standardized Root Mean Square Residual:
##
## SRMR 0.036 0.036
##
## Parameter Estimates:
##
## Standard errors Sandwich
## Information bread Observed
## Observed information based on Hessian
##
## Latent Variables:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## f1 =~
## newhabit01 0.899 0.059 15.246 0.000 0.899 0.731
## newhabit02 1.059 0.043 24.833 0.000 1.059 0.822
## newhabit03 0.765 0.059 12.889 0.000 0.765 0.629
## newhabit04 0.889 0.055 16.220 0.000 0.889 0.689
## newhabit05 0.826 0.053 15.473 0.000 0.826 0.699
## newhabit06 0.726 0.058 12.445 0.000 0.726 0.652
## f2 =~
## newhabit07 0.926 0.062 15.013 0.000 0.926 0.726
## newhabit08 1.005 0.063 15.936 0.000 1.005 0.721
## newhabit09 1.015 0.056 18.172 0.000 1.015 0.788
##
## Covariances:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## f1 ~~
## f2 0.781 0.041 19.242 0.000 0.781 0.781
##
## Intercepts:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .newhabit01 3.404 0.062 55.347 0.000 3.404 2.768
## .newhabit02 3.228 0.063 51.322 0.000 3.228 2.505
## .newhabit03 2.908 0.059 49.068 0.000 2.908 2.393
## .newhabit04 3.083 0.063 49.075 0.000 3.083 2.387
## .newhabit05 3.264 0.057 56.860 0.000 3.264 2.764
## .newhabit06 3.531 0.055 64.504 0.000 3.531 3.173
## .newhabit07 3.516 0.065 54.435 0.000 3.516 2.757
## .newhabit08 3.276 0.070 46.772 0.000 3.276 2.348
## .newhabit09 3.214 0.064 50.514 0.000 3.214 2.495
##
## Variances:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## f1 1.000 1.000 1.000
## f2 1.000 1.000 1.000
## .newhabit01 0.705 0.086 8.239 0.000 0.705 0.466
## .newhabit02 0.539 0.062 8.741 0.000 0.539 0.325
## .newhabit03 0.892 0.091 9.845 0.000 0.892 0.604

```

```
##      .newhabit04      0.877    0.090    9.758    0.000    0.877    0.526
##      .newhabit05      0.713    0.068   10.544    0.000    0.713    0.511
##      .newhabit06      0.712    0.073    9.780    0.000    0.712    0.575
##      .newhabit07      0.769    0.090    8.544    0.000    0.769    0.473
##      .newhabit08      0.936    0.109    8.609    0.000    0.936    0.481
##      .newhabit09      0.629    0.095    6.637    0.000    0.629    0.379
##
## R-Square:
##      Estimate
##      newhabit01      0.534
##      newhabit02      0.675
##      newhabit03      0.396
##      newhabit04      0.474
##      newhabit05      0.489
##      newhabit06      0.425
##      newhabit07      0.527
##      newhabit08      0.519
##      newhabit09      0.621
```

Scale unidimensionality using omega

```
reliability(twofactorfit)
```

```
##      f1      f2
## alpha 0.8531724 0.7874316
## omega 0.8572908 0.7882256
## omega2 0.8572908 0.7882256
## omega3 0.8609405 0.7889081
## avevar 0.5042199 0.5541094
```

When we run a CFA with these two factors, you'll notice that fit improves compared to the one-factor model, as we accounted for the similarities between items. But you'll also notice that the correlation between the two factors is very high ($r = .78$), which suggests that they have a lot of overlap.

Omega has decreased slightly, but remains high at .86. Also notice that we have two sets of omegas: one for f1 and one for f2.

Given the improvement in fit when we split the measure into two factors, but the extensive overlap between factors, we estimated a bifactor model.

Bifactor CFA Model

The goal here is to examine whether a two-factor structure explains any variability in scores that hasn't been accounted for by the general (habit development) factor.

We have to specify three factors: A general factor (g) and our two specific factors (f1 and f2)

```
bifactormodel <- 'g =~ newhabit01 + newhabit02 + newhabit03 + newhabit04 +
                    newhabit05 + newhabit06 + newhabit07 + newhabit08 +
                    newhabit09
                    f1 =~ newhabit01 + newhabit02 + newhabit03 + newhabit04 +
                    newhabit05 + newhabit06
                    f2 =~ newhabit07 + newhabit08 + newhabit09'
```

When fitting the model, all latent variables should be uncorrelated (i.e., orthogonal). To get unstandardized loadings for all items, we can tell lavaan to standardize all factor variances with the code in the cfa fitting function `cfa(..., std.lv = T)` This will automatically free the first indicators of all latent variables.

```
bifactorfit <- cfa(bifactormodel,
  data = habit,
  missing = "fiml",
  estimator = "mlr",
  orthogonal=T, # Making all latent variables uncorrelated
  std.lv=T # Standardized factor variances, freely-estimated loadings
)
```

```
summary(bifactorfit,
  standardized = TRUE,
  fit.measures = TRUE,
  rsquare = TRUE)
```

```
## lavaan 0.6-20.2265 ended normally after 61 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters      36
##
##                                     Used      Total
##      Number of observations          430        432
##      Number of missing patterns       40
##
## Model Test User Model:
##
##                                     Standard      Scaled
##      Test Statistic                 42.020      33.984
##      Degrees of freedom              18          18
##      P-value (Chi-square)            0.001      0.013
##      Scaling correction factor
##      Yuan-Bentler correction (Mplus variant)
##
## Model Test Baseline Model:
##
##      Test statistic                 1554.087      1043.454
##      Degrees of freedom              36          36
##      P-value                        0.000      0.000
##      Scaling correction factor
##
## User Model versus Baseline Model:
##
##      Comparative Fit Index (CFI)      0.984      0.984
##      Tucker-Lewis Index (TLI)        0.968      0.968
##
##      Robust Comparative Fit Index (CFI)      0.987
##      Robust Tucker-Lewis Index (TLI)        0.974
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)      -5206.468      -5206.468
```



```

##      Scaling correction factor                      1.263
##      for the MLR correction
##      Loglikelihood unrestricted model (H1)      -5185.458   -5185.458
##      Scaling correction factor                      1.254
##      for the MLR correction
##
##      Akaike (AIC)                                10484.936   10484.936
##      Bayesian (BIC)                              10631.233   10631.233
##      Sample-size adjusted Bayesian (SABIC)       10516.990   10516.990
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                                0.056       0.045
##      90 Percent confidence interval - lower      0.034       0.023
##      90 Percent confidence interval - upper      0.078       0.066
##      P-value H_0: RMSEA <= 0.050                0.307       0.612
##      P-value H_0: RMSEA >= 0.080                0.035       0.002
##
##      Robust RMSEA                                0.053
##      90 Percent confidence interval - lower      0.018
##      90 Percent confidence interval - upper      0.083
##      P-value H_0: Robust RMSEA <= 0.050          0.406
##      P-value H_0: Robust RMSEA >= 0.080          0.072
##
## Standardized Root Mean Square Residual:
##
##      SRMR                                0.022       0.022
##
## Parameter Estimates:
##
##      Standard errors                        Sandwich
##      Information bread                      Observed
##      Observed information based on          Hessian
##
## Latent Variables:
##      Estimate  Std.Err  z-value  P(>|z|)  Std.lv  Std.all
##      g =~
##      newhabit01      0.882   0.082   10.725   0.000    0.882    0.717
##      newhabit02      1.050   0.063   16.587   0.000    1.050    0.816
##      newhabit03      0.743   0.070   10.612   0.000    0.743    0.612
##      newhabit04      0.878   0.055   16.067   0.000    0.878    0.680
##      newhabit05      0.833   0.071   11.784   0.000    0.833    0.705
##      newhabit06      0.835   0.072   11.640   0.000    0.835    0.749
##      newhabit07      0.754   0.068   11.119   0.000    0.754    0.591
##      newhabit08      0.737   0.069   10.628   0.000    0.737    0.528
##      newhabit09      0.780   0.063   12.393   0.000    0.780    0.605
##      f1 =~
##      newhabit01      0.304   0.167    1.817   0.069    0.304    0.247
##      newhabit02      0.320   0.271    1.179   0.238    0.320    0.248
##      newhabit03      0.209   0.110    1.901   0.057    0.209    0.172
##      newhabit04      0.103   0.158    0.652   0.514    0.103    0.080
##      newhabit05     -0.114   0.095   -1.195   0.232   -0.114   -0.096
##      newhabit06     -0.611   0.499   -1.224   0.221   -0.611   -0.549
##      f2 =~

```

```

##      newhabit07      0.502    0.099    5.078    0.000    0.502    0.394
##      newhabit08      0.749    0.101    7.417    0.000    0.749    0.537
##      newhabit09      0.640    0.105    6.066    0.000    0.640    0.496
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      g ~~
##      f1      0.000      0.000    0.000    0.000    0.000
##      f2      0.000      0.000    0.000    0.000    0.000
##      f1 ~~
##      f2      0.000      0.000    0.000    0.000    0.000
##
## Intercepts:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      .newhabit01    3.403    0.061   55.398    0.000    3.403    2.767
##      .newhabit02    3.231    0.063   51.311    0.000    3.231    2.510
##      .newhabit03    2.908    0.059   48.998    0.000    2.908    2.393
##      .newhabit04    3.083    0.063   49.095    0.000    3.083    2.387
##      .newhabit05    3.265    0.057   56.858    0.000    3.265    2.765
##      .newhabit06    3.532    0.055   64.735    0.000    3.532    3.171
##      .newhabit07    3.519    0.064   54.609    0.000    3.519    2.760
##      .newhabit08    3.275    0.070   46.746    0.000    3.275    2.349
##      .newhabit09    3.215    0.064   50.569    0.000    3.215    2.494
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##      .newhabit01    0.643    0.093    6.901    0.000    0.643    0.425
##      .newhabit02    0.452    0.103    4.392    0.000    0.452    0.273
##      .newhabit03    0.880    0.098    8.950    0.000    0.880    0.596
##      .newhabit04    0.887    0.093    9.529    0.000    0.887    0.532
##      .newhabit05    0.688    0.102    6.726    0.000    0.688    0.493
##      .newhabit06    0.170    0.613    0.278    0.781    0.170    0.137
##      .newhabit07    0.805    0.092    8.735    0.000    0.805    0.495
##      .newhabit08    0.840    0.141    5.944    0.000    0.840    0.432
##      .newhabit09    0.643    0.129    4.995    0.000    0.643    0.387
##      g      1.000      1.000    1.000    1.000    1.000    1.000
##      f1      1.000      1.000    1.000    1.000    1.000    1.000
##      f2      1.000      1.000    1.000    1.000    1.000    1.000
##
## R-Square:
##      Estimate
##      newhabit01    0.575
##      newhabit02    0.727
##      newhabit03    0.404
##      newhabit04    0.468
##      newhabit05    0.507
##      newhabit06    0.863
##      newhabit07    0.505
##      newhabit08    0.568
##      newhabit09    0.613

```

```

standardizedSolution(bifactorfit, ci = TRUE) # Get CI's if needed

```

```

##      lhs op      rhs est.std      se      z pvalue ci.lower ci.upper

```

## 1	g =~ newhabit01	0.717	0.061	11.656	0.000	0.596	0.837
## 2	g =~ newhabit02	0.816	0.043	18.966	0.000	0.732	0.900
## 3	g =~ newhabit03	0.612	0.054	11.277	0.000	0.505	0.718
## 4	g =~ newhabit04	0.680	0.037	18.408	0.000	0.607	0.752
## 5	g =~ newhabit05	0.705	0.053	13.390	0.000	0.602	0.808
## 6	g =~ newhabit06	0.749	0.056	13.330	0.000	0.639	0.860
## 7	g =~ newhabit07	0.591	0.047	12.469	0.000	0.498	0.684
## 8	g =~ newhabit08	0.528	0.046	11.376	0.000	0.437	0.619
## 9	g =~ newhabit09	0.605	0.044	13.747	0.000	0.519	0.692
## 10	f1 =~ newhabit01	0.247	0.136	1.820	0.069	-0.019	0.513
## 11	f1 =~ newhabit02	0.248	0.210	1.180	0.238	-0.164	0.661
## 12	f1 =~ newhabit03	0.172	0.091	1.903	0.057	-0.005	0.350
## 13	f1 =~ newhabit04	0.080	0.122	0.652	0.514	-0.160	0.320
## 14	f1 =~ newhabit05	-0.096	0.080	-1.199	0.231	-0.254	0.061
## 15	f1 =~ newhabit06	-0.549	0.448	-1.223	0.221	-1.427	0.330
## 16	f2 =~ newhabit07	0.394	0.076	5.196	0.000	0.245	0.542
## 17	f2 =~ newhabit08	0.537	0.070	7.648	0.000	0.400	0.675
## 18	f2 =~ newhabit09	0.496	0.081	6.093	0.000	0.337	0.656
## 19	newhabit01 =~ newhabit01	0.425	0.063	6.781	0.000	0.302	0.548
## 20	newhabit02 =~ newhabit02	0.273	0.063	4.363	0.000	0.150	0.395
## 21	newhabit03 =~ newhabit03	0.596	0.061	9.712	0.000	0.476	0.717
## 22	newhabit04 =~ newhabit04	0.532	0.054	9.941	0.000	0.427	0.637
## 23	newhabit05 =~ newhabit05	0.493	0.075	6.554	0.000	0.346	0.641
## 24	newhabit06 =~ newhabit06	0.137	0.493	0.278	0.781	-0.830	1.104
## 25	newhabit07 =~ newhabit07	0.495	0.057	8.646	0.000	0.383	0.608
## 26	newhabit08 =~ newhabit08	0.432	0.074	5.858	0.000	0.288	0.577
## 27	newhabit09 =~ newhabit09	0.387	0.076	5.069	0.000	0.237	0.537
## 28	g =~ g	1.000	0.000	NA	NA	1.000	1.000
## 29	f1 =~ f1	1.000	0.000	NA	NA	1.000	1.000
## 30	f2 =~ f2	1.000	0.000	NA	NA	1.000	1.000
## 31	g =~ f1	0.000	0.000	NA	NA	0.000	0.000
## 32	g =~ f2	0.000	0.000	NA	NA	0.000	0.000
## 33	f1 =~ f2	0.000	0.000	NA	NA	0.000	0.000
## 34	newhabit01 ~1	2.767	0.100	27.646	0.000	2.571	2.964
## 35	newhabit02 ~1	2.510	0.086	29.224	0.000	2.342	2.678
## 36	newhabit03 ~1	2.393	0.075	31.863	0.000	2.246	2.540
## 37	newhabit04 ~1	2.387	0.078	30.546	0.000	2.234	2.540
## 38	newhabit05 ~1	2.765	0.098	28.330	0.000	2.573	2.956
## 39	newhabit06 ~1	3.171	0.132	23.998	0.000	2.912	3.430
## 40	newhabit07 ~1	2.760	0.114	24.253	0.000	2.537	2.983
## 41	newhabit08 ~1	2.349	0.085	27.527	0.000	2.182	2.516
## 42	newhabit09 ~1	2.494	0.092	27.260	0.000	2.315	2.674
## 43	g ~1	0.000	0.000	NA	NA	0.000	0.000
## 44	f1 ~1	0.000	0.000	NA	NA	0.000	0.000
## 45	f2 ~1	0.000	0.000	NA	NA	0.000	0.000

Notice that when we fit the bifactor model, all loadings on the general factor fall between .53-.82 and are significantly different from zero, which suggests a general common factor. Moving on to the group factors, the first group factor (items 1-6) is not significant when we account for the general factor.

Scale unidimensionality using omega

We can see the *omega hierarchical* using the reliability function of the semTools package. The first omega is the reliability controlling for the other factors. Here, we can see that the omega for the general factor g is

about .90, and omegas for the specific factors are .01 and .61 after controlling for the general factor's omega. All omegas provide similar information, but they differ in the different methods used to calculate item-total variances.

```
reliability(bifactorfit) # Omega hierarchical is the third omega. The second omega is based on the mode
```

```
##           g           f1           f2
## alpha  0.8828567 0.853172366 0.7874316
## omega  0.9032614 0.011809432 0.6097963
## omega2 0.8535159 0.001433806 0.3245055
## omega3 0.8518051 0.001436161 0.3248798
## avevar      NA           NA           NA
```

You can also get omega hierarchical for each item using this the `Omega_H` function of the `BifactorIndicesCalculator` package (corresponds to the omega based on the observed covariance matrix; the third omega):

```
lambda <- inspect(bifactorfit,what="std")$lambda
theta <- inspect(bifactorfit,what="std")$theta
Omega_H(lambda, theta)
```

```
##           g           f1           f2
## 0.8609644218 0.0005048811 0.3220548248
```

When it comes to omega hierarchical, values above .75 for the general factor and below .50 for the specific factors suggest a dominant general factor and little reliable variance attributable to specific factors. That is because omega hierarchical indexes how much variance in the composite of items is attributable to the each factor (Reise, Bonifay, & Haviland, 2013).

Scale unidimensionality using explained common variance (ECV)

The same `BifactorIndicesCalculator` package will give you ECVs. ECV is a dimensionality index. `ECV_SG` is the proportion of all common variance explained by that factor. For the general factor, this is simply “ECV,” or the proportion of common variance explained by the general factor. For specific factors, the `ECV_S` computes the strength of a specific factor relative to all explained variance of all items, even those not loading on the specific factor of interest.

(from Dueber, D. M. (2017). Bifactor Indices Calculator: A Microsoft Excel-based tool to calculate various indices relevant to bifactor CFA models. <https://dx.doi.org/10.13023/edp.tool.01> [Available at <http://sites.education.uky.edu/apslab/resources/>]).

You can also get ECV's from Dueber's calculator.

```
ECV_SG(lambda)
```

```
##           g           f1           f2
## 0.77831783 0.08966431 0.13201786
```

`ECV_SS` is the proportion of all common variance explained by that factor. For the general factor, this is simply “ECV.” For specific factors, this `ECV_S` computes the strength of a specific factor relative to all explained variance only of the items loading on that specific factor.

Because the factors in the bifactor model are orthogonal, the ECV values all sum to 1.0. Values of .70 or higher for the general factor suggest a unidimensional structure (Rodriguez et al., 2016).

```
ECV_SS(lambda)
```

```
##           g           f1           f2
## 0.7783178 0.1323167 0.4095474
```

The omegas, ECV and loadings clearly point to a dominant general factor. The fact that the second factor, though small, doesn't go away when the general factor is modeled is a strong argument against using alpha to index internal consistency.

Practice

How would you specify a model with a higher order factor f3 and two lower-order factors f1 and f2? If you're getting error messages (e.g., negative variance, model is not identified), try to troubleshoot the error. Is it coming from the lower part of the model (with factors f1 and f2 and their indicators) or the upper part (with factors f1, f2, and f3)? Based on what you've estimated so far and what you've seen in lecture, why is the issue coming from ____ part of the model? How can you fix this issue?

Try to answer: * What are the loadings of f1 and f2 on f3? * How do you interpret the meaning of R-square for f1 and f2? How is it different from R-square for the observed items? * Is this a well-fitting model? Why are some indices better than others? (going back to the formulas for each fit index may help) * Consider how fit in nested models refers to the changes in parameters between the two models (recall what modification indices mean, for example). How does the fit between this model and the two correlated-factors model differ? What part of the model does the change in fit refer to?

##References Dueber, D. M. (2017). Bifactor Indices Calculator: A Microsoft Excel-based tool to calculate various indices relevant to bifactor CFA models. <https://dx.doi.org/10.13023/edp.tool.01> [Available at <http://sites.education.uky.edu/apslab/resources/>]

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Rodriguez, A., Reise, S. P., & Haviland, M. G. (2016). Evaluating bifactor models: Calculating and interpreting statistical indices. *Psychological Methods*, 21, 137-150. <https://doi.org/10.1037/met0000045>