

Treasury supply shocks and the term structure of interest rates in the UK*

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Abstract

Abstract: How does the additional debt issued by the government affect the term structure of interest rates? In this paper we identify Treasury supply shocks using intraday high-frequency data, by exploiting the institutional setup of the UK government bond primary market. We find that supply shocks have positive effects on interest rates. Most of the reaction is due to the risk premia rather than the expectations component of yields. We argue both theoretically and empirically that supply shocks transmit via the repricing of duration and inflation risks in the economy. We also document stronger and localized effects during market stress.

Keywords: Term structure, Government debt, Bond risk premia, High-frequency identification

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1 Introduction

How does the additional issuance of government debt affect the term structure of interest rates? The question is both important and topical given the rapid increases in government deficits and debt levels across the globe due to the Covid-19 pandemic. At the same time, the interest rate environment is in upheaval, due to central banks' efforts to fight the inflationary pressure of the recovery. The resulting increased public debt service cost requires the active management of the debt and a good understanding of the financial market effects of debt issuance.

The effect of government debt issuance on interest rates is not well established in the empirical literature. Surveys on the effects of fiscal deficits on interest rates by Gale and Orszag (2003) and Engen and Hubbard (2004) found around the same number of papers with positive and significant effects as the number of papers with insignificant effects. The reason is that identification is difficult. While interest rates are available at every point in time, budgetary variables are only available annually or at best quarterly frequency. Reverse causality, common factors, and anticipation effects are all complicating the problem. For example, agents often anticipate and price public policies, such as new debt issuance, in advance, making it difficult to time and measure the true causal effect of the policy. Moreover, factors that affect both interest rates and the deficit can lead to finding a spurious relationship: while the central bank cuts the policy rate in a recession, the fiscal deficit and the debt issuance increase. Our goal in this paper is to get around these issues and uncover the true causal effect of debt supply on interest rates.

The policy relevance of understanding the impact of changes in Treasury supply is twofold. The ever-increasing financing needs of governments provide a steady new supply of bonds to markets. Fiscal authorities need a thorough understanding of how their funding decisions affect financial markets, as it has a direct impact on their own funding costs and the interest rate environment firms and households face. Second, central banks are currently in the process of winding down their large asset portfolios accumulated after the Global Financial

Crisis and the Covid pandemic. To the extent that this is done via outright Treasury sales (otherwise known as quantitative tightening or QT), this means additional bond supply to be absorbed by markets. Our results can provide quantitative estimates of the potential effects of QT on financial markets. As the Bank of England is currently about to become the first major central bank to sell back part of its bond portfolio to the market, our study has operational relevance.

Our first contribution is to propose a novel identification of government bond supply shocks. The identification exploits the institutional features of the Debt Management Office (DMO) of the United Kingdom. We focus on the *announcements* of the supply of upcoming bond auctions. We follow intraday bond futures price movements in a narrow event window around the announcements to capture the information content of the announcements. This information content is high, as the UK DMO does not provide information about the volume of the upcoming auctions before these releases.¹ Furthermore, the announcements contain information solely on the supply side of the bond market. This institutional framework provides an ideal setting to apply the high-frequency identification scheme (Kuttner (2001)). Price movements in a narrow event window around the announcements can be related to information about future bond supply. We interpret these price movements as shocks to the supply of government bonds.

The second contribution of the paper is to study the effects of debt supply shocks on the term structure of interest rates. This adds to the empirical literature that studies the effect of government debt supply on interest rates. The fiscal policy literature tends to relate these two variables at the quarterly or annual frequency. We, on the other hand, estimate this relation at the daily frequency. Moving higher in frequency has the advantage of allowing for an (arguably) cleaner identification, but it comes at the expense of being more restricted in the range of addressable research questions.

We find that debt issuance has significant positive effects on nominal interest rates: a

¹This is in contrast with other countries like the US, where the Treasury provides estimated future auction sizes every quarter.

positive standard deviation bond supply shock increases nominal rates by 1-1.5 basis points. Longer maturities respond more, so the slope of the yield curve increases. This effect spills over to equity and corporate bond markets. To give more intuition on the size of the effect, we provide a back-of-the-envelope exercise. On the 11th of March 2020, the UK government announced a Covid-19 support package of £12bn. According to our estimates, an unexpected debt issuance announcement of this size would raise nominal yields by 13-19 basis points. These estimates are in line with actual changes in benchmark yields on the day of the announcement.

Next, we study the mechanism of how supply shocks transmit to the term structure of interest rates. We find that a standard deviation supply shock increases real rates by 1-1.2 basis points. This implies that the main driver of the effect on nominal rates is not higher inflationary concerns. To investigate further, we decompose yields into expectations and risk premia components with the Affine Term Structure Model (ATSM) of Abrahams, Adrian, Crump, Moench and Yu (2016). We find that over two-thirds of the response of long-term yields are attributed to risk premia components. Additional government debt supply raises both the real term premium and the inflation risk premium. Interestingly, expected inflation is unaffected. In other words, the supply shock mostly operates via the “risk-taking channel” i.e., by investors repricing risks in the economy (Borio and Zhu (2012)).

We illustrate our empirical findings in an equilibrium term structure model with supply effects. The model extends the framework of Greenwood and Vayanos (2014) with exogenous inflation. It has two types of bonds: nominal and inflation-linked. The supply of nominal bonds is stochastic. Shocks to this supply are absorbed by risk averse investors, holding more inflation and interest rate risks in their equilibrium portfolio. As investors become more exposed to these risk factors, they require higher compensation to hold these risks in their portfolio. This drives up risk premia, and consequently, yields. Inflation-linked bonds are unaffected by inflation risk, so their yield is less affected by the shock compared to nominal bond yields, consistent with our empirical finding.

The mechanism in the model is closely linked to investors’ limited risk bearing capacity. Supply effects are stronger when investors are more risk averse. We test this prediction empirically, by exploring state dependence in the effects of the high-frequency supply shock. Consistent with the model’s prediction, we find stronger reactions of yields in times of market stress, driven by rising risk premia. In these periods supply shocks have localized effects, consistent with the findings of Vayanos and Vila (2021) and Droste, Gorodnichenko and Ray (2021). We also explore the effect of supply shocks in times when the policy rate is at the effective lower bound (ELB). Short-term rates respond less, while long-term rates react stronger to supply shocks in these periods. The main driver here is again rising risk premia and not higher expected short rates.

The rest of the paper is organised as follows. Section 2 connects our paper to the literature. Section 3 explains our identification in two steps. Section 3.1 describes the institutional framework of the UK government bond primary market, while Section 3.2 explains how we exploit this to construct the supply shock. Section 4 analyses the effect of the supply shock on yields. Section 4.2 demonstrates that the supply shock transmits by affecting risk prices. Section 4.3 presents an equilibrium asset pricing model where we illustrate this effect. Section 5 investigates the role of non-linearities. Lastly, Section 6 concludes.

2 Related literature

We identify government bond supply shocks using the high-frequency identification (HFI) method. HFI was developed initially to study the effects of monetary policy shocks (Kuttner (2001), Gürkaynak, Sack and Swanson (2005)). It has recently been applied to identify oil price shocks (Känzig (2021a)), carbon policy shocks (Känzig (2021b)), and Treasury demand shocks (Droste et al. (2021), Lengyel and Giuliadori (2022)). The latter is the application most similar to ours. Droste et al. (2021) identifies Treasury *demand* shocks of large institutional investors by following Treasury futures prices around Treasury auction

result releases. We focus, on the other hand, on the *announcements* of auctions. In this aspect, the paper by Simon (1991) is closely related to ours. Simon (1991) analyses the announcements of US government cash-management bills in an event study. The focus of both Droste et al. (2021) and Simon (1991) is on the segmentation of Treasury markets and the localized effects of supply and demand conditions. While we do find some evidence for segmentation, our main focus is on risk pricing and the transmission of government debt supply shocks. Our paper is also similar to D’Amico and Seida (2020), in the sense that our identification isolates the expected and the unexpected components of the announcements of changes in the supply of bonds to see the reaction of yields.

During the finalization of our results, we became aware of Phillot (2021). Similar to us, he proposes the identification of US Treasury supply shocks by following futures price movements around auction supply announcements. In line with our results, he finds empirically that the supply shock is followed by a positive shift in the yield curve, higher inflation compensation, rising stock prices, and corporate bond yields. The main differences compared to our paper are the following. Firstly, we follow *intraday* futures price movements in a one-hour event window around announcements, while Phillot (2021) records daily price differences between the price on the announcement day and the price on the previous day. Our narrower event window means that our shock series is less affected by potential confounding factors contaminating the results. What allows us to go higher in frequency is focusing on the United Kingdom instead of the US. In the UK the exact publication time of the announcements is known, while in the US only the date is known. This allows us to zoom in on intraday price movements in a short event window around the release time of the announcements. An additional benefit of focusing on the UK is that UK auction announcements have higher information content about bond supply compared to the US, as explained more in detail in Section 3.1. Apart from the identification, an important difference between our papers is that our analysis focuses more on the mechanism of how debt supply affects the term structure of interest rates. We first break down yields into their components with an

empirical ATSM and study the reaction of each component separately. Then, we illustrate these empirical findings in an equilibrium asset pricing model.

Supply and demand conditions in Treasury markets have gained much attention with central banks' QE operations. The general finding is that official demand for bonds decreased the level and slope of the yield curve (Hamilton and Wu (2012), Li and Wei (2013), McLaren, Banerjee and Latto (2014) and others). The underlying mechanism is explained by the preferred-habitat theory of interest rates (Modigliani and Sutch (1966), Vayanos and Vila (2021)). This theory argues that changes in the demand and supply conditions are transmitted through bond risk premia. Risk premia has a positive relationship with the slope of the term structure, as long-term bonds are more exposed to risks. Our study brings evidence in line with this literature, connecting bond supply with the level and slope of the yield curve, and bond risk premia. However, in contrast with the empirical literature on QE, we do not focus on changes in central bank demand for bonds, but on the supply from the Treasury. Nevertheless, the mechanism we explain our findings is the mirror image of the one used to explain the effects of QE (Vayanos and Vila (2021), Greenwood and Vayanos (2014)). Our results are more useful for analysing the effects of debt-financed fiscal expansions or the effects of central bank QT programs.

We find that changes in Treasury supply transmit to yields by affecting bond risk premia. Therefore, the spending and financing decisions of the government have direct effects on risk pricing. In this regard, our paper is connected to the strand of literature that establishes a connection between measures of fiscal policy and risk pricing. Studies have found that bond risk premia is affected by the level of fiscal expenditures and the uncertainty around it (Bretscher, Hsu and Tamoni (2020), Horvath, Kaszab and Marsal (2021), Kučera, Kočenda and Maršál (2022), Bayer, Born and Luetticke (2020)), the government debt ratio (Alesina, De Broeck, Prati and Tabellini (1992), Greenwood and Vayanos (2014), Nguyen (2018)) and the maturity structure of the debt (Chadha, Turner and Zampolli (2013), Greenwood and Vayanos (2014), Corhay, Kind, Kung and Morales (2021)). The government debt

ratio was also found to influence equity- and credit-risk premia (Gomes, Michaelides and Polkovnichenko (2013), Liu (2019)) as well as the liquidity premium (Krishnamurthy and Vissing-Jorgensen (2012), Bayer et al. (2020), Reis (2021)). Our paper contributes to this literature, by linking Treasury debt issuance with the term premium and the inflation risk premium.

3 Constructing the supply shock measure

In this section, we explain our identification in two steps. First, we briefly describe the institutional framework of the UK Debt Management Office and the bond issuing process. For more details see DMO (2021). Then, we outline how we apply HFI and isolate supply shocks in this setting.

3.1 Description of the UK primary bond market

The DMO is the institution responsible for the UK government’s debt management policy. It carries out this duty by issuing debt securities denominated in pound sterling. The securities with maturity within a year are called bills, while the securities with maturity over a year are called “gilts” or “gilt-edged securities”. Gilts make up the largest proportion of government debt, around 86%.² The DMO issues two types of gilts: Conventional and index-linked. Conventional gilts are nominal bonds i.e., interest payments and coupon repayments are fixed in nominal terms. They constitute around three-quarters of the gilts issued by the DMO. Index-linked gilts are securities with coupon and final redemption payments linked to inflation, more specifically to the UK Retail Price Index (RPI). They constitute around a quarter of the debt issued by the DMO. The primary means of issuing gilts is through regular auctions, with over 75% of the overall gilt sales. The remaining part is issued through syndicated gilt offerings or mini gilt tenders. The annual financing remit, set by the UK

²See Figure B1 in the Online Appendix.

Treasury, outlines the gilt sales required from the DMO for the upcoming financial year. The document specifies the total amount of gilt sales and the breakdown between index-linked gilts and conventional gilts in different maturity buckets. It is published every year in mid-March as the financial year runs from the 1st of April until the 31st of March. Occasionally the remit is revised in April when the central government's final net cash requirement for the previous financial year is published. Furthermore, the remit is usually revised in November or December when the UK government publishes its budget together with forecasts of public finances. The remit contains the Gilt Auction Calendar, stating the dates of the auctions in the next financial year. Furthermore, the document states the amount of gilts to be issued and the number of planned auctions in four categories. The four categories are index-linked gilts, and three conventional gilt maturity buckets: short, medium, and long conventional gilts with 0-7, 7-15, and 15+ years to maturity. Therefore, the information in the remit gives investors an idea about the average size of the coming auctions in each category. An example of the DMO Financing Remit is displayed in Figure B2 in the Online Appendix.

The DMO announces its auction plan for the next quarter on the last business days of March, May, August, and November in an operations calendar. An operations calendar is shown in Figure B3 in the Online Appendix for an example. This calendar publishes the dates of the coming auctions, mini-tenders, and syndicated issuances in the next quarter. The document also specifies the maturity year and the interest rate of the issuance. Importantly, it does not provide information about the size of the auction. This is in contrast with the US, where the Treasury gives preliminary estimates of future auction volumes every quarter.³

The auction announcements are published at 3:30 pm, usually on the Tuesday in the week preceding the auction. This press release contains all the pertinent information about the issuance. Importantly, this is the time investors learn the exact size of the auction. Additional information released in the statement are ISIN, SEDOL codes, coupon payments, and the terms and conditions of the auction. An example announcement of a 10-year gilt

³See the US Quarterly Refunding Press Conference: <https://home.treasury.gov/policy-issues/financing-the-government/quarterly-refunding>.

auction published on the 21st of April 2015 is displayed in Figure B4 in the Online Appendix. Progress reports on the financing remit are often included in these announcements. These contain information on the remaining amount of gilts to be issued and the number of auctions to be held in the rest of the fiscal year. See Figure B5 in the Online Appendix for an example.

3.2 High-frequency surprises

To study the effects of debt issuance on interest rates, one can regress daily yield changes on the announced volumes. However, most of the announced new debt either covers the refinancing of maturing bonds or finances public expenditures that are known before the announcement. In other words, a large share of new issuances is anticipated by markets. Then, the effect is already priced in by the time of the announcement and the regression coefficients will not reflect the true causal effect.

A second option would be to use the surprise component of the announcements in the regression. Unfortunately, surprises are not observable. What is available is the required average future auction size to meet the DMOs' yearly financing remit. This quantity is published in the auction announcement press releases.⁴ It is calculated as: $\frac{\text{Remaining gilt sales}_t}{\text{Number of auctions remaining}_t}$. We can use this as a proxy for investors' expectations of the announcement. We subtract this from the actual announced volume and label it as the *surprise volume*. While this is arguably a better measure, it is still prone to the issue of anticipation, as investors form their expectations based on much more information than the DMOs' progress. Therefore, the surprise volume series cannot be a true shock, which is confirmed by the fact that the series is autocorrelated.⁵ To overcome these difficulties and capture unexpected changes in the supply of bonds, we opt for high-frequency identification. Nevertheless, below we will make use of the announced volume and the surprise volume series to support the validity of our identification.

We use high-frequency identification to isolate anticipated and unanticipated policy

⁴See Figure B5 in the Online Appendix for an example.

⁵See Figure B6 in the Online Appendix.

changes, as in the monetary policy literature (Kuttner (2001), Nakamura and Steinsson (2018)). Most similar to our application is Droste et al. (2021), who identify Treasury *demand* shocks by following futures price movements around the publication of US auction results. In contrast, we identify Treasury *supply* shocks by following futures price movements around announcements of bond issuance volumes. We restrict our attention to announcement days with conventional nominal gilt announcements only (and no tenders or index-linked gilt auctions).

Data on auction announcements are sourced from the DMO. The dataset starts on the 15th of May 2001 (the date of the first announcement on the DMOs’ website) and ends on the 31st of December 2019. It contains 400 auctions over 360 announcement days. As explained in Section 3.1, these announcement days are usually, but not always the Tuesdays of the week preceding the auction. First, we collect auction dates from the auction results section of the DMOs’ website. Then, we match each auction with the corresponding press release of the announcement. This document contains the announced volume, as well as a progress report with the remaining issuance volume and the remaining number of auctions in the fiscal year. In the few cases when the press release is not available, we use the dates and times specified in the DMOs’ operations calendar and obtain the announced volumes from the auction results.

We record high-frequency gilt futures price movements around the announcements, as it is conventional in the high-frequency identification literature. Futures prices have many advantages for this application compared to spot prices or when-issued prices. Futures contracts trade on exchanges, while bonds trade over the counter. Therefore, the quality and the availability of price data is much better. Futures are also much more liquid than their cash counterparts, and futures markets tend to lead price discovery ahead of the spot (Garbade and Silber (1983)).

We use intra-day gilt futures front contract prices to identify supply shocks, purchased from tickdatamarket.com. The contracts are traded on the London ICE exchange. There are

four futures contracts written on UK government bonds: short, medium, long, and ultra-long. These can be satisfied with bonds with remaining maturities of 1.5–3.25, 4–6.25, 8.75–13, and 28–37 years, respectively. Data on the short and the medium contract are available from 2010 onward, while the long contract is available from 2001 onward. The ultra-long contract is much less liquid and we have data only between 2014 and 2016. Results are unchanged if we leave out the ultra-long contract from the analysis.

As described above in detail in Section 3.1, the DMO releases precise information about an upcoming action usually on Tuesday of the preceding week at 3.30 pm. These occasions are the first time the DMO discloses information about the size of the auction. Prior to this, investors can only speculate on the volume based on the remaining issuance volume and the number of auctions left for the year. The announcements contain information about (among other things) the volume, the coupon, and the exact maturity of the upcoming issuance. An example announcement is displayed in Figure B4. In other words, the announcements contain information solely about the supply side of the market. Price changes in a narrow window around the announcement should reflect revisions in investors’ bond supply expectations. We interpret these as the supply shocks.⁶

The supply shock $S_t^{(m)}$ on announcement day t in maturity segment m is measured as the difference between the (log) futures price after and before the publication of the press release. More explicitly:

$$S_t^{(m)} = \left(\ln(P_{t,post}^{(m)}) - \ln(P_{t,pre}^{(m)}) \right) \times 100 \quad m \in \{\text{short, medium, long, ultra-long}\} \quad (1)$$

where $P_{t,post}^{(m)}$ is the futures price 30-minutes after the announcement and $P_{t,pre}^{(m)}$ is the

⁶Assuming liquidity premia does not change in the narrow event window. While liquidity conditions of Treasury futures are systematically priced, the liquidity premium is considered to move at lower frequencies (see Piazzesi and Swanson (2008) and Nakamura and Steinsson (2018)).

futures price 30-minutes before the announcement.⁷ We use the five-minute centered moving average of the price to smooth out noise in the data. In minutes with no trading activity, we use the midquote: the average of the lowest bid price and the highest ask price. We record the price difference in Equation (1) for all four futures contracts, regardless of the maturity of the bond announced. Ideally, we would like to have time series that track shifts in the supply at every maturity point of the term structure. However, we can only proxy the shifts by price movements at the four points where futures contracts are available.

An illustrative example is the 10-year conventional gilt auction held on the 29th of April 2015. The exact size of the auction was published at 3:30 pm on the 21st of April (see the press release in Figure B4 in the Online Appendix). The volume was £3000 million, which was 10% larger than the average future auction size implied by the DMOs' progress report, published a week earlier (see the medium bucket in Figure B5). The release of this information about lower supply was followed by a marked increase in the price of all futures contracts, as displayed in Figure 1.

The time series of the four supply shocks are displayed in Figure 2. The four $S_t^{(m)}$ series are highly correlated, so we found it convenient to compress these series into one variable by extracting the first (probabilistic) principal component. We label this series S_t without a superscript. The interpretation of S_t is an unexpected, non-maturity-specific shift in the supply of government bonds. The mean of S_t is 0.001 with a standard deviation of 0.132. We normalize it to have zero mean and unit variance and use it in our regression analysis as our explanatory variable.⁸ The dependent variables in the regressions are daily yield changes. By moving from intraday to daily frequency, we intend to capture responses that might take longer to materialize than the one-hour length of the event window.

Table 1 reports the descriptive statistics of the supply shocks. The means are very close to zero, suggesting that the shocks are not systematic. The ultra-long contract has

⁷Our results are robust to both narrower and wider event window specifications. These results are available upon request.

⁸Results using the maturity-specific surprises $S_t^{(m)}$ are similar and available upon request.

a positive mean, most likely due to the short sample and the low liquidity of the contract. The standard deviations increase with the maturity of the contracts. Table 1 shows that the series are strongly correlated. The ACF in Figure 3 shows no serial autocorrelation. This assures us that the shocks are not just due to shifts in the timing of the DMO’s issuance plan.

It is important to make sure that no other relevant information is released around the announcements that could contaminate the identification of the supply shock. The most important drivers of Treasury yields are macroeconomic news releases, monetary policy decisions, and government bond auction results according to Fleming and Remolona (1997). The times of these events are all outside of our event window, but our results are robust to omitting announcement days that coincide with either one of these events.⁹ Our event window starts at 3.00 pm. Macroeconomic data releases are published at 7:30 am or 9:00 am by the statistical office. Monetary policy announcements are published at 12:00, with a press conference held at 12:30. The DMO is also very transparent about releasing public announcements. Market-sensitive information is usually announced between 7.30 am and 8.00 am. On auction days, the bidding process closes at 10.00 am or 10.30 am, and the results are published shortly after. Post Auction Option Facility¹⁰ results are published at the end of the take-up window closure at 1.00 pm or 2.00 pm. For more information, see DMO (2021).

We identify the supply shock S_t as price movements within a narrow event window around announcements. The assumption is that these price movements are the equilibrium responses to underlying shifts in the supply. To verify that these market responses are related to actual changes in the supply, we link our high-frequency shock to observable movements in supply. We can use two available observable measures from the auction announcement press release documents. The announced volume, and the “surprise volume” series. In Section 3 we

⁹These results are available upon request.

¹⁰Since the 1st of June 2009, all successful UK gilt auction bidders have the option to purchase up to 10-15% of the bond they have bought, at the published average auction price.

discussed that these series are not ideal to analyse the effects of variations in the supply. Nevertheless, we can still use them to validate our high-frequency identification, by relating them with S_t .

First, we regressed the announced volumes on the high-frequency supply shock S_t but did not find a significant relationship between the two variables. Next, we regressed the “surprise volume” series on S_t . The estimated coefficient is significant and negative, implying that higher-than-expected supply is associated with a decrease in the futures price within the event window. Table 2 displays these results in the left column. The right column reports the results when we use a one-day event window as in Phillot (2021), instead of the one-hour window of S_t . The insignificant coefficients imply that using a narrower event window captures better the price movements that are related to the surprise component of the announced volumes. Furthermore, a regression of the daily surprise on the intraday surprise (reported in Table 3) yields a very low R^2 , suggesting that there must be other important drivers of prices on announcement days other than the press release. These results underline our argument to use intraday supply shocks instead of daily supply shocks. This is underlined by Kerstenfischer and Schmeling (2022), illustrating how multiple different news events drive yields within a day.

4 Bond supply effects on the term structure of interest rates

4.1 Effect on nominal and real yields

To assess how unexpected shifts in the supply of bonds affect interest rates, we regress the supply shock S_t onto interest rates at each maturity:

$$\Delta R_t^{(m)} = a^{(m)} + b^{(m)} S_t + \varepsilon_t^{(m)} \quad (2)$$

Where $\Delta R_t^{(m)} = R_t^{(m)} - R_{t-1}^{(m)}$ is the change in the Bank of England zero-coupon-curve at maturity m relative to the previous day. The coefficients of interest are the estimated $b^{(m)}$, which capture the effect of the supply shock on the term structure.

The responses to an unexpected standard deviation increase in the (non-maturity-specific) supply of government bonds are displayed in Figure 4. The blue line shows that an increase in the supply of bonds raises nominal interest rates between 1 and 1.5 basis points. Rates at longer maturities respond stronger, implying an increase in the slope of the yield curve. The effect persists in benchmark rates until the next week when the announced auction takes place (see Figure 5). The magnitude of the effect is similar to the responses to the demand shock, found by Droste et al. (2021) in the US and Lengyel and Giuliodori (2022) in Germany and Italy. This is in line with D’Amico and Seida (2020), who found that Treasury yields reacted similarly to the FED’s QE and QT announcements. Figure B7 in the Online Appendix presents similar IV results, where S_t is instrumented by the announced volume made on day t and the “surprise volume”. Our results are also robust to adding control variables, such as the short-term interest rate and inflation (implied by the model in Section 4.3) or weekday dummies.¹¹

To offer some intuition on the size of this effect, we provide a back-of-the-envelope calculation on a fiscal expansion announcement during the Covid-19 pandemic. On March 11, 2020, the UK government announced a fiscal stimulus package of £12bn.¹² We can translate this quantity into a high-frequency futures price surprise, using the regression results of Table 2. Then, we can obtain an estimate of the reaction of the term structure to an unexpected change in the supply of bonds of the size of the package with the results in Figure 4. These imply that an unexpected £12bn issuance of nominal bonds is associated with a $12 \times -0.15 = -1.8$ change in the bond futures price, a roughly 13 standard deviation event. This in turn would increase nominal yields by around 13 – 19 basis points. While this is a huge out-of-sample exercise, actual changes in long-term yields on the announce-

¹¹These results are available upon request.

¹²See: <https://www.gov.uk/government/speeches/budget-speech-2020>.

ment day were in the ballpark, between 4-14 basis points. It is important to note, however, that our calculation assumes that the announced package is fully unanticipated and financed entirely by new debt issuance. In reality, the announcement was at least partially anticipated by the press, implying that some of the effects have already been priced in before the announcement.¹³

What could be the reason behind the reaction of the yield curve? Ang, Bekaert and Wei (2008) finds that about 80% of the variations in nominal yields are attributable to changes in expected inflation and the inflation risk premium. The sum of the two is called the inflation compensation, the additional return investors require for being exposed to inflation. To assess if the reactions in nominal yields are due to a change in the inflation compensation, we regress S_t onto the *real* zero-coupon-curve of the Bank of England. The real term structure is constructed using inflation-linked bonds and is available for maturities over 25 months. The spread between a (comparable maturity) nominal and inflation-linked bond is called the breakeven inflation rate. This is a market-based measure of the inflation compensation. Figure 4 shows the response of real rates in red, and the response of breakeven rates in grey. Real rates react with increases of 1-1.2 basis points. This implies moderate, 0-4 basis points increases in breakeven rates and inflation compensation. Inflation swap rates, a different market-based measure of inflation compensation, show similar responses.¹⁴

These results suggest that the reason behind the reaction of nominal yields to the supply shock is not a change in investors' inflation outlook. Therefore, to get a better understanding of the transmission of the shock, we break down nominal yields into their components in the next section and analyse how each component reacts to the shock.

¹³See: <https://www.reuters.com/article/uk-britain-sterling-close-idUKKBN20W2IV>.

¹⁴See Figure B8 in the Online Appendix. An inflation swap contract exchanges a fixed rate against the realized average inflation rate at maturity. It is a market-based measure of the inflation compensation, which is less affected by market liquidity conditions (ECB (2018)).

4.2 Supply effects on expected short rates and risk premia

According to the expectations hypothesis, the strong response of long-term rates could be the result of either higher expected future short rates or higher risk premia. Using quarterly data and recursive identification, Dai and Philippon (2005) found risk premia to account for one third of the reaction of long-term rates to a shock to the fiscal deficit. Laubach (2011), at the same frequency, found that fiscal deficits mostly affect the short rate and inflation, with small movement in risk premia. Similar investigations in the empirical monetary policy literature suggests that monetary policy shocks primarily influence expected short rates, with some effect on term premia at longer horizons (Hanson and Stein (2015), Abrahams et al. (2016), Nakamura and Steinsson (2018)). To get a better understanding of why government debt issuance affects interest rates, we decompose yields into the average expected nominal short rate and the nominal term premium. Then, to shed light on the role of inflation, we further decompose the nominal short rate into the real short rate and expected inflation, and the nominal term premium into real term premium and inflation risk premium. We assess how each term is affected by the supply shock.

4.2.1 Decomposing nominal and real yields

We use an affine term structure model (ATSM) to jointly price nominal and inflation-linked bonds, following Abrahams et al. (2016). Several papers conducted this type of analysis in the context of monetary policy and analysed how high-frequency monetary policy surprises affect the expectations and risk premium components of yields (see Abrahams et al. (2016), Nakamura and Steinsson (2018), Tillmann (2020) and Kaminska, Mumtaz and Šustek (2021)). This paper, on the other hand, traces the effects of high-frequency debt supply shocks.

The model assumes that bond yields and the market price of risks are affine functions of the state variables, which are assumed to be observable. Hence, the log prices of a nominal ($P_t^{(\tau)}$) and an inflation-linked ($P_{t,R}^{(\tau)}$) zero-coupon risk-free bonds with remaining time to

maturity τ follows:

$$\log P_t^{(\tau)} = A_\tau + B'_\tau X_t \qquad \log P_{t,R}^{(\tau)} = A_{\tau,R} + B'_{\tau,R} X_t$$

under the pricing measure, where X_t is the vector of pricing factors, assumed to follow an autoregression. Bond prices and yields are related through

$$y_t^{(\tau)} = -\frac{\log P_t^{(\tau)}}{n} \qquad y_{t,R}^{(\tau)} = -\frac{\log P_{t,R}^{(\tau)}}{n}$$

By imposing no arbitrage, expressions for the pricing coefficients A . and B . can be obtained, where the pricing coefficients are non-linear, recursive functions of the parameters driving the factors, the short rate, inflation, and the risk prices. For details of the model and the estimation see Section B.1 in the Online Appendix and Abrahams et al. (2016).

A τ -period nominal bond yield can be decomposed into the average expected nominal short rate over the next τ periods and the nominal term premium $\text{TP}_t^{(\tau)}$. More explicitly:

$$y_t^{(\tau)} = \frac{1}{\tau} \sum_{i=0}^{\tau} E_t r_{t+i} + \text{TP}_t^{(\tau)} \tag{3}$$

This can be further decomposed into the average expected real short rate, the average expected inflation, real term premium $\text{TP}_{t,R}^{(\tau)}$ and inflation risk premium $\text{IRP}_t^{(\tau)}$:

$$y_t^{(\tau)} = \frac{1}{\tau} \sum_{i=0}^{\tau} E_t (r_{t+i,R} + \pi_{t+i}) + \text{TP}_{t,R}^{(\tau)} + \text{IRP}_t^{(\tau)} \tag{4}$$

The interpretation of $\text{TP}_{t,R}^{(\tau)}$ is the compensation investors require today to hold (real) interest rate risk for the next τ periods, while the interpretation of $\text{IRP}_t^{(\tau)}$ is the compensation investors require to hold inflation risk for the next τ periods.

The elements of Equations (3) and (4) can be obtained as the following. Setting the price of risk parameters to zero, one can obtain the risk-adjusted counterparts of the pricing

recursion coefficients \tilde{A} and \tilde{B} . Bond yields calculated with these coefficients are interpreted as the time t expectation of average future short rates over the next τ periods. This would be the prevailing yield if all investors were risk neutral. The difference between the risk-adjusted expected nominal and the risk-adjusted expected real short rate is the average expected future inflation over the next τ periods. The nominal (real) term premium can be obtained by subtracting the nominal (real) expected short rate from the fitted yield. The inflation risk premium is obtained as the difference between the fitted breakeven inflation and the inflation expectation.

4.2.2 Reaction of yield components

Which components account for the strong response to the supply shock? To answer this question, we regress the supply shock on each component obtained above. First, we look at the response of expected nominal short-term rates and the nominal term premium. Then, expected real short-term rates, expected inflation, the real term premium, and the inflation risk premium.

The top panel of Figure 6 shows the reaction of the nominal term premium and expected nominal short rates to the supply shock. The response of yields is given by the sum of the two bars. The figure shows that a standard deviation increase in bond supply raises 10-year yields by about 1.4 basis points. Around 1-basis point increase comes from the reaction of the term premium, and 0.4 basis point increase comes from higher expected short rates. Next, to shed more light on the role of inflation, we look at the average real short rate, the expected average inflation, the real term premium, and the inflation risk premium. The reaction of each component to the supply shock is displayed in the bottom panel of Figure 6. It shows that most of the reaction of the nominal term premium is due to the response of the real term premium. Interestingly, inflation expectations are unaffected, while the inflation risk premium displays a modest increase. In other words, additional government debt issuance raises the compensation investors require to hold interest rate and inflation

risks. Phillot (2021) finds that supply shocks raise breakeven inflation rates. Our results demonstrate that this reaction is not due to higher inflation expectations, but due to higher inflation risk premium.

Apart from duration and inflation risks, the behaviour of credit risk is also of interest. Excessive debt issuance by the government can lead to debt repayment issues, raising the credit risk of the government. When markets price higher credit risk, it is reflected in higher credit default swap (CDS) rates. CDS rates can be interpreted as the insurance premium paid to insure against the default of the bond issuing entity. We assess if this is the case by regressing the high-frequency supply shock on daily CDS rate changes written on UK Treasuries from Refinitiv. Table 4 shows that increased bond supply does not have a positive effect on CDS rates, implying no increase in the credit risk of the UK government priced in CDS rates.

Next, we look at the behaviour of corporate bond yields and corporate yield spreads at various maturity buckets from Refinitiv. We found spillover effects of the government bond supply shock into corporate bond markets, consistent with the effects of demand shock, found in Droste et al. (2021) and Lengyel and Giuliadori (2022). Corporate bonds react strongly, with yields increasing between 0.7 and 1.5 basis points in all maturity segments.¹⁵ Figure 7 shows the reaction of AAA, AA, A and BBB rated corporate bonds, categorized into maturity buckets of 3-5, 5-7, 7-10, 10-15 and 15+ years to maturity. Corporate bonds with better rating react more to changes in the supply of government bonds. In terms of remaining maturity, bonds that mature between 7 and 15 years have the strongest reaction to the supply shock, which is the same segment where Treasury bonds respond the most. The reaction of BBB-AAA spreads is reported in Table 5. The regression coefficients are insignificant in all maturity buckets. These results on CDS and corporate bonds suggest that repricing of credit risk is unlikely to be a transmission channel of the supply shock on interest rates.

¹⁵At the same time, stock prices increase, as the FTSE 100 index gains 0.166 (0.067) percent.

Overall, our results imply that an important transmission channel of the effects of government debt issuance is the “risk-taking channel” (Borio and Zhu (2012)). The bond supply shock affects markets’ perception of duration and inflation risks, and this changes the equilibrium price of these risks. This is similar to the transmission of monetary policy shocks (Hanson and Stein (2015), Abrahams et al. (2016)). However, one interesting difference is that monetary policy shocks co-move negatively with the inflation risk premium, while bond supply shocks co-move positively. This is because a positive monetary policy shock is contractionary and disinflationary, while a positive bond supply shock indicates a more expansionary and inflationary stance of fiscal policy.

4.2.3 Adjusting for inflation-linked bond illiquidity

The liquidity of inflation-linked bonds and nominal bonds tend to differ, and the relative liquidity is systematically priced (Pflueger and Viceira (2016)). If the inflation-linked bond relative liquidity effect is priced, it can contaminate the response of inflation-related indicators, as they are derived using the spread between nominal and inflation-linked bonds. According to Joyce, Lildholdt and Sorensen (2010), the liquidity premium is unlikely to have had a big influence on UK yield curve dynamics over the period of 1992-2008 and ignores it (together with Evans (1998), Risa (2001), Abrahams et al. (2016) and others). Others quantified it to be low and decreasing over time but jumping higher in crisis periods (see Kaminska, Liu, Relleen and Vangelista (2018), Bekaert and Ermolov (2021)).

We attempt to account for this effect as a robustness exercise, by expanding the state space X_t , by including a liquidity factor L_t , as in Abrahams et al. (2016). Working at the daily frequency substantially reduces the number of potential liquidity proxies to use. We follow Pflueger and Viceira (2016), Kaminska et al. (2018) and Bekaert and Ermolov (2021) and use the 5-year inflation-swap spread ISS_t^{5Y} as our liquidity proxy.¹⁶ We standardize it

¹⁶This is constructed as the difference between the inflation swap rate ISR_t^{5Y} and the breakeven inflation rate $ISS_t^{5Y} = ISR_t^{5Y} - (y_t^{5Y} - y_{t,R}^{5Y})$. Liquidity premium in inflation swap rates is considered to be negligible, therefore, the rates only represent expected inflation and inflation risk premium (see ECB (2018) and Bekaert and Ermolov (2021)). In the absence of liquidity risk premium in breakeven rates, the spread between the

and add to each observation the negative of the minimum of the series to ensure the positivity of the index. Then, before calculating inflation-related variables in the model, we subtract the effect associated with L_t from yields. The availability of inflation swap data is from the 29th of June 2007 to the 31st of December 2019, and it is displayed in the bottom right panel of Figure B9 in the Online Appendix. It shows a steep increase during the financial crisis and elevated levels during the European debt crisis.

We estimate the effect of the supply shock on the new series and Figure 8 reports the results. Once we account for liquidity effects, almost all of the reaction of yields is attributed to movements in the nominal term premium. The reaction of the real term premium dominates with a minor effect on inflation risk premium. The expectation component only reacts at short horizons. The reason for the muted response of the expectations variables is that they are obtained by setting the prices of risks to zero in the ATSM model when calculating the risk-neutral yields and breakevens. In this exercise, we quantified the price of an additional risk factor: liquidity risk.

4.3 A term structure model of nominal and real bonds with supply effects

To summarize our empirical results, the additional supply of nominal bonds raises nominal and real yields, mostly due to increases in risk premia. When investors are faced with a higher supply of government bonds, they require higher compensation for holding interest rate and inflation risks. In the next section, we examine this effect through the lens of a theoretical framework. The aim of the model is to illustrate the mechanism rather than provide a complete structural explanation of the mechanism.

The model builds on the Greenwood and Vayanos (2014) version of the Vayanos and Vila (2021) model. This section provides the intuition, while the complete model is spelled out in the Appendix. We extend the Greenwood and Vayanos (2014) model with inflation risk and

swap rate and the breakeven rate should be zero.

include two types of bonds: a continuum of nominal bonds and a continuum of inflation-linked bonds. These are supplied by the government in a price inelastic manner. Marginal investors in the model are short-lived risk averse arbitrageurs. They absorb shocks to the supply of bonds and ensure that the term structure of interest rates is smooth and arbitrage free. Arbitrageurs require additional returns for holding the bonds compared to the risk-free short rate, as unexpected shocks can result in the bonds underperforming relative to the short rate.

To the best of our knowledge, there are two papers with similar setups. Saúl (2012) derives the breakeven inflation rate in a model with preferred-habitat investors, as in Vayanos and Vila (2021). Bond prices are determined through the interaction between arbitrageurs and preferred-habitat investors. Preferred-habitat demand for bonds is non-stochastic. This is in contrast to our setup of exogenous bond supply, which is subject to shocks. Our focus is specifically on this additional stochastic risk factor, and we analyse how equilibrium bond prices are affected by this supply risk. Diez de los Rios (2020) constructs a discrete-time version of the Greenwood and Vayanos (2014) model, with both nominal and real bonds in fixed supply. Inflation is endogenous, determined by a Taylor-rule type equation. The focus is on demonstrating how an increase in the bond supply can lead to higher inflation. Our continuous time model has exogenous inflation, to illustrate how additional bond issuance transmits to yields, by altering the price of inflation and duration risks.

Bond yields in the model in Section 6 react positively to the supply shock, with the effect stronger at long horizons, just like in our empirical results. The supply shock raises both the duration risk premium and the inflation risk premium. Furthermore, we also find a positive relationship between the supply shock and the breakeven inflation rate. The intuition is the following. In equilibrium, risk prices are increasing in the sensitivity of investors' portfolios to the risk factors. Expected excess returns of bonds are, therefore, also increasing in this sensitivity. As the outstanding amount of nominal bonds increases, the amount of duration and inflation risks borne by arbitrageurs also increases. The higher

sensitivity of their portfolio to the risk factors raises the price of these factors and the risk premiums. This in turn raises the term premium and the inflation risk premium, raising bond yields. As inflation-linked bonds are free from inflation risk, their yield does not rise as much as nominal yields, resulting in higher breakeven inflation rates.

This mechanism is linked to the limited risk bearing capacity of investors. When risk aversion is high in the model, yields and risk prices become more responsive to the supply shock. In the next section, we test this prediction empirically and explore further state dependencies in the effects of the shock.

5 Non - linearities

The model in Section 4.3 suggests that the response of yields to the supply shock is higher in states when risk aversion is high. This is the same result found in Greenwood and Vayanos (2014) and Vayanos and Vila (2021). We test this non-linearity empirically. He and Krishnamurthy (2013) suggest that risk aversion is higher in a crisis and periods of financial market stress. Therefore, we use a country-level composite indicator of systemic stress in the financial system: the CISS index of Hollo, Kremer and Lo Duca (2012). We construct a financial stress indicator variable I_t , that is equal to one when the CISS index is above its 75th percentile and zero otherwise.¹⁷ The indicator is displayed in Figure 9. We estimate the state-dependent version of Equation 2:

$$\Delta R_t^{(m)} = I_t[a_1^{(m)} + b_1^{(m)}S_t] + (1 - I_t)[a_0^{(m)} + b_0^{(m)}S_t] + \varepsilon_t^{(m)} \quad (5)$$

The findings are reported in Figure 10. In normal times a standard deviation supply shock raises nominal yields up to 1.2 basis points. On the other hand, during market stress periods the reaction is as high as 1.9 basis points at long maturities. The reason behind this is that in turbulent times, the term premium becomes much more responsive to the supply shock.

¹⁷Our results are robust to a wide range of this threshold and they are available upon request.

Long-term bonds are more sensitive to risks and market stress periods are characterized by a steep increase in risk prices. This is consistent with the findings on Treasury demand shocks, which are documented to have stronger effects in times of market stress (Droste et al. (2021), Lengyel and Giuliadori (2022)).

The preferred-habitat theory of bond yields by Vayanos and Vila (2021) and Droste et al. (2021) predicts that when risk aversion is low, demand shocks affect interest rates similarly across the maturity space. However, when investors' risk aversion is high, a shock at a specific maturity segment has more concentrated effects at nearby maturities. In other words, the shock has a localized effect. We test this prediction, by first restricting the announcements sample to only include announcements of short- and medium-maturity bonds (0-15 years according to the DMOs' classification) and estimating Equation (5). Then, we restrict the announcements sample to only include announcements of long-maturity bonds (15+ years) and estimate again Equation (5). The results are reported in Figure 11. It shows that when markets are calm, the effect of the shock is similar across maturities. However, when markets are under stress, short- and medium-maturity bond announcements have a larger effect on the short end of the yield curve, while long-maturity bond announcements have a larger effect on the long end of the curve. This localization effect of bond supply changes is in line with the findings of McLaren et al. (2014) on QE programs in the UK, and Droste et al. (2021) in the US.

Next, we analyse the effects of the supply shock in periods when the monetary policy rate is at the effective lower bound.¹⁸ We construct a dummy variable that takes the value one when the Bank of England policy rate was below 0.5%, and zero otherwise. Figure 9 shows the time series of the variable. The estimation results are reported in Figure 12. Short-maturity rates show a weaker response to the supply shock when they are constrained by the lower bound. At the same time, long-maturity rates react more stronger. The main reason is that average expected short rates do not react as much as in normal periods. Risk

¹⁸We did not find differences in the effect of the supply shock based on the sign of the shock.

premium on the other hand, is more responsive: around 85% of the reaction is due to these components. It is important to note, that ELB periods were often characterized by elevated market levels, which might drive these results.

Overall, in the sub-sample that is characterized by market stress and the ELB, the term premium and the inflation risk premium are more responsive, and short rates are less responsive to supply shocks. As long-term bonds are more sensitive to risk premia, the slope of the yield curve becomes steeper after an increase in government debt issuance. Furthermore, in states of high risk aversion, the localization effect of supply shocks can be observed.

6 Conclusion

In this paper, we identify government bond supply shocks by recording intraday price movements around government bond auction volume announcements. We apply this high-frequency identification to the UK Debt Management Offices announcements and study how additional debt issuance affects the term structure of interest rates. We find that a standard deviation bond supply shock increases nominal yields by 1-1.5 basis points. Real rates rise by 1-1.2 basis points, implying a modest reaction of the inflation compensation.

To study the transmission of the shock, we decompose yields into expected short rates and risk premia. We find that the shock mostly affects risk premia components, with smaller effects on future expected average short-term rates and no effect on expected inflation. Both the real term premium and the inflation risk premium react positively to higher bond supply. We reconcile these results in an equilibrium term structure model, where risk averse investors absorb shocks to the supply of nominal bonds. Their equilibrium portfolio becomes more sensitive to duration and inflation risks, driving up the price of these risk factors. This in turn raises risk premia and yields. As inflation-linked bonds are unaffected by inflation risk, the breakeven inflation rate goes up.

The model also predicts that when risk aversion is high, the effects of the supply shock are more pronounced. In line with this, we find empirically that yields react stronger to the supply shock during times of financial market stress and at the effective lower bound. The increase is driven by higher risk premia, consistent with the equilibrium model. Furthermore, we find evidence for the localization of the effect during market stress periods.

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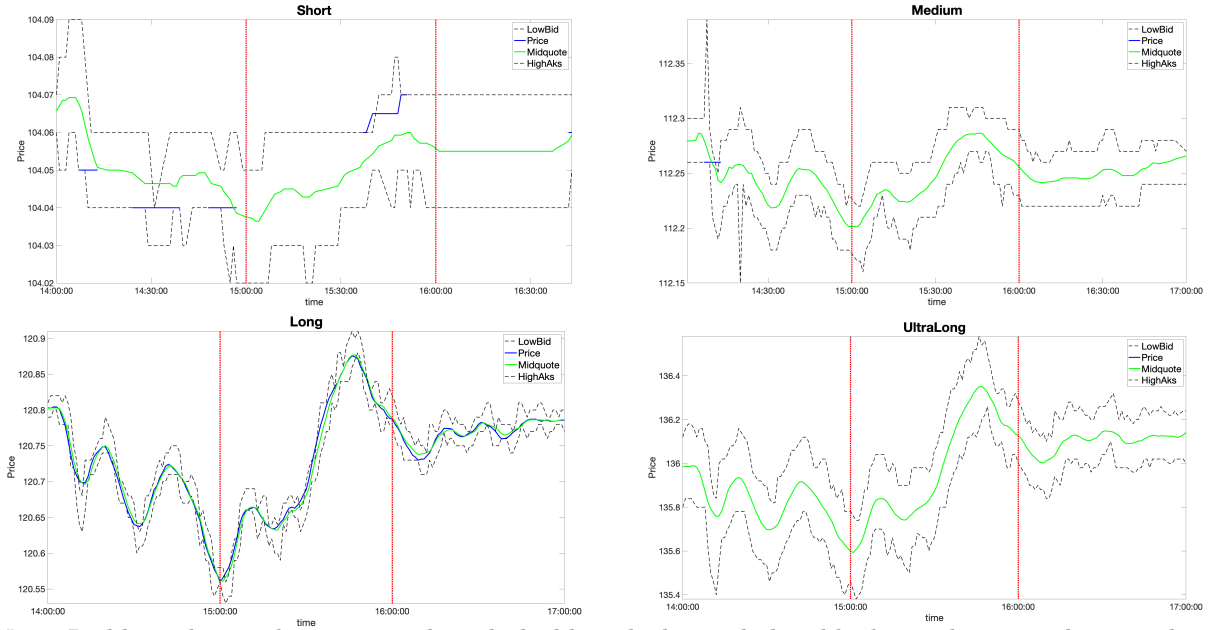
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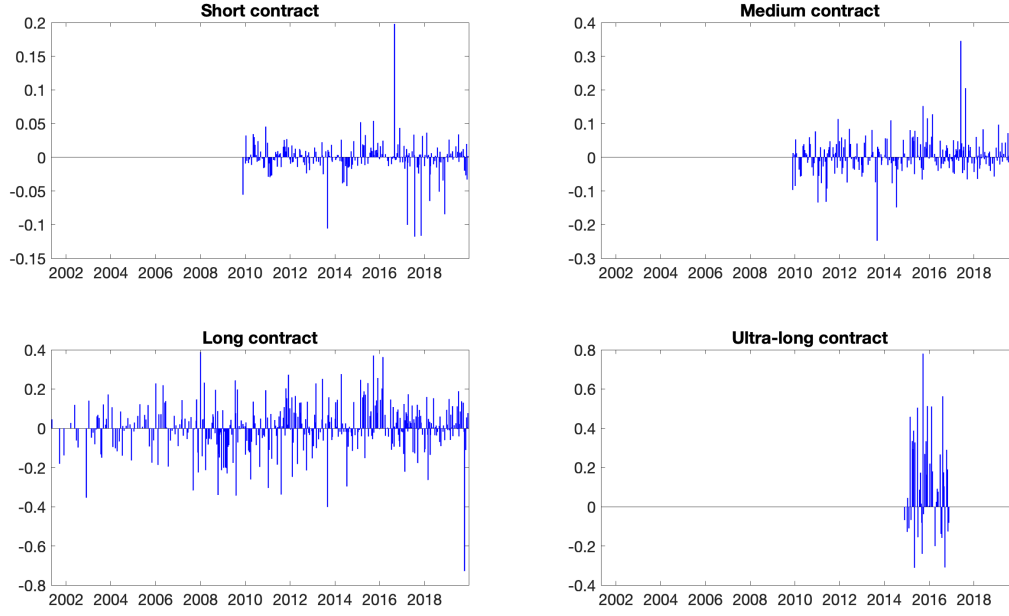
Figures

Figure 1: Futures price movement around the event window on the 21th of April 2015



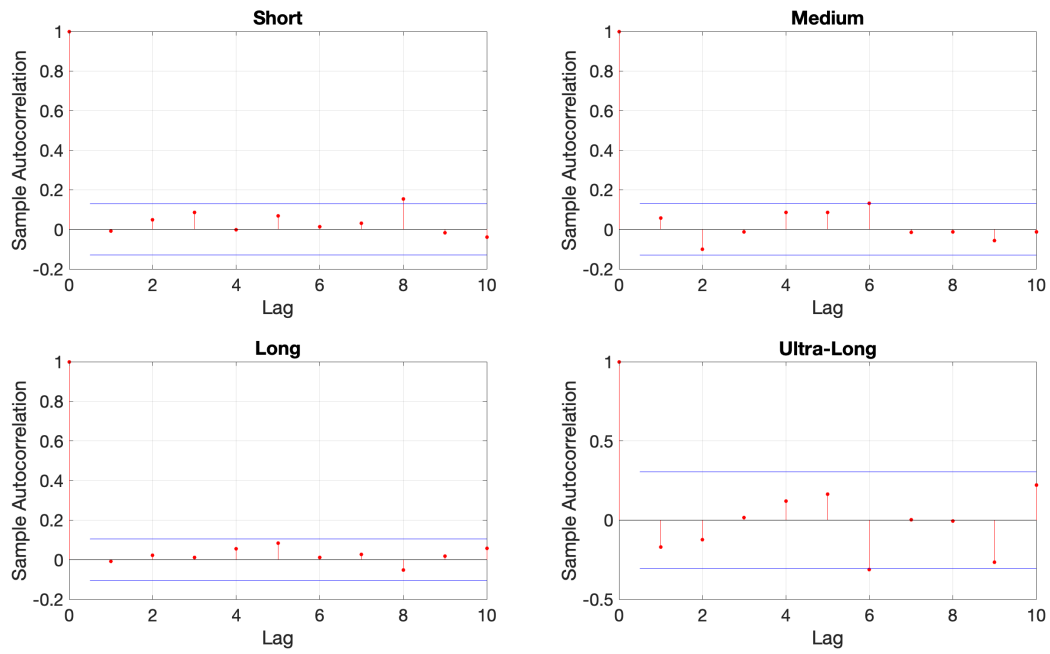
Note: Red lines denote the event window, dashed line the lowest bid and highest ask price. The green line is the 5-minutes moving average of the midquote, blue line is 3-minutes moving average of the recorded traded price, if actual trading took place. Announcements are made at 15:30.

Figure 2: Time series of the supply shocks



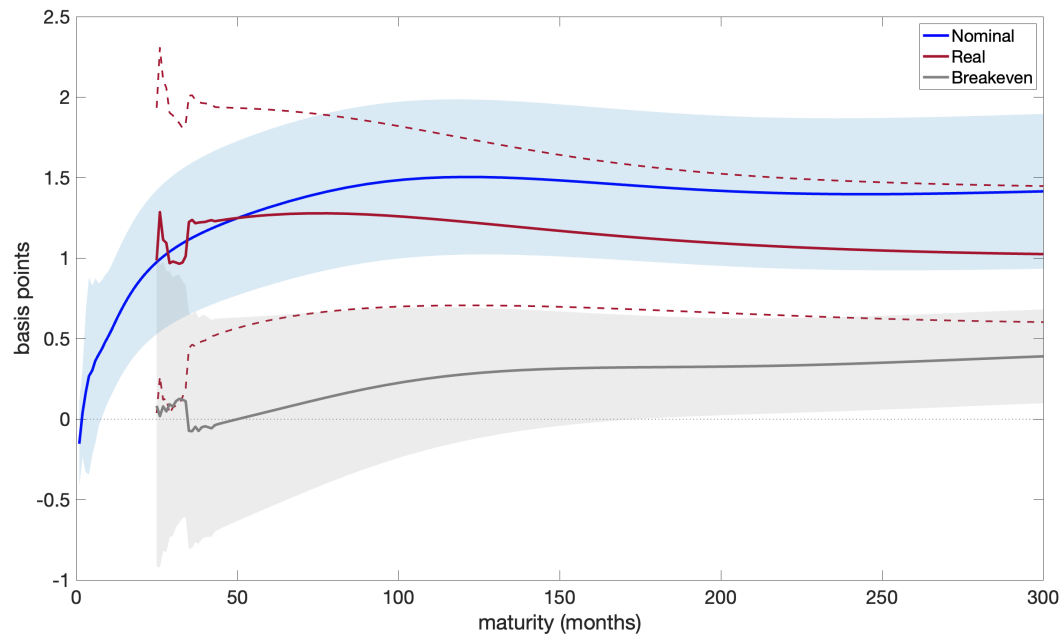
Note: Time series of S_t^{Short} (top left), S_t^{Medium} (top right), S_t^{Long} (bottom left), $S_t^{Ultra-long}$ (bottom right).

Figure 3: Sample autocorrelation function of the shock series



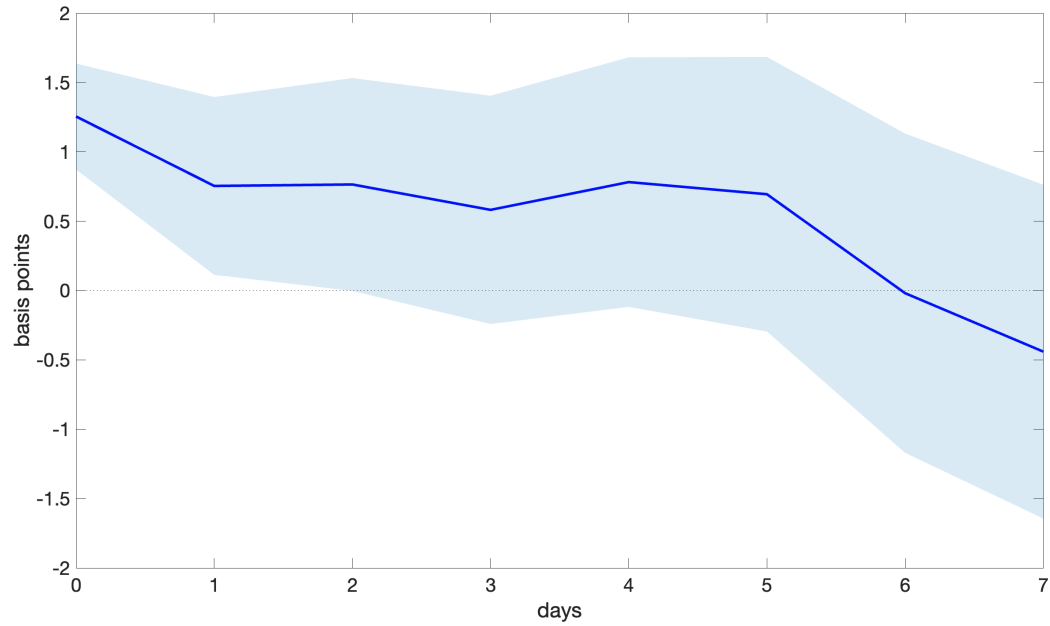
Note: Sample autocorrelation function of the shock series up to ten lags. Blue lines represent two standard errors confidence intervals.

Figure 4: Reactions of the term structure of nominal, real, and breakeven inflation to the bond supply shock



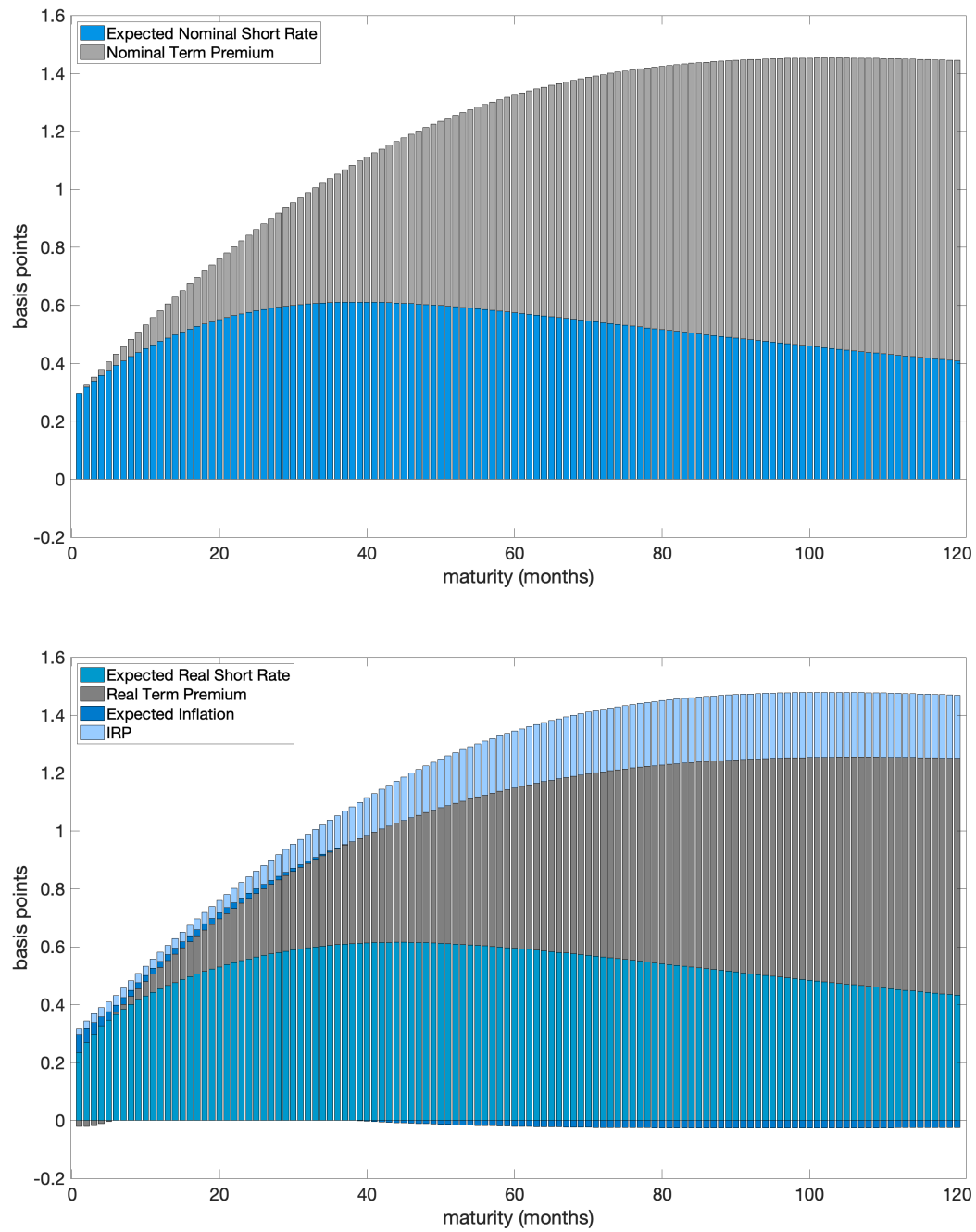
Note: Estimated $b^{(m)}$ coefficients from equation (2). Dashed lines and shaded areas are 95% (Newey-West, 10 lags) confidence intervals. Sample: 31.03.2001-31.12.2019.

Figure 5: Impulse response of the 10-year benchmark nominal rate



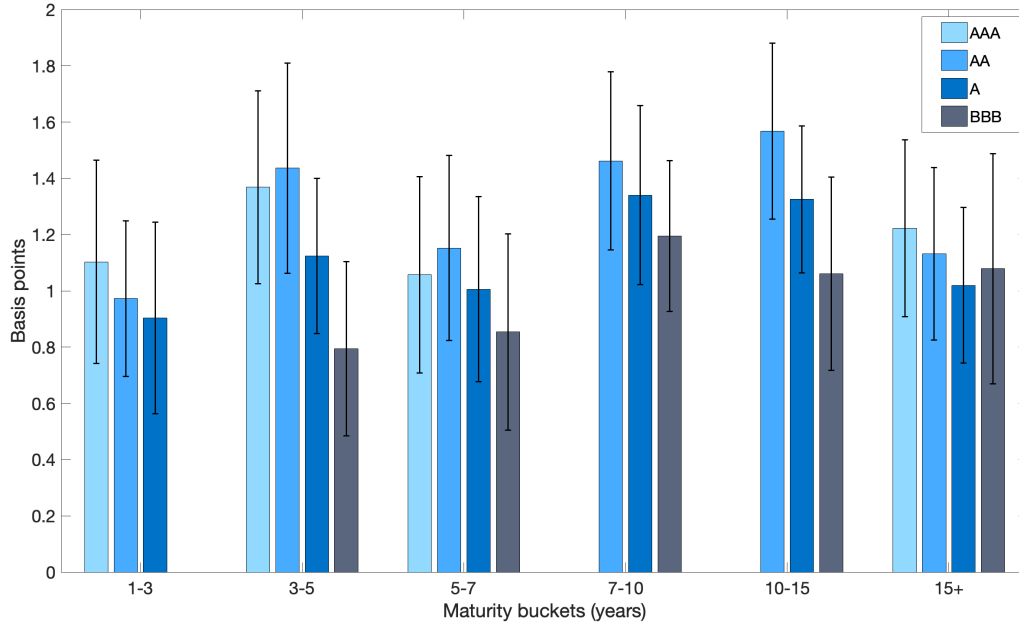
Note: Impulse response of 10-year benchmark rates from long difference regressions, where the dependent variable is $Y_{t+h} - Y_{t-1}$ and h are days. Shaded area is 90% Newey-West (10 lags) confidence interval. Sample: 31.03.2001-31.12.2019.

Figure 6: Reactions of the expected nominal short rates and the nominal term premium to the supply shock



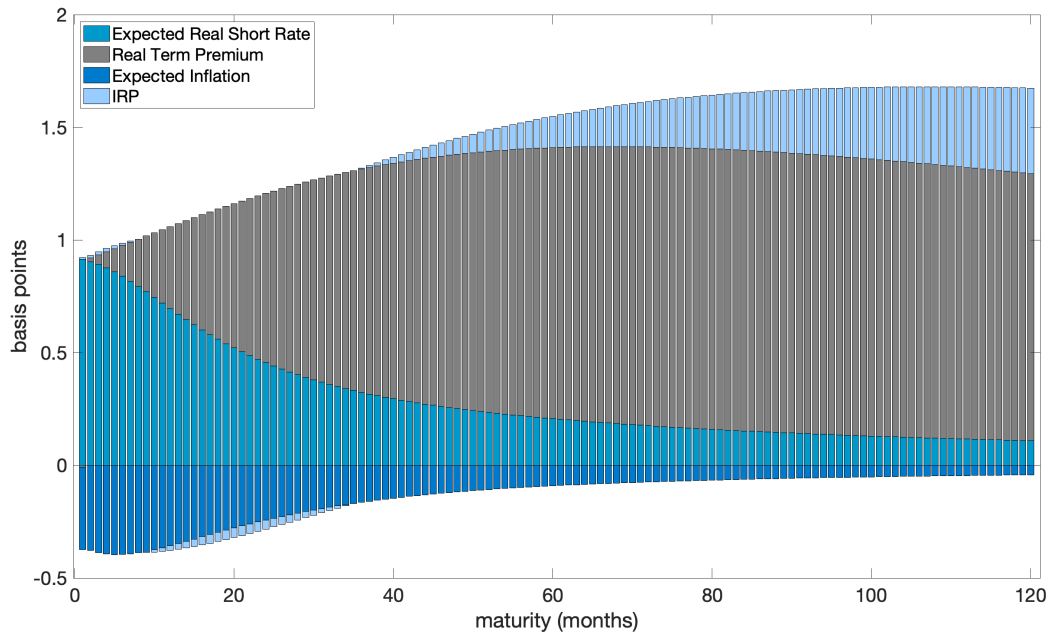
Note: Bars are the estimated $b^{(m)}$ coefficients from equation (2). Top panel: the dependent variables are the average expected nominal short rates and the nominal term premium. Bottom panel: the dependent variable is the average expected real short rate, expected inflation, the real term premium and inflation risk premium.

Figure 7: Reactions of corporate bond indices to the supply shock



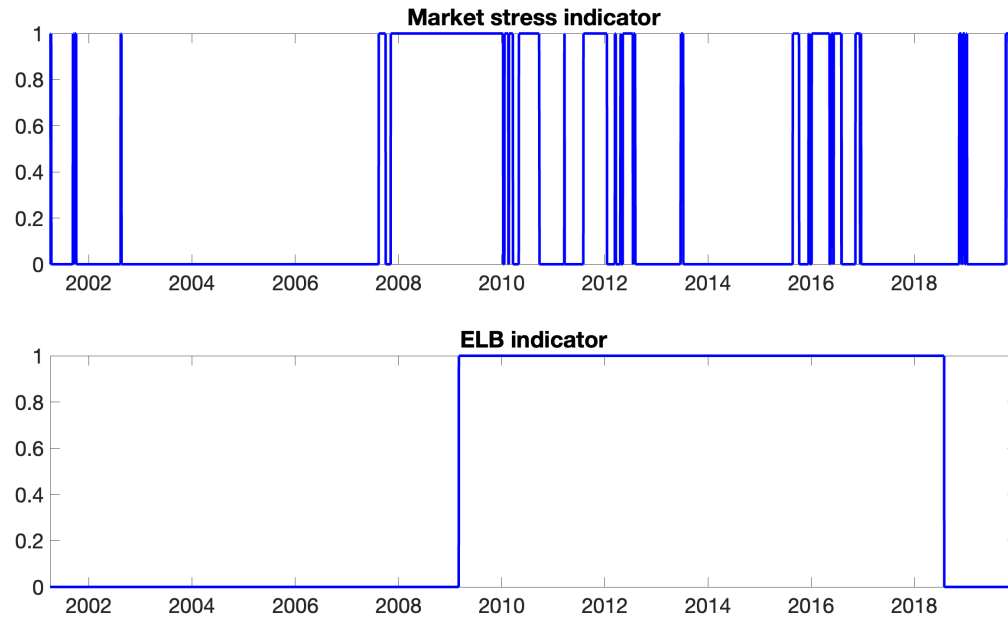
Note: Bars are the estimated $b^{(m)}$ coefficients from equation (2), when the dependent variables are AAA, AA, A and BBB rated corporate indices, with remaining maturities between 1-3, 3-5, 5-7, 7-10, 10-15 and 15+ years, compiled by Refinitiv. Error bands are 95% (Newey-West, 10 lags) confidence intervals.

Figure 8: 10-year breakeven inflation rate decomposition at daily frequency to the supply shock, adjusted for liquidity effects



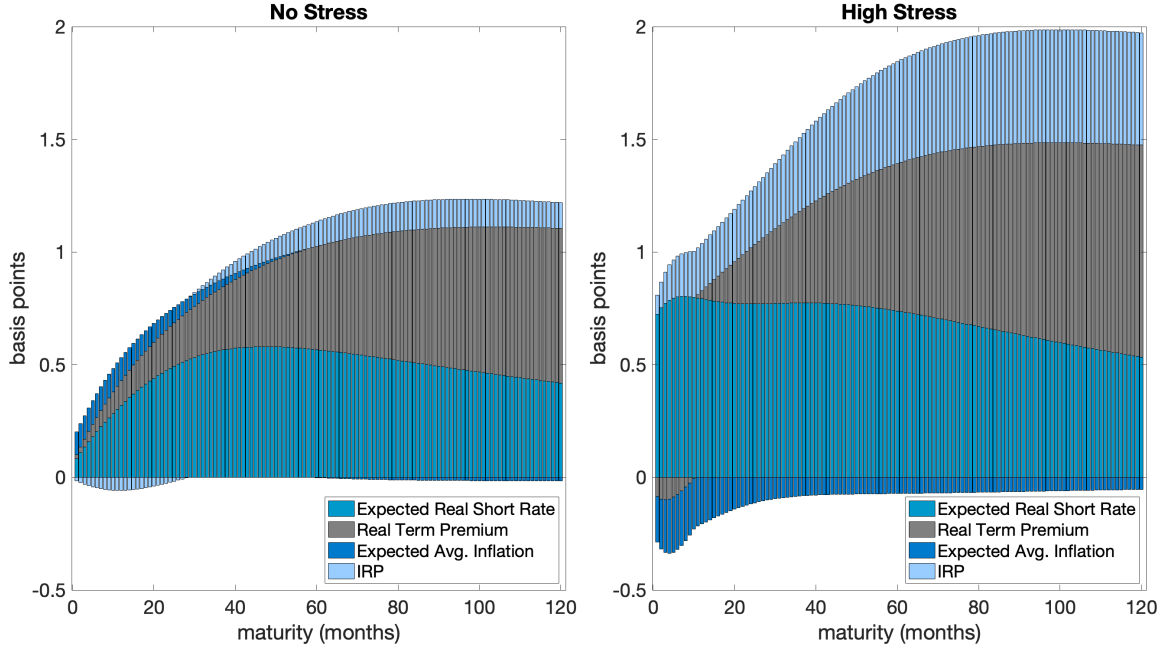
Note: Bars are the estimated $b^{(m)}$ coefficients from equation (2), where the dependent variables are the average expected real short rates, the expected inflation, the real term premium, and the inflation risk premium obtained via the ATSM. The state space of pricing factors in the ATSM is extended with a liquidity proxy: the inflation swap, breakeven inflation rate spread.

Figure 9: Times series of state indicators for the non-linear estimation



Note: Time series of the state indicator variables. The financial stress indicator takes the value one when the CISS index is above its 75th percentile. The ELB periods indicator takes the value one when the Bank of England bank rate is below 0.5%.

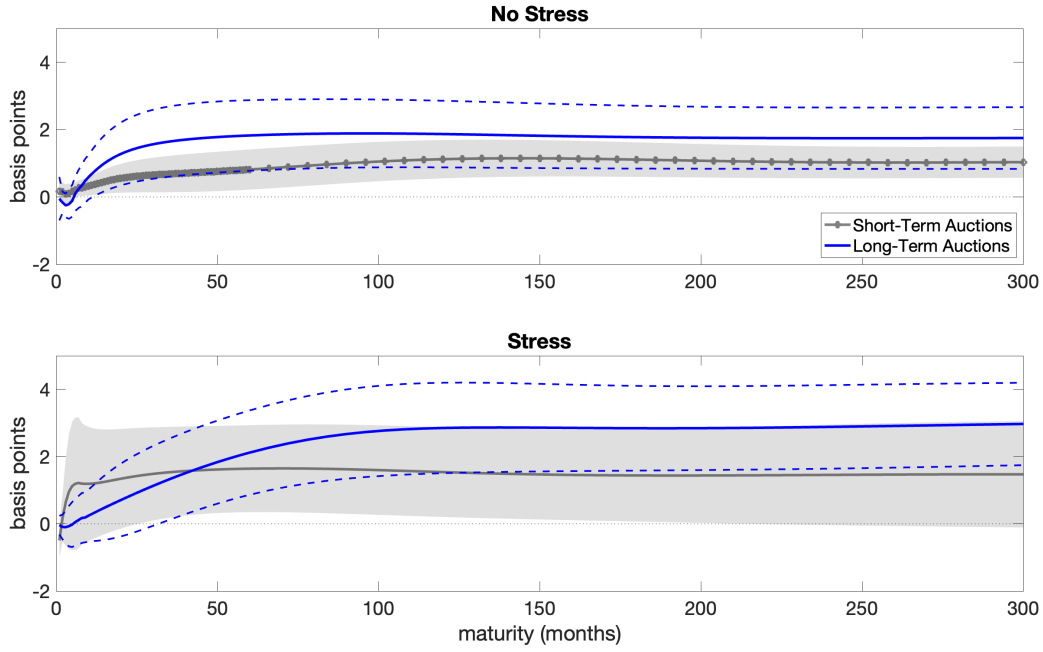
Figure 10: Reactions of yield components in normal times and in high-stress periods



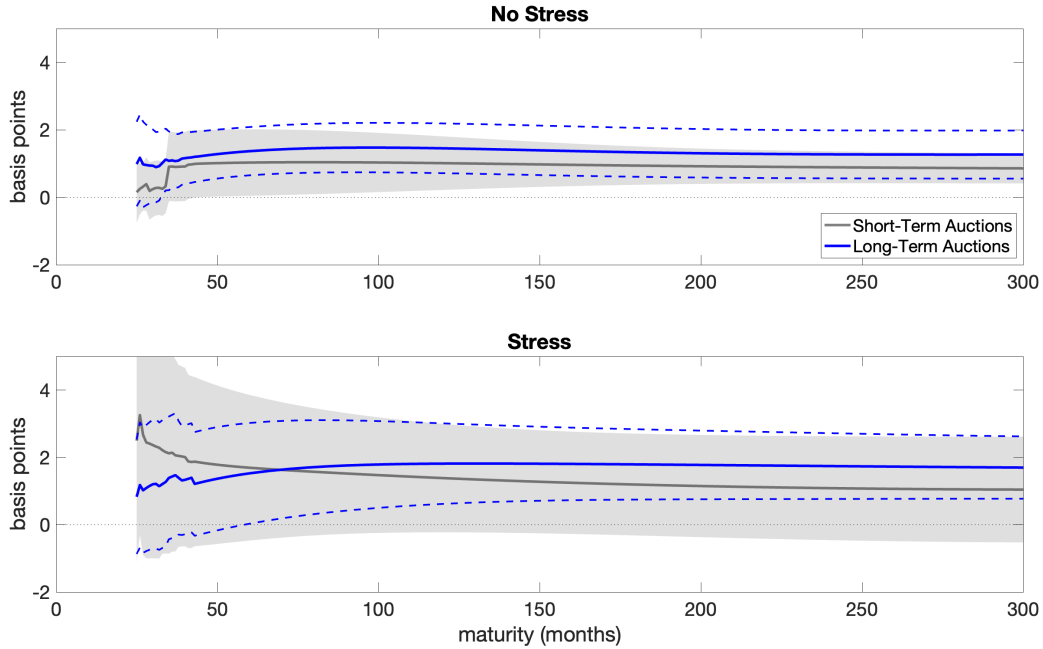
Note: Bars are estimated $b_1^{(m)}$ and $b_0^{(m)}$ coefficients from Equation 5 on yield components obtained via the ATSM. I_t indicates periods with the CISS index above its 75th percentile. Dependent variables are the average expected real short rate, the expected inflation, the real term premium, and the inflation risk premium.

Figure 11: Localization of the effect of the supply shock during market stress

Panel A: Nominal yields

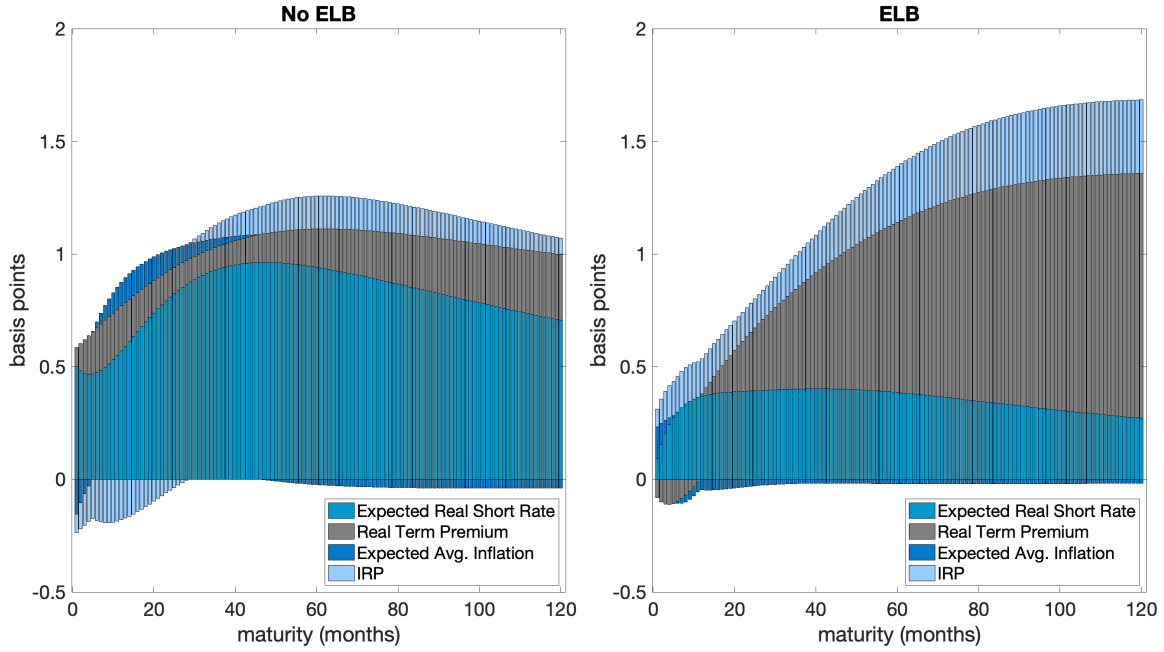


Panel B: Real yields



Note: Estimated $b_1^{(m)}$ and $b_0^{(m)}$ coefficients from Equation 5. Panel A. shows the result of nominal yields, Panel B. shows the results of real yields. The top chart of each panel shows the results during normal times, the bottom chart shows the results in market stress, characterized by the CISS index over its 75th percentile. Grey lines show the results when the announcements sample is restricted to the DMOs' short- and medium-maturity bucket (0-15 years), blue lines show the results when the announcements sample is restricted to the DMOs' long-maturity bucket (15+ years). Dashed lines and shaded areas are 95% (Newey-West, 10 lags) confidence intervals.

Figure 12: Reactions of yield components during normal times and in ELB periods



Note: Bars are estimated $b_1^{(m)}$ and $b_0^{(m)}$ coefficients from Equation 5 on yield components obtained via the ATSM. I_t indicates periods with the BoE Bank Rate below 0.5%. Dependent variables are the average expected real short rate, the expected inflation, the real term premium, and the inflation risk premium.

Tables

Table 1: Descriptive statistics of the high-frequency shocks

	Sample	N	Mean	Std.	Correlations				
					$S_t^{(Short)}$	$S_t^{(Med.)}$	$S_t^{(Long)}$	$S_t^{(U.long)}$	S_t
$S_t^{(Short)}$	24.11.09-31.12.19	238	-0.001	0.026					
$S_t^{(Med.)}$	24.11.09-31.12.19	238	0.004	0.053	0.271				
$S_t^{(Long)}$	15.05.01-31.12.19	360	0.001	0.125	0.293	0.949			
$S_t^{(U.long)}$	25.11.14-15.11.16	45	0.120	0.256	0.280	0.819	0.852		
S_t	15.05.01-18.02.20	360	0.001	0.132	0.311	0.959	0.999	0.853	

Table 3: Regression of the daily surprise on the intraday surprise S_t

	Daily surprises
Intraday surprise S_t	1.110
S.E.	(0.180)
P-value	0.000
R^2	0.111
N	345

Note: Dependent variable is the price surprise on announcement days with one-day event window. Independent variable is the intraday price surprise on announcement days with one-hour event window (S_t). Sample period: 15.05.2001-31.12.2019.

Table 2: Regression of the volume surprise at auctions on the high-frequency shock

	Intraday window	Daily window
Panel (A): Announced volume		
Volume (bn £)	0.002	0.003
S.E.	(0.006)	(0.019)
P-value	0.709	0.883
R^2	0.000	0.000
N	314	314
Panel (B): Surprise volume		
Surprise volume (bn £)	-0.150**	-0.082
S.E.	(0.059)	(0.202)
P-value	0.014	0.686
R^2	0.019	0.000
N	314	314

Note: Dependent variables are the high-frequency price movements in the event window on auction days. Left column: one-hour event window (S_t), right column: one-day event window. Panel (A): independent variable is the announcement volume. Panel (B) independent variable is the difference between the actual announced volume minus the implied average remaining auction size that is required to meet the DMOs' remit. Sample: 04.04.2006-31.12.2019.

Table 4: Reaction of credit default swaps to the Treasury supply shock

	2 years	5 years	10 years	30 years
Coeff.	-0.003	-0.004	-0.003	-0.003
S.E.	(0.004)	(0.003)	(0.002)	(0.002)
P-value	0.417	0.206	0.217	0.250

Note: Each column is a separate regression of credit default swaps written on 2-years, 5-years, 10-years and 30-years UK government bonds on the high-frequency supply shock, respectively. Sample period from 21.07.2008 to 31.12.2019.

Table 5: Reaction of BBB-AAA corporate bond spreads to the Treasury supply shock

	1-3 years	3-5 years	5-10 years	15+ years
Coeff.	1.229	-0.498	-0.239	0.011
S.E.	(1.399)	(0.412)	(0.189)	(0.149)
P-value	0.380	0.227	0.207	0.940

Note: Each column is a separate regression of UK corporate bond spreads with remaining maturities between 1-3 years, 3-5 years, 5-10 years, and 15+ years compiled by Refinitiv, on the high-frequency supply shock, respectively. Sample period from 31.03.2001 to 31.12.2019.

Appendix: An equilibrium term structure model of nominal and real yields with supply effects

A.1 The setup

The model is set in continuous time with two types of assets: nominal and inflation-linked (or real) zero-coupon bonds. These bonds have maturities τ in the interval $(0; T]$. An inflation-linked bond with maturity τ pays one unit of wealth at time $t + \tau$. Its time t price is denoted by $P_t^{R,(\tau)}$. A nominal bond with maturity τ pays one unit of currency at time $t + \tau$. Its time t price is denoted by $P_t^{N,(\tau)}$. The bond's spot yields are denoted by $y_t^{R,(\tau)}$ and $y_t^{N,(\tau)}$. They are related to the prices by:

$$y_t^{N,(\tau)} = -\frac{\log P_t^{N,(\tau)}}{\tau} \quad (\text{A.1a})$$

$$y_t^{R,(\tau)} = -\frac{\log P_t^{R,(\tau)}}{\tau} \quad (\text{A.1b})$$

The instantaneous risk-free real rate is denoted by r_t . It is defined as $\lim_{\tau \rightarrow 0} y_t^{R,(\tau)} = r_t$ and it follows the Ornstein-Uhlenbeck process

$$dr_t = \kappa_r(\bar{r} - r_t)dt + \sigma_r dB_{r,t} \quad (\text{A.2})$$

This rate can be interpreted as the return of a linear and instantaneously riskless production technology. Instantaneous inflation is also assumed to follow the Ornstein-Uhlenbeck process

$$d\pi_t = \kappa_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{\pi,t} \quad (\text{A.3})$$

where $\bar{r}, \kappa_r, \sigma_r, \bar{\pi}, \kappa_\pi, \sigma_\pi > 0$ are constants and $B_{r,t}$ and $B_{\pi,t}$ are independent Brownian motions. Parameters \bar{r} and $\bar{\pi}$ are the long-run means of the processes, κ_r and κ_π are the mean-reverting parameters. The volatility parameters are σ_r and σ_π .

Bonds are issued by a government and traded by arbitrageurs and other investors that are not modelled explicitly. Following Greenwood and Vayanos (2014), we treat the supply and demand of the government and other investors as price inelastic.

The amount of bonds supplied by the government, net of other investors' demand, is exogenous. The supply of nominal bonds $s_t^{N,(\tau)}$ is given by a one factor model, as in Greenwood and Vayanos (2014). For simplicity, inflation-linked bond supply $s_t^{R,(\tau)}$ is fixed.

$$s_t^{N,(\tau)} = \zeta^N(\tau) + \theta^N(\tau)\beta_t \quad (\text{A.4a})$$

$$s_t^{R,(\tau)} = \zeta^R(\tau) \quad (\text{A.4b})$$

The functions $\zeta^N(\tau)$, $\zeta^R(\tau)$, and $\theta^N(\tau)$ are deterministic functions of the maturity of the bonds. The variable β_t is a stochastic nominal bond supply factor that follows the Ornstein-Uhlenbeck process. Its long-run mean is zero, similar to our high-frequency futures price shock series.

$$d\beta_t = -\kappa_\beta \beta_t dt + \sigma_\beta dB_{\beta,t} \quad (\text{A.5})$$

The function $\zeta^N(\tau)$ gives the average supply of bonds at maturity τ , while $\theta^N(\tau)$ measures the sensitivity of the nominal bond supply to the supply factor β_t . We assume that $\theta^N(\tau)$ has the following properties:

Assumption 1. *The functions $\theta^N(\tau)$ satisfies*

- (i) $\int_0^T \theta^N(\tau) \geq 0$;
- (ii) *There exists $\tau^* \in [0; T)$ such that $\theta^N(\tau) < 0$ for $\tau < \tau^*$ and $\theta^N(\tau) > 0$ for $\tau > \tau^*$*

The first point of the assumption ensures that an increase in β_t does not decrease the total value of bonds supplied to arbitrageurs. The second point allows the possibility that after an increase in β_t the supply of some shorter maturity bonds can decrease, while the total supply of bonds does not decrease. These assumptions ensure that an increase in β_t makes arbitrageurs' equilibrium portfolios more sensitive to inflation and duration risks. We assume $B_{\beta,t}$ is independent of $B_{r,t}$ and $B_{\pi,t}$. Greenwood and Vayanos (2014) considers a case where $B_{\beta,t}$ is correlated with $B_{r,t}$ which is reasonable given their empirical measure of supply. In our case, assuming independence corresponds more our to high-frequency supply shock in Section 3.

A.2 Arbitrageurs

Arbitrageurs are assumed to be mean-variance maximizers of their real wealth. They select their portfolio by solving:

$$\max_{\{x_t^{N,(\tau)}, x_t^{R,(\tau)}\}_{\tau \in (0, T]}} E_t[dW_t] - \frac{a}{2} V_t[dW_t] \quad (\text{A.6})$$

W_t denotes arbitrageurs' real wealth, a is the coefficient of risk aversion. $x_t^{N,(\tau)}$ and $x_t^{R,(\tau)}$ are the units of wealth invested in the nominal bond and the inflation-linked bond with maturity of τ . Vayanos and Vila (2021) gives the interpretation for this setting that there are overlapping generations of arbitrageurs living over infinitesimal periods. A generation,

born in t with wealth W_t , invests from t to $t + dt$ and then consumes and dies at $t + dt$. The corresponding budget constraint to the problem is given by:

$$dW_t = \int_0^T \left(x_t^{N,(\tau)} \frac{dP_t^{N,(\tau)}}{P_t^{N,(\tau)}} + x_t^{R,(\tau)} \frac{dP_t^{R,(\tau)}}{P_t^{R,(\tau)}} \right) d\tau - \left(\int_0^T x_t^{N,(\tau)} d\tau \right) \pi_t dt + \left(W_t - \int_0^T (x_t^{N,(\tau)} + x_t^{R,(\tau)}) d\tau \right) r_t dt \quad (\text{A.7})$$

The first expression is the return from investing in bonds, as $\int_0^T x_t^{N,(\tau)} d\tau$ and $\int_0^T x_t^{R,(\tau)} d\tau$ are the amount of wealth invested in nominal bonds and real bonds respectively. The second term $\left(\int_0^T x_t^{N,(\tau)} d\tau \right) \pi_t dt$ deflates the return from nominal bonds. Finally, the last expression is the return gained by investing the remaining wealth in the risk-free rate.

A.3 Solving the model

The model is solved by first conjecturing and later verifying that equilibrium spot rates are affine functions of the risk factors. The price of the nominal bond $P_t^{N,(\tau)}$, and the price of the inflation-linked bond $P_t^{R,(\tau)}$ are:

$$P_t^{N,(\tau)} = e^{-[A_r^N(\tau)r_t + A_\beta^N(\tau)\beta_t + A_\pi^N(\tau)\pi_t + C^N(\tau)]} \quad (\text{A.8a})$$

$$P_t^{R,(\tau)} = e^{-[A_r^R(\tau)r_t + A_\beta^R(\tau)\beta_t + C^R(\tau)]} \quad (\text{A.8b})$$

Lemma 1. *The dynamics of the nominal bond prices and the inflation-linked bond prices are given by*

$$\frac{dP_t^{N,(\tau)}}{P_t^{N,(\tau)}} = \mu_t^{N,(\tau)} dt - A_r^N(\tau)\sigma_r dB_{r,t} - A_\beta^N(\tau)\sigma_\beta dB_{\beta,t} - A_\pi^N(\tau)\sigma_\pi dB_{\pi,t} \quad (\text{A.9a})$$

$$\frac{dP_t^{R,(\tau)}}{P_t^{R,(\tau)}} = \mu_t^{R,(\tau)} dt - A_r^R(\tau)\sigma_r dB_{r,t} - A_\beta^R(\tau)\sigma_\beta dB_{\beta,t} \quad (\text{A.9b})$$

where instantaneous expected returns $\mu_t^{N,(\tau)}$ and $\mu_t^{R,(\tau)}$ are given by equations (B.3a) and (B.3b) in the Online Appendix B.3.

Having derived the price dynamics of the two assets, we can substitute into the budget constraint (A.7) and solve the arbitrageurs' optimization problem (A.6). This is derived in Section B.3 in the Online Appendix.

Lemma 2. *The first order conditions are given by:*

$$\mu_t^{N,(\tau)} - \pi_t - r_t = A_r^N(\tau)\lambda_{r,t} + A_\beta^N(\tau)\lambda_{\beta,t} + A_\pi^N(\tau)\lambda_{\pi,t} \quad (\text{A.10a})$$

$$\mu_t^{R,(\tau)} - r_t = A_r^R(\tau)\lambda_{r,t} + A_\beta^R(\tau)\lambda_{\beta,t} \quad (\text{A.10b})$$

where coefficients $\lambda_{i,t}$ are given by:

$$\lambda_{i,t} = a\sigma_i^2 \int_0^T x_t^{N,(\tau)} A_i^N(\tau) + x_t^{R,(\tau)} A_i^R(\tau) d\tau \quad \text{for } i = r, \beta \quad (\text{A.11})$$

$$\lambda_{\pi,t} = a\sigma_\pi^2 \int_0^T x_t^{N,(\tau)} A_\pi^N(\tau) d\tau \quad (\text{A.12})$$

Equations (A.10a) and (A.10b) are also the no-arbitrage conditions in the model. No arbitrage requires the existence of prices of each risk factor. Then, the expected excess return of each asset over the short rate is equal to the asset's sensitivity to the risk factors times the risk factor's price, summed across all risk factors. The coefficients $\lambda_{i,t}$ are the prices of the risk factors, measuring the expected excess return per unit of sensitivity to each factor. They are determined through equilibrium conditions. Note, that $\lambda_{i,t}$ are proportional to how sensitive the arbitrageurs' portfolio is to factor i . For example, the sensitivity of the portfolio to the short rate is $\int_0^T x_t^{N,(\tau)} A_r^N(\tau) + x_t^{R,(\tau)} A_r^R(\tau) d\tau$. As inflation-linked bonds shield investors from inflation, the inflation risk factor only loads on the nominal bond. Note, that even the real return of the nominal bond is sensitive to inflation risk.

A.4 Equilibrium term structures

In equilibrium, the supplied amount of bonds will be equal to the investment of the arbitrageurs.

$$x_t^{N,(\tau)} = s_t^{N,(\tau)} \quad (\text{A.13a})$$

$$x_t^{R,(\tau)} = s_t^{R,(\tau)} \quad (\text{A.13b})$$

We can use the market clearing (A.13a), (A.13b), the supply (A.4a), (A.4b) and (B.3a), (B.3b) to substitute into the first order conditions (A.10a), (A.10b). This yields two functions that are affine in the risk factors r_t , β_t and π_t , verifying our initial conjecture.

Setting linear terms in r_t , β_t and π_t equal to zero gives five ordinary differential equations (ODEs) in $A_r^N(\tau)$, $A_\beta^N(\tau)$, $A_\pi^N(\tau)$, $A_r^R(\tau)$ and $A_\beta^R(\tau)$. The solutions to these ODEs are stated in Theorem 1 and derived in Section B.3 in the Online Appendix.

Theorem 1. *The nominal bond sensitivities $A_r^N(\tau)$, $A_\beta^N(\tau)$ and $A_\pi^N(\tau)$ are given by*

$$A_r^N(\tau) = \frac{1 - e^{-\kappa_r \tau}}{\kappa_r} \quad (\text{A.14a})$$

$$A_\beta^N(\tau) = \frac{Z_r}{\kappa_r} \left[\frac{1 - e^{-\hat{\kappa}_\beta \tau}}{\hat{\kappa}_\beta} - \frac{e^{-\hat{\kappa}_\beta \tau} - e^{-\kappa_r \tau}}{\kappa_r - \hat{\kappa}_\beta} \right] + \frac{Z_\pi}{\kappa_\pi} \left[\frac{1 - e^{-\hat{\kappa}_\beta \tau}}{\hat{\kappa}_\beta} - \frac{e^{-\hat{\kappa}_\beta \tau} - e^{-\kappa_\pi \tau}}{\kappa_\pi - \hat{\kappa}_\beta} \right] \quad (\text{A.14b})$$

$$A_\pi^N(\tau) = \frac{1 - e^{-\kappa_\pi \tau}}{\kappa_\pi} \quad (\text{A.14c})$$

The real bond sensitivities $A_r^R(\tau)$ and $A_\beta^R(\tau)$ are given by

$$A_r^R(\tau) = \frac{1 - e^{-\kappa_r \tau}}{\kappa_r} \quad (\text{A.15a})$$

$$A_\beta^R(\tau) = \frac{Z_r}{\kappa_r} \left[\frac{1 - e^{-\hat{\kappa}_\beta \tau}}{\hat{\kappa}_\beta} - \frac{e^{-\hat{\kappa}_\beta \tau} - e^{-\kappa_r \tau}}{\kappa_r - \hat{\kappa}_\beta} \right] \quad (\text{A.15b})$$

Where Z_r , Z_π and $\hat{\kappa}_\beta$ are given by equations (B.6a), (B.6b) and (B.7) respectively in the Online Appendix B.3. The functions $C^N(\tau)$ and $C^R(\tau)$ are given in Section B.3 in the Online Appendix.

A.5 Analysis of supply effects

In our empirical analysis, we found that the high-frequency supply shock raises both nominal and real yields. In the model bond yields are given by:

$$\begin{aligned} y_t^{N,(\tau)} &= \frac{1}{\tau} [A_r^N(\tau)r_t + A_\beta^N(\tau)\beta_t + A_\pi^N(\tau)\pi_t + C^N(\tau)] \\ y_t^{R,(\tau)} &= \frac{1}{\tau} [A_r^R(\tau)r_t + A_\beta^R(\tau)\beta_t + C^R(\tau)] \end{aligned}$$

Therefore, we need to show that $\partial y_t^{N,(\tau)} / \partial \beta_t = A_\beta^N(\tau) / \tau$ and $\partial y_t^{R,(\tau)} / \partial \beta_t = A_\beta^R(\tau) / \tau$ are positive.

Proposition 1. *The effect of a shock to the supply factor β_t on nominal and real yields is positive.*

Proposition 2. *The effect of a shock to the supply factor on duration and inflation risk prices is positive.*

Proposition 3. *The effect of a shock to the supply factor on yields and risk prices increases with risk aversion.*

The proofs follow from Lemma A.1. and Lemma A.2. of Greenwood and Vayanos (2014), and we present it in the Online Appendix. The intuition is the following. An increase in the supply factor increases the amount of nominal bonds held by investors in equilibrium. This increases the sensitivity of their portfolio to duration and inflation risks, raising the prices of these factors. The increase in duration risk premium and inflation risk premium raises both nominal and inflation-linked bond yields. As inflation risk loads positively only on nominal bonds, the spread between the two type of bonds widen, consistent with our empirical findings.

Online Appendix (Not for publication)

B.1 The ATSM of Abrahams et al. (2016)

The $K \times 1$ vector of pricing factors follows an autoregression under the physical measure (\mathbb{P}):

$$X_{t+1} - \mu_X = \Phi(X_t - \mu_X) + \nu_{t+1}, \quad \nu_{t+1} \sim i.i.d.N(0_{K \times 1}, \Sigma) \quad (\text{B.1})$$

The stochastic discount factor is written as:

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \Sigma^{-1/2} \nu_{t+1}\right)$$

where r_t is the nominal short rate and λ_t is $K \times 1$ the vector of risk prices. These are related to the pricing factors as: $\lambda_t = \Sigma^{-1/2}(\lambda_0 + \lambda_1 X_t)$. The short rate follows $r_t = \delta_0 + \delta_1 X_t$. Abrahams et al. (2016) define the parameters of the pricing factor dynamics under the risk neutral measure (\mathbb{Q}) as:

$$\begin{aligned} \tilde{\mu} &= (I_K - \Phi)\mu_X - \lambda_0 \\ \tilde{\Phi} &= \Phi - \lambda_1 \end{aligned}$$

The model assumes that bond yields are affine functions of the state variables, which are assumed to be observable. Therefore, under the pricing measure, the log price, $P_t^{(\tau)}$, of a nominal zero-coupon risk-free bond with remaining time to maturity τ follows $\log P_t^{(\tau)} = A_\tau + B_\tau' X_t$. The log price of an inflation-linked bond follows similarly $\log P_{t,R}^{(\tau)} = A_{\tau,R} + B_{\tau,R}' X_t$. The price of such a bond also satisfies:

$$\begin{aligned} \log P_{t,R}^{(\tau)} &= E_t^{\mathbb{Q}} \left[\exp(-r_t - \dots - r_{t+\tau-1}) \frac{Q_{t+\tau}}{Q_t} \right] \\ &= E_t^{\mathbb{Q}} \left[\exp(-r_t - \dots - r_{t+\tau-1} + \pi_{t+1} + \dots + \pi_{t+\tau}) \right] \end{aligned} \quad (\text{B.2})$$

where $E^{\mathbb{Q}}$ is the expectation operator under the risk neutral measure. Q_t is the price index at time t , and $\pi_t = \ln(Q_t/Q_{t-1})$ is the one period log inflation, related to the pricing factors as $\pi_t = \pi_0 + \pi_1' X_t$.

The system of recursive linear restrictions of the bond pricing parameters can be obtained once no-arbitrage is imposed (see Ang and Piazzesi (2003)):

$$\begin{aligned} A_\tau &= A_{\tau-1} + B_{\tau-1}' \tilde{\mu} + \frac{1}{2} B_{\tau-1}' \Sigma B_{\tau-1} - \delta_0 \\ B_\tau' &= B_{\tau-1}' \tilde{\Phi} - \delta_1' \\ A_0 &= 0, \quad B_0 = 0_{K \times 0} \end{aligned}$$

Risk neutral counterparts are obtained by setting the price of risk parameters λ_0 and λ_1 to

zero. Then, the pricing recursion modifies to:

$$\begin{aligned}\tilde{A}_\tau &= \tilde{A}_{\tau-1} + \tilde{B}'_{\tau-1}(I_K - \Phi)\mu - \delta_0 \\ \tilde{B}'_\tau &= \tilde{B}'_{\tau-1}\Phi - \delta'_1 \\ \tilde{A}_0 &= 0, \quad \tilde{B}_0 = 0_{K \times 0}\end{aligned}$$

The inflation-linked bond recursion can be obtained by writing Equation B.2 in terms of an inflation-linked bond purchased one period ahead. Taking logs, calculating the expectation, and matching coefficients in the expression for the log bond price yields the recursion:¹⁹

$$\begin{aligned}A_{\tau,R} &= A_{\tau-1,R} + B^{\pi}_{\tau-1,R}'\tilde{\mu} + \frac{1}{2}B^{\pi}_{\tau-1,R}'\Sigma B^{\pi}_{\tau-1,R} - \delta_{0,R} \\ B^{\pi}_{\tau,R}' &= B^{\pi}_{\tau-1,R}'\tilde{\Phi} - \delta'_1 \\ A_{0,R} &= 0, \quad B_{0,R} = 0_{K \times 0}\end{aligned}$$

where $\delta_{0,R} = \delta_0 - \pi_0$ and $B^{\pi}_{\tau,R} = B_{\tau,R} + \pi_1$. Similar to nominal bonds, the risk neutral counterparts are given by:

$$\begin{aligned}\tilde{A}_{\tau,R} &= \tilde{A}_{\tau-1,R} + \tilde{B}^{\pi}_{\tau-1,R}'(I_K - \Phi)\mu - \delta_{0,R} \\ \tilde{B}^{\pi}_{\tau,R}' &= \tilde{B}^{\pi}_{\tau-1,R}'\Phi - \delta'_1 \\ \tilde{A}_{0,R} &= 0, \quad \tilde{B}_{0,R} = 0_{K \times 0}\end{aligned}$$

where $\tilde{B}^{\pi}_{\tau,R} = \tilde{B}_{\tau,R} + \pi_1$.

The elements of yields can be obtained as the following. We use the risk adjusted counterparts of the pricing recursion coefficients \tilde{A} , and \tilde{B} , to calculate the risk adjusted fitted yields. These yields are interpreted as the time t expectation of average future short rates over the next τ periods:

$$\frac{1}{\tau} \sum_{i=0}^{\tau} E_t r_{t+i} = -\frac{1}{\tau} [\tilde{A}_\tau + \tilde{B}'_\tau X_t], \quad \frac{1}{\tau} \sum_{i=0}^{\tau} E_t r_{t+i,R} = -\frac{1}{\tau} [\tilde{A}_{\tau,R} + \tilde{B}'_{\tau,R} X_t]$$

The difference between the nominal and the real expected short rates is the average expected future inflation over the next τ periods:

$$\frac{1}{\tau} \sum_{i=0}^{\tau} E_t(\pi_{t+i}) = -\frac{1}{\tau} [\tilde{A}_\tau + \tilde{B}'_\tau X_t] - \left(-\frac{1}{\tau} [\tilde{A}_{\tau,R} + \tilde{B}'_{\tau,R} X_t] \right)$$

Term premiums can be obtained by subtracting the expectation component from the fitted yields:

$$TP_t^{(\tau)} = -\frac{1}{\tau} [A_\tau + B'_\tau X_t] - \left(-\frac{1}{\tau} [\tilde{A}_\tau + \tilde{B}'_\tau X_t] \right)$$

¹⁹For details see Abrahams et al. (2016)

for nominal term premium, and similarly for the real term premium $TP_{t,R}^{(\tau)}$. The inflation risk premium is obtained as the difference between the fitted breakeven inflation and the inflation expectation:

$$IRP_t^{(\tau)} = -\frac{1}{\tau} [A_\tau + B'_\tau X_t] - \left(-\frac{1}{\tau} [A_{\tau,R} + B'_{\tau,R} X_t] \right) - \frac{1}{\tau} \sum_{i=0}^{\tau} E_t(\pi_{t+i})$$

The model parameters are estimated following Adrian, Crump and Moench (2013) and Abrahams et al. (2016). First, we estimate the risk neutral dynamics of the pricing factors by an autoregression. Then, we estimate the sensitivities of bond excess returns to current and past values of the pricing factors. Lastly, we do cross-sectional regressions of excess return sensitivities to lagged pricing factors onto excess return sensitivities to current pricing factors.

State variables are extracted principal components from yields. Following Abrahams et al. (2016), we extract three principal components from month-end zero coupon nominal yields and two principal components from orthogonalized real yields.²⁰ The factors are shown in Figure B9 in the Online Appendix. The short rate is the Bank of England's official bank rate, inflation is calculated with the monthly RPI series from the Office of National Statistics. We calculate excess returns on eleven nominal maturities of $\tau = 6, 12, 24, \dots, 120$ months and eight real maturities of $\tau = 60, 66, 72, \dots, 120$ months. For maturities that the Bank of England does not publish data, we interpolate it with cubic spline method. We do the decomposition up to 10 years, as the fit of the model deteriorates at higher maturities. In the baseline model, we do not account for the relative liquidity of inflation-linked bonds due to the lack of good liquidity proxies. Nevertheless, in the paper we also present a robustness exercise where we try also to take this into account.

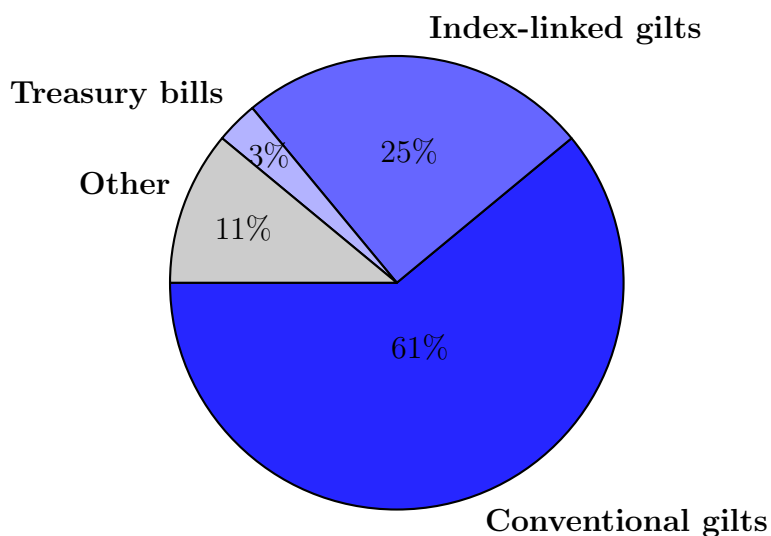
The ATSM model parameters are estimated on monthly data from 03.1997 to 12.2019. Our goal is to decompose yields at the daily frequency, so we follow Adrian et al. (2013) and use the monthly model parameters on factors extracted from daily yield curve data from 31.03.1997 to 31.12.2019 to obtain the yield decomposition at the daily frequency. Model fit diagnostics are summarized in Tables B1 and B2 in the Online Appendix. The fit of the model at 10-years maturity is displayed in Figure B10 at the monthly frequency, and in Figure B11 at the daily frequency in the Online Appendix. The mean pricing errors are somewhat larger than in Abrahams et al. (2016), while the standard deviations are similar. Consistent with the relationship between yield and return pricing errors, we find a strong serial correlation in yield pricing errors but not in return pricing errors (see Adrian et al. (2013) for more details). The decomposed 10-year expected nominal short rate and nominal term premium are displayed in Figure B13. The decomposition of the 10-year breakeven inflation rate into inflation expectations and inflation risk premium are displayed in Figure B14. The trends in the estimated 10-year nominal term premium are in line with the estimates of Bianchi, Mumtaz and Surico (2009), Malik and Meldrum (2016), Abrahams et al. (2016) and Kaminska et al. (2018). The series fluctuates close to 1% at the beginning of the sample and rises after the Global Financial Crisis. It moves into negative territory towards the end of the sample, during the asset purchase programs of the Bank of England.

²⁰Orthogonalized yields are obtained by purging inflation-linked yields from the nominal principal components, to reduce collinearity among the pricing factors.

Expected inflation and inflation risk premium are close to the estimates of Abrahams et al. (2016), Kaminska et al. (2018) and Bekaert and Ermolov (2021). Expected 10-year average inflation is rather stable, fluctuating close to 3%. The inflation risk premium shows more variation. It stays mostly within the 0-1% range but drops into negative territory in the early 2000s, the Global Financial Crisis, and the European debt crisis.

B.2 Figures and Tables

Figure B1: Composition of UK central government sterling debt in December 2019



Source: UK DMO

Figure B2: DMO Financing Remit for the 2015-16 financial year, published the 18th of March 2015

DMO FINANCING REMIT 2015-16: 18 MARCH 2015

1. The DMO's financing remit for 2015-16 has been published today as part of the Budget 2015 announcements. The main points are summarised below.

A) Debt issuance by the DMO

2. The DMO plans to raise £140.4¹ billion in 2015-16, split as follows:

- Outright gilt sales: £133.4 billion.
- Net Treasury bill sales (via tenders): £7.0 billion.

B) Planned gilt sales

3. It is intended that the gilt sales plans will be met through a combination of:
 - £105.2 billion of issuance in 39 auctions; and
 - additional supplementary gilt sales of £28.2 billion (21.1% of total issuance) via a combination of syndicated offerings and, subject to demand, mini-tenders. This will comprise a minimum £24.2 billion via a syndication programme. Any additional sales via syndication can only be of long conventional or index-linked gilts but mini-tenders can be used for issuance of conventional and index-linked gilts across the curve.
4. The planned split of issuance by maturity and type of gilt to be sold via auctions and syndicated offerings is as follows:

Conventional:

- Short: £33.9 billion (25.4%) in 8 auctions
- Medium: £26.7 billion (20.0%) in 8 auctions
- Long: £37.4 billion (28.0%) in 12 auctions and via syndicated offerings (aiming to raise £28.1 billion by auctions and a current planning assumption of a minimum of £9.3 billion via syndication).

Index-linked: £31.4 billion (23.5%) in 11 auctions and via syndicated offerings (aiming to raise £16.5 billion by auctions and a current planning assumption of a minimum of £14.9 billion via syndication).

5. The issuance methods to achieve the syndication and mini-tender plans are based on current assumptions. In particular, total financing achieved through each supplementary issuance method will be dependent on market and demand conditions at the time transactions are conducted.

¹ Sales figures in this announcement are in cash terms unless otherwise indicated.

Figure B3: Gilts Operations Calendar for April-May 2015, published the 31st of March 2015



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31 March 2015

PRESS NOTICE

GILT OPERATIONS CALENDAR: APRIL- JUNE 2015

PLANNED SYNDICATED OFFERING OF AN INDEX-LINKED GILT WITH A MATURITY IN THE 30 YEAR AREA OR LONGER IN JUNE 2015

The UK Debt Management Office ("the DMO") is announcing today that the first syndicated offering of the 2015-16 programme will be the sale of an Index-linked gilt with a maturity in the 30 year area or longer. The DMO expects that, subject to market conditions, the sale will take place in the second half of June 2015. Further details of the sale, including the composition of the syndicate, will be announced in due course.

The DMO also announces that in the period April-June 2015 it plans to hold ten outright gilt auctions as well as the syndicated offering, as set out below.

Auction date	Gilt	Further details announced ¹
Wednesday 8 April	2% Treasury Gilt 2020	Tuesday 31 March
Thursday 16 April	0½% Index-linked Treasury Gilt 2040	Tuesday 7 April
Tuesday 21 April	3½% Treasury Gilt 2045	Tuesday 14 April
Wednesday 29 April	2% Treasury Gilt 2025	Tuesday 21 April
Thursday 14 May	2% Treasury Gilt 2020	Tuesday 5 May
Thursday 21 May	4¾% Treasury Gilt 2030	Tuesday 12 May
Wednesday 27 May	0¼% Index-linked Treasury Gilt 2058	Tuesday 19 May
Tuesday 2 June	2% Treasury Gilt 2025	Tuesday 26 May
Tuesday 9 June	0¼% Index-linked Treasury Gilt 2024	Tuesday 2 June
Thursday 11 June	3½% Treasury Gilt 2045	Tuesday 2 June

¹ Further to the announcement on 29 January 2015, as of 31 March 2015 the DMO will no longer be declaring a "When Issued" (WI) trading period in cases where the stock being auctioned is a re-opening of an existing line of gilts with a pre-existing ISIN code. Accordingly, the DMO will no longer be issuing a separate WI ISIN code for new tranches of existing gilts. The 29 January announcement can be found at: <http://www.dmo.gov.uk/documentview.aspx?docName=/gilts/press/sa290115.pdf>

Figure B4: Auction announcement of the auction of £3,000 million of 2% Treasury Gilt 2025, published the 21st of April 2015



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21 April 2015

PRESS NOTICE

AUCTION OF BRITISH GOVERNMENT STOCK

Auction Details

Auction Date	Wednesday, 29 April 2015
Issue and Settlement Date	Thursday, 30 April 2015
Bidding Convention	Fully paid Bid Price (see Note 1)
Accrued Interest payable with bid	£0.222826 per £100 nominal
Auction Close	10:30am London Time

Details of Security

Title	2% Treasury Gilt 2025
Amount (nominal) for auction	£3,000 million (fungible with previous issue) (see Note 4)
Nominal outstanding after auction	£6,024.9 million
Maturity Date	7 September 2025 at par
Interest Dates	7 March – 7 September
ISIN Code	GB00BTHH2R79
SEDOL Code	B-THH-2R7
Strippable	From 30 April 2015 (see Note 2)
Interest Payable	Gross (see Note 3)
Next Interest Date	7 September 2015 - £0.929348 per £100 nominal (Short First Coupon)

Note 1: Bids may be made on either a competitive or a non-competitive basis. Details of the bidding procedures are set out in the prospectus and in the Information Memorandum. Gilt-edged Market Makers may bid by means of the Bloomberg Bond Auction System to the DMO not later than 10.30 am on Wednesday, 29 April 2015.

Note 2: Following the issue of this further amount of the Gilt, 2% Treasury Gilt 2025 may be stripped and holdings of the Gilt reconstituted: the provisions relating to strips contained in the Information Memorandum will therefore apply except that the minimum stripping unit will be £1,000,000 nominal until the payment of the non-standard first coupon on 7 September 2015. The SEDOL and ISIN codes for the new principal strip are B-WXB-PL9 and GB00BWXBPL93 respectively.

Note 3: Holders may elect to have United Kingdom income tax deducted from interest payments, should they so wish, on application to the Registrar, Computershare Investor Services PLC.

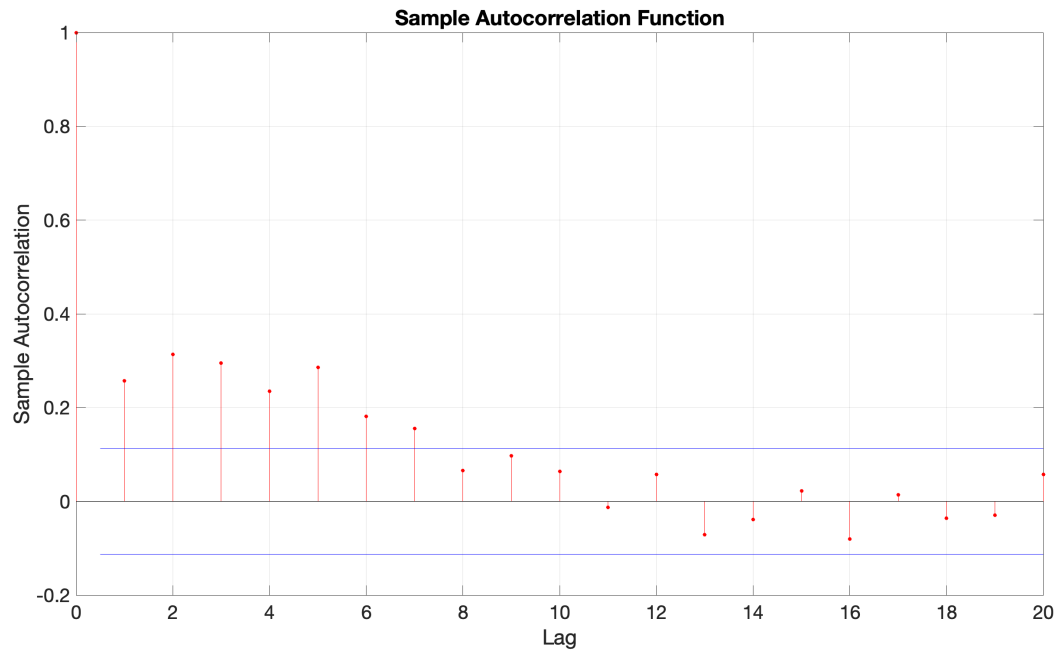
Figure B5: A progress report of the 2015-16 Financing Remit, published on the 14th of April 2015

Remit 2015-16

Gilt sales of £133.4 billion (cash) are planned in 2015-16 and progress against the remit is summarised in the table below (which may not include the amount of gilts issued under the Post Auction Option Facility for the most recent auction, if any).

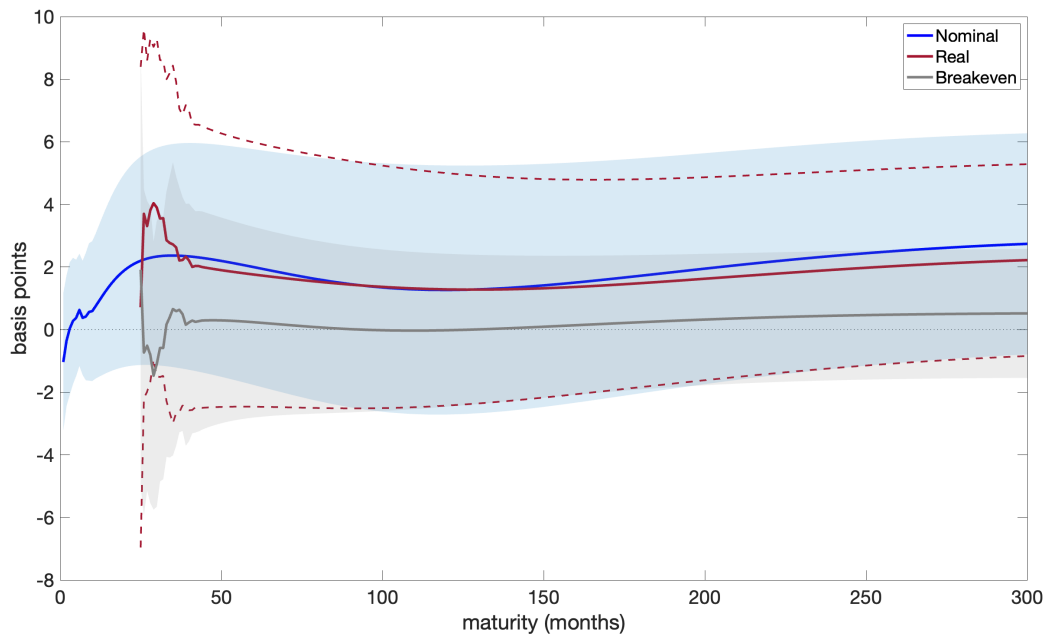
Gilt sales relative to remit plans 14 April 2015 (£ millions)					
	Conventional Gilts			Index-linked gilts	Total
	Short	Medium	Long		
Auction proceeds to-date	4,166	0	0	0	4,166
PAOF proceeds to-date	10	0	0	0	10
Auction and PAOF proceeds to-date	4,177	0	0	0	4,177
Syndication sales to-date	0	0	0	0	0
Mini-tender sales to date	0	0	0	0	0
Total gilt sales to date	4,177	0	0	0	4,177
Auction sales required to meet plans	29,723	26,700	28,100	16,500	101,023
Number of auctions remaining	7	8	12	11	38
Currently required average auction sizes	4,246	3,338	2,342	1,500	
Planned gilt sales at auctions	33,900	26,700	28,100	16,500	105,200
Number of auctions scheduled	8	8	12	11	39
Minimum syndication sales plan	0	0	9,300	14,900	24,200
Syndication sales required to meet minimum plan	0	0	9,300	14,900	24,200
Balance of supplementary gilt sales					28,200
Total planned supplementary gilt sales					28,200
Total planned gilt sales					133,400

Figure B6: Autocorrelation function of the surprise volume series



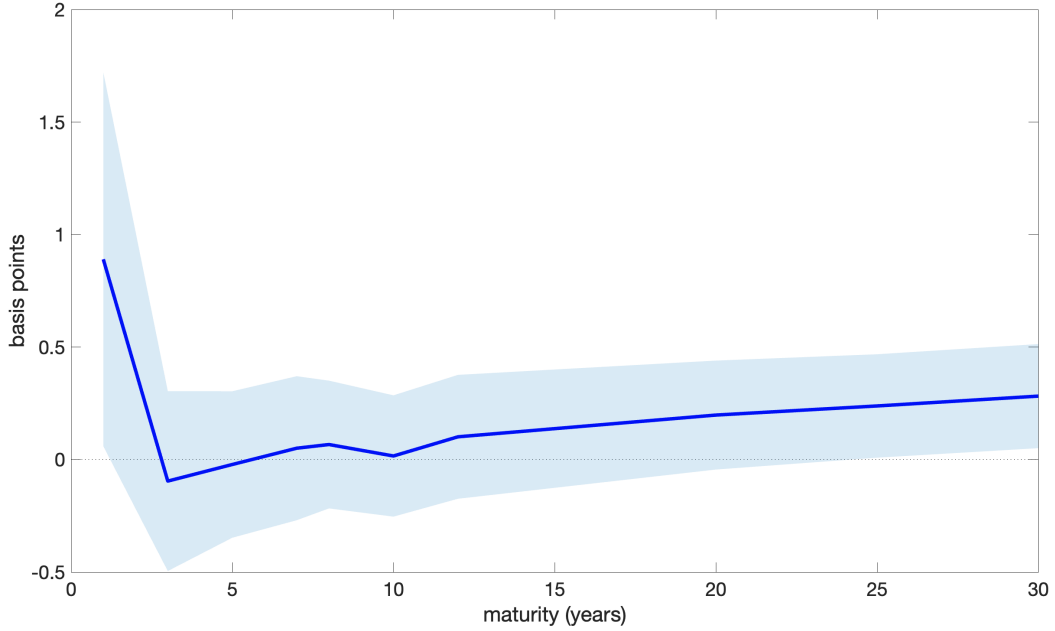
Note: Sample autocorrelation function of the shock series up to ten lags. Blue lines indicate two standard errors confidence bounds.

Figure B7: Reaction of the nominal and real and the breakeven inflation term structures to the bond supply shock - IV regression



Note: Instrumental variable estimation of (2), where S_t is instrumented by the announced volumes and the surprise component of the announced volume. Shaded areas and dotted lines are two standard deviation confidence bands. Sample: 02.05.2006-31.12.2019.

Figure B8: Reaction of the inflation swap curve to the supply shock



Note: Nodes are the estimated $b^{(m)}$ coefficients from equation (2). Dependent variables are inflation swap rates from Refinitiv, with maturities of 1, 3, 5, 7, 8, 10, 12, 20, 25, and 30 years. Dashed lines are 90% confidence intervals. Sample: 01.05.2009-31.12.2019.

Figure B9: Time Series of the ATSM Pricing Factors

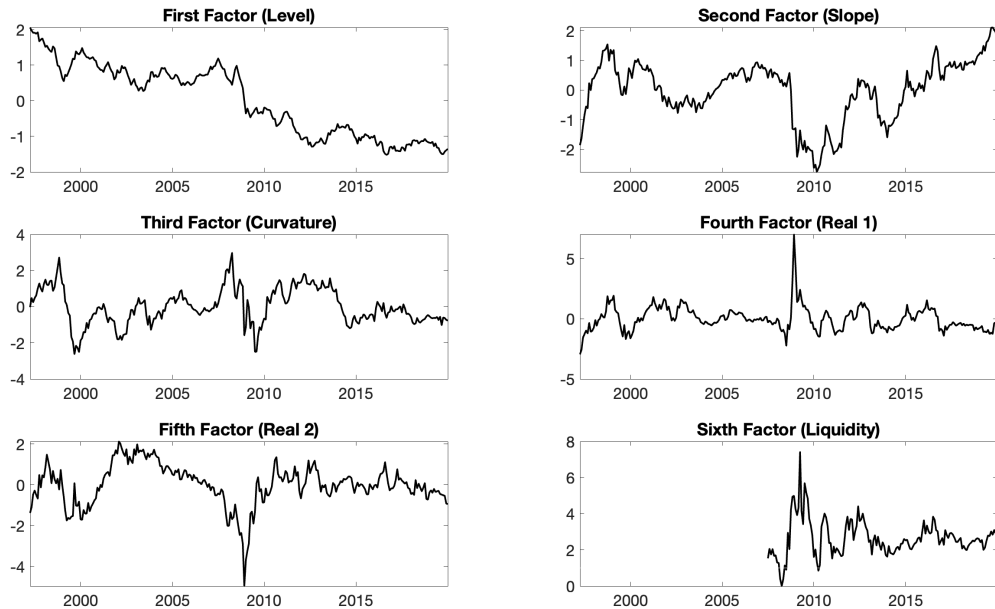


Figure B10: ATSM model fit at 10-years, monthly frequency - nominal yield (left), real yield (right)

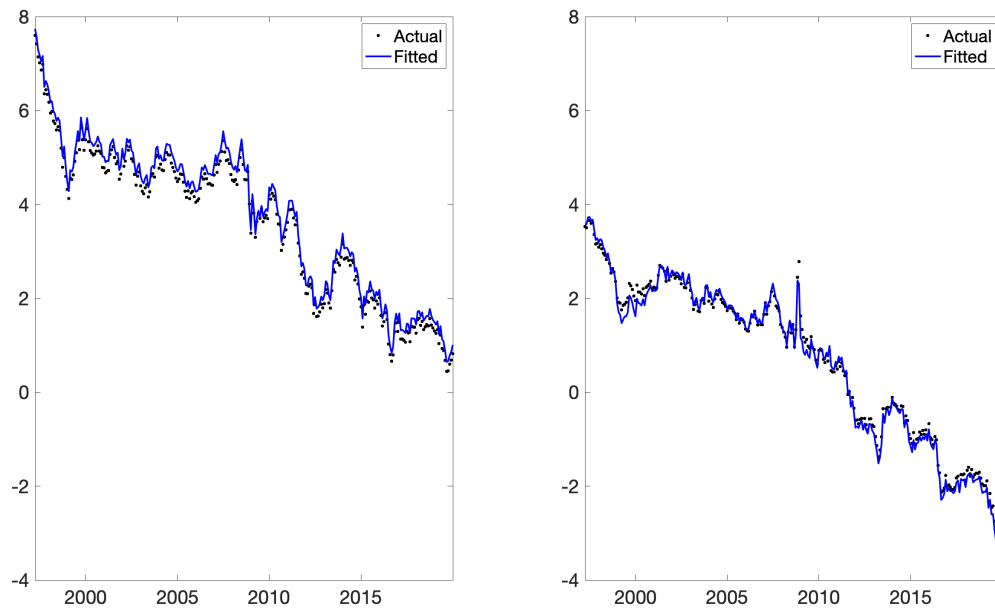


Figure B11: ATSM model fit at 10-years, daily frequency - nominal yield (left), real yield (right)

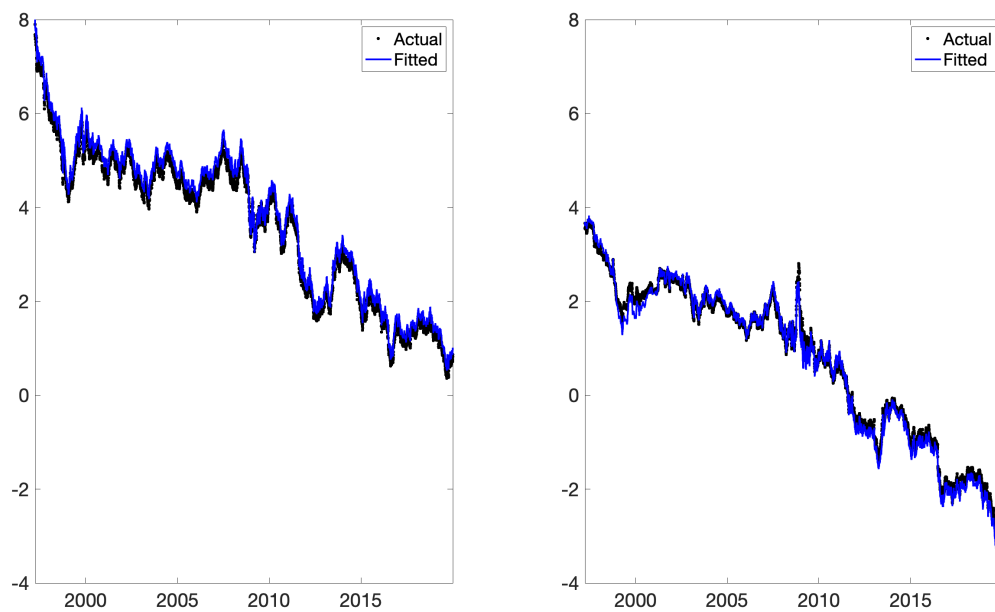
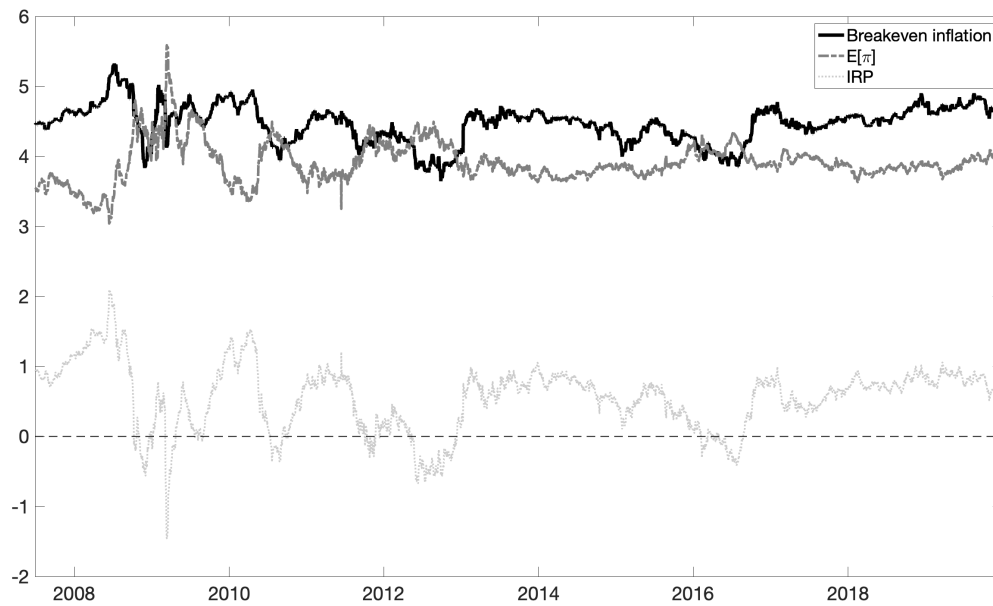


Figure B12: 10-year breakeven inflation rate decomposition at daily frequency, adjusted for liquidity



Note: Decomposition of the ATSM model implied 10-year breakeven inflation rates into expected average inflation and inflation risk premium, where the state space of pricing factors is extended with a liquidity proxy: the inflation swap, breakeven inflation rate spread.

Figure B13: 10-year nominal yield decomposition (left) and real yield decomposition (right) at the daily frequency

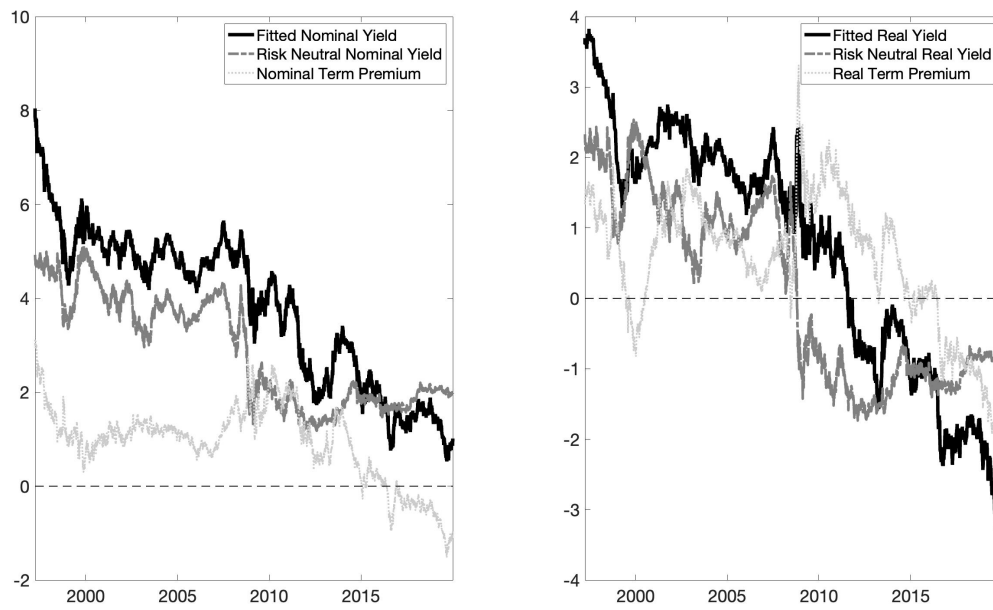


Figure B14: 10-year breakeven inflation rate decomposition at daily frequency

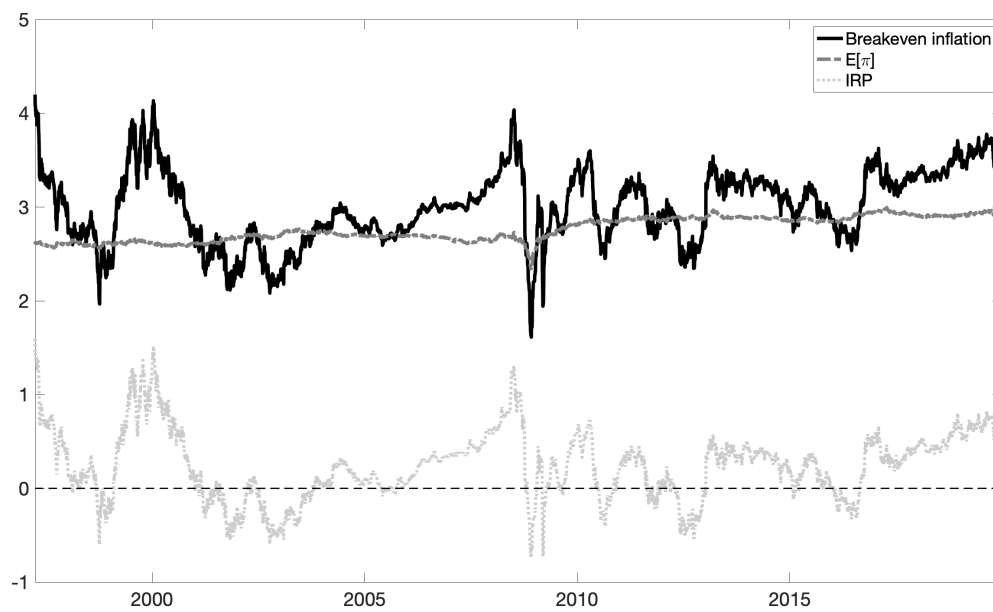


Table B1: Fit Diagnostics of the ATSM model on monthly data

	n = 36	n = 60	n = 84	n = 120
Nominal Yield Pricing Errors				
mean	0.297	0.221	-0.257	0.189
std	0.049	0.034	0.050	0.037
ρ^y	0.952	0.796	0.936	0.791
ρ^{xr}	0.220	0.129	0.048	-0.021
Real Yield Pricing Errors				
mean		-0.110	-0.200	0.046
std		0.160	0.127	0.118
ρ^y		0.796	0.936	0.791
ρ^{xr}		0.129	0.048	-0.021

Note: Time series properties of the ATSM pricing errors implied by the monthly estimation. “Mean” and “std” refers to the sample mean and standard deviation of yield pricing errors; ρ^y denotes first order sample autocorrelation coefficient of the yield pricing errors, ρ^{xr} denotes first order sample autocorrelation coefficient of the excess return pricing errors. Sample period: 1997:03 - 2019:12.

Table B2: Fit Diagnostics of the ATSM model on daily data

	n = 36	n = 60	n = 84	n = 120
Nominal Yield Pricing Errors				
mean	0.304	0.229	-0.248	-0.179
std	0.055	0.039	0.039	0.044
ρ^y	1.000	0.997	1.000	0.997
ρ^{xr}	0.369	0.264	0.205	0.154
Real Yield Pricing Errors				
mean		-0.108	-0.197	0.050
std		0.161	0.131	0.124
ρ^y		0.997	1.000	0.997
ρ^{xr}		0.264	0.205	0.154

Note: Time series properties of the ATSM pricing errors implied by the daily decomposition. “Mean” and “std” refers to the sample mean and standard deviation of yield pricing errors; ρ^y denotes first order sample autocorrelation coefficient of the yield pricing errors, ρ^{xr} denotes first order sample autocorrelation coefficient of the excess return pricing errors. Sample period: 1997:03:31 - 2019:12:31.

B.3 Proof of theoretical results

Proof of Lemma 1. Applying Ito's lemma to (A.8a) and (A.8b) using (A.2), (A.5) and (A.3), we get:

$$\begin{aligned}
\frac{dP_t^{N,(\tau)}}{P_t^{N,(\tau)}} &= [\dot{A}_r^N(\tau)r_t + \dot{A}_\beta^N(\tau)\beta_t + \dot{A}_\pi^N(\tau)\pi_t + \dot{C}^N(\tau)]dt - A_r^N(\tau)dr_t - A_\beta^N(\tau)d\beta_t - A_\pi^N(\tau)d\pi_t \\
&\quad + \frac{1}{2}[(A_r^N(\tau)dr_t)^2 + (A_\beta^N(\tau)d\beta_t)^2 + (A_\pi^N(\tau)d\pi_t)^2] \\
&= [\dot{A}_r^N(\tau)r_t + \dot{A}_\beta^N(\tau)\beta_t + \dot{A}_\pi^N(\tau)\pi_t + \dot{C}^N(\tau)]dt + A_r^N(\tau)[\kappa_r(r_t - \bar{r})dt - \sigma_r dB_{r,t}] \\
&\quad + A_\beta^N(\tau)[\kappa_\beta\beta_t dt - \sigma_\beta dB_{\beta,t}] + A_\pi^N(\tau)[(\pi_t - \bar{\pi})dt - \sigma_\pi dB_{\pi,t}] \\
&\quad + \frac{1}{2}[(A_r^N(\tau))^2\sigma_r^2 + (A_\beta^N(\tau))^2\sigma_\beta^2 + (A_\pi^N(\tau))^2\sigma_\pi^2] \\
\frac{dP_t^{R,(\tau)}}{P_t^{R,(\tau)}} &= [\dot{A}_r^R(\tau)r_t + \dot{A}_\beta^R(\tau)\beta_t + \dot{C}^R(\tau)]dt - A_r^R(\tau)dr_t - A_\beta^R(\tau)d\beta_t \\
&\quad + \frac{1}{2}[(A_r^R(\tau)dr_t)^2 + (A_\beta^R(\tau)d\beta_t)^2] \\
&= [\dot{A}_r^R(\tau)r_t + \dot{A}_\beta^R(\tau)\beta_t + \dot{A}_\pi^R(\tau)\pi_t + \dot{C}^R(\tau)]dt + A_r^R(\tau)[\kappa_r(r_t - \bar{r})dt - \sigma_r dB_{r,t}] \\
&\quad + A_\beta^R(\tau)[\kappa_\beta\beta_t dt - \sigma_\beta dB_{\beta,t}] + \frac{1}{2}[(A_r^R(\tau))^2\sigma_r^2 + (A_\beta^R(\tau))^2\sigma_\beta^2]
\end{aligned}$$

where we arrive to (A.9a) and (A.9b) if we define $\mu_t^{N,(\tau)}$ and $\mu_t^{R,(\tau)}$ as:

$$\begin{aligned}
\mu_t^{N,(\tau)} &= \dot{A}_r^N(\tau)r_t + \dot{A}_\beta^N(\tau)\beta_t + \dot{A}_\pi^N(\tau)\pi_t + \dot{C}^N(\tau) + A_r^N(\tau)\kappa_r(r_t - \bar{r}) + A_\beta^N(\tau)\kappa_\beta\beta_t \\
&\quad + A_\pi^N(\tau)\kappa_\pi(\pi_t - \bar{\pi}) + \frac{\sigma_r^2}{2}(A_r^N(\tau))^2 + \frac{\sigma_\beta^2}{2}(A_\beta^N(\tau))^2 + \frac{\sigma_\pi^2}{2}(A_\pi^N(\tau))^2
\end{aligned} \tag{B.3a}$$

$$\begin{aligned}
\mu_t^{R,(\tau)} &= \dot{A}_r^R(\tau)r_t + \dot{A}_\beta^R(\tau)\beta_t + \dot{C}^R(\tau) + A_r^R(\tau)\kappa_r(r_t - \bar{r}) + A_\beta^R(\tau)\kappa_\beta\beta_t \\
&\quad + \frac{\sigma_r^2}{2}(A_r^R(\tau))^2 + \frac{\sigma_\beta^2}{2}(A_\beta^R(\tau))^2
\end{aligned} \tag{B.3b}$$

□

Proof of Lemma 2. Simplifying terms yields (A.9a) and (A.9b). Substituting these into the budget constraint (A.7):

$$\begin{aligned}
dW_t &= \left[W_t r_t + \int_0^T x_t^{N,(\tau)} [\mu_t^{N,(\tau)} - r_t - \pi_t] + x_t^{R,(\tau)} [\mu_t^{R,(\tau)} - r_t] d\tau \right] dt \\
&\quad - \sigma_r \int_0^T x_t^{N,(\tau)} A_r^N(\tau) + x_t^{R,(\tau)} A_r^R(\tau) d\tau dB_{r,t} - \sigma_\beta \int_0^T x_t^{N,(\tau)} A_\beta^N(\tau) + x_t^{R,(\tau)} A_\beta^R(\tau) d\tau dB_{\beta,t} \\
&\quad - \sigma_\pi \int_0^T x_t^{N,(\tau)} A_\pi^N(\tau) d\tau dB_{\pi,t}
\end{aligned}$$

Then, the optimization problem can be written as:

$$\begin{aligned} \max_{\{x_t^{N,(\tau)}, x_t^{R,(\tau)}\}_{\tau \in (0, T]}} & \int_0^T x_t^{N,(\tau)} [\mu_t^{N,(\tau)} - r_t] + x_t^{R,(\tau)} [\mu_t^{R,(\tau)} - r_t] d\tau - \frac{a\sigma_r^2}{2} \left(\int_0^T x_t^{N,(\tau)} A_r^N(\tau) + x_t^{R,(\tau)} A_r^R(\tau) d\tau \right)^2 \\ & - \frac{a\sigma_\beta^2}{2} \left(\int_0^T x_t^{N,(\tau)} A_\beta^N(\tau) + x_t^{R,(\tau)} A_\beta^R(\tau) d\tau \right)^2 - \frac{a\sigma_\pi^2}{2} \left(\int_0^T x_t^{N,(\tau)} (A_\pi^N(\tau) \pi_t (2 + \sigma_\pi + 1)) d\tau \right)^2 \end{aligned}$$

Point-wise maximization gives the first order conditions (A.10a) and (A.10b). \square

Proof of Theorem 1. Setting linear terms to zero in (A.10a) and (A.10b) yields ordinary differential equations that we solve with the initial conditions $A_r^N(0) = A_r^R(0) = A_\beta^N(0) = A_\beta^R(0) = A_\pi^N(0) = C^N(0) = C^R(0) = 0$. Identifying terms in r_t gives:

$$\dot{A}_r^N(\tau) + \kappa_r A_r^N(\tau) - 1 = 0 \quad (\text{B.4a})$$

$$\dot{A}_r^R(\tau) + \kappa_r A_r^R(\tau) - 1 = 0 \quad (\text{B.4b})$$

Identifying terms in β_t gives:

$$\dot{A}_\beta^N(\tau) + \hat{\kappa}_\beta A_\beta^N(\tau) = Z_r A_r^N(\tau) + Z_\pi A_\pi^N(\tau) \quad (\text{B.5a})$$

$$\dot{A}_\beta^R(\tau) + \hat{\kappa}_\beta A_\beta^R(\tau) = Z_r A_r^R(\tau) \quad (\text{B.5b})$$

where

$$Z_r = a\sigma_r^2 \int_0^T \theta^N(\tau) A_r^N(\tau) d\tau \quad (\text{B.6a})$$

$$Z_\pi = a\sigma_\pi^2 \int_0^T \theta^N(\tau) A_\pi^N(\tau) d\tau \quad (\text{B.6b})$$

And $\hat{\kappa}_\beta$ solves

$$\hat{\kappa}_\beta = \kappa_\beta - a\sigma_\beta^2 \int_0^T \theta^N(\tau) A_\beta^N(\tau) d\tau \quad (\text{B.7})$$

Equilibria in the model exist if the arbitrageurs' risk-aversion coefficient a is below a threshold $\bar{a} > 0$. As in Greenwood and Vayanos (2014), we focus on that case, and select the equilibrium corresponding to the largest solution for $\hat{\kappa}$. For more details see Greenwood and Vayanos (2014).

Identifying terms in π leads to:

$$\dot{A}_\pi^N(\tau) + \kappa_\pi A_\pi^N(\tau) - 1 = 0 \quad (\text{B.8})$$

The solutions to (B.4a) and (B.4b) are (A.14a) and (A.15a). The solutions to (B.5a) and (B.5b) are given by (A.14b) and (A.15b), with Z_r , Z_π and $\hat{\kappa}_\beta$ are given by equations (B.6a), (B.6b) and (B.7). The solution to (B.8) is (A.14c). Identifying constant terms in (A.10a)

yields

$$\begin{aligned}
\dot{C}^N(\tau) &= A_r^N(\tau)\kappa_r\bar{r} - A_\pi^N(\tau)\kappa_\pi\bar{r} + \frac{\sigma_r^2}{2}(A_r^N(\tau))^2 + \frac{\sigma_\beta^2}{2}(A_\beta^N(\tau))^2 + \frac{\sigma_\pi^2}{2}(A_\pi^N(\tau))^2 \\
&= a\sigma_r^2 A_r^N(\tau) \int_0^T \zeta^N(\tau)A_r^N(\tau) + \zeta^R(\tau)A_r^R(\tau)d\tau \\
&+ a\sigma_\beta^2 A_\beta^N(\tau) \int_0^T \zeta^N(\tau)A_\beta^N(\tau) + \zeta^R(\tau)A_\beta^R(\tau)d\tau \\
&+ a\sigma_\pi^2 A_\pi^N(\tau) \int_0^T \zeta^N(\tau)A_\pi^N(\tau)
\end{aligned}$$

The solution to $\dot{C}^N(\tau)$ is

$$\begin{aligned}
C^N(\tau) &= \hat{Z}_r \int_0^\tau A_r^N(\tau')d\tau' + \hat{Z}_\beta \int_0^\tau A_\beta^N(\tau')d\tau' + \hat{Z}_\pi \int_0^\tau A_\pi^N(\tau')d\tau' \\
&- \frac{\sigma_r^2}{2} \int_0^\tau (A_r^N(\tau'))^2 d\tau' - \frac{\sigma_\beta^2}{2} \int_0^\tau (A_\beta^N(\tau'))^2 d\tau' - \frac{\sigma_\pi^2}{2} \int_0^\tau (A_\pi^N(\tau'))^2 d\tau' \quad (B.9)
\end{aligned}$$

with \hat{Z}_r , \hat{Z}_β and \hat{Z}_π given by

$$\begin{aligned}
\hat{Z}_r &= \kappa_r\bar{r} + a\sigma_r^2 \int_0^T \zeta^N(\tau)A_r^N(\tau) + \zeta^R(\tau)A_r^R(\tau)d\tau \\
\hat{Z}_\beta &= a\sigma_\beta^2 \int_0^T \zeta^N(\tau)A_\beta^N(\tau) + \zeta^R(\tau)A_\beta^R(\tau)d\tau \\
\hat{Z}_\pi &= \kappa_\pi\bar{\pi} + a\sigma_\pi^2 \int_0^T \zeta^N(\tau)A_\pi^N(\tau)d\tau
\end{aligned}$$

Identifying constant terms in (A.10b) yields

$$\dot{C}^R(\tau) = A_r^R(\tau)\hat{Z}_r + A_\beta^R(\tau)\hat{Z}_\beta - \frac{\sigma_r^2}{2}(A_r^R(\tau))^2 - \frac{\sigma_\beta^2}{2}(A_\beta^R(\tau))^2$$

with the solution

$$\begin{aligned}
C^R(\tau) &= \hat{Z}_r \int_0^\tau A_r^R(\tau')d\tau' + \hat{Z}_\beta \int_0^\tau A_\beta^R(\tau')d\tau' \\
&- \frac{\sigma_r^2}{2} \int_0^\tau (A_r^R(\tau'))^2 d\tau' - \frac{\sigma_\beta^2}{2} \int_0^\tau (A_\beta^R(\tau'))^2 d\tau' \quad (B.10)
\end{aligned}$$

□

Proof of Proposition 1. The effect of a shock to the supply factor to nominal yields is

given by:

$$\frac{\partial y_t^{N,(\tau)}}{\partial \beta_t} = \frac{A_\beta^N(\tau)}{\tau} = \frac{Z_r}{\tau \kappa_r} \left[\frac{1 - e^{-\hat{\kappa}_\beta \tau}}{\hat{\kappa}_\beta} - \frac{e^{-\hat{\kappa}_\beta \tau} - e^{-\kappa_r \tau}}{\kappa_r - \hat{\kappa}_\beta} \right] + \frac{Z_\pi}{\tau \kappa_\pi} \left[\frac{1 - e^{-\hat{\kappa}_\beta \tau}}{\hat{\kappa}_\beta} - \frac{e^{-\hat{\kappa}_\beta \tau} - e^{-\kappa_\pi \tau}}{\kappa_\pi - \hat{\kappa}_\beta} \right]$$

First, we show that Z_r and Z_π are positive. Then we show that the expression in the brackets are positive.

From Equation (B.4a) (B.8) $A_r^N(\tau)$ and $A_\pi^N(\tau)$ are positive and they are increasing as:

$$\frac{\partial A_r^N(\tau)}{\partial \tau} = e^{-\kappa_r \tau} > 0$$

Then, we show that $\int_0^T A_r^N(\tau) \theta^N(\tau) d\tau > 0$ as it can be written as:

$$\begin{aligned} \int_0^T A_r^N(\tau) \theta^N(\tau) d\tau &= \int_0^{\tau^*} A_r^N(\tau) \theta^N(\tau) d\tau + \int_{\tau^*}^T A_r^N(\tau) \theta^N(\tau) d\tau \\ &> A_r^N(\tau^*) \int_0^{\tau^*} \theta^N(\tau) d\tau + A_r^N(\tau^*) \int_{\tau^*}^T \theta^N(\tau) d\tau \\ &= A_r^N(\tau^*) \int_0^T \theta^N(\tau) d\tau \geq 0, \end{aligned}$$

where we used Part (ii) of Assumption 1 in the second step and Part (i) if Assumption 1 in the third step. Therefore $Z_r = a\sigma_r^2 \int_0^T A_r^N(\tau) \theta^N(\tau) d\tau > 0$ and analogously for Z_π . Then, we can write $A_\beta^N(\tau)$ as:

$$A_\beta^N(\tau) = Z_r \int_0^\tau \frac{1 - e^{-\kappa_r \hat{\tau}}}{\kappa_r} e^{-\hat{\kappa}_\beta(\tau - \hat{\tau})} d\hat{\tau} + Z_\pi \int_0^\tau \frac{1 - e^{-\kappa_\pi \hat{\tau}}}{\kappa_\pi} e^{-\hat{\kappa}_\beta(\tau - \hat{\tau})} d\hat{\tau}$$

which is positive as Z_r and Z_π are both positive. The proof for $A_\beta^R(\tau)$ is analogous.

□

Proof of Proposition 2. The effect of the supply factor on duration risk is given by:

$$\frac{\partial \lambda_{r,t}}{\partial \beta_t} = a\sigma_r^2 \int_0^T \theta^N(\tau) A_r^N(\tau) d\tau$$

which is positive as $\int_0^T \theta^N(\tau) A_r^N(\tau) d\tau > 0$, as shown in Proof of Proposition 1. The effect of the supply factor on interest rate risk is analogous.

□

Proof of Proposition 3. This can be seen immediately from (A.11), (A.12), (B.5a) and (B.5b) as Z_r and Z_π both increase in a .

□