Real-time Simulation of Granular Matter using Smoothed Particle Hydrodynamics

Masterarbeit

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vorgelegt von

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Erklärung

Ich versichere, dass ich die vorliegende Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

	Ja	Nein
Mit der Einstellung der Arbeit in die Bibliothek bin ich einverstanden.	☐	☐

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(Ort, Datum)   (Unterschrift)

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4.1 Smoothed Particle Hydrodynamics

Density neighbors

$$\rho_i = \sum_j V_j \rho_j W(x_i - x_j, h) = \sum_j m_j W(x_i - x_j, h) \tag{1}$$

$W_{ij} \equiv W(x_i - x_j, h)$

Density boundaries

$$\rho_i = \sum_b V_b \rho_0 W_{ib} \tag{2}$$

Unilateral incompressibility

$$\rho_i \leq \rho_0 \perp p \geq 0 \tag{3}$$

Velocity relative to the air

$$v_{i,rel}^2 = |v_a - v_i|^2 \, \frac{v_a - v_i}{|v_a - v_i|} \tag{4}$$

Drag

$$F_i^{drag} = \frac{1}{2} \rho_a v_{i,rel}^2 C_{D,i} \omega A_i \tag{5}$$

Velocity gradient

$$\nabla u_i = \sum_j V_j \nabla W_{ij} u_j^T \tag{6}$$

Strain

$$\varepsilon = \frac{1}{2} (\nabla u_i + \nabla u_i^T) \tag{7}$$

D

$$D_i = \frac{2m_i^2 \Delta t}{\rho_i^2} \sum_j \frac{1}{\rho_j} \nabla W_{ij} \nabla W_{ij}^T \tag{8}$$

$\nabla W_{ij} \nabla W_{ij}^T \equiv W_{ij} \otimes W_{ij}$
Stress

$$s_i = D^{-1} \varepsilon \tag{9}$$

$$s_{i,hydrostatic} = \frac{1}{2} \operatorname{Tr} s_i \tag{10}$$

$$s_{i,deviatoric} = s_i - s_{i,hydrostatic} \tag{11}$$

s_i refers to $s_{i,deviatoric}$ for further equations

Drucker-Prager Yield Criterion

$$||s_i|| \le p_i \sqrt{2} \sin \Theta \tag{12}$$

Friction Force

$$F_i^f = -m_i \sum_{j \ne i} m_j \left(\frac{s_i}{\rho_i^2} + \frac{s_j}{\rho_j^2}\right) \nabla W_{ij} \tag{13}$$

$$F_i^f = -m_i \sum_b V_b \rho_0 \left(\frac{s_i}{\rho_i^2}\right) \nabla W_{ib} \tag{14}$$

Todo: EOS Pressure Force

$$F_i^p = -m_i \sum_{j \ne i} m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right) \nabla W_{ij} \tag{15}$$

$$F_i^p = -m_i \sum_b V_b \rho_0 \left(\frac{p_i}{\rho_i^2}\right) \nabla W_{ib} \tag{16}$$

Non pressure forces

$$F_i^{adv} = F^g + F_i^f + f_i^{drag} \tag{17}$$

intermediate velocity

$$v_i^{adv} = v_i + \Delta t \frac{F_i^{adv}}{m_i} \tag{18}$$

intermediate density

$$\rho_i^{adv} = \rho_i + \Delta t \sum_j m_j v_{ij}^{adv} \nabla W_{ij} \tag{19}$$

pressure

$$p_i^{l+1} = (1 - \omega) p_i^l + \omega \frac{1}{a_{ii} * \Delta t^2} (\rho_0 - \rho_i^{adv} - \Delta t^2 \psi) \tag{20}$$

$\omega = 0.5$

$$\psi = \sum_j m_j \left(\sum_j d_{ij} p_j^l - d_{jj} p_j^l - \sum_{k \ne i} d_{jk} p_k^l\right) \nabla W_{ij} \tag{21}$$

$$\sum_{k \ne i} d_{jk} p_k^l = \sum_k d_{jk} p_k^l - d_{ji} p_i^l \tag{22}$$

$$\rho_i^{l+1} = |p * a_{ii} * \Delta t^2 - (\rho_0 - \rho_i^{adv} - \Delta t^2 \psi)| + \rho_0 \qquad (23)$$

$$a_{ii} = \sum_j m_j (d_{ii} - d_{ji}) \nabla W_{ij} \qquad (24)$$

Integration

$$v_i(t + \Delta t) = v_i^{adv} + \Delta t \frac{F_i^p}{m_i} \qquad (25)$$

$$x_i(t + \Delta t) = x_i + \Delta t v_i(t + \Delta t) \qquad (26)$$

Symbol	Unit	Name
ρ_i	kg/m^3	Density of particle i at position x for time t
ρ_0	kg/m^3	Rest density of the simulated material
V_i	m^3	Volume of particle i at position x for time t

4.2 Vulkan

5 Implementation

5.1 Libraries & NewTechnologies

5.2 Datastructures

5.3 Algorithm

Algorithm 1 Full Simulation Frame

1: find neighbors
2: compute ρ
3: compute drag
4: compute s
5: compute v advection
6: compute ρ advection
7: $l = 0$
8: **while** $l < 2 \ || \ \rho_{avg} - \rho_0 < \eta$ **do**
9:     compute $d_{ij} p_j$
10:     compute p^{l+1}
11:     $p(t) = p^{l+1}$
12:     $l = l + 1$
13: **end while**
14: compute F^p
15: integrate
16: advect HR particles

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