Real-time Simulation of Granular Matter using Smoothed Particle Hydrodynamics

Masterarbeit

zur Erlangung des Grades Master of Science (M.Sc.)
im Studiengang Computervisualistik

vorgelegt von

André Neder

Erstgutachter:	Prof. Dr.-Ing. Stefan Müller (Institut für Computervisualistik, AG Computergraphik)
Zweitgutachter:	Alexander Maximilian Nilles, M.Sc. (Institut für Computervisualistik, AG Computergraphik)

Koblenz, im Dezember 2023

Erklärung

Ich versichere, dass ich die vorliegende Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

	Ja	Nein
Mit der Einstellung der Arbeit in die Bibliothek bin ich einverstanden.	☐	☐

....

(Ort, Datum) (Unterschrift)

Inhaltsverzeichnis

1 Introduction

2 Related Work

3 Basics

3.1 Granular Matter

4 Method

4.1 Smoothed Particle Hydrodynamics

Density neighbors

$$\rho_i = \sum_j V_j \rho_j W(x_i - x_j, h) = \sum_j m_j W(x_i - x_j, h) \tag{1}$$

$W_{ij} \equiv W(x_i - x_j, h)$

Density boundaries

$$\rho_i = \sum_b V_b \rho_0 W_{ib} \tag{2}$$

Unilateral incompressibility

$$\rho_i \leq \rho_0 \perp p \geq 0 \tag{3}$$

Velocity relative to the air

$$v_{i,rel}^2 = |v_a - v_i|^2 \ \frac{v_a - v_i}{|v_a - v_i|} \tag{4}$$

Drag

$$F_i^{drag} = \frac{1}{2} \rho_a v_{i,rel}^2 C_{D,i} \omega A_i \tag{5}$$

Velocity gradient

$$\nabla u_i = \sum_j V_j \nabla W_{ij} u_j^T \tag{6}$$

Strain

$$\varepsilon = \frac{1}{2} (\nabla u_i + \nabla u_i^T) \tag{7}$$

D

$$D_i = \frac{2 m_i^2 \Delta t}{\rho_i^2} \sum_j \frac{1}{\rho_j} \nabla W_{ij} \nabla W_{ij}^T \tag{8}$$

$\nabla W_{ij} \nabla W_{ij}^T \equiv W_{ij} \otimes W_{ij}$
Stress

$$s_i = D^{-1} \varepsilon \tag{9}$$

$$s_{i,hydrostatic} = \frac{1}{2}\operatorname{Tr} s_i \tag{10}$$

$$s_{i,deviatoric} = s_i - s_{i,hydrostatic} \tag{11}$$

s_i refers to $s_{i,deviatoric}$ for further equations

Drucker-Prager Yield Criterion

$$||s_i|| \leq p_i\sqrt{2}\sin\Theta \tag{12}$$

Friction Force

$$F_i^f = -m_i \sum_{j\neq i} m_j(\frac{s_i}{\rho_i^2} + \frac{s_j}{\rho_j^2})\nabla W_{ij} \tag{13}$$

$$F_i^f = -m_i \sum_b V_b\rho_0(\frac{s_i}{\rho_i^2})\nabla W_{ib} \tag{14}$$

Todo: EOS Pressure Force

$$F_i^p = -m_i \sum_{j\neq i} m_j(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2})\nabla W_{ij} \tag{15}$$

$$F_i^p = -m_i \sum_b V_b\rho_0(\frac{p_i}{\rho_i^2})\nabla W_{ib} \tag{16}$$

Non pressure forces

$$F_i^{adv} = F^g + F_i^f + f_i^{drag} \tag{17}$$

intermediate velocity

$$v_i^{adv} = v_i + \Delta t\frac{F_i^{adv}}{m_i} \tag{18}$$

intermediate density

$$\rho_i^{adv} = \rho_i + \Delta t\sum_j m_j v_{ij}^{adv}\nabla W_{ij} \tag{19}$$

pressure

$$p_i^{l+1} = (1-\omega)p_i^l + \omega\frac{1}{a_{ii} * \Delta t^2}(\rho_0 - \rho_i^{adv} - \Delta t^2\psi) \tag{20}$$

$\omega = 0.5$

$$\psi = \sum_j m_j(\sum_j d_{ij}p_j^l - d_{jj}p_j^l - \sum_{k\neq i} d_{jk}p_k^l)\nabla W_{ij} \tag{21}$$

$$\sum_{k\neq i} d_{jk}p_k^l = \sum_k d_{jk}p_k^l - d_{ji}p_i^l \tag{22}$$

$$\rho_i^{l+1} = |p * a_{ii} * \Delta t^2 - (\rho_0 - \rho_i^{adv} - \Delta t^2 \psi)| + \rho_0 \tag{23}$$

$$a_{ii} = \sum_j m_j (d_{ii} - d_{ji}) \nabla W_{ij} \tag{24}$$

Integration

$$v_i(t + \Delta t) = v_i^{adv} + \Delta t \frac{F_i^p}{m_i} \tag{25}$$

$$x_i(t + \Delta t) = x_i + \Delta t v_i(t + \Delta t) \tag{26}$$

4.2 Vulkan

5 Implementation

5.1 Libraries & NewTechnologies

5.2 Datastructures

5.3 Algorithm

Algorithm 1 Full Simulation Frame

```
find neighbors
compute ρ
compute drag
compute s
compute v advection
compute ρ advection
l = 0
while l < 2 || ρ_avg − ρ_0 < η do
    compute d_ij p_j
    compute p^{l+1}
    p(t) = p^{l+1}
    l = l + 1
end while
compute F^p
integrate
advect HR particles
```

5.3.1 Neighborhood Search

5.3.2 Density & Pressure

5.3.3 Stress & Strain

5.3.4 Force

5.3.5 Rigidbody Interactions

5.3.6 Integration

5.3.7 Upscaling

5.3.8 Visualization

5.3.9 User Interface

6 Evaluation

6.1 Performance

7 Conclusion & Future Work

Literatur

[1] M. Ihmsen, A. Wahl und M. Teschner, „High-Resolution Simulation of Granular Material with SPH," in *Workshop on Virtual Reality Interaction and Physical Simulation*, J. Bender, A. Kuijper, D. W. Fellner und E. Guerin, Hrsg., The Eurographics Association, 2012, ISBN: 978-3-905673-96-8. DOI: 10.2312/PE/vriphys/vriphys12/053-060.

[2] I. Alduan, Á. Tena und M. A. Otaduy, „Simulation of High-Resolution Granular Media," in *CEIG 09 - Congreso Espanol de Informatica Grafica*, C. Andujar und J. Lluch, Hrsg., The Eurographics Association, 2009, ISBN: 978-3-905673-72-2. DOI: 10.2312/LocalChapterEvents/CEIG/CEIG09/011-018.

[3] M. Ihmsen, N. Akinci, M. Gissler und M. Teschner, „Boundary Handling and Adaptive Time-stepping for PCISPH," Jan. 2010, S. 79–88. DOI: 10.2312/PE/vriphys/vriphys10/079-088.

[4] N. Bell, Y. Yu und P. J. Mucha, „Particle-Based Simulation of Granular Materials," in *Proceedings of the 2005 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, Ser. SCA '05, Los Angeles, California: Association for Computing Machinery, 2005, S. 77–86, ISBN: 1595931988. DOI: 10.1145/1073368.1073379. Adresse: https://doi.org/10.1145/1073368.1073379.

[5] I. Alduán und M. A. Otaduy, „SPH Granular Flow with Friction and Cohesion," in *Proceedings of the 2011 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, Ser. SCA '11, Vancouver, British Columbia, Canada: Association for Computing Machinery, 2011, S. 25–32, ISBN: 9781450309233. DOI: 10.1145/2019406.2019410. Adresse: https://doi.org/10.1145/2019406.2019410.

[6] B. Solenthaler und R. Pajarola, „Predictive-Corrective Incompressible SPH," in *ACM SIGGRAPH 2009 Papers*, Ser. SIGGRAPH '09, New Orleans, Louisiana: Association for Computing Machinery, 2009, ISBN: 9781605587264. DOI: 10.1145/1576246.1531346. Adresse: https://doi.org/10.1145/1576246.1531346.

[7] R. Narain, A. Golas und M. C. Lin, „Free-Flowing Granular Materials with Two-Way Solid Coupling," in *ACM SIGGRAPH Asia 2010 Papers*, Ser. SIGGRAPH ASIA '10, Seoul, South Korea: Association for Computing Machinery, 2010, ISBN: 9781450304399. DOI: 10.1145/1866158.1866195. Adresse: https://doi.org/10.1145/1866158.1866195.

[8] C. Gissler, S. Band, A. Peer, M. Ihmsen und M. Teschner, „Generalized Drag Force for Particle-Based Simulations," *Comput. Graph.*, Jg. 69, Nr. C, S. 1–11, Dez. 2017, ISSN: 0097-8493. DOI: 10.1016/j.cag.2017.09.002. Adresse: https://doi.org/10.1016/j.cag.2017.09.002.

[9] T. Probst und M. Teschner, „Monolithic Friction and Contact Handling for Rigid Bodies and Fluids Using SPH," *Computer Graphics Forum*, Jg. 42, Jan. 2023. DOI: `10.1111/cgf.14727`.

[10] J. Bender, T. Kugelstadt, M. Weiler und D. Koschier, „Volume Maps: An Implicit Boundary Representation for SPH," in *Proceedings of the 12th ACM SIGGRAPH Conference on Motion, Interaction and Games*, Ser. MIG '19, Newcastle upon Tyne, United Kingdom: Association for Computing Machinery, 2019, ISBN: 9781450369947. DOI: `10.1145/3359566.3360077`. Adresse: `https://doi.org/10.1145/3359566.3360077`.

[11] Z.-B. Wang, R. Chen, H. Wang, Q. Liao, X. Zhu und S.-Z. Li, „An overview of smoothed particle hydrodynamics for simulating multiphase flow," *Applied Mathematical Modelling*, Jg. 40, Nr. 23, S. 9625–9655, 2016, ISSN: 0307-904X. DOI: `https://doi.org/10.1016/j.apm.2016.06.030`. Adresse: `https://www.sciencedirect.com/science/article/pii/S0307904X16303419`.

[12] S. Band, C. Gissler, M. Ihmsen, J. Cornelis, A. Peer und M. Teschner, „Pressure Boundaries for Implicit Incompressible SPH," *ACM Trans. Graph.*, Jg. 37, Nr. 2, Feb. 2018, ISSN: 0730-0301. DOI: `10.1145/3180486`. Adresse: `https://doi.org/10.1145/3180486`.

[13] M. Fujisawa und K. T. Miura, „An Efficient Boundary Handling with a Modified Density Calculation for SPH," *Computer Graphics Forum*, Jg. 34, Nr. 7, S. 155–162, 2015. DOI: `https://doi.org/10.1111/cgf.12754`. Adresse: `https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.12754`.

[14] T. Takahashi, Y. Dobashi, T. Nishita und M. C. Lin, „An Efficient Hybrid Incompressible SPH Solver with Interface Handling for Boundary Conditions," *Computer Graphics Forum*, 2018, ISSN: 1467-8659. DOI: `10.1111/cgf.13292`.

[15] M. Macklin, M. Müller, N. Chentanez und T.-Y. Kim, „Unified Particle Physics for Real-Time Applications," *ACM Trans. Graph.*, Jg. 33, Nr. 4, Juli 2014, ISSN: 0730-0301. DOI: `10.1145/2601097.2601152`. Adresse: `https://doi.org/10.1145/2601097.2601152`.

[16] M. Weiler, D. Koschier, M. Brand und J. Bender, „A Physically Consistent Implicit Viscosity Solver for SPH Fluids," *Computer Graphics Forum*, Jg. 37, Nr. 2, S. 145–155, 2018. DOI: `https://doi.org/10.1111/cgf.13349`. Adresse: `https://onlinelibrary.wiley.com/doi/abs/10.1111/cgf.13349`.

[17] A. Sommer, U. Schwanecke und E. Schoemer, „Interactive High-Resolution Simulation of Granular Material,“ *Journal of WSCG*, Jg. 30, Nr. 1-2, S. 9–15, 2022, ISSN: 1213-6964. DOI: `10.24132/jwscg.2022.2`. Adresse: `http://dx.doi.org/10.24132/JWSCG.2022.2`.