Project Thesis
Bananas and Troika

Isak Hammer

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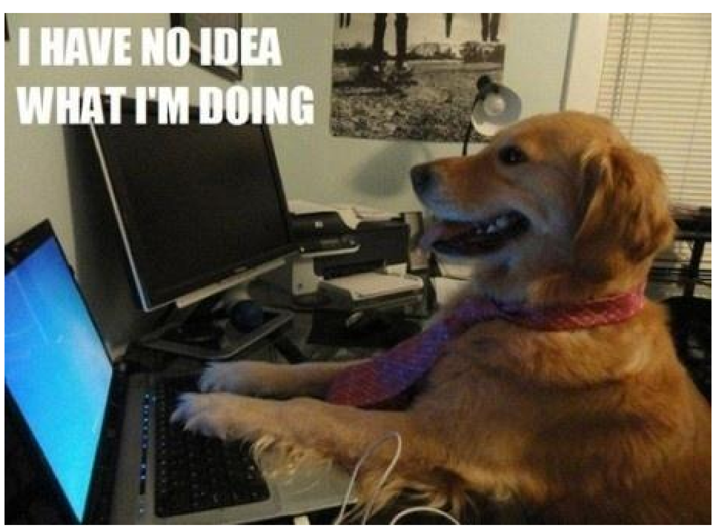

1 Introduction

This guy [1] has done a good job.

2 C^0 Interior Penalty Method

We want to solve the equation on the form

$$\Delta^2 u = f \quad \text{in } \Omega \tag{1}$$

$$u = \frac{\partial u}{\partial n} = 0 \tag{2}$$

$$. \tag{3}$$

2.1 Weak Formulation

The weak formulation of (1) has the form

$$a\left(u,v\right) = \int_{\omega} fvdx \quad u,v \in V. \tag{4}$$

where

$$a\left(w,v\right) = \int_{\Omega} \nabla^2 w : \nabla^2 v dx. \tag{5}$$

Here do we define $\nabla^2 w : \nabla^2 v$ to be the inner product of the Hessian matrix of w and v, and V is a closed subspace of the sobolev space $H^2\left(\Omega\right)$.

C^0 Interior penalty method says that

$$\begin{aligned} a_h\left(w,v\right) = &\sum_{T \in \mathfrak{T}_h} \int_T \left(\nabla^2 w : \nabla^2 v\right) dx + \sum_{e \in \mathfrak{E}_h^i} \int_e \left\{\!\!\left\{ \frac{\partial w}{\partial n_e} \right\}\!\!\right\} \left[\!\!\left[\frac{\partial v}{\partial n_e} \right]\!\!\right] ds \\ &+ \sum_{e \in \mathfrak{E}_h^i} \int_e \left\{\!\!\left\{ \frac{\partial^2 v}{\partial n_e^2} \right\}\!\!\right\} \left[\!\!\left[\frac{\partial w}{\partial n_e} \right]\!\!\right] + \sum_{e \in \mathfrak{E}_h^i} \frac{\sigma}{|e|} \int_e \left[\!\!\left[\frac{\partial w}{\partial n_e} \right]\!\!\right] \left[\!\!\left[\frac{\partial v}{\partial n_e} \right]\!\!\right] ds. \end{aligned}$$

where

$$\begin{aligned} \left[\!\!\left[\frac{\partial v}{\partial n_e} \right]\!\!\right] &= -n_e \nabla \nu_T, \quad \nu_T = \nu|_T \\ \left\{\!\!\left\{ \frac{\partial^2 u}{\partial n_e^2} \right\}\!\!\right\} &= \frac{\partial^2 u}{\partial n_e^2} \\ \left\{\!\!\left\{ \frac{\partial w}{\partial n_e^2} \right\}\!\!\right\} &= \frac{1}{2}\left(\frac{\partial^2 w_-}{\partial n_e^2} + \frac{\partial^2 w_+}{\partial n_e^2} \right) \\ \left\{\!\!\left\{ \frac{\partial^2 w}{\partial n_e^2} \right\}\!\!\right\} &= \frac{\partial^2}{\partial n_e^2}, \quad \text{on edges.} \end{aligned}$$

References

[1] Edmund Brekke. *Fundamentals of Sensor Fusion.* Not published. Third edition, August 2, 2021.