4. IIR, harmadfokú Butterworth

$f_c = 2\,kHz \quad n=3$

$f_s = 20\,kHz$

Butterworth átviteli függvénye: $H(s) = \dfrac{1}{\prod_{i=1}^{n}(s-s_i)}$, ahol $s_i = e^{j\pi[(2i+n-1)/2n]}$

$n=3$

$s_1 = e^{j\pi\cdot\frac{4}{6}} = e^{j\pi\cdot\frac{2}{3}} = \cos\frac{2\pi}{3} + j\cdot\sin\frac{2\pi}{3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$s_2 = e^{j\pi} = \cos\pi + j\cdot\sin\pi = -1$

$s_3 = e^{j\pi\cdot\frac{8}{6}} = e^{j\pi\frac{4}{3}} = \cos\left(\frac{4\pi}{3}\right) + j\sin\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{4\pi}{3}-\pi\right) - j\sin\left(\frac{4\pi}{3}-\pi\right) =$

$= -\cos\left(\frac{\pi}{3}\right) - j\cdot\sin\frac{\pi}{3} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

ω_c vágási frekvenciára az átviteli függvény:

$$H_{\omega_c}(s) = \frac{K\cdot\prod_{i=1}^{m}(s-\omega_c\cdot z_i)}{\omega_c^{(m-n)}\cdot\prod_{i=1}^{n}(s-\omega_c\cdot p_i)}$$

m - zérusok száma

n - pólusok száma

K - erősítés

$$H_{\omega_c}(s) = \frac{\omega_c^3}{(s-\omega_c\cdot p_1)\cdot(s-\omega_c\cdot p_2)\cdot(s-\omega_c\cdot p_3)}$$

$p_1 = s_1 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$p_2 = s_2 = -1$

$p_3 = s_3 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$$H_{\omega_c}(s) = \frac{\omega_c^3}{\left[s-\omega_c\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right]\cdot(s+\omega_c)\cdot\left[s-\omega_c\cdot\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\right]}$$

$m=0$

$n=3$

Részlettörtre bontás, A_1, A_2, A_3 kiszámítása:

$$\frac{A_1}{s-\omega_c\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)} + \frac{A_2}{s+\omega_c} + \frac{A_3}{s-\omega_c\cdot\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)} = \frac{A_1\cdot(s+\omega_c)\cdot\left[s-\omega_c\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right] + A_2\cdot\left[s-\omega_c\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right]\cdot\left[s-\omega_c\cdot\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\right] + A_3\cdot\left[s-\omega_c\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right]\cdot(s+\omega_c)}{\left[s-\omega_c\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right]\cdot(s+\omega_c)\cdot\left[s-\omega_c\cdot\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\right]}$$

számláló: $A_1\cdot\left[s^2 - s\cdot\omega_c\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right) + s\cdot\omega_c - \omega_c^2\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right] +$

$+A_2\cdot\left[s^2 - s\omega_c\cdot\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right) - s\omega_c\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right) + \omega_c^2\cdot 1\right] + A_3\cdot\left[s^2 + s\cdot\omega_c -\right.$

$\left.- s\cdot\omega_c\cdot\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right) - \omega_c^2\cdot\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\right] =$

$$= A_1 \cdot \left[s^2 + s\cdot\omega_c\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) - \omega_c^2\cdot\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right] + A_2\cdot\left[s^2 + s\cdot\omega_c +\right.$$

$$\left.+\,\omega_c^2\right] + A_3\cdot\left[s^2 + s\cdot\left(\frac{3}{2}\omega_c - j\omega_c\frac{\sqrt{3}}{2}\right) - \omega_c^2\cdot\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

Egyenlővé teszem ω_c^3-al

$$\begin{cases} A_1 + A_2 + A_3 = 0 \\ A_1\cdot\omega_c\cdot\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) + A_2\cdot\omega_c + A_3\cdot\omega_c\cdot\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) = 0 \quad /:\omega_c \\ -A_1\cdot\omega_c^2\cdot\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + A_2\cdot\omega_c^2 - A_3\cdot\omega_c^2\cdot\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \cancel{0}\,\omega_c^3 \quad /:\omega_c^2 \end{cases}$$

$$\begin{cases} A_1 + A_2 + A_3 = 0 & \mathrm{I} \\ A_1\cdot\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) + A_2 + A_3\cdot\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) = 0 & \mathrm{II} \\ -A_1\cdot\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + A_2 - A_3\cdot\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \omega_c & \mathrm{III} \end{cases}$$

$$\mathrm{II} - \mathrm{III} \Rightarrow \frac{3}{2}A_1 + \cancel{A_1\cdot j\frac{\sqrt{3}}{2}} + A_2 + \frac{3}{2}A_3 - \cancel{A_3\cdot j\frac{\sqrt{3}}{2}} - \frac{1}{2}A_1 - j\frac{\sqrt{3}}{2}A_1 - A_2 -$$

$$-\frac{1}{2}A_3 + \cancel{A_3\cdot j\frac{\sqrt{3}}{2}} = -\omega_c$$

$$A_1 + A_3 = -\omega_c$$

$$\mathrm{I} \quad A_1 + A_2 + A_3 = 0$$

$$\boxed{A_2 = \omega_c}$$

$$\begin{cases} A_1 + A_3 = -\omega_c & \Rightarrow A_1 = -\omega_c - A_3 \\ A_1\cdot\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) + A_3\cdot\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) = -\omega_c \\ +A_1\cdot\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + A_3\cdot\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = 0 \end{cases}$$

$$(\mathrm{II}) \quad (-\omega_c - A_3)\cdot\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) + A_3\cdot\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) = -\omega_c$$

$$-\frac{3}{2}\omega_c - j\omega_c\cdot\frac{\sqrt{3}}{2} - \cancel{\frac{3}{2}A_3} - A_3\cdot j\frac{\sqrt{3}}{2} + \cancel{\frac{3}{2}\cdot A_3} - A_3\cdot j\frac{\sqrt{3}}{2} = -\omega_c$$

$$-A_3\cdot j\cdot\sqrt{3} = \overbrace{-\omega_c + \frac{3}{2}\omega_c}^{\frac{1}{2}\omega_c} + j\omega_c\cdot\frac{\sqrt{3}}{2} \quad /:j\sqrt{3}$$

$$-A_3 = \frac{\frac{1}{2}\omega_c + j\omega_c \cdot \frac{\sqrt{3}}{2}}{j\sqrt{3}} \cdot j\sqrt{3} = \frac{j\frac{\sqrt{3}}{2}\omega_c - \omega_c \cdot \frac{3}{2}}{-3}$$

$$A_3 = -\omega_c \cdot \frac{3}{6} + j \cdot \frac{\sqrt{3}}{6}\omega_c = -\frac{1}{2}\omega_c + j \cdot \frac{\sqrt{3}}{6}\omega_c$$

(I) $A_1 + A_3 = -\omega_c$

$$A_1 - \frac{1}{2}\omega_c + j\frac{\sqrt{3}}{6}\omega_c = -\omega_c$$

$$A_1 = -\omega_c + \frac{1}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c$$

$$\boxed{\begin{aligned} A_1 &= -\frac{1}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c \\ A_2 &= +\omega_c \\ A_3 &= -\frac{1}{2}\omega_c + j\frac{\sqrt{3}}{6}\omega_c \end{aligned}}$$

$$H(z) = \sum_{k=1}^{n} \frac{A_k}{1 - \exp(s_k' \cdot T) \cdot z^{-1}}$$

$$H(z) = \frac{A_1}{1 - \underbrace{\exp\left(\omega_c \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) \cdot T\right)}_{e_1} \cdot z^{-1}} +$$

$$+ \frac{A_2}{1 - \underbrace{\exp(-\omega_c \cdot T)}_{e_2} \cdot z^{-1}} + \frac{A_3}{1 - \underbrace{\exp\left(\omega_c \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \cdot T\right)}_{e_3} \cdot z^{-1}} =$$

$$= \frac{A_1 \cdot (1 - e_2 \cdot z^{-1}) \cdot (1 - e_3 \cdot z^{-1}) + A_2 \cdot (1 - e_1 \cdot z^{-1}) \cdot (1 - e_3 \cdot z^{-1}) + A_3 \cdot (1 - e_1 \cdot z^{-1}) \cdot (1 - e_2 \cdot z^{-1})}{(1 - e_1 \cdot z^{-1}) \cdot (1 - e_2 \cdot z^{-1}) \cdot (1 - e_3 \cdot z^{-1})}$$

$$s_1' = \omega_c \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$s_2' = -\omega_c$$

$$s_3' = \omega_c \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$A_1 = -\frac{1}{2}\omega_c - \frac{\sqrt{3}}{6}\omega_c \cdot j$$

$$A_2 = \omega_c$$

$$A_3 = -\frac{1}{2}\omega_c + \frac{\sqrt{3}}{6}\omega_c \cdot j$$

$$\omega_c \cdot T = \frac{4000\pi}{20.000} = \frac{\pi}{5}$$

$$e_1 = \exp\left(\frac{\pi}{5} \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right) = \exp\left(-\frac{\pi}{10} + j\frac{\pi\sqrt{3}}{10}\right)$$

$$e_2 = \exp\left(-\frac{\pi}{5}\right)$$

$$e_3 = \exp\left(-\frac{\pi}{10} - j\frac{\pi\sqrt{3}}{10}\right)$$

$$(1 - e_2 \cdot z^{-1}) \cdot (1 - e_3 \cdot z^{-1}) = 1 - e_3 \cdot z^{-1} - e_2 \cdot z^{-1} + e_2 \cdot e_3 \cdot z^{-2} = 1 - z^{-1} \cdot (e_3 + e_2) +$$
$$+ e_2 \cdot e_3 \cdot z^{-2} = 1 - z^{-1} \cdot \left[\exp\left(-\frac{\pi}{10} - j\frac{\pi\sqrt{3}}{10}\right) + \exp\left(-\frac{\pi}{5}\right)\right] + \left[\exp\left(-\frac{3\pi}{10} - j\frac{\pi\sqrt{3}}{10}\right)\right] \cdot z^{-2}$$

$$(1 - e_1 \cdot z^{-1}) \cdot (1 - e_3 \cdot z^{-1}) = 1 - e_3 \cdot z^{-1} - e_1 \cdot z^{-1} + e_1 \cdot e_3 \cdot z^{-2} = 1 - z^{-1} \cdot (e_3 + e_1) + e_1 \cdot e_3 \cdot z^{-2} =$$
$$= 1 - z^{-1} \cdot \left[\exp\left(-\frac{\pi}{10} - j\frac{\pi\sqrt{3}}{10}\right) + \exp\left(-\frac{\pi}{10} + j\frac{\pi\sqrt{3}}{10}\right)\right] + z^{-2} \cdot \exp\left(-\frac{\pi}{10} + j\frac{\pi\sqrt{3}}{10} - \frac{\pi}{10} - j\frac{\pi\sqrt{3}}{10}\right) =$$
$$= 1 - z^{-1} \cdot \left[\exp\left(-\frac{\pi}{10} - j\frac{\pi\sqrt{3}}{10}\right) + \exp\left(-\frac{\pi}{10} + j\frac{\pi\sqrt{3}}{10}\right)\right] + z^{-2} \cdot \exp\left(-\frac{\pi}{5}\right)$$

$$(1 - e_1 \cdot z^{-1}) \cdot (1 - e_2 \cdot z^{-1}) = 1 - z^{-1} \cdot (e_2 + e_1) + z^{-2} \cdot e_1 \cdot e_2 = 1 - z^{-1} \cdot \left[\exp\left(-\frac{\pi}{5}\right) + \exp\left(-\frac{\pi}{10} + j\frac{\pi\sqrt{3}}{10}\right)\right] +$$
$$+ z^{-2} \cdot \exp\left(-\frac{3\pi}{10} + j\frac{\pi\sqrt{3}}{10}\right)$$

Számláló kiszámítása:

$$A_1 \cdot \left(1 - z^{-1} \cdot (e_3 + e_2) + e_2 \cdot e_3 \cdot z^{-2}\right) + A_2 \cdot \left(1 - z^{-1} \cdot (e_3 + e_1) + e_1 \cdot e_3 \cdot z^{-2}\right) +$$
$$+ A_3 \cdot \left(1 - z^{-1} \cdot (e_2 + e_1) + z^{-2} \cdot e_1 \cdot e_2\right) = A_1 + A_2 + A_3 + z^{-1} \cdot (-A_1 \cdot e_3 - A_1 \cdot e_2 -$$
$$- A_2 \cdot e_3 - A_2 \cdot e_1 - A_3 \cdot e_2 - A_3 \cdot e_1) + z^{-2} \cdot (A_1 \cdot e_2 \cdot e_3 + A_2 \cdot e_1 \cdot e_3 +$$
$$+ A_3 \cdot e_1 \cdot e_2)$$

Nevező kiszámítása:

$$\left(1-p_1\cdot z^{-1}\right)\cdot\left(1-p_2\cdot z^{-1}\right)\cdot\left(1-p_3\cdot z^{-1}\right)=\left(1-p_2\cdot z^{-1}-p_1\cdot z^{-1}+p_1\cdot p_2\cdot z^{-2}\right)\cdot\left(1-p_3\cdot z^{-1}\right)=$$

$$=1-p_2\cdot z^{-1}-p_1\cdot z^{-1}+p_1\cdot p_2\cdot z^{-2}-p_3\cdot z^{-1}+p_2\cdot p_3\cdot z^{-2}+p_1\cdot p_3\cdot z^{-2}-$$

$$-p_1\cdot p_2\cdot p_3\cdot z^{-3}=1-z^{-1}\cdot\left(p_1+p_2+p_3\right)+z^{-2}\cdot\left(p_1\cdot p_2+p_2\cdot p_3+p_1\cdot p_3\right)-$$

$$-p_1\cdot p_2\cdot p_3\cdot z^{-3}$$

z^{-1}

▶ $\underline{p_1+p_2+p_3}=\exp\left(-\frac{\pi}{10}+j\frac{\pi\sqrt3}{10}\right)+\exp\left(-\frac{\pi}{5}\right)+\exp\left(-\frac{\pi}{10}-j\frac{\pi\sqrt3}{10}\right)=$

$$=\exp\left(-\frac{\pi}{10}\right)\cdot\exp\left(j\frac{\pi\sqrt3}{10}\right)+\exp\left(-\frac{\pi}{10}\right)\cdot\exp\left(-j\frac{\pi\sqrt3}{10}\right)+\exp\left(-\frac{\pi}{5}\right)=$$

$$=\exp\left(-\left(\frac{\pi}{10}\right)\right)\cdot\left[\exp\left(j\frac{\pi\sqrt3}{10}\right)+\exp\left(-j\frac{\pi\sqrt3}{10}\right)\right]+\exp\left(-\frac{\pi}{5}\right)=$$

$$=\boxed{\exp\left[-\frac{\pi}{10}\right]\cdot\left(2\cos\frac{\pi\sqrt3}{10}\right)+\exp\left(-\frac{\pi}{5}\right)}=1.78$$

$$\exp j\frac{\pi\sqrt3}{10}=\cos\frac{\pi\sqrt3}{10}+j\sin\frac{\pi\sqrt3}{10}$$

$$\exp -j\frac{\pi\sqrt3}{10}=\cos\left(\frac{\pi\sqrt3}{10}\right)-j\sin\frac{\pi\sqrt3}{10}$$

z^{-2}

▶ $\underline{p_1\cdot p_2+p_2\cdot p_3+p_1\cdot p_3}=\exp\left(-\frac{\pi}{10}+j\frac{\pi\sqrt3}{10}-\frac{\pi}{5}\right)+\exp\left(-\frac{\pi}{5}-\frac{\pi}{10}-j\frac{\pi\sqrt3}{10}\right)+$

$$+\exp\left(-\frac{\pi}{10}+\cancel{j\frac{\pi\sqrt3}{10}}-\frac{\pi}{10}\cancel{-j\frac{\pi\sqrt3}{10}}\right)=\exp\left(-\frac{3\pi}{10}+j\frac{\pi\sqrt3}{10}\right)+\exp\left(-\frac{3\pi}{10}-j\frac{\pi\sqrt3}{10}\right)+$$

$$+\exp\left(-\frac{\pi}{5}\right)=\exp\left(-\frac{3\pi}{10}\right)\cdot\left[\exp\left(j\frac{\pi\sqrt3}{10}\right)+\exp\left(-j\frac{\pi\sqrt3}{10}\right)\right]+\exp\left(-\frac{\pi}{5}\right)=$$

$$=\exp\left(-\frac{3\pi}{10}\right)\cdot\left(\cos\frac{\pi\sqrt3}{10}+\cancel{j\sin\frac{\pi\sqrt3}{10}}+\cos\frac{\pi\sqrt3}{10}\cancel{-j\sin\frac{\pi\sqrt3}{10}}\right)+\exp\left(-\frac{\pi}{5}\right)=$$

$$=\boxed{\exp\left(-\frac{3\pi}{10}\right)\cdot\left(2\cdot\cos\frac{\pi\sqrt3}{10}\right)+\exp\left(-\frac{\pi}{5}\right)}=\cancel{1.06}\ 0.53$$

z^{-3}

▶ $p_1\cdot p_2\cdot p_3=\exp\left(-\frac{\pi}{10}+\cancel{j\frac{\pi\sqrt3}{10}}-\frac{\pi}{5}-\frac{\pi}{10}\cancel{-j\frac{\pi\sqrt3}{10}}\right)\boxed{=\exp\left(-\frac{2\pi}{5}\right)}=0.28$

Nevező: $1-z^{-1}\cdot\left[\left[\exp\left(-\frac{\pi}{10}\right)\right]\cdot2\cos\frac{\pi\sqrt3}{10}+\exp\left(-\frac{\pi}{5}\right)\right]+z^{-2}\cdot\left[\exp\left(-\frac{3\pi}{10}\right)\cdot\right.$

$$\left.\cdot\left(2\cdot\cos\frac{\pi\sqrt3}{10}\right)+\exp\left(-\frac{\pi}{5}\right)\right]-z^{-3}\cdot\exp\left(-\frac{2\pi}{5}\right)=1-1.78\cdot z^{-1}+0.53\cdot z^{-2}-0.28\cdot z^{-3}$$

z^0

▶ $A_1 + A_2 + A_3 = -\frac{1}{2}\omega_c - \cancel{j\frac{\sqrt{3}}{6}\omega_c} + \omega_c - \frac{1}{2}\omega_c + \cancel{j\frac{\sqrt{3}}{6}\omega_c} = 0$

z^{-1}

▶ $-A_1 \cdot l_3 - A_1 \cdot l_2 - A_2 \cdot l_3 - A_2 \cdot l_1 - A_3 \cdot l_2 - A_3 \cdot l_1 =$

$= l_1 \cdot (-A_2 - A_3) + l_2 \cdot (-A_1 - A_3) + l_3 \cdot (-A_1 - A_2) = \cancel{l_1 \cdot \left(-\frac{3}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c\right) +}$

$\cancel{+ l_2 \cdot \omega_c + l_3 \cdot \left(\frac{1}{2}\omega_c + j\frac{\sqrt{3}}{6}\omega_c\right)} = l_1 \cdot \left(-\omega_c + \frac{1}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c\right) +$

$+ l_2 \cdot \left(\frac{1}{2}\omega_c + \cancel{j\frac{\sqrt{3}}{6}\omega_c} + \frac{1}{2}\omega_c - \cancel{j\frac{\sqrt{3}}{6}\omega_c}\right) + l_3 \cdot \left(\frac{1}{2}\omega_c + j\frac{\sqrt{3}}{6}\omega_c - \omega_c\right) =$

$= l_1 \cdot \left(-\frac{1}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c\right) + l_2 \cdot \omega_c + l_3 \cdot \left(-\frac{1}{2}\omega_c + j\frac{\sqrt{3}}{6}\omega_c\right) =$

(j - s togole kiesnele)

$= \exp\left(-\frac{\pi}{10}\right) \cdot \left[\left(\cos\frac{\pi\sqrt{3}}{10} + j\sin\frac{\pi\sqrt{3}}{10}\right) \cdot \left(-\frac{1}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c\right) + \left(\cos\frac{\pi\sqrt{3}}{10} - j\sin\frac{\pi\sqrt{3}}{10}\right) \cdot \left(-\frac{1}{2}\omega_c + j\frac{\sqrt{3}}{6}\omega_c\right)\right] +$

$+ \exp\left(-\frac{\pi}{5}\right) \cdot \omega_c = \exp\left(-\frac{\pi}{10}\right) \cdot \left[-\frac{1}{2} \cdot \omega_c \cdot \cos\frac{\pi\sqrt{3}}{10} + \frac{\sqrt{3}}{6} \cdot \omega_c \cdot \sin\frac{\pi\sqrt{3}}{10} - \frac{1}{2} \cdot \omega_c \cos\frac{\pi\sqrt{3}}{10} +\right.$

$\left.+ \frac{\sqrt{3}}{6} \cdot \sin\frac{\pi\sqrt{3}}{10}\right] + \exp\left(-\frac{\pi}{5}\right) \cdot \omega_c = \exp\left(-\frac{\pi}{10}\right) \cdot \left[-\omega_c \cdot \cos\frac{\pi\sqrt{3}}{10} + \frac{\sqrt{3}}{3} \cdot \omega_c \cdot \sin\frac{\pi\sqrt{3}}{10}\right] +$

$+ \exp\left(-\frac{\pi}{5}\right) \cdot \omega_c = 1594.43$

z^{-2}

▶ $A_1 \cdot l_2 \cdot l_3 + A_2 \cdot l_1 \cdot l_3 + A_3 \cdot l_1 \cdot l_2 = \omega_c \cdot \exp\left(-\frac{\pi}{5}\right) + \exp\left(-\frac{3\pi}{10}\right) \cdot \left[-\omega_c \cos\frac{\pi\sqrt{3}}{10} - \frac{\sqrt{3}}{3} \cdot \omega_c \cdot \sin\frac{\pi\sqrt{3}}{10}\right] =$

$A_1 \cdot l_2 \cdot l_3 = \left(-\frac{1}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c\right) \cdot \exp\left[-\frac{\pi}{5} - \frac{\pi}{10} - j\frac{\pi\sqrt{3}}{10}\right] = \left(-\frac{1}{2}\omega_c - j\frac{\sqrt{3}}{6}\omega_c\right) \cdot \exp\left(-\frac{3\pi}{10}\right) \cdot \exp\left(-j\frac{\pi\sqrt{3}}{10}\right)$

$A_2 \cdot l_1 \cdot l_3 = \omega_c \cdot \exp\left[-\frac{\pi}{10} + \cancel{j\frac{\pi\sqrt{3}}{10}} - \frac{\pi}{10} - \cancel{j\frac{\pi\sqrt{3}}{10}}\right] = \omega_c \cdot \exp\left(-\frac{\pi}{5}\right)$

$A_3 \cdot l_1 \cdot l_2 = \left(-\frac{1}{2}\omega_c + j\frac{\sqrt{3}}{6} \cdot \omega_c\right) \cdot \exp\left(-\frac{3\pi}{10}\right) \cdot \exp\left(j\frac{\pi\sqrt{3}}{10}\right)$

z^{-2}

▶ $= 1051.06$

$\omega_c = 4000\pi$

$$H_{\omega_c}(z) = \frac{1594.43 \cdot z^{-1} + 1051.06 \cdot z^{-2}}{1 - 1.78 \cdot z^{-1} + 0.53 \cdot z^{-2} - 0.28 \cdot z^{-3}}$$

$$\frac{Y(z)}{U(z)} = \frac{1594.43 \cdot z^{-1} + 1051.06 \cdot z^{-2}}{1 - 1.78 \cdot z^{-1} + 0.53 \cdot z^{-2} - 0.28 \cdot z^{-3}}$$

$Y(z) - 1.78 \cdot z^{-1} \cdot Y(z) + 0.53 \cdot z^{-2} \cdot Y(z) - 0.28 \cdot z^{-3} \cdot y(z) = U(z) \cdot 1594.43 \cdot z^{-1} + 1051.06 \cdot z^{-2}$

↓ z transformacija

$y[k] - 1.78 \cdot y[k-1] + 0.53 \cdot y[k-2] - 0.28 \cdot y[k-3] = 1594.43 \cdot u[k-1] + 1051.06 \cdot u[k-2]$

$y[k] = 1.78 \cdot y[k-1] - 0.53 \cdot y[k-2] - 0.28 \cdot y[k-3] + 1594.43 \cdot u[k-1] +$

$+ 1051.06 \cdot u[k-2]$

II. direkt megvalósítási forma