① 1.1.1

$$\frac{x^{32}\,x^{7}}{x^{9}\cdot x^{2}\cdot x^{2}} = \frac{x^{39}}{x^{13}} = x^{26}$$

② 1.1.2

$$8^2 \cdot 4^x \cdot 2^x = 8^4$$
$$(2^3)^2 \cdot (2^2)^x \cdot 2^x = (2^3)^4$$
$$2^6 \cdot 2^{2x} \cdot 2^x = 2^{12}$$
$$2^{3x} = 2^6$$
$$x = 2$$

③ 1.1.3

$x = 3y \qquad \frac{1}{x^4}\,y^4 = ?$

$$\frac{y^4}{(3y)^4} = \frac{y^4}{3^4 \cdot y^4} = \frac{1}{3^4} = \frac{1}{81}$$

④ 1.1.4

$$\frac{\sqrt{4^{15}}}{\sqrt{16^7}} = \frac{\left(4^{15}\right)^{\frac{1}{2}}}{\left(16^7\right)^{\frac{1}{2}}} = \frac{2 \cdot 4^7}{4 \cdot 16^3} = \frac{1}{2} \cdot \frac{4^7}{4^6} = \frac{4}{2} = 2$$

⑤ 1.1.5

(a) TRUE (c) FALSE
(b) TRUE (d) FALSE

⑥ 1.1.6

$\ln(x) \geq e \rightarrow x \geq e^a$

⑦ 2.2.1.

$$\left.\begin{aligned} c &= (f-32) \times \tfrac{5}{9} \\ c &= f \end{aligned}\right\} \longrightarrow x = (x-32) \times \tfrac{5}{9}$$
$$9x = 5x - 160$$
$$4x = -160$$
$$x = -40$$

→ AT $-40°C$ THE TEMPERATURE IN FAHRENHEIT IS ALSO $-40°F$

⑧ 2.2.2.

$$f(x) = 3x - 12 = 0 \longrightarrow 3x = 12$$
$$x = 3$$

⑨ 2.2.3

$$9^{x^2-6x+2} = 81$$

$$x^2-6x+2 = 2$$

$$x^2-6x = 0$$

$$x(x-6) = 0$$

$x = 0$ $\qquad$ $x = 6$

---

⑩ 2.2.4.

$$x \cdot (1+0{,}03)^n = 3x \longrightarrow \sqrt[n]{3} = 1{,}03$$

AFTER 38 YRS

$$\frac{1}{n} = \log_3 1{,}03 = \frac{\log_{10} 1{,}03}{\log_{10} 3} = \frac{1}{38{,}17}$$

DEPENDING HOW YOU EXACTLY CALCULATE THE GROWTH / BASELINE IT COULD BE 39 TOO.

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⑪ 2.2.5.

$$\log_\pi\left(\frac{1}{\pi^5}\right) = \log_\pi 1 - \log_\pi \pi^5 = 0 - 5 = -5$$

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⑫ 3.1.

$$\sum_{i=1}^{\infty}\left(\frac{1}{5^i} + 0.3^i\right) = \sum_{i=1}^{\infty}\frac{1}{5^i} + \sum_{i=1}^{\infty}\left(\frac{3}{10}\right)^i = \frac{\frac{1}{5}}{\frac{4}{5}} + \frac{\frac{3}{10}}{\frac{7}{10}} = \frac{1}{4} + \frac{3}{7} \approx 0{,}6786$$

$$\frac{ab}{1-b}$$

⑬ 3.2

$$\lim_{x \to 5} \frac{x^2-25}{x-5} = \lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}} = 5+5 = 10$$

(14) 3.3.

$f(x) = x^3 - 4$ → $f'(-2) = \underline{12}$

$f'(x) = 3x^2$ → $f'(-12) = 432$

(15) 3.4.

$$f(x) = \frac{x^5 + 3}{x^2 - 1}$$

$$f'(x) = \frac{5x^4(x^2-1) - 2x(x^5+3)}{(x^2-1)^2}$$

(16) 3.5.

$f(x) = x^9 + 3$

$f'(x) = 9x^8$

$f''(x) = \underline{72x^7}$

(17) 3.6.

DEPENDS IF 0 IS PART OF THE DOMAIN OR NOT.

TO BE CONTINUOUS F(x) NEEDS TO BE CONTINUOUS THROUGHOUT THE DOMAIN.

BECAUSE I ASSUME $x \in \mathbb{R}$, THUS F(x) IS NOT CONTINUOUS.

(18) 3.7.

$f(x) = 4x^3 - 12x$

$f'(x) = 12x^2 - 12 = 0$

$12x^2 = 12$

$x^2 = 1$

$x = \underline{\pm 1}$

(19) 3.8.

$f(x,y) = x^3 - y^2$

$f(2,3) = 2^3 - 3^2 = 8 - 9 = \underline{-1}$

(20) 3.9

$\ln(x)$ IS DEFINED IF x IS GREATER THAN 0.

THUS → $x - 3y > 0$

$\underline{x > 3y}$

(21) 3.10

$\frac{f'}{\partial x} \rightarrow 5x^4y^7 + \frac{2x}{y^3}$

(22) 3.11

① $d_x \rightarrow \frac{y}{2\sqrt{xy}} - 1 = 0$

② $d_y \rightarrow \frac{x}{2\sqrt{xy}} - 1 = 0$

$y = 2\sqrt{xy} \rightarrow y = 4x$

$x = 2\sqrt{xy}$

$\downarrow$

$x = 4x$ ☹

NO MINIMA / MAXIMA / SADDLE POINT.

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(23) 3.12

$\max x^2y^2$ s.t. $2x + y = 9$

$\mathcal{L}(x, y, \lambda) = x^2y^2 - \lambda(2x + y)$

$\frac{d\mathcal{L}}{dx} \rightarrow 2y^2x - 2\lambda = 0$

$\frac{d\mathcal{L}}{dy} \rightarrow 2x^2y - \lambda = 0$

$\frac{d\mathcal{L}}{d\lambda} \rightarrow 2x + y = 9$

$\rightarrow y = 9 - 2x$

$\lambda = 2x^2y \rightarrow 2x^2(9 - 2x)$

$(9 - 2x)(2x(9 - 2x) - 4x^2) = 0$

$9 - 2x = 0$

$\downarrow$

$x_1 = 4{,}5$

$y_1 = 0$

$18x - 8x^2 = 0$

$\downarrow$

$x_2 = 2{,}25$

$y_2 = 4{,}5$

(24) 4.1. 3×2

$$A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}$$ 2×3

$$B \cdot A = \begin{bmatrix} 1\times2 + 0\times2 + 1\times7 & 1\times5 + 0\times1 + 1\times6 \\ 9\times2 + 1\times2 + 5\times7 & 9\times5 + 1\times1 + 5\times6 \end{bmatrix}$$

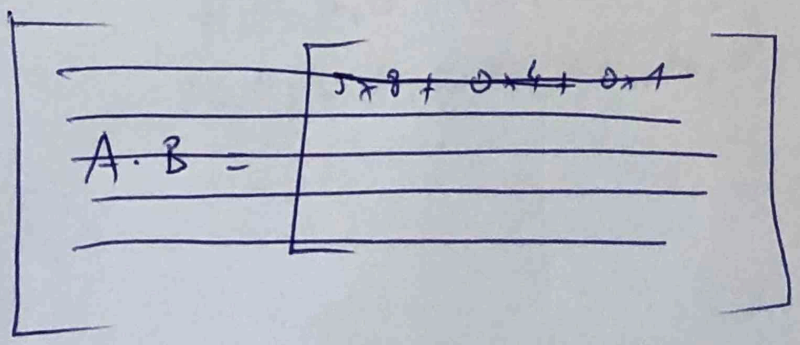
$$B \cdot A = \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}$$

(25) 4.2 3×2

$$A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$$

2×3

$$B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$



$$A \cdot B = \begin{bmatrix} 5\times8 + 3\times2 & 5\times4 + 3\times1 & 5\times0 + 2\times3 \\ 0\times8 + 1\times2 & 0\times4 + 1\times1 & 0\times0 + 1\times2 \\ 1\times8 + 2\times2 & 1\times4 + 2\times1 & 1\times0 + 2\times2 \end{bmatrix}$$

(25) cont.

$$A \cdot B = \begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{bmatrix}$$

(26) 4.3

$$A^T = \begin{bmatrix} e & 2 & 4 \\ 93 & 6,1 & \tilde{u} \\ 4,7 & 4,22 & 0 \end{bmatrix}$$

(27) 4.4.

$$\begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix} \qquad \det(A) = 2 \cdot 8 - 6 \cdot 2 = 16 - 12 = 4$$

(28) 5.1.

$$\Omega = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\} = 6 \times 6 = 36$$

(29) 5.2.

$P(TP) \leftarrow P(\text{TRUE POSITIVE}) = 0{,}1\%$

$P(TN) \leftarrow P(\text{TRUE NEGATIVE}) = 99{,}7\%$

$P(FP) \leftarrow P(\text{FALSE POSITIVE}) = 0{,}3\%$

$P(FN) \leftarrow P(\text{FALSE NEGATIVE}) = 2\%$

$$P\left(\frac{\text{TRUE POSITIVES}}{\text{TOTAL POSITIVES}}\right) = ?$$

$$P(\text{POSITIVE}) = P(TN) \cdot P(FP) + P(TP) \cdot (1 - P(FN)) =$$

$$= 0{,}999 \cdot 0{,}003 + 0{,}001 \cdot 0{,}98 = 0{,}003977$$

$$P\left(\frac{TP}{P}\right) = \frac{0{,}001}{0{,}003977} \approx 25\%.$$

THAT A POSITIVE TEST RESULT INDEED MEANS TRUE POSITIVE.

(30)

$$E(\bar{5}) = 20 \cdot \frac{1}{6} = 3{,}\dot{3}$$