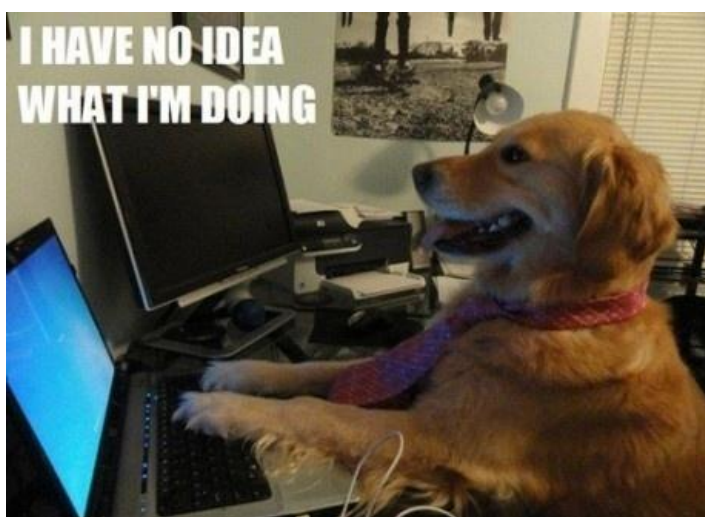


Project Thesis Bananas and Troika

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March 3, 2022



1 Introduction

This guy [1] has done a good job.

2 C^0 Interior Penalty Method

We want to solve the equation on the form

$$\Delta^2 u = f \quad \text{in } \Omega \quad (1)$$

$$u = \frac{\partial u}{\partial n} = 0 \quad (2)$$

$$. \quad (3)$$

2.1 Weak Formulation

The weak formulation of (1) has the form

$$a(u, v) = \int_{\Omega} f v dx \quad u, v \in V. \quad (4)$$

where

$$a(w, v) = \int_{\Omega} \nabla^2 w : \nabla^2 v dx. \quad (5)$$

Here do we define $\nabla^2 w : \nabla^2 v$ to be the inner product of the Hessian matrix of w and v , and V is a closed subspace of the sobolev space $H^2(\Omega)$.

C^0 interior penalty method says that

$$\begin{aligned} a_h(w, v) = & \sum_{T \in \mathfrak{T}_h} \int_T (\nabla^2 w : \nabla^2 v) dx + \sum_{e \in \mathfrak{E}_h^i} \int_e \left\{ \left\{ \frac{\partial w}{\partial n_e} \right\} \right\} \left[\left[\frac{\partial v}{\partial n_e} \right] \right] ds \\ & + \sum_{e \in \mathfrak{E}_h^i} \int_e \left\{ \left\{ \frac{\partial^2 v}{\partial n_e^2} \right\} \right\} \left[\left[\frac{\partial w}{\partial n_e} \right] \right] + \sum_{e \in \mathfrak{E}_h^i} \sigma \frac{1}{|e|} \int_e \left[\left[\frac{\partial w}{\partial n_e} \right] \right] \left[\left[\frac{\partial v}{\partial n_e} \right] \right] ds. \end{aligned}$$

where

$$\begin{aligned} \left[\left[\frac{\partial v}{\partial n_e} \right] \right] &= -n_e \nabla \nu_T \quad \nu_T = \nu|_T \\ \left\{ \left\{ \frac{\partial^2 u}{\partial n_e^2} \right\} \right\} &= \frac{\partial^2 u}{\partial n_e} \\ \left\{ \left\{ \frac{\partial^2 w}{\partial n_e^2} \right\} \right\} &= \frac{1}{2} \left(\frac{\partial^2 w_-}{\partial n_e^2} + \frac{\partial^2 w_+}{\partial n_e^2} \right) \\ \left\{ \left\{ \frac{\partial^2 w}{\partial n_e^2} \right\} \right\} &= \frac{\partial^2 w_T}{\partial n_e^2}, \quad \text{on edges.} \end{aligned}$$

References

- [1] Edmund Brekke. *Fundamentals of Sensor Fusion*. Not published. Third edition, August 2, 2021.