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Deep Reinforcement Learning

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Deep Structured Learning Course
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Outline of the lecture

- **Part I: RL Primer**
  - The RL Problem
  - Markov Decision Process - A Model for RL Problems
  - Optimality & Dynamic Programming
  - Monte Carlo Approaches
  - Temporal Difference Learning
  - The Policy Gradient Theorem

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Outline of the lecture

- **Part II: Deep RL**
  - From RL to Deep RL
  - DQN
  - Deep advantage actor-critic methods
  - Trust region methods

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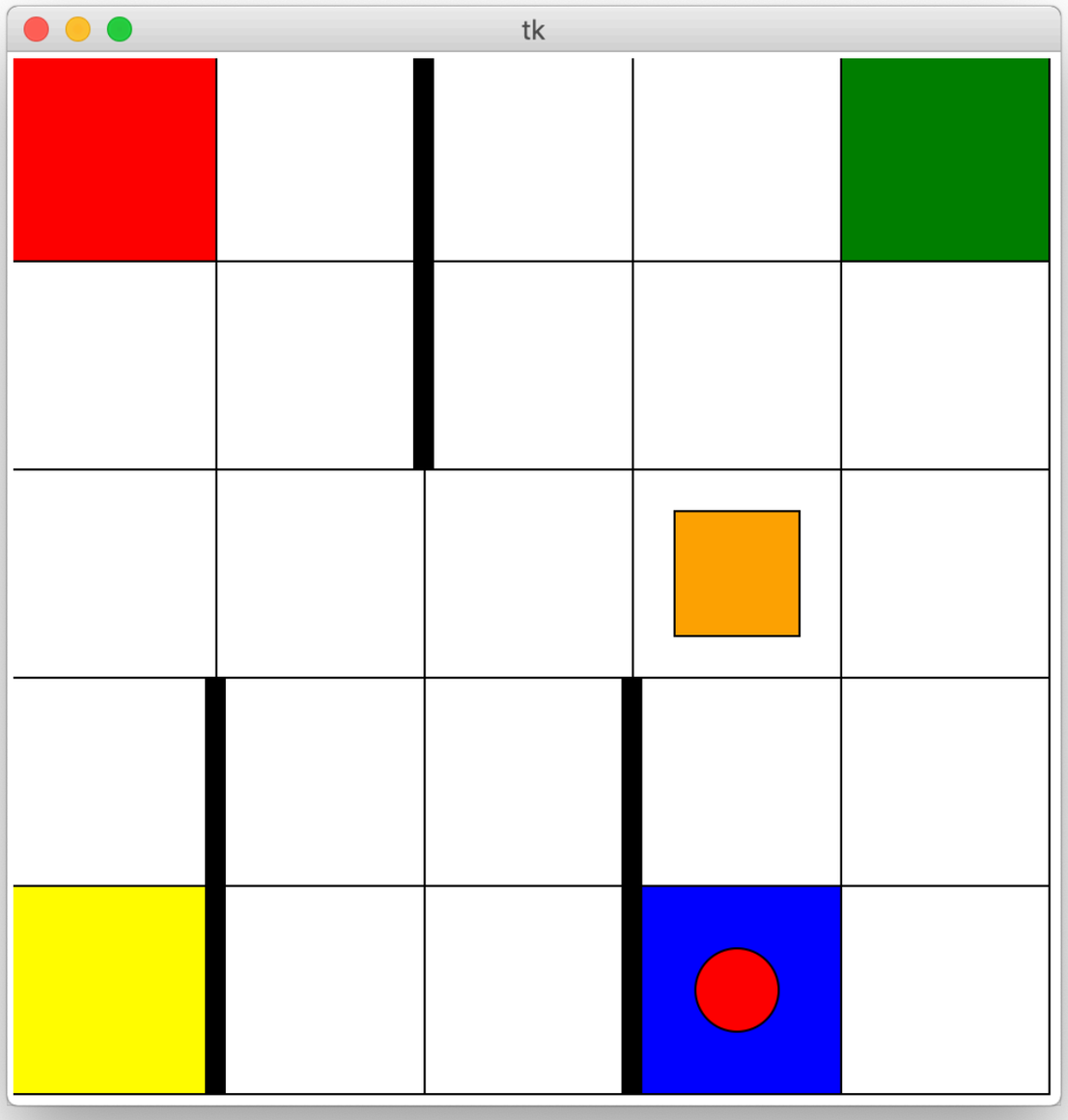
The RL Problem



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The RL Problem

- Ingredients for success:
  - You learned as you played the game
  - You experimented the different actions
  - As soon as you figured out the goal of the game, you stopped experimenting
  - You used the feedback you got (n. of steps) to figure out the goal of the game
  - When pursuing the goal, you had to think ahead to select the actions

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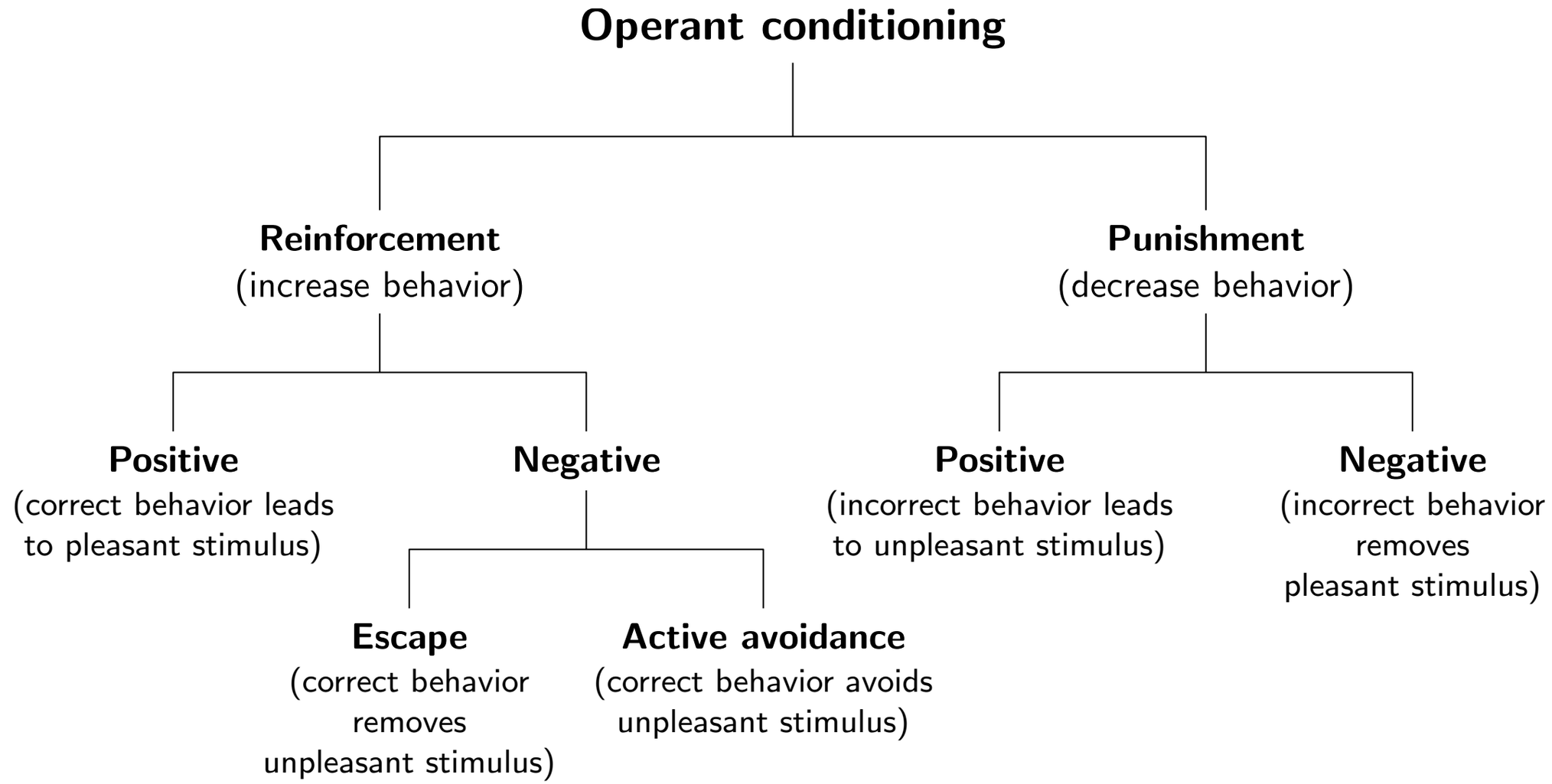
The RL Problem

What is RL?

- Inspired on theory of operant conditioning



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What is RL?

- Computational “counterpart” to operant conditioning
- Class of problems and algorithms to solve those problems
- Learning takes place through the interaction between agent and environment (learning by trial-and-error)
- Learning driven by a “reinforcement signal” rather than examples

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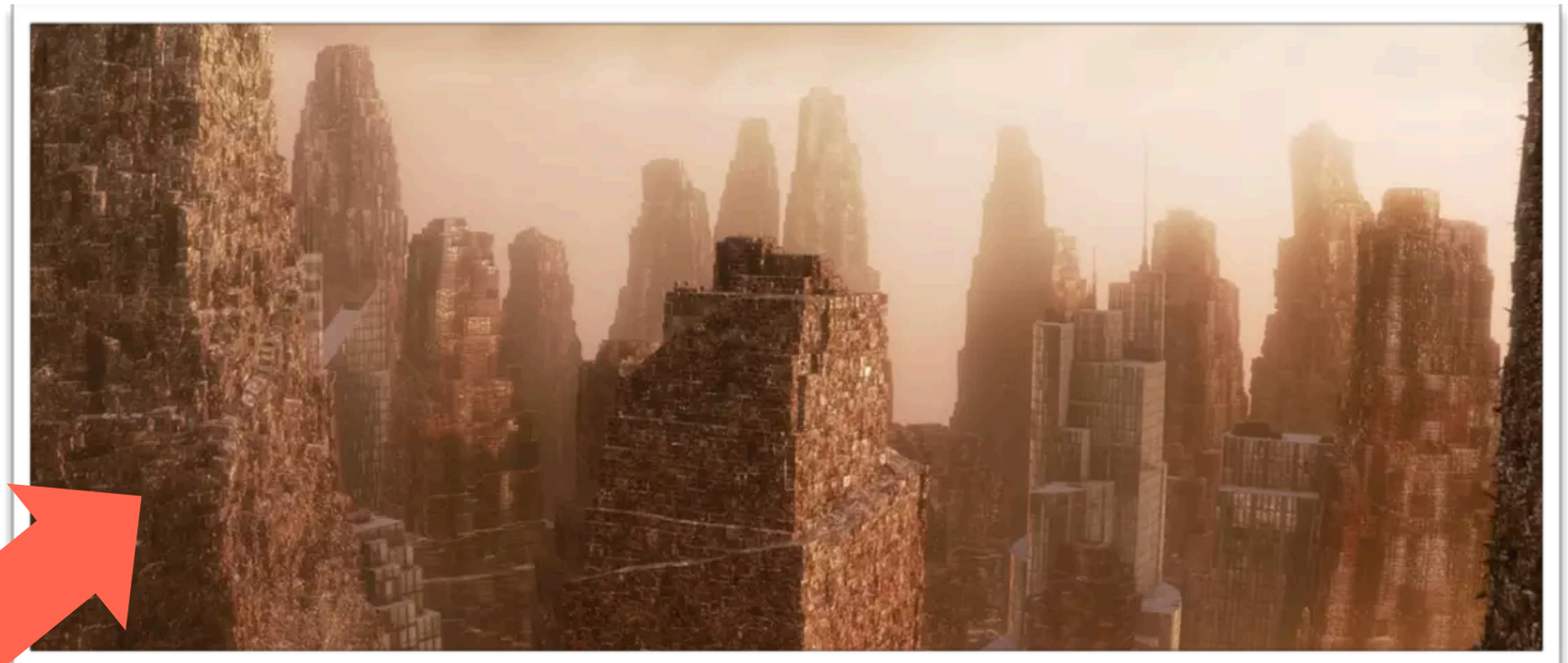
Elements in RL

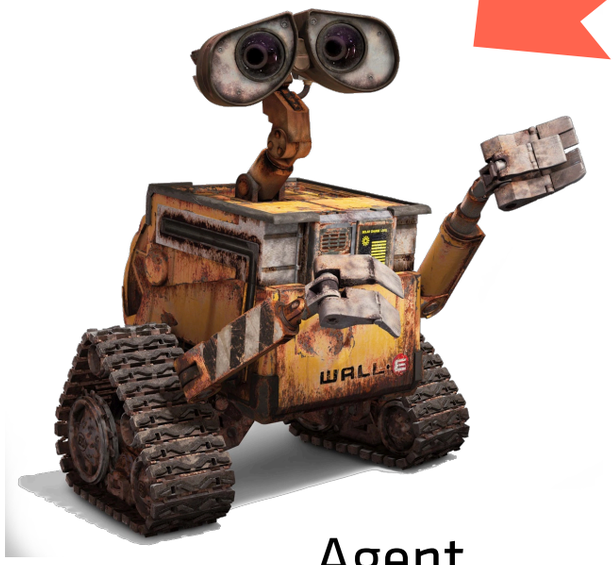
- Key elements in RL:
  - Interactive learning
  - Learning from evaluative feedback
  - Tradeoff between exploration and exploitation
  - Actions impact the future (temporal credit assignment)

Interactive learning

Environment



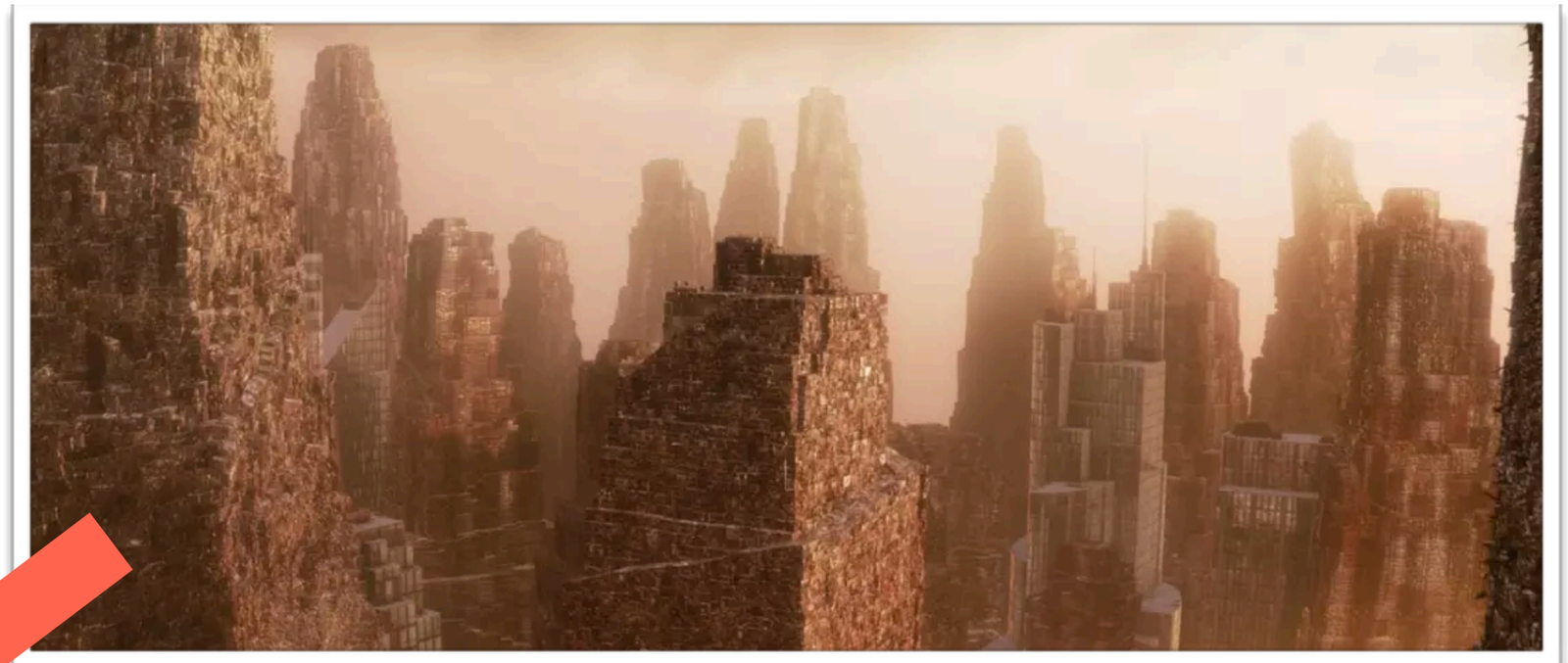
Interaction



Agent

Interactive learning

Environment



State

Agent

Interactive learning

Environment

Action

Agent

Interactive learning

Environment may change state

Reward

Agent

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Markov decision process

- Formalizing the reinforcement learning problem:
  - The state of the world/environment at step *t* is denoted as S_t
  - The state takes values in some set $\mathcal{S}$ (the state space)

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Markov decision process

- Formalizing the reinforcement learning problem:
  - The action of the agent at step *t* is denoted as A_t
  - The action takes values in some set $\mathcal{A}$ (the action space)

Markov decision process

- Formalizing the reinforcement learning problem:
  - Upon performing an action at time step *t*, the agent gets a (random) reward R_t
  - The reward depends on the state S_t and action A_t as

$$\mathbb{E}\left[R_t\right] = r(S_t, A_t)$$

- We call *r* the reward function

Markov decision process

- Formalizing the reinforcement learning problem:
  - As a result of the agent's action at time step t, the state of the environment at time step $t + 1$ may change
- We assume that the evolution of the state verifies the Markov property:

$$\mathbb{P}\left[S_{t+1} = s \mid \boldsymbol{S}_{0:t} = \boldsymbol{s}_{0:t}, \boldsymbol{A}_{0:t} = \boldsymbol{a}_{0:t}\right] = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s_t, A_t = a_t\right]$$

Knowledge of the past...

... is subsumed in the present

Markov decision process

- Formalizing the reinforcement learning problem:
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- We call these the transition probabilities, and write

$$\mathbf{P}(s' \mid s, a) = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s, A_t = a\right]$$

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Markov decision process

- A Markov decision process is defined as a tuple $(\mathcal{S}, \mathcal{A}, \{\mathbf{P}_a, a \in \mathcal{A}\}, r)$
  - $\mathcal{S}$ is the state space
  - $\mathcal{A}$ is the action space
  - For each action $a \in \mathcal{A}$, $\mathbf{P}_a$ is a matrix with entry ss' given by $\mathbf{P}(s' \mid s, a)$
  - r is the reward function

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... so what?

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Optimality

- A Markov decision process is not actually a problem
  - Provides a mere descriptive model for RL problems
  - What does it mean to solve a model??

Objective

Optimality

- We thus formulate a Markov decision problem (MDP) as follows:

Given a Markov decision process and a function

$$J(\{R_t, t = 1, \ldots, \})$$

how can we select the actions $\{A_t\}$ to maximize J?

Policies

- MDPs are formulated in terms of action selection
- A policy is an “action selection rule”:
- Define the history at time step t as

$$H_t = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t\}$$

- It is a random variable
- Depends on the particular action selection

Policies

- A policy is a mapping π between histories and distributions over actions:

$$\pi : \mathcal{H} \rightarrow \Delta(\mathcal{A})$$

Set of all finite histories

Set of distributions over actions

Policies

- **Types of policies:**
  - Deterministic policies - Each history is mapped to exactly one action

$$\pi : \mathcal{H} \to \mathcal{A}$$

  - Markov policies - Depend only on the most recent state (may be time-dependent)

$$\pi_t : \mathcal{S} \to \Delta(\mathcal{A})$$

  - Stationary policies - Depend only on the most recent state (is time-independent)

$$\pi : \mathcal{S} \to \Delta(\mathcal{A})$$

Optimality criteria

- J in the previous formulation is the optimality criterion
- There are several possible optimality criteria in the literature
  - Each has advantages and disadvantages
  - The choice should be problem-driven

Optimality criteria

- **(Expected) immediate reward:**

$$J(\{R_t, t = 1, \dots, \}) = \mathbb{E}\left[R_t\right] = r(S_t, A_t)$$

- Advantages:
  - Simple to optimize:

$$\pi(S_t) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, r(S_t, a)$$

- Disadvantages:
  - Only applicable in very specific problems

Optimality criteria

- **(Expected) total reward:**

$$J(\{R_t, t = 1, \dots, \}) = \mathbb{E}\left[\sum_{t=0}^{\infty} R_t\right]$$

- Advantages:
  - Not myopic
- Disadvantages:
  - Objective not always well-defined (summation may diverge)

Optimality criteria

- **(Expected) average per-step reward:**

$$J(\{R_t, t = 1, \dots, \}) = \lim_{T \to \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T} R_t \right]$$

- Advantages:
  - Not myopic
  - Independent of initial state of the process
- Disadvantages:
  - Sometimes cumbersome to work with

Optimality criteria

- **(Expected) total discounted reward:**

$$J(\{R_t, t = 1, \dots, \}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_t\right]$$

Discount
$0 \leq \gamma < 1$

- Advantages:
  - Not myopic
  - “Economical” interpretation
- Disadvantages:
  - Depends on the initial state of the process

We henceforth focus on this criterion

Markov decision problem (MDP)

- A Markov decision problem is defined as a tuple $(\mathcal{S}, \mathcal{A}, \{\mathbf{P}_a, a \in \mathcal{A}\}, r, \gamma)$
  - $\mathcal{S}$ is the state space
  - $\mathcal{A}$ is the action space
  - For each action $a \in \mathcal{A}$, $\mathbf{P}_a$ is a matrix with entry ss' given by $\mathbf{P}(s' \mid s, a)$
  - r is the reward function
  - γ is the discount

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Solving MDPs

Value function

- Let us consider a fixed stationary policy π
  - Action depends only on current state
  - Invariant through time
- In other words,

$$\pi(a \mid s) = \mathbb{P}\left[A_t = a \mid S_t = s\right]$$

Independent of t

Value function

- The value of J depends on the initial state
- Let

$$v_\pi(s) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t R_t \mid S_0 = s, \right]$$

- $v_\pi(s)$ is the value of J when
  - The agent follows policy π, i.e.,

$$A_t \sim \pi(\cdot \mid S_t)$$

  - The initial state is s

Value function

- The function

$$v_\pi : \mathcal{S} \to \mathbb{R}$$

  is called a value function

- It is the value function associated with π

- It verifies the recursive relation

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

Immediate reward

Future total discounted reward

A computational (parenthesis)

- The relation

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

  offers two possibilities to compute v_π

- Solve the associated (linear) system of equations

- Starting with an arbitrary initial estimate $v^{(0)}$, repeatedly go over the update

$$v^{(k+1)}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v^{(k)}(s') \right]$$

A computational (parenthesis)

- The iterative approach with update

$$v^{(k+1)}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^{(k)}(s') \right]$$

  is known as value iteration

- Computing the value function associated with a policy is usually referred as the prediction problem

- It is a dynamic programming approach that, intuitively, “propagates” reward information back through time

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... moving on...

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Optimal policy

- We say that a policy π^* is optimal if and only if

$$v_{\pi^*}(s) \geq v_{\pi}(s), \forall \pi, \forall s \in \mathcal{S}$$

- That such a policy exists is a central result in the theory of MDPs

Solving MDP = Computing an optimal policy

Value function 2.0

- The value function for the (an) optimal policy is simply denoted as v^*

- It verifies the recursive relation

$$v^*(s) = \max_{a \in \mathcal{A}} \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^*(s') \right]$$

- The optimal policy can be computed from v^* as

$$\pi^*(s) = \operatorname*{argmax}_{a \in \mathcal{A}} \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^*(s') \right]$$

A computational (parenthesis) 2.0

- The relation

$$v^*(s) = \max_{a \in \mathcal{A}} \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^*(s') \right]$$

also offers a possibility to compute v^*

- Starting with an arbitrary initial estimate $v^{(0)}$, repeatedly go over the update

$$v^{(k+1)}(s) \leftarrow \max_{a \in \mathcal{A}} \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^{(k)}(s') \right]$$

- An MDP can thus be solved by computing v^* (and π^* from it)

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...

Value function 3.0

- **Other useful value functions to be considered**
  - Action-value function (or Q-function) associated with a policy:

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

$$q_\pi(s,a)$$

Value function 3.0

- **Other useful value functions to be considered**
  - Action-value function (or Q-function) associated with a policy:

$$q_\pi(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v_\pi(s')$$

  - It verifies the recursive relation

$$q_\pi(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \sum_{a' \in \mathcal{A}} \pi(a' \mid s') q_\pi(s',a')$$

Value function 3.0

- **Other useful value functions to be considered**
  - Optimal action-value function (or Q-function):

$$v^*(s) = \max_{a \in \mathcal{A}} \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v^*(s') \right]$$

$$q^*(s,a)$$

Value function 3.0

- **Other useful value functions to be considered**
  - Optimal action-value function (or Q-function):

$$q^*(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v^*(s')$$

  - It verifies the recursive relation

$$q^*(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \max_{a' \in \mathcal{A}} q^*(s',a')$$

  - Moreover,

$$\pi^*(s) = \operatorname*{argmax}_{a \in \mathcal{A}} q^*(s,a)$$

...

- We can compute qπ and q* using similar iterative approaches

$$q^{(k+1)}(s,a) \leftarrow r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \max_{a' \in \mathcal{A}} q^{(k)}(s',a')$$

$$q^{(k+1)}(s,a) \leftarrow r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \sum_{a' \in \mathcal{A}} \pi(a' \mid s') q^{(k)}(s',a')$$

  which are all collectively known as value iteration

- Computing the optimal Q-function is usually referred as the control problem

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Value function 3.0

- **Other useful value functions to be considered**
  - Advantage function associated with a policy:

$$\mathrm{adv}_\pi(s,a) = q_\pi(s,a) - v_\pi(s)$$

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Wrap up

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Key players in RL

- **Immediate reward**
  - Translates the goal of the agent
  - Instantaneous / myopic
- **Policy**
  - Action selection rule
  - Solving an MDP consists in finding the optimal policy

Key players in RL

- **Value function**
  - "Secondary" reward
  - Long-term evaluation of the states
  - Can be used to compute the policy
- **Model (Markov decision process)**
  - Description of the dynamics of the process (transition probabilities)

Solving RL

- Solving an RL problem consists of solving the associated MDP
- Solving an MDP consists of computing the optimal policy.
- E.g.,
  - Use value iteration to compute v^*

or

  - Use value iteration to compute q^*
  - Use any of the above to compute π^*

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Outline of the lecture

- **Part I: RL Primer**
  - The RL Problem
  - Markov Decision Process - A Model for RL Problems
  - Optimality & Dynamic Programming
  - Monte Carlo Approaches
  - Temporal Difference Learning
  - The Policy Gradient Theorem

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Reinforcement learning

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Reinforcement learning

- Interaction between the agent and the environment
  - Agent observes that $S_t = s$
  - Agent performs an action $A_t = a$
  - Agent gets a reward R_t
  - At the next time step, agent observes $S_{t+1} = s$’
  - …

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Reinforcement learning

- At each step, the agent collects a sample, consisting of a tuple

$$(s, a, r, s')$$

- Each such sample includes information about:
  - The reward, in the triplet $(s,\ a,\ r)$
  - The dynamics, in the triplet $(s,\ a,\ s')$

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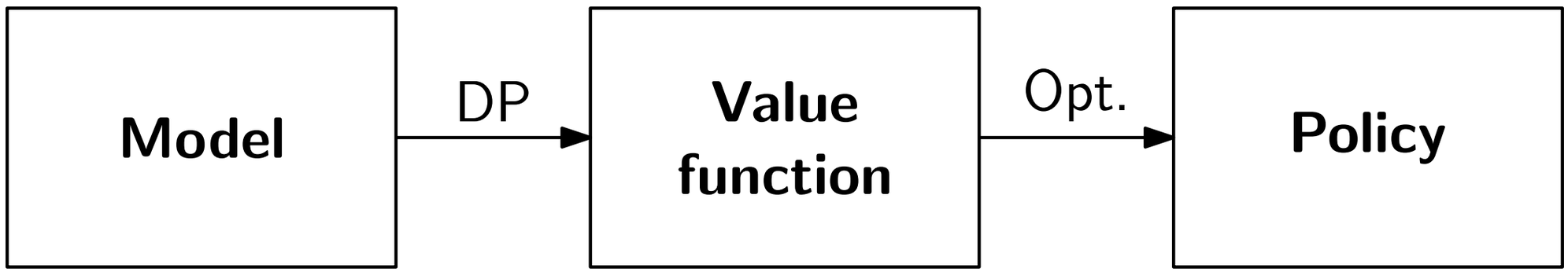
Reinforcement learning

- We'll consider explicitly the two subproblems within RL:
  - The prediction problem (given a policy, compute v_π)
  - The control problem (compute q^*)

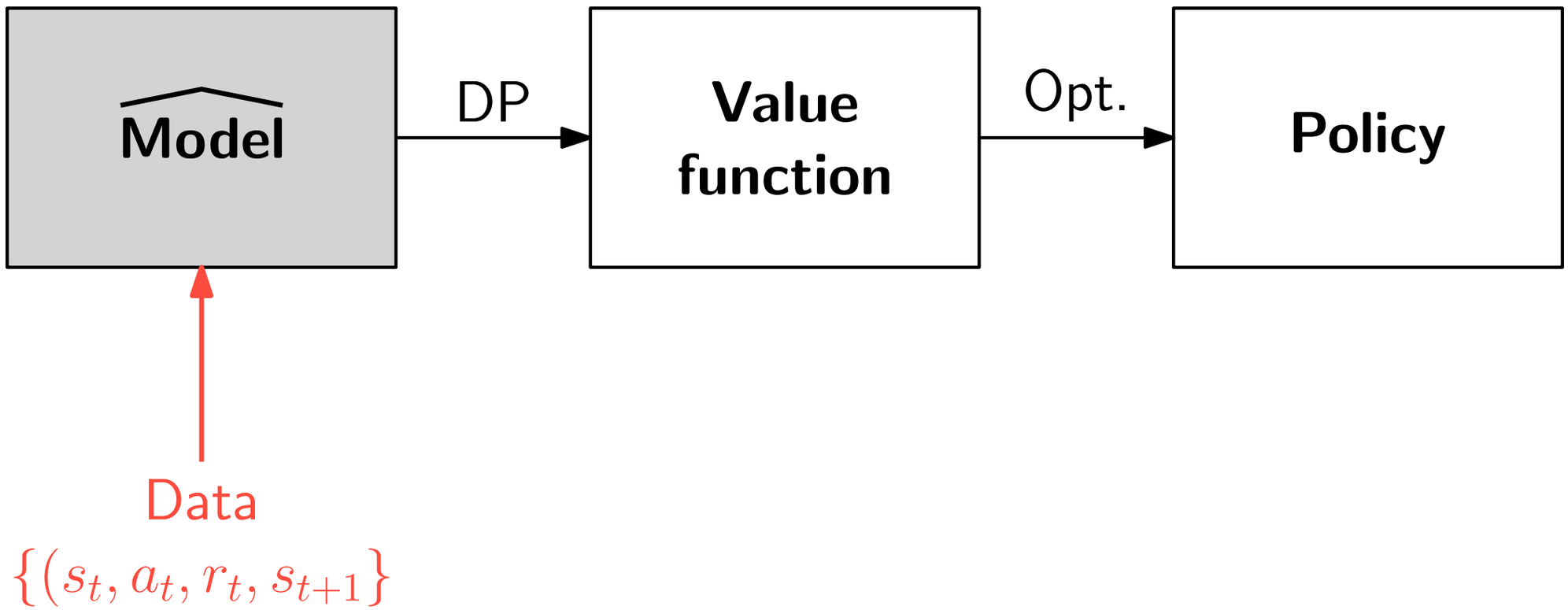
Taxonomy of RL methods

- Solving an MDP:



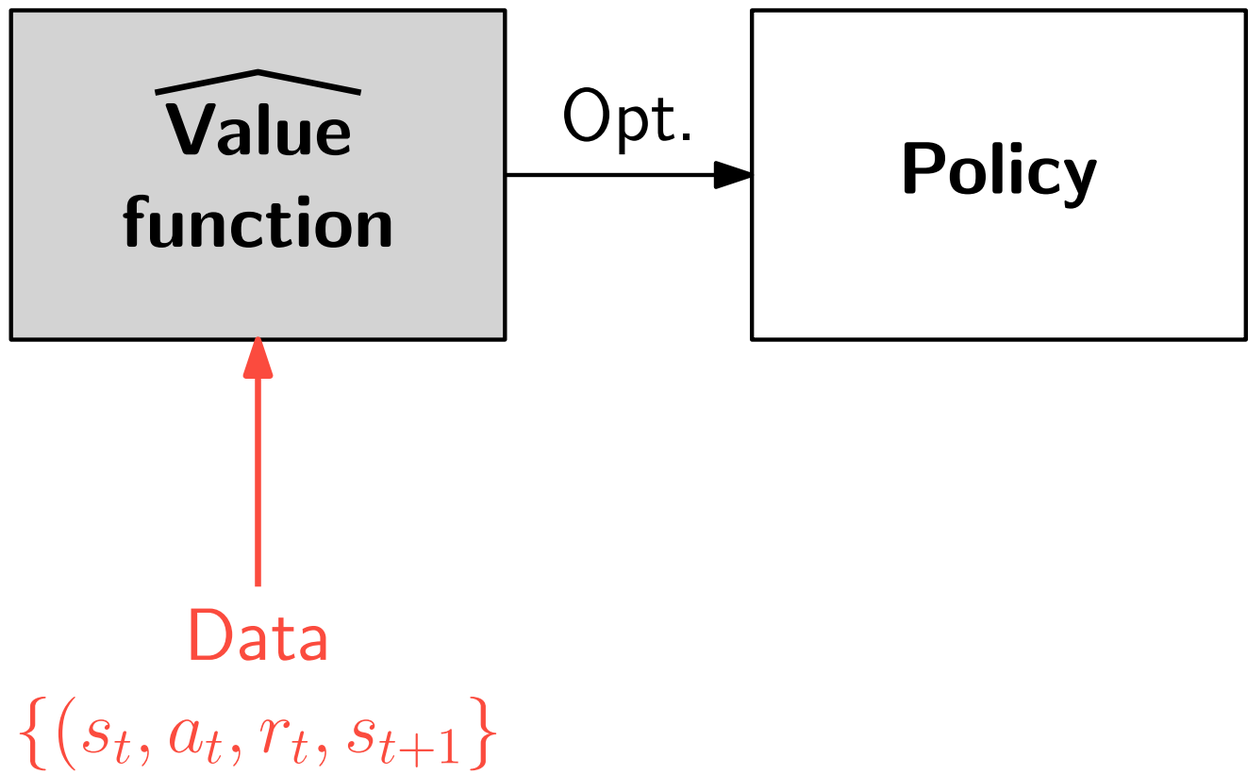
Taxonomy of RL methods

- Model-based methods:



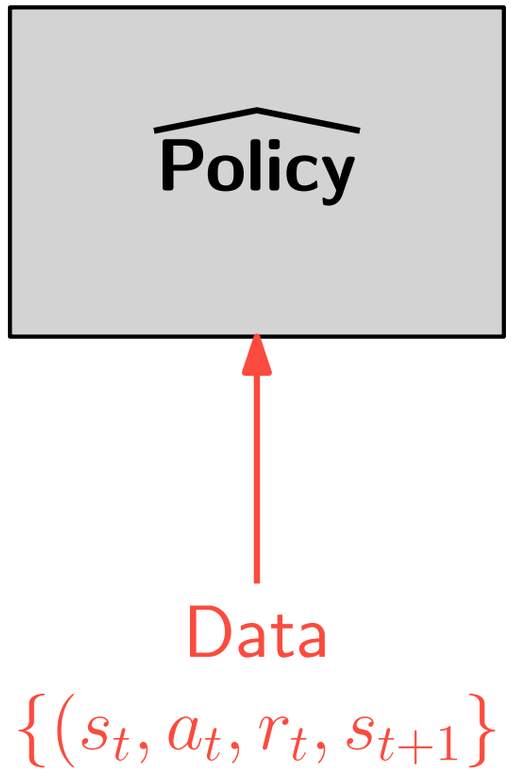
Taxonomy of RL methods

- Value-based methods:

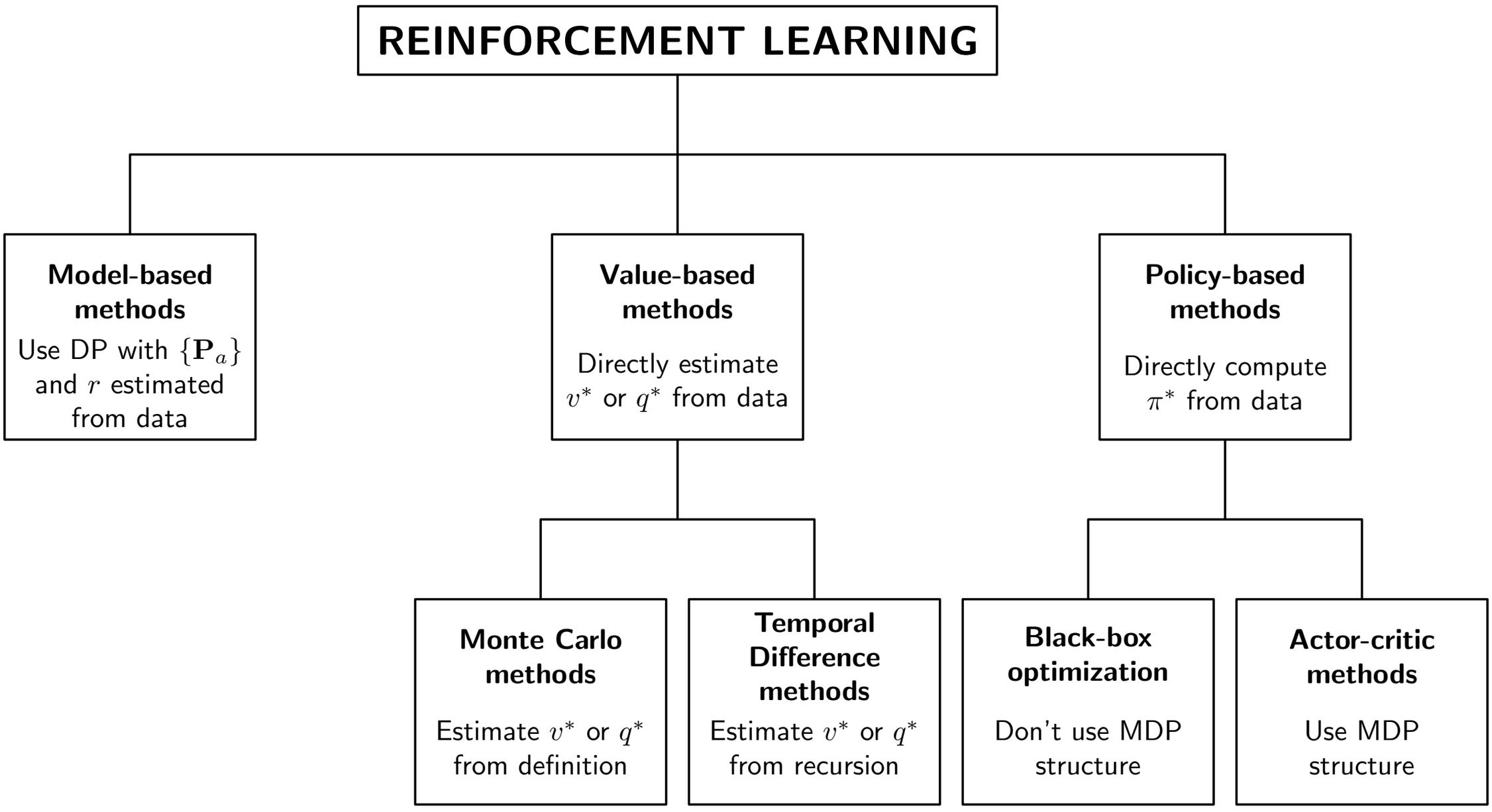


Taxonomy of RL methods

- Policy-based methods:



Taxonomy of RL methods



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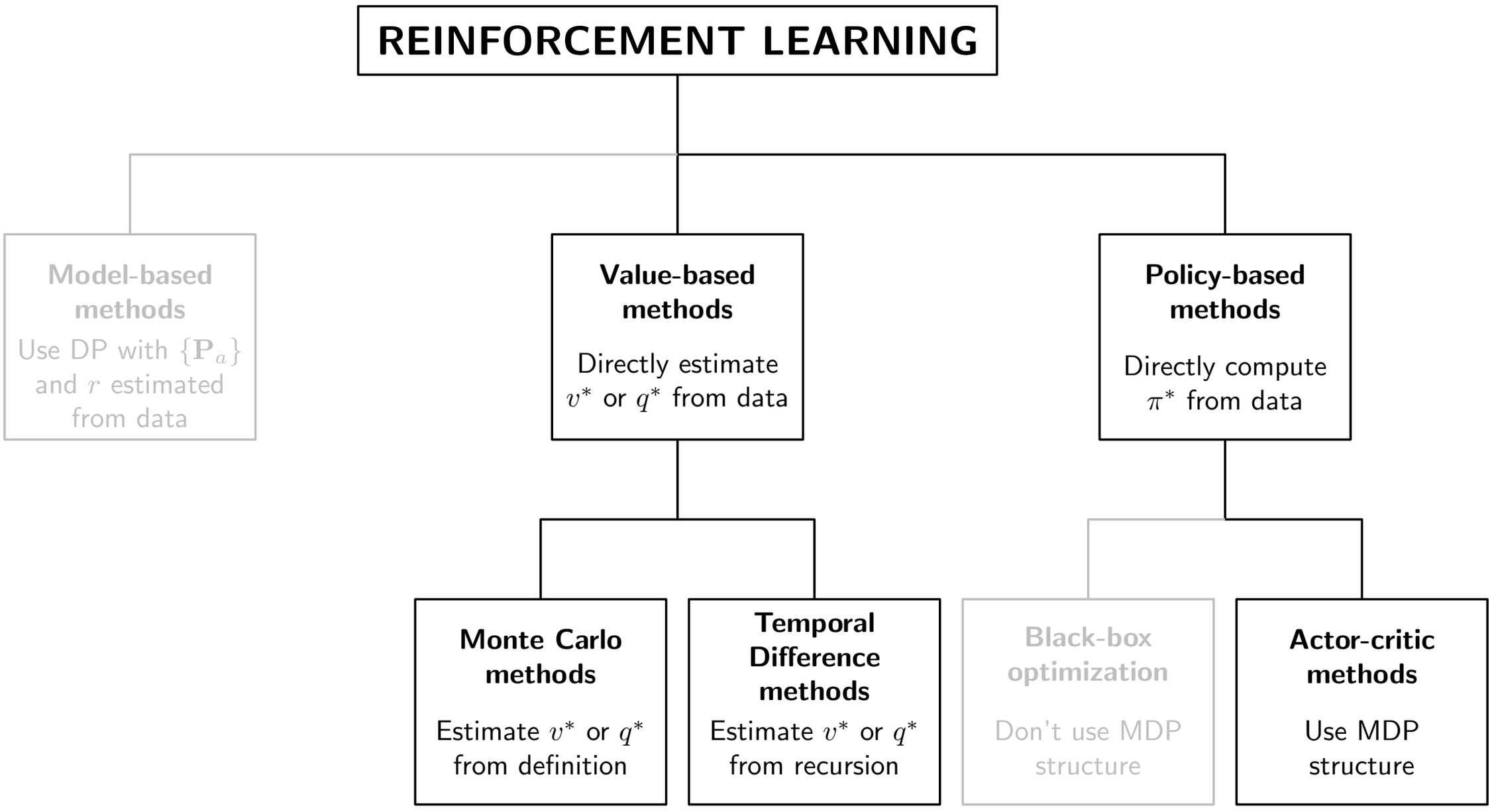

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Monte Carlo approaches

The prediction problem

- We want to estimate v_π

- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

  obtained while following policy π

- We define the return at time step *t* as

$$G_0 = \sum_{t=0}^{T-1} \gamma^t r_t$$

Using the return

- From the definition of v_π,

$$v_\pi(s_0) \approx \mathbb{E}\left[G_0\right]$$

- Then, given *N* trajectories with a common initial state s_0, we can compute

$$\hat{v}(s_0) = \frac{1}{N}\sum_{n=1}^{N} G_{0,n}$$

or, incrementally,

$$\hat{v}(s_0) \leftarrow \hat{v}(s_0) + \frac{1}{N}(G_{0,N} - \hat{v}(s_0))$$

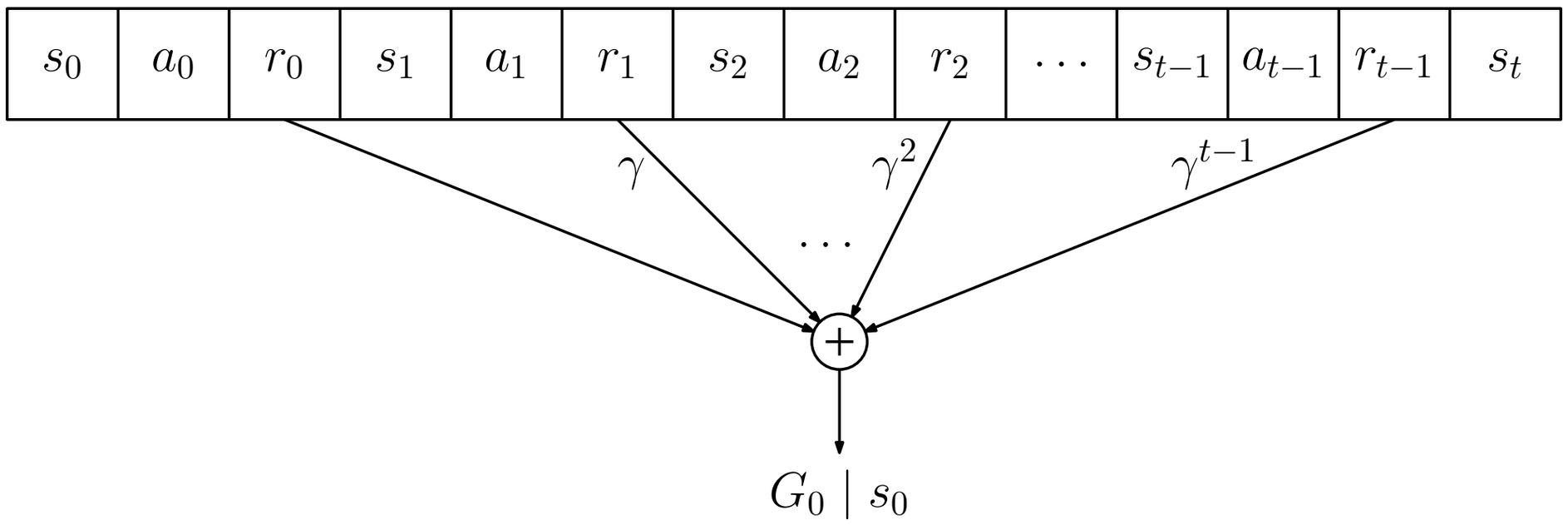
Return for trajectory *N*

Some considerations

- A trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

provides returns for multiple states

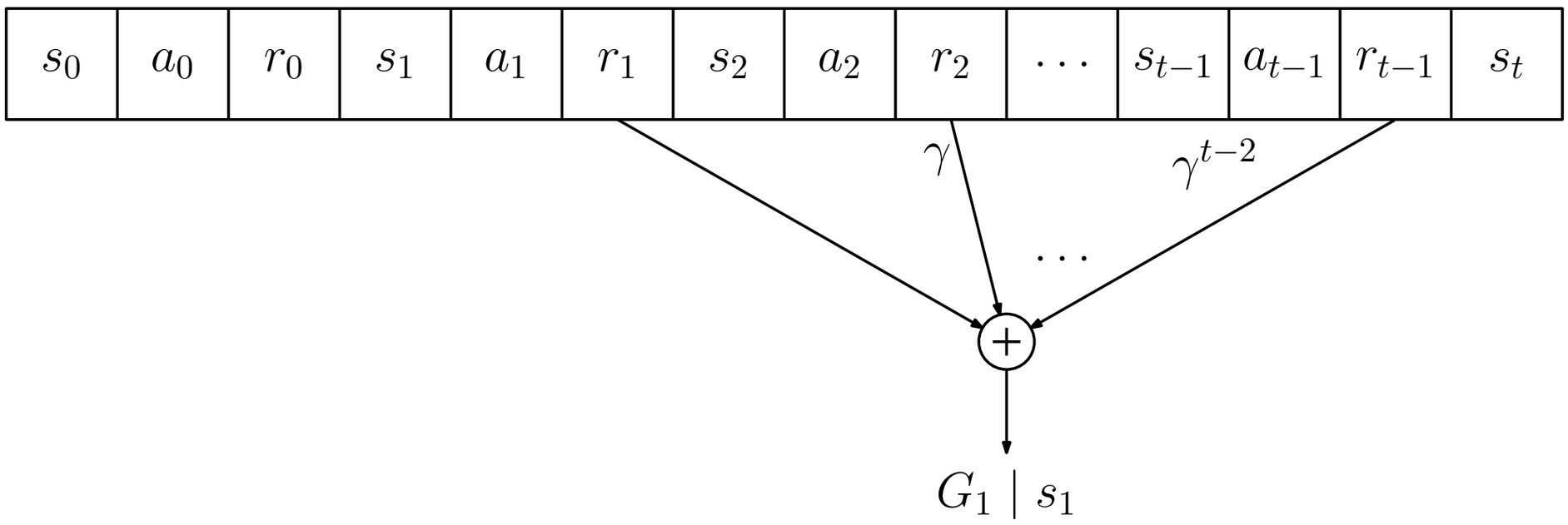


Some considerations

- A trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

provides returns for multiple states



Some considerations

- A trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

  provides returns for multiple states

- Trajectories should visit all states a large number of times

The control problem

- We want to estimate q^*
- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

obtained by selecting a random action a_0 and following a policy $\pi^{(0)}$ thereafter

Using the return

- From the definition of q_π,

$$q_\pi(s_0, a_0) \approx \mathbb{E}\left[G_0\right]$$

- Then, given *N* trajectories with a common initial state s_0 and initial action a_0, we can compute

$$\hat{q}_\pi(s_0, a_0) = \frac{1}{N} \sum_{n=1}^{N} G_{0,n}$$

or, incrementally,

$$\hat{q}(s_0, a_0) \leftarrow \hat{q}(s_0, a_0) + \frac{1}{N}(G_{0,N} - \hat{q}(s_0, a_0))$$

Some considerations

- To estimate the Q-values for all state-action pairs, we need a large number of trajectories starting in each state-action pair

- To compute the optimal Q-values,

  - Start with arbitrary policy $\pi^{(0)}$ and set $k = 0$

  - Generate multiple trajectories, and estimate $q_{\pi^{(k)}}$

  - Compute policy

Improved policy

$$\pi^{(k+1)}(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q_{\pi^{(k)}}(s, a), \forall s$$

  - Set $k = k + 1$ and repeat

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Temporal difference learning

The prediction problem

- We want to estimate v_π
- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

obtained while following policy π

The prediction problem

- We know that

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

or, equivalently,

$$v_\pi(s) = \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

Expectation

The prediction problem

- We know that

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

  or, equivalently,

$$v_\pi(s) = \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

- The value function v_π can be computed iteratively via value iteration using the update

$$v^{(k+1)}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v^{(k)}(s') \right]$$

The prediction problem

- We know that

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left[r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s, a) v_\pi(s') \right]$$

or, equivalently,

$$v_\pi(s) = \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

- The value function v_π can be computed iteratively via value iteration using the update

$$v^{(k+1)}(s) \leftarrow \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v^{(k)}(S_{t+1}) \mid S_t = s \right]$$

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The prediction problem

- We can approximate the update

$$v^{(k+1)}(s) \leftarrow \mathbb{E}_{A_t \sim \pi(S_t)} \left[R_t + \gamma v^{(k)}(S_{t+1}) \mid S_t = s \right]$$

from samples $\{(s, r_n, s'_n)\}$ as

$$v^{(k+1)}(s) \leftarrow \frac{1}{N} \sum_{n=1}^{N} \left(r_n + \gamma v^{(k)}(s'_n) \right)$$

or, incrementally,

$$v^{(k+1)}(s) \leftarrow v^{(k)}(s) + \frac{1}{N} \left(r_n + \gamma v^{(k)}(s'_n) - v^{(k)}(s) \right)$$

Let's turn this into a proper algorithm

TD(0)

- Given a (potentially infinite) trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

generated using policy π, and given an initial estimate $v^{(0)}$ for v_π, TD(0) performs, at each step t, the update

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t \left(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\right)$$

New estimate (only updates component associated with current state s_t)

Old estimate

Step size

Temporal difference

TD(0)

- Given a (potentially infinite) trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

generated using policy π, and given an initial estimate $v^{(0)}$ for v_π, TD(0) performs, at each step *t*, the update

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t\big(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\big)$$

Compare with what we had

$$v^{(k+1)}(s) \leftarrow v^{(k)}(s) + \frac{1}{N}\big(r_n + \gamma v^{(k)}(s'_n) - v^{(k)}(s)\big)$$

The control problem

- We want to estimate q^*
- We start with a similar idea used in MC methods
- We are given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

obtained while following some initial policy π

The control problem

- Repeating the same reasoning,

$$q_\pi(s,a) = \mathbb{E}_{A_{t+1}\sim\pi(S_{t+1})}\left[R_t + \gamma q_\pi(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a\right]$$

leading to the update

$$q^{(k+1)}(s,a) \leftarrow \mathbb{E}_{A_{t+1}\sim\pi(S_{t+1})}\left[R_t + \gamma q^{(k)}(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a\right]$$

The control problem

- Then, given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

generated using a policy π, and given an initial estimate $q^{(0)}$ for q_π, update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t \left(r_t + \gamma q^{(t)}(s_{t+1}, a_{t+1}) - q^{(t)}(s_t, a_t)\right)$$

- After some iterations, compute a new policy

$$\pi(s) \leftarrow \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q^{(t)}(s, a)$$

SARSA

- This approach runs the following cycle:
  - Start with a policy
  - Evaluate it, computing its associated Q-function
  - Update the policy
  - Repeat
- Each update to $q^{(t)}$ uses a sample $(s_t,\ a_t,\ r_t,\ s_{t+1},\ a_{t+1})$
- The algorithm is thus named SARSA

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Can we learn q^* directly?

The control problem

- Let us again repeat the same reasoning

$$q^*(s,a) = \mathbb{E}\left[R_t + \gamma \max_{a \in \mathcal{A}} q_\pi(S_{t+1}, a) \mid S_t = s, A_t = a\right]$$

we get the update

$$q^{(k+1)}(s,a) \leftarrow \mathbb{E}\left[R_t + \gamma \max_{a \in \mathcal{A}} q^{(k)}(S_{t+1}, a) \mid S_t = s, A_t = a\right]$$

Q-learning

- Then, given a (potentially infinite) trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

generated using an arbitrary policy π, and given an initial estimate $q^{(0)}$ for q^*, update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

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Summarizing...

- TD(0) is used to compute the value function for a given policy
- It relies on the update

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t\big(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\big)$$

Summarizing...

- SARSA and Q-learning are used to compute the optimal Q-function
- SARSA relies on the update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t \left(r_t + \gamma q^{(t)}(s_{t+1}, a_{t+1}) - q^{(t)}(s_t, a_t) \right)$$

- SARSA learns the Q-function for the policy used to obtain the samples

☞ On-policy learning

- In order to compute the optimal policy, it must slowly adjust the policy used to obtain the samples

Summarizing...

- Q-learning relies on the update

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

- Q-learning learns the optimal Q-function, independently of the policy used to obtain the samples

☞ Off-policy learning

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The policy gradient theorem

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Policy-based methods

- The goal is to compute π* directly
- We depart from a parameterized family of policies, π_θ

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... however...

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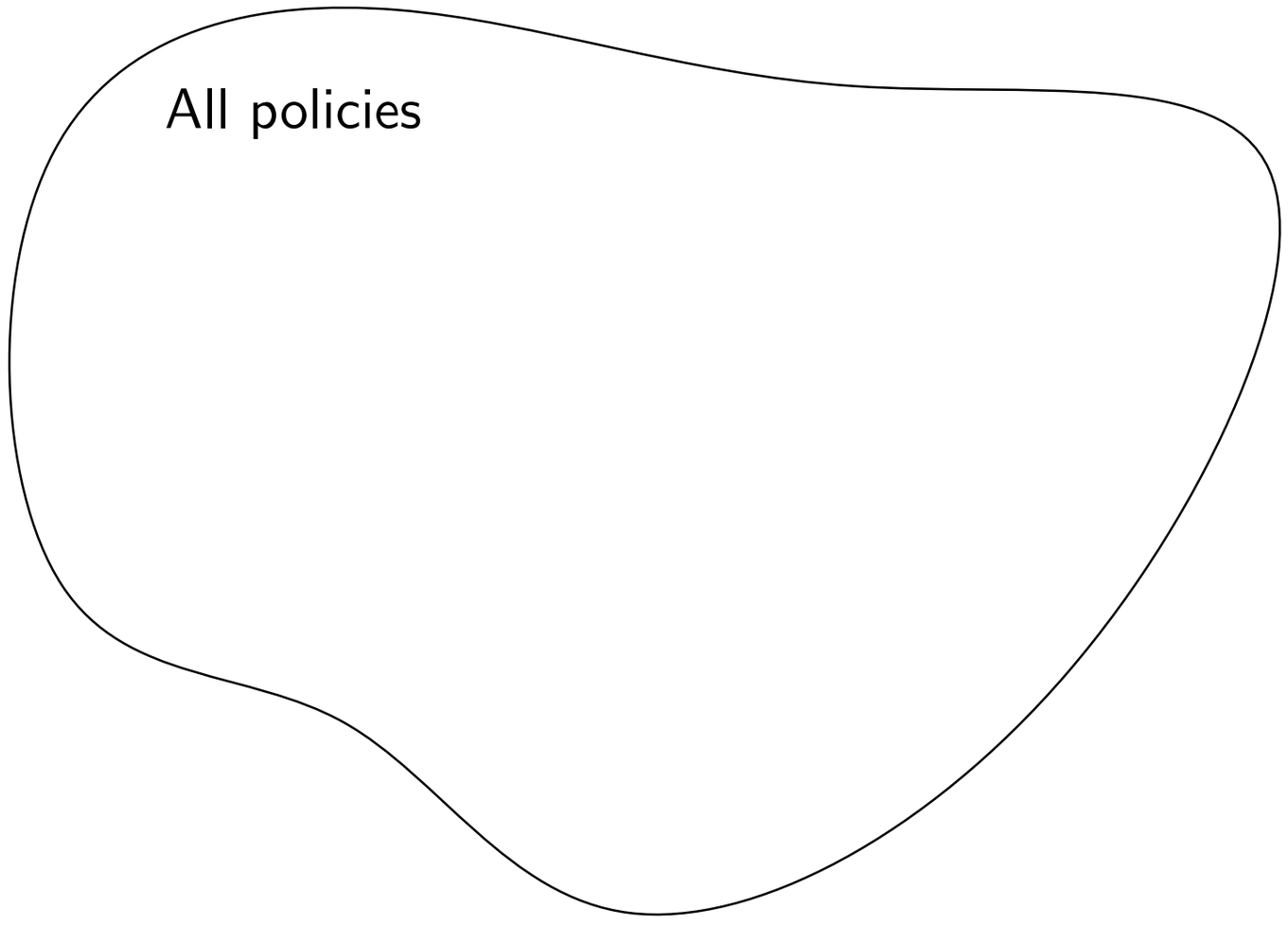
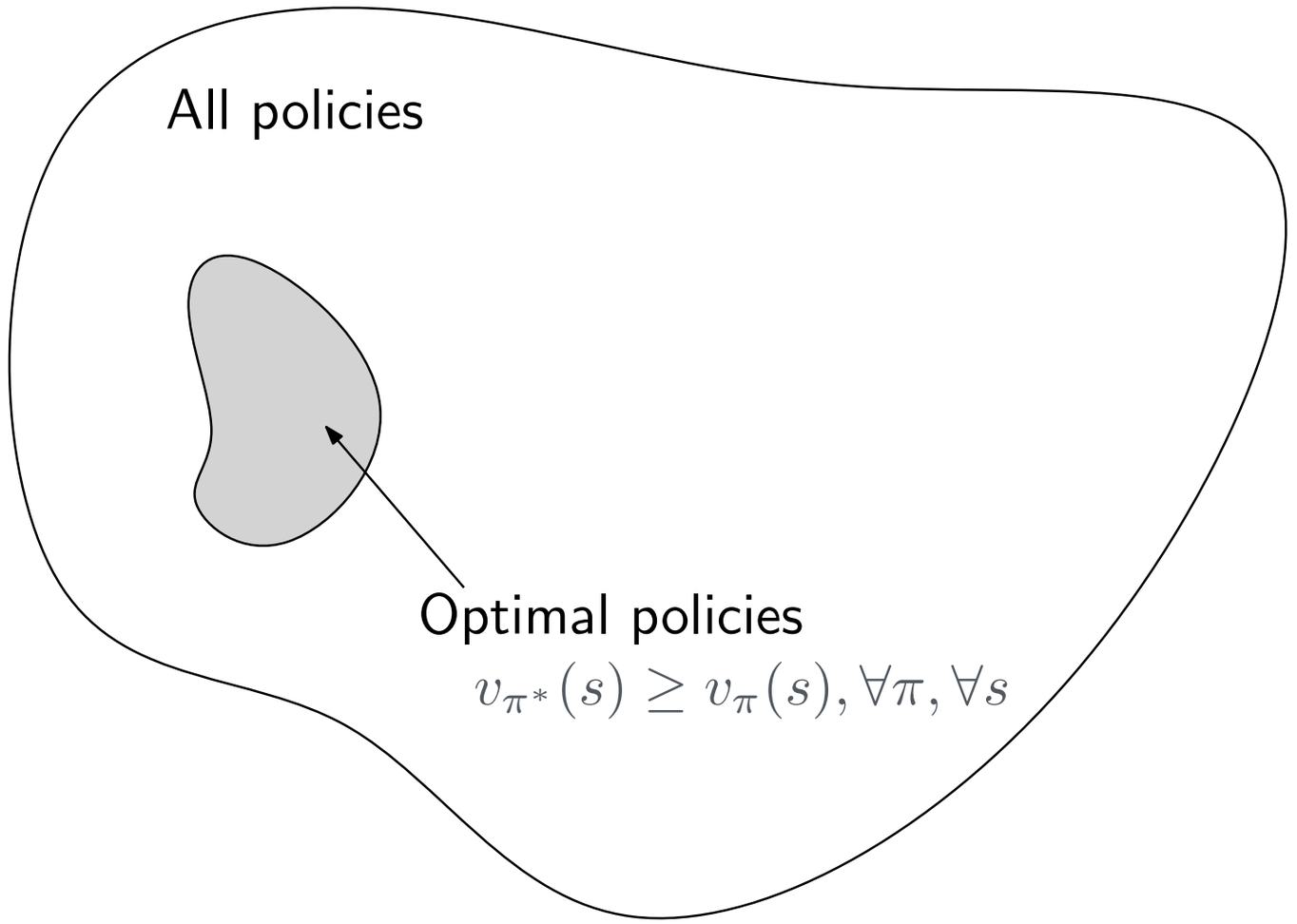
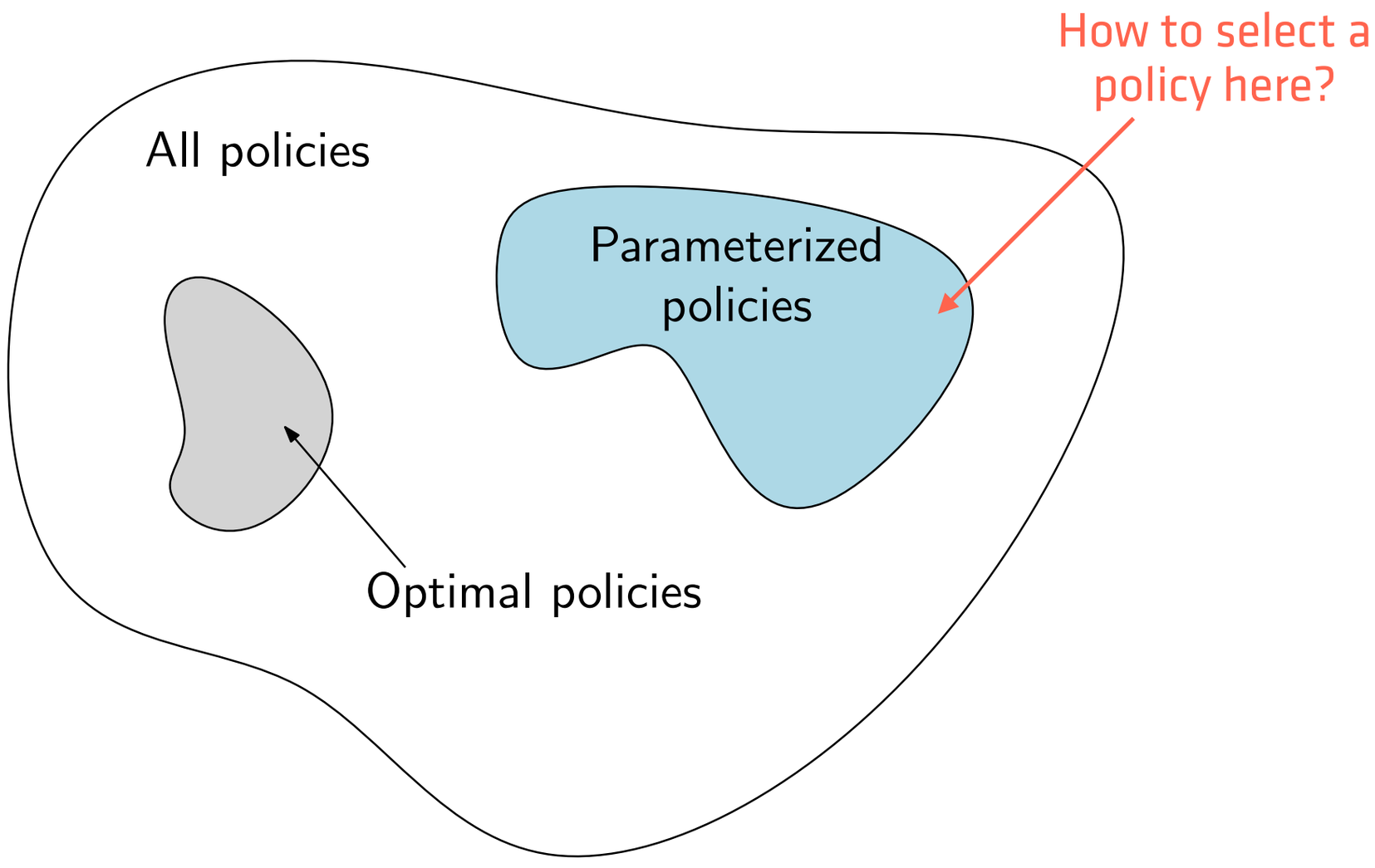
Policy-based methods



Policy-based methods



Policy-based methods



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Revisiting optimality criterion

- When considering the set of all policies, state-wise optimization is possible
- When considering a restricted set of policies, state-wise optimization may not be possible

Revisiting optimality criterion

- Recall that our goal is to maximize

$$J(\{R_t, t = 1, \dots, \}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_t\right]$$

- We consider that the initial state of the MDP follows some initial distribution μ

- To explicitly indicate the dependence of J on the initial distribution μ and the policy π used to generate $\{R_t,\ t = 1,\ ...\}$, we write

$$J(\pi; \mu) \triangleq \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t R_t \mid S_0 \sim \mu\right]$$

Interesting relations

- We have that
  - $v_\pi(s) = J(\pi; \mu)$ when $\mu(s') = \mathbb{I}(s' = s)$
  - Conversely, for an arbitrary distribution μ,

$$J(\pi; \mu) = \sum_{s \in \mathcal{S}} \mu(s) v_\pi(s)$$

RL using gradient ascent

- We can now optimize J with respect to the parameters of the policy
- Using gradient ascent, we get an algorithm

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \nabla_\theta J(\pi_\theta; \mu)$$

Methods based on this idea are globally called “policy-gradient methods”

Policy gradient

- We now compute the policy gradient

$$\nabla_\theta J(\pi_\theta;\mu) = \nabla_\theta \sum_{s\in\mathcal{S}} \mu(s) v_{\pi_\theta}(s)$$

$$= \sum_{s\in\mathcal{S}} \mu(s) \nabla_\theta v_{\pi_\theta}(s)$$

Let us consider this term alone

Policy gradient

- Since

$$v_{\pi_\theta}(s) = \sum_{a \in \mathcal{A}} \pi_\theta(a \mid s) q_{\pi_\theta}(s, a)$$

it holds that

$$\nabla_\theta v_{\pi_\theta}(s) = \sum_{a \in \mathcal{A}} [\nabla_\theta \pi_\theta(a \mid s) q_{\pi_\theta}(s, a) + \pi_\theta(a \mid s) \nabla_\theta q_{\pi_\theta}(s, a)]$$

We now look at this term

Policy gradient

- Since

$$q_{\pi_\theta}(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) v_{\pi_\theta}(s')$$

  it holds that

$$\nabla_\theta q_{\pi_\theta}(s,a) = \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s')$$

Policy gradient

- Putting everything together,

Factoring this out

$$\nabla_\theta v_{\pi_\theta}(s) = \sum_{a \in \mathcal{A}} \left[\nabla_\theta \pi_\theta(a \mid s) q_{\pi_\theta}(s,a) + \gamma \pi_\theta(a \mid s) \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s') \right]$$

$$= \sum_{a \in \mathcal{A}} \pi_\theta(a \mid s) \left[\frac{\nabla_\theta \pi_\theta(a \mid s)}{\pi_\theta(a \mid s)} q_{\pi_\theta}(s,a) + \gamma \sum_{s' \in \mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s') \right]$$

This is just
$\nabla_\theta \log \pi_\theta(a \mid s)$

Policy gradient

- Putting everything together,

$$\nabla_\theta v_{\pi_\theta}(s) = \sum_{a\in\mathcal{A}} \left[\nabla_\theta \pi_\theta(a \mid s) q_{\pi_\theta}(s,a) + \gamma \pi_\theta(a \mid s) \sum_{s'\in\mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s') \right]$$

$$= \sum_{a\in\mathcal{A}} \pi_\theta(a \mid s) \left[\nabla_\theta \log \pi_\theta(a \mid s) q_{\pi_\theta}(s,a) + \gamma \sum_{s'\in\mathcal{S}} \mathbf{P}(s' \mid s,a) \nabla_\theta v_{\pi_\theta}(s') \right]$$

- Recursive relation reminiscent of that for v_π

Plays the role of "reward"

Policy gradient

- Unfolding the recursion finally yields

$$\nabla_\theta J(\pi_\theta; \mu) = \sum_{s \in \mathcal{S}} \mu_\theta(s) \sum_{a \in \mathcal{A}} \pi_\theta(a \mid s) \nabla_\theta \log \pi_\theta(a \mid s) q_{\pi_\theta}(s, a)$$

  or, equivalently,

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S \sim \mu_\theta, A \sim \pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A) \right]$$

- The distribution μ_θ translates the “discounted visitation frequency” under π_θ

- Can be sampled by sampled the MDP while following π_θ

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- The gradient is just the

- Given a trajectory obtained from π_θ and with initial state sampled from μ_θ,

$$\nabla_\theta J(\pi_\theta; \mu) \approx \sum_{t=0}^{T} \gamma^t G_{n,t} \nabla_\theta \log \pi_\theta(a_t \mid s_t)$$

Estimate of
$q_\pi(s_t, a_t)$

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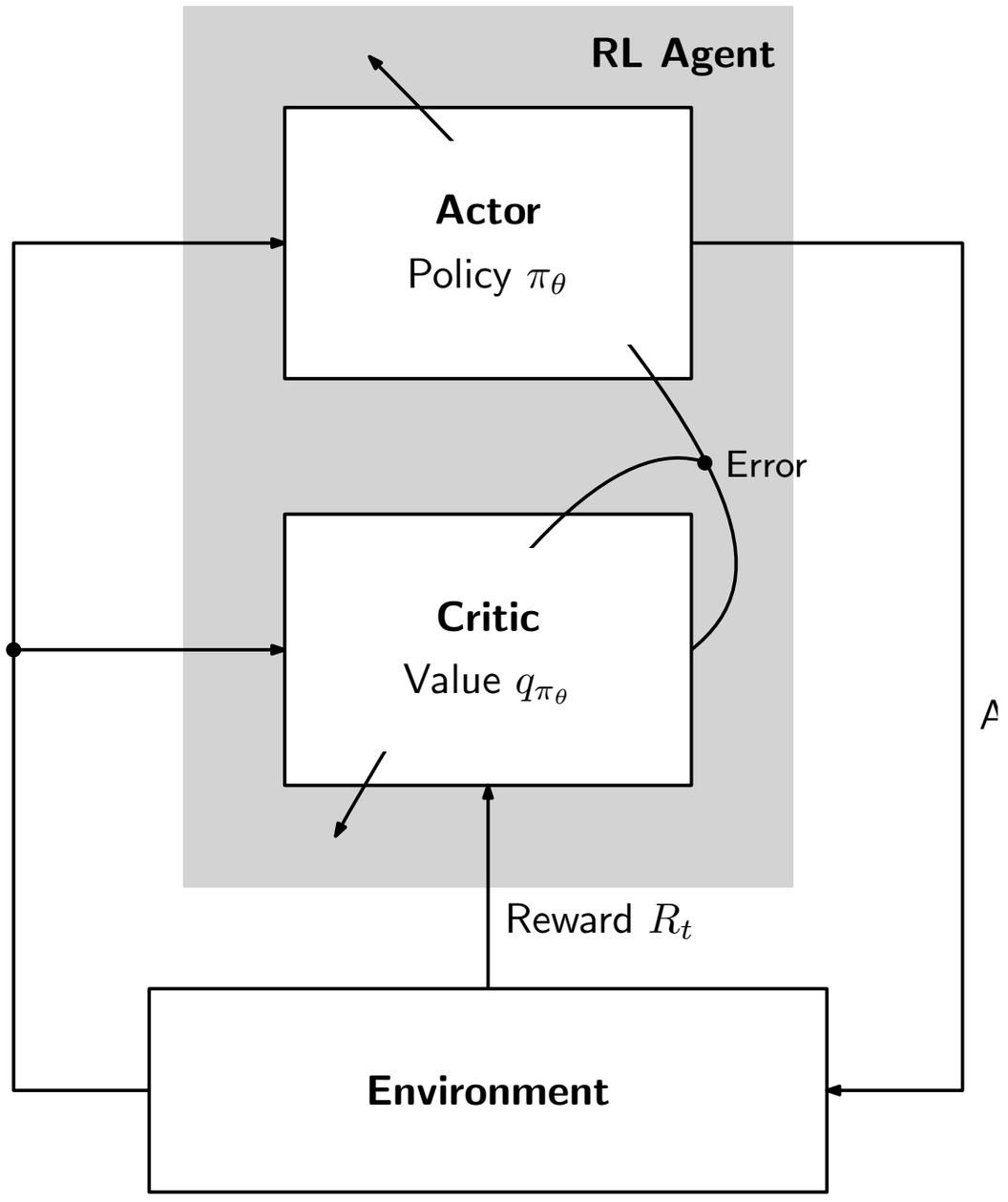
Actor-critic architecture

- To compute the gradient, we require an estimate of the Q-values
- REINFORCE uses a simple Monte Carlo approach to build such estimate
- However, other approaches can be used (e.g., temporal-difference learning)

Actor-critic architecture

- The RL algorithm comprises two components:
  - An actor, responsible for executing the policy π_θ
  - A critic, responsible for evaluating the policy (computing q_π)

Actor-critic architecture



TD-based actor-critic

- For example, we can have an actor-critic based on TD-learning:
  - Given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, \ldots\}$$

  - Update the Q-value estimates as

$$q^{(t+1)}(s_t, a_t) = q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma q^{(t)}(s_{t+1}, a_{t+1}) - q^{(t)}(s_t, a_t))$$

  - Update gradient term

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} + \beta_t \gamma^t q^{(t+1)}(s_t, a_t) \nabla_\theta \log \pi_\theta(s_t, a_t)$$

Considerations

- PG/AC architectures are convenient with function approximation
  - Gradient does not depend on q_π but on a projection thereof
- Variations of the gradient (e.g., natural gradient) can also be used:
- Discount is cumbersome to deal with
  - Many PG/AC applications instead adopt the average per-step reward
- Fully incremental approaches suffer from high variance and are seldom used

Adding a baseline

- Consider once again the gradient expression

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A)\right]$$

- Gradient estimated from samples
- Estimates plagued by high variance (sensitivity to the particular samples)

Adding a baseline

- Result from theory of Monte Carlo integration:
  - Use of a baseline can often improve variance of sample-based estimates

$$\mathbb{E}\left[f(X)\right] \approx \frac{1}{N}\sum_{n=1}^{N} f(x_n)$$

$$\mathbb{E}\left[f(X) - g(X)\right] \approx \frac{1}{N}\sum_{n=1}^{N}(f(x_n) - g(x_n)) \longrightarrow \text{Less variance}$$

Baseline
($\mathbb{E}\left[g(X)\right]$ known)

Adding a baseline

- Consider an arbitrary function

$$b : \mathcal{S} \to \mathbb{R}$$

- Then,

$$\sum_{a \in \mathcal{A}} \nabla_\theta \pi_\theta(a \mid s) b(s) = \text{?}$$

Adding a baseline

- But then

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A) - \nabla_\theta \log \pi_\theta(A \mid S) b(S)\right]$$

or, equivalently,

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) (q_{\pi_\theta}(S, A) - b(S))\right]$$

Best baseline:
$v_{\pi_\theta}(S)$

Adding a baseline

- But then

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A) - \nabla_\theta \log \pi_\theta(A \mid S) b(S)\right]$$

or, equivalently,

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) \boxed{(q_{\pi_\theta}(S, A) - v_{\pi_\theta}(S))}\right]$$

Advantage
$\mathrm{adv}_\pi(S, A)$

Adding a baseline

- But then

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) q_{\pi_\theta}(S, A) - \nabla_\theta \log \pi_\theta(A \mid S) b(S)\right]$$

or, equivalently,

$$\nabla_\theta J(\pi_\theta; \mu) = \mathbb{E}_{S\sim\mu_\theta, A\sim\pi(\cdot|S)} \left[\nabla_\theta \log \pi_\theta(A \mid S) \mathrm{adv}_{\pi_\theta}(S, A)\right]$$

☞ This is the underlying form of most current AC algorithms

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Outline of the lecture

- **Part I: RL Primer**
  - The RL Problem
  - Markov Decision Process - A Model for RL Problems
  - Optimality & Dynamic Programming
  - Monte Carlo Approaches
  - Temporal Difference Learning
  - The Policy Gradient Theorem

Outline of the lecture

- **Part II: Deep RL**
  - From RL to Deep RL
  - DQN
  - Deep advantage actor-critic methods
  - Trust region methods

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RL in large domains

- **Plan:**
  - Revisit temporal difference learning in large domains
  - Revisit policy-gradient methods in large domains

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Temporal difference learning revisited

TDL in large domains

- Temporal difference learning methods require explicit updates:

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t\big(r_t + \gamma v^{(t)}(s_{t+1}) - v^{(t)}(s_t)\big)$$

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

Component s_t is explicitly updated

Component $(s_t,\ a_t)$ is explicitly updated

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TDL in large domains

- For large domains, function approximation is necessary
  - We can no longer compute v_π or q^* exactly
  - Instead, we consider parameterized families of functions

TDL in large domains

- Example: TD-learning with linear function approximation
  - We consider the family of functions of the form

$$v(s; \boldsymbol{w}) = \boldsymbol{w}^\top \boldsymbol{\phi}(s)$$

    where $\boldsymbol{w}$ is a vector of parameters

- We update the parameters $\boldsymbol{w}$ as

$$\boldsymbol{w}^{(t+1)} \leftarrow \boldsymbol{w}^{(t)} + \alpha_t \boldsymbol{\phi}(s_t)(r_t + \gamma v(s_{t+1}; \boldsymbol{w}^{(t)}) - v(s_t; \boldsymbol{w}^{(t)}))$$

Compare

$$v^{(t+1)}(s_t) \leftarrow v^{(t)}(s_t) + \alpha_t \left(r_t + \gamma v^{(t)}(s_t) - v^{(t)}(s_t)\right)$$

TDL in large domains

- Another example: Q-learning with linear function approximation
  - We consider the family of functions of the form

$$q(s, a; \boldsymbol{w}) = \boldsymbol{w}^\top \boldsymbol{\phi}(s, a)$$

where $\boldsymbol{w}$ is a vector of parameters

- We update the parameters $\boldsymbol{w}$ as

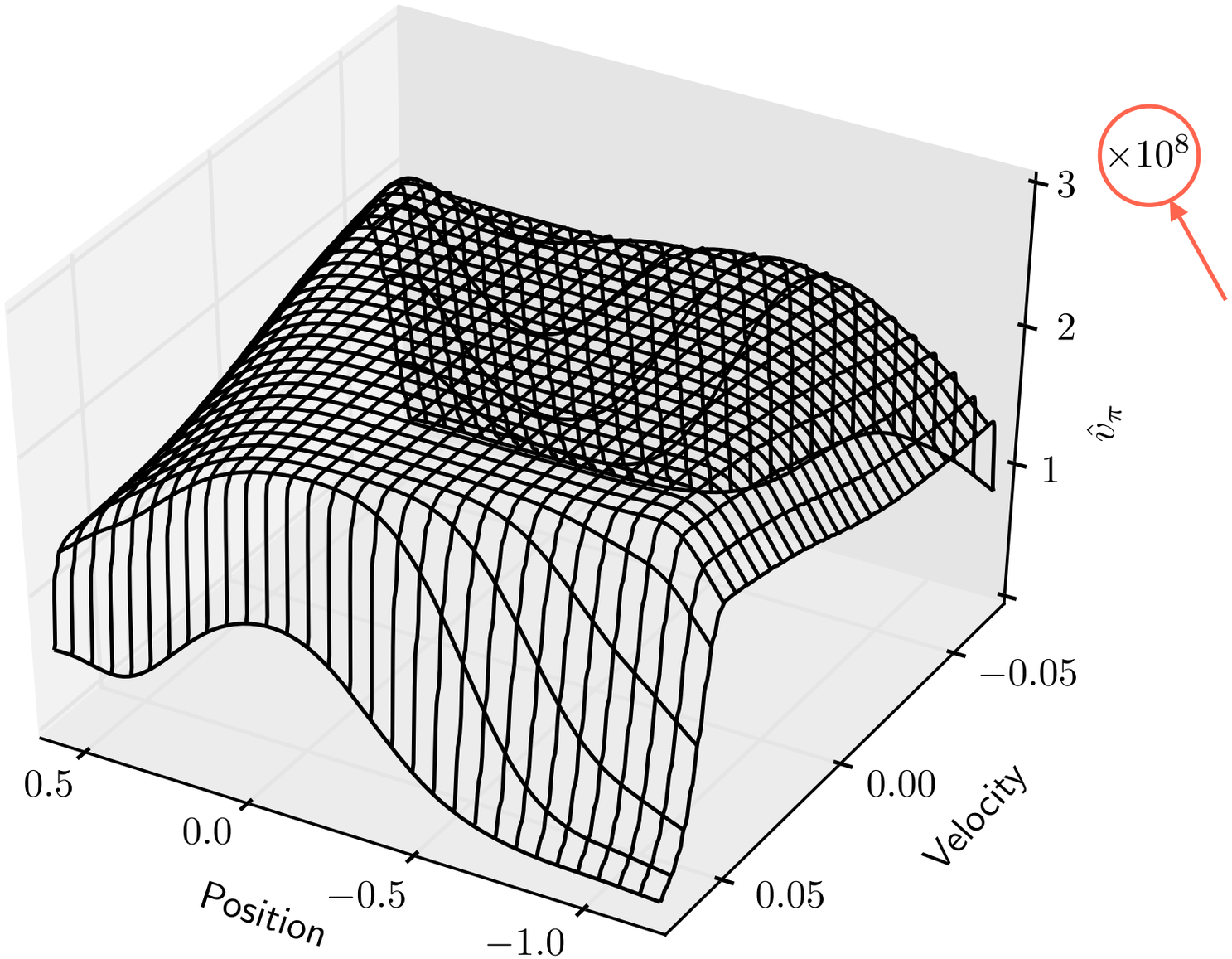
$$\boldsymbol{w}^{(t+1)} \leftarrow \boldsymbol{w}^{(t)} + \alpha_t \boldsymbol{\phi}(s_t, a_t)(r_t + \gamma \max_{a \in \mathcal{A}} q(s_{t+1}, a; \boldsymbol{w}^{(t)}) - q(s_t, a_t; \boldsymbol{w}^{(t)}))$$

Compare

$$q^{(t+1)}(s_t, a_t) \leftarrow q^{(t)}(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a \in \mathcal{A}} q^{(t)}(s_{t+1}, a) - q^{(t)}(s_t, a_t))$$

The problem of function approximation

- Unfortunately, temporal-difference methods may diverge with function approximation



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The problem of function approximation

- Issues with function approximation in RL:
  - Bootstrapping - the target is built from current estimate
  - Sample correlation - samples come from a trajectory

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Given the previous difficulties, how can we combine ANNs with RL?

Combining ANNs and RL

- We address directly the control problem
- Three ideas:
  - Create a replay buffer to avoid sample correlation
  - Use an auxiliary estimate for q^* (a target network) to avoid bootstrapping
  - Turn the trajectory data into supervised learning data

1. Build replay buffer

- Given a trajectory

$$\mathcal{T} = \{s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T\}$$

create a set of transitions (replay buffer)

$$\mathcal{T}' = \{(s_t, a_t, r_t, s_{t+1}), t = 0, \ldots, T-1\}$$

↓

At training time, we
select random transitions
from the replay buffer

↓

Goal: minimize
sample correlation

2. Build targets

- At training time, given a sample (s_t, a_t, r_t, s_{t+1}) from the replay buffer, build target

$$y_t = r_t + \max_{a \in \mathcal{A}} \hat{q}(s_{t+1}, a)$$

where $\hat{q}$ is an estimate of q^*

Auxiliary estimate (target network)

- We thus build a dataset

$$\mathcal{D} = \{(s_{t_k}, a_{t_k}, y_{t_k}), k = 1, \ldots, K\}$$

3. Train

- The error associated with sample t_k is now

$$\varepsilon_k = (y_{t_k} - q(s_{t_k}, a_{t_k}; \boldsymbol{w}))^2$$

with gradient

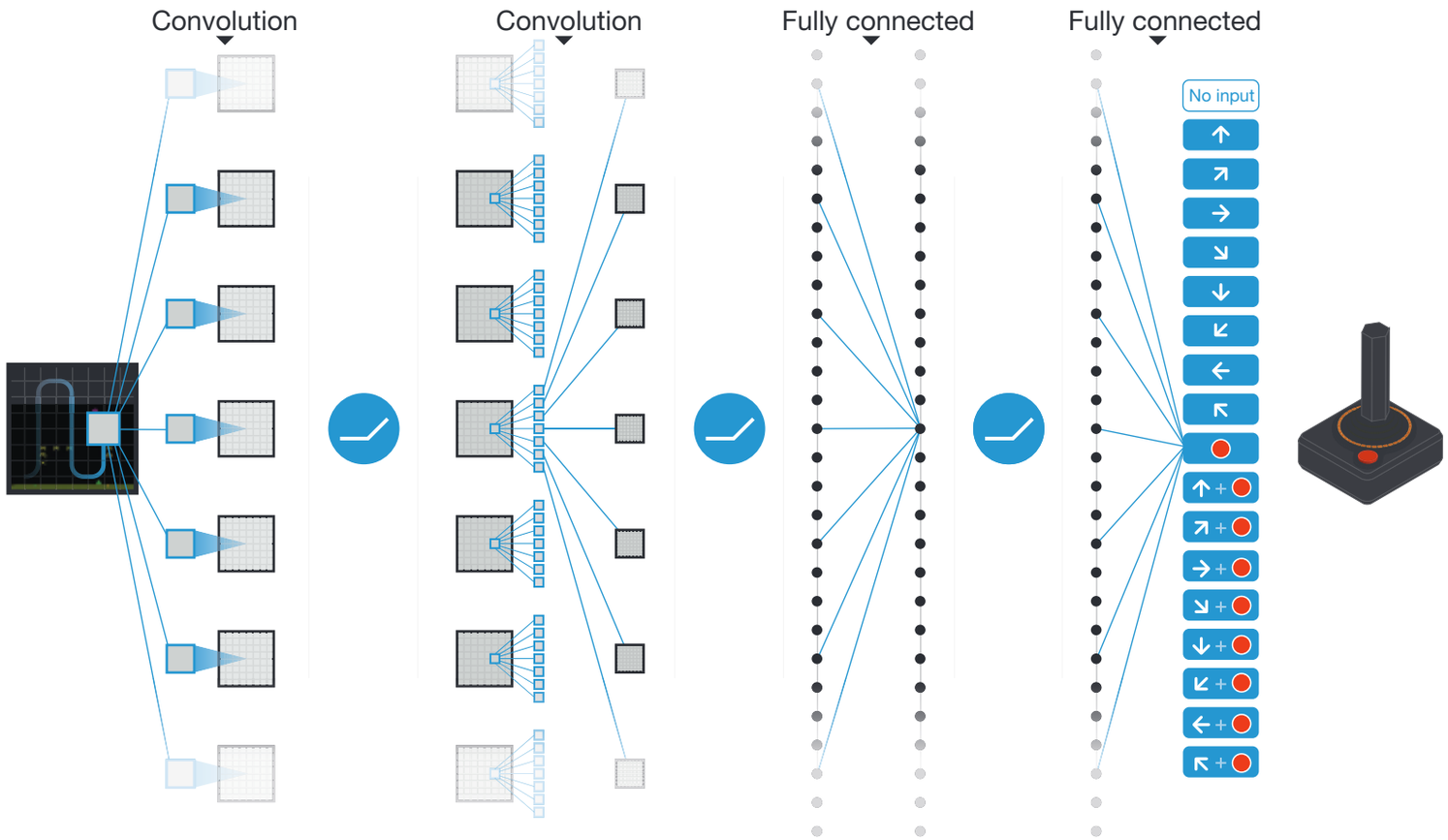
$$\nabla_{\boldsymbol{w}} \varepsilon_k = -2\nabla_{\boldsymbol{w}} q(s_{t_k}, a_{t_k}; \boldsymbol{w})(y_{t_k} - q(s_{t_k}, a_{t_k}; \boldsymbol{w}))$$

$$= \boxed{-2\nabla_{\boldsymbol{w}} q(s_{t_k}, a_{t_k}; \boldsymbol{w})(r_{t_k} + \gamma \max_{a \in \mathcal{A}} \hat{q}(s_{t+1}, a) - q(s_{t_k}, a_{t_k}; \boldsymbol{w}))}$$

Resembles Q-learning update

DQN

- The resulting approach is known as a Deep Q-Network (DQN)
- It was the approach used in the ATARI deep RL paper



V. Mnih. “Human-level control through deep reinforcement learning.” *Nature*, 518:529-533, 2015

DQN

- **Some considerations:**
  - The DQN network takes the state as input and has one output per action
  - The target network is a copy of the DQN, i.e.,

"Old" parameters

$$\hat{q}(s, a) = q(s, a; \boldsymbol{w}^-)$$

  - It is updated every C steps with the weights of the main DQN

Variations: DDQN

- The targets in DQN are computed as

$$y_t = r_t + \max_{a \in \mathcal{A}} q(s_{t+1}, a; \boldsymbol{w}^-)$$

where the target network seeks to avoid bootstrapping

- We can further decouple:
  - … the computation of the maximizing action; and
  - … the value of the maximizing action.

Variations: DDQN

- The targets in double DQN (DDQN), the targets are computed as

$$y_t = r_t + q(s_{t+1}, \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q(s_{t+1}, a; \boldsymbol{w}); \boldsymbol{w}^-)$$

Target network is used to compute the maximizing value

Original network is used to compute the maximizing action

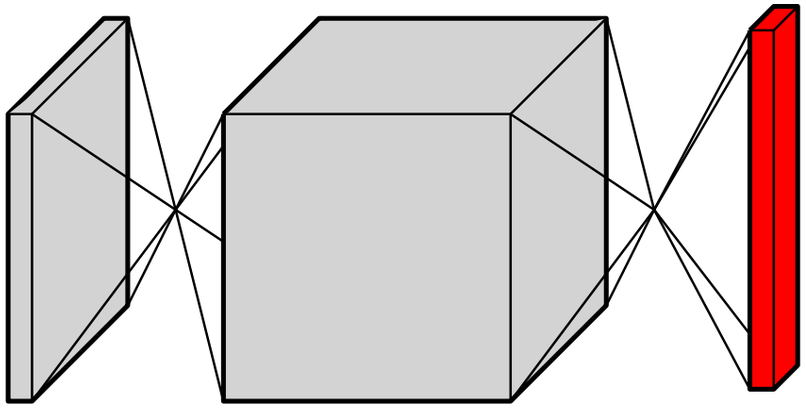
More variations

- Prioritized replay:
  - Transitions are sampled from the replay memory with a probability that increases with the associated error:

$$\varepsilon_k = \left(y_{t_k} - q(s_{t_k}, a_{t_k}; \boldsymbol{w})\right)^2$$

More variations

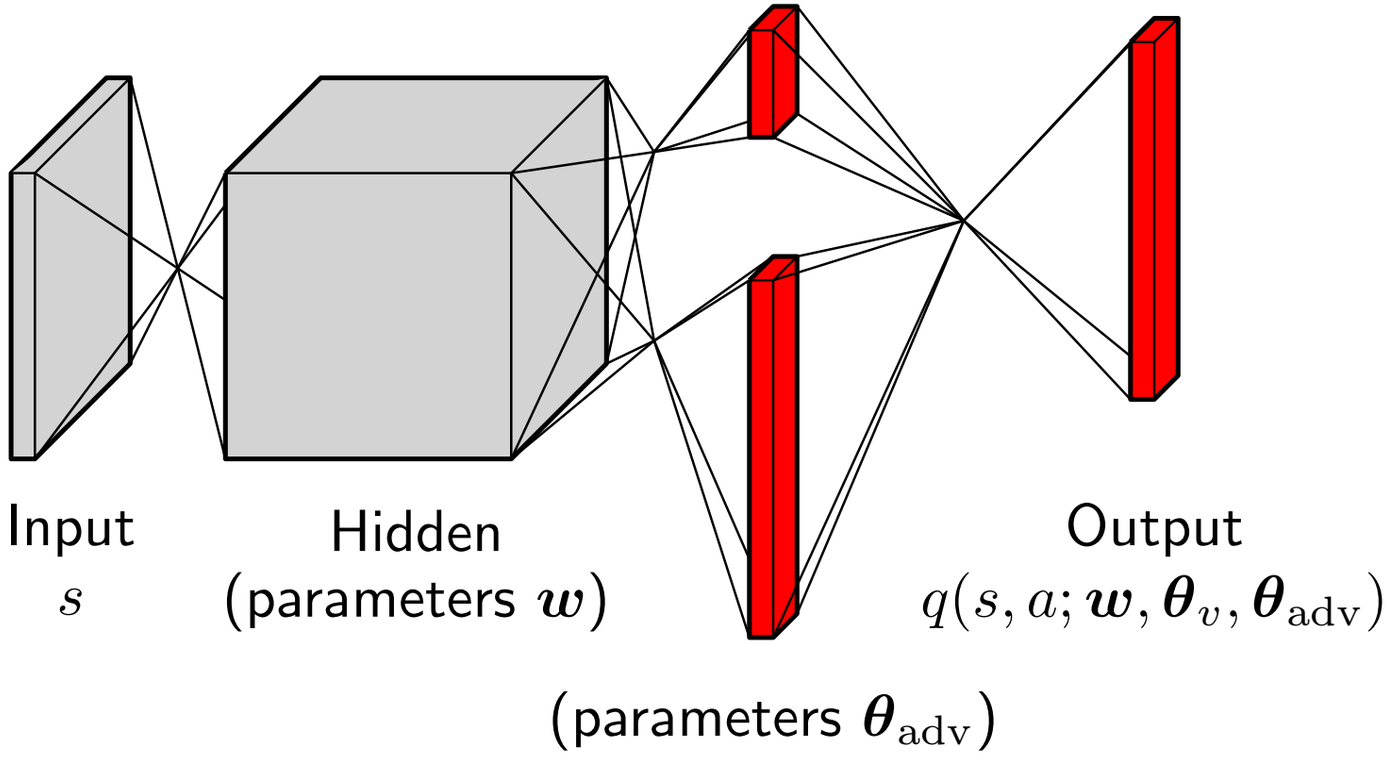
- Dueling network:
  - Instead of the “standard” DQN architecture



More variations

- Dueling network:
  - Instead of the "standard" DQN architecture, dueling networks propose

(parameters $\boldsymbol{\theta}_v$)

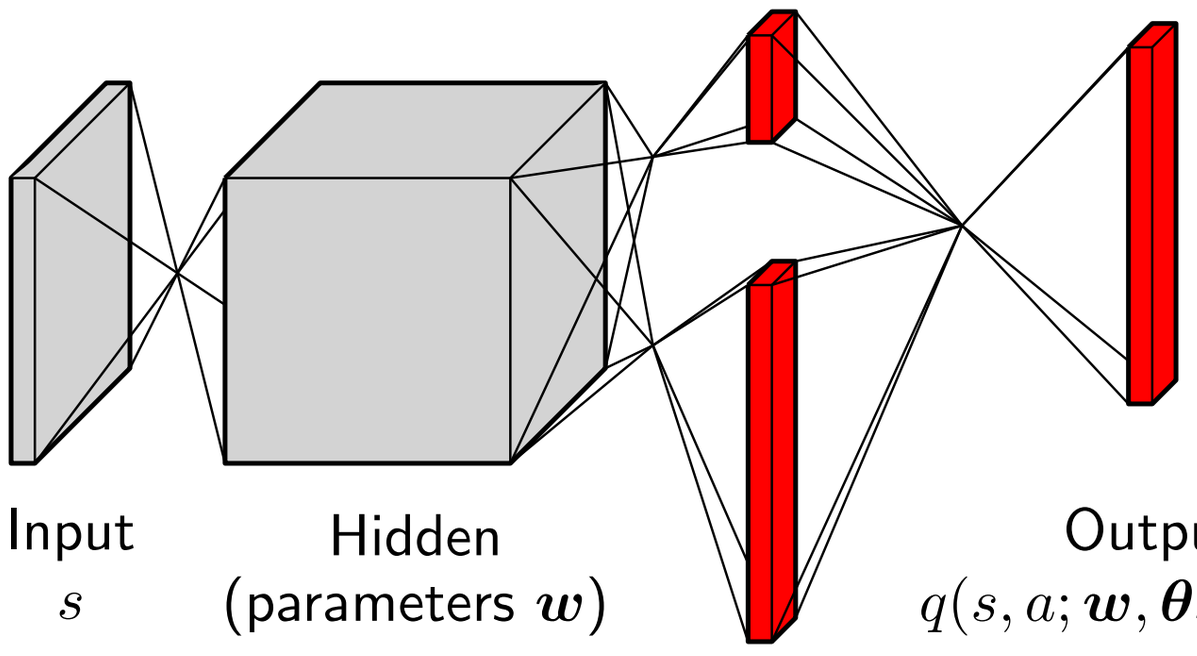


More variations

- Dueling network:

Estimate of $v^*(s)$

(parameters $\boldsymbol{\theta}_v$)



Input
s

Hidden
(parameters $\boldsymbol{w}$)

Output
$q(s, a; \boldsymbol{w}, \boldsymbol{\theta}_v, \boldsymbol{\theta}_{\text{adv}}) = v(s; \boldsymbol{w}, \boldsymbol{\theta}_v) + \text{adv}(s, a; \boldsymbol{w}, \boldsymbol{\theta}_{\text{adv}})$

(parameters $\boldsymbol{\theta}_{\text{adv}}$)

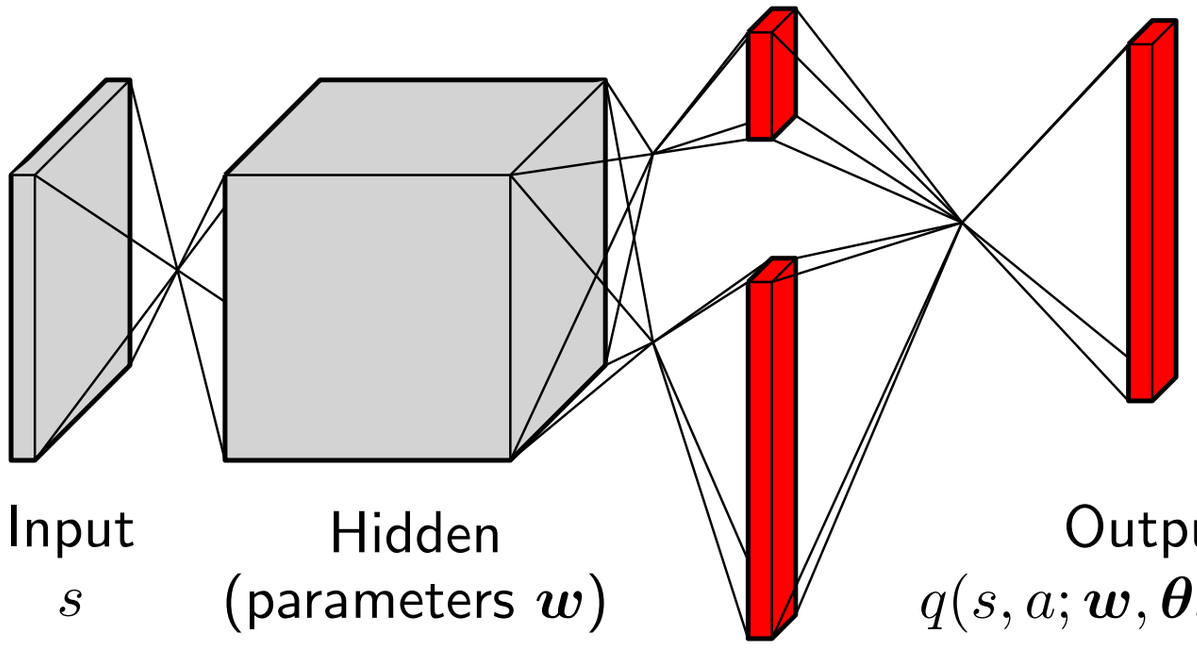
Estimate of $\text{adv}^*(s, a)$

More variations

- Dueling network:

Estimate of $v^*(s)$

(parameters $\boldsymbol{\theta}_v$)



Input
s

Hidden
(parameters $\boldsymbol{w}$)

Output
$q(s, a; \boldsymbol{w}, \boldsymbol{\theta}_v, \boldsymbol{\theta}_{\text{adv}}) = v(s; \boldsymbol{w}, \boldsymbol{\theta}_v) + \text{adv}(s, a; \boldsymbol{w}, \boldsymbol{\theta}_{\text{adv}})$
$-b(s)$

(parameters $\boldsymbol{\theta}_{\text{adv}}$)

Estimate of $\text{adv}^*(s, a)$

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Considerations

- Different variations offer different advantages:
  - DDQN - more stable learning than DQN
  - Prioritized replay - better use of memory (faster learning)
  - Dueling DQN - better performance, particularly in domains where actions only relevant in some states
- Different variations are mostly orthogonal, and can be combined

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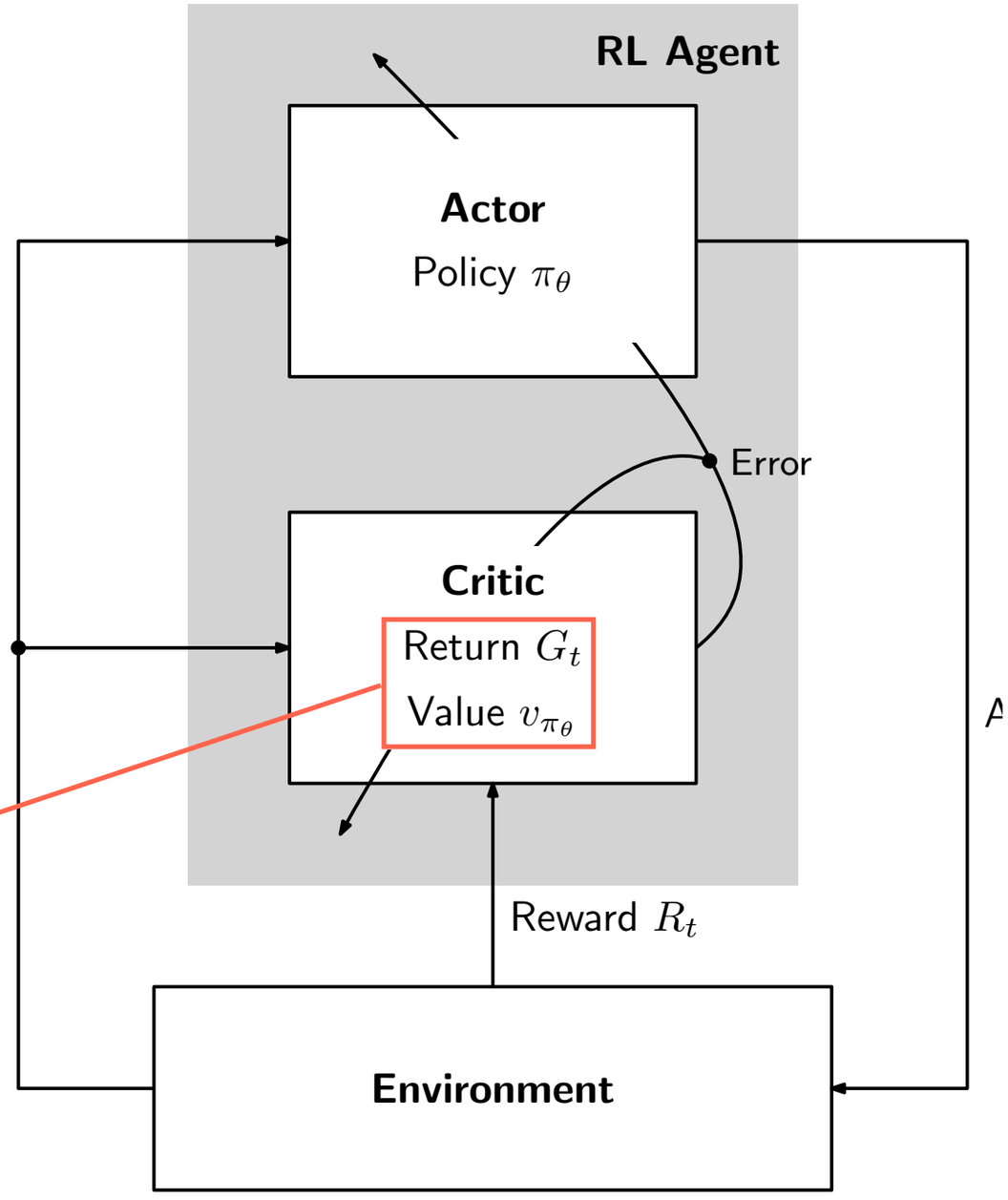
Policy gradient methods revisited

Actor-critic architecture

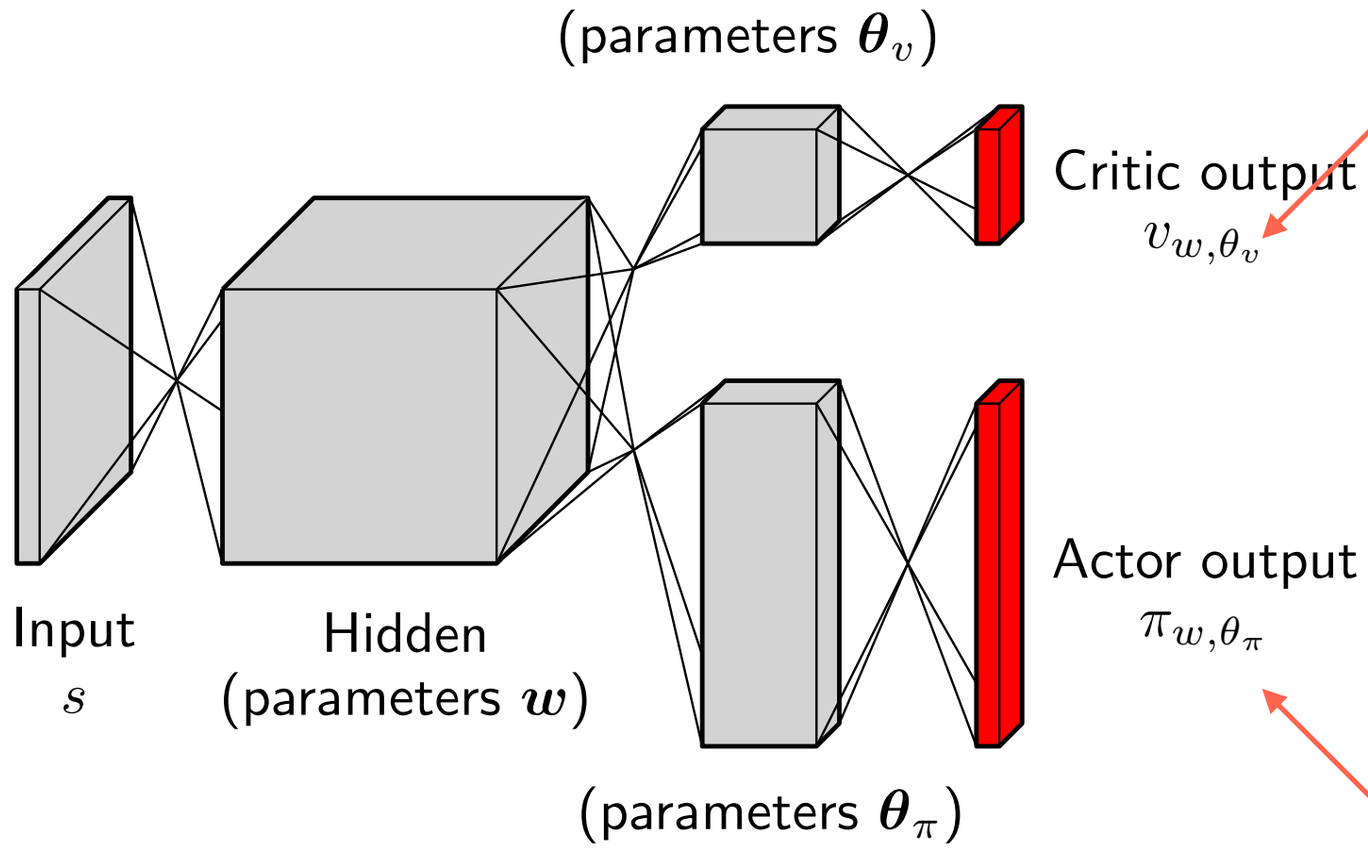
- The AC architecture comprises two components:
  - An actor, responsible for executing the policy π_θ
  - A critic, responsible for evaluating the policy (computing adv_π)



The two components are used to estimate the advantage

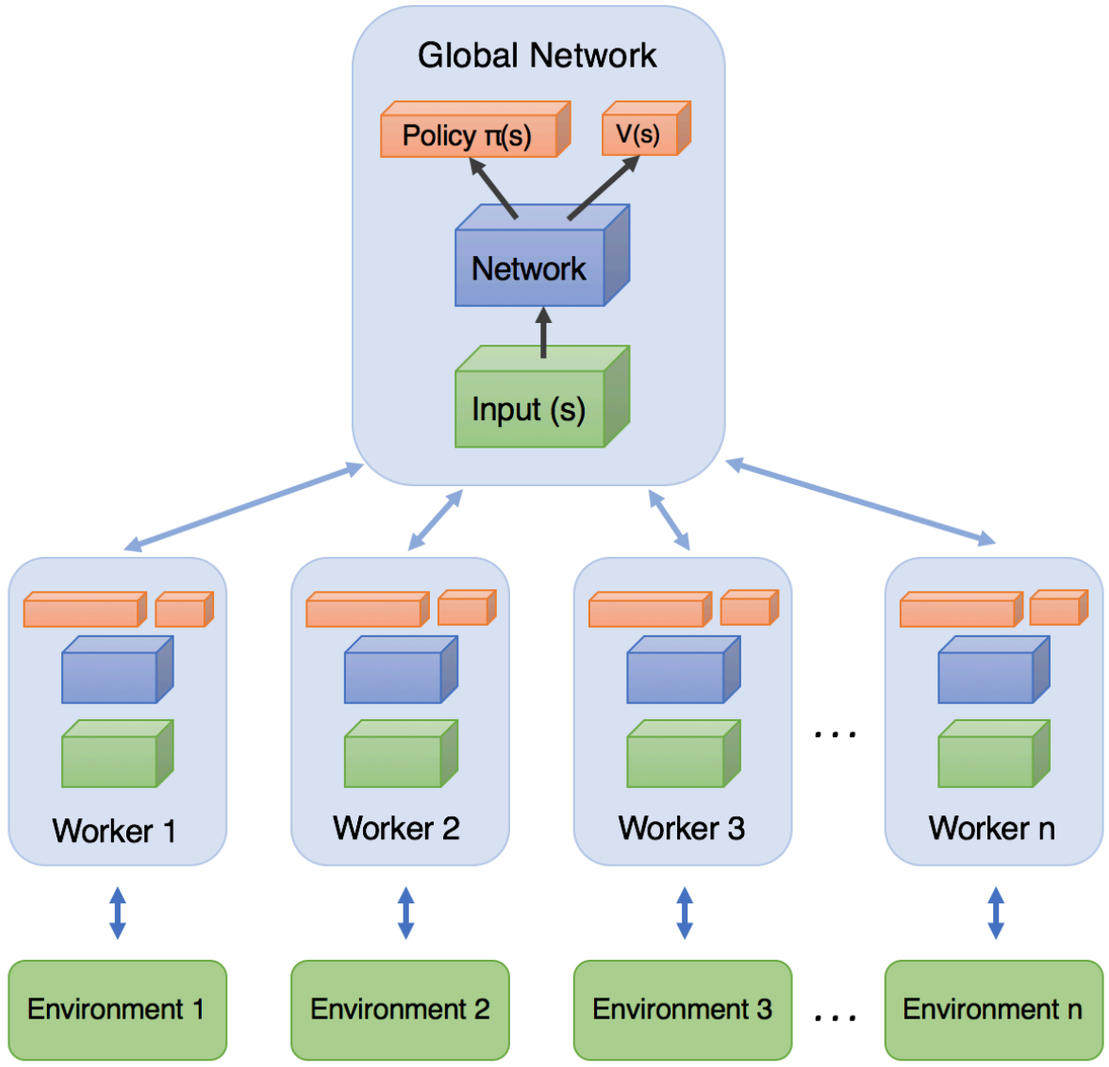
Advantage Actor-Critic

$$\nabla_{w,\theta_v} v_{w,\theta_v}(G_t - v_{w,\theta_v}(s_t))$$



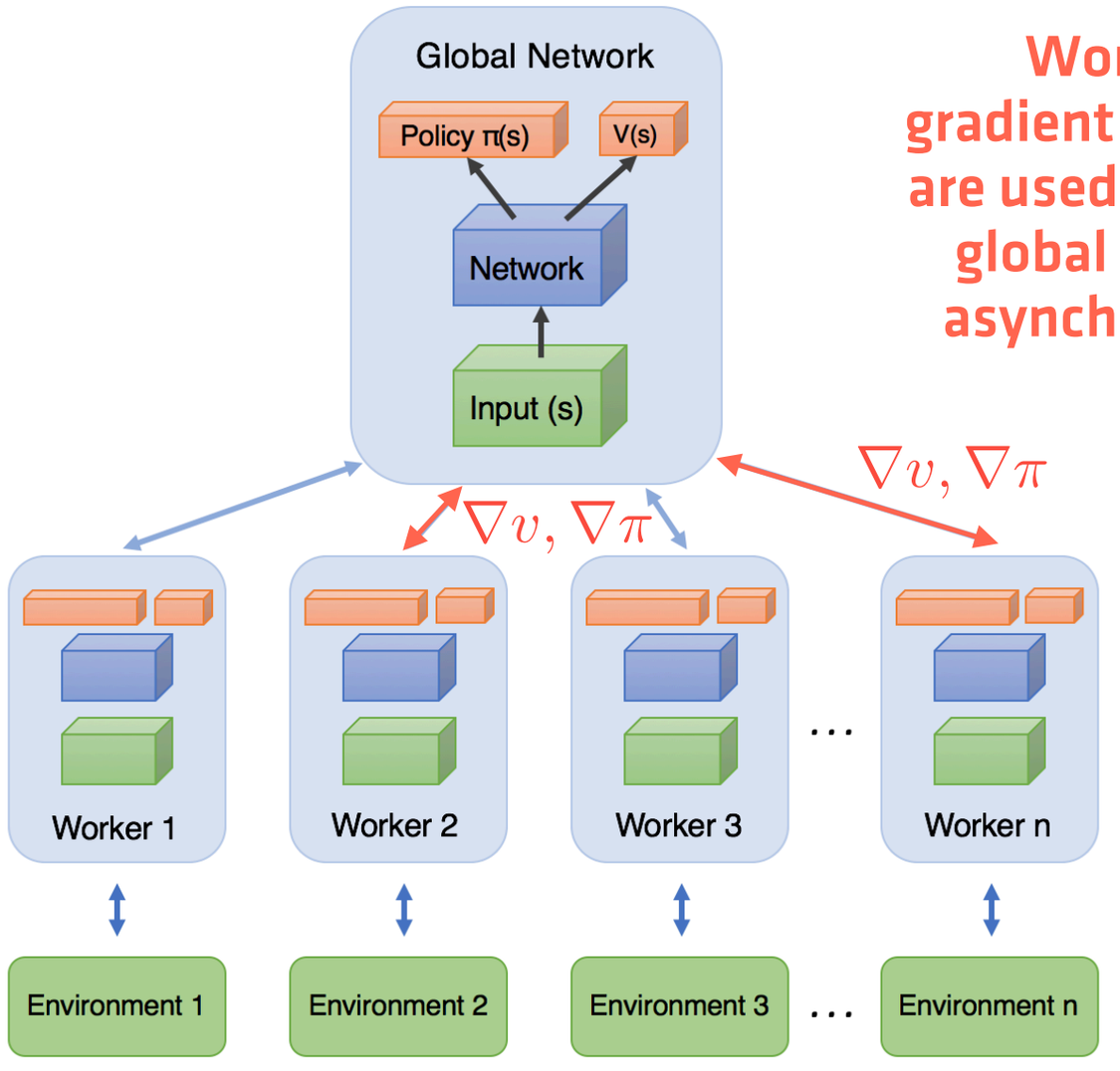
$$\gamma^t \nabla_{w,\theta_\pi} \log \pi_{w,\theta_\pi}(a_t \mid s_t)(G_t - v_{w,\theta_v}(s_t))$$

Asynchronous Advantage Actor-Critic (A3C)



Workers collect data and compute gradient updates

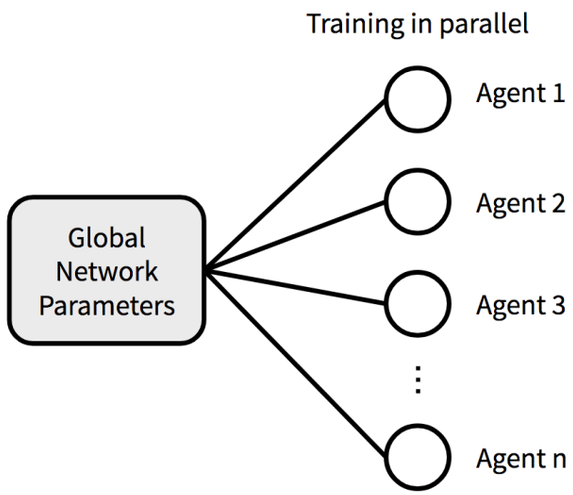
Asynchronous Advantage Actor-Critic (A3C)



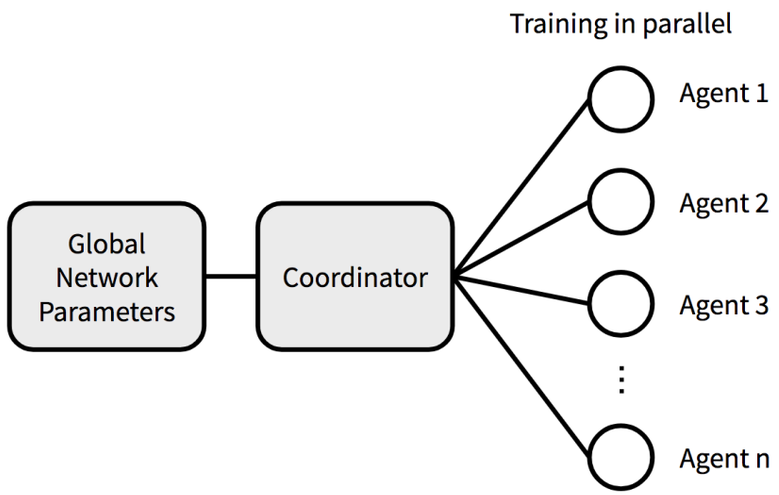
Worker's gradient estimates are used to update global network asynchronously

Asynchronous Advantage Actor-Critic (A3C)

- It is not clear that asynchrony brings an advantage
  - Ongoing work to compare A3C with its synchronous version (A2C)
  - A2C includes a coordinator module that ensures that gradient updates are synchronized



A3C (Async)



A2C (Sync)

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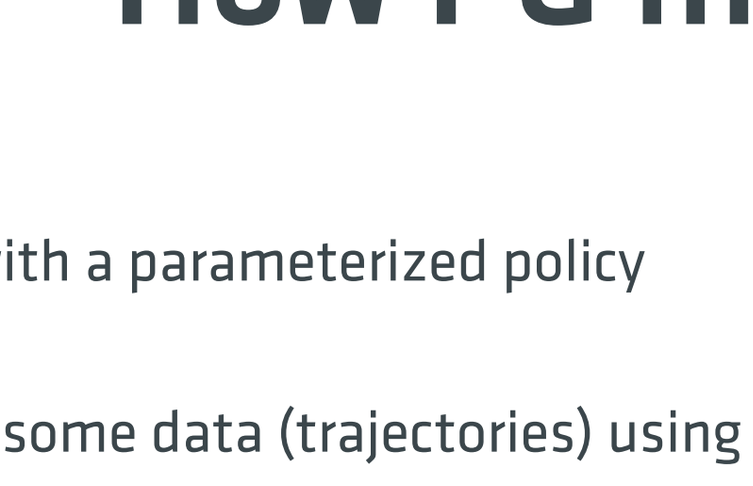
Let’s take a step back…

How PG methods work

- Start with a parameterized policy
- Gather some data (trajectories) using that policy
- Use the data to estimate the advantage
- Update policy parameters using the gradient
- Repeat



How PG methods work

- Old data is “discarded”
  - Old trajectories may be unlikely under the updated policy
  - Old trajectories provide poor estimate to the advantage under updated policy

Not very data efficient

Alternative optimization

- Recall that policy gradient methods arise from the optimization of $J(\pi;\ \mu)$
- Given two policies, π_θ and $\pi_{\theta'}$, it is possible to show that

$$J(\pi_{\theta'};\mu) = J(\pi_\theta;\mu) + \mathbb{E}_{S\sim\mu_\theta}\left[\sum_{a\in\mathcal{A}} \pi_{\theta'}(a \mid S)\mathrm{adv}_{\pi_\theta}(S,a)\right]$$

Trajectories using π_θ

Advantage weighted by $\pi_{\theta'}$

Alternative optimization

- Recall that policy gradient methods arise from the optimization of $J(\pi;\ \mu)$
- Given two policies, π_θ and $\pi_{\theta'}$, it is possible to show that

$$J(\pi_{\theta'};\mu) = J(\pi_\theta;\mu) + \mathbb{E}_{S\sim\mu_\theta}\left[\sum_{a\in\mathcal{A}} \pi_{\theta'}(a \mid S)\mathrm{adv}_{\pi_\theta}(S,a)\right]$$

  if π_θ and $\pi_{\theta'}$ are "close"

- We can thus optimize $J(\pi_{\theta'};\mu)$ by maximizing the expectation on the r.h.s.

Trust region policy optimization

- TRPO thus consists of solving the optimization problem

$$\max_{\theta} \quad \mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\sum_{a\in\mathcal{A}}\pi_\theta(a\mid S)\text{adv}_{\pi_{\theta_{\text{old}}}}(S,a)\right]$$

$$\text{subject to} \quad \mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\text{KL}(\pi_{\theta_{\text{old}}}(\cdot\mid S),\pi_\theta(\cdot\mid S))\right]<\delta$$

Trust region

- Can be solved using, e.g., Lagrange multipliers

- How do we compute the expectation?

Estimating the expectation

- We have that

$$\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\sum_{a\in\mathcal{A}}\pi_\theta(a\mid S)\text{adv}_{\pi_{\theta_{\text{old}}}}(S,a)\right]=\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}},A\sim\pi_{\theta_{\text{old}}}}\left[\frac{\pi_\theta(A\mid S)}{\pi_{\theta_{\text{old}}}(A\mid S)}\text{adv}_{\pi_{\theta_{\text{old}}}}(S,A)\right]$$

Same trajectories used in standard PG algorithms

Importance sampling weight

Estimating the expectation

- We have that

$$\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\sum_{a\in\mathcal{A}}\pi_\theta(a\mid S)\text{adv}_{\pi_{\theta_{\text{old}}}}(S,a)\right]=\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}},A\sim\pi_{\theta_{\text{old}}}}\left[\frac{\pi_\theta(A\mid S)}{\pi_{\theta_{\text{old}}}(A\mid S)}\text{adv}_{\pi_{\theta_{\text{old}}}}(S,A)\right]$$

- Right hand side can be estimated from the trajectories

- Interesting fact:

  - If you differentiate the r.h.s. with respect to θ, you get

$$\mathbb{E}_{S\sim\mu_{\theta_{\text{old}}},A\sim\pi_{\theta_{\text{old}}}}\left[\frac{\nabla\pi_\theta(A\mid S)}{\pi_{\theta_{\text{old}}}(A\mid S)}\text{adv}_{\pi_{\theta_{\text{old}}}}(S,A)\right]_{\theta=\theta_{\text{old}}}=\nabla_\theta J(\theta_{\text{old}};\mu)$$

Relation to PG

- If instead of KL divergence we use an Euclidean constraint, i.e.

$$\begin{aligned} \max_{\theta} \quad & \mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}}\left[\sum_{a\in\mathcal{A}}\pi_\theta(a\mid S)\text{adv}_{\pi_{\theta_{\text{old}}}}(S,a)\right] \\ \text{subject to} \quad & \|\theta-\theta_{\text{old}}\|_2^2<\delta \end{aligned}$$

we recover standard policy gradient

Proximal policy optimization

- Turn the TRPO optimization problem into an unconstrained optimization problem

$$L(\theta) = \mathbb{E}_{S\sim\mu_{\theta_{\text{old}}}, A\sim\pi_{\theta_{\text{old}}}}\left[\frac{\pi_\theta(A \mid S)}{\pi_{\theta_{\text{old}}}(A \mid S)}\text{adv}_{\pi_{\theta_{\text{old}}}}(S, A) - \beta\text{KL}(\pi_{\theta_{\text{old}}}(\cdot \mid S), \pi_\theta(\cdot \mid S))\right]$$

- We can now run SGD on the loss above

- Similar network architecture than standard PG/AC methods